W4260: Problem Set #2

Due: Tuesday, February 10

Problem 1

In the last question of problem set 1, you evaluated an analytic function which gave you the ratio of the obscured to unobscured flux of a star during a planetary transit. This was done for a star with a uniform brightness across it's disk. However, most stars are actually brighter in their centers and dimmer at their edges – a feature known as limb darkening.[1] This limb darkening can be parameterized with the function $I(r)$, which is the specific intensity as a function of r (or alternately θ – see Fig. 1a of Problem set 1).

This makes the transit calculation more complicated[2] and we need to evaluate the ratio of two integrals:

$$F(p,z) = \frac{\int_0^1 I(r)\,[1-\delta(p,r,z)]\,2r\mathrm{dr}}{\int_0^1 I(r)2r\mathrm{dr}}$$

where

$$\delta(p,r,z) = \begin{cases} 0 & r \geq z+p \quad \text{or} \quad r \leq z-p, \\ 1 & r+z \leq p \\ \pi^{-1}\arccos[(z^2-p^2+r^2)/(2zr)] & \text{otherwise} \end{cases}$$

Write code to compute these integrals using a simple "rectangle"-rule: in other words approximate it as a sum with a small step in r. For example, you can write an integral of the form:

$$\int_{x_0}^{x_1} f(x)dx \approx \sum_{i=0}^{N-1} f(x_i)\Delta x$$

where $\Delta x = (x_1 - x_0)/N$ and $x_i = x_0 + i\Delta x$.

Evaluate $F(p,z)$ using no limb-darkening (i.e. $I(r) = 1$). Do this for the values $p = 0.2$ and $z = 0.9$ and try using different numbers of steps to calculate the integral: $N = 10, 10^2, 10^3, 10^4, 10^5$. How does the fractional error decrease as N increases? You can use the analytic formula in the first problem set to check your result.

Problem 2

Use (extended) Simpson's rule to integrate the formula above (for the same p and z values). Do this for a range of step sizes, corresponding to the same N values as in the first problem set. Determine how the "error" decreases with decreasing step size – be quantitative and try to determine a relation between the "error" and the step size.

[1] This is due to the fact that we actually see into the outer layers of a star, rather than a hard surface. When we look at the center (as opposed to the edge), we see deeper into the star, where the gas is hotter and therefore brighter

[2] See Mandel & Agol, 2002, ApJ, 580, L171 for more details.

Problem 3

Use a Monte-Carlo integration to evaluate the same integral as above (again assuming $I(r) = 1$). To do this, generate N random x and y values that go from -1 to 1 so that you are picking random points inside a box that covers the unit circle. Reject points that lie outside the unit circle (i.e. for which $x^2 + y^2 > 1$). Call the number of accepted points N_1. In addition, count how many of the accepted points lie within the eclipsing planet disk (i.e. for which $(x-z)^2 + y^2 < p^2$) and call that number N_2. Then an estimate for $F(p,z)$ is the ratio of points inside the star's disk that do not lie in the planet's disk to the number of points inside the star's disk (without the planet): $F(p,z) \approx (N_1 - N_2)/N_1$.

Evaluate this for the same p and z values as above. Again repeat for the same range of N values and determine how the error decreases with N. What "order" is this method?

To generate a random number from -1 to 1 in python[3], use:

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import random
x = random.uniform(-1, 1)
```

[3]You could also use numpy to generate an array of random values.