UNIVERSITY OF MORATUWA
Faculty of Information Technology
Department of Computational Mathematics

Honours Degree of Bachelor of Science in Information Technology & Management
Level 2 - Semester 2 - Examination

CM 2130 – STATISTICAL DISTRIBUTIONS AND ESTIMATION

Time Allowed: 3 hours

April 2015

INSTRUCTIONS TO CANDIDATES

1. This paper contains 5 questions on 4 pages (including the cover page).
2. The total marks obtainable for this examination is 100. The marks assigned for each question & sections thereof are included in square brackets.
3. This examination accounts for 60% of the module assessment.
4. This is a closed-book examination.
5. Answer **ALL** questions.
6. Start to answer a new question on a new page.
7. **All the necessary steps** for the answers should be **clearly indicated**.
8. Rough work sheets may be attached separately.
9. Calculators are **ALLOWED**.

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Question 1 [*Total Marks allocated: 20 Marks*]

XYZ company introduced a computer assisted ordering system for the easy use of their customers. The number of orders received per hour varies from 10 to 16 with the following probabilities:

x = number of orders	10	11	12	13	14	15
P(X=x)	0.08	0.15	0.1	0.2	0.1	0.07

(a) Let X be the number of orders received per hour. Find the probability mass function of X. [*4 Marks*]

(b) Determine the cumulative distribution function of X. [*4 Marks*]

(c) Determine the Mean and Standard Deviation of the number of messages received per hour. [*6 Marks*]

(d) Express $P(11.5 \leq X \leq 13.5)$ in terms of $F_X(x)$ [3 *Marks*]

(e) Using the results in part (d), Calculate $P(11.5 \leq X \leq 13.5)$ [3 *Marks*]

Question 2 [*Total Marks allocated: 15 Marks*]

Contamination is a problem in the manufacture of magnetic storage disks. Assume that the number of particles of contamination that occur on a disk surface has a Poisson distribution, and the average number of particles per square centimetre of media surface is 0.01. The area of a disk under study is 100 square centimetres.

(a) Determine the probability that 12 particles occur in the area of a disk under study. [*7 Marks*]

(b) The disk operates only if there are no particles present on its surface. Find the probability that the disk operates without causing any problem. [*8 Marks*]

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Question 3 [*Total Marks allocated: 15 Marks*]

The fill volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 fluid ounces and a standard deviation of 0.1 fluid ounces.

(a) What is the probability that a fill volume is less than 12 fluid ounces? [*6 Marks*]

(b) If all cans which are less than 12.1 ounces or more than 12.6 ounces are scrapped, what proportion of cans is scrapped? [*6 Marks*]

(c) Write down the rule you applied (if any) to find the above probabilities. [*3 Marks*]

Question 4 [*Total Marks allocated: 20 Marks*]

(a) **Briefly** explain the difference between each pair of the followings:

(i) Estimator and Estimate [*2 Marks*]

(ii) Point estimation and Interval estimation [*2 Marks*]

(b) *The number* of lions seen on a *1-day safa*ri seems to have a Poisson distribution with the parameter λ. Let $X_1, X_2, \ldots, X_n$ be the numbers of lions seen by the tourists on n randomly selected days.

(i) Derive two moment estimators for λ. [*10 Marks*]

(ii) If {1, 5, 4, 3, 2, 5, 4, 2, 4, 2} are 10 such observations,

(1) Estimate λ using two estimators which you found in (b) (i). [*4 Marks*]

(2) What can you say about the average number of lions seen on that 1-day safari? [*2 Marks*]

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Question 5 [*Total Marks allocated: 30 Marks*]

Let $X_1, X_2, \ldots, X_n$ be a random sample from a Bernoulli population with parameter θ. θ is the probability of success.

(a) Obtain a maximum likelihood estimator for θ.

[*10 Marks*]

(b) Show that maximum likelihood estimator which you found above is unbiased.

[*5 Marks*]

(c) Find the maximum likelihood estimator of the variance of the above mentioned population.

[*5 Marks*]

(d) Tossing a coin 10 times and equating heads to value 1 and tails to value 0, we obtained the following values:

0 1 1 0 1 0 1 1 1 0

(1) Obtain a maximum likelihood estimate for the probability of getting head (probability of success).

[*5 Marks*]

(2) What is the probability of exactly obtaining 9 heads out of 10 tosses of this coin?

[*5 Marks*]

End of Paper