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# Mathematics

## Quarter 1 – Module 2
## Arithmetic Means and Arithmetic Series

**DepED**
DEPARTMENT OF EDUCATION
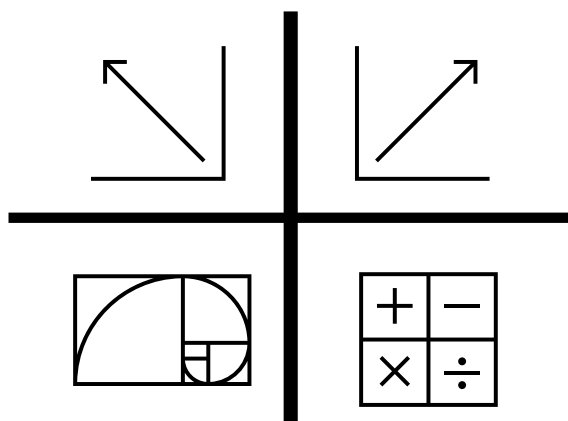
## *About the Module*

This module was designed and written with you in mind. It is here to help you master patterns and sequences. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module is divided into the following lessons:

This module was designed and written with you in mind. It is here to help you master about **Arithmetic Means and Arithmetic Series**. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module is divided into four lessons, namely:

Lesson 1 – Arithmetic Means

Lesson 2 – Arithmetic Series: Finding the Sum using $S_n = \frac{n}{2} (a_1 + a_n)$

Lesson 3 – Arithmetic Series: Finding the Sum using $S_n = \frac{n}{2} [ 2a_1 + ( n-1 ) d]$

Lesson 4 – Arithmetic Series in Real Life

After going through this module, you are expected to:
1. find arithmetic means between two nonconsecutive terms;
2. solve for the sum of finite arithmetic series;
3. apply arithmetic series in real life problem.

## *What I Know (Pre-Test)*

*Instructions*:
Choose the letter of the correct answer. Write your chosen answer on a separate sheet of paper.

1. How do we call the numbers between 5 and -10 in the sequence **10, 5, 0, -5, -10** ?
   - A.  arithmetic means
   - B.  Fibonacci means
   - C.  harmonic means
   - D.  geometric means

2. What type of series is **13 + 16 + 19 + 22 + 25 + 28** ?
   - A. finite arithmetic series
   - B. finite geometric series
   - C. infinite arithmetic series
   - D. infinite geometric series

3. Complete this arithmetic sequence: **1 , $\frac{3}{2}$, \_\_\_\_, \_\_\_\_, 3., ...**
   - A.  $\frac{3}{4}$, $\frac{1}{8}$
   - B.  $2$ , $\frac{5}{2}$
   - C.  $\frac{9}{4}$, 5
   - D.  $2$ , $\frac{2}{5}$

4. What is the value of $x$ if $3x - 1$ is the arithmetic mean between $x + 1$ and $2x + 3$?
   - A.  5
   - B.  3
   - C.  2
   - D.  1

5. What is the sum of the first 10 terms of an arithmetic series if its first term is 3 and last term is 21?
   - A.  60
   - B.  96
   - C.  120
   - D.  260

For items 6 – 8 use this **arithmetic series: 5 + 9 + 13 + ... + 101**

6. What is the common difference?
   - A.  5
   - B.  4
   - C.  3
   - D.  2

7. What is the sum of the given arithmetic series?
   - A.  1325
   - B.  1255
   - C.  1050
   - D.  850

8.  How many terms are there between 5 and 101?
   - A.  22
   - B.  23
   - C.  24
   - D.  25

9. What is the sum of the terms of an arithmetic sequence from $a_1 = 9$ to $a_9 = 33$?
   - A.  540
   - B.  189
   - C.  180
   - D.  93

10. What is the common difference of the arithmetic sequence from $a_1 = 9$ to $a_9 = 33$?
   - A.  2
   - B.  3
   - C.  5
   - D.  6

11. What is $n$ if the given arithmetic sequence has $a_1 = -3$, $a_n = 11$ and $S_n = 32$ ?
    A.  2              B.  4              C.  6              D,    8

12. What is $d$ if the sum of the first eleven terms of an arithmetic series is 286 and its first term is 11 ?

    A.  3              B.  5              C.  9              D.  11

13. Which of the following arithmetic series have a sum greater than 200?

        I.     $a_1 = -6$     $a_{10} = 12$

        II.    $a_1 = 8$       $d = 3$        $n = 8$

        III.   $1 + 9 + 17 + \dots + 65$

    A.  I              B.  II              C.  III              D.  I, III

For items 14 & 15. Refer to the given problem below.

> Chairs for an orchestra are positioned in a curved form with the conductor at the center. The front row has 16 chairs and each successive row has 4 more chairs.

14. How many chairs are in the 10th row?
    A.  20              B.  52              C.  56              D.  64

15. What is the total number of chairs in first to tenth row?
    A.  320           B.  340       C.  456          D.  564

## *What I Need to Know*

At the end of this lesson, you are expected to:
- find the arithmetic means between two nonconsecutive terms of an arithmetic sequence.

## *What's In*

The previous lesson focused on identifying the arithmetic sequence and finding the nth term of an arithmetic sequence. Here is a short activity from the previous lesson.

### *Identifying Arithmetic Sequence:*

Decide whether the sequence is arithmetic. Explain why or why not.

_____1.     2, 5, 8, 11, 14, …
_____2.     1, 4. 16, 64, 256, …
_____3.     1, 0, - 1, - 2, - 3, …
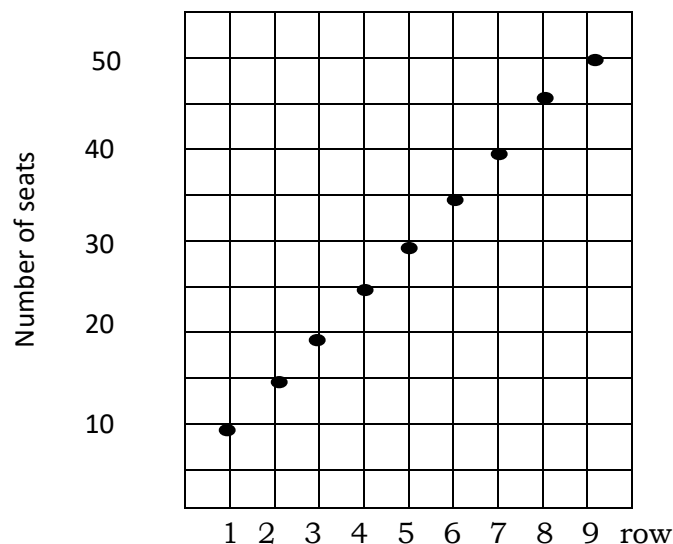_____4.     $\frac{1}{2}$ , 1, $\frac{3}{2}$ , 2, $\frac{5}{2}$ , …

## *What's New*

A school auditorium has 270 seats.
The table below shows the number of seats in some of the rows.

| row | 1st | 2nd | 5th | 9th |
|---|---|---|---|---|
| number of seats | 10 | 15 | 30 | 50 |

Here is the graph of the sequence shown in the table.



Note: The sequence is graphed in a Scatter plot (series of dots).

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➢ What are the number of seats between 2nd and 5th rows?
➢ Give the number of seats between 5th and 9th rows
➢ Is the given sequence an Arithmetic?

# *What Is It*

| 5th row | 6th row | 7th row | 8th row | 9th row |
|---------|---------|---------|---------|---------|
| 30 seats | 35 seats | 40 seats | 45 seats | 50 seats |

The given sequence above is an arithmetic and the terms 35, 40, and 45 are the arithmetic means between 30 and 50.

What are Arithmetic Means?

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**Arithmetic means** are the terms between any two nonconsecutive terms of an arithmetic sequence.

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☞  **21**, <u>24</u>, <u>27</u>, <u>30</u>, <u>33</u>, <u>36</u>, <u>39</u>, **42**

**There are 6 *arithmetic means between 21 and 42***

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**Example 1**

Find the four arithmetic means between 7 and 32.

$$7, \underline{\quad}, \underline{\quad}, \underline{\quad}, \underline{\quad}, 32$$

$$a_1 = 7 \qquad\qquad a_6 = 32$$

Step 1: *Solve for **d***

**Use**    $a_n = a_1 + (n-1)d$
$$a_6 = 7 + (6-1)d$$
$$32 = 7 + 5d$$
$$32 - 7 = 5d$$
$$25 = 5d$$
$$\frac{25}{5} = \frac{5d}{5}$$

$$d = 5$$

Step 2: Now use the value of **d** to find the four arithmetic means.

$7 + 5 = ⑫$ ← 2nd term         $17 + 5 = ㉒$ ← 4th term

$12 + 5 = ⑰$ ← 3rd term         $22 + 5 = ㉗$ ← 5th term

The four arithmetic means are 12, 17, 22, and 27.

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**Example 2**

Insert five arithmetic means between $\frac{1}{3}$ and $\frac{7}{3}$.

$$a_1 = \frac{1}{3} \quad \text{and} \quad a_7 = \frac{7}{3}$$

$a_n \;=\; a_1 + (\,n-1\,)\,d$

$a_7 \;=\; \frac{1}{3} + (\,7-1\,)\,d$     The five arithmetic means are

$\frac{7}{3} \;=\; \frac{1}{3} + 6d$     $\frac{2}{3}$ , 1 , $\frac{4}{3}$ , $\frac{5}{3}$ , 2

$\frac{7}{3} - \frac{1}{3} \;=\; 6d$

$2 \;=\; 6d$

$d \;=\; \frac{1}{3}$

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**Example 3**

What is the arithmetic mean between 6 and $-8$ ?

6 , ____ , - 8

$$\frac{6 + -8}{2} = \frac{-2}{2} = -1$$

Arithmetic mean between 6 and - 8 is -1

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**Example 4**

What are the two arithmetic means between 6x + 2 and 9x − 7 ?

6x + 2, _____, _____, 9x − 7

$a_1$ = 6x + 2         $a_4$ = 9x -7

Solve for **d**

$a_n = a_1 + (n-1)d$
$9x - 7 = (6x + 2) + [(4-1)d]$     The two arithmetic means
$9x - 7 = (6x + 2) + 3d$     are   7x − 1  and  8x − 4.
$(9x - 7) - (6x + 2) = 3d$
$9x - 7 - 6x - 2 = 3d$
$9x - 6x - 7 - 2 = 3d$
$3x - 9 = 3d$

$$\frac{3x-9}{3} = \frac{3d}{3}$$

$x - 3 = d$

$\boxed{d = x - 3}$

**Example 5**

Insert three arithmetic means between $5\sqrt{3}$ and $13\sqrt{3}$

$5\sqrt{3}+1$ ____ , ____ , ____ , $13\sqrt{3}+17$

Solve for d

$a_n = a_1 + (n-1)d$

$a_5 = (5\sqrt{3}+1) + (5-1)d$

$13\sqrt{3}+17 = (5\sqrt{3}+1) + 4d$

$(13\sqrt{3}+17) - (5\sqrt{3}+1) = 4d$

$13\sqrt{3}+17 - 5\sqrt{3} - 1 = 4d$

$13\sqrt{3} - 5\sqrt{3} + 17 - 1 = 4d$

$8\sqrt{3} + 16 = 4d$

$\dfrac{8\sqrt{3}+16}{4} = \dfrac{4d}{4}$

The arithmetic means are

$7\sqrt{3} + 5$   and   $9\sqrt{3} + 9$

$d = 2\sqrt{3} + 4$

# What I Need To Remember

*Arithmetic means* are the terms between any two nonconsecutive terms of an arithmetic sequence.

# What's More

**Give this a try! Apply what you've learned.**
**Activity 1**

A. Find the arithmetic means in each sequence.
1.    - 8, ___, ___, ___, 0
2.    $\dfrac{5}{3}$ , ___ , ___ , ___ , ___ , **5**
3.    $\dfrac{1}{2}$ , ___ , ___ , $\dfrac{5}{2}$
4.    $5\sqrt{2}$ , ___ , $-3\sqrt{2}$
5.    0.25 , ___ , ___ , ___ , 2.25

B.  Solve the following problems.
 1.  Find the three arithmetic means between – 4 and 16.
 2.  What are the first and last terms of an arithmetic sequence when its arithmetic means are 35, 15 and – 5?
 3.  If $a_1$ = - 6 and $a_6$ = 9, what are the arithmetic means between -6 and 9?
 4.  Find the value of $x$ when the arithmetic mean of $x+2$ and $4x+5$ is $3x+2$.

| Lesson 2 Tuesday | **ARITHMETIC SERIES:** **Finding a Sum using** $S_n = \dfrac{n}{2}(a_1 + a_n)$ |
|---|---|

## *What I Need to Know*

At the end of this lesson, you are expected to:
- define arithmetic series;
- find the sum of a finite arithmetic series given the first and the last terms of the sequence.

## *What's In*

Insert two arithmetic means between the first and last terms of the given arithmetic sequence.
1.  $a_1 = 4$      $a_n = -8$
2.  $a_1 = -1.2$    $a_n = -1.8$
3.  $a_1 = 6.6$    $a_n = 8.4$

## *What's New*

Mark, an OFW was asked to return to his country and stopped working because of the pandemic. To support his family's daily needs he thought of selling "biko" online. On the 1st day, he sold six tubs of biko. On the 2nd day, 11 tubs were sold. Because of the good feedback from costumers, the number of orders increased. On the 10th day he sold 51 tubs.

How many tubs of biko were sold during the first ten days of his online business?

The table shows the number of tubs sold per day:

| day | 1st | 2nd | 3rd | 4th | 5th | 10th |
|---|---|---|---|---|---|---|
| number of tubs sold | 6 | 11 | 16 | 21 | 27 | 51 |

- ➢ The sequence **6, 11, 16, 21, 27, ..., 51** is an arithmetic.
- ➢ To find the number of tubs sold for the first ten days, add
     6 + 11 + 16 + 21 + 27 + ... + 51
- ➢ This is an example of an Arithmetic series

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# *What Is It*

**Arithmetic series** is the sum of the terms of an arithmetic sequence.

$$a_1 + a_2 + a_3 + a_4 + \dots + a_n$$

| Arithmetic Sequence | Arithmetic Series |
|---|---|
| **2, 5, 8, 11, ...** | **2 + 5 + 8 + 11 + ...** |
| **-5, -3, -1, 1,...** | **-5 + -3 + -1 + 1 + ...** |

$S_n$ represents the sum of the first n terms of a series. To find a rule for $S_n$, consider the series below:

$$S_7 = 2 + 5 + 8 + 11 + 14 + 17 + 20$$

Write $S_7$ in two different ways and add the two equations

$$
\begin{array}{rl}
S_7 = & 2 + \phantom{0}5 + \phantom{0}8 + 11 + 14 + 17 + 20 \\
(+)\quad S_7 = & 20 + 17 + 14 + 11 + \phantom{0}8 + \phantom{0}5 + \phantom{0}2 \\
\hline
2S_7 = & 22 + 22 + 22 + 22 + 22 + 22 + 22
\end{array}
$$

$$2S_7 = 7\,(\,22\,)$$

| 7 is the number of terms |
|---|

$$S_7 = \frac{7\,(22)}{2}$$

| 22 is the sum of the first term and the last term |
|---|

☆ A simple formula for finding the sum of an arithmetic sequence with a definite number of terms.

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**The Sum of a Finite Arithmetic Series**

If $S_n$ represents the sum of an arithmetic sequence with terms

$$\mathbf{a_1, a_2, a_3, \dots, a_n} \text{ then } \quad \mathbf{S_n} = \frac{n}{2}\,\mathbf{(a_1 + a_n)}$$

where $a_1$ is the first term, $a_n$ is the last term and $n$ is the number of terms.

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Here are examples of finding the sum of finite arithmetic sequence:

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**Example 1**  Use this information to answer the question

| day | 1st | 2nd | 3rd | 4th | 5th | 10th |
|---|---|---|---|---|---|---|
| number of tubs sold | 6 | 11 | 16 | 21 | 27 | 51 |

How many tubs of biko were sold during the first ten days of Mark's online busines?    $a_1 = 6$,   $a_{10} = 51$,   $n = 10$

$$
\begin{aligned}
S_{10} &= \frac{n}{2}\,(a_1 + a_n) \\
&= \frac{10}{2}\,(6 + 51) \qquad\qquad \text{There were 285 tubs} \\
&= 5\,(57) \qquad\qquad\quad\; \text{sold in 10 days.} \\
S_{10} &= 285
\end{aligned}
$$

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**Example 2**

Find the sum of an arithmetic sequence with 20 terms
if the first term is 5 and the last term is 62.

Given:   $a_1$ = 5 ;  $a_n$ = 62 ; $n$ = 20

$S_n = \frac{n}{2}$ ($a_1$ + $a_n$)

$S_{20} = \frac{20}{2}$ (5 + 62)

    = 10  (67)

$S_{20}$ = 670

                  The sum of the first 20 terms is 670.

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**Example 3**

Find the sum of the sequence   5, 8, 11, 14, 17, ..., 62

Given:                   $a_1$ = 5;  $d$ = 3 ; $a_n$ = 62

Step 1: Solve for $n$

$$a_n = a_1 + ( n - 1)\, d$$
$$62 = 5 + ( n - 1 )\, 3$$
$$62 = 5 + 3n - 3$$
$$62 = 3n + 2$$

Subtract 2 to both sides $\longrightarrow$   $62 - 2 = 3n + 2 - 2$
$$60 = 3n$$

Divide 3 to both sides $\longrightarrow$   $3n = 60$
$$n = 20$$

**62 is the 20th term in the sequence**

Step 2: Solve for the sum

$$S_n = \frac{n}{2}\,(a_1 + a_n)$$
$$S_{20} = \frac{20}{2}\,(5 + 62)$$
$$= 10\,(67)$$
$$S_{20} = 670$$

The sum of the first 20 terms is 670.

**Example 4**    Find the first three terms of an arithmetic series
if $a_1 = 6$,  $a_n = -51$ and $S_n = -450$.

Step 1    Find $\boldsymbol{n}$

Use  $S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{n}{2}(a_1 + a_n)$

$-450 = \frac{n}{2}(6 + -51)$

$-450 = \frac{n}{2}(-45)$

$-450 = \frac{n}{2}(-45)$

$10 = \frac{n}{2}$

$20 = n$

$\boldsymbol{n = 20}$

Step 2  Find $\boldsymbol{d}$

$a_n = a_1 + (n-1)d$

$a_{20} = a_1 + (20-1)d$

$-51 = 6 + (19)d$

$-51 - 6 = 19d$

$\frac{-57}{19} = d$

$-3 = d$

$\boldsymbol{d = -3}$

Step 3    Use $\boldsymbol{d}$ to determine $a_2$ and $a_3$
The first three terms are 6, 3, 0

## *What I Need To Remember*

***Arithmetic series*** is the sum of the terms of an arithmetic sequence.
$a_1 + a_2 + a_3 + a_4 + \ldots + a_n$

If $S_n$ represents the sum of an arithmetic sequence with

terms $\boldsymbol{a_1, a_2, a_3, \ldots, a_n}$ then,

$$\boldsymbol{S_n = \frac{n}{2}(a_1 + a_n)}$$

where  $\boldsymbol{a_1}$ is the first term, $\boldsymbol{a_n}$ is the last term and $\boldsymbol{n}$ is the number of terms.

## *What's More*
**Now, it's your turn to solve for the sum of the finite arithmetic series.**

**Activity 2**

A. Fill in the blanks with the sum of the terms $\boldsymbol{S_n}$ , number of terms $\boldsymbol{n}$ , first term $\boldsymbol{a_1}$ and the last term $\boldsymbol{a_n}$ .

| Sum of the terms $S_n$ | Number of terms $n$ | First term $a_1$ | Last term $a_n$ |
|---|---|---|---|
| | 10 | 5 | 50 |
| | 9 | $\frac{1}{2}$ | $\frac{9}{2}$ |
| 132 | 8 | | 27 |
| 253 | | 8 | 38 |

B. Find the first three terms of each arithmetic series, given:

1. $a_1 =$ 2 $a_n =$ 32 $S_n =$ 187
2. $n =$ 12 $a_n =$ 51 $S_n =$ 348

| Lesson 3 Wednesday | ARITHMETIC SERIES: Finding the sum using $S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$ |
|---|---|

# What I Need to Know

At the end of this lesson, you are expected to:
- find the sum of a finite arithmetic series given the first term and the common difference.

# What's In

Find the sum of each arithmetic series.

1. $a_1 =$ 2 $a_n = -48$ $n = 11$
2. $a_1 =$ 0.5 $a_n = -1.3$ $n = 10$
3. $a_1 =$ x $a_n = -13x + 42$ $n = 15$

# What's New

| How do Arithmetic Series apply to Facebook Friend Requests? |
|---|
| Rene uploaded a video of his ECQ experiences to Facebook and tagged his friends, he immediately got 1 friend request. The following day he received 3 friend requests, the 3rd day 5 friend requests, and on the 4th , 7 friend requests and it continued on the following days What is the total number of friend requests for the first 5 days? first 30 days? |

The table below shows the number of friend requests Rene received everyday.

| day | 1 | 2 | 3 | 4 | 5 | ... | 30 |
|---|---|---|---|---|---|---|---|
| number of friend requests | 1 | 3 | 5 | 7 | 9 | | ? |

To find the sum of the number of friend requests for the first 5 days, we should add $1 + 3 + 5 + 7 + 9$

➢ How about finding the total number of the friend requests for the first 30 days?
➢ Do you know of any shorter method to solve the problem?

# *What Is It*

If the last term of an arithmetic sequence is not given, here is another formula to use in finding the sum of a finite arithmetic series.

Use rule  $a_n = a_1 + (n - 1)d$ to find the nth term of an arithmetic sequence.

Using $\qquad\qquad S_n = \frac{n}{2}(a_1 + a_n)$

But $a_n = a_1 + (n - 1)d$, therefore

$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$S_n = \frac{n}{2}[a_1 + \{a_1 + (n - 1)d\}]$$
$$S_n = \frac{n}{2}[a_1 + a_1 + (n - 1)d]$$
$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

We have $\qquad S_n = \frac{n}{2}\Big[2a_1 + (n-1)d\Big]$

where:

$S_n = $ *the sum of the first n terms*
$n = $ *number of terms*
$a_1 = $ *first term*
$d = $ *common difference*

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**Example 1**
Find the sum of the first 40 terms of the arithmetic sequence 2,5,8,11, …

Given:  $a_1$ = 2; $d$ = 3 ; $n$ = 40

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$S_{40} = \frac{40}{2}[2(2) + (40 - 1)3]$$

$$= 20[4 + 39(3)]$$

$$= 20(121)$$

$$S_{40} = 2420$$

➡ The sum of the first 40 terms is 2420

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**Example 2**

In the arithmetic series, **3 + 8 + 13 + 18 + …., its $S_n = 366$,** what is **n** ?

Given:  $S_n = 366$     $a_1 = 3$     $d = 5$

Use    $S_n$    =  $\frac{n}{2}$ **[ $2a_1$ + ( n-1 ) d]**  to solve for **n**

Substitute:        $S_n = 366$        $a_1 = 3$          $d = 5$

$S_n$    =  $\frac{n}{2}$ [ $2a_1$ + ( n-1 ) d]

366    =  $\frac{n}{2}$ [ 2(3) + ( n-1 ) 5 ]

366    =  $\frac{n}{2}$ [ 6 + 5n - 5 ]

366    =  $\frac{n}{2}$ [ 5n + 1 ]

( 2 ) 366    =  $\frac{n}{2}$ [ 5n + 1 ] ( 2 )    ⟵   *multiply both sides by 2*

( 2 **)** 366    =  $\frac{n}{\cancel{2}}$ [ 5n + 1 ] ($\cancel{2}$)

732    =   $5n^2$ + n

732 -  732    =   $5n^2$ + n - 732    ⟵   *subtract both sides by 732*

0    =    $5n^2$ + n - 732

$5n^2$ + n – 732  =     0    ⟵   *write in standard form*

( 5n + 61 ) ( n – 12 )  **=     0**    ⟵   *factor $5n^2$ + n – 73*

5n + 61 =    0          n – 12 = 0

5n    = - 61          n = 12  ✓

n   =  $\frac{-61}{5}$

*not possible*          *choose positive solution*

There are 12 terms to be added to get a sum of 366.

**Example 3**

An auditorium has 20 seats in the first row, 24 seats in the second row, 28 in the third row, and so on, for 15 rows. How many seats are there in the last row? How many seats are there in the auditorium?

$$a_1 = 20 \qquad d = 4 \qquad n = 15$$

Solve for $a_{15}$:

$a_{15} = a_1 + (n-1)d$

$a_{15} = 20 + (15-1)4$

$\quad = 20 + (14)4$

$\quad = 20 + 56$

$a_{15} = 76$

Solve for $S_{15}$:

$S_{15} = \dfrac{n}{2} ( a_1 + a_n )$

$\quad = \dfrac{15}{2} ( 20 + 76 )$

$\quad = \dfrac{15}{2} ( a_1 + a_n )$

$S_{15} = 720$

There are 76 seats in 15th row and there are a total of 720 seats in an auditorium.

# What I Need to Remember

- **Arithmetic series** is the sum of the terms of an arithmetic sequence.

  $\quad a_1 + a_2 + a_3 + a_4 + \ldots + a_n$

- If $S_n$ represents the sum of an arithmetic sequence with terms **$a_1$, $a_2$, $a_3$,..., $a_n$** then,

$$S_n = \frac{n}{2}\left[ 2a_1 + (n-1)d \right]$$

where **$a_1$** is the first term, **$d$** is the common difference and **$n$** is the number of terms

# What's More

**Activity 3**

A. Find the sum of the first 10 terms of the arithmetic series.
1.         $5 + 9 + 13 + \ldots$
2.         $-17 + -12 + -7 + \ldots$
3.         $4 + 9 + 14 + 19 + \ldots$
4.         $-16 + -20 + -24 + \ldots$

.

B. Find the **$n$** for the given sum of the arithmetic series.

1.         $3 + 11 + 19 + 27 + \ldots$      $S_n = 564$
2.         $9 + 12 + 15 + 18 + \ldots$      $S_n = 1575$

# Arithmetic Series in Real Life

## *What I Need to Know*

For this activity, you are expected to:
* solve for the sum of the finite arithmetic sequence and apply the concept in solving real life problems.

## *What I can do*

**Activity 4**

**A. Reality Series**

> ### "More face masks than jellyfish": coronavirus ends up in ocean
>
> Conservationists have warned that the coronavirus pandemic could spark a surge in ocean pollution – adding to a glut of plastic waste that already threatens marine life – after finding disposable masks floating like jellyfish and waterlogged latex gloves scattered across sea beds.
>
> This was posted two months ago in **"The Guardian"** website.
>
> https://www.theguardian.com/environment/2020/jun/08/more-masks-than-jellyfish-coronavirus-waste-ends-up-in-ocean

Assuming that the number of disposable face masks thrown in the ocean within the month of May is in the sequence below:

| day | Month of May | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | ... |
| number of face mask thrown in the ocean | 50 | 75 | 100 | 125 | 150 | 175 | 200 | 225 | |

Answer the following
1.      How many masks were thrown in the ocean on the 30th day?

2.      What is the total number of face masks thrown in the ocean for the whole month of May?

3. As a student, how will you be of help to our environment amidst this pandemic we are facing?

B. Fill in the blank with the possible equation and answer of the given problem.

| A | B | C |
|---|---|---|
| PROBLEM | EQUATION | FINAL ANSWER |
| 1. What is the sum of the first 20 terms of this sequence {5, 15, 25,...} ? | . $S_{20} = 10[10+ 19(10)]$ | |
| 2. What is the sum of the first 15 terms of this sequence { 5, 9, 13,...} ? | | 495 |
| 3. In an arithmetic series, find the sum of the first 20 terms if the first term is -12 and the common difference is -5. | $S_{20} = 10(-24 - 95)$ | |
| 4. Find the sum of the series 14+11+8+...+ - 82 | $S_{33} = \frac{33}{2} (14 - 82)$ | |
| 5. How many terms of the arithmetic sequence {1,3,5,7,...} will give a sum of 961? | | 31 |
| 6. In an arithmetic series, $a_1$= -14 and $a_5$= 30. Find the sum of the first 20 terms. | $S_{20} = 10[ -24 - 19(11)]$ | |
| 7. How many terms of the arithmetic sequence {2,4,6,8,...} will give a sum of 600? | $600 = \frac{n}{2} [4+(n-1)2]$ | |

# Arithmetic Means & Arithmetic Series: Assessment

## Assessment (Post Test)

*Instructions*:
Choose the letter of the correct answer. Write your chosen answer on
a separate sheet of paper.

1. What do you call the numbers between 6 and 30 in the given set of numbers
   6 , 12 , 18 , 24 , 30 ?
   - A. arithmetic means
   - B. Fibonacci means
   - C. geometric means
   - D. harmonic means

2. What type of series is  3 + 16 + 29 + 42 + 55 + 68 ?
   - A. finite arithmetic series
   - B. finite geometric series
   - C. infinite arithmetic series
   - D. infinite geometric series

3. Complete this arithmetic sequence: 29, 21, ____, ____, -3, -11
   - A.  16, 8
   - B.  16, 5
   - C.  13, 5
   - D.  13, 8

4. What is the value of the 3rd term if  $3x - 1$  is the arithmetic mean between $x + 1$
   and  $2x + 3$?
   - A.  2
   - B.  3
   - C.  5
   - D.  7

5. What is the sum of the first 10 terms of an arithmetic series if its first term is
   - 3 and last term is 42 ?
   - A.  360
   - B.  225
   - C.  195
   - D.  160

For items 6 - 8
   **Arithmetic Series: 10 + 13 + 16 + ....+ 37**?

6. What is the common difference?
   - A.  2
   - B.  3
   - C.  4
   - D.  5

7. What is the sum of the given arithmetic series?
   - A.  125
   - B.  235
   - C.  450
   - D.  550

8.  How many terms are there between 10 and 37?
   - A.  4
   - B.  6
   - C.  8
   - D.  10

9. What is the sum of the terms of an arithmetic sequence from $a_1 = 7$ to $a_{10} = 79$?
   - A.  630
   - B.  430
   - C.  405
   - D.  360

10. What is the common difference of the arithmetic sequence from $a_1 = -7$ $a_{10} = 47$?
    A. 4                   B. 6                   C. 8                   D. 10

11. What is $n$ if the given arithmetic sequence has $a_1 = 9$ , $a_n = 27$ and $S_n = 180$ ?
    A. 5                   B. 6                   C.  9                   D.  10

12. What is $d$ if the sum of the first eleven terms of an arithmetic series is  275
    and its first term is -10  ?

    A. 5                   B. 7                   C. 9                   D.  11

13. Which of the following arithmetic series have a sum lesser than 200?

            I.      $a_1 = -6$      $a_{10} = 12$

            II.     $a_1 = 8$        $d = 3$       $n = 8$

            III.    $1 + 9 + 17 + ... + 65$

    A.  I,II               B.  II, III            C.  I, III             D.  I, II, III

For items 14 & 15

> Chairs for an orchestra are positioned in a curved form with the conductor
> at the center. The front row has 20 chairs and each successive row has 5
> more chairs.

14. How many chairs are in the 10th row?
    A.  20                 B.  55                 C.  60                 D.  65

15. What is the total number of chairs in first to tenth row?
    A.  320                B.  340                C.  425                D.  565

![key icon] *Answer Key*

*Remember*: This portion of the module contains all the answers. Your **HONESTY** is required.

**Lesson 1**

**What's In**

1. Arithmetic Sequence, it has a common difference of 3
2. Not arithmetic Sequence, it has no common difference
3. Arithmetic Sequence, it has a common difference of - 1
4. Arithmetic Sequence, it has a common difference of $\frac{1}{2}$

**Activity 1**

A.

1.) 2, 4, 6,
2.) $\frac{13}{3}$, $\frac{11}{3}$, 3, $\frac{7}{3}$
3.) $\frac{7}{6}$, $\frac{11}{6}$
4.) $\sqrt{2}$
5.) 1.75 , 1.25 , 0.75

B.

1.) 11, 6, 1
2.) − 25 , 55
3.) 6 , 3 , 0 , - 3
4.) x = 3

**Lesson 2**

**What's In**

1. 0, -4
2. -1.4 , - 1.6
3. 7.8, 7.2,

19

**Activity 2**

Lesson 2 Activity 2 -A

| Sum of the terms $S_n$ | Number of terms $n$ | First term $a_1$ | Last term $a_n$ |
|---|---|---|---|
| **275** | 10 | 5 | 50 |
| $\dfrac{45}{2}$ | 9 | $\dfrac{1}{2}$ | $\dfrac{9}{2}$ |
| 132 | 8 | **6** | 27 |
| 253 | **11** | 8 | 38 |

B.   1.   2 , 5 , 8        2.   7 , 11 , 15

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**Activity 3**

A.                                                     B.

1.   $S_{10}$ = 230                    1.   n = 12
2.   $S_{10}$ = 55                      2.   n = 30
3.   $S_{10}$ = 265
4.   $S_{10}$ = - 340

**Lesson 3**
**What's In**
1.   $S_{11}$ = -253
2.   $S_{10}$ = - 4
3.   $S_{15}$ = -90x + 315

# *References*

**Text Book**

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# *Congratulations!*

You are now ready for the next module. Always remember the following:

1.  Make sure every answer sheet has your
    - *Name*
    - *Grade and Section*
    - *Title of the Activity or Activity No.*
2.  Follow the date of submission of answer sheets as agreed with your teacher.
3.  Keep the modules with you AND return them at the end of the school year or whenever face-to-face interaction is permitted.