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Evaluation of Uncertainty Visualization Techniques for Information Fusion

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Abstract— This paper highlights the importance of uncertainty visualization in information fusion, reviews general methods of representing uncertainty and presents perceptual and cognitive principles from Tufte, Chambers and Bertin as well as users experiments documented in the literature. Examples of uncertainty representations in information fusion are analyzed using these general theories. These principles can be used in future theoretical evaluations of existing or newly developed uncertainty visualization techniques before usability testing with actual users.

Keywords: uncertainty, information visualization, human computer interaction (HCI), decision support.

I. INTRODUCTION

Many information fusion applications process and present huge quantities of data in order to enable an operator to make effective decisions. Since there are things that are unknowable [1] or may not be knowable with precision [2], there are errors in the acquisition of the data (e.g. measuring devices), processing phases (e.g. data mining techniques), performance reasons, or even in the graphical representation of the information [3], some degree of uncertainty is almost always associated [4]. If visualization is used to communicate the content of the data or to explore it, the uncertainty needs to be visualized as well [5]. This means that the user should be aware of the nature and the degree of uncertainty of the displayed information, otherwise, the data can be misinterpreted, leading to inaccurate conclusions [6].

Even though the need for visualizing uncertainty associated with the data is now widely accepted [7], [8] and [9], most of the visualization research community has ignored the issue or separated the presentation of the uncertainty from the data [10]. Part of the reasons are: it is not easy to include additional uncertainty information into an existing visualization while maintaining comprehensibility ([4], [9]) and there is a lack of methods that present uncertainty along with data ([3], [11]).

Most of the developed techniques to represent uncertainty do not include a perceptual and cognitive analysis or user evaluations that validate its usefulness. In this paper, general principles from Tufte, Chambers and Bertin as well as user experiments documented in the literature are used in order to theoretically evaluate the weakness and strengths of the uncertainty visualizations representations deployed in information fusion applications. This paper provides a general overview

on uncertainty representations techniques and explains why the recognition of uncertainty plays an important role in decision making and thus also is relevant for research in information fusion.

The major contributions of this article are (1) to highlight the importance of uncertainty visualization to support decision-making, (2) to briefly review relevant uncertainty visualization techniques to date, (3) to propose general theories and results of user experiments for their theoretical analysis, (4) to suggest that techniques developed in information visualization can be applied in information fusion and (5) to outline how information fusion research might proceed from here regarding this matter.

II. HCI AND INFORMATION VISUALIZATION IN INFORMATION FUSION

The development of fully automated computer systems has grown in the past decades. In almost any fairly complex system, like nuclear reactors and aircrafts, manual tasks are in many cases being replaced by automated functions [12]. Nevertheless, most information fusion applications are designed for human decision makers but perhaps without enough consideration for the final users. The lack of research in HCI related issues has been acknowledged by several authors in the information fusion community (e.g [13]–[15]). The traditional approach, typically, as illustrated by the Joint Directors of Laboratories (JDL) model, shows that data flows from sensors (source) toward the human (receiver). This could be a very simplistic interpretation, given that the human is actually involved in each step of the fusion process and is not only an information consumer. Using this basic orientation, rich information from multiple sensors is compressed for display on a two dimensional computer screen (referred as the “HCI bottleneck” problem by the authors [16, chap. 19]).

In order to overcome the HCI bottleneck in the information fusion process and account for functions for information representation and human machine interaction, Hall, Hall and Tate, [13], proposed the introduction of a new level in the JDL model, level 5: cognitive refinement. The proposed level accounts for functions to support a human decision-maker in the loop, users in collaborative environments and cognitive aids. Examples of functions for level 5 processing are (adapted from [16, chap. 19]):

- Cognitive aids: functions to aid and assist human understanding and exploitation of data.
- Negative reasoning enhancement: humans have a tendency to seek for information which supports their hypothesis and ignore negative information. Techniques to overcome the tendency to seek confirmatory evidence could be developed.
- *Uncertainty representation*: methods and techniques to improve the representation of uncertainty.
- Time compression/expansion: time compression and time expansion replay techniques could assist the understanding of evolving tactical situations, on account of human capabilities to detect changes.
- Focus/defocus of attention: techniques to assist in directing the attention of an analyst to consider different aspects of data.
- Pattern morphing methods: methods to translate patterns of data into forms that are more easy for an human to interpret

The effectiveness of a general and non-fully automatic information system highly depends on the user's performance. More research is needed to understand information access preferences, how users perceive and process information, interact with the system and make decisions [16, chap. 21]. Additionally, Waltz and Llinas [17] suggest that the overall effectiveness of a data fusion system is affected by the HCI efficacy. New advances should enhance the link between effective human cognition and the information fusion system, considering the human as the center of the fusion process.

In most real-world semi-automatic applications, the interface between an operator/user and a computer or computer system includes a display/s device/-s, audio capabilities and haptic devices [15, chap. 9]. They are the interface between the information fusion system, the environment and the user/operator who perceives, process and makes a decision (a commonly used model for decision making is the OODA-loop: observe, orient, decide and act [18]). A key element in information fusion is the adequate visualization of the data that guides decision-making processes efficiently. Technological and cognitive advances in information visualization can be applied in order to enhance human capabilities and overcome human deficiencies. Techniques developed in information visualization to convey uncertainty, its degree and its nature, are crucial in order to comprehend (situation assessment), project (impact assessment) and make efficient decisions.

The representation of uncertainty is an ongoing unresolved problem in the information visualization community too, which has been acknowledged by many authors (examples can be found in [5], [11], [19]–[22]). Griethe and Shumann [3] highlight the importance of uncertainty annotation for high level tasks like decision making, and specially when large amounts of data are analyzed. Additionally, Johnson and Sanderson in [11] suggested that a formal theoretical framework for visualizing uncertainty and error should be developed:

We see the need to create a formal, theoretical error and uncertainty visualization framework and to investigate and explore new visual representations for characterizing error and uncertainty.

Such a framework will be fundamental to a better understanding of the data with dubious origin or quality and as a result it will facilitate the decision making process.

Regarding the evaluation of uncertainty visualization techniques, Zuk and Carpendale in [9] claim that few authors have applied perceptual theories and Harrower [23] emphasizes that the need for this testing can be argued from both a theoretical and practical perspective. Additionally, Tory and Möller in [24] argue that, traditionally, cognition and perception and in general, human factors, have been forgotten in information visualization research, both in the design and evaluation of visual systems.

III. UNCERTAINTY

Uncertainty is a complex concept and there are many kinds of uncertainty that decision makers must face [25]. It covers a broad range of concepts like inconsistency, doubtfulness, reliability, inaccuracy or error (unknown or not quantified error). Hence, it is difficult to give a generally accepted definition of uncertainty.

According to [10], uncertainty includes statistical variations or spread, errors and differences, minimum-maximum range values and noisy or missing data. The authors consider three types of uncertainty in their discussion:

- 1) *statistical*: distribution of the data or estimated mean and standard deviation (confidence interval)
- 2) *error*: an absolute valued error among estimates or differences between a known correct datum and an estimate
- 3) *range*: an interval in which the data exists (and cannot be quantified into either the statistical or error definitions)

In [25], uncertainty means inaccuracy that is not known objectively (otherwise it would be considered as error). In [26, p. 26] it is simply described as a "*quantitative statement about the probability of error*", where inaccurate measurements, estimates or predictions are associated with large uncertainty.

Despite the attempts to develop typologies of uncertainty (e.g. [27]), its components, their relations across domains of practice, users and information needs are not completely understood [25].

A. Sources of Uncertainty

A key element in the representation of uncertain information is the identification of sources and degree of uncertainty [28]. A general model is described in [10]. The visualization pipeline shows three major blocks as possible sources of uncertainty (see figure 1):

- 1) Introduction of data uncertainty from models and measurements
- 2) Derived uncertainty from transformation processes

3) Visualization (representation) of uncertainty from the visualization process

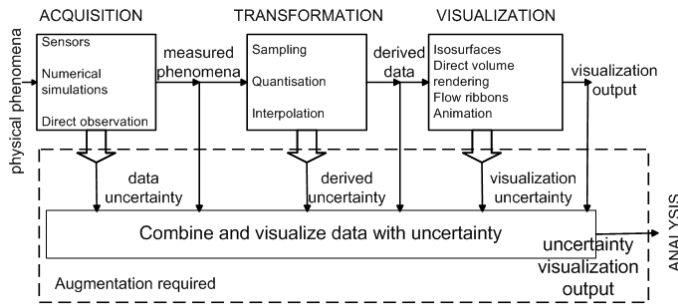


Fig. 1. Uncertainty is introduced from measurements and models, from multiple transformation processes, and from the visualization process itself [10]. Figure adapted from [29].

For example, one can identify the following possible sources of uncertainty in a generic information system ([30]):

- The sensors have limited resolution, their readings contain noise, their positions may be uncertain, sampling is sparse in time and space.
- The process of converting the raw data into suitable input for numerical models may involve operations like averaging, interpolation, sampling, etc.
- The numerical models are also approximations, further, discrete computation introduces errors.
- The visualization of the results introduces quantisation errors (data is interpolated, additional numerical integration may be used).

IV. UNCERTAINTY VISUALIZATION TECHNIQUES

Most of the previous work in uncertainty visualization has been developed in the area of Geographic Information systems, GIS (for example, see [31] for a survey of methods). Pang, Wittenbrink and Lodha [10] present a classification for uncertainty representation techniques. Seven categories are described: add glyphs, add geometry, modify geometry, modify attributes, animation, sonification and psycho-visual.

The following is a list of examples of techniques for displaying uncertainty (adapted from the classification made by Griethe and Schumann [5]):

- *Utilization of free graphical variables*: color (e.g figure 2 and 3), size, position, focus, clarity, fuzziness, saturation, transparency (e.g. figure 4) and edge crispness (e.g. figure 5).
- *Additional objects*: labels, images or glyphs. For example, Wittenbrink, Pang and Lodha, in [29], propose the use of glyphs to represent uncertainty in vector fields. Their approach is to include uncertainty in the magnitude, direction and length in glyphs (see figure 6 and 7). In [10] new ways of modifying glyphs in order to represent uncertainty are presented.
- *Animation*: the uncertainty is mapped to animation parameters such as speed or duration, motion blur, range or extent of motion.

- *Interactive representation*: e.g. uncertainty can be discovered by mouse interaction. An example can be found in [32].
- *Sonification and psycho-visual*: incorporation of acoustics, changes in pitch, volume, rhythm, vibration, or flashing textual messages. See e.g. [33].

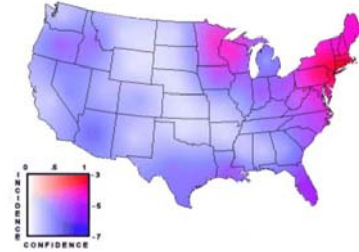


Fig. 2. Color indicates different levels of uncertainty.

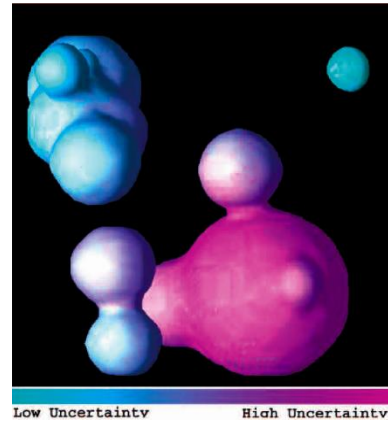


Fig. 3. Color indicates uncertain surface [34]. Reproduced with permission.

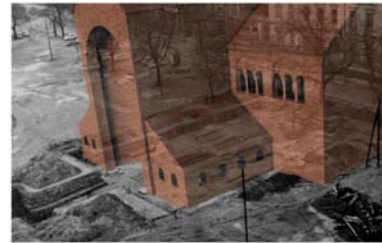


Fig. 4. Castle reconstruction. Uncertainty about the true architecture is encoded into transparency [35]. Reproduced with permission.

Statistical properties can be plotted using standard box-and-whisker plots (they encode the minimum, maximum, mean, median, and quartile information of a distribution). One interesting extension of the use of box-plots over 2D distributions can be found in [36] (see figure 8). The objective is to reduce the visual clutter when a box-plot is depicted over each grid location on the 2D map.

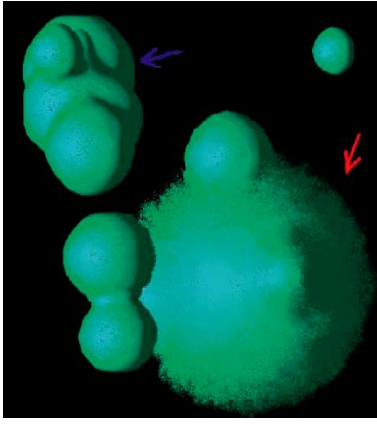


Fig. 5. Uncertain surface indicates uncertainty [34]. Reproduced with permission.

	$d\theta$	dm
	X	X
	X	
	X	X
		X
	X	X
	X	X

Fig. 6. Variety of glyphs representing angular and magnitude uncertainty, presented in [29]. Reproduced with permission.

V. THEORETICAL EVALUATION OF UNCERTAINTY VISUALIZATIONS

Many techniques have been proposed to represent uncertainty. However, almost none of them include a perceptual and/or cognitive evaluation after the development of the technique. An exception is the work by [29], which utilizes Tufte's theories to analyse the proposed method.

This section provides a brief summary of perceptual theories that can be used to evaluate, theoretically, uncertainty representation techniques. Among perceptual design theories, Tufte's, Chambers' and Bertin's are considered for being highly influential in visualization research.

A. Tufte ([37], [38], [39], [9])

Tufte defines two graphical principles that lead to good visualizations: *graphical excellence* and *graphical integrity*.

- Graphical excellence ("give the viewer the greatest number of ideas in the shortest time with the least ink in the smallest space"). Tufte has specified guidelines to encourage graphical clarity, precision and efficiency and achieve graphical excellence. Basically they include: avoid distorting what the data shows; encourage comparison among the data; present a large amount of data in a small space; reveal multiple levels of detail; closely integrate statistical and text descriptions into the data.

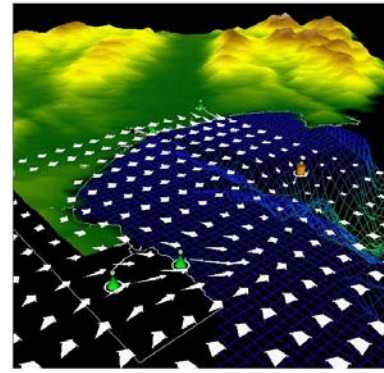


Fig. 7. Uncertainty glyphs (from [29]). Reproduced with permission. www.cse.ucsc.edu/research/slv/uglyph.html.

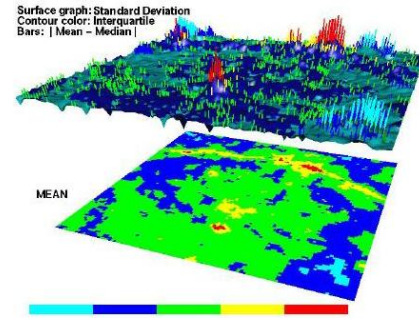


Fig. 8. Extension of the 2D box-plot, by [36]. The bottom plane is the mean. In the upper plane, the surface is deformed by the standard deviation field and colored by the interquartile range and the heights of the vertical bars represent the difference between the mean and median fields colored according to the mean field (bottom plane). Reproduced with permission.

- Graphical integrity. The principles that ensure graphical integrity are: the representation of numbers should be directly proportional to the numerical quantities represented; clear and detailed labeling should be used to defeat ambiguity and distortion; show data variations and not design variations; the number of information carrying dimensions should not exceed the data dimensions; show deflated and standardized units in time-series displays of money; graphics must not present data out of context; convincing graphics should demonstrate cause and effect.

Tufte also advises that one should present the largest amount of data with the least amount of ink (*data-ink maximization principle*). Moreover, the ratio amount of data elements divided by the graphical area must be appropriate. If it is too low, the area should be reduced (*data density principle*).

B. Chambers ([40])

In [40, chap. 8], the authors analyze how the brain, eye and picture interact in order to organize displays, so the potentially most important patterns are associated with the most easily perceived visual aspects in the display. Chambers et al. conjecture that the eye is able to perceive: location along an axis more easily than other graphical aspects, e.g. size; straight lines more clearly than curves; simple patterns more quickly than complex; large or dark objects, or clusters

of objects, with greater impact than small, light or isolated ones; symmetry (especially bilateral and circular symmetry). Moreover two more points are added regarding the mental process: we can perceive several different aspects in one plot simply by switching attention from one aspect to another and accumulated visual evidence is roughly additive.

Following these ideas, Chambers et al. compile general techniques for plot construction. The guidelines are grouped in three categories: reducing clutter (the amount of uninformative detail and clutter in a plot should be minimized); removing gross structure (increase the informativeness of a graph by removing structure from the data once we have identify it) and labeling (effective use of labels).

C. Bertin ([41], [42], [9])

In [41], Bertin presents a classification of visual variables: planar dimensions (x,y) and visible marks over the plane (size, value, grain, color, orientation and shape).

The analysis that Bertin suggests is based on the potential of each variable for: immediate perceptual group selection, natural perceptual ordering (not learned), perceptual grouping characteristics, number of discernible elements that can be represented in the set (length) and ability for quantitative comparisons.

A variable is called *selective* when it is perceived immediately without considering individual marks sequentially. The length of a variable must be greatly reduced in order to use it for selective processing.

Another possible set of perceptual theories that can be used in the evaluation are those compiled by Ware in [43] (his work includes many explanations from cognitive science/psychology, e.g. Gestalt laws).

VI. INSIGHTS FROM USER TESTS

The principles presented in section V can be complemented with formal studies regarding what impact the representation of uncertainty has on users or how the methods compare to each other. Leitner and Battenfield [44] tested saturation, value and texture as means of depicting uncertainty over maps by looking at time, accuracy and confidence within a spatial decision support system. Their results regarding saturation support the studies by Schweizer and Goodchild in [45], where saturation is found to be not especially effective (a more detailed study regarding the use of color to represent uncertainty appears in [46]). On the other hand, they found that if finer texture or lighter value is chosen to depict certainty data the number of correct responses for an easy decision is increased. They also found that response times decreased with the inclusion of certainty information for easy tasks, but no such difference was observed for more complex tasks. Four methods of displaying data quality were compared by Evans [47]: static separate maps, static integrated displays, animated non-controllable flicker maps and interactive toggle maps. The results show that users performed best with the static integrated display and the flicker map. Andre and Cutler [48] studied the use of rings to display the uncertainty

associated with a military-style entity position (the size of the ring indicates the degree of uncertainty). The results show that, using this technique in a collision avoidance task, the performance was improved. A significant contribution regarding coloring uncertainty appears in [46]. Through empirical studies, Jiang, Brown and Ormeling tested the use of five different kinds of color scales and four bivariate color schemes for the representation of uncertainty and data combined with uncertainty respectively. One of their conclusions state that a lightness scale starting with white is perceived as one of the best solutions.

VII. EXAMPLES: COGNITIVE AND THEORETICAL ANALYSIS OF UNCERTAIN VISUALIZATIONS IN INFORMATION FUSION

In this section uncertainty visualization techniques are analyzed based on the perceptual theories introduced in section V and the user experiments presented in section VI. The techniques were selected from information fusion related applications¹.

A. Probabilistic demand prediction for traffic flow decision support [28]

Masalonis, Mulgund, Song, Wanke and Zobell in [28] present visualization concepts and requirements for the display of uncertainty in decision support systems for air traffic flow management (TFM). Interviews with expert operators provide guidance for the development of a human machine interface prototype shown in figure 9 and 10. Based on probabilistic alert levels (depicted using a three-color scheme: green/yellow/red²) operators must decide. The hypothetical display provides varying levels of detail and meta-data related to the predicted alert when the mouse is positioned on a particular cell (see figure 9 right side). This work, still in progress, was not evaluated, neither theoretically nor empirically by its authors (a theoretical evaluation was recently presented in [9] by Zuk and Carpendale).

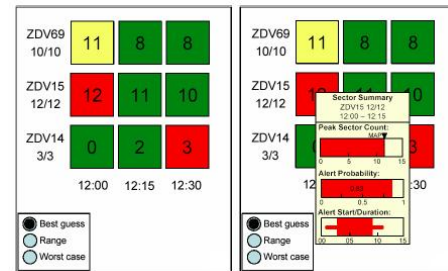


Fig. 9. Hypothetical probabilistic Center Monitor [28]. Basic display on the left and sector summary via mouse rollover on the right side. Reproduced with permission, ©2004 MITRE Corporation.

¹The selected applications are not representative samples of different uncertainty visualization methods (unfortunately, few applications within information fusion display uncertainty).

²In order to avoid confusion with the overall display (figure 10), the scheme green/yellow/red was later replaced by magenta/purple/gray (in conversations with Craig Wanke).

Considering Tufte's recommendation, the ratio of data elements divided by the graphical area is quite low (figure 10) and hence, more data could be displayed. Three colors represent the probability of an alert over this display. Color, defined by Bertin as one of the visible marks over the plane³, has long length so more alert levels could be shown varying it. Following the recommendations of [46], a lightness scale starting with white could be used to depict the probabilistic alert levels. Moreover, the numbers over the color background could be removed if more color levels were used (figure 9). The use of black text numbers over high saturated color background might cause visual stress. Lower saturated colors could reduce the luminance ratio of the black text on the used colors, thus avoiding visual stress. The display follows Chambers' recommendations regarding symmetry, simplicity and straightness (figure 10). Moreover, the hover queries using the mouse allow correct labeling (figure 9), although the pop-up window might hide important changes in alert levels on the general display.

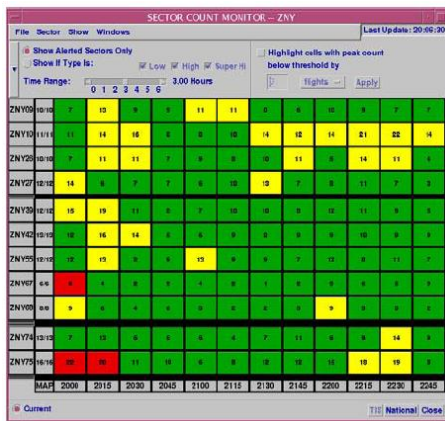


Fig. 10. Center monitor display [28]. Reproduced with permission, ©2004 MITRE Corporation.

B. Graphical formats to convey uncertainty in a decision making task [8]

Finger and Bisantz [8] presented a novel work on the evaluation of how the presentation of the information may affect the decision-maker. Blended and degraded icons were used to represent uncertainty regarding the identity of a radar contact as hostile or friendly. The first part of the study showed that participants could sort, order and rank five different sets of icons (see figure 11) conveying different levels of uncertainty (see figure 12). In the second part of the study, three of the pairs of icons were used in an application in which participants should identify the status of contacts as friendly or hostile (see figure 13).

Blur is a selective (Bertin) visual variable. It gives fuzzy appearance to the icons and for instance, can depict uncertainty. Not all the selected symbols follow Chambers recommendations of symmetry, simplicity and straightness (e.g. dove and

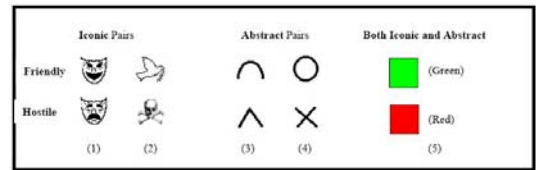


Fig. 11. Pairs of icons representing hostile or friendly objects [7]. Reproduced with permission, ©1999 ISIF.



Fig. 12. The blurred icons represent the probability that an object is hostile or friendly, from probability 1 of being friendly to probability 1 of being hostile [7]. Reproduced with permission, ©1999 ISIF.

skull). Color (changes in value and saturation) was found not to be especially effective for displaying uncertainty in user evaluations ([44], [45]). Moreover, it should be noted that color is defined by Bertin as a variable without implicit order (not learned) and hence not a good choice when an ordering task must be performed. One suggestion is the use of transparency to encode uncertainty. Transparency is considered a color and value hybrid (redundant encoding) and it facilitates perception [9].

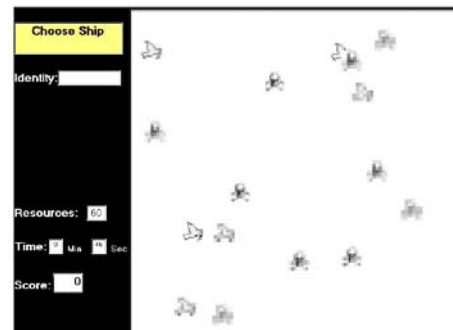


Fig. 13. Graphical display used in the dynamic decision making experiment [8]. Reproduced with permission, ©1997 ISIF.

Perceiving icons with very large magnitude of uncertainty (extremely blurred icons) becomes difficult and its interpretation might be ambiguous. This may be the desired effect, but if the application requires a background map, blur or fog may be associated to low quality mapping rather than low quality data (see [46]). In this case, the integration of text for the symbols might facilitate comprehension (Tufte's principle of close integration of text and graphics). The data density ratio (Tufte) of the application is quite low (e.g. no background information is presented indicating the position of the object) so text labeling and iterative queries on the application might facilitate comprehension. Following Tufte's integrity principle, "do not show data out of context", a low-saturated color map as a background might help to solve complex tasks (figure 13).

³In Bertin's system of visual variables, color refers only to *hue*.

C. DSS prototype display: a critical decision analysis of aspects of naval anti-air warfare [49]

Freeman and Cohen [49] evaluated the effects on tactical decision making of a prototype decision support system (DSS) display developed by the Space and Naval Warfare Systems Center (see figure 14). The strategic use of graphics was intended to support rapid decision making based on pattern recognition (e.g. weapons range rings and task management graph bars). The DSS display improved the ability to think critically under uncertainty. In this case, color coded flags and annotations pop-up on the geographical display (geoplot) to direct the attention of the officers. The geoplot display allows the officer to concentrate on a specific element, while the overview frame provides a general view of the whole situation [50]. Threat values assigned to individual tracks are presented to the decision maker as a sorted list from the most threatening, left side, to the least, right side (see bottom figure 14).

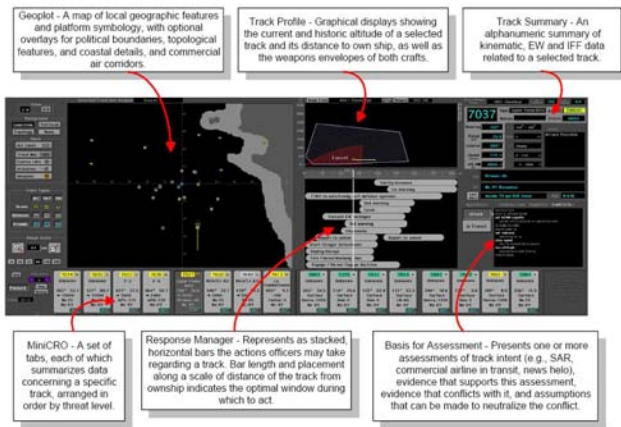


Fig. 14. DSS display features [49]. Reproduced with permission.

Tufte's data density measure appears to be quite high considering the whole display. Multiple displays (physical and functional) account for different granularity on the information, maintaining both background awareness (big picture) and foreground awareness (details and particular goals) (Tufte's guideline of excellence revealing data on several levels of detail). Large regions of the display use low saturated colors, e.g gray, avoiding visual stress. In this application, the threat values are sorted and presented to the user as part of a ordered list avoiding ambiguity (highest priority on the left and lowest on the right). Thus, the uncertain threat values are sorted and easily perceived by the user. However, as situations evolve, the reorganization of the buttons might generate information overload when the decision maker is under stress. Related attention and scanning problems can be analyzed using theories by Wickens [51]. Following Tufte's integrity principle of context, the elements are displayed over a geographical map maintaining a general view of the decision-maker's problem space. Considering the recommendations of Chambers regarding labeling, the buttons display critical identification and kinematic information that allow monitoring without any additional interaction with the system.

VIII. FUTURE WORK AND CONCLUSIONS

Visualization of uncertainty for decision making is an interdisciplinary problem [25] and many authors have pointed out that further research regarding the representation of uncertain information should be done [3], [10]. Few studies have been carried out to determine what impact the display of uncertainty has on users or how various methods compare to each other [23]. Similarly, in information fusion research the representation of uncertainty should not be overlooked due to the high degree of uncertainty that many times is associated with the information handled by a decision maker.

Three general research challenges in uncertainty visualization that are highly relevant to information fusion are: (1) the development of uncertainty visualization techniques for three dimensional (3D) representations [11] and the extension of the existing studies in two dimensions to three, (2) the development of representation and evaluation methods for depicting multiple forms of uncertainty in the same display and (3) the development of methods and tools for interacting with uncertainty representations [25].

The evaluation of uncertainty visualization techniques should include theoretical cognitive/perceptual analysis and usability tests with actual users. In this report, a small set of cognitive and perceptual theories as well as empirical studies documented in the literature have been presented. They can be used in theoretical evaluations of existing and newly developed techniques for displaying uncertain information, providing insights into their weakness and strengths. The preliminary study presented here can support future empirical research regarding the validity of these theoretical guidelines.

To conclude, information fusion can certainly benefit from developments in information visualization research. At the same time information fusion applications provide a valuable source of case studies for researchers within the field of information visualization and human-computer interaction.

ACKNOWLEDGMENTS

This work was supported by the Information Fusion Research Program (University of Skövde, Sweden) in partnership with the Swedish Knowledge Foundation under grant 2003/0104 (<http://www.infofusion.se>) and carried out in collaboration with Saab Microwave Systems (Gothenburg, Sweden). I would like to thank Tom Ziemke and Göran Falkman for their useful comments.

REFERENCES

- [1] H. Couclelis, "The certainty of uncertainty: GIS and the limits of geographic knowledge," *Transactions in GIS*, vol. 7, no. 2, pp. 165–175, 2003.
- [2] P. A. Longley, M. F. Goodchild, D. Maguire, and D. W. Rhind, Eds., *Geographical Information Systems: Principles, Techniques, Management, and Applications*. New York: Wiley, 1999, ch. Models of uncertainty in spatial data (Fischer, P. F.), pp. 191–205.
- [3] H. Griethe and H. Schumann, "Visualizing uncertainty for improved decision making," in *Proceedings of the 4th International Conference on Perspectives in Business Informatics Research (BIR 2005)*, University of Skövde, Sweden, 2005.

- [4] A. Cedilnik and P. Rheingans, "Procedural annotation of uncertain information," in *11th IEEE Visualization 2000 (VIS'00)*. Los Alamitos, CA, USA: IEEE Computer Society Press, 2000, pp. 77–83.
- [5] H. Griethe and H. Schumann, "The visualization of uncertain data: Methods and problems," in *Proceedings SimVis'06*, Magdeburg, Germany, 2006.
- [6] C. Olston and J. D. Mackinlay, "Visualizing data with bounded uncertainty," in *INFOVIS '02: Proceedings of the IEEE Symposium on Information Visualization (InfoVis'02)*. Washington, DC, USA: IEEE Computer Society, 2002, p. 37.
- [7] A. M. Bisantz, R. Finger, Y. Seong, and J. Llinas, "Human performance and data fusion based decision aids," in *Proceedings of the 2nd International Conference on Information Fusion. International Society of Information Fusion.*, Sunnyvale, CA USA, 1999.
- [8] R. Finger and A. Bisantz, "Utilizing graphical formats to convey uncertainty in a decision making task," *Theoretical Issues in Ergonomics Science*, vol. 3, no. 1, pp. 1–25, 1997.
- [9] T. Zuk and S. Carpendale, "Theoretical analysis of uncertainty visualizations," in *Visualization and Data Analysis, Proc. of SPIE-IS&T Electronic Imaging, SPIE*, R. F. Erbacher, J. C. Roberts, M. T. Gröhn, and K. Börne, Eds., vol. 6060, 2006.
- [10] A. Pang, C. Wittenbrink, and S. Lodha, "Approaches to uncertainty visualization," *The Visual Computer*, vol. 13, no. 8, pp. 370–390, November 1997.
- [11] C. R. Johnson and A. R. Sanderson, "A next step: Visualizing errors and uncertainty," *IEEE Computer Graphics and Applications*, vol. 23, no. 5, pp. 6–10, 2003.
- [12] C. J. Matheus, M. M. Kokar, and K. Baclawski, "A core ontology for situation awareness," in *Proceedings of Sixth International Conference on Information Fusion, Cairns Australia*, pp. 545–552, 2003.
- [13] M. J. Hall, S. A. Hall, and T. Tate, "Removing the HCI bottleneck: How the human computer interaction (HCI) affects the performance of data fusion system," in *2000 MSS National Symposium on Sensor and Data Fusion*, 2000, pp. 89–104.
- [14] E. P. Blasch and S. Plano, "JDL level 5 fusion model: user refinement issues and applications in group tracking," in *Proc. SPIE Vol. 4729, p. 270-279, Signal Processing, Sensor Fusion, and Target Recognition XI, Ivan Kadar; Ed., I. Kadar, Ed., July 2002*, pp. 270–279.
- [15] D. Hall and S. A. H. McMullen, Eds., *Mathematical Techniques in Multisensor Data Fusion*, 2nd ed. Norwood, MA: Artech House, Inc., 2004.
- [16] D. Hall and J. Llinas, Eds., *Handbook of multisensor data fusion*. Boca Raton: CRC Press, 2000.
- [17] E. Waltz and J. Llinas, Eds., *Multisensor Data Fusion*. Norwood, MA: Artech House, Inc., 1990.
- [18] B. Brehmer, "The dynamic OODA loop: amalgamating Boyd's OODA loop and the Cybernetic approach to command and control," in *Proc. of 10th international command and control research and technology symposium*, 2005.
- [19] T.-M. Rhyne, W. L. Hibbard, C. Johnson, C. Chen, and S. Eick, "Panel 1: Can we determine the top unresolved problems of visualization?" in *IEEE Visualization*, 2004, pp. 563–566.
- [20] C. Johnson, "Top scientific visualization research problems," *IEEE Computer Graphics and Applications*, vol. 24, no. 3, pp. 13–17, 2004.
- [21] N. Gershon, "Visualization of an imperfect world," *IEEE Comput. Graph. Appl.*, vol. 18, no. 4, pp. 43–45, 1998.
- [22] J. Thomson, E. Hetzler, A. MacEachren, M. Gahegan, and M. Pavel, "A typology for visualizing uncertainty," in *Visualization and Data Analysis 2005*, R. F. Erbacher, J. C. Roberts, M. T. Grohn, and K. Borner, Eds., vol. 5669, no. 1. SPIE, 2005, pp. 146–157.
- [23] M. Harrower, "Representing uncertainty: Does it help people make better decisions?" in *UCGIS Workshop: Geospatial Visualization and Knowledge Discovery Workshop, National Conference Center, Landsdowne, VA., 2003*, (invited white paper).
- [24] M. Tory and T. Möller, "Human factors in visualization research," *IEEE Transactions on Visualization and Computer Graphics*, vol. 10, no. 1, pp. 72–84, 2004.
- [25] A. M. MacEachren, A. Robinson, S. Hopper, S. Gardner, R. Murray, M. Gahegan, and E. Hetzler, "Visualizing geospatial information uncertainty: What we know and what we need to know," in *Cartography and Geographic Information Science*, vol. 32, no. 3, pp. 139–160, 2005.
- [26] G. M. Foody and P. Atkinson, Eds., *Uncertainty in Remote Sensing and GIS*. Sussex, England: Wiley, 2002.
- [27] B. Battenfield and R. Weibel, "Visualizing the quality of cartographic data," in *Third International Geographic Information Systems Symposium, GIS/LIS 88*, San Antonio, Texas, 1988.
- [28] A. Masalonis, S. Mulgund, L. Song, C. Wanke, and S. Zobell, "Using probabilistic demand predictions for traffic flow management decision support," in *AIAA Guidance, Navigation, and Control Conference and Exhibit*, Providence, Rhode Island, 2004.
- [29] C. M. Wittenbrink, A. T. Pang, and S. K. Lodha, "Glyphs for visualizing uncertainty in vector fields," *IEEE Transactions on Visualization and Computer Graphics*, vol. 2, no. 3, pp. 266–279, 1996.
- [30] K. Jää-Aro, "Lecture notes: Visualisation of uncertainty," 2006, KTH. Stockholm.
- [31] G. J. Hunter and M. F. Goodchild, "Managing uncertainty in spatial databases: Putting theory into practice," *URISA J.*, vol. 5, no. 2, pp. 55–62, 1993.
- [32] F. J. M. van der Wel, L. C. van der Gaag, and B. G. H. Gorte, "Visual exploration of uncertainty in remote-sensing classification," *Computers & Geosciences*, vol. 24, no. 4, pp. 335–343, 1998.
- [33] P. Fisher, "On animation and sound for the visualization of uncertain spatial information," *Visualization in Geographical Information Systems*, pp. 181–185, 1994.
- [34] G. Grigoryan and P. Rheingans, "Point-based probabilistic surfaces to show surface uncertainty," *IEEE Transactions on Visualization and Computer Graphics*, vol. 10, no. 5, pp. 564–573, 2004.
- [35] T. Isenberg, "Visualisierung von modellierungsentscheidungen und unsicherheiten in virtuellen rekonstruktionen." Department of Simulation and Graphics, Otto-von-Guericke University Magdeburg, Germany, Tech. Rep., 1999.
- [36] D. Kao, A. Luo, J. L. Dungan, and A. Pang, "Visualizing spatially varying distribution data," *Proceedings of the Sixth International Conference on Information Visualisation (IV'02)*, 2002.
- [37] E. R. Tufte, *The Visual Display of Quantitative Information*, 2nd ed. Cheshire, CT: Graphics Press, 2001.
- [38] E. R. Tufte, *Visual explanations. Images and Quantities, Evidence and Narrative*. Cheshire, Connecticut: Graphics Press, 1997.
- [39] E. R. Tufte., *Envisioning Information*. Cheshire, Connecticut: Graphics Press, 1994.
- [40] J. M. Chambers and W. S. Cleveland and B. Kleiner, and P. A. Tukey, *Graphical Methods for Data Analysis*. New York: Chapman and Hall, 1983.
- [41] Bertin, J., *Semiology of Graphics*. Madison, WI: University of Wisconsin Press, 1983, (trans. W. Berg).
- [42] Bertin, J., *Graphics and Graphics Information Processing*. Berlin-New York: Walter de Gruyter, 1981, (trans. W. Berg and P. Scott).
- [43] C. Ware, *Information Visualization: Perception for Design*. San Francisco, CA.: Morgan Kaufmann, 2000.
- [44] M. Leitner and B. P. Battenfield, "Guidelines for the display of attribute certainty," *Cartography and Geographic Information Science*, vol. 27, no. 1, pp. 3–14, 2000.
- [45] D. Schweizer and M. Goodchild, "Data quality and choropleth maps: An experiment with the use of color," in *Proceedings, GIS/LIS*, vol. 2, 1992, pp. 686–699.
- [46] B. A. Jiang B. and F. J. Ormeling, "Some perceptual aspects of colouring uncertainty," *Advances in GIS Research II*, pp. 477–490, 1996.
- [47] B. Evans, "Dynamic display of spatial data-reliability: does it benefit the map user?" *Computers & Geosciences*, vol. 23, no. 4, pp. 409–422, 1997.
- [48] A. Andre and H. Cutler, "Displaying uncertainty in advanced navigation systems," in *Human Factors and Ergonomics Society 42nd Annual Meeting*, Chicago, IL, USA, 1998, pp. 31–35.
- [49] J. T. Freeman and M. S. Cohen, "A critical decision analysis of aspects of naval anti-air warfare," Cognitive Technologies, Inc., Tech. Rep. 98-2, 9 June 1998.
- [50] J. Roy, R. Breton, and S. Paradis, "Human-computer interface for the study of information fusion concepts in situation analysis and command decision support systems," in *Proc. SPIE, Signal Processing, Sensor Fusion, and Target Recognition X, I. Kadar, Ed., vol. 4380*, no. 1, 2001, pp. 361–372.
- [51] C. D. Wickens and J. G. Hollands, *Engineering Psychology and Human Performance (3rd Edition)*. Prentice Hall, September 1999.