CSC 503 Homework Assignment 13

Out: November 6, 2015
Due: November 13, 2015

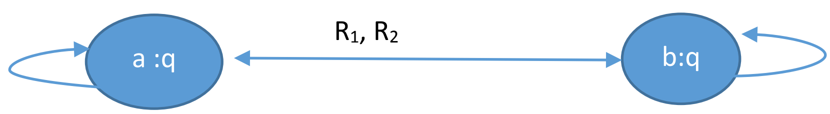
rsandil

Q1. [30 points total] Let ϕ be the sentence $E_Gp \rightarrow E_GE_Gp$, where p is an atomic proposition and G is the set {1, 2}.

(a) [10 points]: Give a formal $KT45^2$ (two-agent) Kripke model I = (W, R_1, R_2, L) in which ϕ is true in every world.

Solution:

Consider the model M = $\{W, R_1, R_2, L\}$ with the following representations. The Kripke modal frame diagram is:



W = {a, b} is the set of worlds
R_1 = {(a, b),(b, a)} is accessibility function for agent K_1
R_2 = {(a, b),(b, a)} is accessibility function for agent K_2
L = {L(a) = q, L(b) = q} is the set of labelling functions

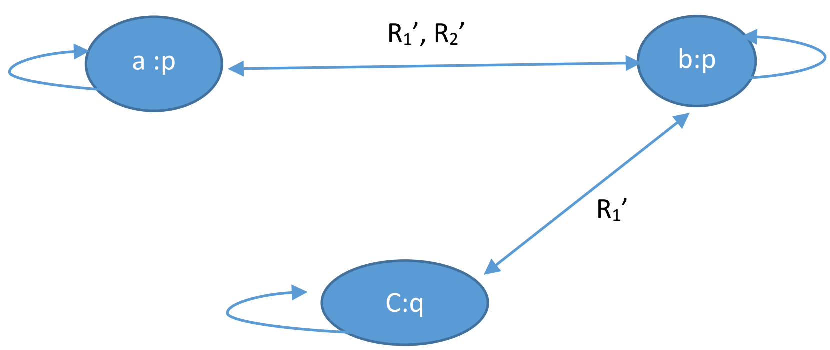
(b) [5 points]: Briefly explain why I makes ϕ true at every world

Solution:

The above kripke diagram satisfies the $KT45^2$ since we have self-loop on both the worlds which satisfies the reflexive property. Also the relations are symmetric since the link is bi-directional. Since both the world a and b satisfies $\Diamond q \rightarrow \Box\Diamond q$ thus Euclidean property is also satisfied. Thus $KT45^2$ model is validated. Interpretation I makes true for every world since LHS of the implication E_Gp is always false for the above interpretation making $E_Gp \rightarrow E_GE_Gp$ always True. This is so because World '**a**' does not force k_1p or k_2p, similarly world **'b'** does not force k_1p or k_2p thus E_Gp is false for both the worlds making $E_Gp \rightarrow E_GE_Gp$ True for all the worlds.

(c) [10 points]: Give a formal $KT45^2$ (two-agent) Kripke model J = (W' , R1' , R2' , L') in which ϕ is false at some world.

Solution:



$W = \{a, b, c\}$ is the set of worlds
$R_1' = \{(a, b),(b, a), \{b, c\}, \{c, b\}\}$ is accessibility function for agent K_1
$R_2' = \{(a, b),(b, a)\}$ is accessibility function for agent K_2
$L = \{L(a) = p, L(b) = p, L(c) = q \}$ is the set of labelling functions

d) Briefly explain why J makes ϕ false at some world.

Solution: The above kripke diagram satisfies the $KT45^2$ since we have self-loop on all the worlds which satisfies the reflexive property. Also the relations are symmetric since the link is bi-directional. Also the Euclidean property is satisfied validating the $KT45^2$ model. In the world 'a', E_Gp is true since 'a' forces k_1p and k_2p. 'a' forces k_2p (since 'a' is accessible by 'b' through relation R1' and 'b' forces p). E_GE_Gp is false in 'a' because 'b' does not force E_Gp. it is because 'b' is accessible by a world 'c' which does not force p. Thus LHS is True and RHS is false for $E_Gp \to E_GE_Gp$ making it false overall. Thus Interpretation J makes ϕ false at some world.

Q2. [30 points] Find a natural deduction proof using basic or derived rules in the modal logic $KT45^n$ for the statement $\neg p \to K_2\neg K_2K_1p$

Solution:

We prove the validity of given expression by:

1	$\neg p$	assumption
2	K_2K_1p	assumption
3	K_1p	KT, 2
4	P	KT, 3
5	$\perp$	¬e, 4, 1
6	$\neg K_2K_1p$	¬i, 2–5
7	$K_2\neg K_2K_1p$	K5, 6
8	$\neg p \to \neg K_2K_1p$	→i, 1–7

Q3. [40 points] Find a natural deduction proof using basic or derived rules in the modal logic $KT45^n$ for the sequent $C(\neg p \to q)$, $C(\neg p \to K_2\neg p)$, $K_1\neg K_2 q$ `$\vdash K_1 p$.

Solution:

1		$C(\neg p \to q)$	Premise
2		$C(\neg p \to K_2\neg p)$	Premise
3		$K_1\neg K_2 q$	Premise
4		$K_1(\neg p \to q)$	CK, 1
5		$K_1(\neg p \to K_2\neg p$	CK, 2
6	K_1	$\neg K_2 q$	Ke, 3
7		$(\neg p \to q)$	Ke, 4
8		$(\neg p \to K_2\neg p)$	Ke, 5
9		$\neg p$	assumption
10.		$K_2\neg p$	$\to$e, 9, 8
11	K_2	$\neg p$	Ke, 10
12		q	$\to$e, 11, 7
13		$K_2 q$	Ki, 11–12
14		$\bot$	$\neg$e, 13, 6
15		$\neg\neg p$	$\neg$i, 9–14
16		p	$\neg\neg$e, 15
17		$K_1 p$	Ki, 6–16