



GENERAL APTITUDE

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What is **Aptitude** ?

It is **your natural ability** to learn or excel in a certain area.

For example, you could have an **aptitude** for math and logic.

Key to **success**

1. Problem Recognition
2. Speed
3. Practice



Link for English Basics

- <https://www.myenglishpages.com/english/exercises.php>
- <https://www.grammarbank.com/>
- <https://www.really-learn-english.com/english-grammar-exercises.html>
- <https://www.really-learn-english.com/english-reading-comprehension-text-and-exercises.html>
- Practice Synonyms and Antonyms regularly.
- Read Idioms and Phrases.
- Book - Word Power Made Easy by Norman Lewis
- Book - English Grammar by Wren and Martin



Basic MATHS

- Tables at least from 1-25
 - Squares from 1-25
 - Prime numbers from 1-100
 - Divisibility rules for 1-20
- Methods for typical multiplications & divisions
 - Methods for finding HCF & LCM
 - Methods for finding squares & square roots

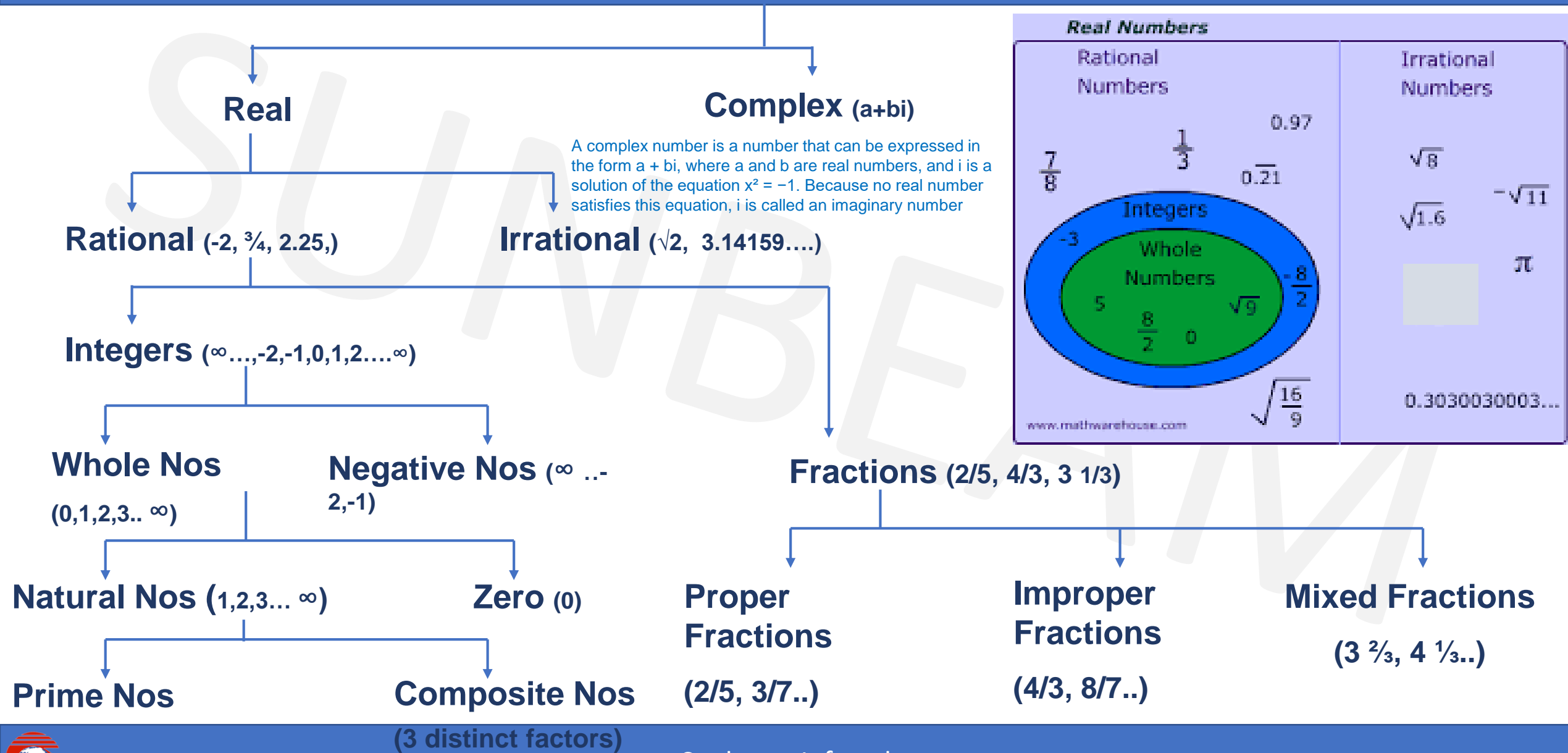


Topic Wise Test Plan

TEST NAME	TOPICS
APT I 1	Numbers + LCM + HCF + Ages + Averages
APT I 2	Percentages + Alligations & Mixtures + Profit & Loss
APT I 3	Time & Work + Pipes & Cisterns + Chain Rule
APT I 4	Time & Distance + Trains + Boats + Interest
APT I 5	Clock + Calendar + Probability + Permutation Combination



Numbers



What is the Difference Between Rational Numbers and Irrational Numbers?

Rational Numbers	Irrational Numbers
Numbers that can be expressed as a ratio of two numbers (p/q form) are termed as a rational number.	Numbers that cannot be expressed as a ratio of two numbers are termed as an irrational number.
Rational Number includes numbers, which are finite or are recurring in nature.	These consist of numbers, which are non-terminating and non-repeating in nature.
If a number is terminating number or repeating decimal, then it is rational. e.g: $1/2 = 0.5$	If a number is non-terminating and non-repeating decimal, then it is irrational. e.g: 0.31545673...
Example:- $1/2$, $3/4$, $11/2$, 0.45, 10, etc.	example:-Pi (π) = 3.14159...., Euler's Number (e) = (2.71828...), and $\sqrt{3}$, $\sqrt{2}$.



Basic MATHEMATICAL operations

- BODMAS

- B - Bracket () , { } , []
- O - Order
- D - Division
- M - Multiplication
- A - Addition
- S - Subtraction.



BASIC FORMULAE

- 1. $(a + b)^2 = a^2 + b^2 + 2ab$
- 2. $(a - b)^2 = a^2 + b^2 - 2ab$
- 3. $(a + b)^2 - (a - b)^2 = 4ab$
- 4. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
- 5. $(a^2 - b^2) = (a + b)(a - b)$
- 6. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- 7. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
- 8. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
- 9. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
- 10. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$



Basic NUMBER Representation

- Place Value : Units, Tens, Hundreds,
- Value of a 2 digit no. 'ab' where both a & b are natural numbers = $10(a) + b$
- The number with reversed digits will be 'ba' & the value of the number will be = $10(b) + a$



Numbers

Q. A number consists of two digits.

Sum of the digits is 9. If 63 is subtracted from the number its digits are interchanged. Find the number.

- A. 72
- B. 90
- C. 63
- D. 81

Solution :

Let the tens digit be a & units digit be b

$$a + b = 9 \quad \text{.....(1)}$$

$$10a + b - 63 = 10b + a$$

$$9a - 9b = 63 \quad \text{.....(2)}$$

$$(1) \times 9$$

$$9a + 9b = 81 \quad \text{.....(3)}$$

$$(2) + (3)$$

$$18a = 144 \rightarrow a = 8, b = 1$$

$$\text{Number} = 81$$

Ans: D



Sum of Natural Numbers

- **Rule 1** : Sum of first n natural numbers = $\frac{n(n+1)}{2}$

e.g. sum of 1 to 74 = $74 \times (74+1)/2 = 2775$.

- **Rule 2** : Sum of first n odd numbers = n^2

e.g. sum of first seven odd numbers

= $(1+3+5+7+9+11+13) = 49 = 7^2$.

- **Rule 3** : Sum of first n even numbers = $n(n+1)$

e.g. sum of first 9 even numbers

= $(2+4+6+8+10+12+14+16+18) = 90$

= $9(9+1) = 9 \times 10 = 90$



Sum of Natural Numbers

- **Rule 4** : Sum of squares of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$

e.g. sum of squares of first 8 natural numbers

$$= (1 + 4 + 9 + 16 + 25 + 36 + 49 + 64) = 204$$

$$= 8 (8+1)(16+1) / 6 = 8 \times 9 \times 17 / 6 = 204$$

- **Rule 5** : Sum of cubes of first n natural numbers = $[n(n+1)/ 2]^2$

e.g. sum of cubes of first 4 natural numbers

$$= (1 + 8 + 27 + 64) = 100$$

$$= [4 (4+1)/2]^2 = 100$$



DIVISION

- DIVISION by ZERO is NOT POSSIBLE
- If two numbers are divisible by a number then their sum & difference is also divisible by the number.
- E.g. For 63 is divisible by 9. 27 is also divisible by 9.
- So $63 + 27 = 90$ is also divisible by 9
- And $63 - 27 = 36$ is also divisible by 9



DIVISIBILITY RULES

- 2 : Unit place is even or zero(last digit should be divisible by 2)
- 3 : Sum of the digits is divisible by 3. e.g : 324
- 4 : Last 2 digits are divisible by 4 or last 2 digits are 0. e.g : 324
- 5 : Unit digit is 5 or 0
- 6 : **Divisible by co primes 2 & 3.** e.g : 324
- 8 : Number formed by last 3 digits is divisible by 8 or last 3 digits are 0.
e.g : 1088
- 9 : Sum of all digits is divisible by 9. e.g : 324
- 10: Units digit is 0.



DIVISIBILITY RULES

- **11** : Difference between sum of digits in odd & even places should either be zero or divisible by 11

e.g: 8283

e.g : 918071

- **12** : Divisible by co primes 3 & 4 e.g : 324
- **14** : Divisible by co primes 2 & 7
- **15** : Divisible by co primes 3 & 5
- **16** : No formed by last 4 digits divisible by 16/ last 4 digits 0.
- **18** : Divisible by co primes 2 & 9
- **20** : Units digit 0 & tens digit is even.



DIVISIBILITY RULES

- **7** : The difference between the two alternate groups taking 3 digits at a time should either be zero or multiple of 7.

eg- 550500006

eg- 7370356

- **13** : The difference between the two alternate groups taking 3 digits at a time should either be zero or multiple of 13.

eg- 200174



DIVISIBILITY RULES

- **17: A number is divisible by 17 if you multiply the last digit by 5 and subtract that from the rest. If that result is divisible by 17, then your number is divisible by 17.**
 - For example, for 986, then : $98 - (6 \times 5) = 68$.
 - Since, 68 is divisible by 17, then 986 is also divisible by 17.
 - Also, 876 is not divisible by 17 because $87 - (6 \times 5) = 57$ and 57 is not divisible by 17.
- **19: To determine if a number is divisible by 19, take the last digit and multiply it by 2. Then add that to the rest of the number. If the result is divisible by 19, then the number is divisible by 19.**
 - For example, 475 is divisible by 19 because $47 + (5 \times 2) = 57$, and 57 is divisible by 19.
 - But , 575 is not divisible by 19 because $57 + (5 \times 2) = 67$, and 67 is not divisible by 19.



PROPERTIES OF DIVISIBILITY

To find a number completely divisible by another :

A) Greatest 'n' digit number exactly divisible by a Number :

Method : By subtracting the remainder

e.g a) Greatest 3 digit number divisible by 13

Greatest 3 digit number = 999. $999/13$ gives remainder 11.

$999 - 11 = 988$ = Greatest 3 digit number divisible by 13

B) Least 'n' digit number exactly divisible by a Number :

Method : By adding the (divisor – remainder)

e.g b) Least 3 digit number divisible by 13

Least 3 digit number = 100. $100/13$ gives remainder 9

$100 + (13 - 9) = 104$ = Least 3 digit number divisible by 13



PROPERTIES OF DIVISIBILITY

Q. On dividing a number by 999, the quotient is 366 and the remainder is 103. The number is:

A.364724 B.365387 C.365737 D.366757 E. None of these

Soln-

$$\text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}$$

$$\begin{aligned}\text{Required number} &= 999 \times 366 + 103 \\ &= (1000 - 1) \times 366 + 103 \\ &= 366000 - 366 + 103 \\ &= 365737\end{aligned}$$

Ans: C



PROPERTIES OF DIVISIBILITY

Q. A number when divided by 5 leaves 3 as remainder. If the square of the same number is divided by 5, the remainder obtained is :

A. 9

B. 4

C. 1

D. 3

Soln:

number when divided by 5 leaves a remainder 3

Let the given number = $5n + 3$ ---> using dividend = divisor quotient + remainder

Square of the number = $(5n + 3)^2$

$$= 25n^2 + 30n + 9 \rightarrow (a + b)^2 = a^2 + 2ab + b^2$$

$$= 5 \times 5n^2 + 5 \times 6n + 5 + 4$$

$$= 5(5n^2 + 6n + 1) + 4$$

Required remainder = 4

Ans: B



PRIME NUMBERS

- A number that is divisible only by itself and 1 (e.g. 2, 3, 5, 7, 11).
- There are **25** prime numbers between 1 - 100
- *1 is neither prime nor composite number.*
- **2 is the only prime number which is even.**
- A number having more than 2 factors is a composite number
- Find prime numbers between 101 and 200??
- There are **21** prime numbers between 101 - 200



Co-Prime

- When two numbers (they may not be prime) do not have any common factor other than one between them they are called co-prime or relatively prime.
- It is obvious that two prime numbers are always co-prime. e.g : 17 and 23
- Two composite numbers can also be co-prime. e.g: 16 & 25 do not have any common factor other than one.
- Similarly 84 and 65 do not have any common factor and hence are co-prime.



Prime Number

- **Sieve of Eratosthenes** is the fastest technique to find whether given number is prime or composite number.
- Let **p** be a given number and **n** be the smallest counting number such that $n^2 \geq p$.
- Ex: check 811 is prime or not. $29^2 > 811$.
- check if 811 is divisible by any prime number below 29 (2,3,5,7,11,13,17,19,23,29).
- none of the prime numbers divides 811.
- 811 is a prime number.



Prime Number

Q. Find whether 467 is prime or not

Step 1 : Sq root of 467 → Between 21 (441) and 22 (484)

Step 2 : 467 is not divisible by 2, 3, 5, 7, 11, 13, 17, 19. Next prime is 23 which exceeds the square limit.

Therefore 467 is prime.



Prime Number(Assignment)

Q. Which of the following is a prime number?

A. 303

B. 477

C. 113

D. None of these

Ans : C



Numbers(Assignment)

Which of the following is the output of $57 \times 57 + 43 \times 43 + 2 \times 57 \times 43$?

A. 10000

B. 5700

C. 4300

D. 1000

Ans : A



Numbers(Assignment)

Q. Which of the following is the output of 6894×99 ?

A. 685506

B. 682506

C. 683506

D. 684506

Ans: B



Numbers(Assignment)

Q. What is the unit digit in $584 \times 428 \times 667 \times 213$?

A. 2

B. 3

C. 4

D. 5

Ans: A



Numbers(Assignment)

Q. The sum of reciprocals of two consecutive numbers is $\frac{15}{56}$. The first number is

- A. 8 B. 7 C. 6 D. 15.

Ans : B



Divisibility (Assignment)

Q. What percentage of the numbers from 1 to 50 have squares ending in the digit 1?

A. 1 B. 10 C. 11 D. 20

Ans : D



Numbers(Assignment)

Q. If $64^2 - 36^2 = 20 \times A$, then $A = ?$

- A. 70 B. 120 C. 180 D. 140 E. None of these

Ans: D



Numbers(Assignment)

Q. On dividing a number by 19 the difference between quotient and remainder is 9. The number is?

A. 370

B. 371

C. 361

D. 352

Ans : B



Numbers(Assignment)

Q. $(112 \times 5^4) = ?$

A. 67000

B. 70000

C. 76500

D. 77200

E. None of these

Ans: B



Numbers(Assignment)

Q. Which of the following is a prime number?

A. 143

B. 289

C. 117

D. 359

Ans : D



HCF & LCM

HCF / GCF(Highest/Greatest Common Factor)

- HCF of two or more numbers is the greatest / largest / highest/biggest number which can divide those two or more numbers exactly.

Factors of 6 : 1, 2, 3, 6

Factors of 8 : 1, 2, 4, 8

Common 1 & 2 Highest & Common 2

• LCM(Least Common Multiple)

- The LCM of two or more numbers is the smallest / lowest / least number which is exactly divisible by those two or more numbers.

Multiples of 6 : 6, 12, 18, 24, 30, 36, 42, 48, 54,...

Multiples of 8 : 8, 16, 24, 32, 40, 48, 56, 64....

Common 24, 48, Lowest & common 24



HCF (Factorization method)

- Eg. HCF for 136, 144, 168

2	136	144	168
2	68	72	84
2	34	36	42
	17	18	21

↓
NO FURTHER COMMON FACTOR

So HCF = $2 \times 2 \times 2 = 8$

Note : HCF is always \leq the smallest of given numbers



HCF (Factorization method) - (Assignment)

- HCF of 54, 72, 126 (factorization method)

A. 21 B. 18 C. 36 D. 54

Ans : B



HCF (Difference Method)

- Find HCF of 203,319

Keep smaller here



- (203, 319)
- (116,203)
- (87,116)
- (29,87)
- (29,58)
- (29,29)



HCF = 29



HCF (Difference Method) - (Assignment)

- HCF of 161,253 (difference method)

A. 27 B. 18 C. 23 D. 17

Ans : C



HCF (Difference Method)

Q. Find HCF of 84,125

- (84,125)
 - (41,84)
 - (41,43)
 - (2,41)
 - (2,39)
-
- If nothing is common then $HCF = 1$ and numbers are said to be co prime numbers.



HCF & LCM

Q. Find the greatest number which can divide 284, 698 & 1618 leaving the same remainder 8 in each case?

- A. 36 B. 46 C. 56 D. 43.

Soln-

Remainder 8 \rightarrow (numbers – 8) would be exactly divisible.

$$\rightarrow 284 - 8 = 276$$

$$\rightarrow 698 - 8 = 690$$

$$\rightarrow 1618 - 8 = 1610$$

\rightarrow Greatest number dividing above 3 = HCF(276, 690, 1610) (difference method)

$$\rightarrow \text{HCF} = 46$$

Ans: B



HCF & LCM

Q. Find the greatest number which can divide 62, 132 & 237 leaving the same remainder in each case?

- A. 35 B. 46 C. 56 D. 43.

Soln:-

If two numbers a & b are divisible by a number n then

→ Their difference (a-b) is also divisible by n.

$$\rightarrow 132 - 62 = 70$$

$$\rightarrow 237 - 132 = 105$$

$$\rightarrow 237 - 62 = 175$$

→ Greatest number dividing above 3 = HCF(70, 105, 175)

$$\rightarrow \text{HCF} = 35$$

Ans: A



HCF & LCM

Q. Find the largest number such that 43,65,108 are divisible by that number and we get the remainder as 1,2,3 respectively in each case?

- A. 21 B. 27 C. 42 D. 63

Soln:

→ (numbers – remainder) would be exactly divisible.

$$\rightarrow 43 - 1 = 42$$

$$\rightarrow 65 - 2 = 63$$

$$\rightarrow 108 - 3 = 105$$

$$\text{HCF}(42, 63, 105) = 21$$

Ans : A



HCF & LCM

Q. A teacher has 25 books, 73 pens & 97 erasers. She wants to distribute them equally to maximum number of students so that after distribution she has equal number of books, pens & erasers left. What is the maximum number of students for such a distribution?

A. 32

B. 21

C. 12

D. 24

Soln:-

If two numbers a & b are divisible by a number n then

→ Their difference (a-b) is also divisible by n.

$$\rightarrow 73 - 25 = 48$$

$$\rightarrow 97 - 73 = 24$$

$$\rightarrow 97 - 25 = 72$$

→ Greatest number dividing above 3 = $\text{HCF}(72, 48, 24)$

$$\rightarrow \text{HCF} = 24$$

Ans: D



HCF & LCM(Assignment)

Q. Find the greatest number which can divide 62, 132 & 237 leaving the same remainder in each case?

- A. 35 B. 46 C. 56 D. 43.

Ans : A



HCF & LCM(Assignment)

Q. Find largest number such that if 45,68 and 113 are divided by that number we get the remainder as 1,2 and 3 respectively.

- A. 21 B. 22 C. 26 D. 24

Ans: B



HCF & LCM(Assignment)

Q. Find the greatest number which can divide 41, 131 & 77 leaving the same remainder in each case?

A. 28

B. 18

C. 36

D. 24

Ans : B



LCM

- Eg. LCM for 18, 28, 108, 105

2	18	28	108	105
2	9	14	54	105
3	9	7	27	105
3	3	7	9	35
3	1	7	3	35
5	1	7	1	35
7	1	7	1	7
Till all quotients are 1	1	1	1	1

So LCM = $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 3780$

Note : LCM is always \geq the greatest of given nos



LCM(Assignment)

Q. LCM for 12,24,20

A. 210

B. 180

C. 120

D. 144

Ans : C



LCM (Assignment)

Q. Find LCM of 72,125

A. 9000 B. 1200 C. 1000 D. 800

Ans : A



Rules to Remember

- Product of two given numbers is equal to the product of their HCF & LCM

$$A \times B = \text{HCF}(A,B) \times \text{LCM}(A,B)$$

- If a, b, c are three numbers that divide a number n to leave the same remainder r, the smallest value of 'n' is

$$n = (\text{LCM of } a, b, c) + r \quad \text{e.g } 3, 4, 5 \text{ \& rem } 1$$



Q. Find LCM of 147 & 231

Soln:-

- As we know,
- **HCF X LCM = product**
- Find HCF by difference method
- Put in the formula,
- $21 \times \text{LCM} = (147 \times 231)$
- 1617

Q. Find LCM of 84 and 125

Soln:-

- As they are co-prime numbers the product is the LCM because $HCF = 1$ (for co-primes)
- $HCF \times LCM = \text{product}$
- $1 \times LCM = 84 \times 125$
- $LCM = 10500$



LCM

Q. Find the least number which when divided by 12,15,24 leaves a remainder of 5 in each case

- **Soln:**
- Find $\text{LCM}(12,15,24) = ?$

If a, b, c are three numbers that divide a number n to leave the same remainder r, the smallest value of 'n' is

$$n = (\text{LCM of } a, b, c) + r \quad \text{e.g } 3,4,5 \text{ \& rem } 1$$

- $\text{LCM} = 120$
- In an LCM problem, if remainder is common then,
Result = LCM + common remainder
 $= 120 + 5 = 125$



Q. Find the smallest number which when divided by 20,36,45 leaves a remainder 15,31 and 40 respectively.

- **Soln:**
- Find LCM(20,36,45)
- In LCM problem , if difference is common(constant) then,
- **Result = LCM – Common difference**

• 20	36	45	} 5
• 15	31	40	

- Result = $180 - 5$
 $= 175$

Q. Four numbers are in the ratio of 10: 12 : 15 : 18. If their HCF is 3, then find their LCM.

A. 420

B. 540

C. 620

D. 680

Ans : B



Q. Find the least number which when divided by 5,6,7 and 8 leaves a reminder of 3 but when divided by 9 leaves no remainder.

A. 1677

B. 2523

C. 3363

D. 1683

Ans: D



HCF/LCM with Decimal point

- Find HCF of 1.08, 0.36 and 0.9

- **Soln:**

1. Convert each of the decimals into like decimals.

1.08, 0.36 and 0.90

2. Write each number without decimal point.

$\text{HCF}(108, 36, 90) = 18$

3. Put decimal point after the numbers which are in like decimals.

Here it is after 2 numbers(digits)

$\text{HCF}(1.\underline{08}, 0.\underline{36} \text{ and } 0.\underline{90}) = 0.\underline{18}$



HCF(Assignment)

Q. In a school of 437 boys & 342 girls it was decided to divide the girls & boys into separate classes. However it was required that each class consist of the same number of students. What would be the number of classrooms required?

A. 41 classrooms B. 14 classrooms C. 17 classrooms D. 26 classrooms

Ans : A

Same Class Size = HCF (Boys, Girls)

→ $\text{HCF}(437, 342) = 19$

→ Boys Classes = $437/19 = 23$

→ Girls Classes = $342/19 = 18$

→ Total Classes = $23 + 18 = 41$



LCM(Assignment)

Q. Find the least number which when divided by 12,15,40 leaves a remainder of 5 in each case

A. 120

B. 125

C. 130

D. 140

Ans : B



LCM(Assignment)

Q. If the product of two numbers is 324 and their HCF is 3, then their LCM will be = ?

A. 972 B. 327 C. 321 D. 108

Ans: D



LCM(Assignment)

Q. Three number are in the ratio of 3 : 4 : 5 and their L.C.M. is 2400. Their H.C.F. is:

A. 40

B. 80

C. 120

D. 200

Ans: A



LCM(Assignment)

Q. Find the least number which when divided by 16,18,20 and 25 leaves a reminder of 4 but when divided by 7 leaves no remainder.

A. 17004

B. 18000

C. 18002

D. 18004

Ans: D



HCF & LCM(Assignment)

Q. The HCF of two numbers is 8. Which one of the following can never be their LCM ?

- A. 24 B. 48 C. 56 D. 60

Ans: D

If $HCF = 8$ then LCM should have a factor of 8

Going by options 60 does not have a factor 8. So never be their LCM.



HCF & LCM(Assignment)

Q. The LCM of three different numbers is 120. Which of the following cannot be their HCF?

A. 8

B. 12

C. 24

D. 35

Ans: D



Numbers(Assignment)

Q. The number nearest to 43582 divisible by each of 25, 50 and 75 is ?

A. 43500

B. 43550

C. 43600

D. 43650

Ans: D



Numbers(Assignment)

Q. What is the smallest 5 digits number which is divisible by 12, 15, and 18?

A.10010

B. 10015

C.10020

D. 10080

Ans: D



Rules to Remember

- **Fractions :**

LCM = LCM of Numerators / HCF of Denominators

HCF = HCF of Numerators / LCM of Denominators

LCM of $25/12$ & $35/18$

LCM = $175/6$

HCF of $25/12$ & $35/18$

HCF = $5/36$



HCF & LCM Fractions(Assignment)

- Find HCF & LCM of $\frac{5}{9}$ and $\frac{25}{36}$
- Ans : HCF = $\frac{5}{36}$ and LCM = $\frac{25}{9}$



Properties of Square Numbers

- A square can't end with odd number of zeroes. The number of 0's of perfect square is always even and the non-zero part should also be a perfect square.

- A square can't end with 2, 3, 7 or 8.

1	2	3	4	5
6	7	8	9	0

- Square of **odd** no. is **odd** & **even** no. is **even**
- Whenever last digit of square is 6, then second last digit is always odd.
- Whenever last digit of square is 5, then second last digit is always 2.
- Whenever last digit of square is 1,4,9, then second last digit is always even.



Squares

Q. A man plants his orchard with 15876 trees & arranges them so that there are as many rows as there are trees in each row. How many rows does the orchard have?

- A. 124 B. 134 C. 126 D. 136

- **Soln:-**

- No of trees = No. of rows x no of trees/row
- $15876 = n \times n$
- $n = \sqrt{15876}$
- $n = \sqrt{9 \times 1764}$
- $= \sqrt{9 \times 9 \times 196}$
 $= ?$
 $= 9 \times 14$
 $= 126$

- **Ans C**



Squares(Assignment)

Q. Find a positive number x , such that the difference between the square of this number and 21 is the same as the product of 4 times the number?

- A. 9 B. 27 C. 7 D. 13

Ans : C



Progression

- Arithmetic Progression :

- If quantities increase or decrease by a common difference then they are said to be in AP e.g. 3, 5, 7, 9, 11,
- If a is first term, d is the common difference, l is the last term then
- General form : $a, a+d, a+2d, a+3d, \dots, a+(n-1)d$
- n^{th} term $T_n = a + (n-1)d$, **$n = 1, 2, \dots$**
- Sum of first n terms $S_n = \frac{n}{2} [2a + (n-1)d]$
 $S_n = \frac{n}{2} (a + l)$



Progression

- Prove that the sum S_n of n terms of an Arithmetic Progress (A.P.) whose first term 'a' and common difference 'd' is
- $S = n/2[2a + (n - 1)d]$
- Or, $S = n/2[a + l]$, where l = last term = $a + (n - 1)d$
- **Proof:**
- $a, a+d, a+2d, a+3d, \dots, a(n-2)d, a(n-1)d$, as l = last term
- $a, a+d, a+2d, a+3d, \dots, l-d, l$
- $S = a + a+d + a+2d + a+3d + \dots + l-d + l$ -----1
- Writing equation 1 in reverse order(sum remains same even if we write in reverse order)
- $S = l + l-d + l-2d + l-3d + \dots + a+d + a$ -----2
- Adding equation 1 and 2
- $2S = (a + l) + (a + l) + (a + l) + \dots + (a + l) + (a + l)$
- So for n terms,
- $2S = n(a + l)$
- $S = \frac{n}{2} (a + l)$



Progression

Q. The sum of all two digit numbers divisible by 3 is

A. 550

B. 1550

C. 1665

D. 1680

Soln

Two digit numbers divisible by 3 are :

12, 15, 18, 21,, 96, 99.

This is an A.P. with $a = 12$, $d = 3$, $l = 99$

Let n be the number of terms.

Last term $= a + (n-1)d$

$$99 = 12 + (n-1) \times 3$$

$$3n = 90, \quad n = 30$$

$$\begin{aligned} \text{Sum} &= n/2 (a + l) = 30/2 \times (12 + 99) \\ &= 1665 \end{aligned}$$

Ans: C



Progression

Q. Find the sum of all natural numbers between 10 and 200 which are divisible by 7

A. 2835

B. 2865

C. 2678

D. 2646

Soln:

Two digit numbers divisible by 7 are :

14, 21, 28, 35,, , 196.

This is an A.P. with $a = 14$, $d = 7$, $l = 196$

Last term $= a + (n-1)d$

$$196 = 14 + (n-1) \times 7$$

$$196 - 14 = (n-1) \times 7$$

$$n-1 = 26$$

$$n = 27$$

$$\text{Sum} = n/2 (a + l)$$

$$= 27/2 \times (14 + 196)$$

$$= 27 \times 210 / 2$$

$$= 27 \times 105$$

$$= 2835$$

OR

$$n = \frac{\text{LastTerm} - \text{FirstTerm}}{d} + 1$$

Ans: A



Progression(Assignment)

Q. Find the sum of the series 3,8,13,18,,93

A. 912

B. 925

C. 998

D. 936

Ans : A



Progression

- Geometric Progression :
- If quantities increase or decrease by a constant factor then they are said to be in GP e.g. 4, 8, 16, 32,
- If a is first term, r is the common ratio, then
- General form : a, ar, ar², ar³,, arⁿ⁻¹
- nth term $T_n = ar^{(n-1)}$
- Sum of first n terms $S_n = \frac{a(r^n - 1)}{(r - 1)}$



Geometric Progression of n terms :

- To prove that the sum of first n terms of the Geometric Progression whose first term 'a' and common ratio 'r' is given by-

- $S = a + ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1} \quad \text{-----} \quad 1$

- Multiply both sides of this equation by r

- $Sr = ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1} + ar^n \quad \text{-----} \quad 2$

- - - - -

- Eq 2 - Eq 1

- $Sr - S = ar^n - a$

- $S(r - 1) = a(r^n - 1)$

- $S = \frac{a(r^n - 1)}{(r - 1)}$



Geometric Progression

Q. Find the 10th term of the series: 4, 16, 64, 256, 1024,

- A. 4^{10} B. 4^8 C. 4^9 D. 1022480

Soln:

The given series is in geometric progression

Where $a = 4$, $r = 4$

$$\begin{aligned}\text{So } T_{10} &= a \times r^{(10-1)} \\ &= 4 \times 4^{(10-1)} \\ &= 4^{10}\end{aligned}$$

Ans: A



Progression

- What is the difference between arithmetic progression and geometric progression?
- A sequence is a set of numbers, called terms, arranged in some particular order. An arithmetic sequence is a sequence with the difference between two consecutive terms constant. The difference is called the common difference. A geometric sequence is a sequence with the ratio between two consecutive terms constant.



