CSC 503 Homework Assignment 2

Out: August 26, 2015
Due: September 2, 2015
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Prove the validity of the following sequents. Use only the basic rules of natural deduction (no derived rules).

1. [10 points] $p \rightarrow (q \rightarrow r), p \rightarrow q \vdash p \rightarrow r$

   Answer:

   Answer

1	$p \rightarrow (q \rightarrow r)$	premise
2	$p \rightarrow q$	premise
3	$\quad p$	assumption
4	$\quad q$	$\rightarrow$e, 2, 3
5	$\quad q \rightarrow r$	$\rightarrow$e, 1, 3
6	$\quad r$	$\rightarrow$e, 4, 5
7	$p \rightarrow r$	$\rightarrow$i, 3, 6

2. [20 points] $(p \vee (q \rightarrow p)) \wedge q \vdash p \vee r$

   Answer

1	$(p \vee (q \rightarrow p)) \wedge q$	premise
2	q	$\wedge$e$_2$, 1
3	$(p \vee (q \rightarrow p))$	$\wedge$e$_1$, 1
4	$\quad p$	assumption
5	$\quad (q \rightarrow p)$	assumption
6	$\quad p$	$\rightarrow$e, 5, 2
7	p	$\vee$e, 3, 4–4, 5–6
8	$p \vee r$	$\vee$i, 7

3. [30 points] $(p \wedge q) \rightarrow r, r \rightarrow s, q \wedge \neg s \vdash \neg p$

   Answer

1	$(p \wedge q) \rightarrow r$	premise
2	$r \rightarrow s$	premise
3	$q \wedge \neg s$	premise
4	q	$\wedge e_1, 3$
5	$\neg s$	$\wedge e_2, 3$
6	p	assumption
7	$p \wedge q$	$\wedge i, 4, 6$
8	r	$\rightarrow e, 1, 7$
9	s	$\rightarrow e, 2, 8$
10	$\bot$	$\neg e, 6{-}9$
11	$\neg p$	$\neg i, 6{-}10$

4. [40 points] $(q \wedge u) \rightarrow t, r \wedge \neg s, r \wedge \neg (s \vee q) \rightarrow p \vdash (u \wedge \neg t) \rightarrow p$

Answer

1	$(q \wedge u) \to t$	premise
2	$r \wedge \neg s$	premise
3	$r \wedge \neg(s \vee q) \to p$	premise
4	r	$\wedge e_1, 2$
5	$\neg s$	$\wedge e_2, 2$
6	$u \wedge \neg t$	assumption
7	u	$\wedge e_1, 6$
8	$\neg t$	$\wedge e_2, 6$
9	q	assumption
10	$q \wedge u$	$\vee e, 9, 7$
11	t	$\to e, 1, 10$
12	$\bot$	$\neg e, 8\text{--}11$
13	$\neg q$	$\neg i, 9\text{--}12$
14	$s \vee q$	assumption
15	s	assumption
16	$\bot$	$\neg e, 5, 15$
17	q	assumption
18	$\bot$	$\neg e, 13, 17$
19	$\bot$	$\vee e, 14, 15\text{--}16, 17\text{--}18$
20	$\neg(s \vee q)$	$\neg i, 14\text{--}19$
21	$r \wedge (\neg(s \vee q)$	$\wedge i, 4, 20$
22	p	$\to e, 21, 3$
23	$(u \wedge \neg t) \to p$	$\to i, 6, 22$

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