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Introduction

((To be written later))

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Problem Analysis

2.1 Problemstatement

What are the theories and models behind the Rubik's cube and how can they be implemented in an application in order to solve the Rubik's cube?

- Which mathematical models and theories describe the Rubik's cube?
- What is the origin of these theories and models and how have they evolved?
- How can the models and theories be implemented in an overall algorithm for solving the Rubik's cube?
  - How can this algorithm be improved with respect to processing power?

2.2 Problem Limitations

Chapter 3

Development of the Cube

Ernö Rubik is the inventor behind the world famous Rubik's cube. He was born in Budapest, Hungary in 1944. In college he studied sculpture. After his graduation he started studying architecture. Once he graduated in architecture he stayed at the college to teach interior design.

Rubik got the idea for the cube when he wanted to make a 3D design with blocks that could move individually but many at the same time. Rubik initially tried to make a cube that was held together with rubber bands but failed. Then he got the idea that the cubes had to hold each other in place, which resulted in a 2x2x2 cube that could twist each face individually. Rubik got the inspiration for the cube from the Magic Puzzle (see chapter 5). At first the cube was named the Magic Cube. The company Ideal Toy bought exclusive rights for the Magic Cube, but changed the name of the cube to Rubik's cube within a year in order to get trademark protection.

Rubik mentioned that the most important feature of the cube was that the parts of the cube was able to stay together as opposed to many other puzzles. Rubik also expressed his fondness for the cube's ability to move several pieces at once and the fact that the cube is three dimensional.

In January 1975 he applied for patent for his invention in Hungary. Two years later in 1977 he got the patent on the Rubik's cube.

At that time there were also two others applying for patent for products similar to the Rubik's cube. One of them was an American named Doctor Larry D. Nichols, and his cube was a 2x2x2 cube which was held together with magnets. The other one who applied for patent was a Japanese man named Terutoshi Ishige. He applied for patent a year after Rubik. Terutoshi Ishige's cube was almost identically to the Rubik's cube.

3.1 The Nichols Cube Puzzle

Dr. Larry D. Nichols has studied chemistry on DePauw University in Greencastle, Indiana, before moving to Massachusetts to attend Harvard Graduate

School. He is a lifelong puzzle enthusiast and inventor who began developing a twist cube puzzle with six colored faces in 1957. It was made of eight smaller cubes assembled to a 2x2x2 cube. The eight cubes were held together by magnets.

On April 11 1972 he was granted U.S. Patent 3,655,201 on behalf of Moleculon Research Corp. U.S. Patent 3,655,201 covered Nichols Cube and the possibility for making larger versions later. This was two years before Ernö Rubik took out the patent for his Rubik's cube.

In 1982 Moleculon Research corp. Sued Ideal Toy Company that had the U.S. Patent 4,378,116 for Rubik's cube because they believed that Ideal Toy Company violated their patent, but the U.S. District Court ruled in Ideal Toy Company favor. In 1986 the Court of Appeals ruled that the Pocket Rubik's cube 2x2x2 was guilty of infringement but not the 3x3x3 Rubik's cube.

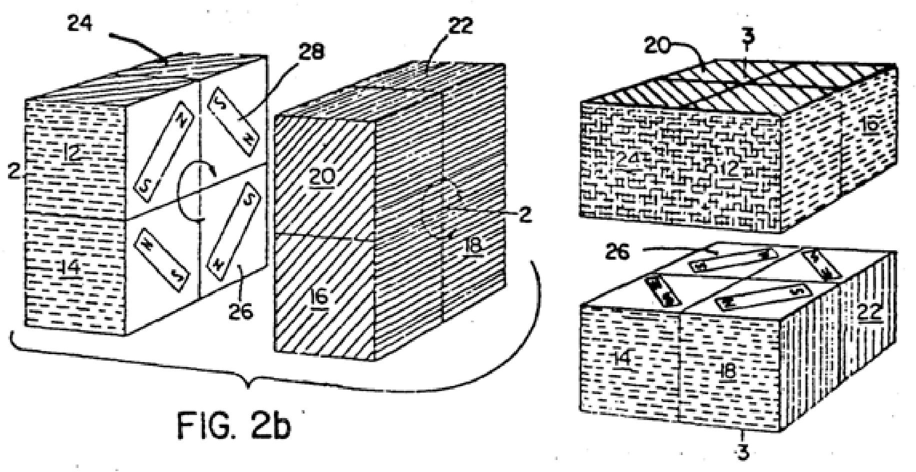

Figure 3.1: *Figure of Nichols Patent.*

Chapter 4

Community

The defintion of a cuber?
The Rubik's cube community consist of both a real life community handling competitions and events and a online community where cubers can find the real life competitions, improve their skills and talk together. The majority of the community is focussed on the speed cubing aspect. In the following chapter both the online and offline community will be discussed. Note that the two sides of the community often interfere with each other.

4.1 The online community

In the world of the Rubik's cube there is a large online community. The community consist of everything from forums and guides to competitions and bragging. Cubers, as they often refer to them selves as, have a place that is the online community to express and compare their abilities, experiences and skills with each other.

4.1.1 Forums

Forums give the cubers a place for sharing their knowledge and experience(2)(1)(10). Forums or specific Rubik's cube sites allow visitors to find information concerning the cube. The main focus of forums is on how to solve the Rubik's cube or similar puzzles in the least amount of time. Forums are often split into several divisions. Some concerning the hardware used and maintenance techniques for making the Rubik's cube spin with greater ease giving the cuber a slightly better solve time. A major part is reserved for the theory and the algorithms used to solve the cube – most of which strive for an efficient solve both in respect to time and number of twists. Another part of the community is focused on competitions of various kinds. The "offline" competitions are held in cooperations with the World Cube Association(WCA) (10), which are further discussed in section 4.2. Beside the WCA-competitions some forums hold weekly online competitions where the forum members can upload their solve times for a Rubik's cube or similar puzzles.

Beside the forums the online community offers a wide variety of sites containing guides, solutions and algorithms for solving the cube. The majority of the Rubik's sites contain the beginner's guide(6) whose target group is the beginner who may recently have got their first cube and wants to learn how to solve it.

4.2 Competitions

Speed cubing competitions are held on a regular basis(11). These competitions have different disciplines for various puzzles related to the original 3x3x3 Rubik's cube. All the official competitions are held in cooperation the World Cube Association (WCA). The WCA governs the official regulations for speed cubing and holds annual world and regional championships. The first World championship in speed cubing was held in 1982 in Budapest, Hungary. WCA governs the official rankings and records for solving the Rubik's cube. In total WCA keeps regulation, ranking and records for 19 different types of competitions. All 19 competitions include puzzles and games which are related to or based on the original Rubik's cube.

This page contain following cites: (2) (1) (10) (11) (6)

Chapter 5

Recreational Mathematics

It is important to understand what recreational mathematics is in order to get a better understanding of the Rubik's cube. This chapter presents a definition of recreational mathematics and a few examples of recreational mathematical puzzles other than the Rubik's cube.

5.1 Definition

Recreation means to do something which is amusing or relaxing. Mathematics is somewhat harder to give a precise definition, due to the vast amount of subjects that fall under this term. Most people do however have a common idea what mathematics is.

Recreational mathematics is hereby defined as mathematical problems, puzzles or games which are fun and interesting to common people. (8) (9, 18)

5.2 Puzzles

This project is dedicated to the Rubik's cube and the cube will be covered in detail later in this report. This section will instead describe some puzzles related to the Rubik's cube.

5.2.1 Magic Square

The Magic Square origins from ancient China. It was said that the people near the river Lo made offerings. Every time they made an offering a tortoise emerge from the river. On the back of the tortoise there was said to be a Magic Square.

The Magic Square from this tale was a 3 order normal magic square. This is not the only order in which a magic square can be created; it is possible to make an "n" order normal magic square. Although it has been proven that it is not possible to make a second order magic square.

6	1	8	15
7	5	3	15
2	9	4	15
15	15	15	45

Table 5.1: *A Magic Square of the order 3, by adding the three numbers in any row, column or diagonal, the magic constant is seen to be 15*

In order to solve the Magic Square, it is needed to know the magic constant – the constant which every row, line and diagonal adds up to for the given order n. This constant can be computed with the formula in 5.1.

$$M(n) = \frac{n \cdot (n^2 + 1)}{2} \tag{5.1}$$

The proof of this formula is quite straight forward. As the table 5.1 illustrates, a Magic Square of the order 3 contains the numbers from 1 to 9. Generally a Magic Square of the order n contains the numbers from 1 to n^2.

The sum of the numbers of a row in a Magic Square is equal to the magic constant. If the magic constant is multiplied by the order n it would be equal to the sum of alle the integers, since each number only occurs once in a Magic Square.

$$n \cdot M(n) = \sum_{i=1}^{n^2} i = 1 + \cdots + n^2 \tag{5.2}$$

The equation 5.2 can be rewritten into the equation 5.3(See proof of the right hand side transcription in appendix X).

$$n \cdot M(n) = \frac{n^2 \cdot \left(n^2 + 1\right)}{2} \tag{5.3}$$

$$M(n) = \frac{n \cdot \left(n^2 + 1\right)}{2} \tag{5.4}$$

The equation 5.4 shows the function which gives the magic constant for a Magic Square of the order n.

5.2.2 Magic Cube

A Magic Cube(3) is created from squares put on top of each other so they make up a cube form. This makes it clear that there is a connection between Magic Squares and Magic Cubes. An example of this can be seen on figure 5.1.

7	11	24
23	9	10
12	22	8

Top layer

15	25	2
1	14	27
26	3	13

Middle layer

20	6	16
18	19	5
4	17	21

Button layer

Figure 5.1: *This is a magic cube split up into 3 magic squares.*

Both of them have a magic constant, which can be the sum of each row, colon and diagonal if it is a Magic Cube or normal Magic Square. This i where the similarity ends.

We have shown how to calculate the magic constant in a Magic Square. In a Magic Cube there is not a big differens in the formula to calculate the magic constant.

$$M(n) = \frac{n \cdot (n^3 + 1)}{2} \tag{5.5}$$

As shown in the formula the only diffenrence is the power of n that is changed from 2 to 3. This will be show in appendix A.1 why it is so.

To create a Magic Cube, there is some parts that needs to be explanied. All these basics is shown on figure 5.2.

Because of all these different parts there is a lot of different ways to define Magic Cubes. The simplets of them all is a simpel Magic Cube, the only requerements to make such is the following:

- All 9- rows, columns and pillars must equal the magic constant.
- All 4 triagonals must also equal the magic constant.

5.2.3 Magic Puzzle

The Magic Puzzle is also known as the 15-puzzle (5, pp. 48-50). It's a puzzle that consist of a tray with 15 square tiles and an empty square arranged in a 4x4 contraption. ((Correct choice of word?))

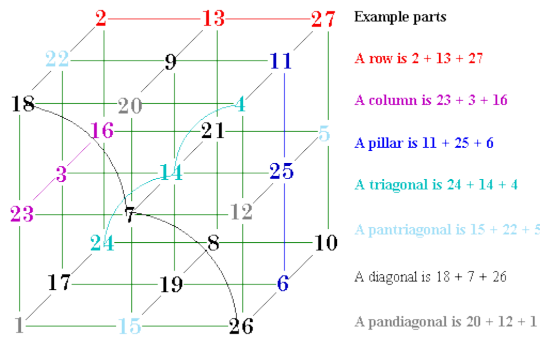

Figure 5.2: *This is a Magic Cube where the colors show all of the parts.*

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Figure 5.3: *Figure of Magic Puzzle.*

The tiles are often numbered or labeled with small pictures which correctly assembled form a larger picture.

It has never been discovered who actually invented the Magic Puzzle, but Samuel Loyd who were an American chess player and puzzle author claimed that he invented the Magic Puzzle and therefore he get the credit. This is turned down by a research of Jerry Slocum. He discovered that there was a wooden version of the game already in 1865, this was manufactured by the Embossing Co. Jerry Slocum searched for the patent and found it, US 50.608 and was applied by a Henry May.

Jerry Slocum also found a patent by Ernest U. Kinsey that was published August 20th 1878. This version by Ernest U. Kinsey is a 6x6 version of the puzzle which also prevented the tiles from being lifted out.

Permutations

The tiles in a Magic Puzzle can be arranged in 16! different positions (7). This limit can not be reached because you have to make a permutation to switch the tiles. The permutation must be an even or odd number(correct word?) of transpositions depending on where the position of the empty square is.

For instance we got the figure above (need reference) and want to switch the tiles to be positioned like this (need picture). This permutation requires an odd transposition of the seven pairs (1,15), (2,14), (3,13), (4,12), (5,11), (6,10) and (7,9). This permutation isnt possible because it requires an even number (amount?) of transpositions to get the empty square at the same position. If we color the contraption like a chess board (need picture), then we can se that every odd transposition makes the empty square change color and with every even transposition the empty square lands on the same kind of color.

Therefore the number of different positions is $\frac{16!}{2}$. But if the empty square has to be in a fixed position then the possible permutations is down to $\frac{15!}{2}$.

Part II

Appendix

Appendix A

Proofs

A.1 Proof of Magic Constant

A hyper cube (4) is a term that covers both the Magic Square and the Magic Cube. In theory the numbers of dimensions of a hyper cube can be any positive integer, the illustration of a hyper cube of any dimension higher than 3 has to be an absraction. It is still posible to compute the magic constant of a hyper cube of en dimension d of the order n using the function in equation A.1:

$$M\left(n,d\right)=\frac{n^{d}\cdot\left(n+1\right)}{2} \tag{A.1}$$

The proof of this function resembles that of the function for 2 dimensions – which is the magic Square(See section 5.2.1).

First of all a hyper cube of d dimensions and the order n, contains the integers from 1 through n^d.

The magic constant of the given hyper cube can be obtain by calculating the sum of any line of numbers (be that a row, column, pilar or any other line that is appropriate for the given dimension). This sum can than be multiplied by n^{d-1} which is the same as adding all the numbers in the hyper cube together, since you add one dimension's magic constant's together every time n is multiplied. Therefore we can write:

$$n^{d-1}\cdot M\left(n\right)=\sum_{i=1}^{n^d} i=1+\cdots+n^{d} \tag{A.2}$$

$$n^{d-1}\cdot M\left(n\right)=\frac{n^{d}\cdot\left(n^{d}+1\right)}{2} \tag{A.3}$$

$$M\left(n\right)=\frac{n\cdot\left(n^{d}+1\right)}{2} \tag{A.4}$$

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