CS 61A Fall 2020

Final Review

Discussion 14: December 2, 2020

1 Recursion

1.1 (Adapted from Fall 2013) Fill in the blanks in the implementation of `paths`, which takes as input two positive integers `x` and `y`. It returns a list of paths, where each path is a list containing steps to reach `y` from `x` by repeated incrementing or doubling. For instance, we can reach 9 from 3 by incrementing to 4, doubling to 8, then incrementing again to 9, so one path is `[3, 4, 8, 9]`

```
def paths(x, y):
    """Return a list of ways to reach y from x by repeated
    incrementing or doubling.
    >>> paths(3, 5)
    [[3, 4, 5]]
    >>> sorted(paths(3, 6))
    [[3, 4, 5, 6], [3, 6]]
    >>> sorted(paths(3, 9))
    [[3, 4, 5, 6, 7, 8, 9], [3, 4, 8, 9], [3, 6, 7, 8, 9]]
    >>> paths(3, 3) # No calls is a valid path
    [[3]]
    """
```

```
def paths(x, y):
    """Return a list of ways to reach y from x by repeated
    incrementing or doubling.
    >>> paths(3, 5)
    [[3, 4, 5]]
    >>> sorted(paths(3, 6))
    [[3, 4, 5, 6], [3, 6]]
    >>> sorted(paths(3, 9))
    [[3, 4, 5, 6, 7, 8, 9], [3, 4, 8, 9], [3, 6, 7, 8, 9]]
    >>> paths(3, 3) # No calls is a valid path
    [[3]]
    """
    if x == y: # 从最后的doctest中推测出的
        return [[y]]
    elif x > y: # 返回空list, 做加法也无所谓
        return []
    else:
        a = paths(x+1, y)
        b = paths(x*2, y)
        return [[x] + each for each in a + b] # 注意这里a+b本身也是嵌套的list
```

1.2 We will now write one of the faster sorting algorithms commonly used, known as *merge sort*. Merge sort works like this:

1. If there is only one (or zero) item(s) in the sequence, it is already sorted!
2. If there are more than one item, then we can split the sequence in half, sort each half recursively, then merge the results, using the `merge` procedure described below. The result will be a sorted sequence.

Using the algorithm described, write a function `mergesort(seq)` that takes an unsorted sequence and sorts it.

Recall the `merge` procedure is as follows:

```
def merge(s1, s2):
    """ Merges two sorted lists """
    if len(s1) == 0:
        return s2
    elif len(s2) == 0:
        return s1
    elif s1[0] < s2[0]:
        return [s1[0]] + merge(s1[1:], s2)
    else:
        return [s2[0]] + merge(s1, s2[1:])
```

```
def mergesort(seq):
```

```
def mergesort(seq):
    if len(seq) == 1 or len(seq) == 0:
        return seq
    else:
        length = len(seq)
        left = mergesort(seq[:length//2])
        right = mergesort(seq[length//2:])
        return merge(left, right)
```

2 Trees

2.1 Implement `long_paths`, which returns a list of all *paths* in a tree with length at least `n`. A path in a tree is a linked list of node values that starts with the root and ends at a leaf. Each subsequent element must be from a child of the previous value's node. The *length* of a path is the number of edges in the path (i.e. one less than the number of nodes in the path). Paths are listed in order from left to right. See the doctests for some examples.

```
def long_paths(tree, n):
    """Return a list of all paths in tree with length at least n.

    >>> t = Tree(3, [Tree(4), Tree(4), Tree(5)])
    >>> left = Tree(1, [Tree(2), t])
    >>> mid = Tree(6, [Tree(7, [Tree(8)]), Tree(9)])
    >>> right = Tree(11, [Tree(12, [Tree(13, [Tree(14)])])])
    >>> whole = Tree(0, [left, Tree(13), mid, right])
    >>> for path in long_paths(whole, 2):
    ...     print(path)
    ...
    <0 1 2>
    <0 1 3 4>
    <0 1 3 4>
    <0 1 3 5>
    <0 6 7 8>
    <0 6 9>
    <0 11 12 13 14>
    >>> for path in long_paths(whole, 3):
    ...     print(path)
    ...
    <0 1 3 4>
    <0 1 3 4>
    <0 1 3 5>
    <0 6 7 8>
    <0 11 12 13 14>
    >>> long_paths(whole, 4)
    [Link(0, Link(11, Link(12, Link(13, Link(14)))))]
    """
    # 注意path是链表表示的
    paths = []
    if tree.is_leaf() and n <= 0: # 注意这个不是边界，而是满足depth的单个节点加入
        paths.append(Link(tree.label))
    for b in tree.branches: # 边界隐含在这里了，如果没有branch就不会执行该循环
        for path in long_paths(b, n - 1): # 非常巧妙，for循环用递归
            paths.append(Link(tree.label, path))
    return paths
```

2.2 Write a function that takes a `Tree` object and returns the elements at the depth with the most elements.

In this problem, you may find it helpful to use the second optional argument to `sum`, which provides a starting value. All items in the sequence to be summed will be concatenated to the starting value. By default, start will default to 0, which allows you to sum a sequence of numbers. We provide an example of sum starting with a list, which allows you to concatenate items in a list.

```
def widest_level(t):
    """
    >>> sum([[1], [2]], [])
    [1, 2]
    >>> t = Tree(3, [Tree(1, [Tree(1), Tree(5)]),
    ...              Tree(4, [Tree(9, [Tree(2)])])])
    >>> widest_level(t)
    [1, 5, 9]
    """
    levels = []
    x = [t]

    while ____________________________________________________:

        ____________________________________________________

        __________ = sum(_________________________________, [])

    return max(levels, key=_____________________________________)
```

```
def widest_level(t):
    """
    >>> sum([[1], [2]], [])
    [1, 2]
    >>> t = Tree(3, [Tree(1, [Tree(1), Tree(5)]), Tree(4, [Tree(9, [Tree(2)])])])
    >>> widest_level(t)
    [1, 5, 9]
    """
    levels = []
    x = [t] # 用来存放每一层的树节点

    while x:
        levels.append([t.label for t in x]) # 把这一层的所有节点放入level的一个元素中
        x = sum([each.branches for each in x], []) # 妙极，将所有下一层的节点以list的单个元素放入进去

    return max(levels, key=len) # key表示经过该函数对levels中的每个元素处理后再进行max
```

3 Mutation

3.1 For each row below, fill in the blanks in the output displayed by the interactive Python interpreter when the expression is evaluated. Expressions are evaluated in order, and expressions may affect later expressions.

```
>>> cats = [1, 2]
>>> dogs = [cats, cats.append(23), list(cats)]
>>> cats
```

```
>>> dogs[1] = list(dogs)
>>> dogs[1]
```

```
>>> dogs[0].append(2)
>>> cats
```

```
>>> cats[1::2]
```

```
>>> cats[:3]
```

```
>>> dogs[2].extend([list(cats).pop(0), 3])
>>> dogs[3]
```

```
>>> dogs
```

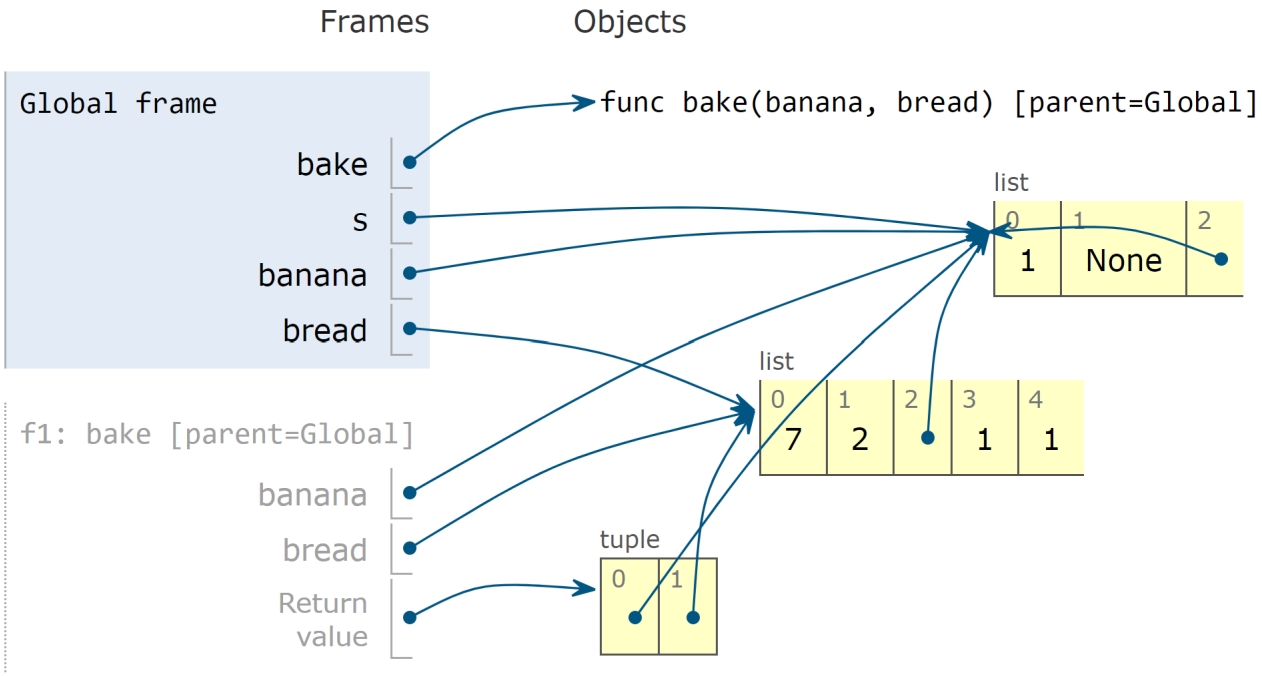
```
>>> cats = [1, 2]
>>> dogs = [cats, cats.append(23), list(cats)]
>>> cats
[1, 2, 23]
>>> dogs[1] = list(dogs)
>>> dogs[1]
[[1, 2, 23], None, [1, 2, 23]]
>>> dogs[0].append(2)
>>> cats
[1, 2, 23, 2]
>>> cats[1::2]
[2, 2]
>>> cats[:3]
[1, 2, 23]
>>> dogs[2].extend([list(cats).pop(0), 3])
>>> dogs[3]
Traceback (most recent call last):
  File <string>, line 1, in <module>
IndexError: list index out of range
>>> dogs
[[1, 2, 23, 2], [[1, 2, 23, 2], None, [1, 2, 23, 1, 3]], [1, 2, 23, 1, 3]]
```

3.2 Fill in the following lines so that the code creates the environment diagram shown below. **You may only use** `append`, `extend`, `1`, `banana`, **and bread in your solution.**

```
def bake(banana, bread):
    _____________(___________(__________)) # This line is Multiple Choice
    # Select all correct answers for the blank above
    # A. banana.append(bread.append(1))
    # B. bread.append(banana.append(1))
    # C. banana.extend([bread.extend([1])])
    # D. bread.extend([banana.extend([1])])
    bread += banana[: (len(_____________) - _____________)]
    banana._____________(bread[___________[_____________]])
    return ___________, _____________
```

```
bread += banana[: (len(banana) - 1)]
banana.append(bread[bread[1]])
return banana, bread
```

```
s = [1]
banana, bread = bake(s, [7, 2, s])
```



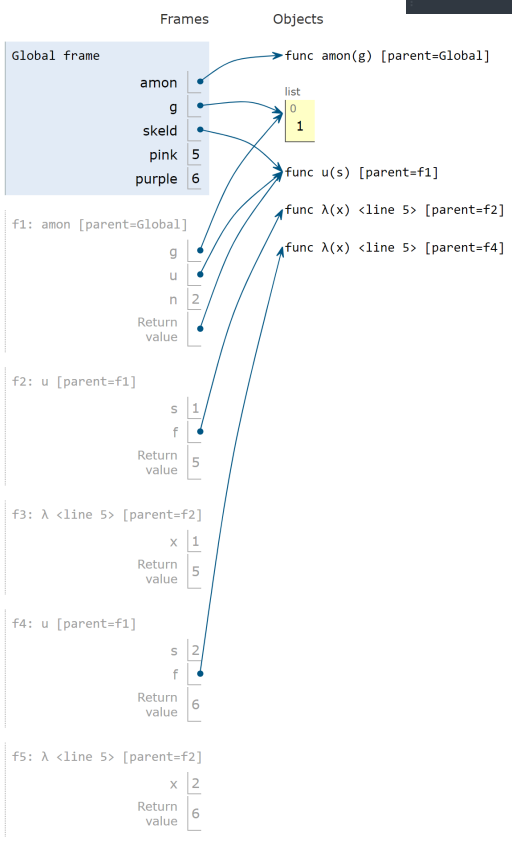
3.3 Fill in the following lines so that the code creates the environment diagram shown below.

```
def amon(g):
    ____________________
            (a)
    def u(s):
        ______________________
                (b)
        f = lambda x: x + g._________ + n
                                (c)
        _________________
                (d)
        return f(s)
    return u

g = [1, 2, 3]
skeld = amon(g)
pink = skeld(1)
purple = skeld(2)
```

```
def amon(g):
    n = 0
    def u(s):
        nonlocal n
        f = lambda x: x + g.pop() + n
        n += 1
        return f(s)

    return u
```



4 OOP

4.1 Fill in the classes `Emotion`, `Joy`, and `Sadness` below so that you get the following output from the Python interpreter.

```
>>> Emotion.num
0
>>> joy = Joy()
>>> sadness = Sadness()
>>> Emotion.num # number of Emotion instances created
2
>>> joy.power
5
>>> joy.catchphrase() # Print Joy's catchphrase
Think positive thoughts
>>> sadness.catchphrase() #Print Sad's catchphrase
I'm positive you will get lost
>>> sadness.power
5
>>> joy.feeling(sadness) # When both Emotions have same power value, print "Together"
Together
>>> sadness.feeling(joy)
Together
>>> joy.power = 7
>>> joy.feeling(sadness) # Print the catchphrase of the more powerful feeling before the less
    powerful feeling
Think positive thoughts
I'm positive you will get lost
>>> sadness.feeling(joy)
Think positive thoughts
I'm positive you will get lost
```

```
class Emotion(_______):

    def __init__(self):

    def feeling(self, other):

class Joy(_______):

    def catchphrase(self):

class Sadness(_______):

    def catchphrase(self):
```

```python
class Emotion(object):
    Emotion.num = 0
    def __init__(self):
        Emotion.num += 1
        self.power = 5

    def feeling(self, other):
        if self.power == other.power:
            print("Together")
        elif self.power > other.power:
            self.catchphrase()
            other.catchphrase()
        else:
            other.catchphrase()
            self.catchphrase()

class Joy(Emotion):

    def catchphrase(self):
        print("Think positive thoughts")

class Sadness(Emotion):

    def catchphrase(self):
        print("I'm positive you will get lost")
```

5 Mutable Linked Lists and Trees

5.1 Write a function that takes a sorted linked list of integers and mutates it so that all duplicates are removed.

```
def remove_duplicates(lnk):
    """
    >>> lnk = Link(1, Link(1, Link(1, Link(1, Link(5)))))
    >>> remove_duplicates(lnk)
    >>> lnk
    Link(1, Link(5))
    """
```

```
def remove_duplicates(lnk):
    """
    >>> lnk = Link(1, Link(1, Link(1, Link(1, Link(5)))))
    >>> remove_duplicates(lnk)
    >>> lnk
    Link(1, Link(5))
    """
    exists = [lnk.first]
    before = lnk
    while lnk is not Link.empty:
        if not (lnk.first in exists):
            exists.append(lnk.first)
            before = lnk
            lnk = lnk.rest
        else:
            before.rest = lnk.rest
            lnk = lnk.rest
```

6 Generators

6.1 Write a generator function that yields functions that are repeated applications of a one-argument function `f`. The first function yielded should apply `f` 0 times (the identity function), the second function yielded should apply `f` once, etc.

```
def repeated(f):
    """
    >>> double = lambda x: 2 * x
    >>> funcs = repeated(double)
    >>> identity = next(funcs)
    >>> double = next(funcs)
    >>> quad = next(funcs)
    >>> oct = next(funcs)
    >>> quad(1)
    4
    >>> oct(1)
    8
    >>> [g(1) for _, g in
    ...  zip(range(5), repeated(lamb
    [1, 2, 4, 8, 16]
    """
```

```
def repeated(f):
    """
    >>> double = lambda x: 2 * x
    >>> funcs = repeated(double)
    >>> identity = next(funcs)
    >>> double = next(funcs)
    >>> quad = next(funcs)
    >>> oct = next(funcs)
    >>> quad(1)
    4
    >>> oct(1)
    8
    >>> [g(1) for _, g in zip(range(5), repeated(lambda x: 2 * x))]
    [1, 2, 4, 8, 16]
    """
    g = lambda x:x
    while True:
        yield g
        g = (lambda g: lambda x: f(g(x)))(g)
        # 上一行非常精彩，下题便给出了错误的例子，必须在这里将此时的g赋值进去
        # 不然会不断递归下去，而f则不变所以无所谓
```

```
    g = ____________________________________________________________________

    while True:

        ____________________________________________________________________

        ____________________________________________________________________
```

6.2 Ben Bitdiddle proposes the following alternate solution. Does it work?

```
def ben_repeated(f):
    g = lambda x: x
    while True:
        yield g
        g = lambda x: f(g(x))
```

不行。这样会无限递归下去。

6.3 Implement `accumulate`, which takes in an `iterable` and a function `f` and yields each accumulated value from applying `f` to the running total and the next element.

```
from operator import add, mul

def accumulate(iterable, f):
    """
    >>> list(accumulate([1, 2, 3, 4, 5], add))
    [1, 3, 6, 10, 15]
    >>> list(accumulate([1, 2, 3, 4, 5], mul))
    [1, 2, 6, 24, 120]
    """
    it = iter(iterable)


    ____________________________________________________________________________


    ____________________________________________________________________________


    for ____________________________________________________________________:

        ____________________________________________________________________


        ____________________________________________________________________
```

```
def accumulate(iterable, f):
    """
    >>> list(accumulate([1, 2, 3, 4, 5], add))
    [1, 3, 6, 10, 15]
    >>> list(accumulate([1, 2, 3, 4, 5], mul))
    [1, 2, 6, 24, 120]
    """
    try:
        it = iter(iterable)
        now = next(it)
        res = now
        for __ in range(len(iterable)):
            yield res
            res = f(res, next(it))
    except Exception:
        pass

    # 这里得用try, 不然过不了doctest
```

7 Scheme

7.1 Write a function that takes a procedure and applies to every element in a given nested list.

The result should be a nested list with the same structure as the input list, but with each element replaced by the result of applying the procedure to that element.

Use the built-in `list?` procedure to detect whether a value is a list.

```
(define (deep-map fn lst)
        (cond ((eq? lst nil) nil)
              ((list? (car lst)) (cons (deep-map fn (car lst)) (deep-map fn (cdr lst))))
              (else (cons (fn (car lst)) (deep-map fn (cdr lst))))

        )
)
```

```
scm> (deep-map (lambda (x) (* x x)) '(1 2 3))
(1 4 9)
scm> (deep-map (lambda (x) (* x x)) '(1 ((4) 5) 9))
(1 ((16) 25) 81)
```

8 SQL

(Adapted from Fall 2019) The scoring table has three columns, a player column of strings, a points column of integers, and a quarter column of integers. The players table has two columns, a name column of strings and a team column of strings. Complete the SQL statements below so that they would compute the correct result even if the rows in these tables were different than those shown. Important: You may write anything in the blanks including keywords such as WHERE or ORDER BY. Use the following tables for the questions below:

```
CREATE TABLE scoring AS
    SELECT "Donald Stewart" AS player, 7 AS points, 1 AS quarter UNION
    SELECT "Christopher Brown Jr.", 7, 1 UNION
    SELECT "Ryan Sanborn", 3, 2 UNION
    SELECT "Greg Thomas", 3, 2 UNION
    SELECT "Cameron Scarlett", 7, 3 UNION
    SELECT "Nikko Remigio", 7, 4 UNION
    SELECT "Ryan Sanborn", 3, 4 UNION
    SELECT "Chase Garbers", 7, 4;

CREATE TABLE players AS
    SELECT "Ryan Sanborn" AS name, "Stanford" AS team UNION
    SELECT "Donald Stewart", "Stanford" UNION
    SELECT "Cameron Scarlett", "Stanford" UNION
    SELECT "Christopher Brown Jr.", "Cal" UNION
    SELECT "Greg Thomas", "Cal" UNION
    SELECT "Nikko Remigio", "Cal" UNION
    SELECT "Chase Garbers", "Cal";
```

8.1 Write a SQL statement to select a one-column table of quarters in which more than 10 total points were scored.

```
select quarter from scoring
        group by quarter
        having sum(points) > 10;
```

8.2 Write a SQL statement to select a two-column table of the points scored by each team. Assume that no two players have the same name.

```
select b.team, sum(a.points) as total_score from scoring as a, players as b
        where a.player = b.name
        group by b.team;
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.