Contents

Part I

Introduction

Chapter 1

Terminology

1.1 General terminology

- **Cuber**: The self reference for people who are devoted to the community of the Rubik's Cube.

1.2 Cube terminology

- **Face**: A face is an entire side of the cube. A Rubik's Cube has 6 faces.
- **Facet**: The small stickers on the cube. Each face has 9 facets.
- **Corner**: Corner pieces have 3 facets and are placed at the corners.
- **Edge**: Edge pieces have 2 facets and are placed at the edges of each face.
- **Center**: Center pieces have 1 facet and are placed at the center of each face and are immovable unless the cube is turned.
- **Turn**: A turn of the cube is equal to rotating the whole cube 90 degrees(=changing view angle).
- **Twist**: A twist is a rotation of a face.

1.3 Movement notation terminology

A cube consists of 6 faces and the notations of these are the following.

- **Front face**: F – This face faces the cuber.
- **Left face**: L – This face faces the left hand side of the cuber.
- **Right face**: R – This face faces the right hand side of the cuber.

- **Up face**: U – This face faces up.
- **Down face**: D – This face faces down.
- **Back face**: B – This face faces away from the cuber.

A face can be twisted in two directions – clockwise and counterclockwise. When twisting a face the direction is determined as if you were facing the face. A twist in the clockwise direction has the same name as the face. i.e. a clockwise turn of the right face is notated ”R” and pronounced ”right”. A counterclockwise twist of the right face is notated ”R’” and pronounced ”right prime”. This goes for all the faces.

A turn of the cube can be done in six directions. Clockwise and counterclockwise around each of the three axes.

Chapter 2

Problem Analysis

Since 1977, when the Rubik's Cube was initially released for sale, the Rubik's Cube has frustrated, inspired and entertained many people. This 3x3x3 cube has so many possible settings that the solution can not just be guessed out of sheer luck and since 1982 people has been competing each other in solving the Rubik's Cube fastest or by the least number of twist. Because of these competitions, it has been interesting for the competitors to find algorithms for solving the Rubik's Cube in the least number of twists. The development of these algorithms is an ongoing process which has given the latest theory in 2008, that states that an algorithm which can solve the Rubik's Cube in 22 twists, no matter which setting the Rubik's Cube starts in, is possible to create. No such algorithm has been created so far (6).

Now that computers are getting faster and faster, it could be interesting to study the implementation of the solving algorithms in a computer program. The efficiency of these algorithms with respect to the time of calculation and the number of twists is an interesting focus point.

2.1 Problem Statement

How come that it has been proven that it is possible to solve the Rubik's Cube in 22 steps and no such algorithm has been made?

- Which algorithms are there now and how efficient are they?

- How can we create an application which can solve the Rubik's Cube?

  - How efficient can we make this application, with respect to the number of twist?

2.2 Problem Limitations

Because the amount of different algorithms for Rubik's Cube solving, not every algorithm will be covered in this project.

The Rubik's Cube solving algorithm will be primarily for technical use, meaning that usability will not be in focus.

Chapter 3

Development of the Cube

In this chapter the development and the problematics with the patenting and legal issues regarding the cube are described. We will look at the patent to get a better understanding of the cubes made at the time. The purpose of this chapter is to give the reader a basic understanding of the Rubik's Cube. Furthermore we will describe the history behind Ernö Rubik and how he got the idea for the Rubik's Cube to get a better understanding of the Rubik's Cube.

3.1 Ernö Rubik

Ernö Rubik is the inventor behind the world famous Rubik's Cube. He was born in Budapest, Hungary in 1944. In college he studied sculpture. After his graduation he started studying architecture. Once he graduated with a degree in architecture he stayed at the college to teach interior design.

In January 1975 Rubik applied for a patent for his invention in Hungary. Two years later in 1977 he got the patent on the Magic Cube. In the 80s he became professor with full tenure, he started Rubik Studio, which employs a dozen people to design furniture and toys. Rubik has produced several other toys, including Rubik's Snake later the studio began to make computer game. He became the president of the Hungarian Engineering Academy in 1990. Same Year he created the International Rubik Foundation to support especially talented young engineers and industrial designers.

3.2 Rubik's Cube(Magic Cube)

In the 70s Ernö Rubik was teaching Interior Design at Academy of Applied Arts and Crafts and he was trying to find a tool to help his students to understand three dimensional objects as result he made the *Magic Cube*n 1974 and obtained a Hungarian patent HU170062. Rubik got the idea for the cube when he wanted to make a three dimensional design with blocks that could move individually but many at the same time. Rubik initially tried to make a cube that was held together with rubber bands but failed. Then he got the idea that the cubes had to hold each other in place, which resulted in a 3x3x3 cube that could twist each face individually. Rubik got the inspiration for the cube from the Magic Puzzle (see chapter 5).

Rubik described that some of the most important features behind the cube were that the parts of the cube stay together, which many other puzzles do not. He also pointed out that you can move several pieces at once. Also that it is three dimensional.

In the end of 70s a Hungarian Businessman showed the Magic Cube at the Nuremberg toy fair and made it popular in Europe. The company Ideal Toy bought exclusive rights for the Magic Cube, but changed the name of the cube to Rubik's Cube within a year in order to get trademark protection.

At that time there were also two others applying for patent for products similar to the Rubik's Cube. One of them was an American named Doctor Larry D. Nichols, and his cube was a 2x2x2 cube which was held together with magnets. The other one who applied for patent was a Japanese man named Terutoshi Ishige. He applied for patent a year after Rubik. Terutoshi Ishige's cube was almost identically to the Rubik's Cube.

Ideal Toy Company were bought by CBS Toy Company in 1982 and the trademark surpassed with it, but they sold the rights to Rubik's Cube to Seven Towns which is a Toy Company in Great Britain, they are still producing The Rubik's Cube today.

3.3 The Nichols Cube Puzzle

Dr. Larry D. Nichols studied chemistry at DePauw University in Greencastle, Indiana, before moving to Massachusetts to attend Harvard Graduate School. He is a lifelong puzzle enthusiast and inventor who began developing a twist cube puzzle with six colored faces in 1957. It was made of eight smaller cubes assembled to a 2x2x2 cube. The eight cubes were held together by magnets.

On April 11, 1972 he was granted U.S. Patent 3,655,201 on behalf of Moleculon Research Corp. U.S. Patent 3,655,201 covered Nichols Cube and the possibility for making larger versions later. This was two years before Ernö Rubik took out the patent for his Rubik's Cube in Hungary.

In 1982 Moleculon Research corp. Sued Ideal Toy Company that had the U.S. Patent 4,378,116 for Rubik's Cube because they believed that Ideal Toy Company violated their patent, but the U.S. District Court ruled in Ideal Toy Company favor. In 1986 the Court of Appeals ruled that the Pocket Rubik's Cube 2x2x2 was guilty of infringement but not the 3x3x3 Rubik's Cube.

In this chapter it has been described how Ernö Rubik got the idea for the cube. We also stated that Ernö Rubik was not the only one at that time, that came *with the invention of cubes. Ernö Rubik's cube was special since the blocks hold each other together, which is different from the one Doctor Larry D. Nichols applied patent for which is hold together with magnets.*

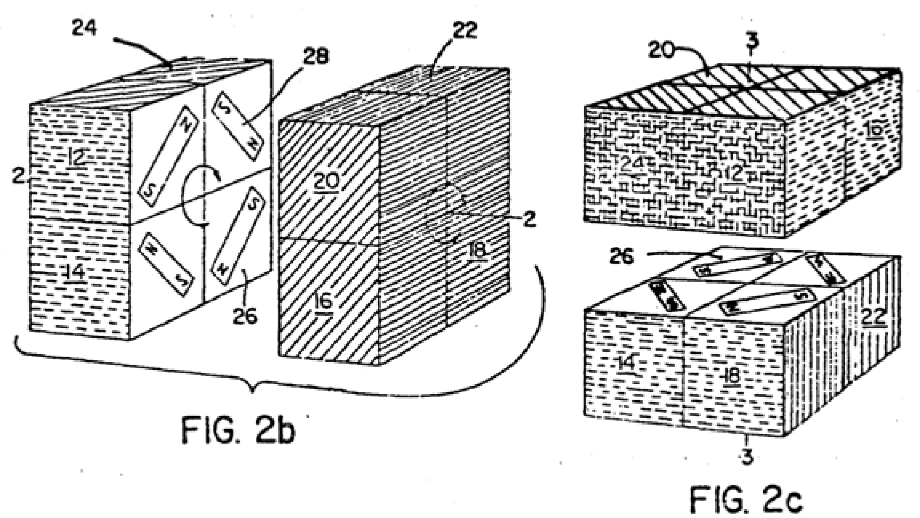

Figure 3.1: *Figure of Nichols Patent.*

Chapter 4

Community

In this chapter the community concerning the original Rubik's Cube and other similar puzzles will be presented and described briefly. This chapter will include a description of the online and offline community and the competitions in which members of the community partake. The community is an important and interesting topic because it is the primary place where solving strategies and algorithms are produced, presented and discussed.

The Rubik's Cube community consists of both a real life community handling competitions and events and an online community where cubers can find the real life competitions, improve their skills and talk to each other. The majority of the community is focused on the speedcubing aspect. In this chapter both the online and offline community will be discussed. Note that the two sides of the community often interfere.

4.1 The Online Community

In the world of the Rubik's Cube there is a large online community. The community consist of everything from forums and guides to competitions and bragging. Cubers, as they often refer to them selves as, have a place that is the online community to express and compare their abilities, experiences and skills with each other.

4.1.1 Forums

Forums give the cubers a place for sharing their knowledge and experience(3)(2)(10). Forums or specific Rubik's Cube sites allow visitors to find information concerning the Rubik's Cube. The main focus of these forums is on how to solve the Rubik's Cube or similar puzzles in the least amount of time. Forums are often split into several divisions. A division could e.g. concern the hardware used and maintenance techniques for making the Rubik's Cube spin with greater ease giving the cuber a slightly better solving time. A major part is reserved for the theory and the algorithms used to solve the Rubik's Cube – most of which strive for an efficient solve both with respect to time and number of twists.

Another part of the community is focused on competitions of various kinds. The offline competitions are held in cooperation with the World Cube Association(WCA) (10), which are further discussed in section 4.2. Beside the WCA-competitions some forums hold weekly online competitions where the forum members can upload and compare their solve times for the Rubik's Cube or similar puzzles.

Other than the forums the online community offers a wide variety of sites containing guides, solutions and algorithms for solving the Rubik's Cube. The majority of the Rubik's Cube sites contain the beginner's guide(5) whose target group is the beginners who may recently have gotten their first cube and want to learn how to solve it.

4.2 Competitions

Speed cubing competitions are held on a regular basis(11). These competitions have different disciplines for various puzzles related to the original 3x3x3 Rubik's Cube. All the official competitions are held in cooperation with the World Cube Association (WCA). The WCA governs the official regulations on speed cubing and holds annual world and regional championships. The first World championship in speed cubing was held in 1982 in Budapest, Hungary. WCA governs the official rankings and records for solving the Rubik's cube. In total WCA keeps regulation, ranking and records for 19 different types of competitions. All 19 competitions include puzzles which are related to or based on the original Rubik's Cube.

In this chapter the community in regards to the Rubik's Cube has been described. It has been stated that the community consists of both an online and an offline aspects. In general the strategies and theories are produced and discussed online and they are for one thing put to use in competitions where the right solving method is crucial in order to finish in a competitive time.

Chapter 5

Recreational Mathematics

It is important to understand what recreational mathematics is in order to get a better understanding of the Rubik's Cube. The Rubik's Cube is related to other recreational mathematical puzzles, which have inspired the Rubik's Cube and are simpler to understand at first grasp. This chapter presents a definition of recreational mathematics and a few examples of recreational mathematical puzzles other than the Rubik's Cube. Different theorems for these puzzles are presented and proved, because similar proofs are used later for the Rubik's Cube.

5.1 Definition

Recreation means to do something which is amusing or relaxing. Mathematics is somewhat harder to give a precise definition of due to the vast amount of subjects that fall under this term. Most people do however have a common idea of what mathematics is.

Recreational mathematics is hereby defined as mathematical problems, puzzles or games which are fun and interesting to laymen. (8) (9, 18)

5.2 Puzzles

This project is dedicated to the Rubik's Cube and the cube will be covered in detail later in this report. This section will instead describe some puzzles related to the Rubik's cube.

5.2.1 Magic Square

A Magic Square is a square which is divided into a number of sub squares. The number of sub squares in any row or column is referred to as the "order" of the Magic Square. In each sub square there is a positive integer. In order for the

Magic Square, to be "magical", the sum of any row, column or diagonal must be the same, this sum is referred to as the magic constant.

The Magic Square(1) hails from ancient China. It was said that the people near the river Lo made offerings. Every time they made an offering a tortoise emerged from the river. On the back of the tortoise there was said to be a Magic Square.

The Magic Square from this tale was of the order 3. This is not the only order in which a Magic Square can be created; it is possible to make an "n" order Magic Square. Generally a Magic Square of the order n contains the numbers from 1 to n^2. It has been proven that it is not possible to make a second order Magic Square. Since simply trying all possible squares with the numbers 1, 2, 3 and 4 inside results no combination that gives the same sum on each row, column and diagonal.

In order to solve the Magic Square, it is needed to know the magic constant – the constant which every row, line and diagonal adds up to for the given order n. This constant can be computed with the formula in 5.1.

$$M(n) = \frac{n \cdot (n^2 + 1)}{2} \tag{5.1}$$

The proof of this formula is quite straight forward. As the table 5.1 illustrates, a Magic Square of the order 3 contains the numbers from 1 to 9.

The sum of the numbers of a row in a Magic Square is equal to the magic constant. If the magic constant is multiplied by the order n it would be equal to the sum of all the integers, since each number only occurs once in a Magic Square.

6	1	8	15
7	5	3	15
2	9	4	15
15	15	15	45

Table 5.1: *A Magic Square of the order 3, by adding the three numbers in any row, column or diagonal, the magic constant is seen to be 15*

The equation 5.2 can be rewritten into the equation 5.3(See proof of the right hand side transcription in appendix X).

$$n \cdot M(n) = \sum_{i=1}^{n^2} i = 1 + \cdots + n^2 \tag{5.2}$$

$$n \cdot M(n) = \frac{n^2 \cdot (n^2 + 1)}{2} \tag{5.3}$$

$$M(n) = \frac{n \cdot (n^2 + 1)}{2} \tag{5.4}$$

The equation 5.4 shows the function which gives the magic constant for a Magic Square of the order n.

Variations of the Magic Square exist. For example the numbers which can be inserted into the sub squares, could exceed n^2. This would change the magic constant. A Magic Squareith the integers 1 to n^2 within its sub squares is called a "Normal Magic Square'.

5.2.2 Magic Cube

A Magic Cube is created from squares put on top of each other so they make up a cube form. This makes it clear that there is a connection between Magic Squares and Magic Cubes. An example of this can be seen on figure 5.1.

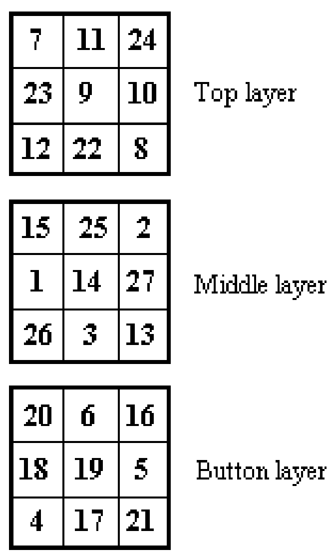

Figure 5.1: *This is a magic cube split up into 3 magic squares.*

Both of them have a magic constant, which can be the sum of each row, colon and pillar. This i where the similarity ends.

We have shown how to calculate the magic constant in a Magic Square. In a Magic Cube there is not a big difference in the formula to calculate the magic constant.

$$M(n) = \frac{n \cdot (n^3 + 1)}{2} \tag{5.5}$$

As shown in the formula the only difference is the power of n that is changed from 2 to 3. See appendix A.1 for en explanation.

To create a Magic Cube, there are some parts that needs to be explained. All these basics are shown on figure 5.2.

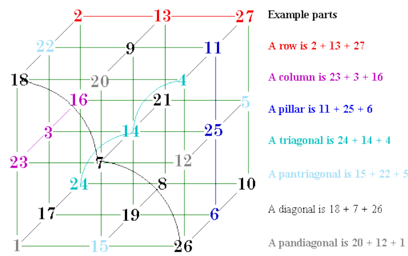

Figure 5.2: *This is a Magic Cube where the colors show all of the parts.*

Because of all these different parts there are a lot of different ways to define Magic Cubes. The simplest of them all is a simple Magic Cube. The only requirements to make such a cube is the following:

- All 9 rows, columns and pillars must be equal to the magic constant.
- All 4 triagonals must also equal the magic constant.

Figure 5.3: This is a Rubiks Cube.

When looking at the Rubik's Cube it is easy to see that it looks a lot like the Magic Cube. There are two differences. The first is that the Magic Cube consists of numbers whereas the Rubik's Cube have colors, which are different on each face. The other difference is that the Magic Cube has a number in the center where the Rubik's Cube does not because it is rotating around the center.

5.2.3 Magic Puzzle

The Magic Puzzle is also known as the 15-puzzle (4, pp. 48-50). It is a puzzle that consists of a tray with 15 square tiles and an empty square arranged in a 4x4 contraption.

It has never been discovered who actually invented the Magic Puzzle, but Samuel Loyd who was an American chess player and puzzle author claimed that he invented the Magic Puzzle and therefore he got the credit. This was disputed by Jerry Slocum who discovered that there was a wooden version of the game back in 1865. This version was manufactured by Embossing Co. Jerry Slocum searched for the patent and found it – US 50.608 applied by Henry May.

Jerry Slocum also found a patent by Ernest U. Kinsey that was published August 20th 1878. This version by Ernest U. Kinsey was a 6x6 version of the puzzle which also prevented the tiles from being lifted out.

Permutations

The tiles in a Magic Puzzle can be arranged in 16! different positions (7). This limit can not be reached because you have to make a permutation to switch the tiles. The permutation must be an even or odd number of transpositions depending on where the position of the empty square is.

The tiles are often numbered or labeled with small pictures which when assembled correctly form a larger picture.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Figure 5.4: *Figure of Magic Puzzle.*

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

Figure 5.5: *Figure of Magic Puzzle with inverse numbers.*

For instance we got the figure 5.4 and want to switch the tiles to be positioned like on figuare 5.5. This permutation requires an odd transposition of the seven pairs (1,15), (2,14), (3,13), (4,12), (5,11), (6,10) and (7,9). This permutation is not possible because it requires an even number of transpositions to get the empty square at the same position. If we color the contraption like a chess board 5.6 we can see that every odd transposition makes the empty square change color and with every even transposition the empty square lands on a square of the same color.

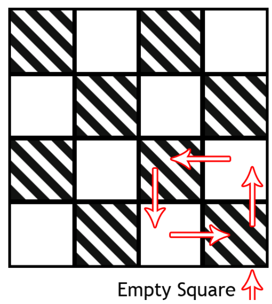

Figure 5.6: *Figure of Empty Square.*

Therefore the number of different positions is $\frac{16!}{2}$. But if the empty square has to be in a fixed position then the possible permutations is $\frac{15!}{2}$.

This chapter has given a definition of recreational mathematics and shown three puzzles, which all relates to the Rubik's Cube; Magic Square, Magic Cube and Magic Puzzle. The Magic Square was the predecessor to Magic Cube, which is in turn the predecessor to the Rubik's Cube. The permutation from the Magic Puzzle inspired the creation of Rubik's Cube, which uses a similar principle for moving the pieces around.

Part II

Appendix

Appendix A

Proofs

A.1 Proof of Magic Constant

Theorem 1 (**Magic Constant**)**.** A hyper cube is a term that covers both the Magic Square and the Magic Cube. In theory the numbers of dimensions of a hyper cube can be any positive integer, the illustration of a hyper cube of any dimension higher than 3 has to be an abstraction. It is still possible to compute the magic constant of a hyper cube of any dimension d of the order n using the function in equation A.1:

$$M\left(n,d\right) = \frac{n^d \cdot (n+1)}{2} \tag{A.1}$$

Proof. The proof of this function resembles that of the function for 2 dimensions – which is the magic Square(See section 5.2.1).

First of all a hyper cube of d dimensions and the order n, contains the integers from 1 through n^d.

The magic constant of the given hyper cube can be obtain by calculating the sum of any line of numbers (be that a row, column, pilar or any other line that is appropriate for the given dimension). This sum can than be multiplied by n^{d-1} which is the same as adding all the numbers in the hyper cube together, since you add one dimension's magic constant's together every time n is multiplied. Therefore we can write:

$$\begin{aligned}
n^{d-1} \cdot M\left(n\right) &= \sum_{i=1}^{n^d} i = 1 + \cdots + n^d \\
n^{d-1} \cdot M\left(n\right) &= \frac{n^d \cdot \left(n^d+1\right)}{2} \\
M\left(n\right) &= \frac{n \cdot \left(n^d+1\right)}{2}
\end{aligned}$$

□

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