

# CSC 503 Homework Assignment 4

Out: September 9, 2015

Due: September 16, 2015

MISSING-ID

Unless directed otherwise, follow the convention of the text and assume that  $a, b, c, d, e$  are constant symbols,  $f, g, h$  are function symbols, and  $w, u, v, x, y, z$  are variable symbols.

## 1. Use the predicates

$C(x, y)$  :  $x$  is a champion of  $y$   
 $F(x, y)$  :  $x$  is a fan of  $y$   
 $Q(x, y)$  :  $x$  is the quarterback of  $y$   
 $R(x, y)$  :  $x$  is a rival of  $y$   
 $S(x, y)$  :  $x$  is the sister of  $y$   
 $T(x)$  :  $x$  is a team

and the constant (nullary function) symbols

$s$  : Serena  
 $t$  : Tom

to translate the following English sentences into predicate logic. You are not allowed to use any predicate, function, or constant symbols other than the above.

- (a) [5 points] Serena is a champion.
  - (b) [5 points] Any team that has Serena for a quarterback has Tom for a fan.
  - (c) [5 points] Tom is a fan of every champion.
  - (d) [5 points] Tom is a fan of Tom.
  - (e) [5 points] Every team has a fan.
  - (f) [5 points] All champions are rivals.
  - (g) [5 points] Only teams have rivals.
  - (h) [5 points] All rivals are teams that have Tom for a quarterback.
  - (i) [5 points] Some sister of some champion is a champion.
  - (j) [5 points] Every sister of every champion is a champion.
2. Let  $c$  and  $d$  be constants,  $f$  a function symbol with two arguments,  $g$  a function symbol with three arguments,  $h$  a function symbol with one argument,  $P$  a predicate symbol with two arguments, and  $Q$  a predicate symbol with three arguments. Indicate, for each of the following strings, which strings are formulas in predicate logic, and specify a reason for failure for strings which are not.
- (a) [5 points]  $\forall x Q(f(d, y), g(h(c, x), d, y), x)$
  - (b) [5 points]  $\forall x P(x, c) \vee g(f(d, x), h(y), y)$
  - (c) [5 points]  $\forall x (Q(z, z, z) \rightarrow P(h(P(z, z)), z))$
  - (d) [5 points]  $Q(h(h(h(c))), d, \neg f(d, d)) \rightarrow P(c, c)$
  - (e) [5 points]  $\forall x \forall y \exists z P(c, d, c)$
3. Let  $P$  be a predicate symbol with arity 2, and let  $\phi$  be the formula

$$\forall y [(\neg P(y, x) \vee P(y, z)) \wedge \exists y \forall z P(y, z)]$$

- (a) [5 points] Indicate, for each occurrence of each variable in  $\phi$ , whether that occurrence is free or bound.

- (b) [5 points] List all variables which occur both free and bound in  $\phi$ .
- (c) [5 points] Compute  $\phi[t/x]$  for  $t = g(f(g(y, y)), z)$ . Is  $t$  free for  $x$  in  $\phi$ ?
- (d) [5 points] Compute  $\phi[t/y]$  for  $t = g(f(g(y, y)), z)$  Is  $t$  free for  $y$  in  $\phi$ ?
- (e) [5 points] Compute  $\phi[t/z]$  for  $t = g(f(g(y, y)), z)$  Is  $t$  free for  $z$  in  $\phi$ ?