CSC 503 Homework Assignment 5

Out: September 11, 2015
Due: September 18, 2015
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In using the Fitch macros to typeset proofs in first order logic, one introduces a dummy variable x by means of the command `\open[x]`.

1. [10 points] Using only the basic natural deduction rules, find a proof for

$$\forall x(P(x) \to Q(x)) \vdash \forall x P(x) \to \forall x Q(x).$$

Answer

1	$\forall x(P(x) \to Q(x)))$	premise
2	$\quad \forall x(P(x)$	assumption
3	$\quad x_0 \quad (P(x_0) \to Q(x_0))$	$\forall$e, 1
4	$\qquad P(x_0)$	$\forall$e, 2
5	$\qquad Q(x_0)$	$\to$e, 3, 4
6	$\quad \forall x Q(x)$	$\forall$i, 3–5
7	$\forall x P(x) \to \forall x Q(x)$	$\to$i, 2–6

2. [20 points] Using only the basic natural deduction rules, find a proof for

$$\forall x \forall y P(x,y) \vdash \forall u \forall v P(u,v).$$

Answer

1	$\forall x \forall y P(x,y)$	premise
2	$x_0 \quad \forall y P(x_0,y)$	$\forall$e, 1
3	$\quad y_0 \quad P(x_0,y_0)$	$\forall$e, 2
4	$\quad \forall v P(x_0,v)$	$\forall$i, 3
5	$\forall u \forall v P(u,v)$	$\forall$i, 2–4

3. [35 points] Using only the basic natural deduction rules, find a proof for

$$\exists x(\neg P(x) \lor Q(x)) \vdash \exists x \neg(P(x) \land \neg Q(x)).$$

Answer

1		$\exists x(\neg P(x) \lor Q(x))$	premise
2	x_0	$\neg P(x_0) \lor Q(x_0)$	$\exists$e, 1
3		$P(x_0) \land \neg Q(x_0)$	assumption
4		$\neg P(x_0)$	assumption
5		$P(x_0)$	$\land$e$_1$, 3
6		$\bot$	$\neg$e, 5, 4
7		$Q(x_0)$	assumption
8		$\neg Q(x_0)$	$\land$e$_2$, 3
9		$\bot$	$\neg$e, 8, 7
10		$\neg(P(x_0) \land \neg Q(x_0))$	$\neg$i, 3, 4–6, 7–9
11		$\exists x \neg(P(x_0) \land \neg Q(x_0))$	$\exists$i, 2–10

4. [35 points] Using only the basic natural deduction rules, find a proof for

$$\forall x \forall y \forall z[S(x,y) \land S(y,z) \to S(x,z)], \forall x \neg S(x,x) \vdash \forall x \forall y[S(x,y) \to (\neg S(y,x) \lor S(x,x))]$$

Answer

1			$\forall x \forall y \forall z[S(x,y) \land S(y,z) \to S(x,z)]$	premise
2			$\forall x \neg S(x,x)$	premise
3	x_0		$\neg S(x_0,x_0)$	$\forall$e, 2
4			$\forall y \forall z[S(x_0,y) \land S(y,z) \to S(x_0,z)]$	$\forall$e, 1
5			$\forall y[S(x_0,y) \land S(y,x_0) \to S(x_0,x_0)]$	$\forall$e, 4
6		y_0	$S(x_0,y_0) \land S(y_0,x_0) \to S(x_0,x_0)$	$\forall$e, 5
7			$S(x_0,y_0)$	assumption
8			$S(y_0,x_0)$	assumption
9			$S(x_0,y_0) \land S(y_0,x_0)$	$\land$i, 7, 8
10			$S(x_0,x_0)$	$\to$e, 6, 9
11			$\bot$	$\neg$i, 3, 10
12			$\neg S(y_0,x_0)$	$\neg$i, 8–11
13			$\neg S(y_0,x_0) \lor S(x_0,x_0)$	$\lor$i, 12
14			$S(x_0,y_0) \to \neg S(y_0,x_0) \lor S(x_0,x_0)$	$\to$i, 7–13
15			$\forall y[S(x_0,y) \to \neg S(y,x_0) \lor S(x_0,x_0)]$	$\forall$i, 6–14
16			$\forall x \forall y[S(x,y) \to \neg S(y,x) \lor S(x,x)]$	$\forall$i, 3–15