



Forecasting on Air Passengers Dataset

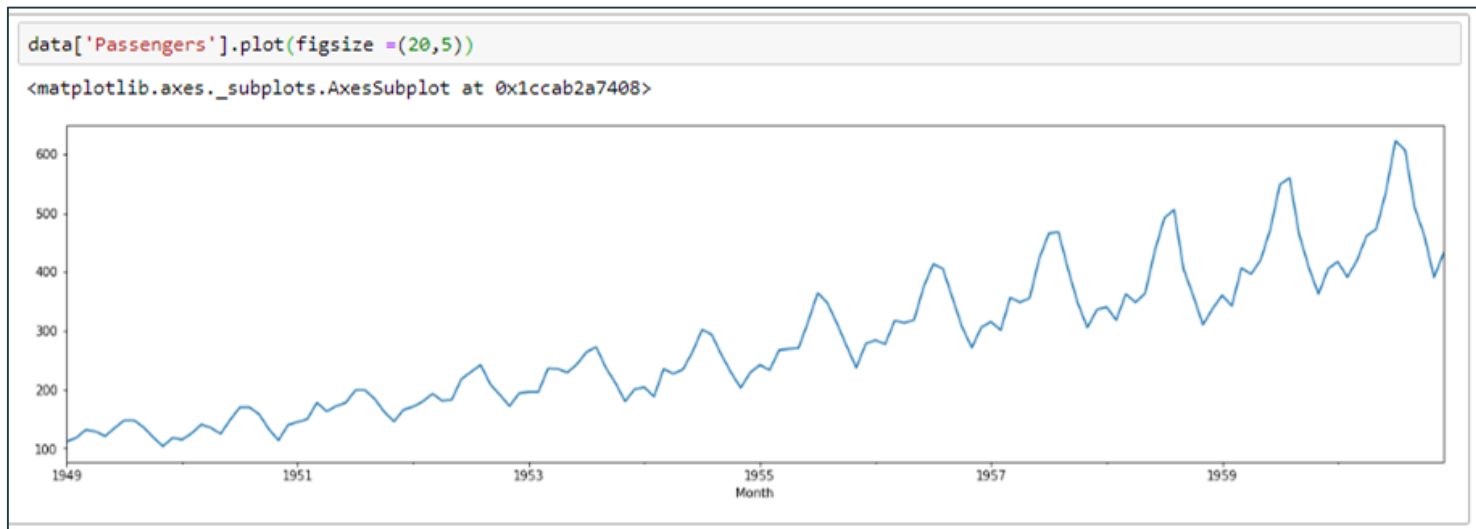
Objective

- Objective - Build a model to forecast the demand(passenger traffic) in Airplanes.

About the Dataset:

- Number of observations: 144
- Variables in the Dataset - Passengers.
- The data is classified in date and the passengers travelling per month.
- There are no missing values in the dataset.

Visualizing Passenger Data:



- From the above plot we can observe that, there is some trend and seasonality in the time series
- X - axis: Months
- Y - axis: Number of Passengers

Checking for Stationarity:

Rolling Mean

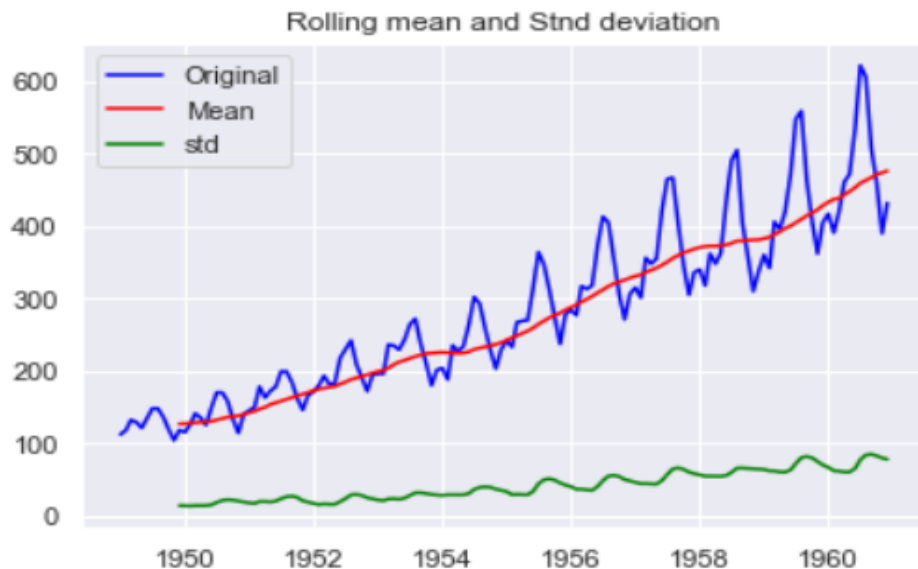
Rolling mean is the test of stationarity in the data.

```
rolmean = IndexedDF.rolling(window=12).mean()  
rolstd = IndexedDF.rolling(window=12).std()  
print(rolmean,rolstd)
```

Month	Passengers
1949-01-01	NaN
1949-02-01	NaN
1949-03-01	NaN
1949-04-01	NaN
1949-05-01	NaN
...	...
1960-08-01	463.333333
1960-09-01	467.083333
1960-10-01	471.583333
1960-11-01	473.916667
1960-12-01	476.166667

As we can see from the diagram that the rolling mean and Standard Deviation increase with time, we can conclude that the time series is not stationary.

```
orig = plt.plot(IndexedDF,color = 'blue',label = 'Original')
mean = plt.plot(rolmean,color = 'red',label = 'Mean')
std = plt.plot(rolstd,color = 'green',label = 'std')
plt.legend(loc = 'best')
plt.title('Rolling mean and Stnd deviation')
plt.show(block = False)
```



Checking for Stationarity:

ACDF Test:

- We can clearly observe that the p value is greater than 0.05, suggesting that the data is not stationary
- The underlying principle is to model or estimate the trend and seasonality in the series and remove those from the series to get a stationary series.

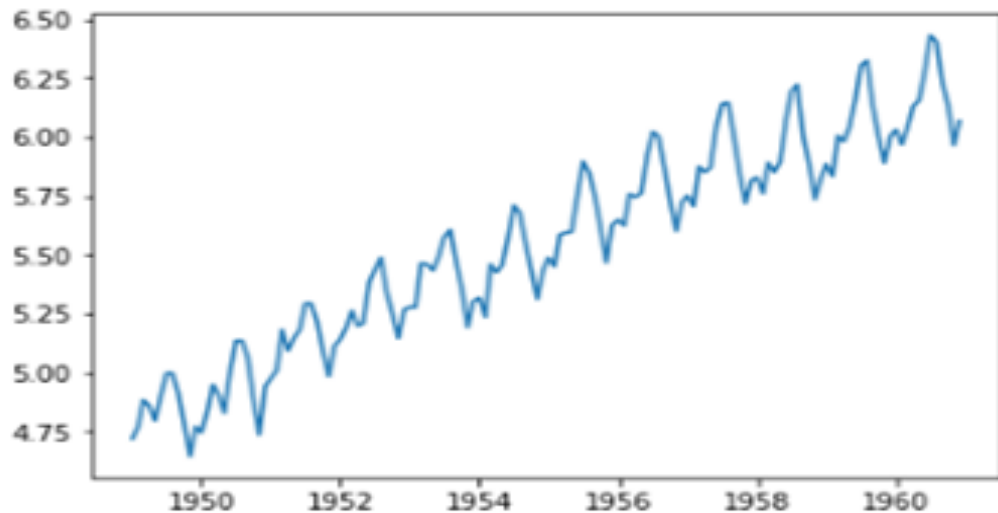
Results of the ACDF test :

Test statistics	0.815369
p-value	0.991880
#lags used	13.000000
# observations used	130.000000
Critical Value (1%)	-3.481682
Critical Value (5%)	-2.884042
Critical Value (10%)	-2.578770
dtype:	float64

Estimating the Trend

```
data_log = np.log(data.Passengers)  
plt.plot(data_log)
```

```
[<matplotlib.lines.Line2D at 0x1cca9d91cc8>]
```

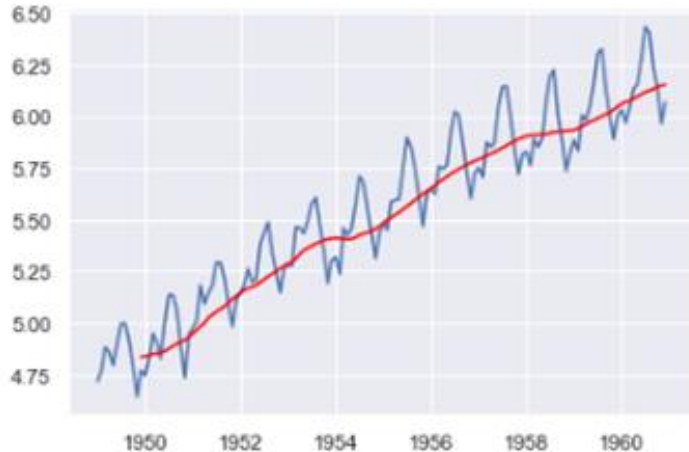


Taking the log of the dependant variable is a simple way of lowering the rate at which rolling mean increases.

Data Stationarity Test - Rolling Mean Method

```
movingAverage = IndexedDF_logScale.rolling(window = 12).mean()  
movingstd = IndexedDF_logScale.rolling(window = 12).std()  
plt.plot(IndexedDF_logScale)  
plt.plot(movingAverage,color = 'red')
```

[<matplotlib.lines.Line2D at 0x1c6dc34b188>]

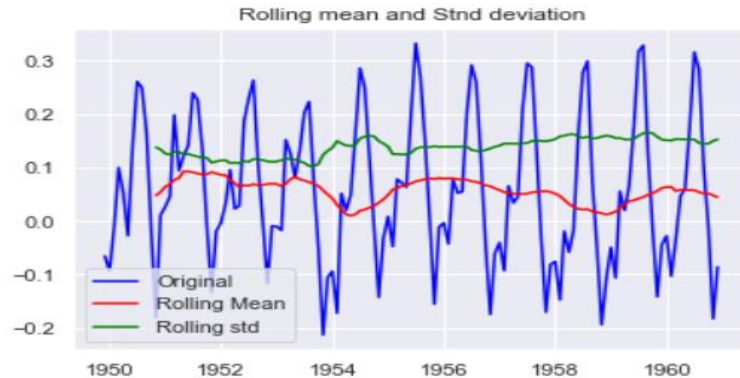


- For the graph on the left, we can see that although rolling mean is not stationary, it is still better than the previous case where no transformation were applied to series.
- We know from the graph that both time series with log scale as well as moving average have a trend component. Subtracting one from the other should remove the trend component in both.


```
datasetLogScaleMinusMovingAverage = IndexedDF_logScale - movingAverage
datasetLogScaleMinusMovingAverage .head(12)
```

#remove the NaN values

```
datasetLogScaleMinusMovingAverage.dropna(inplace = True)
datasetLogScaleMinusMovingAverage .head(10)
```



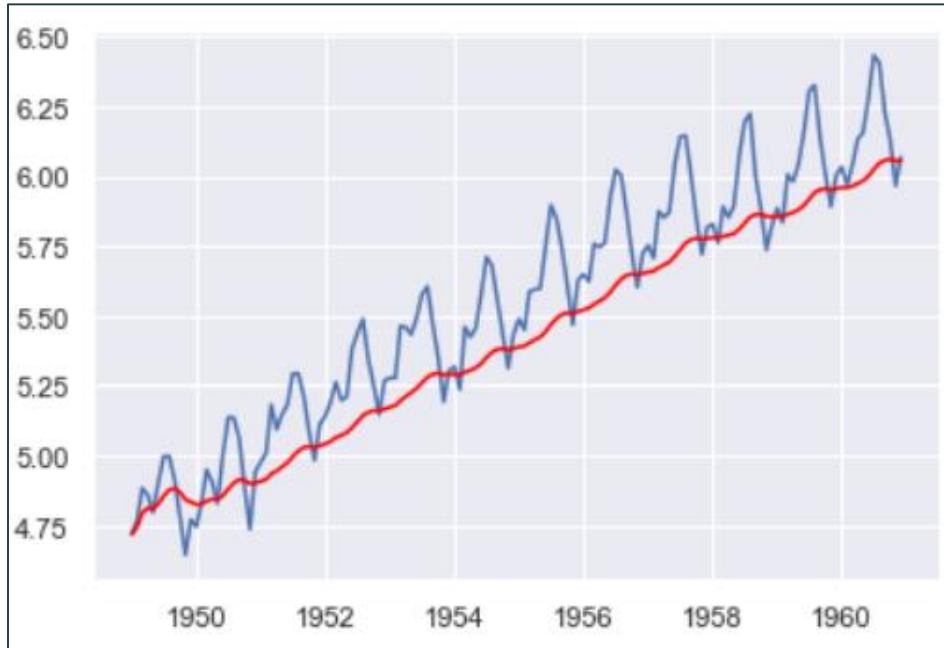
Results of the ACF test :

Test statistics	-3.162908
p-value	0.022235
#lags used	13.000000
# observations used	119.000000
Critical Value (1%)	-3.486535
Critical Value (5%)	-2.886151
Critical Value (10%)	-2.579896
dtype:	float64

- We found that our assumption of subtracting two related series of similar trend components will make the result stationary was true.
 - 1. P value has reduced from 0.99 to 0.022
 - 2. ADF Test Statistic is close to the critical values
-
- Thus, we can say that the given series is stationary.

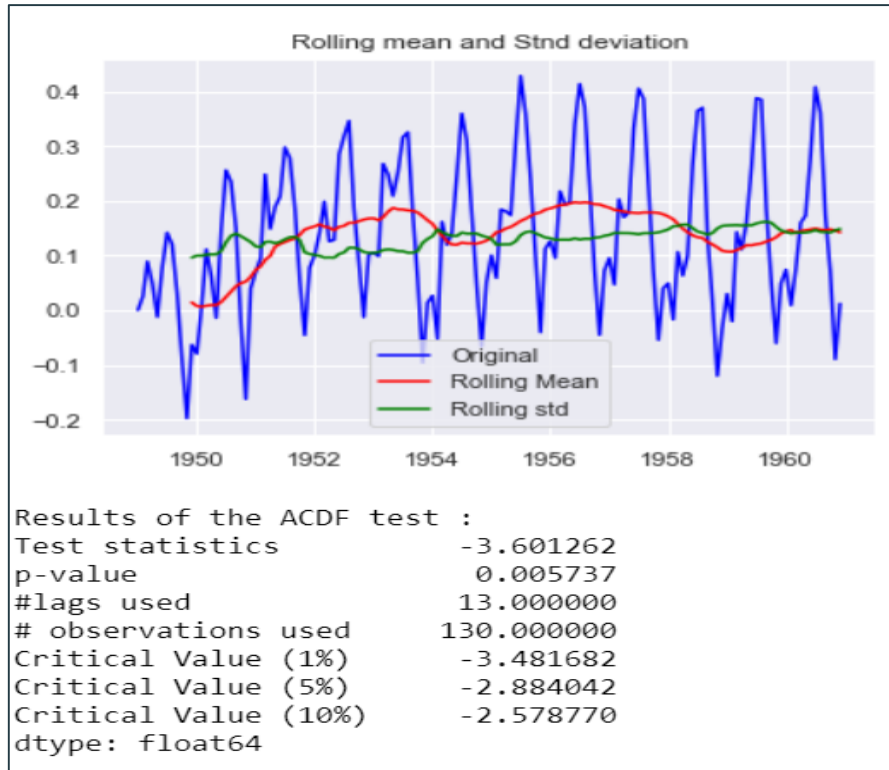
Making Data Stationary - Exponential Decay Method

```
exponentialDecayingWeightedAverage = IndexedDF_logScale.ewm(halflife=12,min_periods=0, adjust = True).mean()  
plt.plot(IndexedDF_logScale)  
plt.plot(exponentialDecayingWeightedAverage, color = 'red')
```



- For the given graph, it seems that the given method is not showing any advantage over log scale as corresponding curves are similar.
- Since no concrete inference can be drawn, we perform ADF test on the decay series.

```
datasetlogScaleMinusexponentialDecayingWeightedAverage = IndexedDF_logScale - exponentialDecayingWeightedAverage
test_stationarity(datasetlogScaleMinusexponentialDecayingWeightedAverage)
```



We observe that the time series is stationary and the series for moving average and std deviation is almost parallel to x-axis, thus they also have no trend.

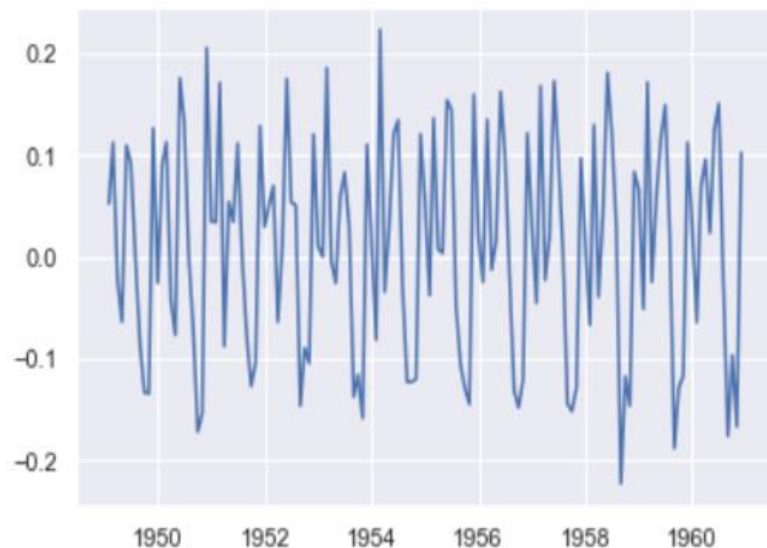
Additionally,

1. P value decreased from 0.022 to 0.005
2. Test statistic is closer to critical values.

Making Data Stationary - Differencing Method

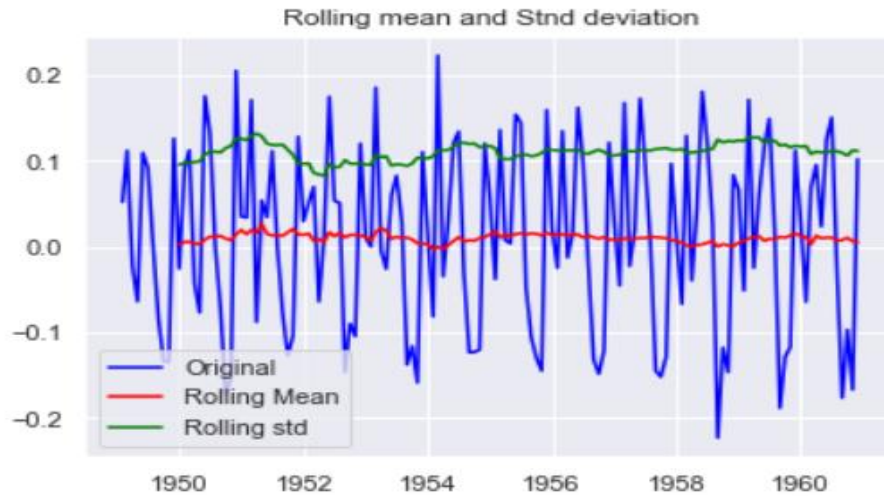
```
datasetlogDiffShifting = IndexedDF_logScale - IndexedDF_logScale.shift()  
plt.plot(datasetlogDiffShifting)
```

[<matplotlib.lines.Line2D at 0x1d7ce504608>]



```
datasetlogDiffShifting.dropna(inplace = True)  
test_stationarity(datasetlogDiffShifting)
```

Month	Passengers
1949-01-01	NaN
1949-02-01	NaN
1949-03-01	NaN
1949-04-01	NaN
1949-05-01	NaN
...	...
1960-08-01	463.333333
1960-09-01	467.083333
1960-10-01	471.583333
1960-11-01	473.916667
1960-12-01	476.166667



Results of the ACF test :

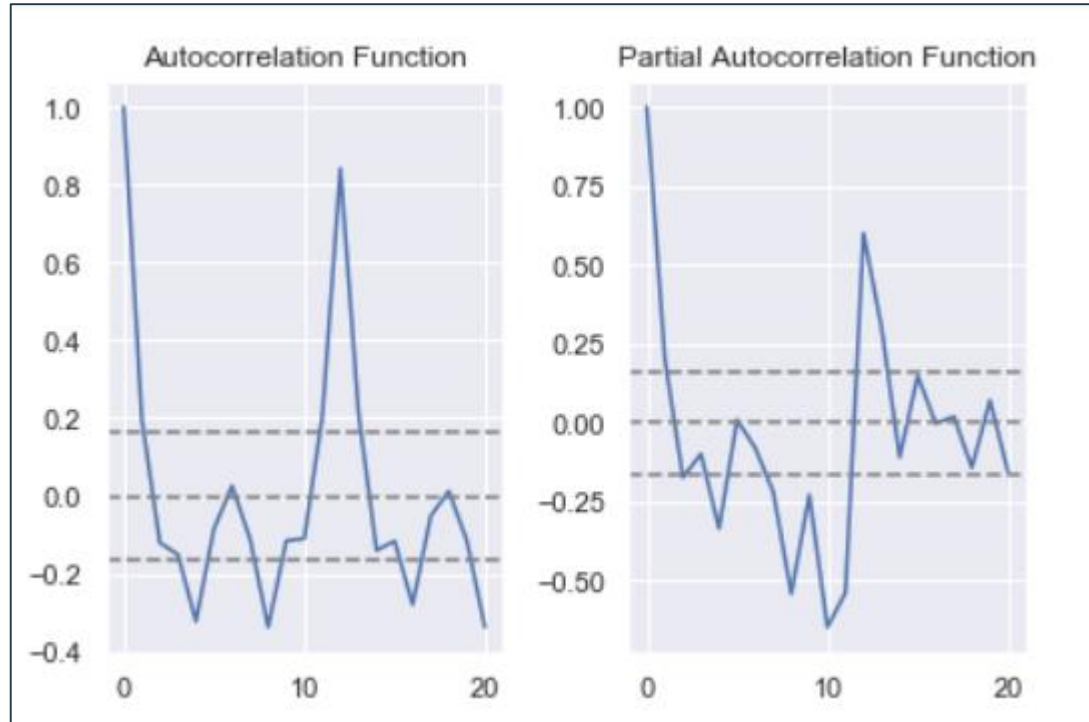
Test statistics	-2.717131
p-value	0.071121
#lags used	14.000000
# observations used	128.000000
Critical Value (1%)	-3.482501
Critical Value (5%)	-2.884398
Critical Value (10%)	-2.578960
dtype: float64	

The ACF Result shows that:

1. P value of 0.07 is not as good as 0.005 of exponential decay.
2. The test statistic value is not as close to the critical values as that for exponential decay.

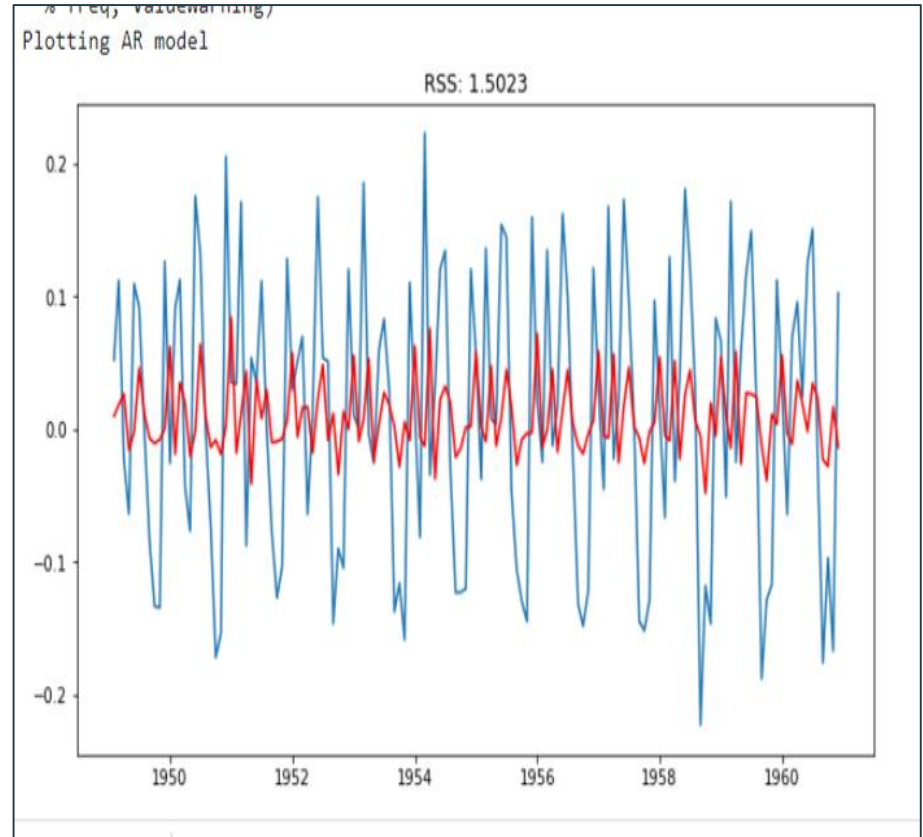
ACF and PACF

- ACF is auto-correlation between the elements of a series and others from the same series separated from them by a given interval
- PACF gives the partial correlation of a stationary time series with its own lagged values
- The value of q from ACF plot is 2
- The value of p from PACF plot is 2



AR Model

- Before, we see an ARIMA model, let us check the results of the individual AR & MA model. These models will give a value of RSS. Lower RSS values indicate a better model.
- **Residual sum of squares (RSS)/sum of squared residuals (SSR)/sum of squared estimate of errors (SSE)** is a measure of the discrepancy between the data and an estimation model.
- A small RSS indicates a tight fit of the model to the data. It is used as an optimality criterion in parameter selection and model selection
- Here in this AR model gives lower RSS of 1.5023 at order 2,1,0.



AR Model Summary

```
print(results_AR.summary())
```

```
=====
                        ARIMA Model Results
=====
Dep. Variable:          D.#Passengers      No. Observations:          143
Model:                  ARIMA(2, 1, 0)      Log Likelihood             122.802
Method:                  css-mle           S.D. of innovations         0.102
Date:                   Wed, 16 Sep 2020    AIC                        -237.605
Time:                   20:38:35           BIC                        -225.753
Sample:                 02-01-1949         HQIC                       -232.789
                  - 12-01-1960
=====

              coef      std err          z      P>|z|      [0.025      0.975]
-----
const                0.0096      0.009      1.048      0.295     -0.008      0.028
ar.L1.D.#Passengers   0.2359      0.083      2.855      0.004      0.074      0.398
ar.L2.D.#Passengers  -0.1725      0.083     -2.070      0.038     -0.336     -0.009
                    Roots
=====
              Real      Imaginary      Modulus      Frequency
-----
AR.1           0.6838      -2.3088j      2.4079      -0.2042
AR.2           0.6838      +2.3088j      2.4079       0.2042
=====
```

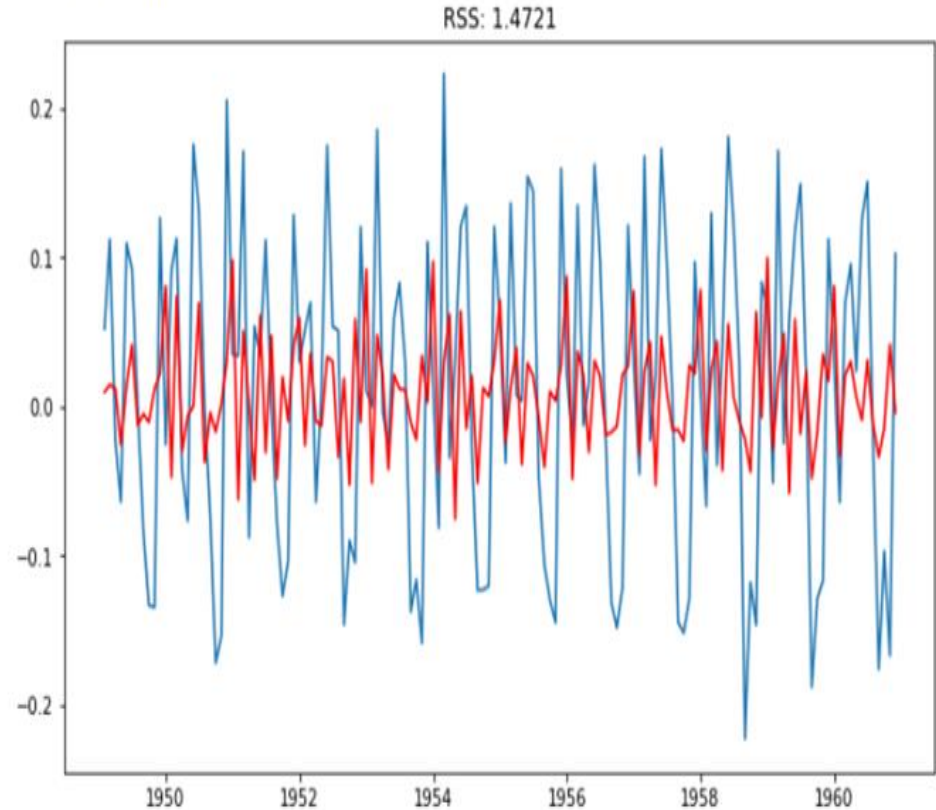
- Here we can see that p values for AR L1 and AR L2 is less than 0.05
- Therefore they seem significant. The value for AIC and BIC values are -237.605 and -225.753 respectively.

MA Model

Moving Average Model (MA)

- Assumes the value of the dependent variable on the current day depends on the previous days error terms
- The MR model gives lower RSS of 1.4721 at order 0,1,2
- As in both the models RSS value is comparatively less.

Plotting MA model



MA Model Summary

```
print(results_MA.summary())
```

```

                    ARIMA Model Results
=====
Dep. Variable:          D.#Passengers      No. Observations:          143
Model:                 ARIMA(0, 1, 2)      Log Likelihood             124.189
Method:                css-mle             S.D. of innovations         0.101
Date:                  Wed, 16 Sep 2020    AIC                        -240.379
Time:                  20:40:31            BIC                        -228.528
Sample:                02-01-1949         HQIC                       -235.563
                  - 12-01-1960
=====

```

	coef	std err	z	P> z	[0.025	0.975]
const	0.0096	0.007	1.314	0.189	-0.005	0.024
ma.L1.D.#Passengers	0.2019	0.120	1.688	0.091	-0.033	0.436
ma.L2.D.#Passengers	-0.3409	0.188	-1.814	0.070	-0.709	0.027

```

                    Roots
=====

```

	Real	Imaginary	Modulus	Frequency
MA.1	-1.4419	+0.0000j	1.4419	0.5000
MA.2	2.0342	+0.0000j	2.0342	0.0000

```

=====

```

Here we can see that p values for MA L1 and MA L2 values are more than 0.05. Therefore they are insignificant. The value for AIC and BIC values are -240.379 and -228.528 respectively.

Now we will combine AR and MA model into ARIMA model and see whether the RSS value has decreased or not. The model with the lowest RSS and AIC & BIC value will be used for predictions. We will also look at whether the all components are significant.

ARIMA Model

```
model_ar2ma = ARIMA(data_log, order=(2, 1, 2))
results_ARIMA = model_ar2ma.fit(dispatch=-1)
plt.plot(data_log_diff)
plt.plot(results_ARIMA.fittedvalues, color='red')
print(results_ARIMA.summary())
```

```

ARIMA Model Results
=====
Dep. Variable:      D.Passengers    No. Observations:      143
Model:              ARIMA(2, 1, 2)  Log Likelihood         149.640
Method:              css-mle        S.D. of innovations    0.084
Date:               Thu, 17 Sep 2020 AIC                        -287.281
Time:               00:02:41        BIC                     -269.504
Sample:             02-01-1949      HQIC                    -280.057
                  - 12-01-1960

=====
              coef    std err          z      P>|z|      [0.025    0.975]
-----
const                0.0096     0.003     3.697     0.000     0.005     0.015
ar.L1.D.Passengers    1.6293     0.039    41.868     0.000     1.553     1.706
ar.L2.D.Passengers   -0.8946     0.039   -23.127     0.000    -0.970    -0.819
ma.L1.D.Passengers   -1.8270     0.036   -51.303     0.000    -1.897    -1.757
ma.L2.D.Passengers    0.9245     0.036    25.568     0.000     0.854     0.995

Roots
=====
              Real      Imaginary      Modulus      Frequency
-----
AR.1          0.9106      -0.5372j      1.0573      -0.0848
AR.2          0.9106      +0.5372j      1.0573       0.0848
MA.1          0.9881      -0.3245j      1.0400      -0.0505
MA.2          0.9881      +0.3245j      1.0400       0.0505
=====

```

- In this case we observe that all our AR and MA components are significant.
- Hence we can consider this as a best fit model.

ARIMA Model	AIC	BIC
(1,1,1)	-241.6	-229.8
(1,1,2)	-265.2	-250.4
(2,1,1)	-270.2	-255.3
(2,1,2)	-287.3	-269.5

Ljung Box test

H0: The model does not show lack of fit

H1: The model exhibits lack of fit

Since the $p\text{-value} < 0.05$. We reject the null hypothesis which means this model exhibits lack of fit.

In order to overcome this we will apply auto arima on our dataset to get a best fit model.

	lb_stat	lb_pvalue
1	0.008330	9.272774e-01
2	5.455342	6.537138e-02
3	5.671274	1.287459e-01
4	11.161776	2.480479e-02
5	13.539662	1.881372e-02
6	23.208052	7.297086e-04
7	23.650007	1.312438e-03
8	36.733352	1.288387e-05
9	38.322395	1.525618e-05
10	52.570518	8.945834e-08
11	54.051012	1.155342e-07
12	129.038668	9.623163e-22

Applying Auto ARIMA

Best model: ARIMA(3,1,3)(0,0,0)[0] intercept
Total fit time: 5.276 seconds

SARIMAX Results

```
=====
Dep. Variable:          y      No. Observations:          144
Model:                SARIMAX(3, 1, 3)  Log Likelihood        146.806
Date:                 Thu, 17 Sep 2020  AIC                 -277.612
Time:                 10:44:43    BIC                   -253.909
Sample:                0      HQIC                  -267.980
                        - 144
```

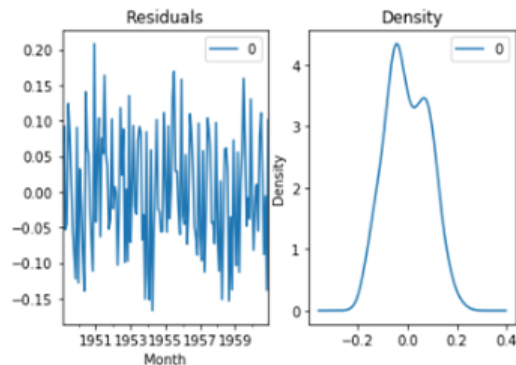
Covariance Type: opg

```
=====
              coef    std err          z      P>|z|      [0.025    0.975]
-----
intercept    0.0049    0.002     3.051     0.002     0.002     0.008
ar.L1        0.5566    0.149     3.724     0.000     0.264     0.849
ar.L2        0.5686    0.137     4.153     0.000     0.300     0.837
ar.L3       -0.6247    0.096    -6.506     0.000    -0.813    -0.437
ma.L1       -0.7073    0.172    -4.122     0.000    -1.044    -0.371
ma.L2       -0.9321    0.067   -14.005     0.000    -1.063    -0.802
ma.L3        0.6961    0.149     4.684     0.000     0.405     0.987
sigma2       0.0081    0.002     5.325     0.000     0.005     0.011
=====
```

```
=====
Ljung-Box (Q):                292.60   Jarque-Bera (JB):                5.93
Prob(Q):                      0.00     Prob(JB):                  0.05
Heteroskedasticity (H):        1.04     Skew:                      0.06
Prob(H) (two-sided):           0.88     Kurtosis:                  2.01
=====
```

Auto Arima model

```
# Plot residual errors
residuals1 = pd.DataFrame(results_ARIMA.resid)
fig, ax = plt.subplots(1,2)
residuals1.plot(title="Residuals", ax=ax[0])
residuals1.plot(kind='kde', title='Density', ax=ax[1])
plt.show()
```



```
acorr_ljungbox(residuals1, lags = 12)

(array([ 0.29499887,  0.53242258,  0.69969247,  2.68826059,  3.81646287,
        11.27093801, 11.52508214, 18.89257608, 19.33691629, 24.91524491,
        26.43116691, 91.63987923]),
 array([5.87034800e-01, 7.66277206e-01, 8.73276280e-01, 6.11270416e-01,
        5.76131853e-01, 8.03551183e-02, 1.17298667e-01, 1.54448292e-02,
        2.24757836e-02, 5.50849239e-03, 5.59591104e-03, 2.37515033e-14]))
```