

Spatial Descriptions and Transformations

Related Exercise and Solution

Matrix Multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

$$\begin{bmatrix} ax_1 & ax_2 & ax_3 & ax_4 \\ ay_1 & ay_2 & ay_3 & ay_4 \\ az_1 & az_2 & az_3 & az_4 \\ aw_1 & aw_2 & aw_3 & aw_4 \end{bmatrix} \times \begin{bmatrix} bx_1 & bx_2 & bx_3 & bx_4 \\ by_1 & by_2 & by_3 & by_4 \\ bz_1 & bz_2 & bz_3 & bz_4 \\ bw_1 & bw_2 & bw_3 & bw_4 \end{bmatrix}$$

=

$$\begin{bmatrix} ax_1*bx_1+ax_2*by_1+ax_3*bz_1+ax_4*bw_1 & ax_1*bx_2+ax_2*by_2+ax_3*bz_2+ax_4*bw_2 \\ ay_1*bx_1+ay_2*by_1+ay_3*bz_1+ay_4*bw_1 & ay_1*bx_2+ay_2*by_2+ay_3*bz_2+ay_4*bw_2 \\ az_1*bx_1+az_2*by_1+az_3*bz_1+az_4*bw_1 & az_1*bx_2+az_2*by_2+az_3*bz_2+az_4*bw_2 \\ aw_1*bx_1+aw_2*by_1+aw_3*bz_1+aw_4*bw_1 & aw_1*bx_2+aw_2*by_2+aw_3*bz_2+aw_4*bw_2 \end{bmatrix}$$

$$\begin{bmatrix} ax_1*bx_3+ax_2*by_3+ax_3*bz_3+ax_4*bw_3 & ax_1*bx_4+ax_2*by_4+ax_3*bz_4+ax_4*bw_4 \\ ay_1*bx_3+ay_2*by_3+ay_3*bz_3+ay_4*bw_3 & ay_1*bx_4+ay_2*by_4+ay_3*bz_4+ay_4*bw_4 \\ az_1*bx_3+az_2*by_3+az_3*bz_3+az_4*bw_3 & az_1*bx_4+az_2*by_4+az_3*bz_4+az_4*bw_4 \\ aw_1*bx_3+aw_2*by_3+aw_3*bz_3+aw_4*bw_3 & aw_1*bx_4+aw_2*by_4+aw_3*bz_4+aw_4*bw_4 \end{bmatrix}$$

Exercise and Solution

Q1: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along Z axis by anti-clock 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution : $[T(3,-5,3) * RotY(60) * RotZ(45) * T(2,3,5)]^{-1} * Q[2,5,7] = P$

Q2: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along X axis by 60 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 45 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution: $[T(3,-5,3) * RotY(45) * RotX(60) * T(2,3,5)]^{-1} * Q[2,5,7] = P$

Q3: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along Z axis by anti-clock 30 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 45 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution: $[T(3,-5,3) * RotY(45) * RotZ(30) * T(2,3,5)]^{-1} * Q[2,5,7] = P$

Q4: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along X axis by 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 30 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution: $[T(3,-5,3) * RotY(30) * RotX(45) * T(2,3,5)]^{-1} * Q[2,5,7] = P$

Solution

Q1: A frame F_{noa} is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]^T. A rotation along Z axis by anti-clock 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution : $[T(3,-5,3) * RotY(60) * RotZ(45) * T(2,3,5)]^{-1} * Q[2,5,7] = P$

$$RotZ(45) * T(2,3,5) = \begin{bmatrix} \cos(45) & -\sin(45) & 0 & 0 \\ \sin(45) & \cos(45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.707 & -0.707 & 0 & 0 \\ 0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 0.707 \times 1 + -0.707 \times 0 + 0 \times 0 + 0 \times 0 & 0.707 \times 0 + -0.707 \times 1 + 0 \times 0 + 0 \times 0 & 0.707 \times 0 + -0.707 \times 0 + 0 \times 1 + 0 \times 0 & 0.707 \times 2 + -0.707 \times 3 + 0 \times 5 + 0 \times 1 \\ 0.707 \times 1 + 0.707 \times 0 + 0 \times 0 + 0 \times 0 & 0.707 \times 0 + 0.707 \times 1 + 0 \times 0 + 0 \times 0 & 0.707 \times 0 + 0.707 \times 0 + 0 \times 1 + 0 \times 0 & 0.707 \times 2 + 0.707 \times 3 + 0 \times 5 + 0 \times 1 \\ 0 \times 1 + 0 \times 0 + 1 \times 0 + 0 \times 0 & 0 \times 0 + 0 \times 1 + 1 \times 0 + 0 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 + 0 \times 0 & 0 \times 2 + 0 \times 3 + 1 \times 5 + 0 \times 1 \\ 0 \times 1 + 0 \times 0 + 0 \times 0 + 1 \times 0 & 0 \times 0 + 0 \times 1 + 0 \times 0 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 0 \times 1 + 1 \times 0 & 0 \times 2 + 0 \times 3 + 0 \times 5 + 1 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & -0.707 & 0 & -0.707 \\ 0.707 & 0.707 & 0 & 3.535 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution

Q1: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along Z axis by anti-clock 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution : $[T(3,-5,3) * RotY(60) * RotZ(45) * T(2,3,5)]^{-1} * Q[2,5,7] = P$

$$T(3,-5,3) * RotY(60) = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(60) & 0 & \sin(60) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(60) & 0 & \cos(60) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.5 & 0 & 0.866 & 0 \\ 0 & 1 & 0 & 0 \\ -0.866 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 \times 0.5 + 0 \times 0 + 0 \times -0.866 + 3 \times 0 & 1 \times 0 + 0 \times 1 + 0 \times 0 + 3 \times 0 & 1 \times 0.866 + 0 \times 0 + 0 \times 0.5 + 3 \times 0 & 1 \times 0 + 0 \times 0 + 0 \times 0 + 3 \times 1 \\ 0 \times 0.5 + 1 \times 0 + 0 \times -0.866 + -5 \times 0 & 0 \times 0 + 1 \times 1 + 0 \times 0 + -5 \times 0 & 0 \times 0.866 + 1 \times 0 + 0 \times 0.5 + -5 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 0 + -5 \times 1 \\ 0 \times 0.5 + 0 \times 0 + 1 \times -0.866 + 3 \times 0 & 0 \times 0 + 0 \times 1 + 1 \times 0 + 3 \times 0 & 0 \times 0.866 + 0 \times 0 + 1 \times 0.5 + 3 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 0 + 3 \times 1 \\ 0 \times 0.5 + 0 \times 0 + 0 \times -0.866 + 1 \times 0 & 0 \times 0 + 0 \times 1 + 0 \times 0 + 1 \times 0 & 0 \times 0.866 + 0 \times 0 + 0 \times 0.5 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0 & 0.866 & 3 \\ 0 & 1 & 0 & -5 \\ -0.866 & 0 & 0.5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution

Q1: A frame F_{noa} is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]^T. A rotation along Z axis by anti-clock 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution : $[T(3,-5,3) * RotY(60) * RotZ(45) * T(2,3,5)]^{-1} * Q[2,5,7] = P$

$$T(3,-5,3) * RotY(60) * RotZ(45) * T(2,3,5) = \begin{bmatrix} 0.5 & 0 & 0.866 & 3 \\ 0 & 1 & 0 & -5 \\ -0.866 & 0 & 0.5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.707 & -0.707 & 0 & -0.707 \\ 0.707 & 0.707 & 0 & 3.535 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 0.5 \times 0.707 + 0 \times 0.707 + 0.866 \times 0 + 3 \times 0 & 0.5 \times -0.707 + 0 \times 0.707 + 0.866 \times 0 + 3 \times 0 \\ 0 \times 0.707 + 1 \times 0.707 + 0 \times 0 + -5 \times 0 & 0 \times -0.707 + 1 \times 0.707 + 0 \times 0 + -5 \times 0 \\ -0.866 \times 0.707 + 0 \times 0.707 + 0.5 \times 0 + 3 \times 0 & -0.866 \times -0.707 + 0 \times 0.707 + 0.5 \times 0 + 3 \times 0 \\ 0 \times 0.707 + 0 \times 0.707 + 0 \times 0 + 1 \times 0 & 0 \times -0.707 + 0 \times 0.707 + 0 \times 0 + 1 \times 0 \end{bmatrix} \begin{bmatrix} 0.5 \times 0 + 0 \times 0 + 0.866 \times 1 + 3 \times 0 & 0.5 \times -0.707 + 0 \times 3.535 + 0.866 \times 5 + 3 \times 1 \\ 0 \times 0 + 1 \times 0 + 0 \times 1 + -5 \times 0 & 0 \times -0.707 + 1 \times 3.535 + 0 \times 5 + -5 \times 1 \\ -0.866 \times 0 + 0 \times 0 + 0.5 \times 1 + 3 \times 0 & -0.866 \times -0.707 + 0 \times 3.535 + 0.5 \times 5 + 3 \times 1 \\ 0 \times 0 + 0 \times 0 + 0 \times 1 + 1 \times 0 & 0 \times -0.707 + 0 \times 3.535 + 0 \times 5 + 1 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3535 & -0.3535 & 0.866 & 6.9765 \\ 0.707 & 0.707 & 0 & -1.465 \\ -0.612262 & 0.612262 & 0.5 & 6.112262 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution

Q1: A frame F_{noa} is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]^T. A rotation along Z axis by anti-clock 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution : $[T(3,-5,3) * RotY(60) * RotZ(45) * T(2,3,5)]^{-1} * Q[2,5,7] = P$

$$[T(3,-5,3) * RotY(60) * RotZ(45) * T(2,3,5)]^{-1} = \begin{bmatrix} 0.3535 & -0.3535 & 0.866 & 6.9765 \\ 0.707 & 0.707 & 0 & -1.465 \\ -0.612262 & 0.612262 & 0.5 & 6.112262 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} 0.35 & 0.707 & -0.61 & -(6.98 \times 0.35 + -1.465 \times (-0.35) + 6.11 \times 0.866) \\ -0.35 & 0.707 & 0.61 & -(6.98 \times (-0.35) + -1.465 \times 0.707 + 6.11 \times 0.61) \\ 0.866 & 0 & 0.5 & -(6.98 \times 0.866 + -1.465 \times 0 + 6.11 \times 0.5) \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.35 & 0.707 & -0.61 & -8.24701 \\ -0.35 & 0.707 & 0.61 & -0.248345 \\ 0.866 & 0 & 0.5 & -9.09968 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution

Q1: A frame Fnoa is located in the position P. After the following transformation the frame position has changed to Q[2,5,7]T. A rotation along Z axis by anti-clock 45 degree but before that a translation along all axis by [2,3,5]. After those two, another rotation along Y axis by 60 degree followed by a translation along all axis by [3,-5,3]. Find the position P with respect to Q.

Solution : $[T(3,-5,3) * RotY(60) * RotZ(45) * T(2,3,5)]^{-1} * Q[2,5,7] = P$

$$\begin{bmatrix} 0.35 & 0.707 & -0.61 & -8.25 \\ -0.35 & 0.707 & 0.61 & -0.25 \\ 0.866 & 0 & 0.5 & -0.91 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.35 \times 2 + 0.707 \times 5 + -0.61 \times 7 + -8.25 \times 1 \\ -0.35 \times 2 + 0.707 \times 5 + 0.61 \times 7 + -0.25 \times 1 \\ 0.866 \times 2 + 0 \times 5 + 0.5 \times 7 + -0.91 \times 1 \\ 0 \times 2 + 0 \times 5 + 0 \times 7 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} -8.285 \\ 6.855 \\ 4.322 \\ 1 \end{bmatrix}$$

Answer: P [-8.3 , 6.86 , 4.3]T

Solution

Q5: A frame Fnoa is located in the position P[1,1,1]T. After the following transformation the frame position has changed to Q. A rotation along Z axis by anti-clock 45 degree followed by a translation along all axis by [2,3,5], after those two, another rotation along Y axis by 45 degree followed by a translation along all axis by [3,3,3]. Find the position Q with respect to P.

Solution: $T(3,3,3) * RotY(45) * T(2,3,5) * RotZ(45) * P[1,1,1] = Q$

$$\begin{aligned}
 T(2,3,5) * RotZ(45) &= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(45) & -\sin(45) & 0 & 0 \\ \sin(45) & \cos(45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.707 & -0.707 & 0 & 0 \\ 0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \\
 &= \begin{bmatrix} 1 \times 0.707 + 0 \times 0.707 + 0 \times 0 + 2 \times 0 & 1 \times -0.707 + 0 \times 0.707 + 0 \times 0 + 2 \times 0 & 1 \times 0 + 0 \times 0 + 0 \times 1 + 2 \times 0 & 1 \times 0 + 0 \times 0 + 0 \times 0 + 2 \times 1 \\ 0 \times 0.707 + 1 \times 0.707 + 0 \times 0 + 3 \times 0 & 0 \times -0.707 + 1 \times 0.707 + 0 \times 0 + 3 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 1 + 3 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 0 + 3 \times 1 \\ 0 \times 0.707 + 0 \times 0.707 + 1 \times 0 + 5 \times 0 & 0 \times -0.707 + 0 \times 0.707 + 1 \times 0 + 5 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 + 5 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 0 + 5 \times 1 \\ 0 \times 0.707 + 0 \times 0.707 + 0 \times 0 + 1 \times 0 & 0 \times -0.707 + 0 \times 0.707 + 0 \times 0 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\
 &= \begin{bmatrix} 0.707 & -0.707 & 0 & 2 \\ 0.707 & 0.707 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

Solution

Q5: A frame Fnoa is located in the position P[1,1,1]^T. After the following transformation the frame position has changed to Q. A rotation along Z axis by anti-clock 45 degree followed by a translation along all axis by [2,3,5], after those two, another rotation along Y axis by 45 degree followed by a translation along all axis by [3,3,3]. Find the position Q with respect to P.

Solution: $T(3,3,3) * RotY(45) * T(2,3,5) * RotZ(45) * P[1,1,1] = Q$

$$\begin{aligned}
 T(3,3,3) * RotY(45) &= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(45) & 0 & \sin(45) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(45) & 0 & \cos(45) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.707 & 0 & 0.707 & 0 \\ 0 & 1 & 0 & 0 \\ -0.707 & 0 & 0.707 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \\
 &= \begin{bmatrix} 1 \times 0.707 + 0 \times 0 + 0 \times -0.707 + 3 \times 0 & 1 \times 0 + 0 \times 1 + 0 \times 0 + 3 \times 0 & 1 \times 0.707 + 0 \times 0 + 0 \times 0.707 + 3 \times 0 & 1 \times 0 + 0 \times 0 + 0 \times 0 + 3 \times 1 \\ 0 \times 0.707 + 1 \times 0 + 0 \times -0.707 + 3 \times 0 & 0 \times 0 + 1 \times 1 + 0 \times 0 + 3 \times 0 & 0 \times 0.707 + 1 \times 0 + 0 \times 0.707 + 3 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 0 + 3 \times 1 \\ 0 \times 0.707 + 0 \times 0 + 1 \times -0.707 + 3 \times 0 & 0 \times 0 + 0 \times 1 + 1 \times 0 + 3 \times 0 & 0 \times 0.707 + 0 \times 0 + 1 \times 0.707 + 3 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 0 + 3 \times 1 \\ 0 \times 0.707 + 0 \times 0 + 0 \times -0.707 + 1 \times 0 & 0 \times 0 + 0 \times 1 + 0 \times 0 + 1 \times 0 & 0 \times 0.707 + 0 \times 0 + 0 \times 0.707 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\
 &= \begin{bmatrix} 0.707 & 0 & 0.707 & 3 \\ 0 & 1 & 0 & 3 \\ -0.707 & 0 & 0.707 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

Solution

Solution: $T(3,3,3) * RotY(45) * T(2,3,5) * RotZ(45) * P[1,1,1] = Q$

$$T(3,3,3) * RotY(45) * T(2,3,5) * RotZ(45) = \begin{bmatrix} 0.707 & 0 & 0.707 & 3 \\ 0 & 1 & 0 & 3 \\ -0.707 & 0 & 0.707 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.707 & -0.707 & 0 & 2 \\ 0.707 & 0.707 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 0.707 \times 0.707 + 0 \times 0.707 + 0.707 \times 0 + 3 \times 0 & 0.707 \times -0.707 + 0 \times 0.707 + 0.707 \times 0 + 3 \times 0 & 0.707 \times 2 + 0 \times 3 + 0.707 \times 5 + 3 \times 1 \\ 0 \times 0.707 + 1 \times 0.707 + 0 \times 0 + 3 \times 0 & 0 \times -0.707 + 1 \times 0.707 + 0 \times 0 + 3 \times 0 & 0 \times 2 + 1 \times 3 + 0 \times 5 + 3 \times 1 \\ -0.707 \times 0.707 + 0 \times 0.707 + 0.707 \times 0 + 3 \times 0 & -0.707 \times -0.707 + 0 \times 0.707 + 0.707 \times 0 + 3 \times 0 & -0.707 \times 2 + 0 \times 3 + 0.707 \times 5 + 3 \times 1 \\ 0 \times 0.707 + 0 \times 0.707 + 0 \times 0 + 1 \times 0 & 0 \times -0.707 + 0 \times 0.707 + 0 \times 0 + 1 \times 0 & 0 \times 2 + 0 \times 3 + 0 \times 5 + 1 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.499849 & -0.499849 & 0.707 & 7.949 \\ 0.707 & 0.707 & 0 & 6 \\ -0.499849 & 0.499849 & 0.707 & 5.121 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution

Q5: A frame Fnoa is located in the position P[1,1,1]T. After the following transformation the frame position has changed to Q. A rotation along Z axis by anti-clock 45 degree followed by a translation along all axis by [2,3,5], after those two, another rotation along Y axis by 45 degree followed by a translation along all axis by [3,3,3]. Find the position Q with respect to P.

Solution: $T(3,3,3) * RotY(45) * T(2,3,5) * RotZ(45) * P[1,1,1] = Q$

$$\begin{bmatrix} 0.5 & -0.5 & 0.707 & 7.95 \\ 0.707 & 0.707 & 0 & 6 \\ -0.5 & 0.5 & 0.707 & 5.12 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \times 1 + -0.5 \times 1 + 0.707 \times 1 + 7.95 \times 1 \\ 0.707 \times 1 + 0.707 \times 1 + 0 \times 1 + 6 \times 1 \\ -0.5 \times 1 + 0.5 \times 1 + 0.707 \times 1 + 5.12 \times 1 \\ 0 \times 1 + 0 \times 1 + 0 \times 1 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} 8.657 \\ 7.414 \\ 5.827 \\ 1 \end{bmatrix}$$

Answer: Q [8.657 , 7.414 , 5.827]T

Keep practicing more and more exercise