W4260: Problem Set #6

Due: Tuesday, March 31

Problem 1

Write an N-body code for integrating solar system dynamics. Use the leap-frog method (or better) to integrate the orbits of N bodies under their mutual self-gravity. You can assume N is small and evaluate the gravitational accelerations by direct summation (you may do this in either 2 or 3 dimensions).

Test the method on a system with $N = 2$ where the mass of the central object (e.g. the sun) is much, much larger than the satellite (e.g. the earth). Use a circular orbit ($v_c = \sqrt{GM/r}$) and check to see how much the radius of the satellite changes after five orbital periods. If this changes too much (i.e. more than a few percent per orbit), you will need to decrease the time step of your integration or improve the accuracy of your integrator. Also, if you can, monitor the total energy (kinetic plus potential) of the system.

Problem 2

Some extrasolar planetary systems observed have Jupiter-mass planets in very elliptical orbits. Set up a hypothetical solar system composed of three bodies: (1) a one solar mass central star, (2) an earth-mass planet with a circular orbit at 1 A.U. (astronomical units), and (3) a jupiter-mass planet with a semi-major axis $a = 4$ A.U. (similar to, but slightly lower than Jupiter's) and an eccentricity of $e = 0.6$.[1] Integrate the system for as many years (i.e. earth orbits) as you can, but at least one hundred. What happens?

[1]It may be helpful to know, as can be confirmed with an elementary dynamics textbook, that when the jupiter-mass planet is at pericenter, $r = (1 - e)a$, it's velocity $v_{\tan} = \sqrt{(GM(1 + e)/(a(1 - e))}$ is entirely tangential.