CSC 503 Homework Assignment 10

Out: October 14, 2015

Due: October 21, 2015

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1. M2, s2 $\models r \rightarrow q$
   Solution:
   In s2, r is true but q is false. Since True → False is False. Thus r → q is False. So does not hold.
2. M2, s1 $\models r \rightarrow q$
   Solution:
   In s1, r is False and q is true. Since False→ True is true. Thus r → q is true. It holds.
3. M2, s3 $\models EF\neg t$
   EF¬t means that there exists at least one path where ¬t becomes true in future starting from s3. One such path is s3, $(s4)^{\infty}$. Hence, given entailment holds.
4. M2, s1 $\models EF\neg t$
   Solution:
   EF¬t means that there exists at least one path where ¬t becomes true in future starting from s1. One such path is s1,s2,$(s4)^{\infty}$. Hence, given entailment holds.
5. M2, s1 $\models \neg EGr$
   Solution:
   ¬EGr means that there does not exists any path from s1 where r is always true for the entire path. In s1 r is false so any path starting from s1 can't have r as always True. Thus entailment holds.
6. M2, s3 $\models \neg EGr$
   Solution:
   ¬EGr means that there does not exists any path from s3 where r is always true for the entire path. In path s3, $(s3)^{\infty}$, r is always true. Thus entailment does not hold.

7. M2, s3 $\models EGAFq$
   Starting from state s3 there exists a path s3,$(s3)^{\infty}$ where q is never True in future. Thus AFq is false. Hence EGAFq is false and entailment does not hold.

8. M2, s1 $\models EGAFq$
   Solution: This means there exists a path starting from s1 where globally AFq is True.
   AFq means q becomes True sometime on all the path starting from s1.
   For all paths starting from s1 as all paths it would have s2 or s3 in next state.
   AFq is true for s2. Both paths that start from s2 will have q True sometime in its future states.
   AFq is false for s3 since for path s3,$(s3)^{\infty}$ q is never True in future (same as in question 7)

   Since we have a path from s2 where AFq is true Ex- s1, s2, $(s4)^{\infty}$

   This means there exists a path starting from s1 where globally AFq is True. Hence, the given entailment holds.

9. M2, s4 ⊨ AXAG(q ∨ t)
   Solution:
   q is true for s4 and so (q ∨ t) also becomes true. There is only one path S4, $(s4)^{\infty}$ that starts from s4. Thus AG(q ∨ t) holds i.e for all paths starting from s4, G(q ∨ t) is true. Since next state is s4 only for all paths starting from s4, XAG(q ∨ t) is True for all paths. Thus AXAG(q ∨ t) is True.
   Hence the entailment holds.
10. M2, s2 ⊨ AXAG(q ∨ t)
    Solution:
    AXAG(q ∨ t) means that for all paths starting from s2, AG(q ∨ t) should be True in the next state.
    For all paths starting from s2, next state is either s1 or s4.
    AG(q ∨ t) is true for s4 because, future states of s4 is s4 only(only paths starting from s4 is S4, $(s4)^{\infty}$) so next state for all paths is s4 where(q ∨ t) is True.
    AG(q ∨ t) is false for s1 because there exists path for example s1, s2, $(s4)^{\infty}$ in which AG(q ∨ t) is False.
    Now, for the next states of s2, AG(q ∨ t) is true for one and false for another.
    Hence, given entailment does not hold.

11. Given, CTL formula.
    E[¬EXq U (AX (p → q) ∧ A [A (p U q) U EGp])]
    The Subformulas for above CTL formulas are:

    - E[¬EXq U (AX (p → q) ∧ A [A (p U q) U EGp])]
    - ¬EXq
    - EXq
    - q
    - AX (p → q) ∧ A [A (p U q) U EGp]
    - AX (p → q)
    - p → q
    - p
    - A [A (p U q) U EGp]
    - A (p U q)
    - EGp