Computerstøttet beregning

Lektion 10. Repetition

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Differentialligning

Find en funktion $y(x)$ så

$$\begin{aligned} y'(x) &= f(x, y(x)) \\ y(x_0) &= y_0, \end{aligned}$$

hvor f er en kendt funktion af to variable og x_0 og y_0 er to reelle tal.

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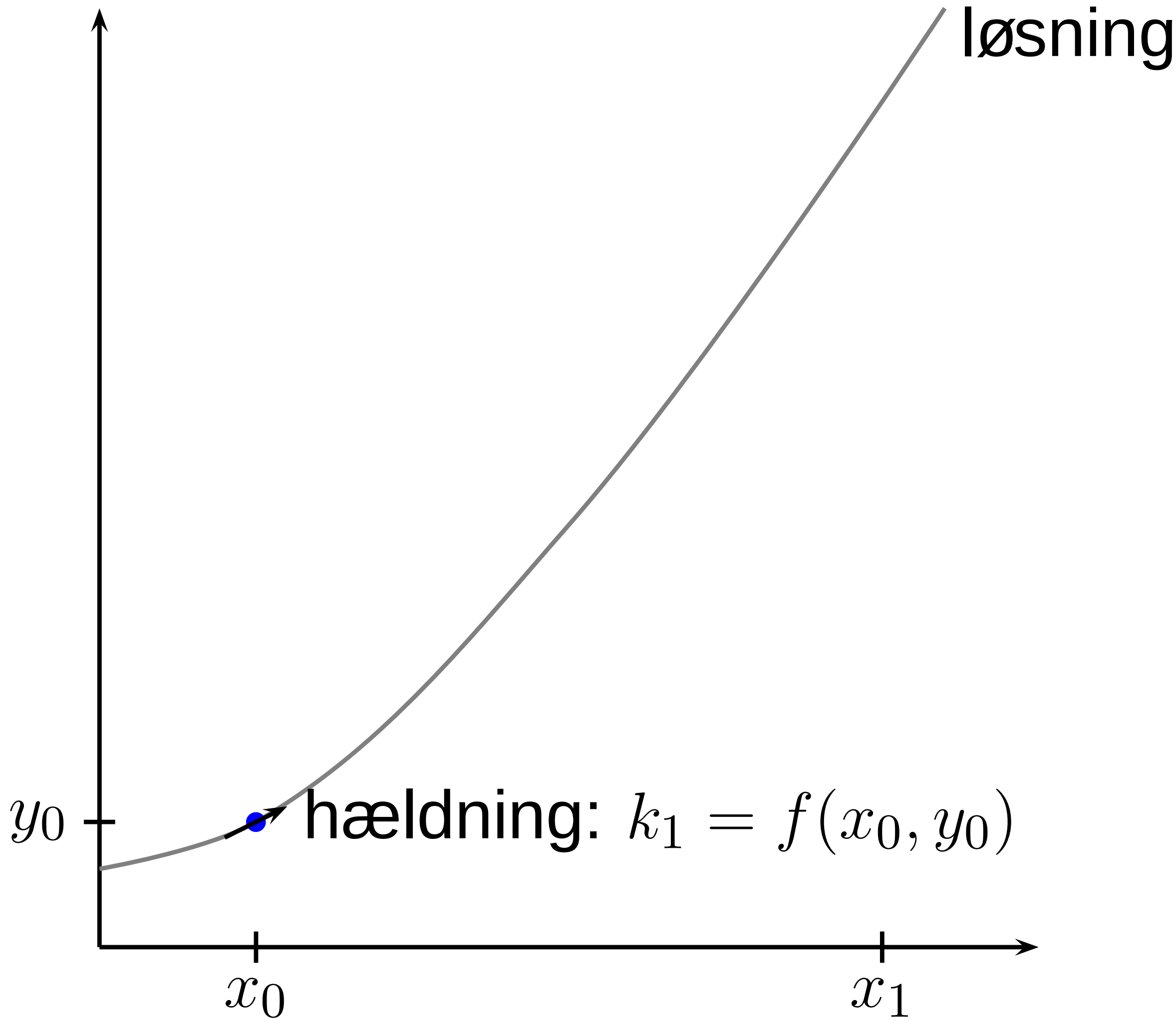
Integralligning:

$$y(x_1) = y_0 + \int_{x_0}^{x_1} f(x, y(x))dx \tag{1}$$

Korrigeret Euler metode



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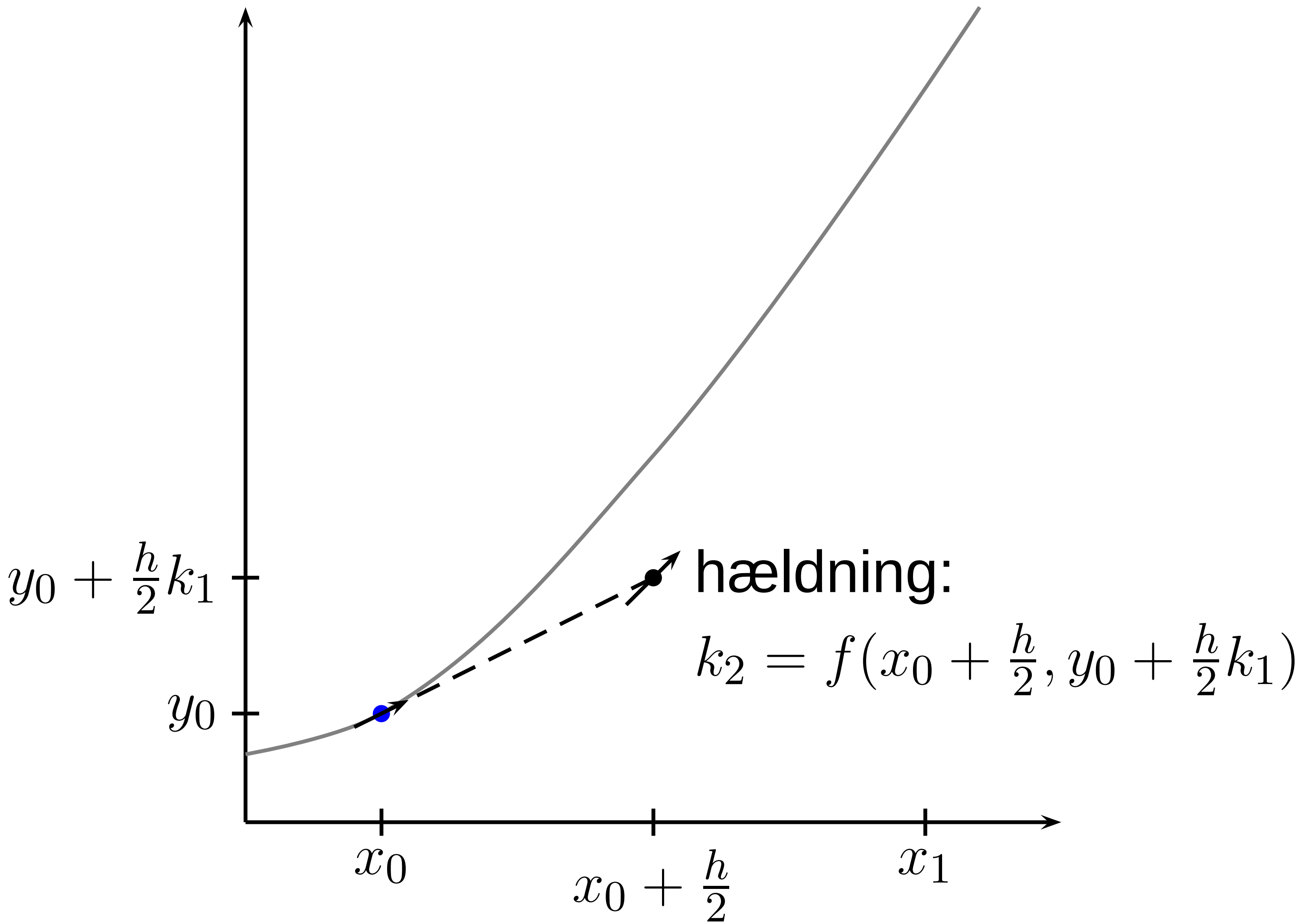

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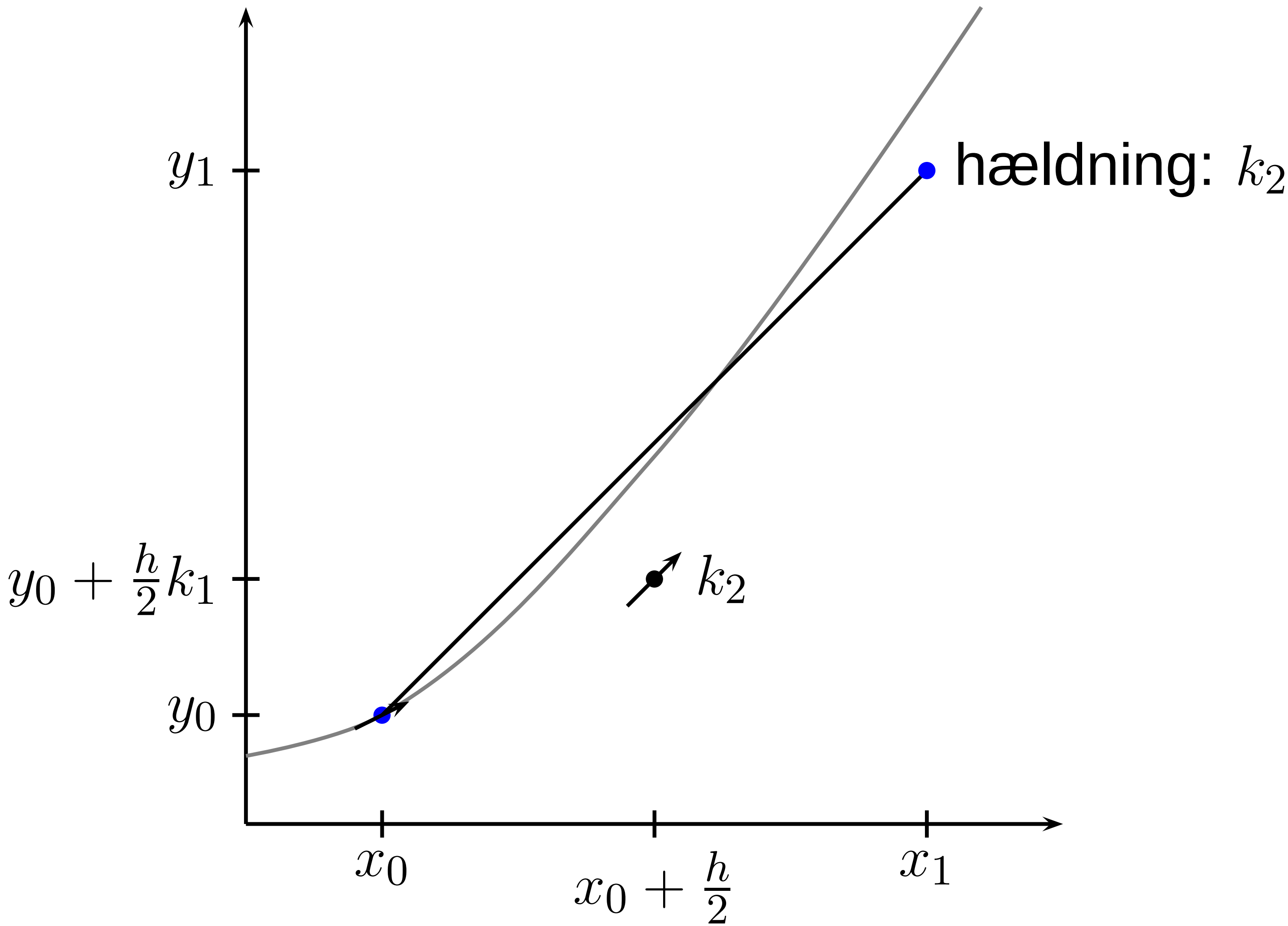


Korrigeret Euler metode

Generelt:

$$x_{n+1} = x_n + h$$
$$y_{n+1} = y_n + h \cdot k_2,$$

hvor $k_1 = f(x_n, y_n)$, $k_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2} \cdot k_1)$.

Svarer til, at integralet i integralligningen (1) approksimeres ved midtpunktsreglen.

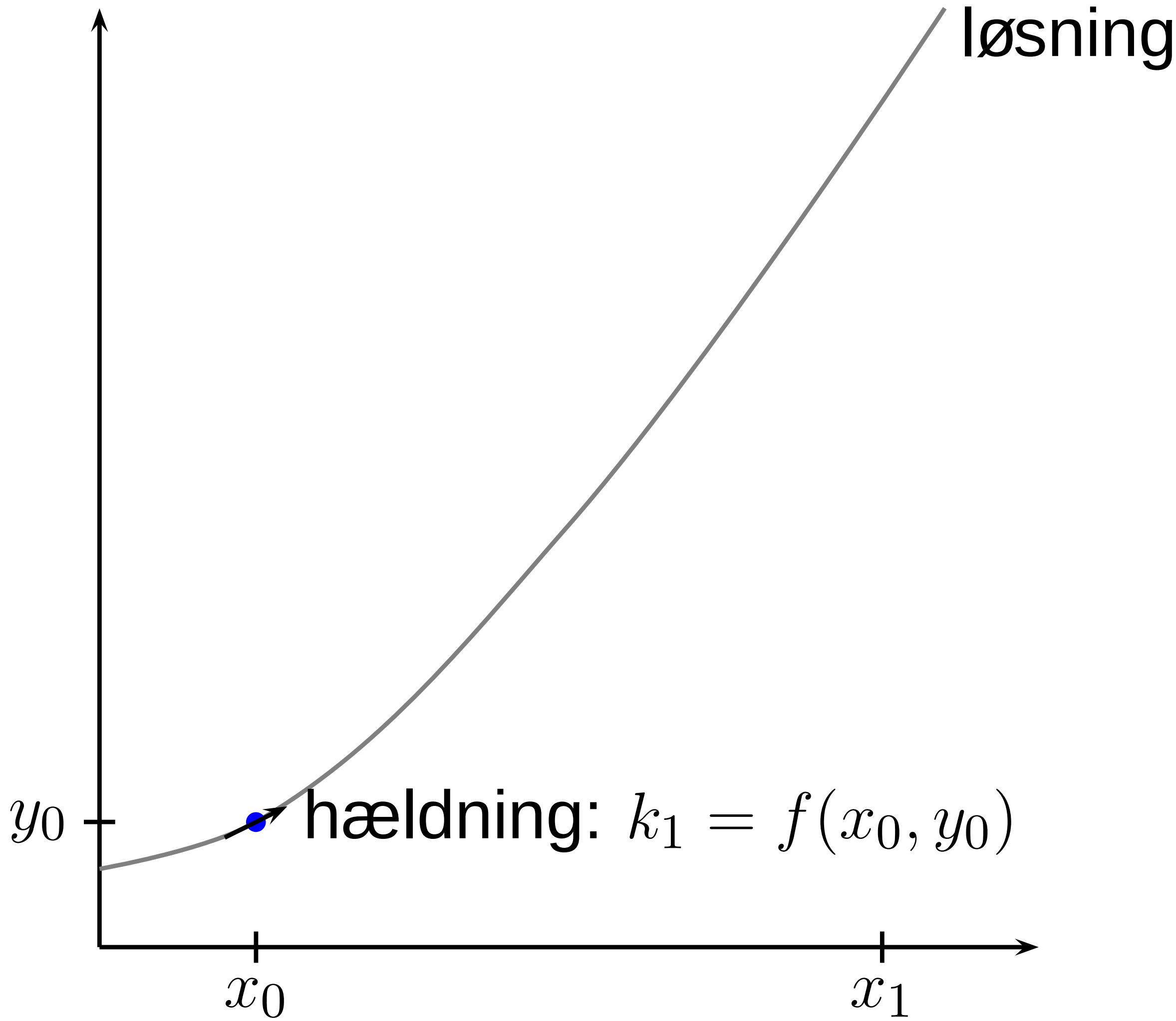
Metoden er en andenordens metode, dvs.

$$E_N = |y_N - y(b)| \leq Ch^2$$

hvor

$$h = \frac{b-a}{N}, \quad x_0 = a.$$

Modificeret Euler metode



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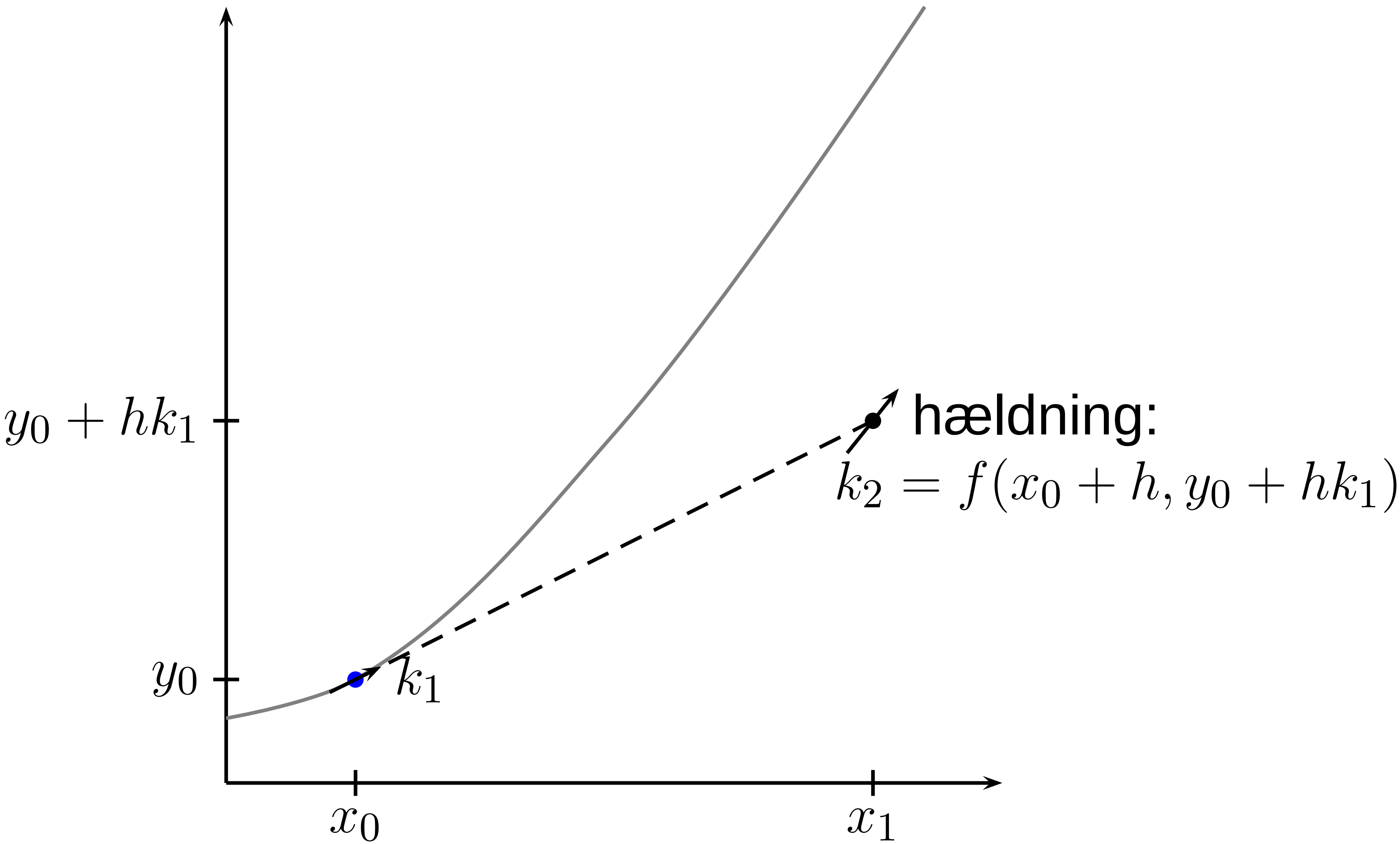
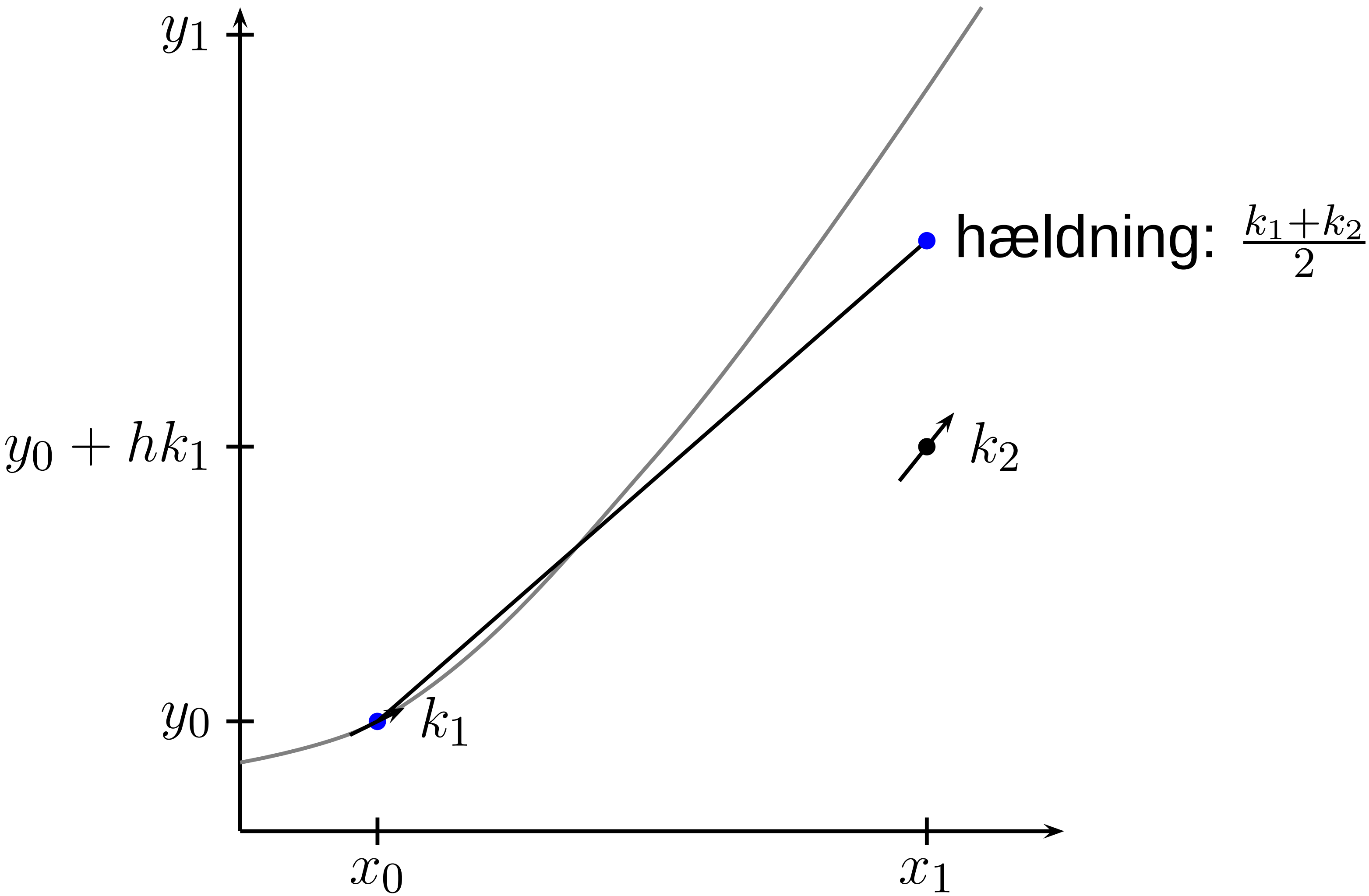


Modificeret Euler metode



Modificeret Euler

Generelt:

$$x_{n+1} = x_n + h$$
$$y_{n+1} = y_n + \tfrac{h}{2}[k_1 + k_2],$$

hvor $k_1 = f(x_n, y_n)$, $k_2 = f(x_{n+1}, y_n + h \cdot k_1)$.

Svarer til, at integralet i integralligningen (1) approksimeres ved Trapezreglen.

Metoden er også en andenordens metode, dvs.

$$E_N = |y_N - y(b)| \leq Ch^2$$

hvor

$$h = \frac{b-a}{N}, \quad x_0 = a.$$

RK4 og Simpsons regel

Integralet i integralligningen approksimeres ved Simpsons regel:

$$y(x_1) \approx y_0 + \frac{h}{6}\Big[f(x_0, y_0) + 4f(m, y(m)) + f(x_1, y(x_1))\Big],$$

hvor $m = x_0 + \frac{h}{2}, \quad h = (x_1 - x_0).$

RK4 og Simpsons regel

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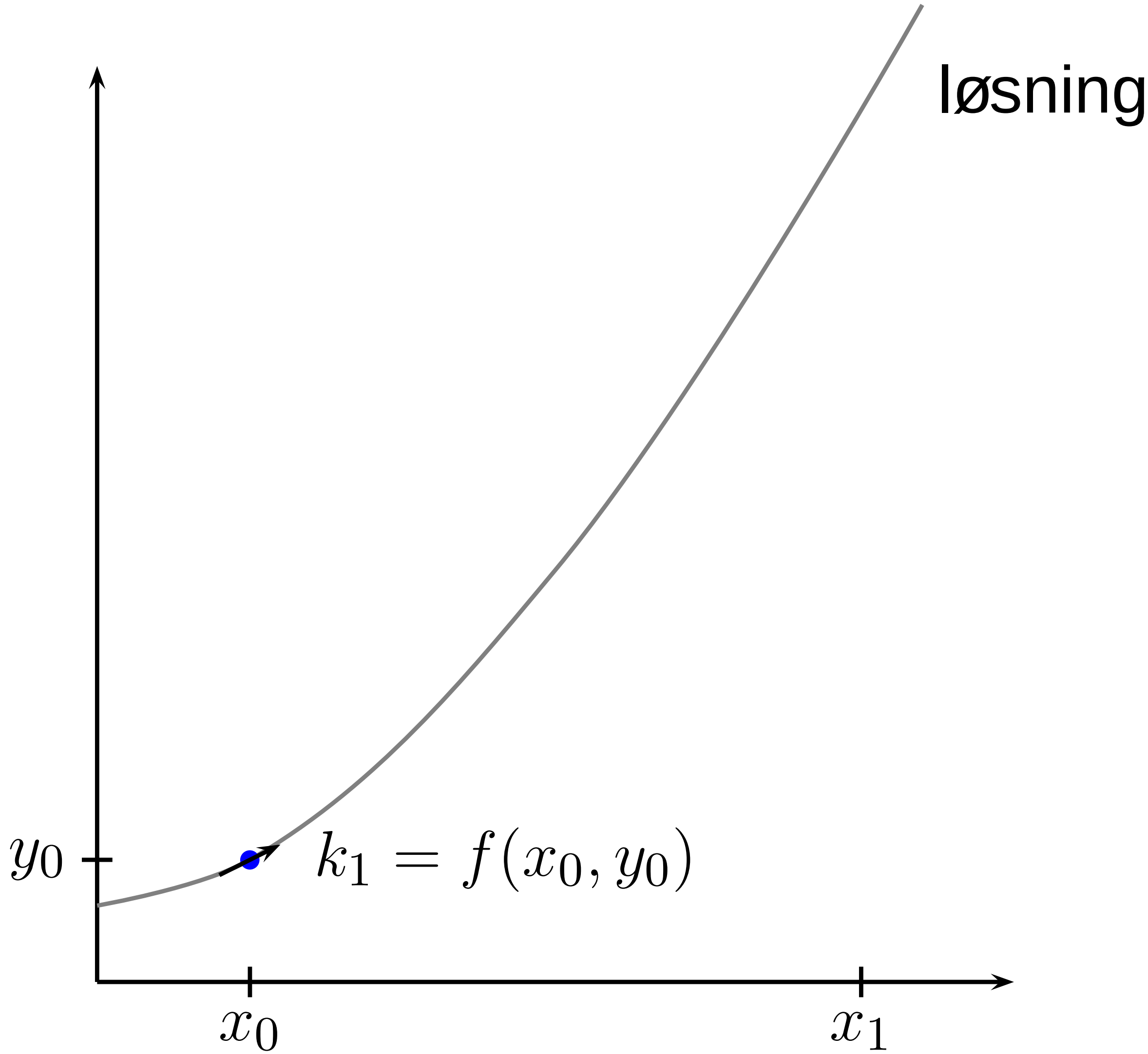
I RK4 approksimeres $f(m, y(m))$ og $f(x_1, y(x_1))$:

$$y(x_1) \approx y_0 + \frac{h}{6}\Big[k_1 + 2\big(k_2 + k_3\big) + k_4\Big]$$

hvor

$$\begin{aligned}
k_1 &= f(x_0, y_0), \\
k_2 &= f\big(x_0 + \tfrac{h}{2}, y_0 + \tfrac{h}{2}k_1\big), \\
k_3 &= f\big(x_0 + \tfrac{h}{2}, y_0 + \tfrac{h}{2}k_2\big), \\
k_4 &= f(x_1, y_0 + hk_3).
\end{aligned}$$

4. ordens Runge-Kutta metode (RK4)



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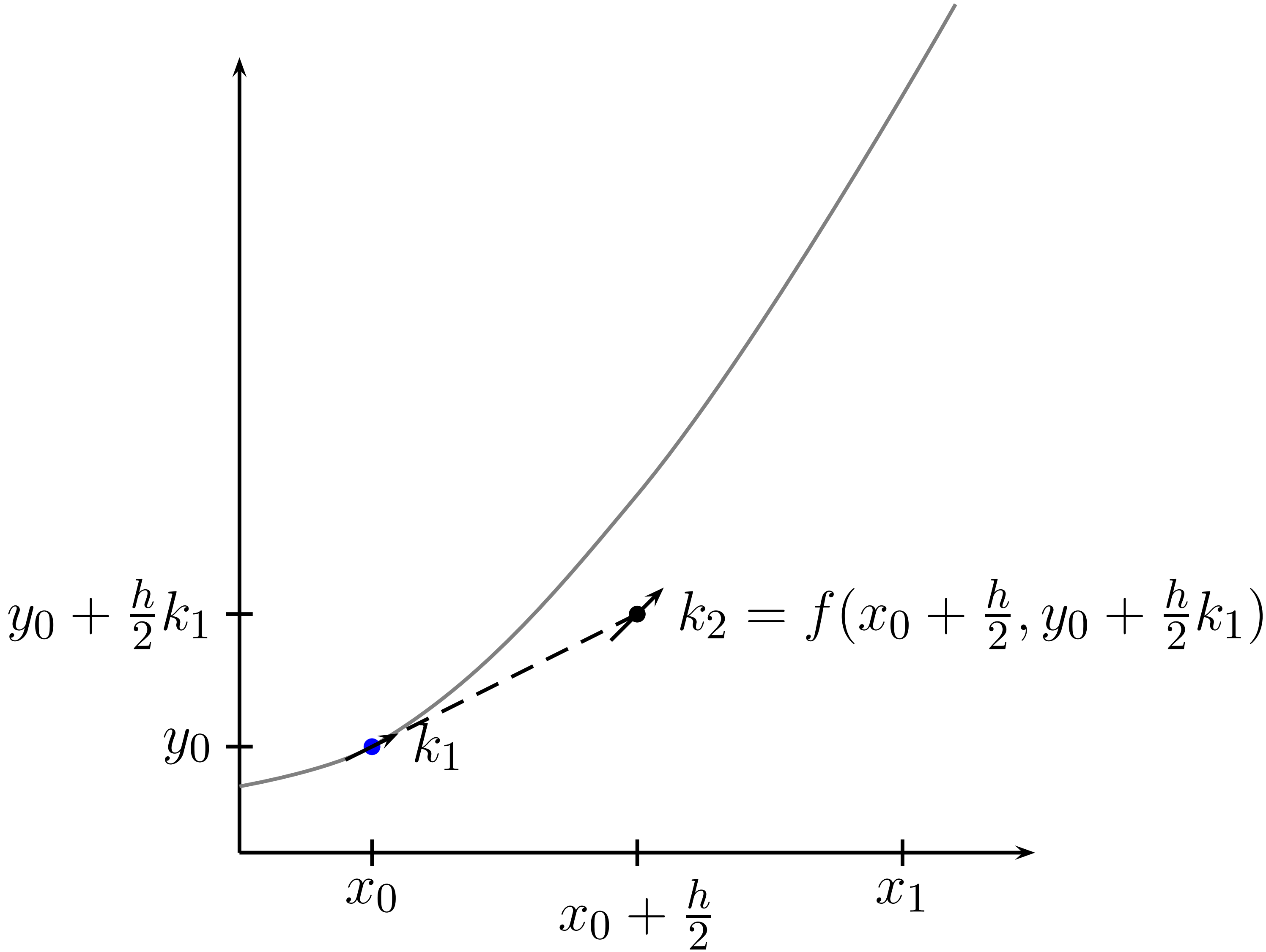

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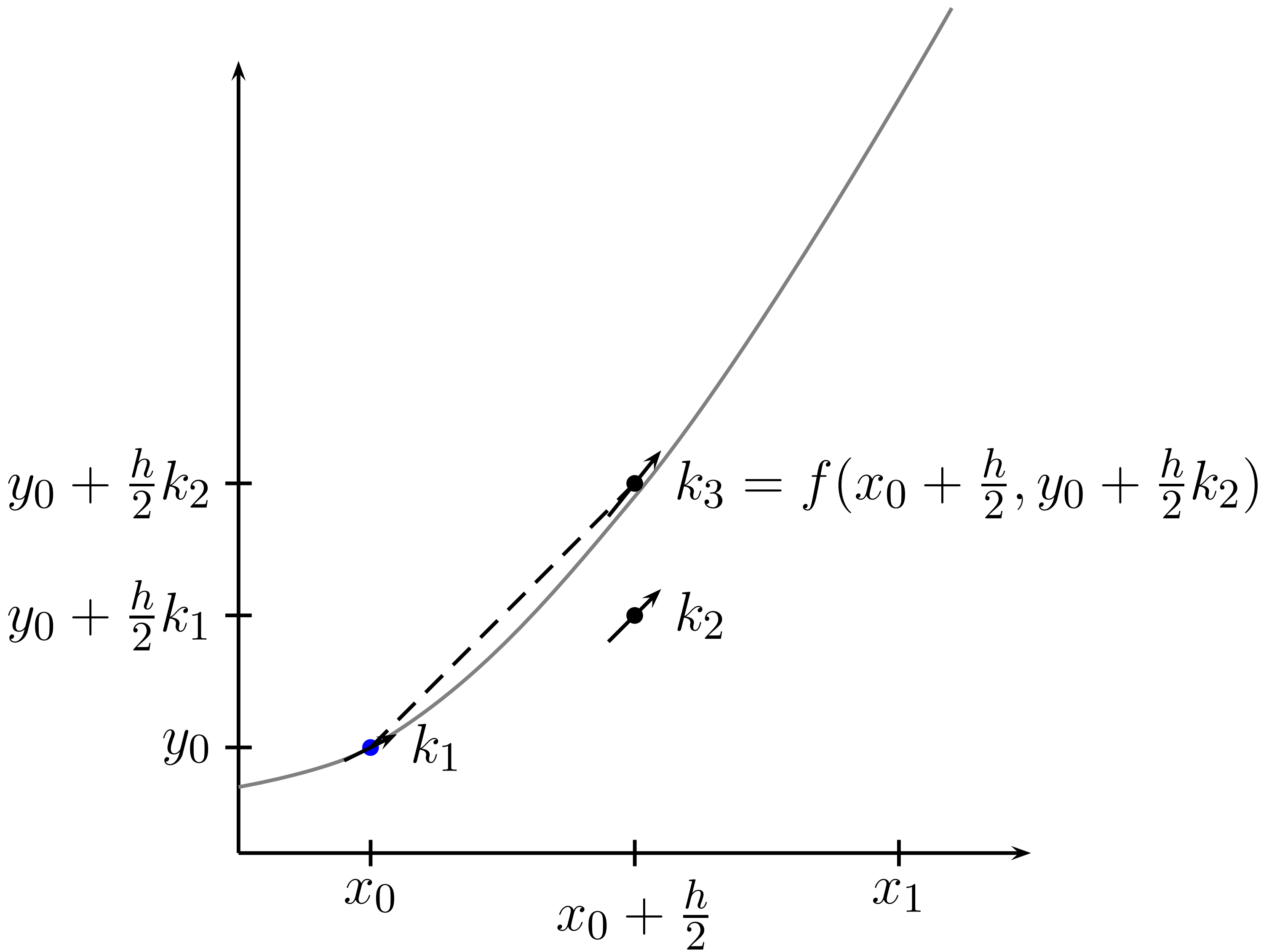

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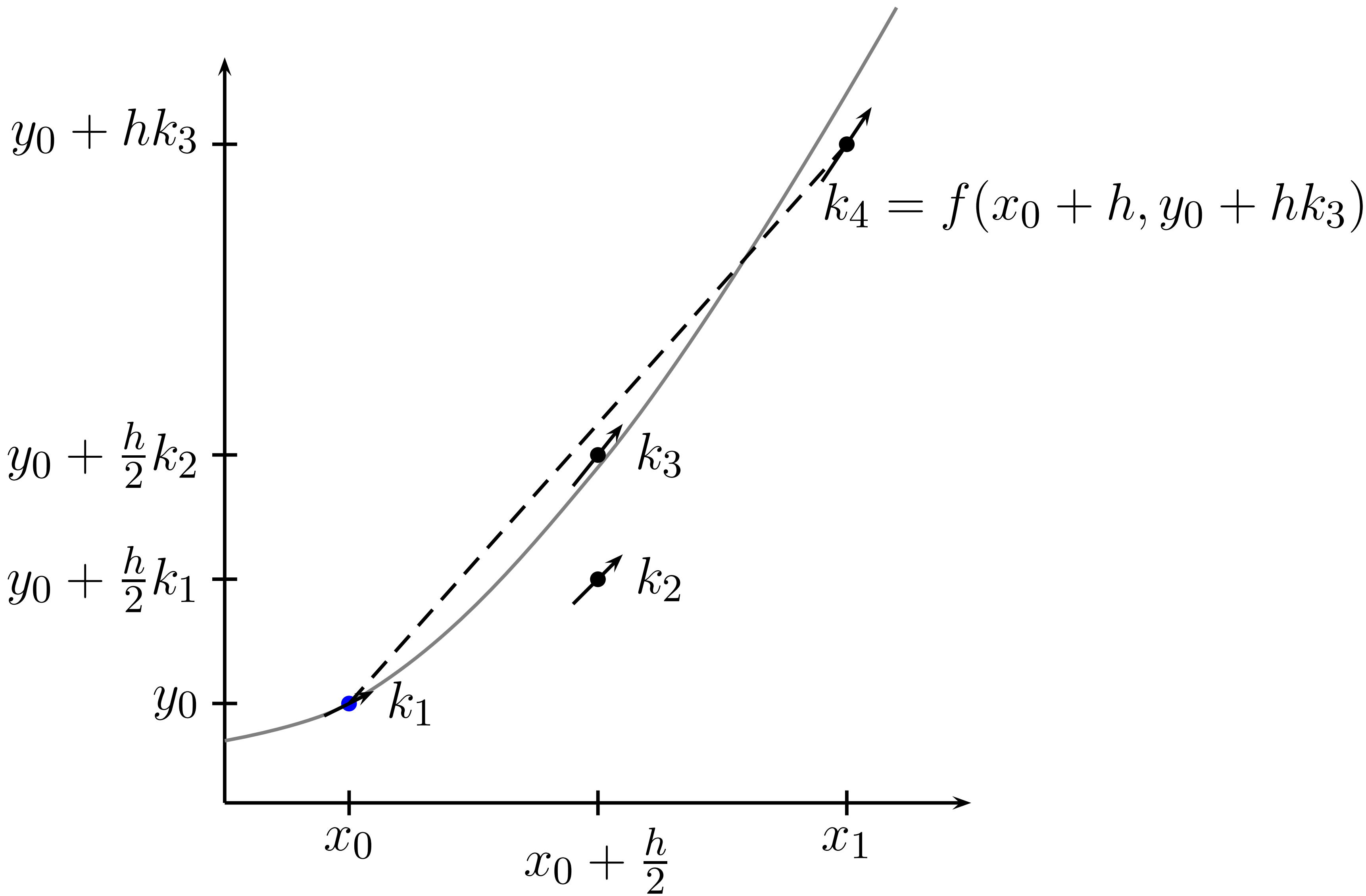

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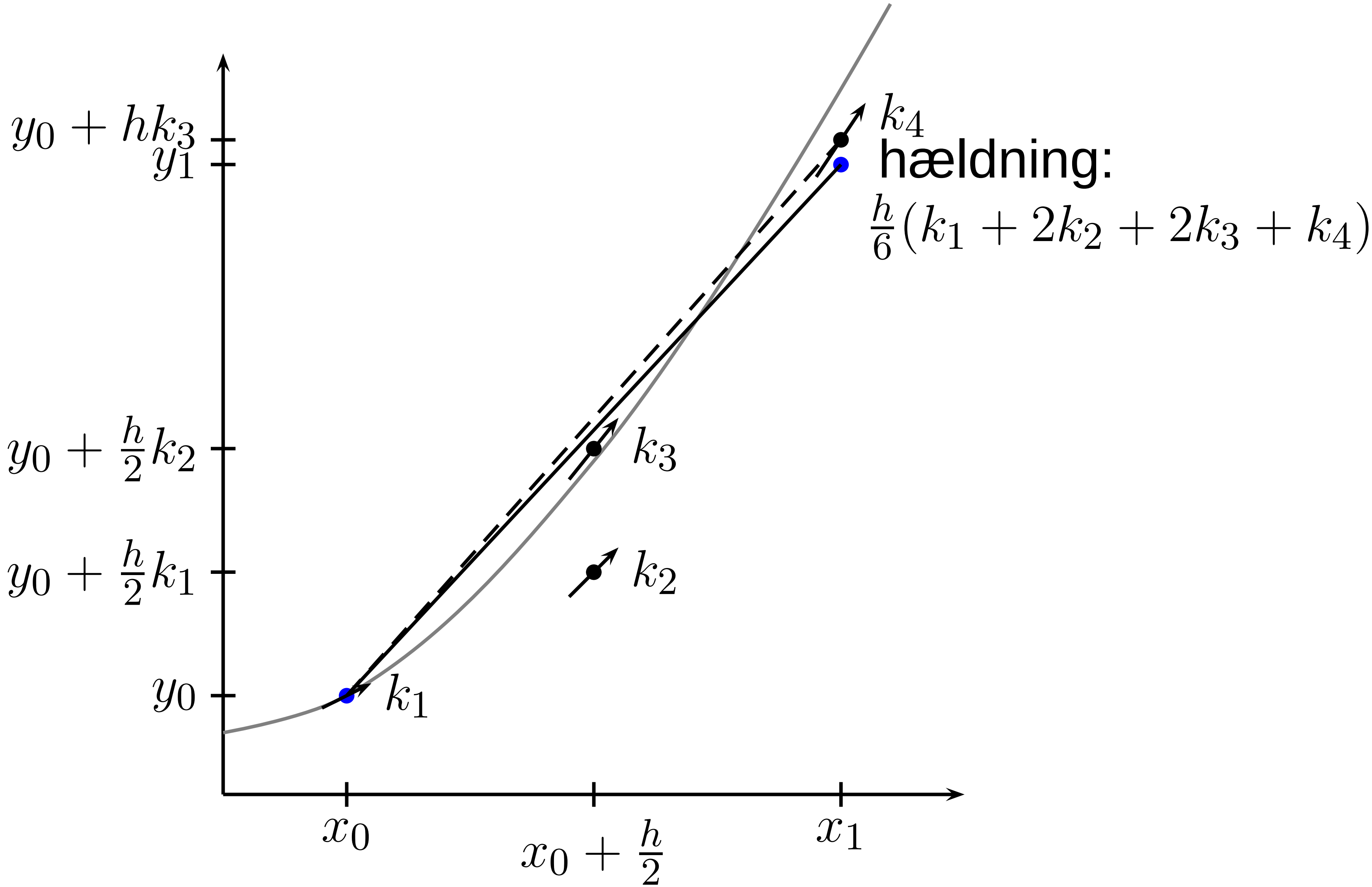

4. ordens Runge-Kutte (RK4)

Generelt:

$$\begin{aligned} x_{n+1} &= x_n + h \\ y_{n+1} &= y_n + \tfrac{h}{6}\big[k_1 + 2(k_2 + k_3) + k_4\big], \end{aligned}$$

hvor

$$\begin{aligned} k_1 &= f(x_n, y_n), \\ k_2 &= f\big(x_n + \tfrac{h}{2}, y_n + \tfrac{h}{2}k_1\big), \\ k_3 &= f\big(x_n + \tfrac{h}{2}, y_n + \tfrac{h}{2}k_2\big), \\ k_4 &= f(x_{n+1}, y_n + hk_3). \end{aligned}$$

Metoden er en fjerdeordens metode, dvs.

$$E_N = |y_N - y(b)| \leq Ch^4$$

hvor $h = \frac{b-a}{N}, \quad x_0 = a.$