CSC 503 Homework Assignment 3

Out: September 4, 2015
Due: September 11, 2015
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1. [20 points] Construct a formula ϕ in **DNF** to match the following truth table:

p	q	ϕ
T	T	T
T	F	F
F	T	T
F	F	F

Answer

From the above truth tables we can derive the DNF with following steps

(a) Lines with truth value as 'T' are $Line1$ and $Line3$.

(b) Thus ϕ can be represented in DNF by conjoining two lines $(Line1 \vee Line3)$. Except for these valuation there are no other valuation for ϕ as T.

(c) After representing each line in literals, formula becomes $\phi : (p \wedge q) \vee (\neg p \wedge q)$

Thus the DNF to match the given truth table is $(p \wedge q) \vee (\neg p \wedge q)$

2. [20 points] Construct a formula ϕ in **CNF** to match the following truth table:

p	q	r	ϕ
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

Answer

(a) Lines with truth value as 'F' are $Line2$, $Line3$, $Line4$ and $Line5$

(b) Thus ϕ can be represented in CNF as $(\neg Line2 \wedge \neg Line3 \wedge \neg Line4 \wedge \neg Line5)$ which means that except for these line values, other values evaluates to ϕ as 'T'

(c) Representing each row by the corresponding values of the literals, the formula is $\phi : (\neg(p \wedge q \wedge \neg r) \wedge \neg(p \wedge \neg q \wedge r) \wedge \neg(p \wedge \neg q \wedge \neg r) \wedge \neg(\neg p \wedge q \wedge r))$

(d) Using De'Morgans Law, the above formula can be reduced to $\phi : ((\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee \neg r))$

Thus the CNF for the given truth table is $((\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee \neg r))$

3. [30 points] Consider the atomic sentences

p = The cow jumped over the moon.
q = The little dog laughed.
r = The dish ran away with the spoon.

Using these, form the three complex statements

(a) If the cow jumped over the moon and the little dog laughed, then the dish ran away with the spoon.

(b) If the little dog laughed, then the cow jumped over the moon.

(c) If the dish ran away with the spoon, then the little dog laughed.

Show that these three complex statements are logically independent of each other by providing, for each of these three complex statements, a truth assignment to the atomic sentences that makes the complex statement false and the other complex statements true.

Answer

The three complex sentences can be represented as follows:

1. $(p \wedge q) \rightarrow r$

2. $q \rightarrow p$

3. $r \rightarrow q$

Proof for 1st complex sentence

To prove : if $(p \wedge q) \rightarrow r$ is false then both $q \rightarrow p$ and $r \rightarrow q$ are true

Let us assume that $(p \wedge q) \rightarrow r$ **is false**

This means $(p \wedge q)$ has to be 'T' and r has to be 'F'

Since $(p \wedge q)$ is 'T', it means p has to be 'T' and q has to be 'T'

Thus $q \rightarrow p$ **is True**

And $r \rightarrow q$ **is True**

Proof for 2nd complex sentence

To prove : if $q \rightarrow p$ is false then both $(p \wedge q) \rightarrow r$ and $r \rightarrow q$ are true

Let us assume that $q \rightarrow p$ **is false**

This means q has to be 'T' and p has to be 'F'

Thus $(p \wedge q)$ has to be 'F'

Now Since $(p \wedge q)$ is 'F', we can conclude that $(p \wedge q) \rightarrow r$ **is True**

Also since q is 'T', we can say that $r \rightarrow q$ **is True**

Proof for 3rd complex sentence

To prove : if $r \rightarrow q$ is false then both $(p \wedge q) \rightarrow r$ and $q \rightarrow p$ are true

Let us assume that $r \rightarrow q$ **is false**

This means that r has to be 'T' and q has to be 'F'

Now since r is 'T', we can say that $(p \wedge q) \rightarrow r$ **is True**

Also since q is 'F', we can say that $q \rightarrow p$ **is True**

Thus we can say that the three complex sentences are logically independent of each other.

4. [30 points] Apply algorithm HORN from page 66 of the textbook to the following Horn formula.

$$
\begin{array}{ll}
(\top \rightarrow q) & \wedge \\
(\top \rightarrow s) & \wedge \\
(w \rightarrow \bot) & \wedge \\
(p \wedge q \wedge s \rightarrow \bot) & \wedge \\
(v \rightarrow s) & \wedge \\
(\top \rightarrow r) & \wedge \\
(r \rightarrow p) & \wedge \\
(p \wedge s \rightarrow s) &
\end{array}
$$

Your answer should list propositional letters in the order in which they are marked as well as giving the overall answer.

Answer

(a) Mark all occurances of $\top$.

(b) Mark q, s, r from $(\top \to q), (\top \to s), (\top \to r)$

$$
\begin{array}{ll}
(\top \to \mathbf{q}) & \wedge \\
(\top \to \mathbf{s}) & \wedge \\
(w \to \bot) & \wedge \\
(p \wedge \mathbf{q} \wedge \mathbf{s} \to \bot) & \wedge \\
(v \to \mathbf{s}) & \wedge \\
(\top \to \mathbf{r}) & \wedge \\
(\mathbf{r} \to p) & \wedge \\
(p \wedge \mathbf{s} \to \mathbf{s}) &
\end{array}
$$

(c) Mark p from $(r \to p)$

$$
\begin{array}{ll}
(\top \to \mathbf{q}) & \wedge \\
(\top \to \mathbf{s}) & \wedge \\
(w \to \bot) & \wedge \\
(\mathbf{p} \wedge \mathbf{q} \wedge \mathbf{s} \to \bot) & \wedge \\
(v \to \mathbf{s}) & \wedge \\
(\top \to \mathbf{r}) & \wedge \\
(\mathbf{r} \to \mathbf{p}) & \wedge \\
(\mathbf{p} \wedge \mathbf{s} \to \mathbf{s}) &
\end{array}
$$

(d) Mark $\bot$ from $(\mathbf{p} \wedge \mathbf{q} \wedge \mathbf{s} \to \bot)$

The order of propositional letters is $q, s, r, p, \bot$. Since we have marked $\bot$, the horn formula is **'Unsatisfiable'**.