CSC 503 Homework Assignment 6

Out: September 23, 2015
Due: September 30, 2015

rsandil

1. Open formula: $P(x, y) \vee (Q(x, y) \wedge \neg R(y, z))$
   Using Distributive law, above formula can be rewritten as
   $(P(x, y) \vee Q(x, y)) \wedge (P(x, y) \vee \neg R(y, z))$

   a. Thus the clausal form using set notation is : $\{\{P(x, y), Q(x, y)\}, \{P(x, y), \neg R(y, z)\}\}$
   b. The above formula can be written so that variable can be standardized apart as :
      $\{\{P(x_1, y_1), Q(x_1, y_1)\}, \{P(x_2, y_2), \neg R(y_2, z_2)\}\}$

2. Using the rules for negation normal form $\exists x \neg(P(x, y) \leftrightarrow \forall y\, Q(x, y))$ can be converted to NNF as per following steps:

   Step 1: $\exists x \neg ((P(x, y) \rightarrow \forall y\, Q(x, y)) \wedge (\forall y\, Q(x, y) \rightarrow P(x, y)))$
   Since $v \leftrightarrow w \Rightarrow (v \rightarrow w) \wedge (v \rightarrow w)$

   Step 2: $\exists x \neg ((\neg P(x, y) \vee \forall y\, Q(x, y)) \wedge (\neg\forall y\, Q(x, y) \vee P(x, y)))$

   Since $u \rightarrow w \Rightarrow \neg u \vee w$
   Step 3: $\exists x \neg ((\neg P(x, y) \vee \forall y\, Q(x, y)) \wedge (\exists y\, \neg Q(x, y) \vee P(x, y)))$

   Since $Qx \neg \forall y\, \phi \Rightarrow Qx\, \exists y \neg \phi$ NOTE: Qx stands for quantifier x.
   Step 4: $\exists x (\neg (\neg P(x, y) \vee \forall y\, Q(x, y)) \vee \neg (\exists y\, \neg Q(x, y) \vee P(x, y)))$

   Since $\neg (u \wedge w) \Rightarrow$ to $\neg u \vee \neg w$ using De-Morgan's law
   Step 5: $\exists x ((P(x, y) \wedge \neg \forall y\, Q(x, y)) \vee (\neg \exists y\, \neg Q(x, y) \vee \neg P(x, y)))$

   Since using De Morgan's Law: $\neg (u \vee w)$ is equivalent to $\neg u \wedge \neg w$

   Step 6: $\exists x ((P(x, y) \wedge \exists y \neg Q(x, y)) \vee (\forall y\, Q(x, y) \wedge \neg P(x, y)))$

3. The formula $\forall x((\exists y\, P(x, y)) \rightarrow Q(x, z)) \wedge \exists x((\forall y\, R(x, y)) \vee Q(x, y))$ can be converted to prenex normal form as per following steps:
   Step 1: $\forall x (\neg (\exists y\, P(x, y)) \vee Q(x, z)) \wedge \exists x ((\forall y\, R(x, y)) \vee Q(x, y))$
   Since $a \rightarrow b \Rightarrow \neg a \vee b$
   Step 2: $\forall x ((\forall y \neg P(x, y)) \vee Q(x, z)) \wedge \exists x ((\forall y\, R(x, y)) \vee Q(x, y))$
   Since $Qx \neg \exists y\, \phi \Rightarrow Qx\, \forall y \neg \phi$
   Step 3: $\forall x\, \forall t ((\neg P(x, t/y)) \vee Q(x, z)) \wedge \exists x\, \forall s (R(x, s/y) \vee Q(x, y))$
   Since $Qx (\forall y\, \phi \vee \psi) \Rightarrow Qx\, \forall z (\phi(z/y) \vee \psi)$
   Step 4: $\forall x\, \forall t\, \forall s\, \exists w ((\neg P(x, t/y)) \vee Q(x, z)) \wedge (R (w/x, s/y) \vee Q(w/x, y))$

4. The formula $\forall x \, \exists y \, P(x, y) \rightarrow \forall x \neg \forall y \, Q(x, y) \wedge \neg \exists x \, \exists y \, P(x, y)$ can be converted in Skolem Normal form as per following steps:
   Step 1: $\neg (\forall x \, \exists y \, P(x, y)) \vee ((\forall x \, \neg \forall y \, Q(x, y)) \wedge (\neg \exists x \, \exists y \, P(x, y)))$

   Since $u \rightarrow w \Rightarrow \neg u \vee w$
   Step 2: $(\exists x \, \neg \exists y \, P(x, y)) \vee ((\forall x \, \exists y \, \neg Q(x, y)) \wedge (\forall x \, \neg \exists y \, P(x, y)))$

   Since $Qx \neg \forall y \, \phi \Rightarrow Qx \, \exists y \, \neg \phi$ and $Qx \, \neg \exists y \, \phi \Rightarrow Qx \, \forall y \, \neg \phi$
   Step 3: $(\exists x \, \forall y \, \neg P(x, y)) \vee ((\forall x \, \exists y \, \neg Q(x, y)) \wedge (\forall x \, \forall y \, \neg P(x, y)))$

   Since $Qx \, \neg \exists y \, \phi \Rightarrow Qx \, \forall y \, \neg \phi$
   Step 4: $(\forall y \, \neg P(f(y), y)) \vee ((\forall x \, \neg Q(x, f(x))) \wedge (\forall x \, \forall y \, \neg P(x, y)))$

5. $\theta = \{f(y)/x, g(z)/y, v/w\}$
   $\sigma = \{a/x, b/y, f(y)/z, w/v, c/u\}$
   $\theta \, \sigma$ **=** $\{f(b)/x, g(f(y))/y, f(y)/z, w/v, c/u\}$