Past Paper: Numerical Analysis-I

Midterm Exam, 2024

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Disclaimer: Use at your own risk. Errors possible.

Question 1.

Convert the following numbers to base 10.

(a) $x = (10010110)_2$

(b) $x = (1011)_2$

(c) $x = (777)_8$

Solution.

(a) $x = (10010110)_2 = 0(1)+1(2)+1(4)+0(8)+1(16)+0(32)+0(64)+1(128) = 999.$

(b) $x = (1011)_2 = 1(1) + 1(2) + 0(4) + 1(8) = 11.$

(c) $x = (777)_8 = 7(1) + 7(8) + 7(64) = 999.$

Question 2.

Determine the LU factorization of the given matrix using Crout's method

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

Solution. We take

$$L = \begin{bmatrix} \ell_{11} & 0 & 0 \\ \ell_{21} & \ell_{22} & 0 \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Then, $A = LU$ gives

$$\begin{gathered} \ell_{11} = 2, \quad \ell_{21} = 8, \quad \ell_{31} = 4, \\ \ell_{11}u_{12} = 1 \implies u_{12} = 0.5, \quad \ell_{11}u_{13} = 4 \implies u_{13} = 2 \\ \ell_{21}u_{12} + \ell_{22} = -3 \implies \ell_{22} = -7, \quad \ell_{31}u_{12} + \ell_{32} = 11 \implies \ell_{32} = 9 \\ \ell_{21}u_{13} + \ell_{22}u_{23} = 2 \implies u_{23} = 2, \quad \ell_{31}u_{13} + \ell_{32}u_{23} + \ell_{33} = -1 \implies \ell_{33} = -27. \end{gathered}$$

Overall,

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix} \begin{bmatrix} 1 & 0.5 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 3.

Apply the Gauss-Seidel method to solve the following system of equations.

$$4x - y + z = 12, \quad x - 2y + 4z = 5 \quad -x + 4y - 2z = -1.$$

Perform two iterations with 5 decimal place digit calculations.

Solution. Solution

Question 4 (a).

Derive the formula for the false-position method.

Solution. Solution

Question 4 (b).

Apply the method of false position to find a real root of the equation $x^4 - 11x + 8 = 0$ within the interval $[1, 2]$. Perform five decimal place digit calculations. Apply two iterations.

Solution. Solution

Question 5 (a).

Derive Newton-Raphson method.

Solution. Solution

Question 5 (b).

Evaluate $\sqrt{29}$ by Newton-Raphson method. Carry out five decimal place digit calculations. Stop the calculations when five decimal digits match with each other. Take an initial guess as $x_0 = 3.3$.

Solution. Take $f(x) = x^2 - 29$. Then, $f(x) = 0 \implies x = \pm\sqrt{29}$. So, we apply the Newton-Raphson method to estimate $f(x) = 0$. The corresponding formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

with $f'(x) = 2x$. As a result, we have

$$\begin{aligned} x_0 &= 3.3 \\ x_1 &= SOMETHING \\ x_2 &= SOMETHING \\ x_3 &= SOMETHING \\ x_4 &= SOMETHING \\ x_5 &= SOMETHING. \end{aligned}$$

Therefore, $\sqrt{29} = 5.ABCDE$ to 5 decimal digits.

Question 6.

Convert the system into an iterative form suitable for Newton-Raphson method then solve the following

$$x - y^3 - 1 = 0, \quad \sin(x) + y - 1 = 0.$$

Use the initial guess as $(x_0, y_0) = (0, 0)$. Apply one iteration only.

Solution. Solution