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Mathematical concepts

Denotions

- **Not (¬)** When we use the character "¬", we mean "*not*". E.g. "¬*sunny*" = "*not sunny*".
- **For all (∀)** When we use the character "∀", we mean "*for all*". E.g. "∀ *cats C, C has four legs*" = "*for all cats C, C has four legs*".
- **In (∈)** The character "∈" is read "*in*" and describe the relationship between two sets. E.g. if we have the set X = { **a**, **b**, **c** }, then we could write "**a** $\in X$" to point out that **a** is a member of the set X. Furthermore, we can write "**d** $\notin X$" to point out that **d** is not a member of the set X.
- **Permutations (S_n)** The set S_n is used to denote all possible permutations of a set of length n. E.g. if we look at the set X = { **a**, **b**, **c** }. Then $S_3(X)$ = { (abc), (acb), (bac), (bca), (cab), (cba) }.
- **Sum (∑)** We will use two ways of summing: *summing by indices* and *summing over elements in a set*. When we sum using indices, the notation looks like this

$$\sum_{k=0}^{5} k = 0 + 1 + 2 + 3 + 4 + 5 = 15$$

  Let's say that we have a set Y = { 5, 8, 3 }. Then we can use the following notation to sum over all the elements

$$\sum_{y \in Y} y = 5 + 8 + 3 = 16$$

- **Union (∪)** We use union to denote the merging of two sets. That is, if we have two sets A = { **a**, **b**, **c** } and B = { **c**, **d**, **e** }. Then A ∪ B = { **a**, **b**, **c**, **d**, **e** }. Note that we only declare **c** once. Union can also be interpreted as "OR" (in this context, OR is usually written in capital letters). Hence A OR B = { **a**, **b**, **c**, **d**, **e** } as well.
- **Intersection (∩)** We use intersection to denote what two sets have in common. That is, if we, again, have the two sets A = { **a**, **b**, **c** } and B = { **c**, **d**, **e** }. Then A ∩ B = { **c** }. Note that we only declare **c** once. Intersection can also be interpreted as "AND" (in this context, AND is usually written in capital letters). Hence A AND B = { **c** } as well.

Caveat Throughout this site, I'm trying my best to use the words probability and likelihood in their proper context. However, there's a lot of content to cover on this site, why there is a risk of me mixing the two together. Therefore, take those notations with a pinch of salt.

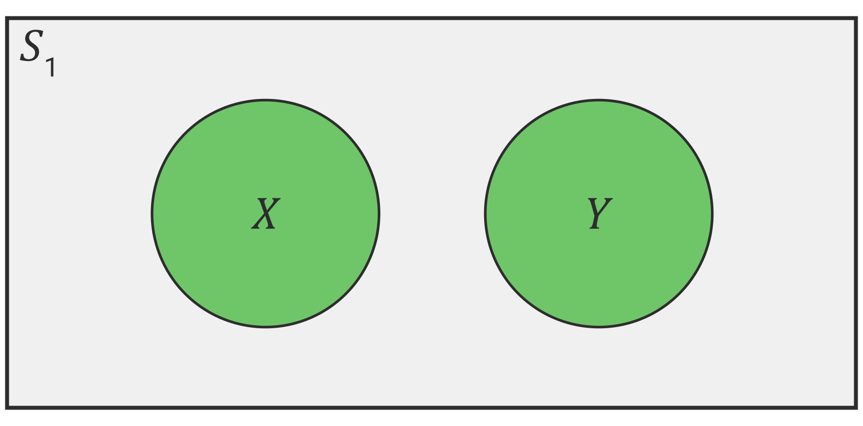
Rule of Sum

- **Mutually exclusive events** If two events are mutually exclusive, they cannot happen at the same time. E.g. if we roll a normal six-sided dice. Let E_1 = { *a four is rolled* } be the first event and E_2 = { *a three is rolled* } the second. Then E_1 and E_2 are mutually exclusive, because both events cannot happen at the same time. Conversely, if we have the events E_3 = { *a 1, 3 or 6 is rolled* } and E_4 = { a *2, 3 or 5 is rolled* } the events are *not* mutually exclusive. Because if we roll a three, both event E_3 and E_4 occur at the same time.

Let X and Y be two mutually exclusive events, then:

$$P(X \cup Y) = P(X) + P(Y)$$

Let's illustrate this further with a Venn diagram where S_1 is the sample space:

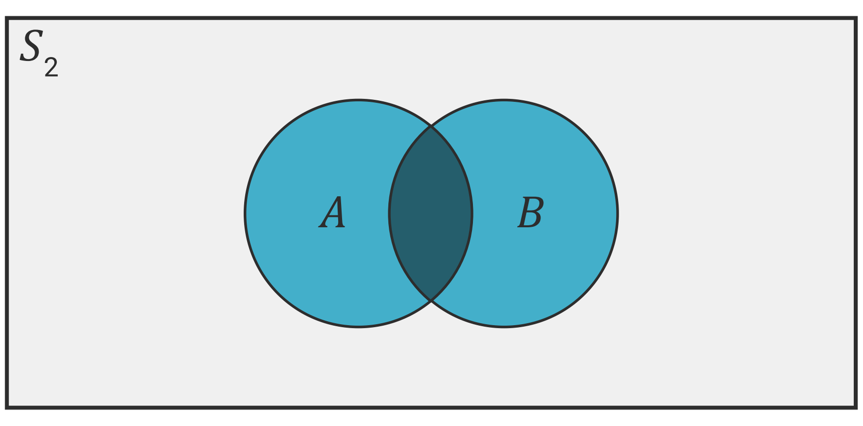


The goal with the sum rule is to calculate the probability that *some* of the events X and Y happens. As we know, X and Y are mutually exclusive, meaning they cannot occur at the same time. Therefore, the probability that some of the events happen is the sum of their individual probability. Hence, $P(X) + P(Y)$.

Now let's say that we have two other events, A and B, which are not mutually exclusive. That is, they can occur at the same time. Then the total probability of some of the events A and B to occur is:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Why $- P(A \cap B)$? Let's have a look at the following Venn diagram:



As we can see in the dark are in the middle, we have accounted for the overlapping section two times. That is, we have counted the scenarios when both A and B happens two times. And since we are only interested in the probability that some of the events happen, we have to subtract for one of those times, hence $- P(A \cap B)$.

Conditional Probability

A conditional probability is a probability that a certain event will occur given information about the outcome of some other event. Let X and Y be two events, then:

$$P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)}$$

That is, the probability that event X occurs given that event Y occurred is equivalent to the probability that both X and Y happens at the same time divided by the probability that Y happens.

Rule of Product

- **Independent events** In the context of probability, we call two events **independent** if the occurrence of one of the events doesn't affect the probability of the other event occurring. E.g. if we roll a six-sided dice twice. Let E_1 = { *a four is obtained on the first roll* } be the first event and E_2 = { *a six is obtained on the second roll* } the second. Then, if we would successfully roll a four at the first throw, the probability of rolling a six on the second throw wouldn't be affected. In a more mathematical context we can write the following: $P(E_2 \mid E_1) = P(E_2)$. That is, the probability that we roll a six on the second throw given that we rolled a four on the first row is equivalent with the probability that we roll a six given no additional information.
- **Dependent events** Conversely from the previous section, if the occurrence of one event does affect the probability of the other event taking place, then we call the events **dependent**. Let's look at an example of two such events. Let E_3 = { *the sun is shining* } be the first event and E_4 = { *Kim wears sunglasses* } be the second. If event E_3 occurs, we can reason that Kim probably will wear sunglasses. Therefore, the events E_3 and E_4 are dependent. If the two events are dependent, then $P(E_4 \mid E_3) \neq P(E_4)$, because the probability of E_4 occurring given that E_3 occurred is higher then if observed E_4 with no additional information.

Let X and Y be two **independent** events, then:

$$P(X \cap Y) = P(X) \times P(Y)$$

That is, we find the probability that both event X and event Y occur at the same time by multiplying the individual probabilities $P(X)$ and $P(Y)$. Let's again look at the two independent events E_1 = { *a four is obtained on the first roll* } and E_2 = { *a six is obtained on the second roll* }. We know that $P(E_1) = 1/6$ and that $P(E_2) = 1/6$. Therefore, the probability that both E_1 and E_2 occurs is $P(E_1 \cap E_2) = P(E_1) \times P(E_2) = 1/6 \times 1/6 = 1/36$.

Let A and B be two **dependent** events, then:

$$P(A \cap B) = P(A) \times P(B \mid A) = P(B) \times P(A \mid B)$$

To better understand why the intersection of A and B is equal to the expression above, check out the conditional probability section. The expression above is a direct derivation from conditional probability.

Bayes' rule

If we think about the example given above, where we have the two events E_3 = { *the sun is shining* } and E_4 = { *Kim wears sunglasses* }. Then we said that if the sun is shining, then Kim probably wear sunglasses. In other words $P(E_4 \mid E_3) \geq P(E_4 \mid \neg E_4)$. But what if we are interested in the probability that the sun is shining? And would information about Kim wearing sunglasses affect that probability? The answer is yes and we can find out the answer using **Bayes' rule**:

$$P(X \mid Y) = \frac{P(Y \mid X)}{P(Y)} \times P(X)$$

As we can see with Bayes' rule, dependent probabilities affects each other in both directions.