Q1.

a) Answer:

p - Jack ran up the hill.

q - Jill ran up the hill.

$(p \wedge q)$

b) Answer:

p - Real Madrid FC ran up the hill

p

c) Answer:

p - NC State has a red and white logo.

p

d) Answer:

P Jack fell down, q jack broke his crown.

$(\neg p) \rightarrow (\neg q)$

e) Answer:

P Jack fell down, q jack broke his crown, r Jill came tumbling after

$(p \wedge q) \wedge r$

Q2.

a) $((\neg p) \vee q) \rightarrow ((\neg(\neg q)) \wedge (\neg r))$

b) $r \rightarrow (((\neg q) \vee p) \rightarrow (q \rightarrow (\neg p) \vee r))$

Q3.

Let F be $((p \rightarrow \neg q) \vee (p \wedge s) \rightarrow r) \wedge \neg r$ and S(F) be the set of subformulas of F

Then

S(F) = $((p \rightarrow \neg q) \vee (p \wedge s) \rightarrow r) \wedge \neg r$, $(p \rightarrow \neg q) \vee (p \wedge s) \rightarrow r$, $(p \rightarrow \neg q) \vee (p \wedge s)$, $(p \rightarrow \neg q)$, $(p \wedge s)$,p, $\neg q$, s,$\neg r$, r

Q4.

The expression $p \vee q \wedge r$ is problematic because both $\wedge$ and $\vee$ have same connective precedence. This will result in conflict or disagreement if interpreted in different ways as follows:

First interpretation: $(p \vee q) \wedge r$

p	q	r	$(p \vee q)$	$(p \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

Second interpretation:
$p \vee (q \wedge r)$

p	q	r	$(q \wedge r)$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

Q5.
The complete truth table of $((p \rightarrow \neg q) \rightarrow (q \rightarrow p)$ can be demonstrated as:

p	q	$\neg q$	$p \rightarrow \neg q$	$q \rightarrow p$	$(p \rightarrow \neg q) \rightarrow (q \rightarrow p)$
T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	F	F	F
F	F	T	T	T	T

Q6.
The Truth table for the formula $(p \rightarrow q) \vee (\neg q \wedge \neg r)$ can be demonstrated as :

p	q	r	$p \rightarrow q$	$\neg q$	$\neg r$	$\neg q \wedge \neg r$	$(p \rightarrow q) \vee (\neg q \rightarrow \neg r)$
T	T	T	T	F	F	F	T
T	T	F	T	F	T	F	T
T	F	T	F	T	F	F	F
T	F	F	F	T	T	T	T
F	T	T	T	F	F	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	F	F	T
F	F	F	T	T	T	T	T

$(p \to q) \vee (\neg q \wedge \neg r)$, thus is satisfiable as some interpretation (here all except one) makes it true but is not valid since every interpretation does not make it true.

Q7.

The Truth table for $\neg p \to (r \vee q), \neg q \wedge p \models p \to q$ is:

p	q	r	$\neg p$	$\neg q$	$r \vee q$	$\neg p \to (r \vee q)$	$\neg q \wedge p$	$p \to q$
T	T	T	F	F	T	T	F	T
T	T	F	F	F	T	T	F	T
T	F	T	F	T	T	T	T	F
T	F	F	F	T	F	T	T	F
F	T	T	T	F	T	T	F	T
F	T	F	T	F	T	T	F	T
F	F	T	T	T	T	T	F	T
F	F	F	T	T	F	F	F	T

The entailment claim is not true due to the valuation highlighted in the table since the truth values of the formulas to the left are T and truth values of the formula to the right are F.

Q8.

The Truth table for $\models (p \vee q) \wedge (\neg q \vee r) \to (p \vee r)$ is:

p	q	r	$\neg q$	$p \vee q$	$\neg q \vee r$	$(p \vee q) \wedge (\neg q \vee r)$	$p \vee r$	$(p \vee q) \wedge (\neg q \vee r) \to (p \vee r)$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	F	T	T
T	F	T	F	T	T	T	T	T
T	F	F	T	T	T	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	T	T	F	F	F	T
F	F	T	F	F	T	F	T	T
F	F	F	T	F	T	F	F	T

From the truth table, it can be seen that the claim holds as it evaluates to T for every valuation.