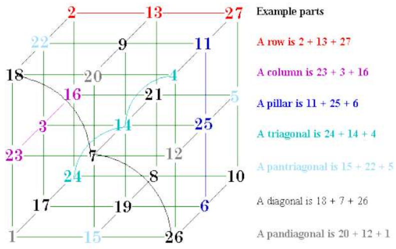


Figure 2.2: *This is a Magic Cube where the colors show all of the parts.*

Because of all these different parts there are a lot of different ways to define Magic Cubes. The simplest of them all is a simple Magic Cube. The only requirements to make such a cube is the following:

- All 9 rows, columns and pillars must be equal to the magic constant.
- All 4 triagonals must also equal the magic constant.

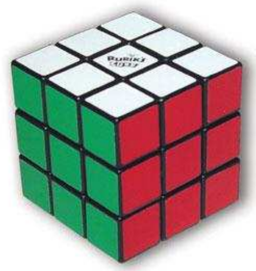


Figure 2.3: *This is a Rubik's Cube.*

When looking at the Rubik's Cube it is easy to see that it looks a lot like the Magic Cube. There are two differences. The first is that the Magic Cube consists of numbers whereas the Rubik's Cube has colors, which are different on each face. The other difference is that the Magic Cube has a number in the center where the Rubik's Cube does not because it is rotating around the center.

2.2.3 Magic Puzzle

The Magic Puzzle is also known as the 15-puzzle [4, pp. 48-50]. It is a puzzle that consists of a tray with 15 square tiles and an empty square arranged in a 4x4 contraption.

It has never been discovered who actually invented the Magic Puzzle, but Samuel Loyd who was an American chess player and puzzle author claimed that

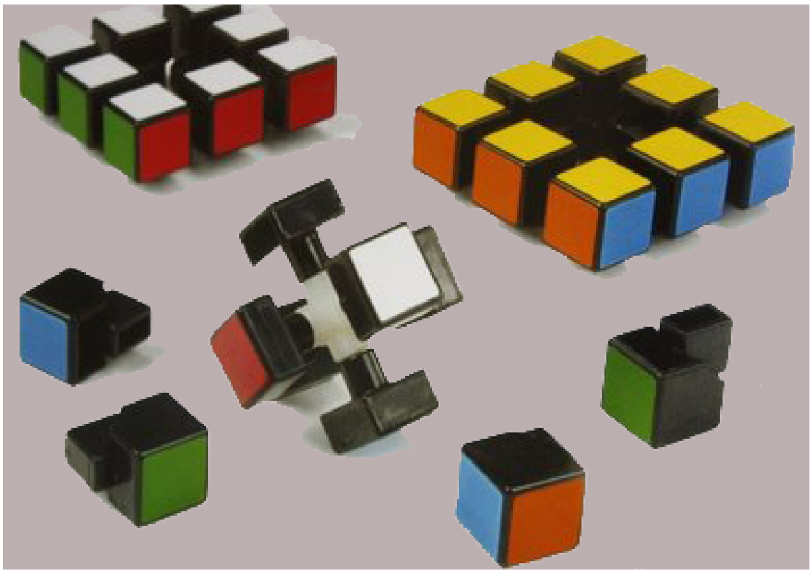


Figure 3.1: *Figure of Rubik's Cube.*

three dimensional objects as result he made the *Magic Cube*n 1974 and obtained a Hungarian patent HU170062. Rubik got the idea for the cube when he wanted to make a three dimensional design with blocks that could move individually but many at the same time. Rubik initially tried to make a cube that was held together with rubber bands but failed. Then he got the idea that the cubes had to hold each other in place, which resulted in a 3x3x3 cube that could twist each face individually. Rubik got the inspiration for the cube from the Magic Puzzle (see chapter 2).

Rubik described that some of the most important features behind the cube were that the parts of the cube stay together, which many other puzzles do not. He also pointed out that you can move several pieces at once. Also that it is three dimensional.

In the end of 70s a Hungarian Businessman showed the Magic Cube at the Nuremberg toy fair and made it popular in Europe. The company Ideal Toy bought exclusive rights for the Magic Cube, but changed the name of the cube to Rubik's Cube within a year in order to get trademark protection.

At that time there were also two others applying for patent for products similar to the Rubik's Cube. One of them was an American named Doctor Larry D. Nichols, and his cube was a 2x2x2 cube which was held together with magnets. The other one who applied for patent was a Japanese man named Terutoshi Ishige. He applied for patent a year after Rubik. Terutoshi Ishige's cube was almost identically to the Rubik's Cube.

Ideal Toy Company were bought by CBS Toy Company in 1982 and the trademark surpassed with it, but they sold the rights to Rubik's Cube to Seven by a single move. Unless a detour is taken trough one or several other positions. The full Rubik's Cube graph consist of approximately $4.33 \cdot 10^{19}$ vertices and all having 18 edges. It is practically impossible to draw this graph. Therefore the graph will be explained with a much simpler graph; the Middle Movement graph.

6.3.1 The Middle Movement Graph

This graph is a Rubik's Cube graph consisting of only the moves that twist the middle sections. This is Rm2 Fm2 Um2. Note: Rm2 = R2 L2. This graph is fairly small, since it only consist of eight vertices and 12 edges. See figure 6.2. [9, pp. 158-167]

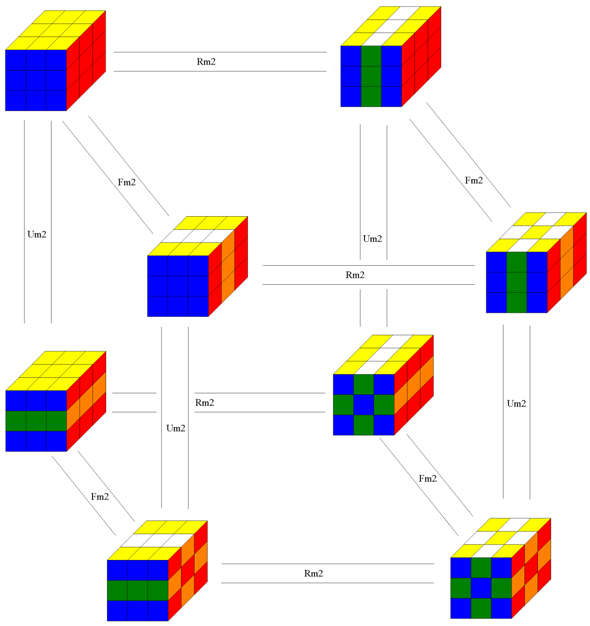


Figure 6.2: *The graph of the middle movement positions.*

Because of this graphs relatively small size the computation of the diameter is a some what simple task. It is possible to calculate the distance from all vertices to all other vertices, but since the graph is clearly symmetrical many calculations can be omitted. It is easy to see that the diameter must go from one corner to an opposite corner e.g. from the solved state to the pons asinorum[1]. The diameter here is 3.

In the full Rubik's Cube group it is believed that the analogous position to the pons asinorum, the superflip position, is the position which has the longest

[1] Pons asinorum is obtained from the solved state of a Rubik's Cube by the move sequence Rm2 Fm2 Um2.

orientation without ruining any part of the cross that may already be assembled:

F' U L' U' or F U' R U

7.1.2 Step 2 - completing the first layer

When the cross is completed the next step is to position the corner pieces of the first layer correctly. The first layer is set as the down face (D). A corner with a facelet of the color of the down face is positioned directly above it's correct position. The correct position is between the three faces that have the same colors as the three facelets of the corner piece. Once the piece is above it's correct position, the cube should be viewed in such an angle that the piece is in the upper right corner of the front face, the following move sequence is repeated until the corner piece is oriented and positioned correctly:

Figure 7.1: *A first layer cross on the green face*

R' D' R D

If the piece is above the correct position the algorithm twists the corner clock-wise and positions it in the correct position. If the piece is in the correct position the algorithm positions the piece above the correct position. The maximum number of repetitions until the piece is oriented and positioned correctly is five, because the piece can be two twists away from it's correct orientation. The move sequence can be performed inverted which twists the corner counter clock-wise and looks as follows:

D' R' D R

If the correct move sequence is used the maximum number of repetitions is three. If number of twists and time used is not of importance it is only necessary to remember one of them.

7.1.3 Step 3 - completing the second layer

The purpose of this step is to position the four edges belonging to the second layer correctly. The move sequence used in this step either swaps the top edge piece of the front face with the left or right edge piece. That is why the edge piece that needs to be moved down to the second layer must be positioned at the top of the face that is next to the face of one of the edges colors, where the second color is the same as the front face's color. There is however a difference in which of the two faces is used as the front face, because if the wrong face is used as front face the piece will only be positioned correctly but not oriented correctly. That facelet of the edge towards the front face must be of the same color as the front face. If the edge piece is to be swapped with the right edge piece of the front face the following move sequence is used:

U R U' R' U' F' U F

If the piece is to be swapped with the left edge piece of the front face the following move sequence is used:

U' L' U L U F U' F'

If none of the edge pieces who belong to the second layer are in the third layer and the first two layers is still not completed. This occurs if the edges are all in the second layer but not positioned or oriented correctly. One of the move sequences can be used to swap an edge piece from the third layer with one of the edges in the second layer which is not positioned or oriented correctly. Now the edge piece belonging to the second layer can be moved to it's correct position.

Figure 7.2: *The first layer completed*

7.1.4 Step 4 - getting the last layer cross

Solving the last layer is divided into four steps. The order of these steps can be different and still yield the same result. We start out by getting the cross in the last layer, which is the same as the cross on the first layer. The move sequence used is however different. The only move sequence used in this step is the following:

F R U R' U' F'

Besides remembering the move sequence it is also important to know how the cube should be oriented.

If the up face colors form a line. The cube can be turned until the line forms a horizontal line. If the move sequence then is performed, the cross will be formed.

Figure 7.3: *The first two layers completed*

(a) *The up face colors form the opposite L* (b) *The up face colors form the horizontal line* (c) *The up face colors form the cross*

Figure 7.4: *The steps in the completion of the last layer cross*

If the up face colors form an opposite L character with the up face colors the move sequence will form the line. The cube must be oriented with the opposite L in the back left corner of the cube.

If the up face colors do not form the opposite L the line or the cross, the move sequence will form the opposite L.

To orient the cross correctly there is one move sequence to remember:

R U R' U R 2U R'

Again it is important to know how to orient the cube. By twisting the upper layer it is possible to position the upper layer so it either has two edge pieces next to each other or directly across from each other.

If the correct edges piece are next to each other. The cube must be oriented with a correct edge piece on the back face and a complete face on the right face. The move sequence will then make it possible to twist the up layer so that all the edge pieces are correctly positioned and oriented.

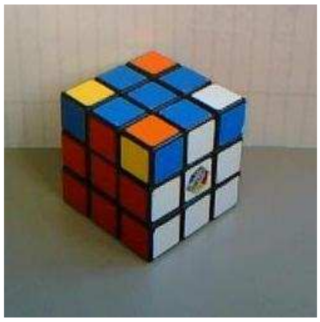


Figure 7.5: *The correct oriented last layer cross.*

If the correct edge pieces are across each other the move sequence can be performed a single time to get two edge pieces next to each other.

7.1.5 Step 5 - completing the last layer

The purpose of this step is to position and orient the corners correctly. Firstly the corners must be positioned correctly. To do so there are two move sequences to remember – one for rotating three corners clockwise and one for rotating them counter-clockwise. It is however only necessary to remember one of them and repeat that one until the corners are positioned correctly, if the number of twists is unimportant to the solver.

U R U' L' U R' U' L
This move sequence will rotate the corners counter-clockwise.

U' L' U R U' L U R'
This move sequence will rotate the corners clockwise.

The orientation is again important to position the corners. If one of the corners already is positioned correctly. That corner is chosen not to be moved.

If the three other corners need to be moved counter-clockwise. The correct corner is positioned as the front right corner. The move sequence will then position the corners correctly.

If the three other corners need to be moved clockwise. The correct corner is positioned as the front left corner. The move sequence will then position the corners correctly.

If there are no correct corners. One of the two move sequences above performed once will make yield a correcly positioned corner.

To orient the corners correctly the move sequence from orienting the corners in the first layer is used:

R' D' R D

But in this step when one corner is completed. Instead of turning the whole cube to the next corner, the cube is locked on one face. Then when going to solve the next corner the upper layer is twisted until the incorrect corner is positioned at the front right corner of the cube and the move sequence is repeated.

Figure 7.6: *The solved cube – e*

7.2 Kociemba's Optimal Solver

Kociamba's optimal solver is an algorithm created with the purpose to find the twist-wise optimal solution to any scrambled Rubik's Cube[3] [12]. The algorithm consists of two phases. The first phase is based on a principle called relabeling, which will be defined at first.

7.2.1 Relabeling

The relabeling process starts with choosing an up face with a corresponding down face. Then choosing a front face with a corresponding back face. Each facelet with the color of the up or down face is marked with the letters “UD”. Every edge piece that is not labeled with ”‘UD”’ is labeled with ”‘FB”’ on the front and back face. Figure 7.7 shows an example. When relabeling a Rubik’s

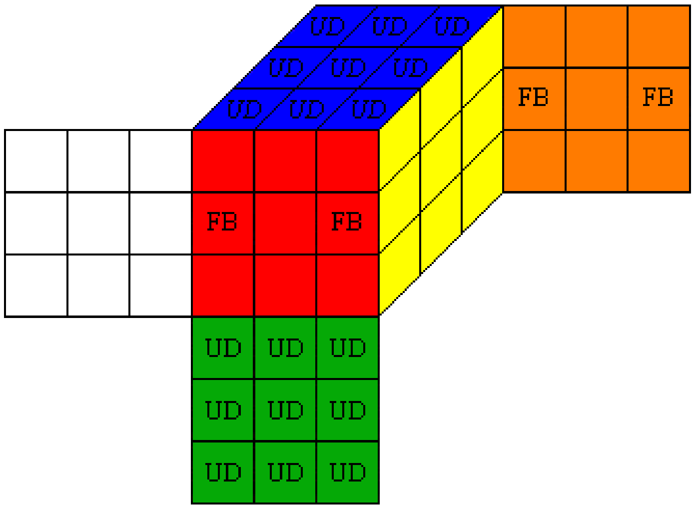


Figure 7.7: *A relabeled Rubik’s Cube with the up and down faces as blue and green and red and orange as front and back.*

Cube in the position s it is written as $r(s)$. A Rubik’s Cube in any position, s, is said to be in the set of positions called H(See subsection 7.2.2) if and only if $r(s) = r(e)$.

7.2.2 The subgroup H

The moves U, U’, U2, D, D’, D2, R2, L2, F2 and B2 is the set of moves A. Using moves from A on a position in H, will always result in a Rubik’s Cube in H. The reason for this can easily be tested with a Rubik’s Cube. If using one of the three up face moves or one of the three down face moves, the “FB” labels are not moved and the “UD” labels are simply rotated on the face that is being twisted, see figure 7.8a. The last four moves are all very similar to each other. If any side face – not up or down – is twisted 180 degrees, the three facelets on the up face is moved to the down face and vice versa and thereby keeping all the “UD” labels on the up and down face and keeping the orientation correct. The two remaining relabeled facelets are swapped which keeps the “FB” labels placed and oriented correctly, see figure 7.8b.

7.2.3 Overall description

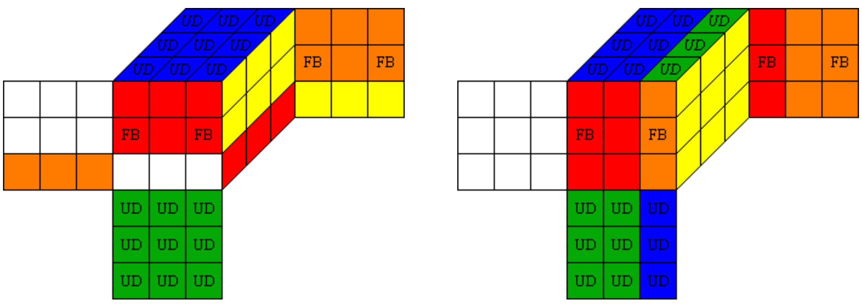
The first phase takes a scrambled cube a and relabels it $r(a)$ then it finds a move sequence b which will transform the relabeled cube into the subgroup H. This move will be denoted: $r(a) \cdot b \in H$. The second phase will then determine the length from the position ab to the unit position e by a table lookup.

Algorithm 1 Kociemba's Algorithm [7]

$d = 0$
$l = \infty$
while $d < l$ **do**
  for $b \in S^d$ **do**
    if $r(ab) \in H$ **then**
      if $d + d_2(ab) < l$ **then**
        $l = d + d_2(ab)$
      end if
    end if
  end for
  $d = d + 1$
end while
b is now the optimal solution

The search distance in the algorithm (d), see figure 7.10 is initially set to zero and the total length (l) is set to infinite. The total length is the amount of moves used to get from a to e, while d is the limited search length to find a move sequence that transforms the Rubik's Cube into a position in H. The **while** loop will run as long as d is less than l.

The **for** loop runs through all move sequences in the range d – recall that S^d is the set containing every move sequence that uses d twists. This search of moves in S^d is called the first phase and is further described in 7.2.4. Then an



(a) *A relabeled cube that has been permutated with the move D.*

(b) *A relabeled cube that has been permutated with the move R2.*

Figure 7.8: *Two positions which have been permuted with a move in A.*