CSC 503 Homework Assignment 12

Out: October 30, 2015

Due: November 6, 2015

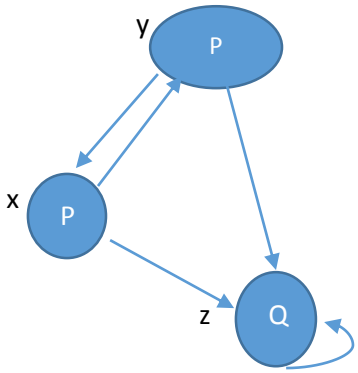
rsandil

Q1. 1. Consider the modal logic formula p → □◊q.

(a) [10 points] Construct a modal frame of interpretation M in which p → □◊q is valid.

Solution:

Consider the Frame (interpretation, model) M = (W, R, L) such that x, y and z are worlds of M.



p → □◊q is valid in above modal frame interpretation M.

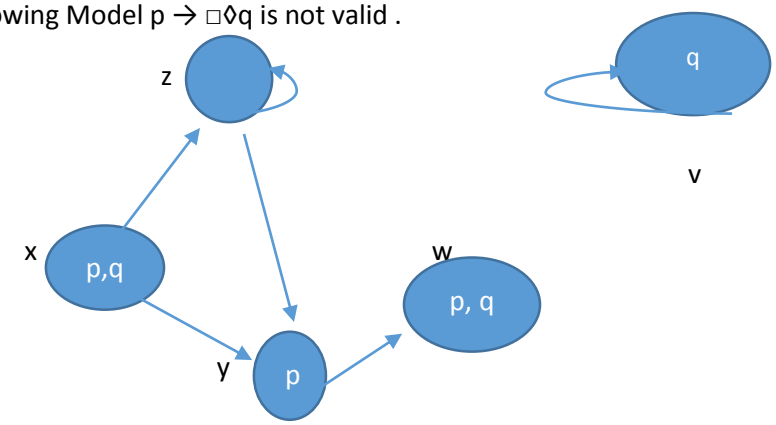
(b) Explain why p → □◊q is valid in M.

Solution:

To prove, p → □◊q as valid in frame M, we need to prove it is valid for all the worlds of M. In the above model all the worlds of M i.e x, y and z satisfies p → □◊q. The world z satisfies p → □◊q since z does not satisfy z ||- p (z ||- p means z forces p). For the world x and y, it satisfies x ||-p and y ||- p so we need to secure x ||- □◊q and y ||- □◊q. But this can be done since for x and y worlds, of this model, we have an immediate successor state a' such that a' ||- q.

c) [10 points] Present a formal modal frame of interpretation M' in which p → □◊q is not valid.

Solution: Consider the Frame (interpretation, model) M' = (W, R, L) such that x, y, z, w and v are worlds of M' In the following Model p → □◊q is not valid .



d) Explain why p → □◊q is not valid in M2

Solution: To prove p → □◊q as not valid in M2, we need to prove that p → □◊q is not valid in all the worlds of M2. Every world in M_2 does not satisfy p → □◊q. For example world x does not satisfy p → □◊q. Thus p → □◊q is not valid in M2

Q2. **Solution:**

We prove the validity of given expression by:

1	(□ (□p → q) ∧ □(q → □r))	assumption
2	□ (□p → q)	∧ e_1 1
3	□(q → □r))	∧ e_2 2
4	□p → q	□ e 2
5	q → □r	□ e 3
6	□p	assumption
7	q	→ e 4, 6
8	□r	→ e 5, 7
9	□p → □r	→ i 6-8
10	□(□p → □r)	□ i 4-9
11	(□ (□p → q) ∧ □(q → □r)) → □(□p → □r)	→ i 1-10

Q3 **Solution:**

We prove the validity of given expression by:

1	¬□¬T	assumption
2	□p	assumption
3	□¬p	Assumption
4.	P	□ e 2
5.	¬p	□ e 3
6	⊥	¬e 4,5
7	¬T	⊥ e, 6
8	□¬T	□ i 4-7
9	⊥	¬e 8,1
10	¬□¬p	¬i 3-9
11	□p → ¬□¬p	→ i 2-9
12	¬□¬T → (□p → ¬□¬p)	→ i 1-11

Note: ¬□¬T → (□p → ¬□¬p) is equivalent to ◊T → (□p → ◊p)