Past Paper: Differential Geometry

Midterm Exam, 2024

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Question 1.

Let $\alpha(t) = (t, t^2, t^3)$ be a curve in the Euclidean space. Calculate the following parameters for the given curve: unit tangent vector, unit normal vector, unit binormal vector, curvature, and torsion.

Solution. Solution

Question 2 (i).

State and prove the Wirtinger's inequality.

Solution. Solution

Question 2 (ii).

Using the Wirtinger's inequality, prove the isoperimetric inequality.

Solution. Solution

Question 3 (i).

For a unit speed curve $\beta(s)$, show that $\beta'' \cdot \beta''' \times \beta'''' = \kappa^5 \frac{d}{ds}\left(\frac{\tau}{\kappa}\right)$, where κ and τ are curvature and torsion of the curve $\beta(s)$.

Solution. Solution

Question 3 (ii).

Reparametrize the curve $\alpha(t) = e^t(\cos t, \sin t, 1)$ by its arc-length.

Solution. Solution

Question 4 (i).

Show that the curve $\beta(s) = \frac{1}{2}\left(s + \sqrt{s^2+1}, \left(s + \sqrt{s^2+1}\right)^{-1}, \sqrt{2}\ln\left(s + \sqrt{s^2+1}\right)\right)$ has unit speed.

Solution. Solution

Question 4 (ii).

Calculate the curvature of the curve $\beta(s)$ given in Q4(i).

Solution. Solution