Problem: 2-1

Demonstrate by means of truth tables the validity of the following identities:

(a) DeMorgan's theorem for three variables: (x+y+z)' = x'y'z' and (xyz)'=x'+y'+z'

(b) The distributive law: x+yz = (x+y)(x+z)

Solution:

(a)

x	y	z	x+y+z	(x+y+z)'
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

x'	y'	z'	x'y'z'
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	0

x	y	z	xyz	(xyz)'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0

x'	y'	z'	x'+y'+z'
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	0

(b)

x	y	z	yz	x+yz
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

x+y	x+z	(x+y)(x+z)
0	0	0
0	1	0
1	0	0
1	1	1
1	1	1
1	1	1
1	1	1
1	1	1

Problem: 2-4

Reduce the following Boolean expressions to the indicated number of literals:

(a) $A'C' + ABC + AC'$ to three literals
(b) $(x'y'+z)' + z + xy + wz$ to three literals
(c) $A'B(D'+C'D) + B(A+A'CD)$ to one literal
(d) $(A'+C)(A'+C')(A+B+C'D)$ to four literals

Solution:

(a) $A'C' + ABC + AC'$

$= A'C' + AC' + ABC$
$= C'(A'+ A) + ABC$
$= C'\cdot 1 + ABC$
$= C' + ABC$
$= (C'+ AB)(C'+C)$ [distributive]
$= AB + C'$

(b) $(x'y'+z)' + z + xy + wz$

$= (x'y'+z)' + z + wz + xy$
$= (x'y'+z)' + z(1+ w) + xy$
$= (x'y'+z)' + z + xy$
$= (x + y)z' + z + xy$ [DeMorgan]
$= (z + (x + y)) \cdot (z + z') + xy$ [distributive]
$= (z + (x + y)) \cdot 1 + xy$
$= x + y + z + xy$
$= x + y + z$ [absorption]

(c) $A'B(D' + C'D) + B(A+A'CD)$

$= A'BD' + \underline{A'BC'D} + AB+ \underline{A'BCD}$
$= A'BD(C+C')+ A'BD'+ AB$
$= A'BD+ A'BD'+ AB$
$= A'B(D+D')+ AB$
$= A'B+ AB$
$= B(A'+ A)$
$= B$

(d) $(A'+C)(A'+C')(A+B+C'D)$

$= (A'+C)(A'+C')(A+B+C'D)$
$= (A' + CC')(A + B + C'D)$
$= A'(A + B + C'D)$
$= A'A + A'B + A'C'D$
$= A'B + A'C'D$
$= A'(B + C'D)$

Problem 2-5:
Find the complement of $F = x + yz$; then show that $FF' = 0$ and $F + F' = 1$

Solution:
$F = x + yz$

The dual of F is: $x \bullet (y + z)$

Complement each literal: $x' \bullet (y' + z') = F'$

$FF' = (x + yz) \bullet (x' \bullet (y' + z')) = (xx' + x'yz) \bullet (y' + z') = x'yz \bullet (y' + z') = x'yy'z + x'yzz' = 0$

$F + F' = (x + yz) + (x' \bullet (y' + z')) = (x + yz + x') + (x + yz + y' + z') = (1 + yz) + (x + yz + y' + z')$
$= 1 + (x + yz + y' + z') = 1$

Problem 2-8:

List the truth table of the function:
$F = xy + xy' + y'z$

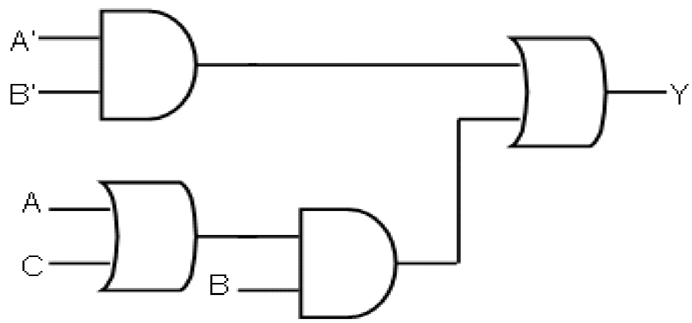
Solution:
The truth table is:

x	y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

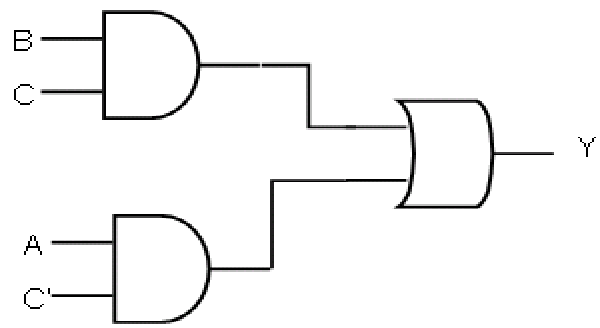
Problem 2-10:

Draw the logical diagrams for the following Boolean expressions:

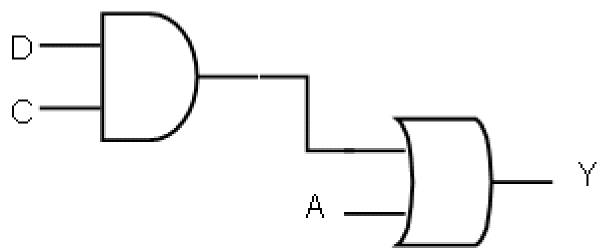
(a) Y=A′B′+B(A+C).



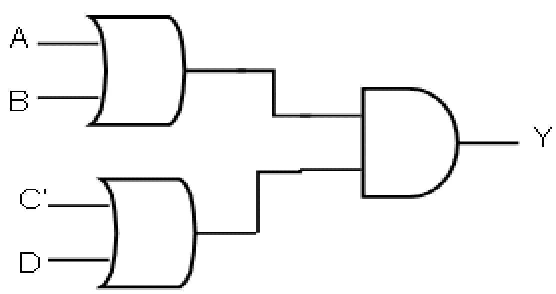
(b) Y=BC+AC′.



(c) Y=A+CD.



(d) Y=(A+B)(C′+D).



Problem 2-15

Given the Boolean function F=xy'z+x'y'z+w'xy+wx'y+wxy.

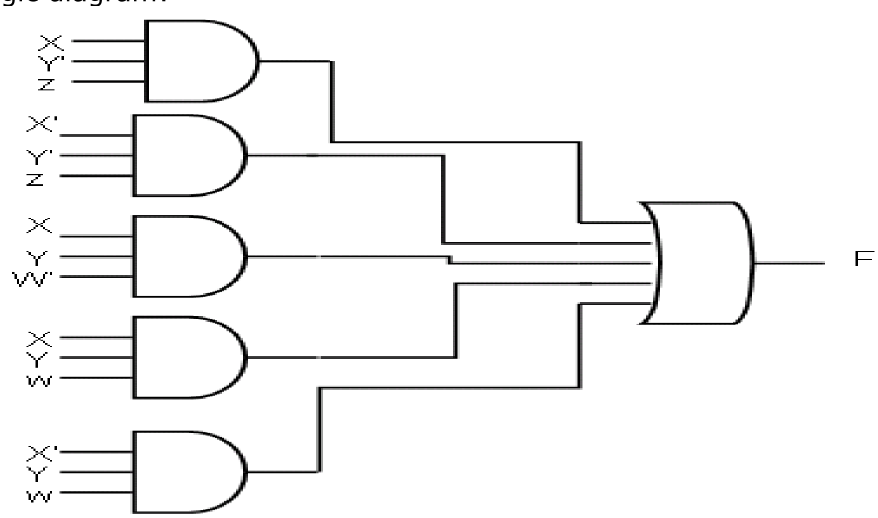
(a) Obtain the truth table of the function.
(b) Draw the logical diagram using the original Boolean expression.
(c) Simplify the function to a minimum number of laterals using Boolean algebra.
(d) Obtain the truth table of the function using the simplified expression.
(e) Draw the logical diagram from the simplified expression and compare the total number of gates with the diagram of part (b).

Solutions:

(a) The truth table of the function:

X	Y	z	w	F	F(simplified
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	1	1
0	0	1	1	1	1
0	1	0	0	0	0
0	1	0	1	1	1
0	1	1	0	0	0
0	1	1	1	1	1
1	0	0	0	0	0
1	0	0	1	0	0
1	0	1	0	1	1
1	0	1	1	1	1
1	1	0	0	1	1
1	1	0	1	1	1
1	1	1	0	1	1
1	1	1	1	1	1

(b) The logic diagram:



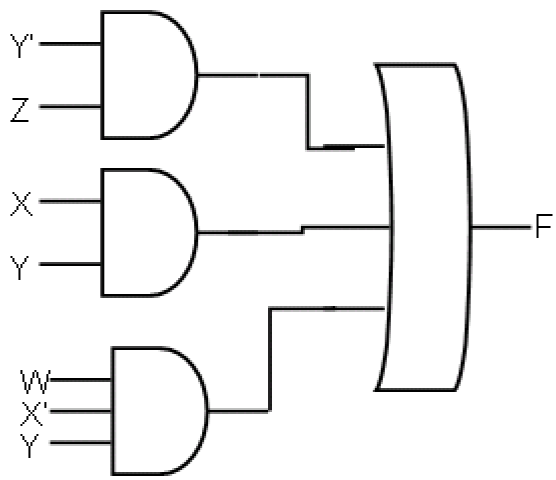
(c) A simplified function:

F =XY’Z+XÝ’Z+W’XY+WXÝ+XYW

=Y’Z (X+X’) +XY (W+W’) +WXÝ= Y’Z+ XY+ WXÝ

(d) See the truth table

(e) Logic circuit for simplified function



For 1st design there is 5 And gates with 3 inputs and 1 OR Gate with 5 inputs.
For 2nd design there is 2 And gates with 2 inputs and 1 OR Gate with 3 inputs and 1 And gates with 3 inputs.

<u>Problem 18:</u>

Convert the following to the other canonical form:

(a) F (x, y , z) = Σ (1,3,7) (b) F(A,B,C,D)= Π (0,1,2,3,4,6,12)

<u>Solution:</u>

(a) F (x, y, z) = Σ (1,3,7) = Π (0,2,4,5,6)

F (x, y, z) = $(x+y+z)\bullet(x+\overline{y}+z)\bullet(\overline{x}+y+z)\bullet(\overline{x}+y+\overline{z})\bullet(\overline{x}+\overline{y}+z)$

(b) F (A,B,C,D) = Π (0,1,2,3,4,6,12) = Σ (5,7,8,9,10,11,13,14,15)

F(A,B,C,D)=

$(\overline{A}B\overline{C}D)+(\overline{A}BCD)+(A\overline{B}\overline{C}\overline{D})+(A\overline{B}\overline{C}D)+(A\overline{B}C\overline{D})+(A\overline{B}CD)+(AB\overline{C}D)+(ABC\overline{D})+(ABCD)$

Problem 21:
Show that the dual of the exclusive-OR is equal to its complement.

XOR

$X \oplus Y = XY' + X'Y$

$= (X + Y') \bullet (X' + Y)$

$= XX' + XY + X'Y' + YY'$

$= XY + X'Y'$

$= (X \oplus Y)' = \text{XNOR}$

Problem 23:
Show that a positive logic NAND gate is a negative logic NOR gate and vice versa.

Solution:

Positive Logic NAND L = 0 H = 1

x	y	Z
0	0	1
0	1	1
1	0	1
1	1	0

x	y	Z
L	L	H
L	H	H
H	L	H
H	H	L

To use the Negative Logic let L = 1, H = 0

x	y	Z
1	1	0
1	0	0
0	1	0
0	0	1

This Truth Table is that of the NOR gate using negative logic.

Another type of problem:

1. The Boolean function Y= AB + CD is to be realized using only 2 input NAND gates. The minimum number of gates required is three. To prove this statement.

   Answer:

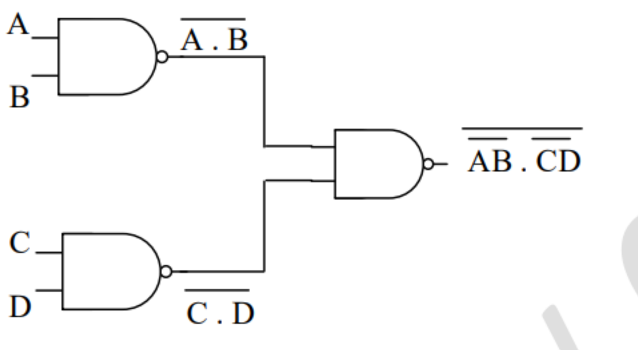
   $\boldsymbol{Y} = \boldsymbol{AB} + \boldsymbol{CD}$

   We double complement either side

   i.e., $\overline{\overline{\boldsymbol{Y}}} = \boldsymbol{Y} = \overline{\overline{\boldsymbol{AB} + \boldsymbol{CD}}}$

   $= \overline{\overline{\boldsymbol{AB}}.\overline{\boldsymbol{CD}}}$

   The logic diagram for the expression is



   So, requires three NAND gates

2. The Minimum number of 2 input NAND gates required to implement the function, $F = (\bar{X} + \bar{Y})(Z + W)$ is 4. To explain it.

   Answer:

   $\boldsymbol{F} = (\bar{\boldsymbol{X}} + \bar{\boldsymbol{Y}})(\boldsymbol{Z} + \boldsymbol{W})$

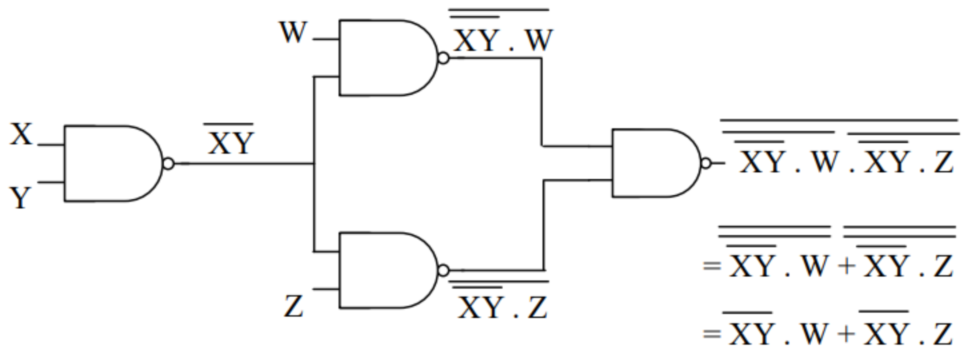
   The Boolean expression is in POS form. It should be converted into SOP and simplified

   $\boldsymbol{F} = (\bar{\boldsymbol{X}} + \bar{\boldsymbol{Y}})(\boldsymbol{Z} + \boldsymbol{W})$

   $= \overline{\boldsymbol{XY}}(\boldsymbol{Z} + \boldsymbol{W})$

   $\boldsymbol{F} = \overline{\boldsymbol{XY}}\boldsymbol{Z} + \overline{\boldsymbol{XY}}\boldsymbol{W}$

   Above expression cannot be simplified further.



3. The minimum number of 2-input NAND gates required to implement the Boolean function $Z = A\bar{B}C$, assuming that A, B, and C are available is three/four/five. To find it.