CSC 503 Homework Assignment 9

Out: October 12, 2015
Due: October 19, 2015
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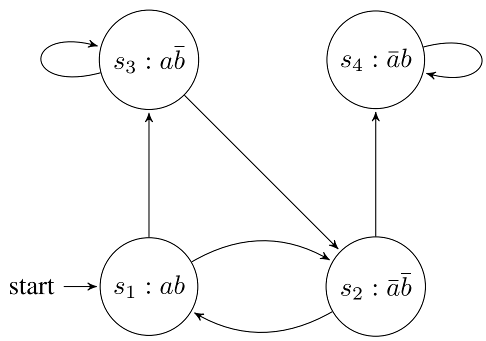
Consider the transition model $\mathcal{M}_1$ depicted in Figure 1.

Figure 1: Model $\mathcal{M}_1$



In answering the following questions, recall that all paths are infinite. To indicate a path that ends with a repeated set of states, put parenthesis around the repeated subsequence (e.g., $(s_1, s_2)^\infty$. To indicate a path in which the initial subsequence s_1, s_2 is followed by any possible continuing path, write "s_1, s_2, (any)".

1. **[8 points]** Find a path from the initial state s_1 which satisfies $\mathsf{G}a$.

   Answer

   There are multiple paths which satisfy $\mathsf{G}a$.
   Path: $s_1 - (s_3)^\infty$
   Explanation: Starting from state s_1 and then repeating on state s_3 infinitely keeps the literal a always true. Hence this path satisfies $\mathsf{G}a$

2. **[8 points]** Determine whether $\mathcal{M}_1, s_1 \models \mathsf{G}a$ and explain why or why not.

   Answer

   $\mathcal{M}_1, s_1 \models \mathsf{G}a$ is False.
   Explanation:$\mathcal{M}_1, s_1 \models \mathsf{G}a$ means that for every path in the model literal 'a' is always true. This is not the case in the given model. Considering the path $(s_1 s_3 s_2)^\infty$, which is loop, we can see that 'a' is true in all the states but not in s_2

3. **[8 points]** Find a path from the initial state s_1 which satisfies $b \; \mathsf{U} \; a$.

   Answer

   There are multiple paths that satisfy $b \; \mathsf{U} \; a$.
   Path: $s_1 - (s_3)^\infty$
   Explanation: At first state s_1 itself literal a becomes true. Thus any path starting from state s_1 will satisfy the condition $b \; \mathsf{U} \; a$.

4. **[8 points]** Determine whether $\mathcal{M}_1, s_1 \models b \; \mathsf{U} \; a$ and explain why or why not.

   Answer

   $\mathcal{M}_1, s_1 \models b \; \mathsf{U} \; a$ is True and satisfies.
   Explanation: For all paths starting from state s_1, $b \; \mathsf{U} \; a$ is true as in the first state itself we get a as true hence state s_1 will satisfy the $b \; \mathsf{U} \; a$. Since the condition of U is satisfied at the first step itself, we need not check further.

5. **[8 points]** Find a path from the initial state s_1 which satisfies $\mathsf{X}a \mathbin{\mathsf{U}} \mathsf{X}(\neg a \wedge \neg b)$.

   Answer

   $\mathsf{X}a \mathbin{\mathsf{U}} \mathsf{X}(\neg a \wedge \neg b)$ is satisfied in one of the many paths.
   Path: $(s_1 - s_2)^\infty$
   Explanation: Starting from s_1 in the path above $(\neg a \wedge \neg b)$ is true in the next state i.e state s_2. since the until condition is satisfied in next step s_2, we need not check further. All the path "s_1, s_2, (any)" will satisfy $\mathsf{X}a \mathbin{\mathsf{U}} \mathsf{X}(\neg a \wedge \neg b)$

6. **[8 points]** Determine whether $\mathcal{M}_1, s_1 \models \mathsf{X}a \mathbin{\mathsf{U}} \mathsf{X}(\neg a \wedge \neg b)$ and explain why or why not.

   Answer

   $\mathcal{M}_1, s_1 \models \mathsf{X}a \mathbin{\mathsf{U}} \mathsf{X}(\neg a \wedge \neg b)$ is False.
   Explanation: Path $s_1 - (s_3)^\infty)$ is one such path where $\mathcal{M}_1, s_1 \models \mathsf{X}a \mathbin{\mathsf{U}} \mathsf{X}(\neg a \wedge \neg b)$ does not hold. This is because s_3 turns into loop so $(\neg a \wedge \neg b)$ does not become true at any further step.

7. **[8 points]** Find a path from the initial state s_1 which satisfies $\mathsf{X}(\neg b \wedge \mathsf{X}(\neg a \rightarrow \mathsf{G}\neg a))$.

   Answer

   One of the multiple paths satisfy $\mathsf{X}(\neg b \wedge \mathsf{X}(\neg a \rightarrow \mathsf{G}\neg a))$.
   Path: $s_1 - s_2 - (s_4)^\infty$.
   Explanation: In the path above starting from s_1,in the next step i.e s_2, $\neg b$ is True. Also in the next step of s_2 i.e s_4, $(\neg a \rightarrow \mathsf{G}\neg a))$ is true,thus $\mathsf{X}(\neg b \wedge \mathsf{X}(\neg a \rightarrow \mathsf{G}\neg a))$ satisfies for above path.

8. **[8 points]** Determine whether $\mathcal{M}_1, s_1 \models \mathsf{X}(\neg b \wedge \mathsf{X}(\neg a \rightarrow \mathsf{G}\neg a))$ and explain why or why not.

   Answer

   $\mathcal{M}_1, s_1 \models \mathsf{X}(\neg b \wedge \mathsf{X}(\neg a \rightarrow \mathsf{G}\neg a))$ does not hold.
   Explanation: In the path "$s_1 s_3 s_2 s_1$(any)" $\mathcal{M}_1, s_1 \models \mathsf{X}(\neg b \wedge \mathsf{X}(\neg a \rightarrow \mathsf{G}\neg a))$ does not hold. Since in transition from state s_2 to s_1 literal $\neg a$ is not true thus $\mathsf{X}(\neg a \rightarrow \mathsf{G}\neg a))$ will not be true.

9. **[8 points]** Find a path from the initial state s_1 which satisfies $\mathsf{X}(\neg a \wedge \neg b) \wedge \mathsf{F}(\neg a \wedge b)$.

   Answer

   Path satisfying above is: **Path**: $s_1 - s_2 - (s_4)^\infty$.

10. **[8 points]** Determine whether $\mathcal{M}_1, s_1 \models \mathsf{X}(\neg a \wedge \neg b) \wedge \mathsf{F}(\neg a \wedge b)$ and explain why or why not.

    Answer

    $\mathcal{M}_1, s_1 \models \mathsf{X}(\neg a \wedge \neg b) \wedge \mathsf{F}(\neg a \wedge b)$ does not hold. Considering the path $s_1 - (s_3)^\infty$, $\neg a \wedge \neg b$ is never achieved as True. So does not satisfy.

11. **[20 points]** List all subformulas of the LTL formula

    $$((\neg \mathsf{X}p) \mathbin{\mathsf{W}} q) \mathbin{\mathsf{U}} (\neg p \rightarrow (q \mathbin{\mathsf{U}} (\mathsf{X}\mathsf{G}r \vee \mathsf{F}\mathsf{X}\neg q)))$$

    Answer

    Subformulas are:

    (a) $((\neg \mathsf{X}p) \mathbin{\mathsf{W}} q) \mathbin{\mathsf{U}} (\neg p \rightarrow (q \mathbin{\mathsf{U}} (\mathsf{X}\mathsf{G}r \vee \mathsf{F}\mathsf{X}\neg q)))$
    (b) $((\neg \mathsf{X}p) \mathbin{\mathsf{W}} q)$
    (c) $\neg \mathsf{X}p$
    (d) $\mathsf{X}p$
    (e) p
    (f) q

(g) $(\neg p \rightarrow (q \; \mathsf{U} \; (\mathsf{XG}r \vee \mathsf{FX}\neg q)))$

(h) $\neg p$

(i) $q \; \mathsf{U} \; (\mathsf{XG}r \vee \mathsf{FX}\neg q)$

(j) $(\mathsf{XG}r \vee \mathsf{FX}\neg q)$

(k) $\mathsf{XG}r$

(l) $\mathsf{G}r$

(m) r

(n) $\mathsf{FX}\neg q$

(o) $\mathsf{X}\neg q$

(p) $\neg q$