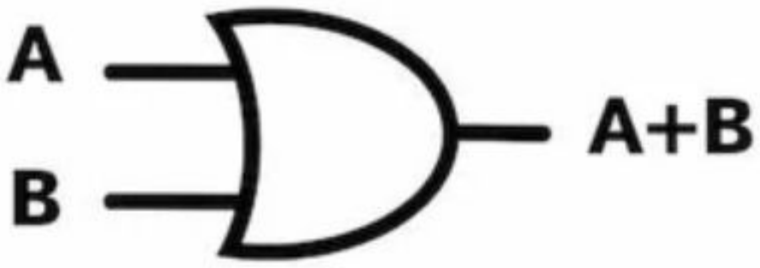
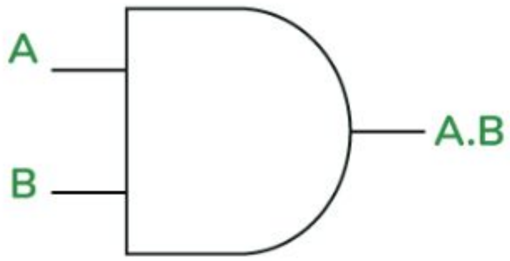
Week 6, Lecture 1 + 2



2 input OR gate

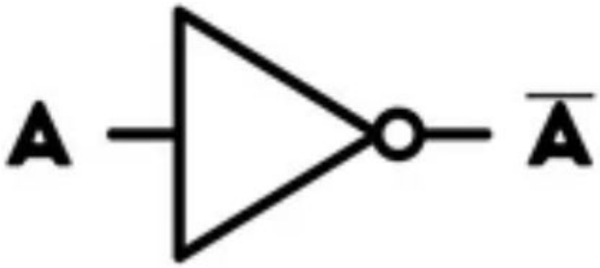
A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

2- Input AND Gate



Truth Table

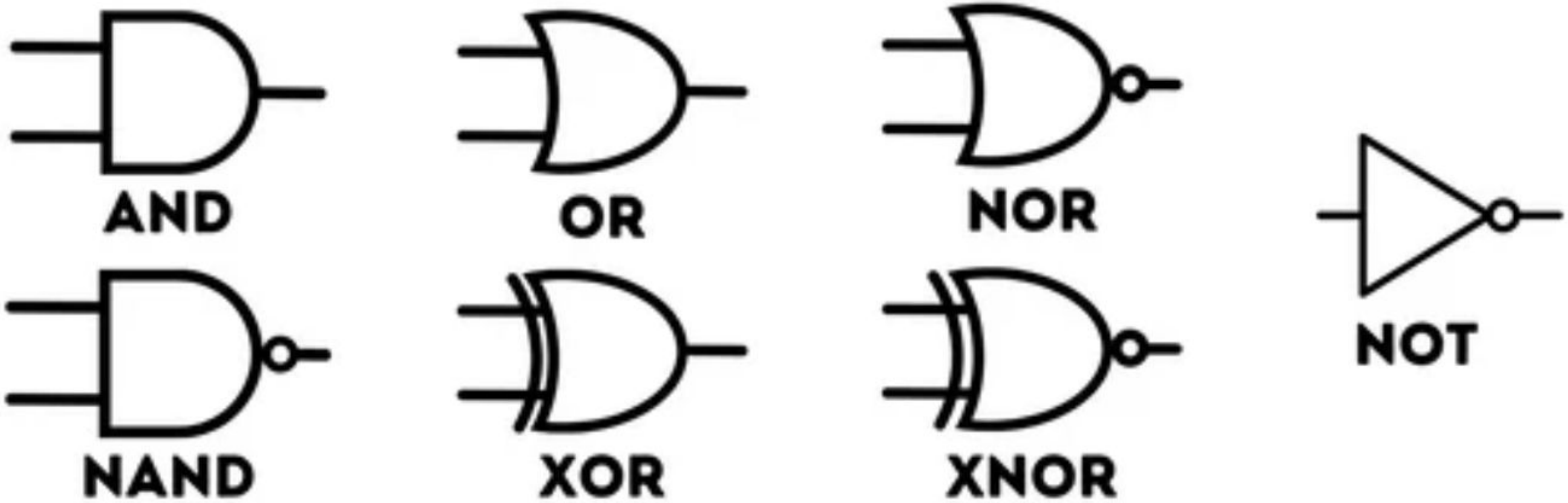
A (Input 1)	B (Input 2)	X = (A.B)
0	0	0
0	1	0
1	0	0
1	1	1

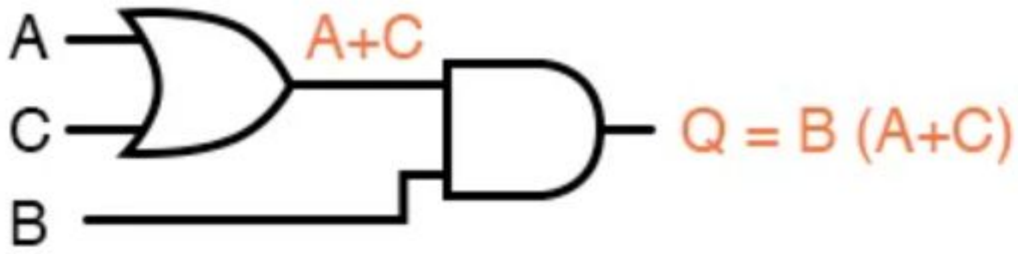


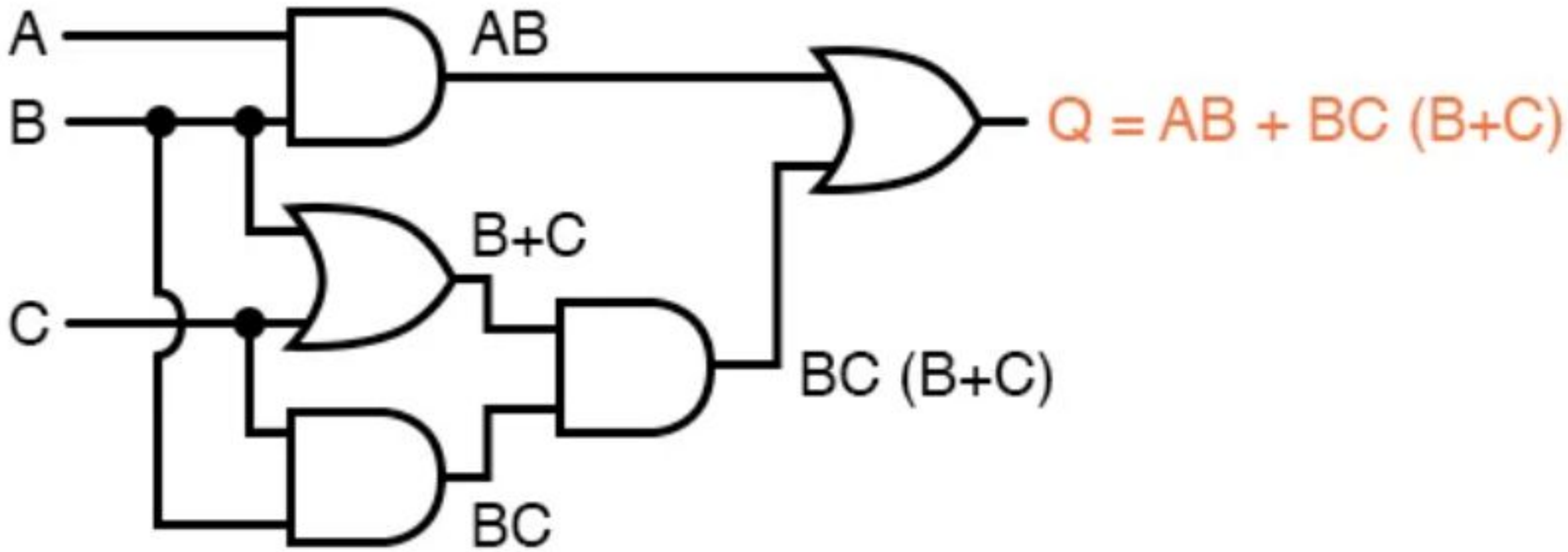
NOT

NOT Gate

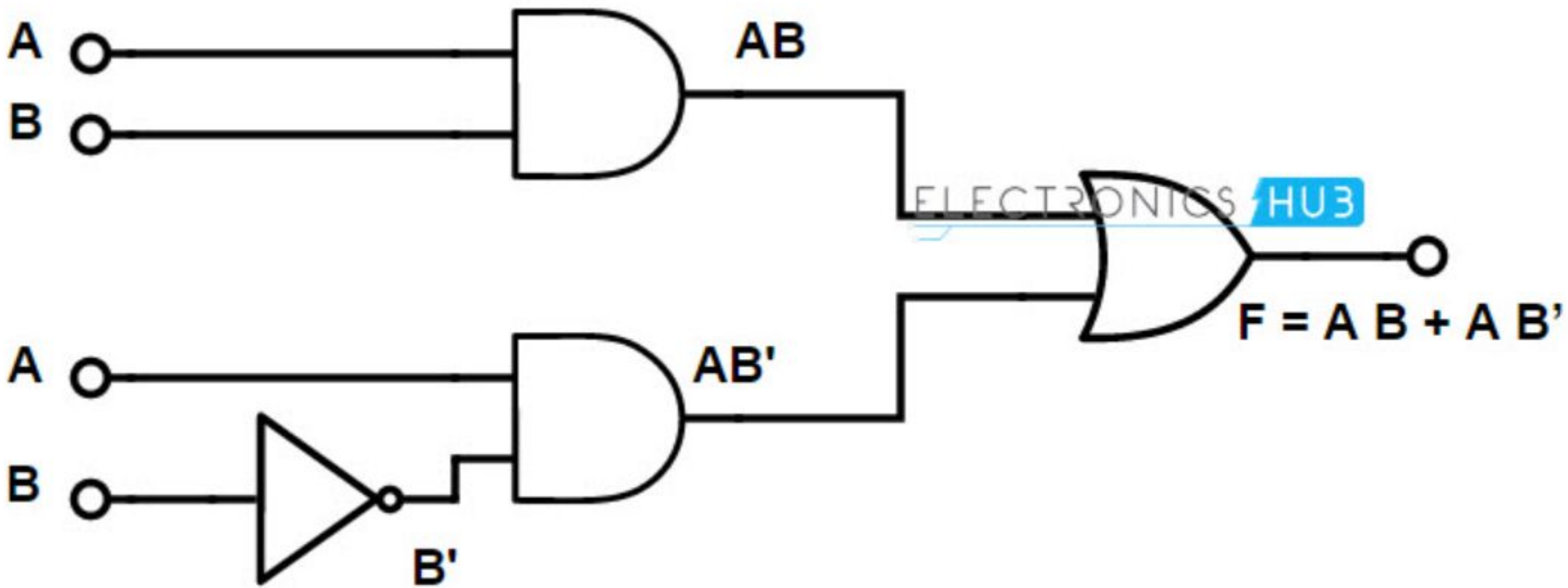
A	$\overline{A}$
0	1
1	0


AND
OR
NOR
NAND
XOR
XNOR
NOT

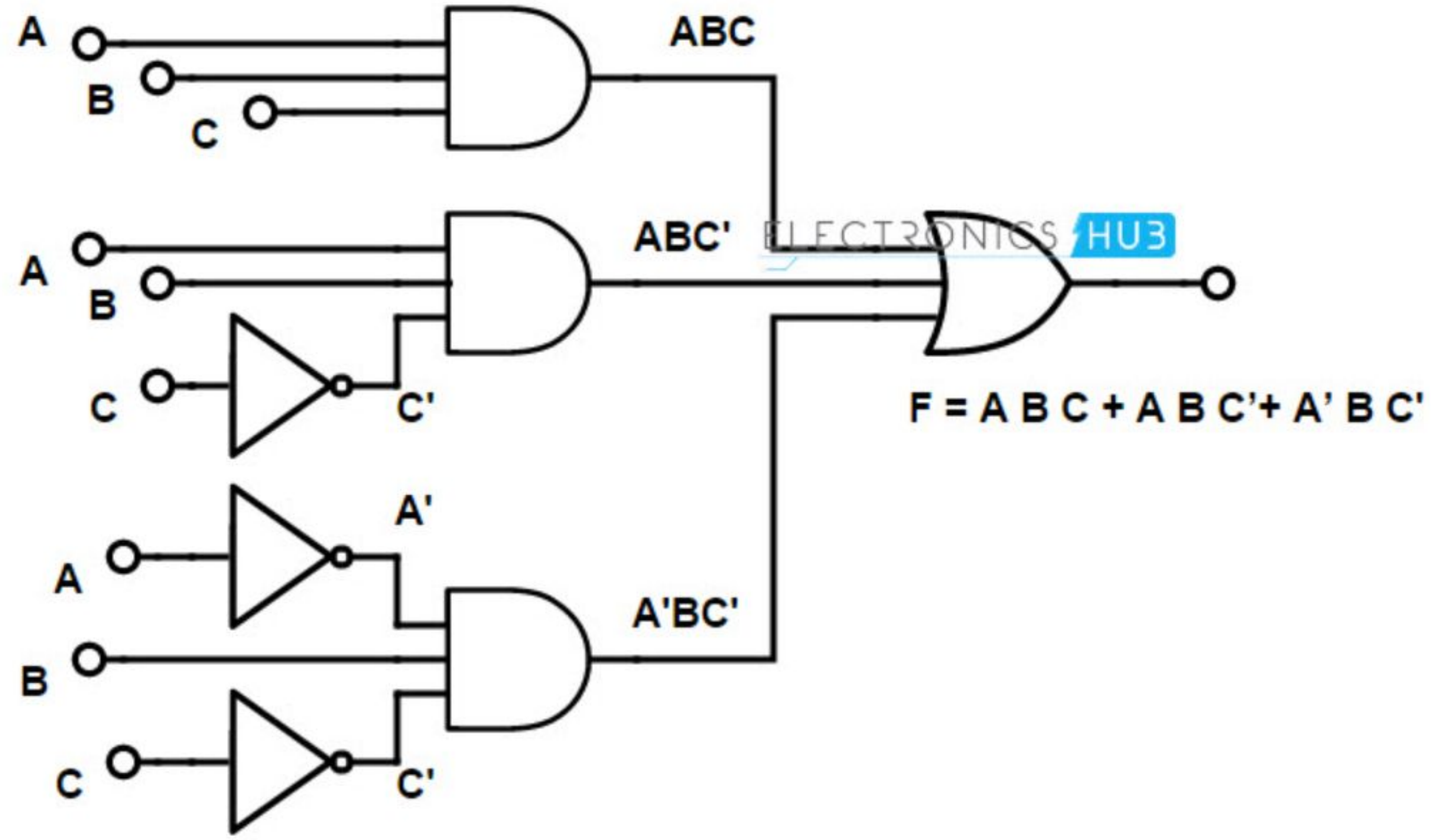

A
C
B
A+C
Q = B (A+C)



A
B
C
AB
B+C
BC
BC (B+C)
Q = AB + BC (B+C)



A
B
AB
A
B
AB'
B'
ELECTRONICS HUB
F = A B + A B’


A
B
C
ABC
A
B
C
C'
ABC'
ELECTRONICS HUB
A'
A
B
C
C'
A'BC'
F = A B C + A B C'+ A' B C'

Name	AND form	OR form
Identity law	$1A = A$	$0 + A = A$
Null law	$0A = 0$	$1 + A = 1$
Idempotent law	$AA = A$	$A + A = A$
Inverse law	$A\overline{A} = 0$	$A + \overline{A} = 1$
Commutative law	$AB = BA$	$A + B = B + A$
Associative law	$(AB)C = A(BC)$	$(A + B) + C = A + (B + C)$
Distributive law	$A + BC = (A + B)(A + C)$	$A(B + C) = AB + AC$
Absorption law	$A(A + B) = A$	$A + AB = A$
De Morgan's law	$\overline{AB} = \overline{A} + \overline{B}$	$\overline{A + B} = \overline{A}\,\overline{B}$

$$AB + BC(B + C)$$

$$\Big\downarrow \quad \text{Distributing terms}$$

$$AB + BBC + BCC$$

$$\Big\downarrow \quad \text{Applying identity } AA = A \text{ to 2nd and 3rd terms}$$

$$AB + BC + BC$$

$$\Big\downarrow \quad \text{Applying identity } A + A = A \text{ to 2nd and 3rd terms}$$

$$AB + BC$$

$$\Big\downarrow \quad \text{Factoring B out of terms}$$

$$B(A + C)$$

The Boolean Expression: A.(A + B)

Multiplying out the brackets gives us:

	A.(A+B)	Start
multiply:	A.A + A.B	Distributive Law
but:	A.A = A	Idempotent Law
then:	A + A.B	Reduction
thus:	A.(1 + B)	Annulment Law
equals to:	A	Absorption Law

	$(A + B)(A + C)$	Start
multiply:	$A.A + A.C + A.B + B.C$	Distributive law
but:	$A.A = A$	Idempotent Law
then:	$A + A.C + A.B + B.C$	Reduction
however:	$A + A.C = A$	Absorption Law
thus:	$A + A.B + B.C$	Distributive Law
again:	$A + A.B = A$	Absorption Law
thus:	$A + B.C$	Result

	$AB(\overline{B}C+AC)$	Start
multiply	$A.B.\overline{B}.C + A.B.A.C$	Distributive Law
again:	$A.A = A$	Idempotent Law
then:	$A.B.\overline{B}.C + A.B.C$	Reduction
but:	$B.\overline{B} = 0$	Complement Law
so:	$A.0.C + A.B.C$	Reduction
becomes:	$0 + A.B.C$	Reduction
as:	$0 + A.B.C = A.B.C$	Identity Law
thus:	ABC	Result

The truth table that shows the verification of De Morgan's First law is given as follows:

A	B	A′	B′	(A.B)′	A′+B′
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

The last two columns show that $(A.B)' = A'+B'$.

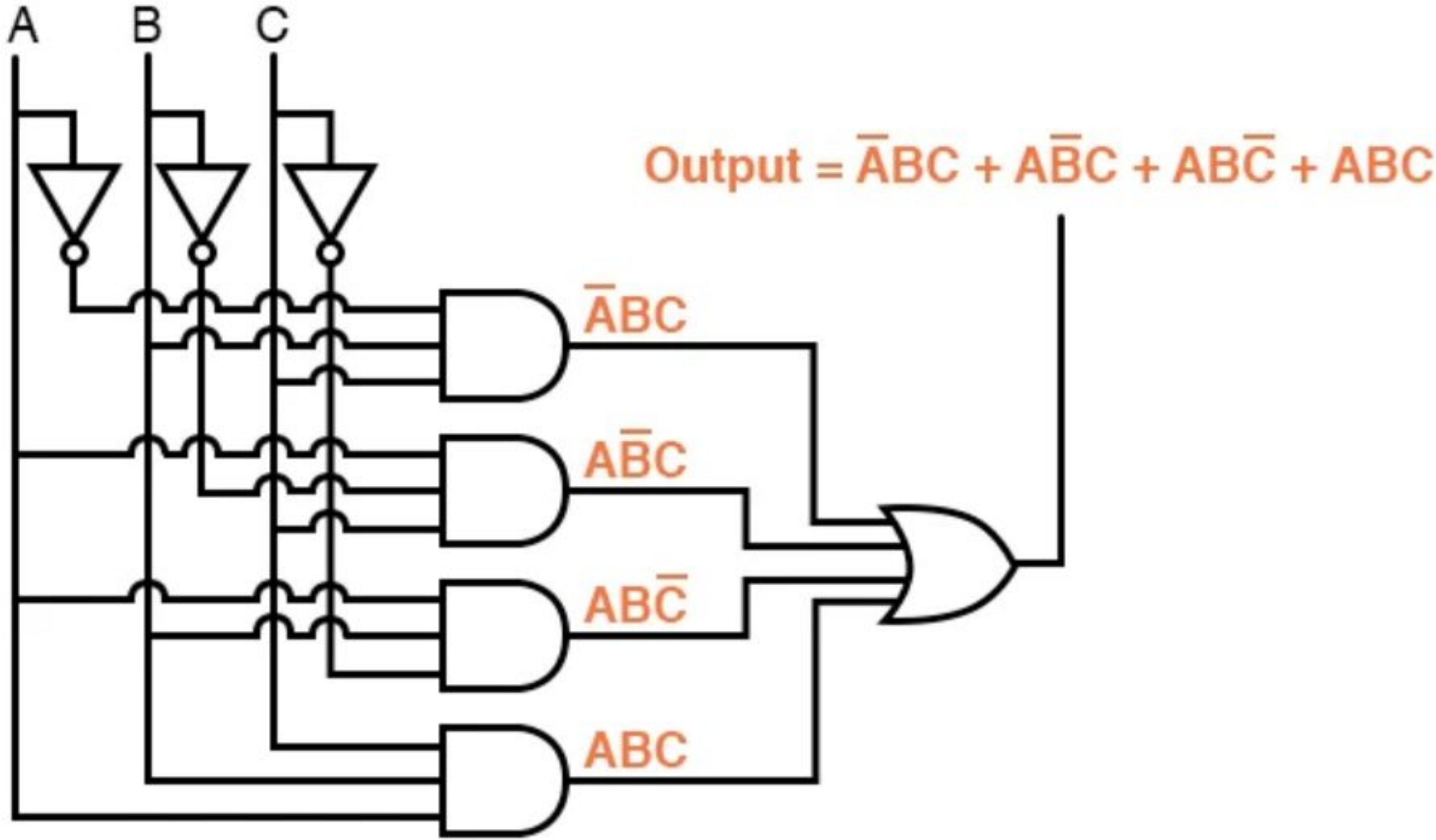
The following truth table shows the proof for De Morgan's second law.

A	B	A′	B′	(A+B)′	A′. B′
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

The last two columns show that $(A+B)' = A'.\,B'$.

Solution: Given expression A(B+D).

A	B	D	B+D	A(B+D)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1


A
B
C
Output = $\bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$
$\bar{A}BC$
$A\bar{B}C$
$AB\bar{C}$
ABC

$$\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

Factoring BC out of 1st and 4th terms

$$BC(\overline{A} + A) + A\overline{B}C + AB\overline{C}$$

Applying identity $A + \overline{A} = 1$

$$BC(1) + A\overline{B}C + AB\overline{C}$$

Applying identity $1A = A$

$$BC + A\overline{B}C + AB\overline{C}$$

Factoring B out of 1st and 3rd terms

$$B(C + A\overline{C}) + A\overline{B}C$$

Applying rule $A + \overline{A}B = A + B$ to the $C + A\overline{C}$ term

$$B(C + A) + A\overline{B}C$$

Distributing terms

$$BC + AB + A\overline{B}C$$

Factoring A out of 2nd and 3rd terms

$$BC + A(B + \overline{B}C)$$

Applying rule $A + \overline{A}B = A + B$ to the $B + \overline{B}C$ term

$$BC + A(B + C)$$

Distributing terms

$$BC + AB + AC$$

or

$$AB + BC + AC$$

Simplified result