fundamental research within the visualization community to address, integrate, and expect uncertainty information. In this chapter, we outline sources and models of uncertainty, give an overview of the state-of-the-art, provide general guidelines, outline small exemplary applications, and finally, discuss open problems in uncertainty visualization.

1.1 Introduction

Visualization is one window through which scientists investigate, evaluate and explore available data. As technological advances lead to better data acquisition methods, higher bandwidth, fewer memory limits, and greater computational power, scientific data sets are concurrently growing in size and complexity. Because of the reduction of hardware limitations, scientists are able to run simulations at higher resolution, for longer amounts of time, using more sophisticated numerical models. These advancements have forced scientists to become increasingly reliant on data processing, feature and characteristic extraction, and visualization as tools for managing and understanding large, highly complex data sets. In addition, there is becoming a greater accessibility to the error, variance, and uncertainty not only in output results but also incurred throughout the scientific pipeline.

With increased size and complexity of data becoming more common, visualization and data analysis techniques are required that not only address issues of large scale data, but allow scientists to understand better the processes that produce the data, and the nuances of the resulting data sets. Information about uncertainty, including confidence, variability, as well as model bias and trends are now available in these data sets, and methods are needed to address the increased requirements of the visualization of these data. Too often, these aspects remain overlooked in traditional visualization approaches; difficulties in applying pre-existing methods, escalating visual complexity, and the lack of obvious visualization techniques leave uncertainty visualization an unsolved problem.

Effective visualizations present information in a manner that encourages data understanding through the appropriate choice of visual metaphor. Data are used to answer questions, test hypotheses, or explore relationships and the visual presentation of data must facilitate these goals. Visualization is a powerful tool allowing great amounts of data to be presented in a small amount of space, however, different visualization techniques are better than others for particular types of data, or for answering specific questions. Using the most befitting visualization method based on the data type and motivated by the intended goals of the data results in a powerful tool for scientists and data analysts.

The effective visualization of uncertainty, however, is not always possible through the simple application of traditional visualization techniques. Often, the visualization of the data itself has a high visual complexity, and the addition of uncertainty, even as a scalar value, complicates the display. Issues of visual clutter, data concealment, conflicts in how the data and the uncertainty are represented, and unintentional that can in principle could be known, but in practice is not. This type of uncertainty is called epistemic and is subjective, such as not knowing the birth date of the last Chinese Emperor. These uncertainties are due to errors that practically cannot be controlled and can be described by non-probabilistic modeling.

1.3.2 Mathematical Modeling of Uncertainty

A variety of types of uncertainties occur in practice, including mixtures. Quantification of uncertainties, including mixtures, requires a unifying mathematical framework, which is very difficult to establish and not yet fully accomplished.

1.3.2.1 Fundamental Setting

From a fundamental standpoint, we are interested in the situation with possible outcomes or occurrences of "events" A,B,C, where A,B, and C are subsets of the set of all elementary events in the universe. The task at hand is to then *measure* the evidence that A ever happened, the degree of truth of that statement "event A happened", and the probability that event A will happen. The question is then, how do we measure and what is measurement?

In mathematics, measurement means to assign real numbers to sets. For example, the classical task in metric geometry is to assign numbers to geometric objects for length, area, or volume. The requirement in the measurement task is that the assigned numbers should be invariant under displacement of the respective objects.

In ancient times, the act of measuring was equivalent to comparing with a standard unit. However, it soon became apparent that measurement was more complicated than initially thought in that it involves finite processes and sets. The first tool to deal with this problem was the Riemann integral which enabled the computation of length, areas, and volumes for complex shapes (as well as other measures). However, the Riemann integral has a number of deficiencies, including its applicability only to functions with a finite number of discontinuities, fundamental operations of differentiation and integration are, in general, not reversible, and limit processes, in general, can not be interchanged. In 1898, Émile Borel developed classical measure theory which includes σ-algebra to define a class of sets that is closed under set union of countably many sets and set complement, and defined as additive measure μ that associates a number $\in \mathbb{R}_0^+$ with each bounded subset in the σ-algebra. Around 1899-1902, Henry Lebesgue defined integrals based on a measure that subsumes the Borel measure, based on a special case. He connected measures of sets and measures of functions.

1.3.2.2 Quantification

Probability measure was then developed in 1933 by Andrey Nikolaevich Kolmogorov, which used classical measure theory and added the measure of 1 assigned to the universal set. This is thought of as classical probability theory.

The classical probability theory has since become the dominant approach to examine uncertainty and randomness. Extensive mathematical studies followed and resulted in highly sophisticated theories. Its foundation rests on the definition of *probability space*, which was Kolmogorov's big achievement. A probability space is a triplet (Ω, F, P). Here Ω is a countable event space containing all possible outcomes of a random event. F is the so-called σ-algebra of Ω and it represents all combinations of the outcomes from Ω. Its construction satisfies:

- It is not empty: $\emptyset \in F$ and $\Omega \in F$.
- If a set $A \in F$, then its complement $A^c \in F$.
- If sets $A_1, A_2, \ldots, \in F$, then $\bigcup_{i=1}^{\infty} A_i \in F$, and $\bigcap_{i=1}^{\infty} A_i \in F$.

P is the well known *probability measure* and it is used to assign a real number, i.e., the probability, on the occurrence of any outcomes of the events (from Ω) and their potential combinations (from F). It satisfies the following important and well known principles.

1. $0 \leq P(A) \leq 1$, for any $A \in F$.
2. $P(\Omega) = 1$. That is, the probabilities of all outcomes add up to one.
3. For $A_1, A_2, \cdots \in F$ and $A_i \cap A_j = \emptyset$, for any $i \neq j$,

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i).$$

About 50 years later, the additivity requirement became a subject of controversy in that it was too restrictive to capture the full scope of measurement. For example, it works well under idealized, error-free measurements, but is not adequate when measurement errors are unavoidable. In 1954, Gustave Choquet developed a (potentially infinite) family of non-additive measures (capacities), and for each given capacity, there exists a dual "alternating capacity". An integral based on these measures is non-additive, can be computed using Riemann or Lebesgue integration and is applied specifically to membership functions and capacities.

In 1967, Arthur P. Dempster introduced imprecise probabilities based on the motivation that the precision required in classical probability is not realistic in many applications. Imprecise probabilities deal with convex sets of probability measures rather than single measures. For each given convex set of probability measures he also introduced 2 types of a non-additive measures: lower and upper probabilities, and super- and supra-additive. This allow probabilities to be represented imprecisely by intervals of real numbers.

In 1976, Glenn Shafer analyzed special types of lower and upper probabilities and call then belief and plausibility measures. The theory based on these measures became known as Dempster-Shafer theory (DST) or evidence theory. DST is capable of dealing with interval-based probabilities, such that belief or probability measures are equal to the ranges of admissible probabilities. As it turns out, belief measures are equivalent to Choquet capacities of order inf and plausibility measures are equivalent to alternating capacities of order inf.

The comparison of membership functions of fuzzy sets and probabilities was investigated in 1978 by Michio Sugeno and found to be not directly possible. This lead to the generalization of additive measures analogous to generalization such that crisp sets generalize to fuzzy set, and additive measures generalize to (non-additive) fuzzy measures or monotone measures. The Sugeno integral was then introduced with respect to a monotone measure. That same year, Lofti Zadeh defined a *possibility function* associated with each fuzzy set that is numerically a membership function, and a *possibility measure* that is a supremum of the possibility function in each set of concern, for both crisp and fuzzy sets. This is one of several interpretations of the "theory of graded possibilities". Its connection to DST is that constant plausibility measures are equivalent to possibility measures and constant belief measures are necessity measures.

In summary, the three most utilized uncertainty theories are the Classical Probability Theory, the Dempster-Shafer Theory, and Possibility Theory and can be divided into two classes. The first class uses additive measures in which the addition equal to the union expresses no interaction between events and can be thought of a classical probability combined with measure theory. The second class uses non-additive measures, in which addition greater than the union expresses positive interaction between events, such as synergy, cooperation, coalition, enhancement or amplification, while addition less then the union expresses negative interaction between events such as incompatibility, rivalry, inhibition, downgrading, or condensation. This class combines one of many uncertainty theories with generalized measure theory.

1.4 evaluation

Visualization research is too often neglected by industry and other potential expert users. One of the reasons is the lack of a proper evaluation of the results. This lack of evaluation was especially obvious in historical visualization fields such as volume rendering or fluid flow visualization. In the more recent domain of uncertainty visualization, researchers have made a significant effort into the assessment of the proposed techniques. The types of evaluation may be classified into three groups:

- Theoretical evaluation: the method is analyzed to see if it follows established graphical design principles,
- Low-level visual evaluation: a psychometric visual user study is performed to evaluate low-level visual effects of the method,
- Task oriented user study: a cognitive, task-based user study is conducted to assess the efficiency or the usability of the method.

1.4.1 Theoretical Evaluation

General guidelines and rules regarding visual depiction of data have been established, that have proven their efficiency. Bertin in [5], later translated in [6], has introduced the concept of visual variables. These include among others the location, size, orientation, shape, focus and realism. Furthermore he defined four visual properties, natural ordering, the ability to quantify, the ability to focus user attention (selectivity) and the ability to associate similar elements (associativity). He studied which of these properties are verified by the visual variables. Tufte in [105], through his concepts of graphical excellence and integrity, has proposed a number of guidelines to enhance the precision and the usability of graphical depiction. Chambers and co-authors in [14] have studied the relative influence of specific patterns on the visual perception, for example straight lines versus curves, dark versus light objects or small versus large patterns. This study leads the authors to define general rules for plot construction.

These graphical design principles may be used to conduct a theoretical evaluation of new uncertainty visualization techniques. As already mentioned in Section 1.2, Zuk and Carpendale in [114] have done such an evaluation for eight uncertainty visualization techniques. The same type of theoretical evaluation was followed by Riveiro in [90] to evaluate three uncertainty visualization techniques in the context of information fusion and decision making.

1.4.2 Low-level Visual Evaluation

Barthelmé in [2] studied the influence on noise uncertainty in a decision-making task. Based on psychometric and simple task experiments, he proved that users can reliably measure the visual uncertainty and use this information in their decision-making. Coninx in [18] conducted psychometric experiments to measure the impact of contrast sensitivity on the visibility of uncertain noisy patterns. He used this information in order to control the visibility of uncertainty data in a visualization technique based on the perturbation of colormaps by Perlin noise.

1.4.3 Task-oriented User Study

Task oriented cognitive user studies are by far the most common way of assessing the efficiency and usability of uncertainty visualization techniques. In this type of evaluation a panel of users is typically asked to perform a task that requires not only low-level visual processing but also high-level cognitive treatment of the visual information. Standard tasks as an example may consist in counting the number of local minima in a dataset, find the location of the maximum or minimum value, find the direction of rotation of a vortex. The task completion time, task completion accuracy, user's rating of efficiency and usability may be recorded. A statistical analysis of the recorded data is done. Typical analyses include analysis of variance (ANOVA), used to check in particular if the difference in the mean value of two distributions is significant. Examples of uncertainty visualization papers with a task-based evaluation include [20, 21, 70, 94].

1.5 Review of Current State of the Art

The goal of visualization is to effectively present large amounts of information in a comprehensible manner, however, most visualizations lack indications of *uncertainty* [43, 44, 64, 86].

1.5.1 Traditional Representations

Tukey [106] proposed graphical techniques to summarize and convey interesting characteristics of a data set not only to facilitate an understanding of the given data but also to further investigation and hypothesis testing. These tested graphical methods, such as the boxplot, histogram, and scatter plot, provide identifiable representations of a data distribution, and their simplicity allows for quick recognition of important features and comparison of data sets. In addition, they can be substituted for the actual display of data, specifically when data sets are too large to plot efficiently.

1.5.1.1 1D

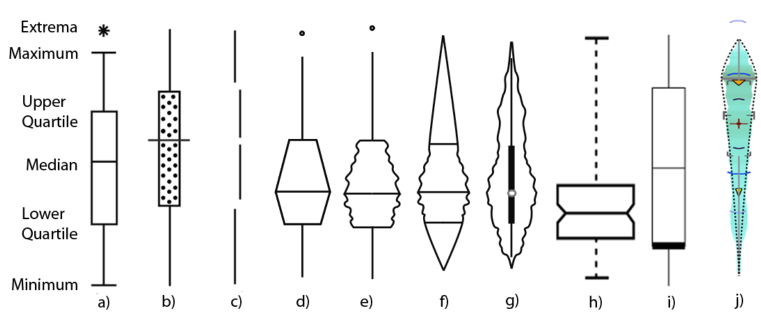

Fig. 1.2 Variations of the boxplot. a) The construction of the boxplot [106] b) Range plot [97] c) Innerquartile plot [105] d) Histplot [4] e) Vaseplot [4] f) Box-percentile plot [26] g) Violin plot [40] h) Variable width notched boxplot [68] i) Skewplot [16] j) Summary plot [85]

One of the most ubiquitous approaches to displaying uncertainty information is the boxplot [28, 35, 97, 106], which is the standard technique for presenting the *five-number summary*, consisting of the minimum and maximum range values, the upper and lower quartiles, and the median, as illustrated in Figure 1.2 a). This collection of values quickly summarizes the distribution of a data set, including range and expected value, and provides a straightforward way to compare data sets. In addition, the reduced representation afforded by the five-number summary provides a concise tool for data analysis, since only these characteristic values need to be analyzed. Figure 1.2 b and c show visual modifications of the boxplot. Surveys on the introduction and evolution of the boxplot can be found in [16, 84].

The box plot is often adapted to include information about the underlying distribution, as demonstrated in Figure 1.2(d-g). The most common modification adds density information, typically through changes to the sides of the plot. The hist plot [4] extends the width of the cross bars at the quartiles and median to express density at these three locations. The vase plot [4] instead varies the "box" continuously to reflect the density at each point in the innerquartile range. Similarly, the box-percentile plot [26] and violin plot [40] show density information for the entire range of the data set. Density can also be shown by adding dot plots [109], which graph data samples using a circular symbol. The sectioned density plot [17] completely reconstructs the box plot by creating rectangles whose colors and size indicate cumulative density, and placement express the location of the quartiles. Sample size and confidence levels can be expressed through changing or notching the width of the plot [68] (Figure 1.2, h) or by using dot-box plots, which overlay dot plots onto box plots [110]. Other descriptors, such as skew and modality, can be added by modifying the width of the median line [68], thickening the quartile lines [16], (Figure 1.2, i) adding beam and fulcrum displays [23] alongside, or overlaying additional glyphs [85] (Figure 1.2, j).

1.5.1.2 2D

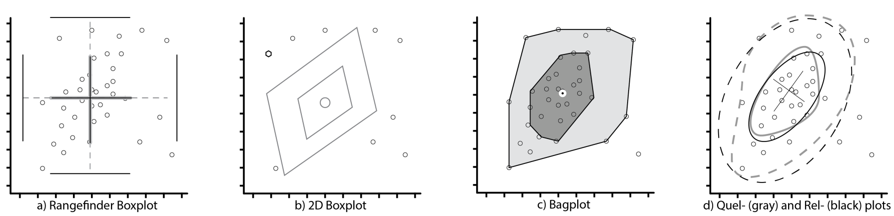

Fig. 1.3 Bivariate extensions of the boxplot. (a) The rangefinder boxplot [3]. (b) The 2D boxplot [102]. (c) The bagplot [91]. (d) The quel- and relplots [33]

Standard implementations of the boxplot focus on univariate data distributions. The five-number summary is a useful descriptor of not only univariate, but also bivariate data distributions. The main challenge in extending the boxplot for use with higher dimensional data is how to translate the five-number summary values, which are vector values in the bivariate case, into visual metaphors with meaningful spatial positions, while maintaining the simplicity of the original boxplot. A rangefinder boxplot [3], as seen as the solid back lines in Figure 1.3(a), is a simple extension of the boxplot into 2D which determines boxplots for the two dimensions independently and draws lines to show the interquartile ranges and extrema of those plots. This idea was further improved upon, as shown as the thick gray lines in Figure 1.3(a), to emphasize the quartiles rather than the range, by moving the perpendicular lines from the extrema values to the upper and lower quartile positions and extending whisker lines to the extrema value of the variables [54]. Other techniques for extending the boxplot into 2D all use the notion of a hinge that encompasses 50% of the data and a fence that separates the central data from potential outliers. The distinctions between each of these methods are the way the contour of the hinge and fence are represented, and the methods used to calculate the contours. The 2D boxplot [102], as seen in Figure 1.3(b), computes a robust line through the data by dividing the data into three partitions, finding the median value of the two outer partitions, and using these points as the line. Depending on the relationship between the slope of the line and each variable, the quartile and fence lines are drawn either parallel to the robust line, or parallel to the variables coordinate axis. The lines not comprising the outer-fence and the inner-hinge boxes are removed. The bagplot [91] uses the concept of halfspace depth to construct a bivariate version of the boxplot, as seen in Figure 1.3(c). The relplot and the quelplot [33] use concentric ellipses to delineate between the hinge and fence regions. Both the relplot and quelplot can be seen in Figure 1.3(d).

1.5.1.3 PDFs

There is a body of research investigating methods for displaying probability distribution functions with spatial positions. Each of these methods takes an exploratory approach to the presentation of the data by filtering down the amount of data, and then providing a user interface for the scientist to explore the data sets. Ehlschlaeger et al. [25] present a method to smoothly animate between realizations of surface elevation. Bordoloi et al. [7] use clustering techniques to reduce the amount of data, while providing ways to find features of the data sets such as outliers. Streamlines and volume rendering have been used by Luo et al. [62] to show distributions mapped over two or three dimensions.

Kao (2002) [49] uses a slicing approach to show spatially varying distribution data. This approach is interesting in that a colormapped plane shows the mean of the PDFs, and cutting planes along two edges allow for the interactive exploration of the distributions. Displaced surfaces as well as isosurfaces are used to enhance the understanding of the density of the PDFs.

Case studies of specific data have been performed by Kao [47, 48]. Their data sets come from NASAs Earth Observing System (EOS) Satellite images and Light

Detection And Ranging (LIDAR) data. The methods used to show this data include encoding the mean as a 2D color map, and using standard deviation as a displacement value. Histograms are also employed to understand better the density of the PDFs. To explore the mode of specific distributions, a small set of PDFs are plotted onto a color mapped spatial surface.

1.5.2 Uncertainty Visualization

Many visualization techniques that incorporate uncertainty information treat uncertainty like an unknown or fuzzy quantity; [78] is a survey of such techniques. These methods employ the meaning of the word uncertainty to create the interpretation of uncertainty or unknown to indicate areas in a visualization with less confidence, greater error, or high variation. Ironically, while blurring or fuzzing a visualization accurately indicates the lowered confidence in that data, it does not lead to more informed decision making. On the contrary, it obfuscates the information that leads to the measure of uncertainty. Because it obscures rather than elucidates the quantitative measures leading to the uncertain classification, such a solution to the problem of adding qualitative information to visualization misses important information.

1.5.2.1 Comparison Techniques

Often, uncertainty describes a comparison that can most clearly be understood visually, such as the difference between surfaces generated using different techniques, or a range of values that a surface might fall in. A simple approach to the visualization of this type of information is a side-by-side comparison of data sets. And example of this type of visualization is presented in Jiao et al [42] use where streamlines computed from various fiber tracking algorithms are interactively displayed along with the global and local difference measures. Another example is the time window, presented in [115], in which temporal uncertainty around archeological sites is displayed, using various visual clues, in an interactive, exploratory system.

However, this approach may not clearly manifest subtle differences when the data are nearly the same, and it becomes harder to perform this comparison as the visualization becomes more complicated. Another simple approach is to overlay the data to be compared [46]. With this technique, the addition of transparency or wire frame can produce a concise, direct comparison of the data sets. A similar approach uses difference images to display areas of variation [111]. These approaches are less effective, however, when the uncertainty can be categorized as more of a range of values rather than just two distinct ones. In such cases, a surface sweep, known as a fat surface [78], can be used to indicate all possible values. Another approach is the integration of isosurface and volume rendering. Here, an opaque isosurface can be used to indicate the most likely value, and a transparent volume rendering surrounding the isosurface can indicate the range of possible values [44]. Uncertainty information for large collections of aggregated data can be presented using hierarchical parallel coordinates [29]. Lee et al [53] visualize differences in location and sub-tree structure between two hierarchies through color and transparency. Finally, bounded uncertainty, while not effectively visualized in 3D, can be expressed through the ambiguation of boundaries and edges of pie charts, error bars, and other 2D abstract graphs [71] or as modifications to line charts [99].

1.5.2.2 Attribute Modification

Another standard method to visualize uncertainty involves mapping it to free variables in the rendering equation or modifying the visual attributes of the data. Such methods include modifying the bidirectional reflectance function (BRDF) to change surface reflectance, mapping uncertainty to color or opacity [66, 94, 100], or pseudo-coloring using a look-up table [78]. This technique has been used as a means for conveying uncertainty in the areas of volume rendering [22, 52, 92], point cloud surface data [80], isosurfacing [46, 82, 83, 89] and flow fields[8], and is often combined with other uncertainty visualization methods. An example technique colormaps flowline curvature onto volume rendered surfaces, highlighting areas in which small changes in isovalue lead to large changes in isosurface orientation and thus indicating areas where the isosurface is a poor representation of material boundary [50]. Another example uses height as free parameter to display uncertainty in 2D vector fields [73]. Texture can be used similarly to convey uncertainty and is also often modified by opacity, hue, or texture irregularities [18, 41, 76]. Sound has also been used as another channel for expressing uncertainty [59].

1.5.2.3 Glyphs

Glyphs are symbols used in visualization to signify data through parameters such as location, size, shape, orientation, and color. Because of the multivariate nature of glyphs, they can be used in visualization to map uncertainty to a free parameter. One such approach uses glyphs to present the distribution of multivariate aggregated data over a range of values [15]. These glyphs show the average, standard deviation, and distribution of three attributes of the data set. Conical glyphs have also been used to portray fiber tracks from DTI, leveraging the radius of the cone to encode uncertainty in the orientation of bundles [45].

An approach that modifies attributes of glyphs already present in the visualization is presented as a procedural generation algorithm [13]. In this work, the data is sampled on a regular grid and the size, color, and placement of glyphs are taken directly from the data samples. The uncertainty is then used to distort the glyphs so that glyphs with low uncertainty are very sharp, with the sharpness level decreasing as the uncertainty level increases. This distortion provides a clear indication of uncertainty and error while not placing heavy emphasis on areas of high uncertainty. In a similar fashion, contours already present in the visualization can be used [87, 88] or modified [72, 95] to express uncertainty.

Because not all data is visualized effectively using glyphs, the addition of glyphs to convey only uncertainty information is often a preferable approach. A specific example is the UISURF system [46], which visually compares isosurfaces and the algorithms used to generate them. In this system, glyphs are used to express positional and volumetric differences between isosurfaces by encoding the magnitude of the differences in the size of the glyphs. Similarly, line, arrow, and ellipsoidal glyphs can be used to depict uncertainty in radiosity solutions, interpolation schemes, vector fields, flow solvers, astrophysical data and animations through variation of placement, magnitude, radii, and orientation [55, 56, 58, 78, 94, 96, 112, 113, 116].

1.5.2.4 Image Discontinuity

Uncertainty visualization often relies on the human visual systems ability to quickly pick up an images discontinuities and to interpret these discontinuities as areas with distinct data characteristics. Techniques that utilize discontinuities rely on surface roughness, blurring, oscillations [13, 34, 57, 111], depth shaded holes, noise, and texture [22], as well as on the translation, scaling, rotation, warping, and distortion of geometry already used to visualize the data [78], to visualize uncertainty. Animation can highlight the regions of distortion or blur or highlight differences in visualization parameters [31, 61, 67]. Such techniques have been applied to multivaritate data displayed through scatter plots or parallel coordinates [27, 37].

1.6 Examples

1.6.1 Medical Visualization

A fundamental task in medical visualization is segmentation, the partitioning of a given image into regions that correspond to different materials, to different anatomical structures, or to tumors and other pathologies. Medical image acquisition typically introduces noise and artifacts, and we may wish to segment structures for which the data itself provides little contrast. This is a source of data uncertainty. In many cases, segmentation also involves complex computational models and numerous parameters, which introduces model uncertainty.

Traditional volume rendering classifies materials based on scalar intensity or feature vectors that account for first and second derivatives [51]. Lundström et al. [61] introduce probabilistic transfer functions that assign material probabilities to model cases in which the feature ranges of different materials overlap. This results in a distribution of materials at each location in space, which is visualized by an animation in which each material is shown for a duration that is proportional to its probability.

More complex segmentation tasks cannot be achieved based on local image properties alone. They require models that account for more global assumptions or more complex prior knowledge. Such models are also more computationally demanding and are typically run as a pre-process of the visualization. Some of them output class probabilities, from which Kniss et al. [52] derive measures that can be used to define transfer functions that enable exploring the risk associated with binary classifications, or to visualize spatial decision boundaries. Figure 1.4 shows the use of such transfer functions in a visualization of a segmented brain.

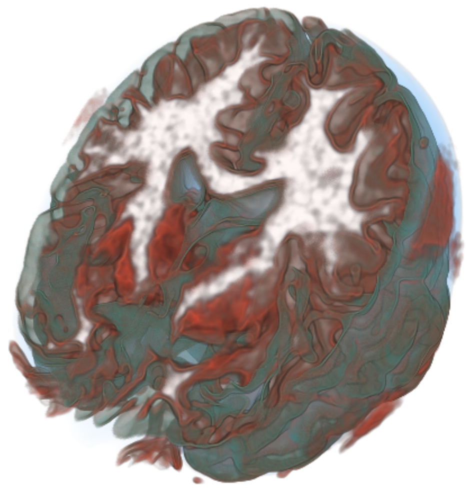

Fig. 1.4 A visualization of the brain using transfer functions that express the risk associated with classification.

The framework of Saad et al. [93] combines volume rendering with tables that list groups of voxels for which the same materials have been found to be most, second most, and third most likely. They demonstrate several examples in which these tuples can be used to detect anomalous subregions within areas that share the most likely material. Follow-up work [92] has concentrated on identifying anomalies or misclassification by considering regions in which the image-based likelihood disagrees with shape and appearance priors.

Finally, work by Torsney-Weir et al. [103] addresses the model uncertainty in segmentation methods by providing a systematic framework to explore the impact of model parameters. This should facilitate finding settings that produce the desired segmentation, and for which the results do not change significantly when slightly changing the exact parameter values.

Fiber tracking, the reconstruction of nerve fiber bundles from diffusion MRI, is another subfield of medical visualization in which uncertainty plays an important role. It is treated in detail in Chapter 8 of this book.

1.6.2 *Weather and Climate*

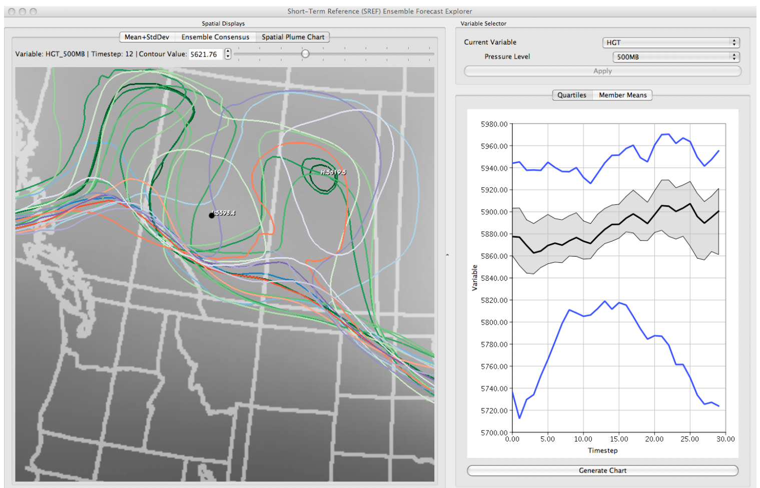

Fig. 1.5 The EnsembleVis tool [87] for exploring short-range weather forecast data.

Uncertainties are prolific in weather and climate applications and arise not only from insufficient models, but also from our inability to accurately measure current weather conditions and obtain precise knowledge on parameter settings. The typical approach for mitigating uncertainties in weather and climate applications is to perform multi-run simulations, often using a collection of models, parameter perturbations, and initial conditions to generate outcome results for multiple variables and time steps. While the variables contained in the output of both weather and climate simulations are similar, the main differences between the two domains are the spatial region of interest and the duration of time covered. Weather applications are typically only interested in a small subsection of the planet, such as North America, and run to cover time steps within the near future. In contrast, climate modeling has a spatial interest of the whole planet and is run over hundreds of years.

The uncertainty resulting from these multi-run simulations are typically captured in what is know as "Ensemble data sets". These ensembles combine the multiple runs such that notions of probability of outcome can be explored. An ensemble consists of multiple simulation realizations and are often generated by pre-defined parameter perturbations. The visualization and analysis of these data sets aims to understand variations between models and effects of parameters and initial conditions, culminating in an understanding of the phenomenon leading to weather and climate events.

An example of a visualization and analysis tool can be seen in Figure 1.5, which shows a screen shot of the EnsembleVis framework for the exploration of short-range weather forecasting data [87]. This tool uses a multiwindow approach to provide a collection of views for the end user, an approach used by other tools [95]. This approach allows the user to see overviews of a single time step, the progression of the data over time, drill downs to explore interesting spatial locations, including direct data display, and finally query-based exploration for more complex analyses.

1.6.3 Security and Intelligence

Security and intelligence uncertainty factors are a natural fit for security visualization, where making well-informed decisions is the primary goal. Enforcing security has become a top priority among a wide range of real-life applications, for instance large corporate or government/military networks. However, the task of decision making is notoriously difficult due to the malicious, hidden nature of attacks, sparse sampling of real-time environment, and time-critical requirements. Therefore, in security analysis uncertainty often exists among decisions at all levels, ranging from global scale such as "is there any malicious activity? to finer scale such as which entities are malicious?" or "in what order did these events actually occur?". The results of these decisions are used to make recommendations which can have significant operational impact, as nodes identified as malicious will be quarantined or removed from the network. Previously, both automated attack mitigation and interactive visualization approaches have been developed for security visualization. These existing techniques serve as a good platform for the integration of uncertainty visualizations and interactions. For example, several visual abstractions have been explored for detecting the sybil attack, which is a coordinated attack that can subvert many types of networks [24]. Sybil attacks are challenging to detect due to their variable attack forms and patterns. Because of this, traditional signature-based or behavior-based methods are ineffective, and security analysts must often find these nodes through manual analysis of their network. Visual abstractions from both adjacency matrix of the network connections [60] and spectral space [38] are explored, which can elucidate the signature patterns of an attack and apply automatic pattern matching algorithms or interactive analysis methods to search for similar patterns. As the short paper (Incorporating Uncertainty in Intrusion Detection to Enhance Decision Making) in this chapter describes, the factors of uncertainty can be introduced to existing detection mechanisms to improve the continuing analytic process. Since uncertainty is prevalent in security applications, the impact of uncertainty should be integrated into the entire procedure of data analysis and interactive exploration. Many current security visualization approaches can and should be augmented with interactions and visualizations for specifying and managing analytic uncertainty. By integrating analytic uncertainty in security visualization, analysts are able to better-informed decisions regarding critical network infrastructure issues.

1.7 Open Problems

1.7.1 Perceptual and Cognitive Implications

Since visualization often relies heavily on the use of colors to convey information, it can be quite challenging for individuals with color vision deficiency. For them, even interpreting visualizations that would pose no problems for individuals with normal color vision can be a difficult task. In this case, however, the resulting ambiguity, and therefore, uncertainty, is inherent to the observer, falling outside the broad sources of uncertainty discussed in Section 1.1.1 (i.e., uncertainty observed in sampled data, uncertainty measures generated by models or simulations, and uncertainty introduced by the data processing or visualization processes). Thus, individuals with color vision deficiency have to constantly deal with uncertainty visualizations and make decisions based on ambiguous information. For those individuals, the display of additional data that tries to express the amount of uncertainty from various sources may even generate further ambiguities. The issues involving uncertainty visualization and color vision deficiency are discussed in Chapter 2.

1.7.2 Comparative Visualizations

The visualization of uncertainty may involve a comparison of different results, such as a weather forecast generated with different parameters. To detect similarities or differences in the results a comparative visualization technique [74] can be employed. In 3D a visualization via fusion [10, 12] is not feasible beyond a small number (2 or 3) of data sets, due to clutter and inter-dependence of the different data sets. An alternative to fusion is a side-by-side view of the data sets. This may be problematic in 3D since it is hard to find corresponding reference points in more than two volumes. As an example to control a 3D comparison Balabanian et al. [1] propose to integrate volume visualization into a hierarchical graph structure. These integrated views provide an interactive side-by-side display of different volumes while the parameter space can be explored through the graph structure. In 2D a blending of different results has basically the same issues as a fusion in 3D [36, 32]. There are techniques which allow a comparative visualization of different data sets in a single image. Urness et al. [107] introduced color weaving for flow visualization to compare different flow fields in a single 2D view. In contrast to blending, each pixel of the resulting image represents an unmodified value from one of the data sets. The generated pattern provides a good overview to detect similar or varying regions in the data sets. To compare certain regions in more detail, e.g., borders, it is better to consider larger comparison areas than individual pixels. In this context it is crucial that data sets which should be compared are visualized next to each other to get a direct comparison for a certain area. For only two data sets a checkerboard pattern can be used to achieve screen door transparency [98]. The white squares show one data set and the black squares show the other data set. The attribute block by Miller [69] allows a simultaneous comparison of four data sets. A repeating 2x2 pattern provides a shared border between all four data sets. An extension to this approach is the comparative visualization technique of Malik et al. [65]. Instead of a rectangular pattern a hexagonal pattern is used to more finely subdivide the image space. This allows the comparison of a larger number of data sets to one central data set since the hexagonal pattern can be subdivided according to the number of data sets to compare. Uncertainty of a measurement, simulation, or process provides an additional data stream which generates further visualization challenges. Uncertainty may be shown at discrete positions through glyphs or icons. For a dense representation of uncertainty, comparative visualization seems to be a promising emerging area. Topics of research will be: integrated views; sparsification of many data sets which shall be shown simultaneously; comparative navigation; visualization of competing, contradictive, or conflicting features.

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