

# CSC 503 Homework Assignment 2

Out: August 26, 2015  
Due: September 2, 2015  
rsandil

Prove the validity of the following sequents. Use only the basic rules of natural deduction (no derived rules).

1. [10 points]  $p \rightarrow (q \rightarrow r), p \rightarrow q \vdash p \rightarrow r$

Answer:

**Answer**

1	$p \rightarrow (q \rightarrow r)$	premise
2	$p \rightarrow q$	premise
3	$p$	assumption
4	$q$	$\rightarrow e, 2, 3$
5	$q \rightarrow r$	$\rightarrow e, 1, 3$
6	$r$	$\rightarrow e, 4, 5$
7	$p \rightarrow r$	$\rightarrow i, 3, 6$

2. [20 points]  $(p \vee (q \rightarrow p)) \wedge q \vdash p \vee r$

Answer

1	$(p \vee (q \rightarrow p)) \wedge q$	premise
2	$q$	$\wedge e_2, 1$
3	$(p \vee (q \rightarrow p))$	$\wedge e_1, 1$
4	$p$	assumption
5	$(q \rightarrow p)$	assumption
6	$p$	$\rightarrow e, 5, 2$
7	$p$	$\vee e, 3, 4-4, 5-6$
8	$p \vee r$	$\vee i, 7$

3. [30 points]  $(p \wedge q) \rightarrow r, r \rightarrow s, q \wedge \neg s \vdash \neg p$

Answer

1	$(p \wedge q) \rightarrow r$	premise
2	$r \rightarrow s$	premise
3	$q \wedge \neg s$	premise
4	$q$	$\wedge e_1, 3$
5	$\neg s$	$\wedge e_2, 3$
6	$p$	assumption
7	$p \wedge q$	$\wedge i, 4, 6$
8	$r$	$\rightarrow e, 1, 7$
9	$s$	$\rightarrow e, 2, 8$
10	$\perp$	$\neg e, 6-9$
11	$\neg p$	$\neg i, 6-10$

4. [40 points]  $(q \wedge u) \rightarrow t, r \wedge \neg s, r \wedge \neg(s \vee q) \rightarrow p \vdash (u \wedge \neg t) \rightarrow p$

**Answer**

1	$(q \wedge u) \rightarrow t$	premise
2	$r \wedge \neg s$	premise
3	$r \wedge \neg(s \vee q) \rightarrow p$	premise
4	$r$	$\wedge e_1, 2$
5	$\neg s$	$\wedge e_2, 2$
6	$u \wedge \neg t$	assumption
7	$u$	$\wedge e_1, 6$
8	$\neg t$	$\wedge e_2, 6$
9	$q$	assumption
10	$q \wedge u$	$\vee e, 9, 7$
11	$t$	$\rightarrow e, 1, 10$
12	$\perp$	$\neg e, 8-11$
13	$\neg q$	$\neg i, 9-12$
14	$s \vee q$	assumption
15	$s$	assumption
16	$\perp$	$\neg e, 5, 15$
17	$q$	assumption
18	$\perp$	$\neg e, 13, 17$
19	$\perp$	$\vee e, 14, 15-16, 17-18$
20	$\neg(s \vee q)$	$\neg i, 14-19$
21	$r \wedge (\neg(s \vee q))$	$\wedge i, 4, 20$
22	$p$	$\rightarrow e, 21, 3$
23	$(u \wedge \neg t) \rightarrow p$	$\rightarrow i, 6, 22$