MIDDLE EAST TECHNICAL UNIVERSITY

DEPARTMENT OF
ELECTRICAL AND ELECTRONICS
ENGINEERING

EE568 - Special Topics on Electrical Machines
Project #1
Torque in a Variable Reluctance Machine

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1. Contents

2. Introduction

Variable Reluctance Machines are a machine that has a rotor which shorts the magnetic field, i.e. reduce the reluctance of the stator when aligned with this magnetic field. Thus it rotates with the field in the stator as the field rotates. The rotor does not have any windings. It generates torque through magnetic reluctance. The inductance of the machine is high when the rotor and stators are aligned, and low when the rotor and stators are not aligned. Many synchronous machines have highly salient rotors. They produce torque by magnetic attraction, but also a much smaller amount just like variable reluctance machines. They can operate as motors without a field applied. Reluctance motors can deliver high power density at low cost, making them attractive for many applications. Disadvantages include high torque ripple (the difference between maximum and minimum torque during one revolution) when operated at low speed, and noise due to torque ripple.

In this project, a variable reluctance machine is analyzed and modeled. The dimensions of the electrical machine are given in Figure 1. The objective is to observe the flux density, torque, inductance reluctance etc. variation with the rotational part position. The analysis procedure has started with analytical calculation. In order to verify the analytical calculations, ANSYS Maxwell is used as FEA tool.

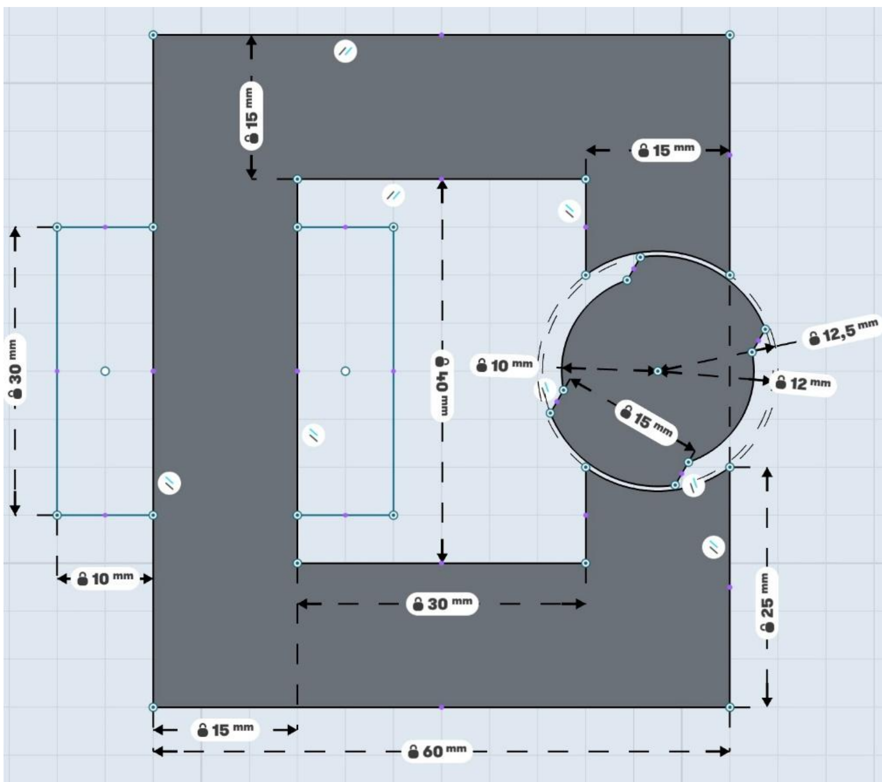


Figure 1. Mechanical dimensions of variable reluctance machine

Other specifications of the machine are listed as:

- Coils are wound within 30mmx10mm rectangle areas
- Each air gap clearance is 0.5mm
- Depth of the core is 20mm

- Number of turns = 250
- Coil Current = 3 A DC

3. Analytical Modeling

3.1. Reluctance calculation

In order to calculate the reluctance, inductance and torque analytically, some assumptions are considered. First of all, the core and rotor material is assumed to be infinitely magnetically permeable. Thus, the only path that has reluctance is air gap. As can be seen from below Fig. 2., the air gap changes during the magnetic flux path. Second assumption is that, the straight line in rotor is ignored for reluctance calculation. It is assumed that the flux flows either through the circle with 12mm radius and the circle with 10mm radius.

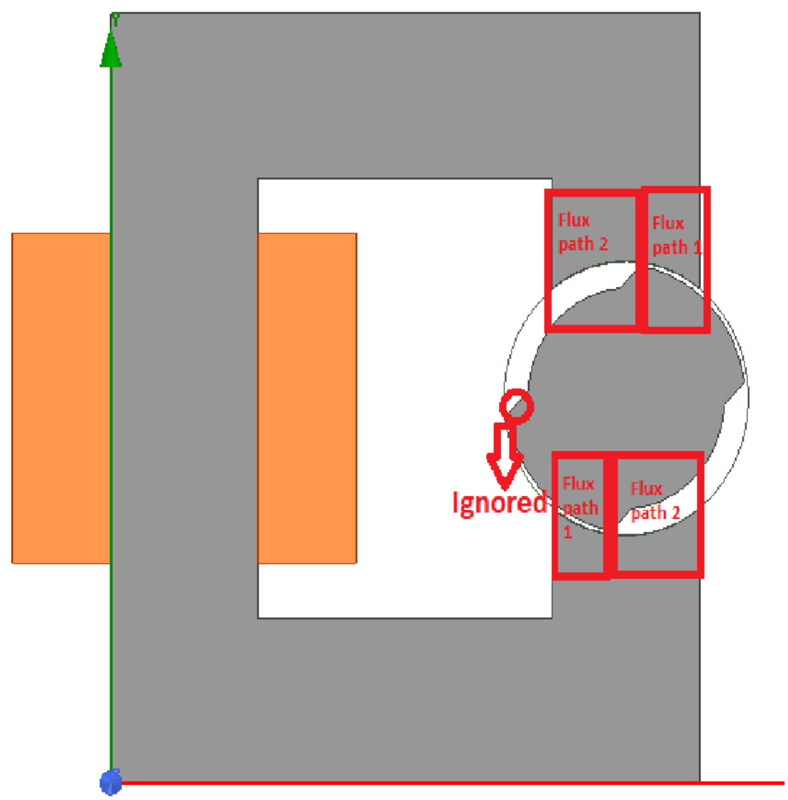
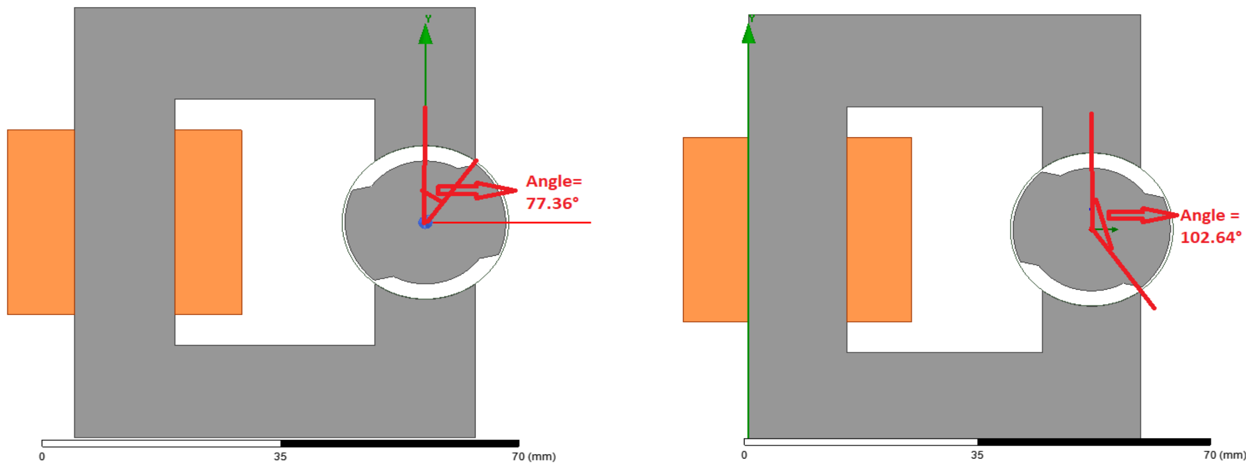


Figure 2. Possible flux paths by considering assumptions

The equivalent reluctance is the parallel of the reluctances of these two flux path shown in Fig. 2. When the rotor is fully aligned with stator, i.e. angle is 0° and 180°, the reluctance is equal to reluctance of flux path 1. As the angle increases, the equivalent reluctance increases because of increased the effective air gap length. As shown below in Fig.3.a., when the angle is equal to 77.36°, the flux path 2, starts to become the possible flux path. As shown below in Fig.3.b., when the angle is equal to 102.64°, two flux path again become the possible flux paths similar to first part.



a) b)

Figure 3. a) Rotor position at angle = 77.36° , b) Rotor position at angle = 102.64°

In part 1, the angle is between 0°-77.36° degrees, the flux is divided between two path. To calculate the equivalent reluctance in this part, it is assumed two parallel reluctance for flux path as mentioned above. For part1, the reluctances of path 1, path 2 and equivalent reluctance are given in equation (1), (2) and (3) respectively.

$$R_{\text{path1\_part1}} = \frac{l_{12mm}}{\mu_0 A_{12mm}} = \frac{2*0.5*10^{-3}}{\mu_0*\frac{(77.36^\circ-\Theta)}{360^\circ}*2\pi*12*10^{-3}*20*10^{-3}} = 527.8*10^3*\left(\frac{360^\circ}{(77.36^\circ-\Theta)}\right)\left(\frac{1}{Henry}\right) \quad (1)$$

$$R_{\text{path2\_part1}} = \frac{l_{10mm}}{\mu_0 A_{10mm}} = \frac{2*2.5*10^{-3}}{\mu_0*\frac{\Theta}{360^\circ}*2\pi*12*10^{-3}*20*10^{-3}} = 3166.3*10^3*\left(\frac{360^\circ}{\Theta}\right)\left(\frac{1}{Henry}\right) \quad (2)$$

$$R_{eq\_part1} = \frac{R_{\text{path1\_part1}}*R_{\text{path2\_part1}}}{R_{\text{path1\_part1}}+R_{\text{path2\_part1}}} \quad \left(\frac{1}{Henry}\right) \quad (3)$$

In part 2, the angle is between 77.36°-102.64° degrees, it is assumed that all the flux is flowing through path2. Therefore, reluctance is constant for this part it does not depends on rotation angle. The reluctance for part2 is given below in equation (4).

$$R_{eq\_part2} = R_{\text{path2}} = \frac{l_{10mm}}{\mu_0 A_{10mm}} = \frac{2*2.5*10^{-3}}{\mu_0*\frac{77.36}{360^\circ}*2\pi*12*10^{-3}*20*10^{-3}} = 14735*10^3\left(\frac{1}{Henry}\right) \quad (4)$$

In part 3, the angle is between 102.64°-180° degrees, similar to the part 1, the flux is divided to two path. For part3, the reluctances of path 1, path 2 and equivalent reluctance are given in equation (5), (6) and (7) respectively.

$$R_{\text{path1}\_part3} = \frac{l_{12mm}}{\mu_0 A_{12mm}} = \frac{0.5*10^{-3}*2}{\mu_0*\frac{(\Theta-102.64^{\circ})}{360^{\circ}}*2\pi*12*10^{-3}*20*10^{-3}} = 527.8*10^3*\left(\frac{360^{\circ}}{(\Theta-102.64^{\circ})}\right)\left(\frac{1}{Henry}\right) \quad (5)$$

$$R_{\text{path2}\_part3} = \frac{l_{10mm}}{\mu_0 A_{10mm}} = \frac{2.5*10^{-3}*2}{\mu_0*\frac{(180^{\circ}-\Theta)}{360^{\circ}}*2\pi*12*10^{-3}*20*10^{-3}} = 3166.3*10^3*\left(\frac{360^{\circ}}{180-\Theta}\right)\left(\frac{1}{Henry}\right) \quad (6)$$

$$R_{eq\_part3} = \frac{R_{\text{path1\_part3}}*R_{\text{path2\_part3}}}{R_{\text{path1\_part3}}+R_{\text{path2\_part3}}} \quad \left(\frac{1}{Henry}\right) \quad (7)$$

The inductances for these three part are also calculated separately by using below formula.

$$\mathrm{L}_{\text{eq\_part(x)}} = \frac{\mathrm{N}^2}{\mathrm{R}_{\text{eq\_part(x)}}} \text{ (Henry)} \quad (8)$$

where, N is the number of turns which is 250.

Finally, resultant torque is calculated by using below formula.

$$\mathrm{T}_{\text{eq}} = 0.5*\mathrm{i}^2*\frac{\mathrm{d}_{\mathrm{L}(\Theta)}}{\mathrm{d}_{\Theta}} \text{ (N.m)} \quad (9)$$

where, i is the current which is 3A. L(Θ) is the resultant inductance function of angle over one full rotation of rotor.

3.2. Analytical Results

The resultant reluctance, inductance and torque versus rotor position angle are calculated based on above equations in MATLAB and given below in Figure 4.

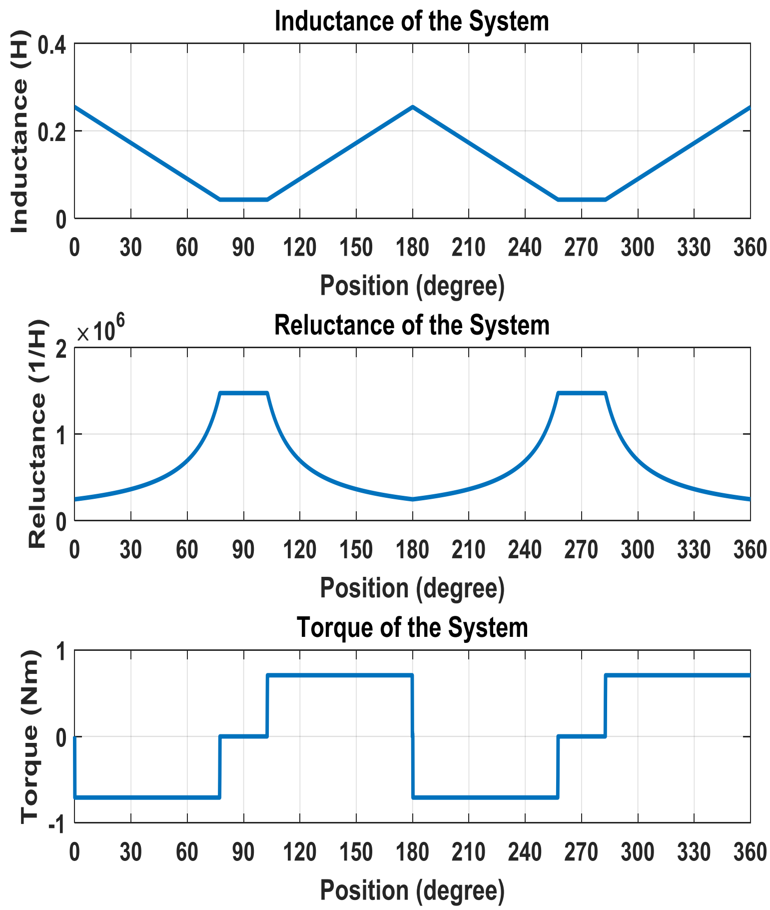


Figure 4. Analytically calculated reluctance, inductance and torque versus rotor position

3.3. Further improvements on analytical model

As stated before, the reluctance path is divided into two path which are two circles one of them is with 10mm radius, other one is 12mm radius. To improve analytical model, reluctance paths can be divided into more paths and straight part of rotor can be taken into account. In summary, the effective air gap can be modeled more accurately to improve analytical model.

Moreover, we assumed the core and rotor with infinite relative permeability but actually they have finite permeability. Also sharp corners, fringing flux can be considered to improve analytical model.

4. 2D FEA Modelling with Linear Magnetic Material

4.1. Flux density distributions at rotor angle 0° - 45° - 90°

In this section, the machine is modeled in Maxwell 2D. As material, iron with relative permeability 4000 has chosen. The resultant magnetic flux vectors for three position of rotor are given in Figure 5.

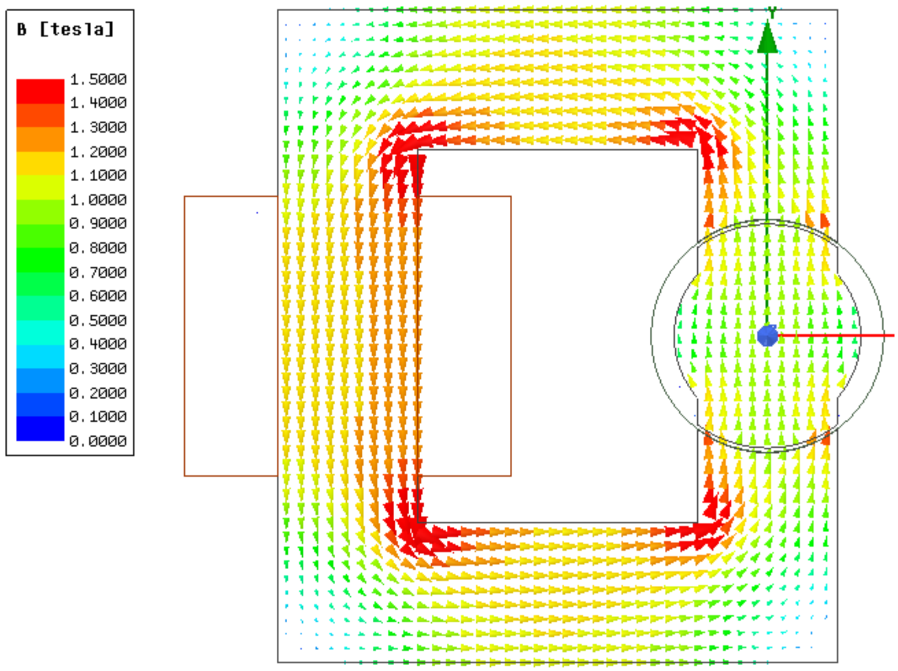
As can be seen from Fig. 5. the magnetic flux distribution for all three cases is not uniform in the core. As expected, the flux wants the flow through minimum reluctance path which result in such a distribution waveforms. In an other words, the flux is concentrated on the corner points of stator and rotor. At rotation angle of 45° where, rotor and stator fully aligned, the reluctance is minimum and hence inductance is maximum point. Therefore, maximum flux density occurs when to rotor is in this position. At rotation angle of 45°, the reluctance is increased and hence the inductance is reduced. As can be seen from Fig. 5.b. in this position, the flux density at the sharp edges of rotor and stator is increased. At rotation angle of 90°, the reluctance is at maximum point and hence inductance at its minimum point. There is almost not flux flowing. Also the torque on the rotor is also zero at this angle.

4.2. Inductance, stored energy and torque at rotor angle 0° - 45° - 90°

Inductance, stored energy and torque results of linear FEA model are given in Table 1. It can be seen that at 0°, the magnetic energy stored in the core is the highest since the inductance is also at maximum value. Also, inductance is decreases as the angle increases. Notice that, the torque is almost zero for angle 0° and 90°.

Table 1: Inductance, stored energy and torque at three different rotor position for linear material FEA results

Rotor angle	Inductance (mH)	Stored Energy (J)	Torque (mNewtonMeter)
0°	29.68	0.1348	7.38
45°	18.67	0.0855	-81.2
90°	8.74	0.0396	3.73



B [tesla]
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1.1000
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0.8000
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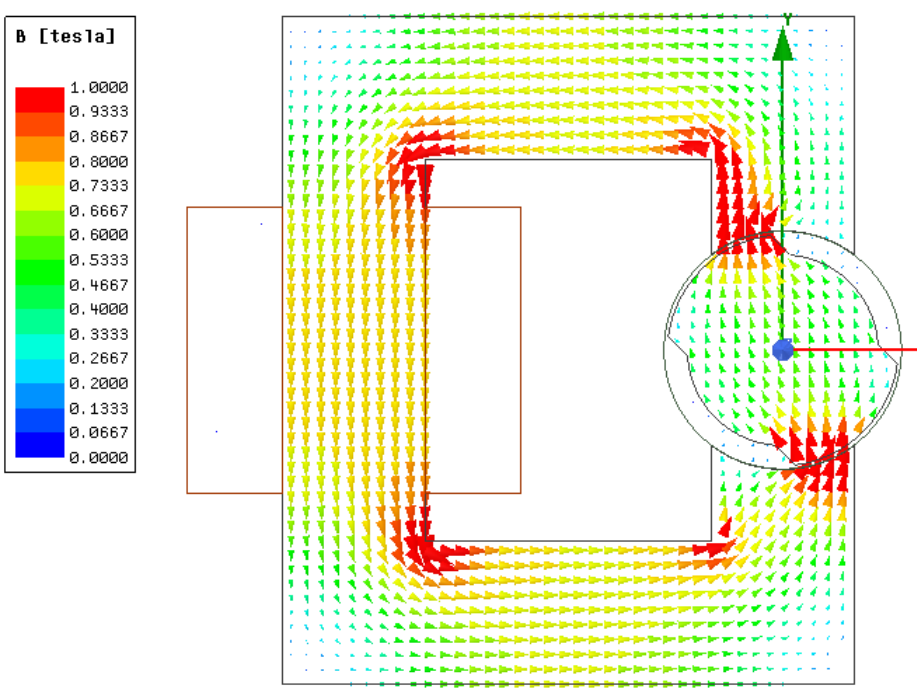
0.4000
0.3000
0.2000
0.1000
0.0000

a)



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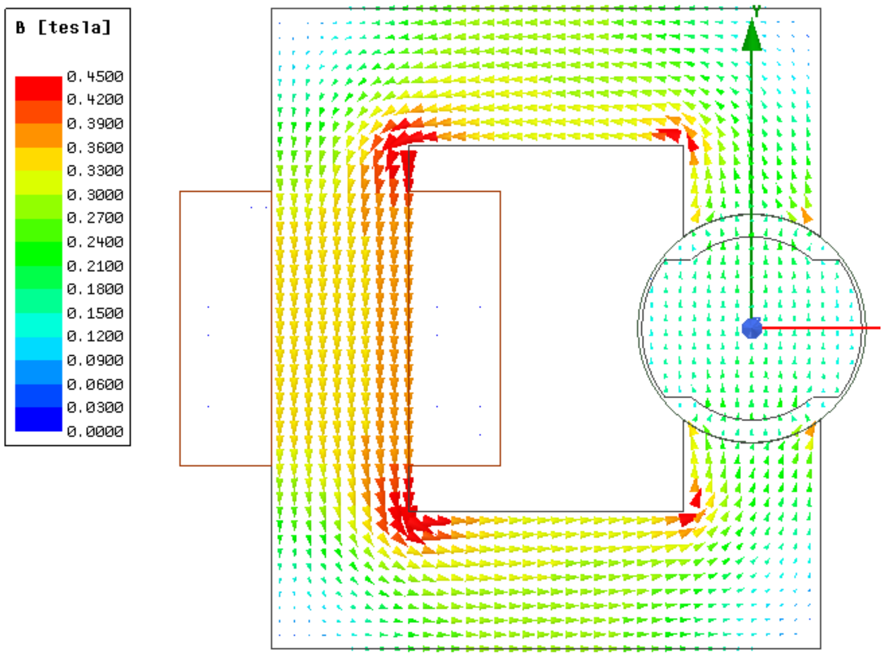
0.0000

b)



c)

Figure 5. Magnetic flux density vectors for three different rotor positions of linear material

a) angle=0° b) angle=45° c) angle=90°

4.3. Analytical results and FEA results comparison

FEA results for linear materials and analytical results are given in Figure 6. The ignorance of straight part, leakage flux, non-homogeneous flux distribution may have lead to the difference in the inductance. Moreover, it can be seen that in FEA results the inductance changes more smoothly when compared to analytical result. Even though, they look similar. FEA results must be more accurate since it calculates it by considering all effects mentioned above.

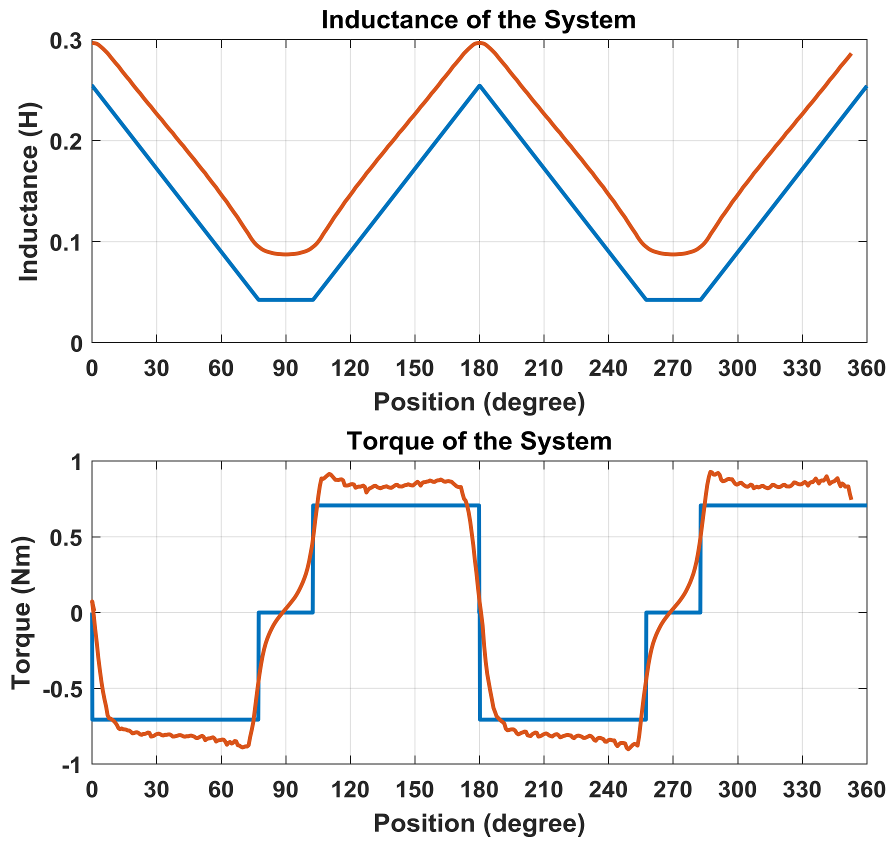


Figure 6. Analytical results and FEA results for linear material (iron)

5. 2D FEA Modelling with Linear Magnetic Material

5.1. Flux density distributions at rotor angle 0° - 45° - 90°

In this section, a material named as steel_1010 is used for analysis. The B-H curve of the material is provided in Figure 7.

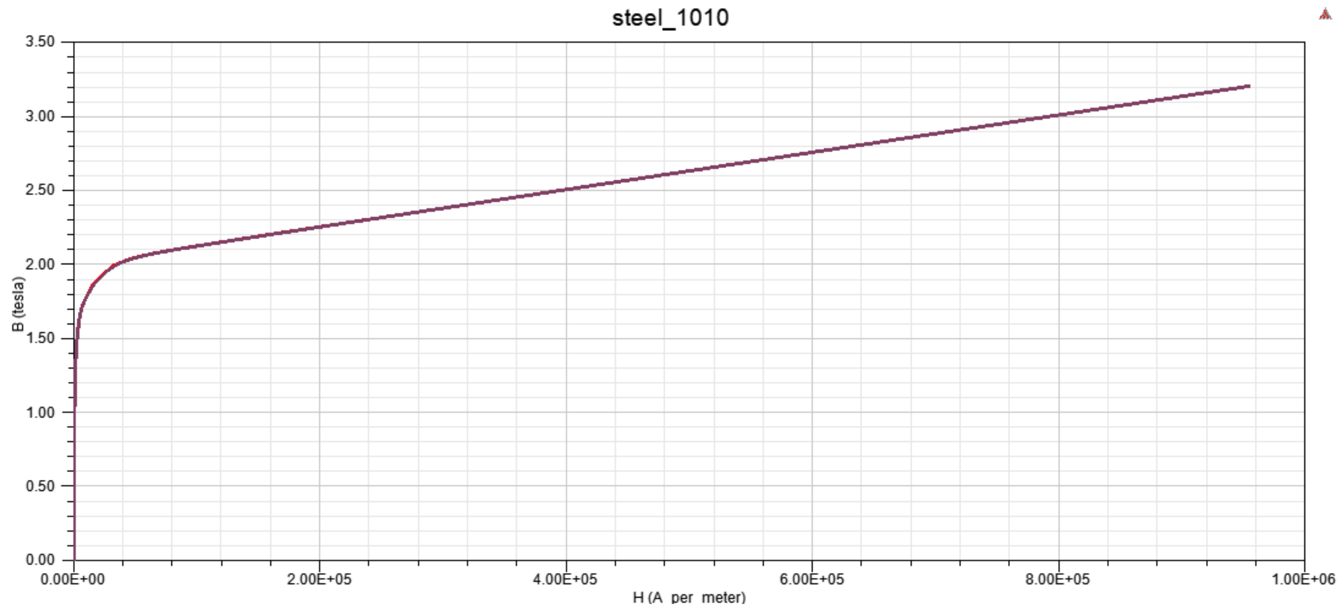
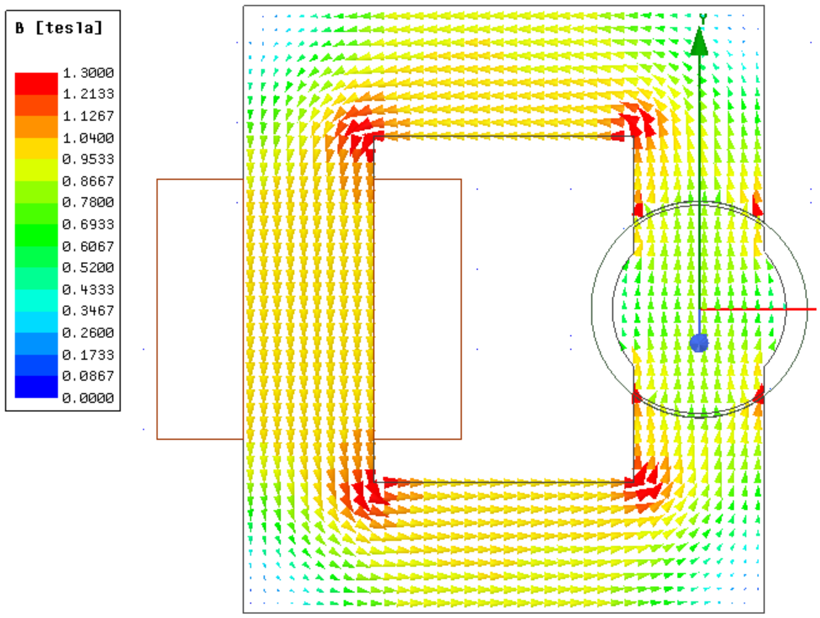


Figure 7. Steel_1010 material B-H curve

Magnetic flux density vector distributions of this non-linear magnetic material for three different rotor position is provided in Figure 8.

As can be seen from Fig. 8. the magnetic flux distribution for all three cases is not uniform in the core. As expected, the flux wants the flow through minimum reluctance path which result in such a distribution waveforms. In an other words, the flux is concentrated on the corner points of stator and rotor. At rotation angle of 45° where, rotor and stator fully aligned, the reluctance is minimum and hence inductance is maximum point. Therefore, maximum flux density occurs when to rotor is in this position. At rotation angle of 45°, the reluctance is increased and hence the inductance is reduced. As can be seen from Fig. 5.b. in this position, the flux density at the sharp edges of rotor and stator is increased. At rotation angle of 90°, the reluctance is at maximum point and hence inductance at its minimum point. There is almost not flux flowing. Also the torque on the rotor is also zero at this angle.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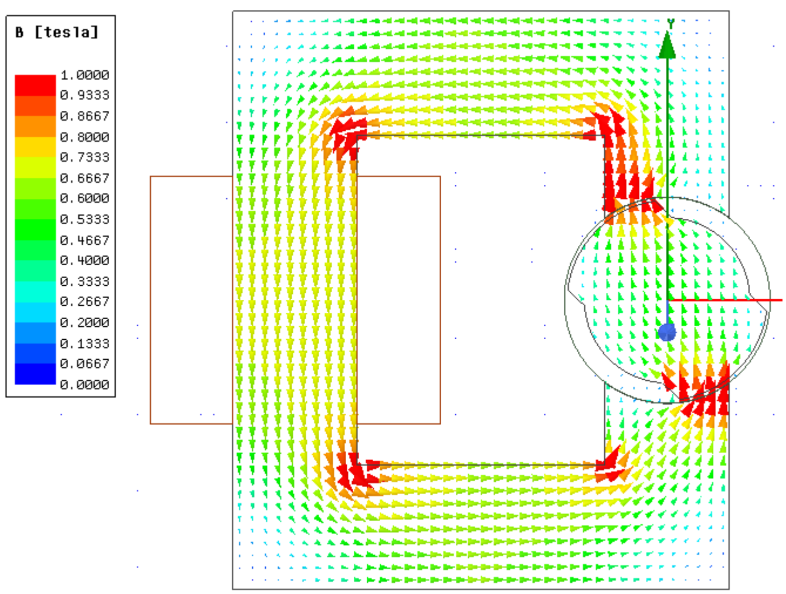
0.3467
0.2600
0.1733
0.0867
0.0000
Y

a)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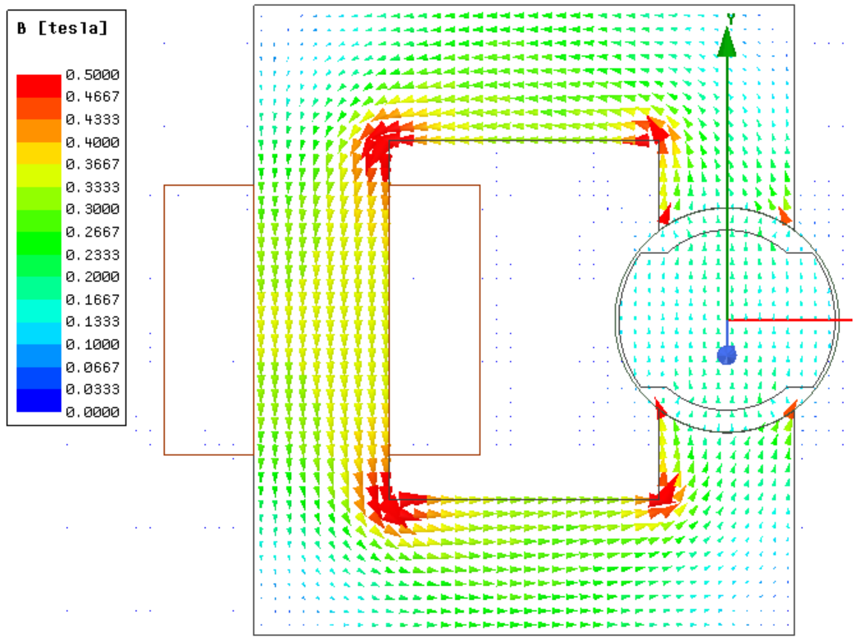
Y

b)



c)

Figure 8. Magnetic flux density vectors for three different rotor positions of nonlinear material

a) angle=0° b) angle=45° c) angle=90°

5.2. Inductance and stored energy at rotor angle 0° - 45° - 90°

Inductance, stored energy and torque results of nonlinear FEA model are given in Table 1. It can be seen that at 0°, the magnetic energy stored in the core is the highest since the inductance is also at maximum value. Also, inductance is decreases as the angle increases. Notice that, the torque is almost zero for angle 0° and 90°.

Table 1: Inductance, stored energy and torque at three different rotor position for nonlinear material FEA results

Rotor angle	Inductance (mH)	Stored Energy (J)	Torque (mNewtonMeter)
0°	25.16	0.1135	7.98
45°	16.95	0.0783	-63.79
90°	8.35	0.0380	2.50

5.3. Analytical results and FEA results comparison

FEA results for nonlinear materials and analytical results are given in Figure 9. The ignorance of straight part, leakage flux, non-homogeneous flux distribution may have lead to the difference in the inductance. Moreover, it can be seen that in FEA results the inductance changes more smoothly when compared to analytical result. Even though, they look similar. FEA results must be more accurate since it calculates it by considering all effects mentioned above.

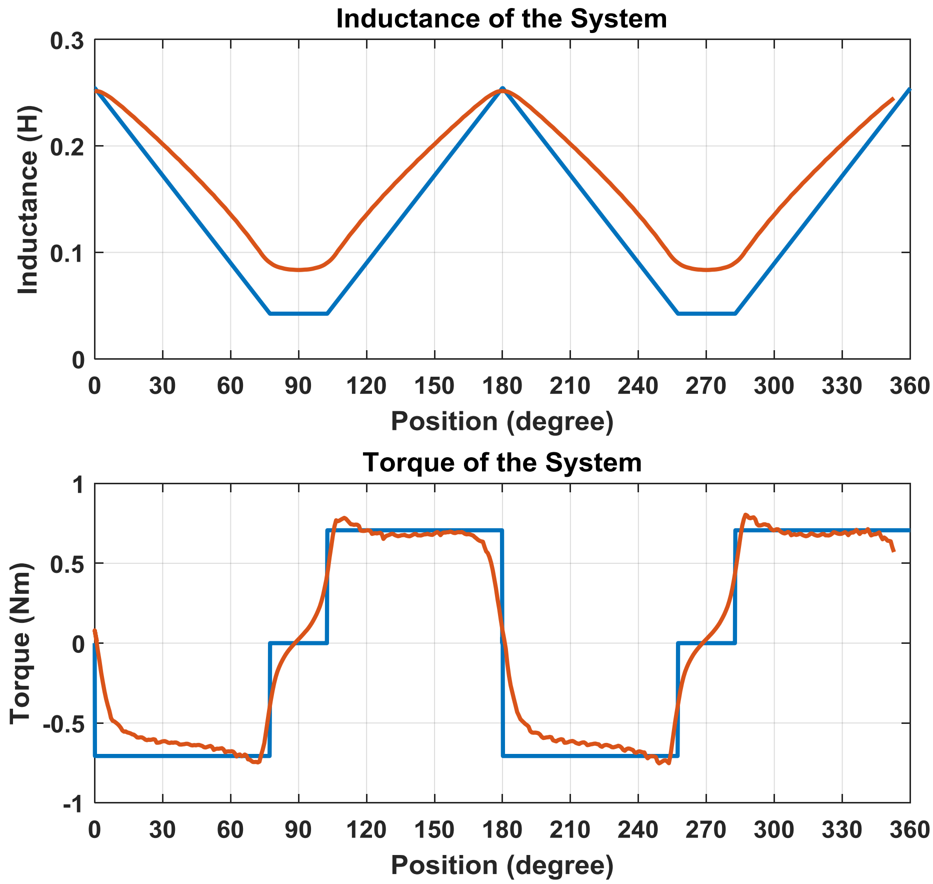


Figure 9. Analytical results and FEA results for nonlinear material (steel_1010)

6. Control Method

In this machine, since there is not excitation on rotor side, there. Therefore, it generates torque through magnetic reluctance. To obtain non-zero average torque and hence full rotation, various excitations can be applied to the windings instead of constant DC current. For example, square wave current can be applied to the windings. By this way, for a on period of square wave, positive current is applied and the rotor is rotated. Then, for off period of square wave, zero current is applied and due to the inertia of the rotor, the rotor continues to rotate. When it goes to stop, positive current is applied again to continue rotation. With this type of periodic excitation signal, non-zero average torque or full rotation is achieved.

7. Motion Animation

It is uploaded to the project repository;

https://github.com/rasitgokmen/EE568-