The Visualization of Uncertain Data: Methods and Problems

Heidrun Schumann
University of Rostock

The Visualization of Uncertain Data: Methods and Problems

Henning Griethe [*]
University of Rostock

Heidrun Schumann [†]
University of Rostock

Abstract

Visualization is a valuable approach to the exploration and communication of large data sets. In different domains these data sets may contain an unavoidable amount of uncertainty that needs to be included in the visualization process to enable the correct cognition of hidden facts. In recent years many different concepts regarding this *uncertainty visualization* were developed but the topic is still gaining interest. Within this quick development it is difficult to keep track of what is there and what is missing.
Therefore this paper provides a statement on the position of uncertainty visualization today: It defines the basic concept of uncertainty and discusses sources and necessary measures. It furthermore explains the basic uncertainty visualization process and systematically presents existing approaches to the acquisition and especially to the display of uncertainty that can be transferred to new fields, e.g. the visualization of uncertainty in structures. This leads to the depiction of limits and open questions that partly already are or that are assumed to be the focus of further research in this field.

1 Introduction

Many applications from different fields ranging from business to research depend on the evaluation of increasingly large data volumes. For this complex task a wealth of methods has been developed – e.g. in the area of data mining. An alternative approach to this problem is provided by visualization. Its aim is to graphically present data in such a way that the relevant characteristics can be spontaneously grasped.
Unfortunately in real world scenarios the used data is not always 100% perfect. Often errors, lacking values in a dataset, deviations or other kinds of uncertainty lower the quality of an analyzed data volume. Reasons for this might be e.g. inaccurate measurement techniques or losses from interpolations. To give a correct impression of the characteristics of the data such uncertainty has to be carefully considered in order not to lead to wrong decisions. So if visualization is used as a means to explore a data volume or to communicate its contents the uncertainty has to be included.
This demand led to the emergence of the research area of uncertainty visualization that generated many new techniques in recent years for the effective integration of uncertainty into visualizations – especially for the areas cartography, flow and volume visualization. Despite achieved successes there is still a high demand for new solutions in this field, e.g.

[*] Business Informatics, 18057 Rostock, Germany
[†] Computer Graphics, 18051 Rostock, Germany

for displaying uncertain abstract data. Thus there is a noticeable movement in this young research area.

To keep track of the developments and isolate limits and open questions, the aim of this paper is to provide a closed and compact overview on the different aspects of the visualization of uncertainty today – from its acquisition and processing to its graphical representation.

This paper is the further development of a work published in [GrSc05]. Therein the aim was to introduce general visualization concepts to an audience from a business informatics environment. The center of interest was laid on graphically supported decision making wherein uncertainty was motivated as an important influence on the decision quality that should therefore be displayed.

With this paper the focus is narrowed on pure uncertainty visualization. The preceding work was greatly extended to form the following far more complete state-of-the-art discussion on the visualization of uncertain data.

For this purpose the paper introduces in section 3 the concept of uncertainty as it is understood in visualization. Thereby relevant causes of uncertainty and how to measure it are discussed. In section 4 the general process of uncertainty visualization is described. Key issues here are the acquisition of uncertainty, challenges of its visualization, a classification and an overview presentation of existing techniques. Moreover, we present an idea to include uncertainties into structure visualizations. In section 5 a number of open questions and limits of uncertainty visualization today are discussed. Finally section 6 concludes with a short summary.

2 Uncertainty – the Concept

2.1 Definition

In the literature today there is no consensus on the perception of uncertainty (often also denoted as “data quality problems”) or on a universal way to represent it ([Pan01]). One of the few closed definitions of uncertainty can be found in [HuGo93] where it is stated as: „degree to which the lack of knowledge about the amount of error is responsible for hesitancy in accepting results and observations without caution”.

In general uncertainty is understood as a composition of different concepts (see e.g. [KlWi99, THM05, GBW94, BBC91, Pan01]) such as:

- error – outlier or deviation from a true value,
- imprecision – resolution of a value compared to the needed resolution (e.g. values are highly accurately given for countries but are needed for states),
- accuracy – size of the interval a value lies in,
- lineage – source of the data (e.g. at first hand or at second hand)
- subjectivity – degree of subjective influence in the data,
- non-specificity – lack of distinctions for objects (e.g. an attribute value is known to be one of several alternatives but not which one)
- noise – undesired background influence.

Other authors distinguish e.g. more application oriented between different geometric uncertainties ([LWS96]) or geospatial, time and theme data uncertainty ([SSP03]).
Some concepts out of this diversity are very different from others and might therefore require specialized treatment. This is why some authors differentiate e.g. error from uncertainty which has the drawback of losing a unified term that subsumes the relevant kinds of distrust in data. Due to this drawback we want to follow e.g. [Pan01] who uses uncertainty as the desired generic term. Nevertheless the intended visualization has to enable the graphical separation of inherently different kinds of uncertainty like error or imprecision to let the user comprehend the contained data quality problems.

2.2 Sources

For an application, different sources of uncertainty may occur at the acquisition of data, but also at the different stages of the visualization pipeline – from filtering the acquired information – over mapping the results onto geometrical data – to finally rendering a picture on a computer screen (see [PWL97], [LoBr98]).
In the acquisition errors in measurements or simulation runs, unsuitable data collection techniques or statistical variations may introduce uncertainty. This can be increased by the conversion of units of measure or by resampling, rescaling or interpolation of data at the filtering stage. Other sources of uncertainty might be unavoidable or for performance reasons intended losses in the mapping process. This is also true for the rendering when e.g. approximations are used for illumination calculations or when animation frames are interpolated.
Having identified relevant sources of uncertainty, it is necessary to assign it to the data of the application to enable its processing in order to ensure a correct interpretation of the presented data.

2.3 Assignment of Uncertainty to the Data

Relevant sources for an application result in different uncertainties in its data. Most often uncertainty is considered for scalar attribute values by e.g. estimating which measured values are missing or how probable a value seems. But it is also used to address distrust in whole datasets representing e.g. multivariate, vector or tensor data, as well as distrust in structure information (e.g. if a relation between two objects is correct). In principle uncertainty can be assigned to any component of a used data model. For example [Hua05] and [SSP03] assign it to “dimensions” (attributes) to give an impression of the average uncertainty of all values belonging to this attribute.
To capture the concept of uncertainty for practical usage in a visualization system suitable measures have to be provided. This will be discussed next.

2.4 Measures of Uncertainty

Most often, uncertainty is quantitatively described by scalar values like probability, error percentage, distance (e.g. from the true value) or standard deviation. For example

[THM05] shows how to intuitively express concepts like error, precision, completeness, consistency, lineage, credibility, subjectivity and interrelatedness with standard deviation measures. [KlWi99] points out that formal concepts other than probability – like fuzzy, plausibility, belief, possibility or necessity measures – are more suitable for certain applications and should therefore be considered. Other possibilities include assignments to defined uncertainty classes, ranges of values or sets of possible values (e.g. to express non-specificity). Also qualitative descriptions like “values are second-hand” or “estimated by three experts” might be used.

These measures may be automatically or interactively attached as metadata to the elements of the application specific data model which forms the basis for a visualization that creates awareness of the underlying uncertainty and thus provides a more complete and realistic visual communication of the context.

3 Uncertainty – Put to Visualization

3.1 Basic Process of Uncertainty Visualization

In principle, uncertainty is part of the raw data an arbitrary visualization is based on that also runs through the transformation process to the final picture on the screen. However, for a better understanding one can consider the visualization of “normal” data and uncertainty as separated in the following way (see figure 1).

The illustration distinguishes two phases – the acquisition of uncertainty and the visualization – which will be discussed in more detail in the next two sections. It also differentiates four different kinds of data flows:

(1) the basic transformation process of data on its way through the visualization pipeline. As mentioned this process can be separated into the pipeline of the certain components of the data and the pipeline of the corresponding uncertainty.

(2) the in- and output of the acquisition of uncertainty data.
Here the argument from the section “Sources” is represented that data from all stages of the visualization process have the potential to carry a relevant degree of uncertainty that needs to be considered.

(3) the dependencies between the visualization of the given raw data and its uncertainty.
The connections in the illustration represent the usual case that specific application data are to be explored or communicated while its uncertainty is recognized here as integral part of this data. Then the decisions on what raw data to focus on and what geometric forms and rendering techniques to choose, influences the according decisions in the processing of uncertainty. For example, if certain information is not interesting at the moment and is therefore filtered out, its uncertainty is also not relevant.

(4) the parameterization of the visualization pipeline.
Filtered uncertainty may serve two purposes. One is –according to (1) – to pass it on through the uncertainty visualization pipeline to display it directly, so that the user can actually “see” the underlying uncertainty. The other purpose is to cause an indirect effect. Here uncertainty is not displayed itself, but used to parameterize the

visualization of the other data instead. For example it could be applied in the filtering stage to decide that parts of the data are too unreliable to be displayed.

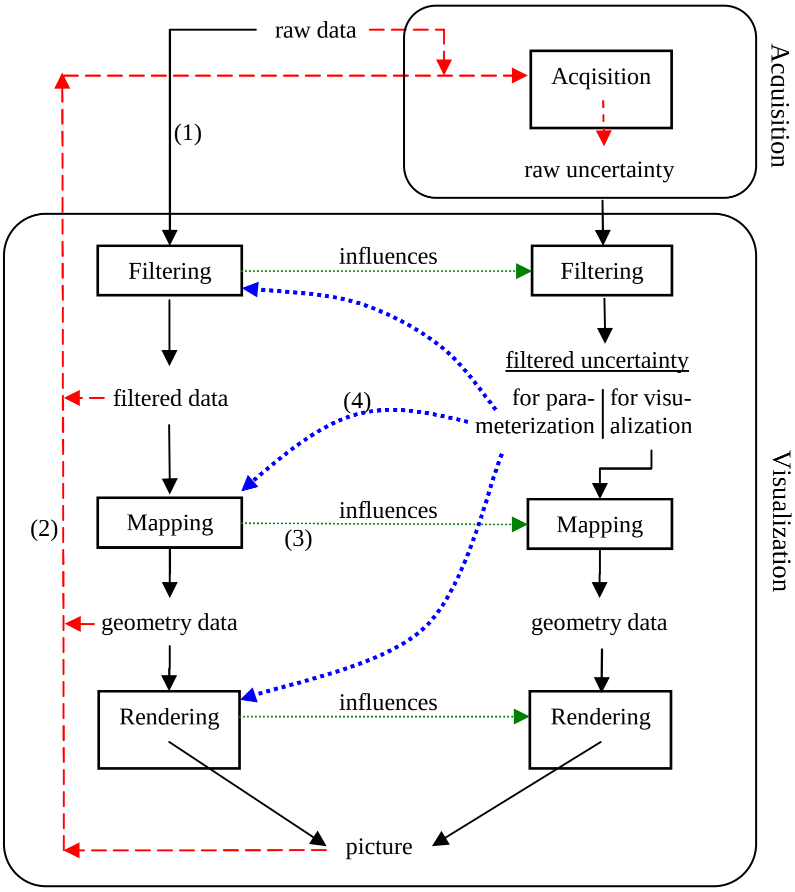

Figure 1: The process of uncertainty visualization.

It shall be remarked here that the different steps and data flows in the pipeline can be executed automatically, or be highly controlled by user interaction. But such a decision on the amount of control and support of the user depends on his experience and the philosophy of the visualization system and will be discussed here no further.
After this short introduction of the basic process, we will now take a closer look at the acquisition and especially at the visualization of uncertainty.

3.2 Acquisition

Often, the given raw data already contain statements on its reliability, e.g. on the precision of used instruments (e.g. standard deviation) or on the completeness of a set of data (e.g. a number of missing values). Further statements may be automatically derived (e.g. by statistical or other analyses) or obtained through user interaction where one selects suitable uncertainty evaluation methods or directly chooses / specifies special uncertainty values due to one's experience.

Different methods determine uncertainty values in various ways. These include e.g. comparisons with former results, specifications or experience ([TaKu93]), comparisons to known or postulated distributions, assumed relations or expected value ranges ([BeBu99]), distances from interpolated to measured values ([CeRh00]) or results from Monte-Carlo-Simulations ([KMB03]).

If this is not possible, distances between approximating and exact methods or between different interpolation or integration methods ([PWL97]), differences according to distinct parameterizations of methods ([PFN94]) or tests of reconstruction techniques for missing values against randomly deleted, known values (cross validation) ([BeBu99]) can be used. Otherwise subjective estimations of uncertainty might also be suitable for certain scenarios.

For the decision of which and how much uncertainty data to collect – as stated in [Noc00] for the acquisition of metadata in general – one has to answer the questions which kinds of uncertainty (metadata) are relevant from the data's perspective, which kinds are helpful from the perspective of the visualization steps and which kinds are practicably extractable from the perspective of the available methods?

The answering of these questions and the acquisition of the relevant uncertainty enables the advancement to the next phase: the visualization of uncertainty.

3.3 Visualization

3.3.1 Challenge

One of the greatest challenges in the visualization of uncertainty is to control the explosion of the amount of output data. Already without the inclusion of uncertainty many visualization systems meet their limits and require very sophisticated methods to let users effectively explore large datasets. But imagine that many or maybe all values of the considered data each have one assigned measure of uncertainty. Or if different kinds of uncertainties are distinguished, it may result in "multi-dimensional uncertainty" (as termed in [SCB04]) so that far more than one measure becomes necessary. This would multiply the amount of data to cope with and most probably prevent a complete presentation of the data and the underlying uncertainty on the screen. Therefore, usually a relevant subset or an aggregation of the different kinds of uncertainty is determined, as described e.g. in [WGG98] with the term "weighted uncertainty", that maps the different kinds to one measure. More information on how to suitably combine different uncertainties within the raw data or on the propagation through the visualization pipeline can be found e.g. in [KlWi99] where methods to calculate with or reason about quantitatively expressed uncertainty are described.

Another challenge still is the development of effective metaphors for the conspicuous display of uncertainty that is clearly distinguishable from other data (see e.g. [THM05]) – even if certain application domains like cartography, flow and volume visualization already brought up many different uncertainty visualization techniques.
An overview on available techniques today and how to principally classify them will be considered next.

3.3.2 Classification

As suggested e.g. in [PWL97] uncertainty visualization techniques can be classified similarly to the visualization techniques for the data the uncertainty is based on. According to this the criterion “value of datum” allows a classification in scalar, vector, tensor or multivariate techniques. Furthermore “location of datum” may serve to differentiate between 0D, 1D, 2D, 3D, time, etc.; “extend of datum” between discrete and continuous domains- and co-domains; “visualization extend” between a display of discrete individual datasets (e.g. by glyphs) and of a continuous range of data (e.g. by surfaces), and “axes mapping” between different choices to map attributes to axes (e.g. to abstract ones) and to combine axes.
This classification is quite detailed but available uncertainty visualization techniques are not very balanced across the different characteristics and concentrate e.g. on scalar values. Therefore the classification is not suitable as a guide to present an overview on existing techniques. Instead a simplified and more intuitive division shall be used that differentiates on how, in principle, uncertainty can be displayed: indirectly by parameterization (as mentioned in section 3.1) or directly. The direct display can be further subdivided in a utilization of free graphical variables, the integration of additional graphical objects, the use of animation, an interactive representation and the addressing of other human senses. Following this classification, we now want to give an overview on uncertainty visualization techniques.

3.3.3 Overview on Existing Techniques

The following overview is intended to systematically describe possible techniques. A thorough discussion on the effectiveness of the different approaches in different contexts and under which conditions will not be included, since corresponding usability studies unfortunately are still quite rare. We judge this to be a very important next step.
One way to include uncertainty into a visualization system is by an indirect approach where it is not explicitly displayed, but used to parameterize the visualization process instead. As described e.g. in [PaBe94] with the term “data quality filter” or in [SCB04] uncertainty may serve to decide what data is reliable enough to be displayed and what should be filtered. But also in later steps of the visualization pipeline uncertainty can be applied as a parameter. [PFN94] suggests the integration of uncertainty as a weight into interpolation and approximation methods so that reliable values have more influence in the filtering or mapping than unreliable ones. Likewise uncertainty may steer the precision of a rendering algorithm so that computation resources are not wasted on an unnecessary accuracy.
In most cases however uncertainty is directly included into the graphical representation. As mentioned this can be reached by:

- the utilization of free graphical variables:
  Here uncertainty is mapped on yet unused graphical attributes (see e.g. [BBC91], [Pan01]) like color, size, position, angle, texture or on focus, clarity, fuzziness, transparency or edge crispness. Figure 2 shows some interesting examples where color (top), surface texture (bottom left) and clarity/blurring (bottom right) are used to encode uncertainty. Especially blurring is recognized as a very intuitive uncertainty metaphor and is therefore often considered in current research – like [BWE05] or [KUS05].

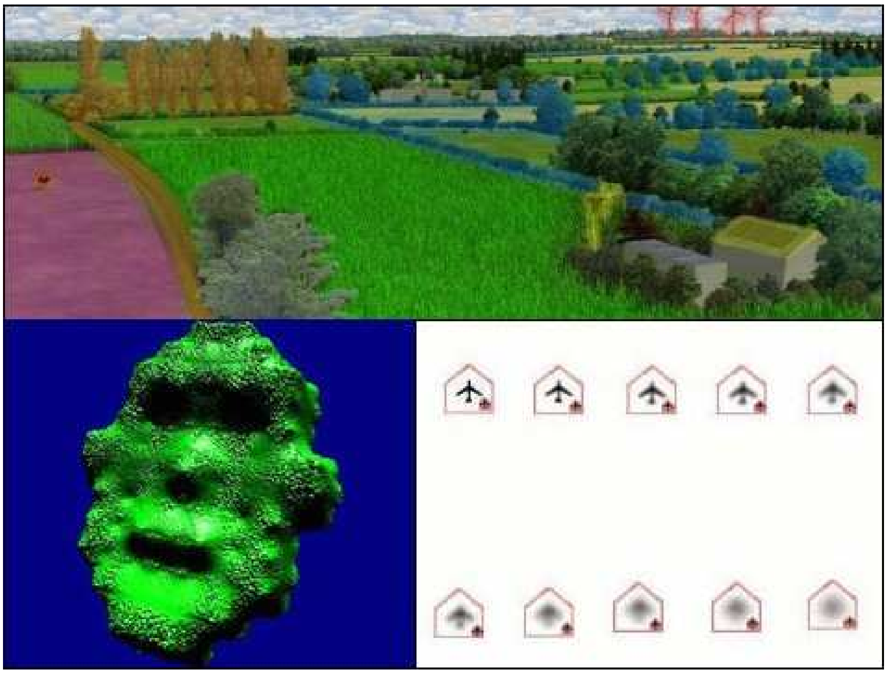

Figure 2: Uncertainty expressed by the utilization of free graphical variables.
Top: Usage of color to indicate different degrees of uncertainty in a future environmental setting (from [App04]). Bottom left: The variation of the surface texture creates this bumpy face according to uncertainty about its true shape determined by the difference between different surface reconstruction methods (from [PWL97]). Bottom right: uncertainty on the kind of an object is encoded into the clarity of an icon – the more blurred the more uncertain (from [BKS02]).

- the integration of additional geometrical objects:
  This approach presents uncertainty by different additional objects. Quite common are glyphs (see e.g. [PWL97] or [SCB04]) or labels but also a volume rendered thickness of an isosurface view ([JoSa03]), error bars ([OlMa02]) or an overlaid grid with varying thickness, sharpness, noise or transparency of grid lines ([CeRh00]) may indicate uncertainty. An interesting variation of the grid inclusion is presented in [KMB03]. There overlaid hierarchical spatial data structures like

quadtrees or hexagonal heptrees indicate uncertainty by the degree of segmentation. Figure 3 illustrates some of the mentioned examples.

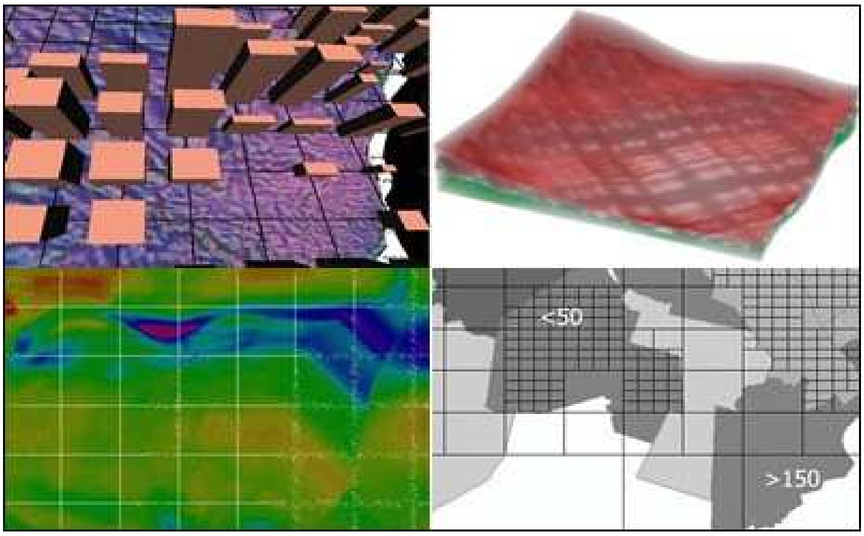

Figure 3: Uncertainty expressed by the integration of additional geometrical objects.
Top left: Box glyphs representing uncertainty on latitude and longitude of underwater objects (from [SCB04]). Top right: Thickness of an artificial volume around an isosurface depicts local uncertainty (from [JoSa03]). Bottom left: Regional uncertainty is indicated by the noise within overlaid gridlines (from [ChRe00]). Bottom right: Here regional uncertainty is presented by rectangle sizes within an overlaid quadtree tessellation (from [KMB03]).

- the use of animation:
  To represent uncertainty in a dynamic way, parameters like speed, blinking, duration, motion blur, range or extend of motion, specific order (what moves when), or oscillation qualify for a mapping ([PWL97]). Figure 4 shows an example for uncertainty coded into the range of motion (3D-distances).

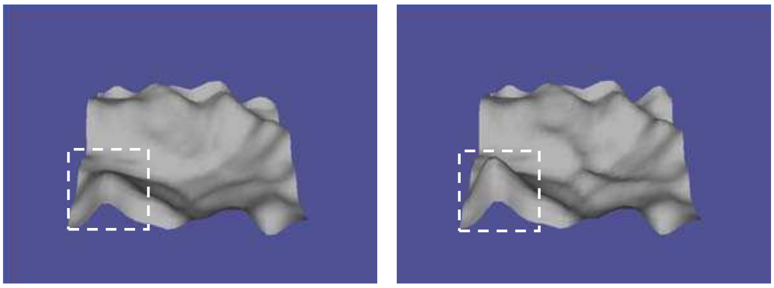

Figure 4: Uncertainty expressed by the use of animation.
Here two frames of an animation demonstrate how the range of the movement in a surface is able to express uncertainty (from [Bro04]).

- an interactive representation:
  An impression on uncertainty can be provided to the user by interaction. A known example is the "clickable map" ([WGG98]) where uncertainty is discoverable by mouse interaction.
- the addressing of other human senses:
  Uncertainty can also be incorporated by the use of acoustic, e.g. pitch, volume or rhythm ([LWS96]), or haptic senses like touch ([SCB04]) or vibration ([BKS02]).

The decision of which of these techniques to choose strongly depends on the intended goal. Shall the user be made aware of the existence of underlying uncertainty or should the amount of uncertainty also be delivered? The first goal is the easier task for which all mentioned techniques are more or less usable within different application scenarios – especially if they create a clear contrast to the other visualized content. This could be e.g. animation or the utilization of free graphical variables like transparency or the blur factor. If it is the goal to provide an impression on the "uncertainty value", more quantitative representations are appropriate – e.g. the integration of additional geometrical objects like grids or annotations.

The presented techniques provided an overview on how, in principle, to integrate uncertainty into visualizations. We believe this view on existing approaches to be general enough to be transferred to application domains which do not appropriately include uncertainty yet. As an example we shortly want to discuss uncertainty in structure visualizations.

3.3.4 An Example: Uncertainty in Structure Visualization

An exemplary application area where uncertainty might be relevant, but is hardly considered, is the visualization of structure information.

Imagine the analysis of a company structure in which many sub firms engage in different kinds of official, but also unofficial relations. For the analyzer certain relations are known for sure but others might be doubtful. Uncertainty on money flows, secret collaborations or relations to bogus companies might occur. For a correct interpretation this uncertainty has to be included into a visualization of the sub firm relationships. For this we now want to sketch the use of *the utilization of free graphical variables* (as also mentioned in [GrSc05]).

Relationships are often represented explicitly by edges in a tree or a graph, or implicitly by e.g. graphically nesting components of a hierarchy. This basic choice for a representation strongly influences how uncertainty can be included therin:

- Explicit visualizations naturally contain node and edge attributes for a mapping. So the inclusion can be realized relatively straight forward – e.g. the lines indicating the relations could be varied. Blurred, distinguished in color, wavy or dotted lines are able to indicate less trusted relationships. If there is non-specificity – e.g. if it is known that a sub firm belongs to one of two organizations, but not to which one – a link might be drawn from the sub-firm-node to some place between the organization-nodes without directly connecting to them (see figure 5 left).
- For implicit representations, like hierarchy visualizations that nest components, this inclusion is more difficult. Here nodes and edges are not so easily accessable in the display. This is especially true for edges that are not mapped to graphical objects. A

possibility for the integration of uncertainty therein is e.g. the variation of the appearance of the nesting. As an example, consider a Tree-map ([JoSh91]) with box lines indicating the hierarchy level: Here e.g. the variation of the box line style as mentioned for edges (see figure 5 top right) or the adaption of the corresponding area (see figure 5 bottom right) results in a simple mapping of uncertainty. The determination of effective metaphors in more challenging situations, and also the avoidance of clutter, are questions that need to be addressed, but are beyond the scope of this paper and shall therefore be left open.

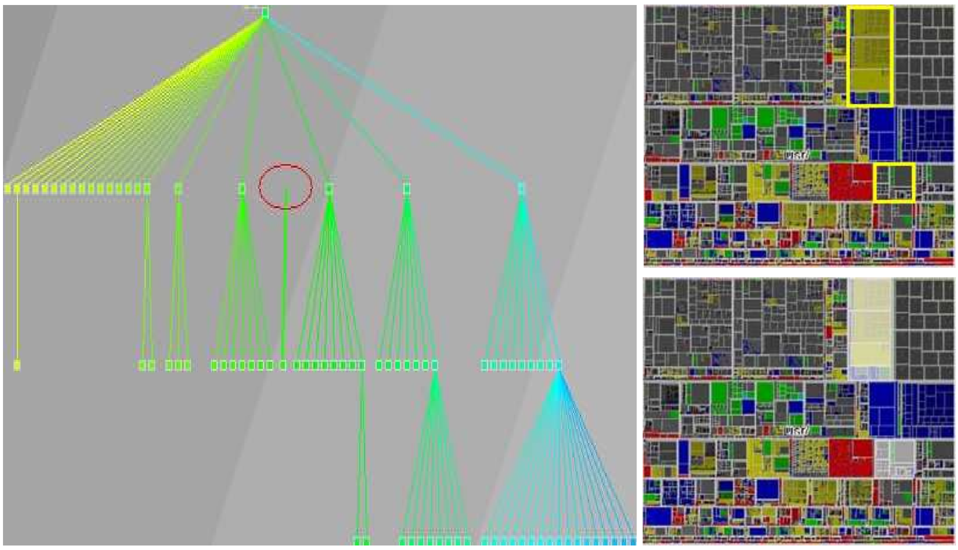

Figure 5: Uncertainty in structure visualizations.
Left: Uncertainty about where to incorporate a dubious sub firm into a company structure (based on the InfoVis Toolkit [Fek04]). Top right: Line color variation in a Tree-map indicates uncertainty in some sections of a hierarchy (adaptation from [FePl02]). Bottom right: The same effect created by an increased brightness.

The presented ideas provide first alternatives to fill a gap in uncertainty visualization: the display of distrust in relationships. Despite of the many different techniques available today this already hints at limits and open research questions that deserve further research.

4 Uncertainty – Limits of Today's Visualization

As already mentioned a great challenge in uncertainty visualization is the potential multiplication of the data volume that needs to be displayed. This leads directly to the question how existing uncertainty visualization techniques scale to large data sets. If the application of a single uncertainty measure already has a high risk to result in a cluttered view, how can higher dimensional uncertainties be represented in general? The ability to

graphically distinguish e.g. error from imprecision is quite desirable, but the prevalent literature primarily focusses on single concepts.
Existing techniques were mainly developed for cartography, flow and volume visualization. There are comparably few articles on this issue in other fields – e.g. on the display of uncertainty in abstract data without a natural time or space dimension. As discussed, the effective integration of uncertainty in structure visualizations also still remains an open question.
Furthermore there exists a demand to evaluate the practical suitability of techniques to help decide when and for what data which techniques are most reasonable. How can techniques even be automatically chosen and parameterized? Is there a difference in their effectiveness if applied to different kinds of uncertainty? What are suggestible metaphors for different kinds of uncertainty?
These questions show that despite impressive advances in the field, there still is a great need for further research on how to convey content from different application areas to the user as truly as possible.

5 Conclusion

The aim of this paper was to present an overview on uncertainty visualization today – on achieved results but also on missing solutions.
For this purpose the main concept of uncertainty was introduced as it is understood in the relevant literature. Then a closed illustration of the process of uncertainty visualization was presented. This guided the following systematic view on existing concepts for the acquisition and especially the visualization of uncertainty.
The given depiction provides a thorough understanding of uncertainty and its consequences on the visualization field. It shows what is possible and available today but also allows an imagination of what is to come in the near future.

Literatur

[App04] K. Appleton. *Representing uncertainty in geovisualisation: a research agenda*. European Science Foundation conference on Geovisualization, Crete, 2004.

[BBC91] M. K. Beard, B. P. Buttenfield and S. B. Clapham. *NCGIA Research Initiative 7: Visualization of Spatial Data Quality*. Technical Paper 91-26, National Center for Geographic Informationand Analysis, 1991.

[BeBu99] M. K. Beard and B. Buttenfield. 1999. *Detecting and Evaluating Errors by Graphical Methods*. In Geographic Information Systems: Principles and Technical Issues. New York: John Wiley Vol. 1, pp. 219-233, 1999.

[BKS02] A. S. Bisantz, T. Kesavadas, P. Scott, D. Lee, S. Basapur, P. Bhide, C. Sharma and E. Roth. *Holistic Battlespace Visualization: Advanced Concepts in Information Visualization and Cognitive Studies.* Technical Report to the Center for Multisource Information Fusion, University of Buffalo, 2002.

[Bro04] R. Brown. *Animated visual vibrations as an uncertainty visualization technique*. Proc. of the 2nd international conference on computer graphics and interactive techniques in Australasia and South East Asia, Singapore, ACM, 2004.

[BWE05] R. P. Botchen, D. Weiskopf and T. Ertl. *Texture-Based Visualization of Uncertainty in Flow Fields*. IEEE Visualization 2*005* Proceedings, 2005.

[CeRh00] A. Cedilnik and P. Rheingans. *Procedural Annotation of Uncertain Information*. Proc. Vis'00, IEEE Visualization, Salt Lake City, UT, 2000.

[Fek04] J.-D. Fekete. *The InfoVis Toolkit*. Proc. of the 10th IEEE Symposium on Information Visualization, 2004.

[FePl02] J.-D. Fekete and C. Plaisant. *Interactive Information Visualization of a Million Items*. Proc. of IEEE Symposium on Information Visualization, Boston, 2002.

[GBW94] M. Goodchild, B. Buttenfield and J. Wood. *On introduction to visualizing data validity*. In H. Hearnshaw and D. Unwin, editors, Visualization in Geographical Information Systems, p. 141-149. Wiley, 1994.

[GrSc05] H. Griethe and H. Schumann. *Visualizing Uncertainty for Improved Decision Making*. Proc. of the 4th International Conference on Business Informatics Research, Skövde, Sweden, 2005.

[Hua05] S. Huang. *Exploratory Visualization of Data with Variable Quality*. Master thesis, Worcester Polytechnic Institute, 2005.

[HuGo93] G. J. Hunter and M.F. Goodchild. *Managing uncertainty in spatial databases: Putting theory into practice*. Journal of Urban and Regional Information Systems Association, 5(2): p. 55-62, 1993.

[JoSa03] C. R. Johnson and A. R. Sanderson. *A Next Step: Visualizing Errors and Uncertainty*. In: Visualization Viewpoints published by IEEE Computer Society, 2003.

[JoSh91] B. Johnson and B. Shneiderman. *Tree-maps: A Space Filling Approach to the Visualization of Hierarchical Information Structures*. In IEEE Visualization '91. IEEE, 1991.

[KlWi99] G. Klir, and M. Wierman. *Uncertainty-Based Information: Elements of Generalized Information Theory*. 2nd edition, Physica-Verlag, 1999.

[KMB03] J. D. Kardos, A. Moore and G. L. Benwell. *Visualising Uncertainty in Spatially-Referenced Attribute Data using Hierarchical Spatial Data Structures.* Proceedings of the 7th International Conference on GeoComputation, UK, 2003.

[KUS05] J. M. Kniss, R. V. Uitert, A. Stephens, G. Li, T. Tasdizen and C. Hansen. *Statistically Quantitative Volume Visualization*. IEEE Visualization 2*005* Proceedings, 2005.

[LoBr98] A. Lopes and K. W. Brodlie. *Accuracy in 3D Particle Tracing*. In: Mathematical Visualizations: Algorithms, Applications and Numerics, Heidelberg, 1998.

[LWS96] S. K. Lodha, C. M. Wilson and R. E. Sheehan. *LISTEN: Sounding Uncertainty Visualization.* Proceedings of the 7th IEEE Visualization '96 Conference, San Francisco, 1996.

[Noc00] T. Nocke. *Metadatengewinnung und -spezifikation für Visualisierungs-entscheidungen*. Master Thesis, University of Rostock, 2000.

[OlMa02] C. Olsten and J. D. Mackinlay. *Visualizing Data with Bounded Uncertainty*. In Proceedings of the IEEE Symposium on Information Visualization, 37-40, 2002.

[PaBe94] J. Paradis and M. K. Beard. *Visualization of Data Quality for the Decision Maker: A Data Quality Filter*. Journal of the Urban and Regional Information Systems Association 6(2): 25-34, 1994.

[Pan01] A. Pang. *Visualizing Uncertainty in Geo-spatial Data.* In Proceedings of the Workshop on the Intersections between Geospatial Information and Information Technology, Arlington, 2001.

[PFN94] A. Pang, J. Furman and W. Nuss. *Data quality issues in visualization.* SPIE Proceedings on Visual Data Exploration and Analysis I, Vol. 2178, pp. 12-23, 1994.

[PWL97] A. Pang, C. M. Wittenbrink and S. K. Lodha. *Approaches to uncertainty visualization.* The Visual Computer 13, 1997.

[SCB04] G. S. Schmidt, S.-L. Chen, A. N. Bryden, M. A. Livingston, B. R. Osborn and L. J. Rosenblum. *Multi-dimensional Visual Representations for Underwater Environmental Uncertainty*. IEEE Computer Graphics & Applications, Special Issue on Visual Analytics, 2004.

[SSP03] H. Shu, S. Spaccapietra, C. Parent and D. Q. Sedas. *Uncertainty of Geographic Information and Its Support in MADS.* 2nd International Symposium on Spatial Data Quality, Hong Kong, 2003.

[TaKu93] B. N. Taylor and C. E. Kuyatt. *Guidelines for evaluating and expressing the uncertainty of NIST measurement results*. Technical report, National Institute of Standards and Technology, Technical Note 1297, Gaithersburg, 1993.

[THM05] J. Thomson, B. Hetzler, A. MacEachren, M. Gahegan and M. Pavel. *A Typology for Visualizing Uncertainty.* In Proceedings of the IS&T/SPIE Symposium on Electronic Imaging, Conference on Visualization and Data Analysis, San Jose, 2005.

[WGG98] F. J. M. van der Wel, L. C. van der Gaag and B. G. H. Gorte. *Visual Exploration of Uncertainty in Remote-sensing Classification.* Computers & Geosciences, v.24 n.4, p.335-343, 1998.