CSC 503 Homework Assignment 4

Out: September 9, 2015
Due: September 16, 2015
rsandil

Unless directed otherwise, follow the convention of the text and assume that a, b, c, d, e are constant symbols, f, g, h are function symbols, and w, u, v, x, y, z are variable symbols.

1. Use the predicates

$$\begin{aligned} C(x,y): &\quad x \text{ is a champion of } y \\ F(x,y): &\quad x \text{ is a fan of } y \\ Q(x,y): &\quad x \text{ is the quarterback of } y \\ R(x,y): &\quad x \text{ is a rival of } y \\ S(x,y): &\quad x \text{ is the sister of } y \\ T(x): &\quad x \text{ is a team} \end{aligned}$$

and the constant (nullary function) symbols

$$\begin{aligned} s: &\quad \text{Serena} \\ t: &\quad \text{Tom} \end{aligned}$$

to translate the following English sentences into predicate logic. You are not allowed to use any predicate, function, or constant symbols other than the above.

(a) [5 points] Serena is a champion.
Answer

$$\exists x(C(s,x))$$

(b) [5 points] Any team that has Serena for a quarterback has Tom for a fan.
Answer

$$\forall x((T(x) \land Q(s,x)) \to F(t,x))$$

(c) [5 points] Tom is a fan of every champion.
Answer

$$\forall x \exists y(C(x,y) \to F(t,x))$$

(d) [5 points] Tom is a fan of Tom.
Answer

$$F(t,t)$$

(e) [5 points] Every team has a fan.
Answer

$$\forall x(T(x) \to \exists y(F(y,x)))$$

(f) [5 points] All champions are rivals.
Answer

$$\forall x \forall y(C(x,y) \to \exists w(R(x,w)))$$

(g) [5 points] Only teams have rivals.
Answer

$$\forall x \forall y (R(x,y) \to (T(y))$$

(h) [5 points] All rivals are teams that have Tom for a quarterback.
Answer

$$\forall x \forall y ((R(x,y) \to (T(x) \wedge Q(t,x)))$$

(i) [5 points] Some sister of some champion is a champion.
Answer

$$\exists x \exists y ((S(x,y) \wedge \exists z (C(y,z)) \wedge \exists w (C(x,w)))$$

(j) [5 points] Every sister of every champion is a champion.
Answer

$$\forall x \forall y ((S(x,y) \wedge \exists z (C(y,z)) \wedge \exists w (C(x,w)))$$

2. Let c and d be constants, f a function symbol with two arguments, g a function symbol with three arguments, h a function symbol with one argument, P a predicate symbol with two arguments, and Q a predicate symbol with three arguments. Indicate, for each of the following strings, which strings are formulas in predicate logic, and specify a reason for failure for strings which are not.

(a) [5 points] $\forall x\ Q(f(d,y), g(h(c,x), d, y), x)$
Answer
This formula is **invalid** as function h has arity of one but here it has been incorrectly used to accept two arguments.

(b) [5 points] $\forall x\ P(x,c) \vee g(f(d,x), h(y), y)$
Answer
This formula is a **invalid** since left hand side of logical connective $\vee$ is a formula whereas right hand side is a term.

(c) [5 points] $\forall x (Q(z,z,z) \to P(h(P(z,z)), z))$
Answer
This formula is **invalid** as $P(z,z)$ is not a term and so all the parameters to the function h are not terms.

(d) [5 points] $Q(h(h(h(c))), d, \neg f(d,d)) \to P(c,c)$
Answer
This formula is a **invalid** formula in predicate logic as negation of $f(d,d)$ is not a term

(e) [5 points] $\forall x \forall y \exists z\ P(c,d,c)$
Answer
This formula is **invalid** formula in predicate logic since the predicate P is incorrectly used to accept three arguments. Predicate P accepts only two arguments.

3. Let P be a predicate symbol with arity 2, and let ϕ be the formula

$$\forall y\ [(\neg P(y,x) \vee P(y,z)) \wedge \exists y \forall z\ P(y,z)]$$

(a) [5 points] Indicate, for each occurrence of each variable in ϕ, whether that occurrence is free or bound.
Answer
The variables in ϕ include x, y and z. The highlighted variables in ϕ are free.

$$\forall y\ [(\neg P(y,\mathbf{x}) \vee P(y,\mathbf{z})) \wedge \exists y \forall z\ P(y,z)]$$

The rest of the variables are bounded.

(b) [5 points] List all variables which occur both free and bound in ϕ.

Answer

The variable z in ϕ are both free and bound. z is bounded by $\exists y \forall z\ P(y, z)$ on right hand side. z is free left hand side.

(c) [5 points] Compute $\phi[t/x]$ for $t = g(f(g(y, y)), z)$. Is t free for x in ϕ?

Answer

t is not free for x in ϕ as the free instance of x on replacement with t will be become bounded as t is function of y.Thus $\phi[t/x]$ will remain ϕ.

(d) [5 points] Compute $\phi[t/y]$ for $t = g(f(g(y, y)), z)$ Is t free for y in ϕ?

Answer

t is not free for y in ϕ as there is no free occurrence of y in ϕ to be replaced by t. Thus $\phi[t/x]$ will remain ϕ.

(e) [5 points] Compute $\phi[t/z]$ for $t = g(f(g(y, y)), z)$ Is t free for z in ϕ?

Answer

t is not free for z in ϕ as the free instance of z on replacement with t will become bounded since t is a function of y .Thus $\phi[t/z]$ will remain ϕ.