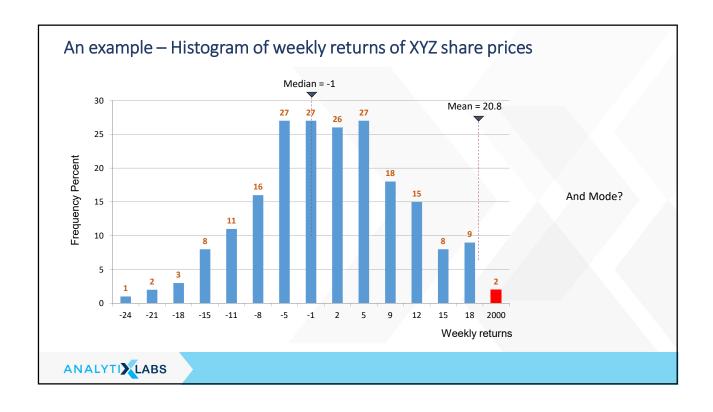


Measures of Central Tendency MEAN **MEDIAN** MODE It is just the average of the data, It is the value in the middle of the data It is the most common value in the data computed as the sum of the data points set, when the data points are arranged divided by the number of points from smallest to largest. Tricky circumstances: Tricky circumstances: If no value occurs more than once, then If there is an even number of data there is no mode points, you will need to take the If two, or more, values occur as average of the two middle values. frequently as each other and more frequently than any other, then there are two, or more, modes. It is the easiest metric to Not very practical since it is Median is a more "robust" to presence of outliers understand and communicate affected by skewness Most real life distributions are Mean is prone to presence of It is more complicated to multimodal outliers communicate Example: A parent wanting to know Example: What is a typical student in Example: To compare performance of whether their child is better or worse the class doing? any single student against group than typical child at his grade level ANALYTI LABS



But are these sufficient?

- There is the man who drowned crossing a stream with an average depth of six inches.
- Say you were standing with one foot in the oven and one foot in an ice bucket. According to the averages, you should be perfectly comfortable.
- Time taken by different modes of transport

	Auto	Office Transport	Own Car
	7	9	1
	6	9	3
	3	9	5
	8	9	7
	12	9	9
	9	9	9
	9	9	9
	13	9	11
	13	9	13
	9	9	15
	10	9	17
Mean	9	9	9
Median	9	9	9
Mode	9	9	9

ANALYTI LABS

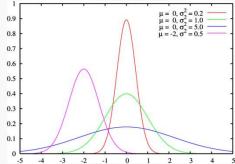
Measures of Variation/Dispersion

Dispersion refers to the spread or variability in the data. It determines how spread out are the scores around the mean.

Why is Dispersion important?

- It gives additional information that enables to judge the reliability of the measure of central tendency
- If data are widely spread the central location is less representative of data as a whole than it would be for data more closely centered around
- Since problems are peculiar to widely dispersed data, dispersion enables to identify and tackle problems accordingly
- This enables to compare dispersions of various samples
- For eg. If a wide spread of values are away from center, this may be
- undesirable or presents a risk, one may avoid choosing that distribution

Distributions with different dispersions

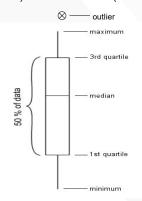


Common Measures of Variation/Dispersion

- Range
- Inter-Quartile Range
- Mean Deviation
- Standard Deviation
- Variance
- Percentiles/Quartiles

- Box-plot
 - · Reveals the spread of the data
 - · Outliers defined using the

Q1 - 1.5(Q3-Q1) and Q3 + 1.5(Q3-Q1)



AN/9LYTIXLABS

Common measures of dispersion

Standard Deviation is a measure of how spread out numbers are

Variance is defined as the average of the squared differences from the Mean

$$\sigma^2 = \frac{\sum (X - \mu)^2}{N} = \frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \dots}{N}$$

Where

X is the value of an observation in the population $\boldsymbol{\mu}\,$ is the arithmetic mean of the population

N is the number of observations in the population

Mean = 394mm $206^{2} + 76^{2} + (-224)^{2} + 36^{2} + (-94)^{2} = 21,704$

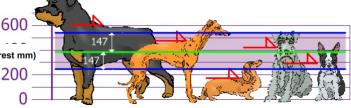
Example: You have just measured the heights of your

dogs (in millimeters). The heights are: 600mm, 470mm, 170mm, 430mm and 300mm.

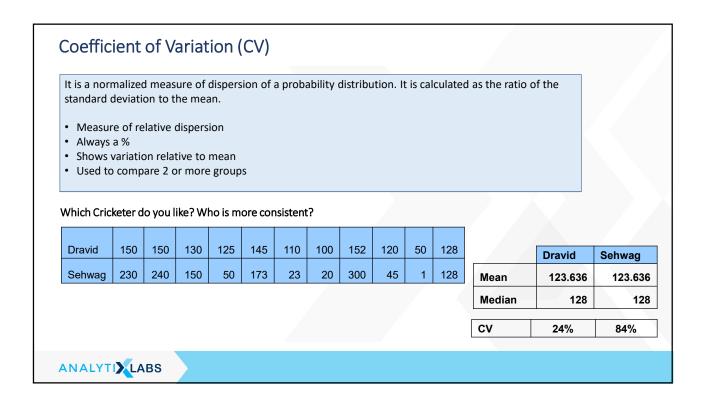
Variance: $\sigma^2 =$

Standard Deviation: $\sigma = \sqrt{21,704} = 147.32... = 147$ (to the nearest mm)

Using the Standard Deviation we have a "standard" way of knowing what is normal, and what is extra large or extra small.



	Auto	Office Transport	Own Car
	7	9	1
	6	9	
	3	9	
	8	9	
	12	9	
	9	9	
	9	9	
	13		
	13	9	
	9	9	
Maan	10		
Mean Median	9	9	
Mode	9	9	
Mode			
Std Dev	3.0	0.0	4.9
Variance	9.2	0.0	24.0

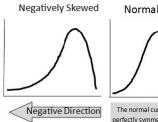


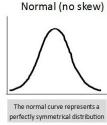
Descriptive Statistics

Central Tendency: is the middle point of distribution. Measures of Central Tendency include Mean, Median and Mode

Dispersion: is the spread of the data in a distribution, or the extent to which the observations are scattered.

Skewness: When the data is asymmetrical ie the values are not distributed equally on both sides. In this case, values are either concentrated on low end or on high end of scale on horizontal axis.







If the trail is to the right or positive end of the scale, the distribution is said to be "positively skewed". If the distribution trails off to the left or negative side of the scale, it is said to be "negatively skewed".



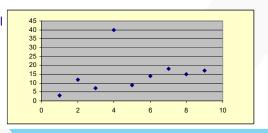
Outliers

An outlier is an observation that is numerically distant from the rest of the data.

An outlying observation, or outlier, is one that appears to deviate markedly from other members of the sample in which it occurs. Outliers can occur by chance in any distribution, but they are often indicative either of measurement error or that the population has a heavy-tailed distribution.

Example: Bill Gates makes \$500 million a year. He's in a room with 9 teachers, 4 of whom make \$40k, 3 make \$45k, and 2 make \$55k a year. What is the mean salary of everyone in the room? What would be the mean salary if Gates wasn't included? Mean With Gates: \$50,040,500 Mean Without Gates: \$45,000

A **Scatterplot** is useful for "eyeballing" the presence of **outliers**.



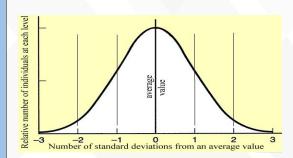
We can also use Standard Deviation to identify Outliers!

Normal Distribution

Normal distribution is a pattern for the distribution of a set of data which follows a **bell shaped curve**. This also called the *Gaussian distribution*.

The normal distribution is a theoretical ideal distribution. Real-life empirical distributions never match this model perfectly. However, many things in life do approximate the normal distribution, and are said to be "normally distributed."

- Normal Distribution has the mean, the median, and the mode all coinciding at its peak
- The curve is concentrated in the center and decreases on either side ie most observations are close to the mean
- The bell shaped curve is symmetric and Unimodal
- It can be determined entirely by the values of mean and std dev
- Area under the curve = 1
- The empirical 68-95-99.7 rule states that for a normal distribution:
 - 68.3% of the data will fall within 1 SD of mean
 - 95.4% of the data will fall within 2 SD's of the mean
 - Almost all (99.7%) of the data will fall within 3 SD's of the mean



ANALYTI LABS

Standard Normal Distribution

Standard Normal distribution is a special case of the Normal distribution which has a mean of 0 and a standard deviation of 1

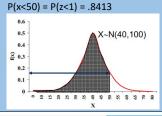
Any normal distribution can be converted to a Standard normal distribution through:

$$Z = \frac{X - \mu}{\sigma}$$

Example: If X is a continuous random variable with a mean of 40 and a standard deviation of 10, what proportion of observations are a) Less than 50 b) Less than 20 c) Between 20 and 50

a) P(x<50)?

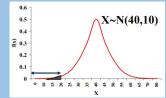
$$Z = \frac{50-40}{10} = 1$$



b) P(x<20)?

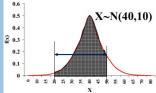
$$Z = \frac{20-40}{12} = -2$$

P(x<20) = P(x<-2) = .0228



c) P(20<x<50)?

$$\frac{20-40}{10}$$
 < Z < $\frac{50-40}{10}$



Standard scores

A standard score (also called Z score) is the number of standard deviations that a given raw score is above or below the mean.

- All normal distributions can be converted into the standard normal curve by subtracting the mean and dividing by the standard deviation.
- Standardizing variables helps to get variables to the same scale, or makes the variable unit free For eg: If we want to input Income (in INR) and Number of calling minutes into the same analysis, we will have to standardize both variables to get them to the same scale

$$Z = \frac{X - \mu}{\sigma}$$

ANALYTI LABS

Practice problem

If birth weights in a population are normally distributed with a mean of 109 oz and a standard deviation of 13 oz

- a. What is the chance of obtaining a birth weight of 141 oz *or heavier* when sampling birth records at random?
- b. What is the chance of obtaining a birth weight of 120 or lighter?

Answer

- a. What is the chance of obtaining a birth weight of 141 oz *or heavier* when sampling birth records at random?
- b. What is the chance of obtaining a birth weight of 120 *or lighter*?

$$Z = \frac{141 - 109}{13} = 2.46$$

$$Z = \frac{120 - 109}{13} = .85$$

From the chart or SAS \rightarrow Z of 2.46 corresponds to a right tail (greater than) area of:

$$P(Z \ge 2.46) = 1 - (.9931) = .0069 \text{ or } .69 \%$$

From the chart \rightarrow Z of .85 corresponds to a left tail area of:

$$P(Z \le .85) = .8023 = 80.23\%$$

ANALYTI LABS

Populations and Samples

So far we have determined the results associated with individual observations or sample means when the true population parameters are known. In reality, the true population parameters are seldom known. We now learn how to infer levels of confidence, or a measure of accuracy on parameters, estimated using samples

POINT ESTIMATOR

- If we take a sample from a population, we can estimate parameters from the population, using sample statistics
- $\bullet \quad$ Example: Sample mean (x) is our best estimate of the population mean (μ)
- Whereas, we really don't know how close the estimate is to the true parameter
- The mean annual rainfall of Melbourne is 620mm per year

INTERVAL ESTIMATOR

- If we estimate a range or interval within which the true population parameter lies, then we are using an interval estimation method
- This is the most common method of estimation. We can also apply a level of how confident we are in the estimate
- In 80% of all years Melbourne receives between 440 and 800 mm rain



Sampling methodologies

Sampling is required because it is seldom possible to measure all the individuals in a population. Researchers hence, use samples and infer their results to the population of interest

Eg: Election polls, market research surveys, etc

For a sample to be a "good sample", it is imperative that there is a good sample size and there is no biasness in the sample.

Simple Random Sample

is one in which every member of the population is equally likely to be measured

Eg: Allocate a number to each member of the population and use a random number generator to determine which individuals will be measured

Stratified Sampling

separates the population into mutually exclusive groups and randomly samples within the groups

Eg: Randomly select a number of people within each demographic cell, while maintaining overall proportions like gender ratio, income ratio, etc

Other methodology

Cluster sampling: is used when there is a considerable variation within each group but the groups are essentially similar to each other. Here we divide the population into groups, or clusters, and then select a random sample of these clusters.



Central Limit Theorem

It is always not possible to get the true information about the population. In this case we have to live with samples. For eg: we don't know the actual average income for India, but can estimate it based on a random sample picked from the Indian population

In this case, the average we have is not the population average μ but an estimate X

If we take a similar second sample, it is extremely unlikely that the average calculated for the second sample will be the same as the average calculated for the first sample. In fact, statisticians know that repeated samples from the same population give different sample means.

They have also proven that the distribution of these sample means will always be normally distributed, regardless of the shape of the parent population. This is known as the Central Limit Theorem.

A distribution with a mean μ and variance σ^2 , the sampling distribution of the mean approaches a normal distribution with a mean (μ) and a variance σ^2/N as N, the sample size increases.

The amazing and counter-intuitive thing about the central limit theorem is that *The distribution of an average tends to be Normal, even when the distribution from which the average is computed is decidedly non-Normal distribution from which the average is computed is decidedly non-Normal.*

As the sample size n increases, the variance of the sampling distribution decreases. This is logical, because the larger the sample size, the closer we are to measuring the true population sample size, the closer we are to measuring the true population parameters.



Standard Error

Since all samples drawn from a population are similar BUT NOT the same as population, we calculate a Standard Error.

Standard Error is the standard deviation of the sample means from the population mean

Also, Standard Error ultimately converges to the Standard Deviation of the population.

Standard Error =
$$\frac{\sigma}{\sqrt{N}}$$

ANALYTI LABS

Confidence Intervals

Because we know the properties of the normal distribution so well, we can use these properties to assist us in applying confidence intervals to estimates. This is essentially the interval estimator range.

For example, we know that 68.3 % of sample means lie within one standard error of the true population mean.

Therefore, if we know the true population variance, we can infer the range within which we can be 68.3% confident that the true population mean lies

Example:

$$X \sim N(\mu, 3.62)$$
 grams, n = 36, x = 25.5

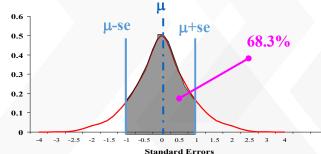
We can be 68.3% confident that $\boldsymbol{\mu}$ is within the interval

x ± SE

= 25.5 \pm 3.6/ $\sqrt{36}$

 $= 25.5 \pm 0.6$

= (24.9 to 26.1) grams



Confidence Intervals

We can extend this principle further:

- We can be 90% confident that the true population mean lies within $x \pm 1.645$ (SE)
- We can be 95% confident that the true population mean lies within $x \pm 1.960(SE)$
- We can be 99% confident that the true population mean lies within $x \pm 2.576$ (SE)

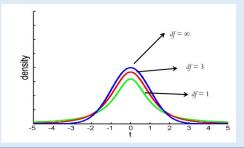
ANALYTI LABS

Student's t - distribution

- While z-distribution is for the population, t-distribution is for the sample distribution.
- $\bullet \quad \text{Hence, the shape of 't' sampling distribution is similar to that of the 'z' sampling distribution in that it is }$
 - a) Symmetrical
 - b) Centered over a mean of zero
 - c) Variance depends on the sample size, more specifically on the degrees of freedom (abbreviated as df)
- As the number of degrees of freedom increases , variance of the t distribution approaches more closely to that of z
- For n ≥ 30, shapes are almost similar
- For n of 30 taken as dividing point between small & large samples
- t-test for population mean is:

$$\frac{\overline{X} - \mu_0}{s / \sqrt{n}}$$

When n < 30



When to use

z-test:

- \bullet σ is known and the population is normal
- \bullet σ is known and the sample size is at least 30. (The population need not be normal)

t-test:

- \bullet Whenever σ is not known
- The population is assumed to be normal
- And n<30
- The correct distribution to use is the 't' distribution with n-1 df

ANALYTI LABS

Hypothesis testing

- In a Test Procedure, to start with, a hypothesis is made.
- The validity of the hypothesis is tested.
- If the hypothesis is found to be true, it is accepted.
- If it is found to be untrue, it is rejected.
- The hypothesis which is being tested for possible rejection is called null hypothesis
- Null hypothesis is denoted by H₀
- The hypothesis which is accepted when null hypothesis is rejected is called Alternate Hypothesis H_a
- Ex. H_o: The drug works –it has a real effect.
 - $\mathbf{H}_{\mathbf{a}}$: The drug doesn't work Any effect you saw was due to chance.

Hypothesis testing

Hypothesis tests consist of the following steps:

- Null Hypothesis
- Alternative Hypothesis
- Confidence Level
- Decision Rule
- Test statistic
- Decision

ANALYTI LABS

Hypothesis testing

- <u>Null hypothesis</u> We always assume the null hypothesis is true, or at least is the most
 plausible explanation before we do the test. The test can only *disprove* the null hypothesis.
- Alternative hypothesis The alternative hypothesis is the hypothesis that we set out to test for. It is the hypothesis that we wish to *prove*.
- Decision Rule After we know the null and alternative hypotheses and the level of confidence associated with the test, we determine the points on the distribution of the test statistic where we will decide when the null hypothesis should be rejected in favor of the alternative hypothesis
- Use the terminology "Reject H₀" or "Do not reject H₀". Never say "Accept Ho"

Type I and Type II Error

Process of testing a hypothesis indicates that there is a possibility of making an error. There are two types of errors:

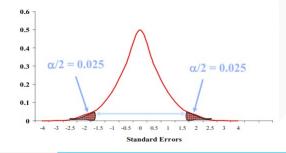
Type I error: The error of rejecting the null hypothesis H₀ even though H₀ was true.

<u>Type II error</u>: The error of accepting the null hypothesis H₀ even though H₀ was false.

ANALYTI LABS

P - value

- Furthermore, the area outside the confidence interval is cumulatively known as $\boldsymbol{\alpha}$ (alpha)
- Confidence Interval = 1α
- Example: for 95% confidence interval, ∝=0.05
- α is also known as p-value.
- Hence, p-value is the probability that a randomly picked sample will have the mean lying outside the confidence interval.

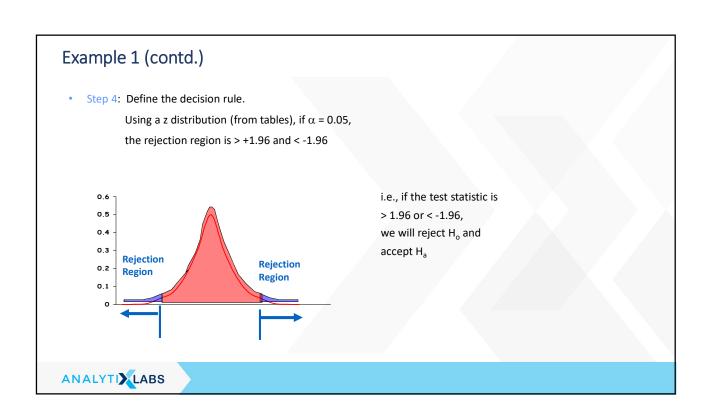


Example 1

Suppose that we have been told that the price of petrol in Melbourne is normally distributed with a mean of 92 cents per litre, and a standard deviation of 3.1 cents/litre. To test whether this price is in fact true, we sample 50 service stations and obtain a mean of 93.6 cents/litre

Solution:

- Step 1: State the null and alternative hypotheses
 - Ho: μ = 92
 - Ha: $\mu \neq 92$
- Step2: Determine the appropriate test statistic and it's distribution
 Because we know the population standard deviation, we can use the z distribution
- Step3: Specify the significance level, Say α = 0.05



Example 1 (contd.)

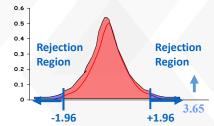
• Step 5: Calculate the test statistic

$$Z = \frac{\overline{X} - \mu_{\overline{x}}}{SE} = \frac{\overline{X} - \mu_{o}}{\sigma/\sqrt{n}}$$

$$Z = \frac{93.6 - 92}{3.1/\sqrt{50}} = 3.65$$

- Step 6: Make a decision and answer the question: As, 3.65 > 1.96, the test statistic is in the rejection region Reject H_o, accept H_a as a more plausible explanation
- Step 7: Write your conclusion in the context of the aims of the study.
 "The average price of petrol in Melbourne was

significantly different to 92 cents/litre"



ANALYTI LABS

Example 1 (contd.) – Importance of sample sizes

• Consider the petrol prices in Melbourne example. If the sample size we had used was only 10, rather than 50, the test statistic would have been;

$$z = \frac{93.6 - 92}{3.1/\sqrt{10}} = 1.63$$

In which case we would not have rejected the H_o

Example 2

A company pays production workers \$630 per week. The union claims that these workers are paid below the industry average for their work. A sample of 15 workers from other sites gives a mean wage of \$670/week with a standard deviation of \$58/week. Is the unions claim justified?

Solution:

Step 1: Ho: μ =< \$630 (industry weekly average is not significantly different to \$630)

Ha: μ > \$630 (The industry weekly average is greater than \$630)

Step2: Test Statistic - As we don't know the population variance, and the sample size is < 30, we shall use the t test.

Step3: Significance level - We will use α = 0.10 (as we want to be liberal rather than conservative)

Step 4: Decision rule - From 't' table, t (0.1, 14df) = 1.345

Non-Rejection Region (1- α = 0.90)

ANALYTI LABS

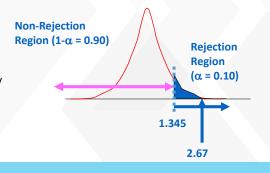
Example 2 (contd.)

Step 5: Calculate test statistic;

$$t = \frac{\overline{X} - \mu_{\bar{x}}}{SE} = \frac{\overline{X} - \mu_{\bar{x}}}{\sqrt[S]{\sqrt{n}}} \qquad t = \frac{670 - 630}{58 / \sqrt{15}} = 2.67$$

Step 6: Make a decision - As 2.67 is > 1.345, we will reject the H_o

Step 7: Conclusion - "Production workers at the company earn an average of \$40 per week less than the industry standard (t = 2.67, df = 14, p < 0.1)"



Comparison of two populations

Hypothesis testing for two samples:

- Difference between independent samples & dependent samples
- Two sample z test for means using independent samples
- Two sample t test for means using independent Samples
- Two sample t tests for means using dependent Samples

ANALYTI LABS

Chi-square test

Two properties are associated if the probability of having one property affects the probability of having another. Sometimes it is not known whether two properties are associated or not. What is required is a test of association, or, what is equivalent, a test of independence.

The Chi-square (χ 2) distribution can be used as a test of independence.

Example:

A psychologist conducted a survey into the relationship between the way in which a calculator was held and the speed with which 10 arithmetical operations were performed. The calculator could be either placed on a table or held in the hand; the sums could be performed in either less than 2 minutes, between 2 and 3 minutes or more than 3 minutes.

The following results were obtained for a sample of 150 children between 12 and 13 years old.

		Mode of Computation	
		On Table	Hand Held
	<2	28	12
Speed Of	2-3	25	35
Computation	>3	21	29

Example (contd.)

Solution:

Step 1: Ho: Speed and mode are independent

Ha: Speed and mode are associated

Step2: In order to determine whether the two variables are associated it is necessary to calculate what the frequencies would be if there was absolutely no connection between them, or as we call them "Expected

Frequencies"

	Table	Hand
<2	40*74/150 = 19.73	40*76/150 = 20.27
2-3	60*74/150 = 29.60	60*76/150 =30.40
>3	50*74/150 =24.67	50*76/150 = 25.33

ANALYTI LABS

Example (contd.)

Step 3: Now we have to use the expected and observed frequencies to calculate a test statistic.

The χ 2 test statistic is determined by $\sum \frac{\left(O_i - E_i\right)^2}{E_i}$

Step 4: In order to compare this with a critical value, we need to know the degrees of freedom of statistic.

v=degrees of freedom=(row number -1)· (column number -1)

Then
$$\chi^2_{test} = 9.329 > \chi^2_{critical} = 5.992$$

Step 5: Therefore, we reject H_0 and accept H_1 .

The result is significant at the 0.05 or 5% level. This means that there is a 5 in 100 probability that the difference between the two conditions could have arisen by chance.

According to these results the way you use your calculator does affect the speed with which you do a calculation.

ANOVA

- Analysis of variance is a statistical technique used for comparing the means of different samples and deciding whether they are drawn from the same population or different populations.
- Main Question: Do the (means of) the quantitative variables depend on which group (given by categorical variable) the individual is in?
- The ANOVA F-statistic is a ratio of the Between Group Variation divided by the Within Group Variation:

$$F = \frac{Between}{Within} = \frac{MSTR}{MSE}$$

• A large F is evidence against H0, since it indicates that there is more difference between groups than within groups.

ANALYTI LABS

Correlation

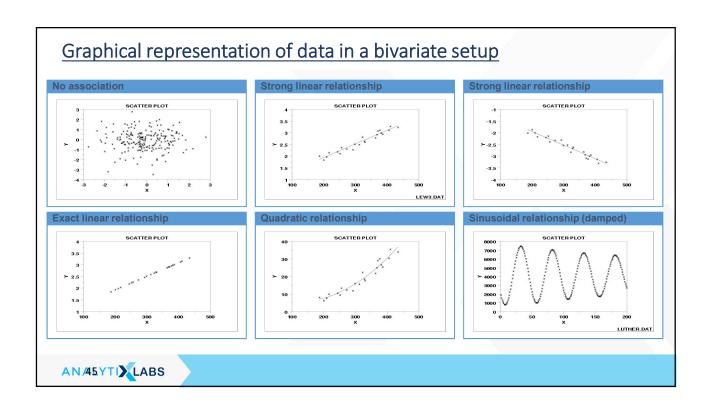
What is the relationship between two variables?

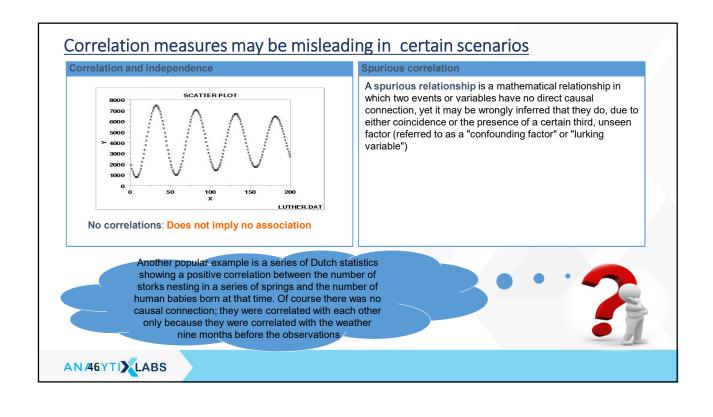
Relationship between hours studying (X) and grades on a midterm (Y)?

Relationship between self-esteem (X) and depression (Y)?

The relationship between two variables over a period, especially one that shows a close match between the variables' movements

Direction and strength of relationship between two variables





Non - Parametric tests

- Deals with enumerative data
- Does not deal with specific population parameters
- Does not require assumptions about specific population distributions (in particular , the assumption of normality)
- Non-Parametric tests ignores the magnitude of information contained in observations
- Use either frequencies or ranks (categorical or ordinal)
- · Non-parametric tests are called "non-parametric" because they do not make any assumption about a population parameter.
- In other words, when we apply a non-parametric test we do not have to make assumptions about mean of a population, its variance or background probability distribution.
- Thus, non-parametric tests are not as powerful as parametric tests they are of more general application and are available when the parametric tests fail.
- Basically, there is at least one non-parametric equivalent for each parametric general type of test
- Non-Parametric tests broadly fall into the following categories:
- Tests of differences between independent samples: The Mann-Whitney U test (t-test for independent samples), The Kruskal-Wallis H test (ANOVA), The Kolmogorov-Smirnov test (t-test for independent samples)
- Tests of differences between dependent samples: Wilcoxon Mann-Whitney Test (t-test for independent samples)



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