CHAPTER 9
Work, Energy, and Simple Machines

Figure 9.1 People on a roller coaster experience thrills caused by changes in types of energy. (Jonrev, Wikimedia Commons)

Chapter Outline

INTRODUCTION Roller coasters have provided thrills for daring riders around the world since the nineteenth century. Inventors of roller coasters used simple physics to build the earliest examples using railroad tracks on mountainsides and old mines. Modern roller coaster designers use the same basic laws of physics to create the latest amusement park favorites. Physics principles are used to engineer the machines that do the work to lift a roller coaster car up its first big incline before it is set loose to roll. Engineers also have to understand the changes in the car's energy that keep it speeding over hills, through twists, turns, and even loops.

What exactly is energy? How can changes in force, energy, and simple machines move objects like roller coaster cars? How can machines help us do work? In this chapter, you will discover the answer to this question and many more, as you learn about work, energy, and simple machines.

9.1 Work, Power, and the Work–Energy Theorem

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe and apply the work–energy theorem
- Describe and calculate work and power

Section Key Terms

energy	gravitational potential energy	joule	kinetic energy	mechanical energy
potential energy	power	watt	work	work–energy theorem

The Work–Energy Theorem

In physics, the term **work** has a very specific definition. Work is application of force, $\mathbf{f}$, to move an object over a distance, d, in the direction that the force is applied. Work, W, is described by the equation

$$W = \mathbf{f}d.$$

Some things that we typically consider to be work are not work in the scientific sense of the term. Let's consider a few examples. Think about why each of the following statements is true.

- Homework *is not* work.
- Lifting a rock upwards off the ground *is* work.
- Carrying a rock in a straight path across the lawn at a constant speed *is not* work.

The first two examples are fairly simple. Homework is not work because objects are not being moved over a distance. Lifting a rock up off the ground is work because the rock is moving in the direction that force is applied. The last example is less obvious. Recall from the laws of motion that force is *not* required to move an object at constant velocity. Therefore, while some force may be applied to keep the rock up off the ground, no net force is applied to keep the rock moving forward at constant velocity.

Work and **energy** are closely related. When you do work to move an object, you change the object's energy. You (or an object) also expend energy to do work. In fact, energy can be defined as the ability to do work. Energy can take a variety of different forms, and one form of energy can transform to another. In this chapter we will be concerned with **mechanical energy**, which comes in two forms: **kinetic energy** and **potential energy**.

- Kinetic energy is also called energy of motion. A moving object has kinetic energy.
- Potential energy, sometimes called stored energy, comes in several forms. **Gravitational potential energy** is the stored energy an object has as a result of its position above Earth's surface (or another object in space). A roller coaster car at the top of a hill has gravitational potential energy.

Let's examine how doing work on an object changes the object's energy. If we apply force to lift a rock off the ground, we increase the rock's potential energy, PE. If we drop the rock, the force of gravity increases the rock's kinetic energy as the rock moves downward until it hits the ground.

The force we exert to lift the rock is equal to its weight, w, which is equal to its mass, m, multiplied by acceleration due to gravity, $\mathbf{g}$.

$$\mathbf{f} = w = m\mathbf{g}$$

The work we do on the rock equals the force we exert multiplied by the distance, d, that we lift the rock. The work we do on the rock also equals the rock's gain in gravitational potential energy, PE_e.

$$W = PE_e = \mathbf{f}m\mathbf{g}$$

Kinetic energy depends on the mass of an object and its velocity, $\mathbf{v}$.

$$KE = \frac{1}{2}m\mathbf{v}^2$$

When we drop the rock the force of gravity causes the rock to fall, giving the rock kinetic energy. When work done on an object increases only its kinetic energy, then the net work equals the change in the value of the quantity $\frac{1}{2}m\mathbf{v}^2$. This is a statement of the **work–energy theorem**, which is expressed mathematically as

$$W = \Delta KE = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2.$$

The subscripts $_2$ and $_1$ indicate the final and initial velocity, respectively. This theorem was proposed and successfully tested by James Joule, shown in Figure 9.2.

Does the name Joule sound familiar? The **joule** (J) is the metric unit of measurement for both work and energy. The measurement of work and energy with the same unit reinforces the idea that work and energy are related and can be converted into one another. 1.0 J = 1.0 N•m, the units of force multiplied by distance. 1.0 N = 1.0 k•m/s^2, so 1.0 J = 1.0 k•m^2/s^2. Analyzing the units of the term (1/2)$m\mathbf{v}^2$ will produce the same units for joules.

Figure 9.2 The joule is named after physicist James Joule (1818–1889). (C. H. Jeens, Wikimedia Commons)

WATCH PHYSICS

Work and Energy

This video explains the work energy theorem and discusses how work done on an object increases the object's KE.

Click to view content (https://www.khanacademy.org/embed_video?v=2WS1sG9fhOk)

GRASP CHECK

True or false—The energy increase of an object acted on only by a gravitational force is equal to the product of the object's weight and the distance the object falls.

a. True
b. False

Calculations Involving Work and Power

In applications that involve work, we are often interested in how fast the work is done. For example, in roller coaster design, the amount of time it takes to lift a roller coaster car to the top of the first hill is an important consideration. Taking a half hour on the ascent will surely irritate riders and decrease ticket sales. Let's take a look at how to calculate the time it takes to do work.

Recall that a rate can be used to describe a quantity, such as work, over a period of time. **Power** is the rate at which work is done. In this case, rate means *per unit of time*. Power is calculated by dividing the work done by the time it took to do the work.

$$P = \frac{W}{t}$$

Let's consider an example that can help illustrate the differences among work, force, and power. Suppose the woman in Figure 9.3 lifting the TV with a pulley gets the TV to the fourth floor in two minutes, and the man carrying the TV up the stairs takes five minutes to arrive at the same place. They have done the same amount of work $(\mathbf{f}d)$ on the TV, because they have moved the same mass over the same vertical distance, which requires the same amount of upward force. However, the woman using the pulley has generated more power. This is because she did the work in a shorter amount of time, so the denominator of the power formula, t, is smaller. (For simplicity's sake, we will leave aside for now the fact that the man climbing the stairs has also done work on himself.)

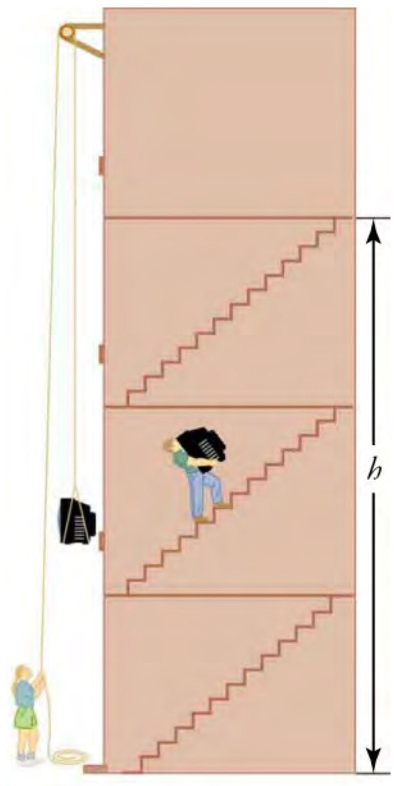

Figure 9.3 No matter how you move a TV to the fourth floor, the amount of work performed and the potential energy gain are the same.

Power can be expressed in units of **watts** (W). This unit can be used to measure power related to any form of energy or work. You have most likely heard the term used in relation to electrical devices, especially light bulbs. Multiplying power by time gives the amount of energy. Electricity is sold in kilowatt-hours because that equals the amount of electrical energy consumed.

The watt unit was named after James Watt (1736–1819) (see Figure 9.4). He was a Scottish engineer and inventor who discovered how to coax more power out of steam engines.

Figure 9.4 Is James Watt thinking about watts? (Carl Frederik von Breda, Wikimedia Commons)

LINKS TO PHYSICS

Watt's Steam Engine

James Watt did not invent the steam engine, but by the time he was finished tinkering with it, it was more useful. The first steam engines were not only inefficient, they only produced a back and forth, or reciprocal, motion. This was natural because pistons move in and out as the pressure in the chamber changes. This limitation was okay for simple tasks like pumping water or mashing potatoes, but did not work so well for moving a train. Watt was able build a steam engine that converted reciprocal motion to circular motion. With that one innovation, the industrial revolution was off and running. The world would never be the same. One of Watt's steam engines is shown in Figure 9.5. The video that follows the figure explains the importance of the steam engine in the industrial revolution.

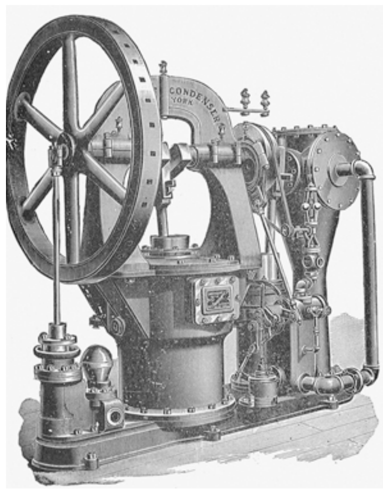

Figure 9.5 A late version of the Watt steam engine. (Nehemiah Hawkins, Wikimedia Commons)

WATCH PHYSICS

Watt's Role in the Industrial Revolution

This video demonstrates how the watts that resulted from Watt's inventions helped make the industrial revolution possible and allowed England to enter a new historical era.

Click to view content (https://www.youtube.com/embed/zhL5DCizj5c)

GRASP CHECK

Which form of mechanical energy does the steam engine generate?

a. Potential energy
b. Kinetic energy
c. Nuclear energy
d. Solar energy

Before proceeding, be sure you understand the distinctions among force, work, energy, and power. Force exerted on an object over a distance does work. Work can increase energy, and energy can do work. Power is the rate at which work is done.

WORKED EXAMPLE

Applying the Work–Energy Theorem

An ice skater with a mass of 50 kg is gliding across the ice at a speed of 8 m/s when her friend comes up from behind and gives her a push, causing her speed to increase to 12 m/s. How much work did the friend do on the skater?

Strategy

The work–energy theorem can be applied to the problem. Write the equation for the theorem and simplify it if possible.

$$W = \Delta \mathrm{KE} = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2$$

$$\text{Simplify to } W = \frac{1}{2}m(\mathbf{v}_2^2 - \mathbf{v}_1^2)$$

Solution

Identify the variables. $m = 50$ kg,

$$\mathbf{v}_2 = 12\frac{\mathrm{m}}{\mathrm{s}}, \text{ and } \mathbf{v}_1 = 8\frac{\mathrm{m}}{\mathrm{s}} \tag{9.1}$$

Substitute.

$$W = \frac{1}{2}50(12^2 - 8^2) = 2,000 \text{ J} \tag{9.2}$$

Discussion

Work done on an object or system increases its energy. In this case, the increase is to the skater's kinetic energy. It follows that the increase in energy must be the difference in KE before and after the push.

TIPS FOR SUCCESS

This problem illustrates a general technique for approaching problems that require you to apply formulas: Identify the unknown and the known variables, express the unknown variables in terms of the known variables, and then enter all the known values.

Practice Problems

1. How much work is done when a weightlifter lifts a 200 N barbell from the floor to a height of 2 m?
   a. 0 J
   b. 100 J
   c. 200 J
   d. 400 J

2. Identify which of the following actions generates more power. Show your work.
   - carrying a 100 N TV to the second floor in 50 s or
   - carrying a 24 N watermelon to the second floor in 10 s?

   a. Carrying a 100 N TV generates more power than carrying a 24 N watermelon to the same height because power is defined as work done times the time interval.
   b. Carrying a 100 N TV generates more power than carrying a 24 N watermelon to the same height because power is defined as the ratio of work done to the time interval.
   c. Carrying a 24 N watermelon generates more power than carrying a 100 N TV to the same height because power is defined as work done times the time interval.
   d. Carrying a 24 N watermelon generates more power than carrying a 100 N TV to the same height because power is defined as the ratio of work done and the time interval.

Check Your Understanding

3. Identify two properties that are expressed in units of joules.
   a. work and force
   b. energy and weight
   c. work and energy
   d. weight and force

4. When a coconut falls from a tree, work W is done on it as it falls to the beach. This work is described by the equation

$$W = Fd = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2. \tag{9.3}$$

Identify the quantities F, d, m, v_1, and v_2 in this event.

a. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the earth, v_1 is the initial velocity, and v_2 is the velocity with which it hits the beach.
b. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the coconut, v_1 is the initial velocity, and v_2 is the velocity with which it hits the beach.
c. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the earth, v_1 is the velocity with which it hits the beach, and v_2 is the initial velocity.
d. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the coconut, v_1 is the velocity with which it hits the beach, and v_2 is the initial velocity.

9.2 Mechanical Energy and Conservation of Energy

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the law of conservation of energy in terms of kinetic and potential energy
- Perform calculations related to kinetic and potential energy. Apply the law of conservation of energy

Section Key Terms

law of conservation of energy

Mechanical Energy and Conservation of Energy

We saw earlier that mechanical energy can be either potential or kinetic. In this section we will see how energy is transformed from one of these forms to the other. We will also see that, in a closed system, the sum of these forms of energy remains constant.

Quite a bit of potential energy is gained by a roller coaster car and its passengers when they are raised to the top of the first hill. Remember that the *potential* part of the term means that energy has been stored and can be used at another time. You will see that this stored energy can either be used to do work or can be transformed into kinetic energy. For example, when an object that has gravitational potential energy falls, its energy is converted to kinetic energy. Remember that both work and energy are expressed in joules.

Refer back to . The amount of work required to raise the TV from point A to point B is equal to the amount of gravitational potential energy the TV gains from its height above the ground. This is generally true for any object raised above the ground. If all the work done on an object is used to raise the object above the ground, the amount work equals the object's gain in gravitational potential energy. However, note that because of the work done by friction, these energy–work transformations are never perfect. Friction causes the loss of some useful energy. In the discussions to follow, we will use the approximation that transformations are frictionless.

Now, let's look at the roller coaster in Figure 9.6. Work was done on the roller coaster to get it to the top of the first rise; at this point, the roller coaster has gravitational potential energy. It is moving slowly, so it also has a small amount of kinetic energy. As the car descends the first slope, its *PE* is converted to *KE*. At the low point much of the original *PE* has been transformed to *KE*, and speed is at a maximum. As the car moves up the next slope, some of the *KE* is transformed back into *PE* and the car slows down.

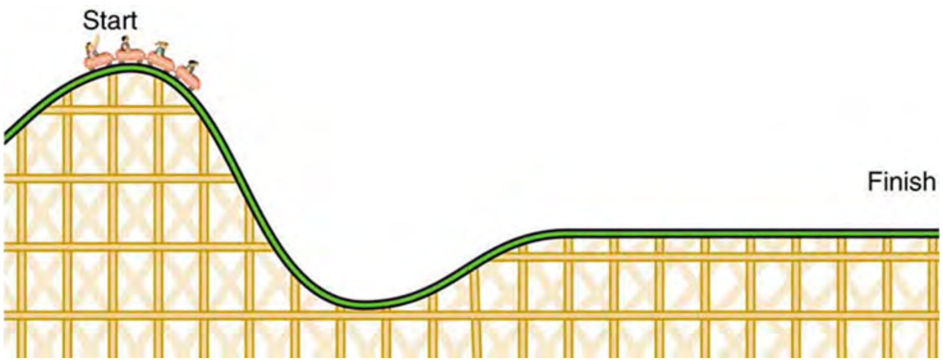

Figure 9.6 During this roller coaster ride, there are conversions between potential and kinetic energy.

Virtual Physics

Energy Skate Park Basics

This simulation shows how kinetic and potential energy are related, in a scenario similar to the roller coaster. Observe the changes in *KE* and *PE* by clicking on the bar graph boxes. Also try the three differently shaped skate parks. Drag the skater to the track to start the animation.

Click to view content (http://phet.colorado.edu/sims/html/energy-skate-park-basics/latest/energy-skate-park-basics_en.html)

GRASP CHECK

This simulation (http://phet.colorado.edu/en/simulation/energy-skate-park-basics (http://phet.colorado.edu/en/simulation/energy-skate-park-basics)) shows how kinetic and potential energy are related, in a scenario similar to the roller coaster. Observe the changes in KE and PE by clicking on the bar graph boxes. Also try the three differently shaped skate parks. Drag the skater to the track to start the animation. The bar graphs show how KE and PE are transformed back and forth. Which statement best explains what happens to the mechanical energy of the system as speed is increasing?

a. The mechanical energy of the system increases, provided there is no loss of energy due to friction. The energy would transform to kinetic energy when the speed is increasing.
b. The mechanical energy of the system remains constant provided there is no loss of energy due to friction. The energy would transform to kinetic energy when the speed is increasing.
c. The mechanical energy of the system increases provided there is no loss of energy due to friction. The energy would transform to potential energy when the speed is increasing.
d. The mechanical energy of the system remains constant provided there is no loss of energy due to friction. The energy would transform to potential energy when the speed is increasing.

On an actual roller coaster, there are many ups and downs, and each of these is accompanied by transitions between kinetic and potential energy. Assume that no energy is lost to friction. At any point in the ride, the total mechanical energy is the same, and it is equal to the energy the car had at the top of the first rise. This is a result of the **law of conservation of energy**, which says that, in a closed system, total energy is conserved—that is, it is constant. Using subscripts 1 and 2 to represent initial and final energy, this law is expressed as

$$KE_1 + PE_1 = KE_2 + PE_2.$$

Either side equals the total mechanical energy. The phrase *in a closed system* means we are assuming no energy is lost to the surroundings due to friction and air resistance. If we are making calculations on dense falling objects, this is a good assumption. For the roller coaster, this assumption introduces some inaccuracy to the calculation.

Calculations involving Mechanical Energy and Conservation of Energy

TIPS FOR SUCCESS

When calculating work or energy, use units of meters for distance, newtons for force, kilograms for mass, and seconds for time. This will assure that the result is expressed in joules.

WATCH PHYSICS

Conservation of Energy

This video discusses conversion of *PE* to *KE* and conservation of energy. The scenario is very similar to the roller coaster and the skate park. It is also a good explanation of the energy changes studied in the snap lab.

Click to view content (https://www.khanacademy.org/embed_video?v=kw_4Loo1HR4)

GRASP CHECK

Did you expect the speed at the bottom of the slope to be the same as when the object fell straight down? Which statement best explains why this is not exactly the case in real-life situations?

a. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is large amount of friction. In real life, much of the mechanical energy is lost as heat caused by friction.
b. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is small amount of friction. In real life, much of the mechanical energy is lost as heat caused by friction.
c. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is large amount of friction. In real life, no mechanical energy is lost due to conservation of the mechanical energy.
d. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is small amount of friction. In real life, no mechanical energy is lost due to conservation of the mechanical energy.

WORKED EXAMPLE

Applying the Law of Conservation of Energy

A 10 kg rock falls from a 20 m cliff. What is the kinetic and potential energy when the rock has fallen 10 m?

Strategy

Choose the equation.

$$KE_1 + PE_1 = KE_2 + PE_2 \tag{9.4}$$

$$KE = \tfrac{1}{2}m\mathbf{v}^2; \quad PE = m\mathbf{g}h \tag{9.5}$$

$$\frac{1}{2}m\mathbf{v}_1^2 + m\mathbf{g}h_1 = \frac{1}{2}m\mathbf{v}_2^2 + m\mathbf{g}h_2 \tag{9.6}$$

List the knowns.

$m = 10\text{ kg}$, $\mathbf{v}_1 = 0$, $\mathbf{g} = 9.80$

$$\frac{\text{m}}{\text{s}^2}, \tag{9.7}$$

$h_1 = 20\text{ m}$, $h_2 = 10\text{ m}$

Identify the unknowns.

KE_2 and PE_2

Substitute the known values into the equation and solve for the unknown variables.

Solution

$$PE_2 = mgh_2 = 10\,(9.80)\,10 = 980\ \text{J} \tag{9.8}$$

$$KE_2 = PE_2 - (KE_1 + PE_1) = 980 - \{[0 - [10\,(9.80)\,20]]\} = 980\ \text{J} \tag{9.9}$$

Discussion

Alternatively, conservation of energy equation could be solved for $\mathbf{v}_2$ and KE_2 could be calculated. Note that m could also be eliminated.

TIPS FOR SUCCESS

Note that we can solve many problems involving conversion between *KE* and *PE* without knowing the mass of the object in question. This is because kinetic and potential energy are both proportional to the mass of the object. In a situation where $KE = PE$, we know that $m\mathbf{g}h = (1/2)m\mathbf{v}^2$.
Dividing both sides by m and rearranging, we have the relationship

$2\mathbf{g}h = \mathbf{v}^2$.

Practice Problems

5. A child slides down a playground slide. If the slide is 3 m high and the child weighs 300 N, how much potential energy does the child have at the top of the slide? (Round g to $10\ \ m/s^2$.)
   a. 0 J
   b. 100 J
   c. 300 J
   d. 900 J

6. A 0.2 kg apple on an apple tree has a potential energy of 10 J. It falls to the ground, converting all of its PE to kinetic energy. What is the velocity of the apple just before it hits the ground?
   a. 0 m/s
   b. 2 m/s
   c. 10 m/s
   d. 50 m/s

Snap Lab

Converting Potential Energy to Kinetic Energy

In this activity, you will calculate the potential energy of an object and predict the object's speed when all that potential energy has been converted to kinetic energy. You will then check your prediction.

You will be dropping objects from a height. Be sure to stay a safe distance from the edge. Don't lean over the railing too far. Make sure that you do not drop objects into an area where people or vehicles pass by. Make sure that dropping objects will not cause damage.

You will need the following:

Materials for each pair of students:

- Four marbles (or similar small, dense objects)
- Stopwatch

Materials for class:

- Metric measuring tape long enough to measure the chosen height
- A scale

Instructions

Procedure

1. Work with a partner. Find and record the mass of four small, dense objects per group.
2. Choose a location where the objects can be safely dropped from a height of at least 15 meters. A bridge over water with a safe pedestrian walkway will work well.
3. Measure the distance the object will fall.
4. Calculate the potential energy of the object before you drop it using $PE = m\mathbf{g}h = (9.80)mh$.
5. Predict the kinetic energy and velocity of the object when it lands using $PE = KE$ and so,
   $m\mathbf{g}h = \frac{m\mathbf{v}^2}{2}$; $\mathbf{v} = \sqrt{2(9.80)h} = 4.43\sqrt{h}$.
6. One partner drops the object while the other measures the time it takes to fall.
7. Take turns being the dropper and the timer until you have made four measurements.
8. Average your drop multiplied by and calculate the velocity of the object when it landed using $\mathbf{v} = \mathbf{a}t = \mathbf{g}t = (9.80)t$.
9. Compare your results to your prediction.

GRASP CHECK

Galileo's experiments proved that, contrary to popular belief, heavy objects do not fall faster than light objects. How do the equations you used support this fact?

a. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term gets cancelled and the velocity is independent of the mass. In real life, the variation in the velocity of the different objects is observed because of the non-zero air resistance.
b. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term does not get cancelled and the velocity is dependent on the mass. In real life, the variation in the velocity of the different objects is observed because of the non-zero air resistance.
c. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy the system, the mass term gets cancelled and the velocity is independent of the mass. In real life, the variation in the velocity of the different objects is observed because of zero air resistance.
d. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term does not get cancelled and the velocity is dependent on the mass. In real life, the variation in the velocity of the different objects is observed because of zero air resistance.

Check Your Understanding

7. Describe the transformation between forms of mechanical energy that is happening to a falling skydiver before his parachute opens.
a. Kinetic energy is being transformed into potential energy.
b. Potential energy is being transformed into kinetic energy.
c. Work is being transformed into kinetic energy.
d. Kinetic energy is being transformed into work.

8. True or false—If a rock is thrown into the air, the increase in the height would increase the rock's kinetic energy, and then the increase in the velocity as it falls to the ground would increase its potential energy.
a. True
b. False

9. Identify equivalent terms for *stored energy* and *energy of motion*.
a. Stored energy is potential energy, and energy of motion is kinetic energy.
b. Energy of motion is potential energy, and stored energy is kinetic energy.
c. Stored energy is the potential as well as the kinetic energy of the system.
d. Energy of motion is the potential as well as the kinetic energy of the system.

9.3 Simple Machines

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe simple and complex machines
- Calculate mechanical advantage and efficiency of simple and complex machines

Section Key Terms

complex machine	efficiency output	ideal mechanical advantage	inclined plane	input work
lever	mechanical advantage	output work	pulley	screw
simple machine	wedge	wheel and axle		

Simple Machines

Simple machines make work easier, but they do not decrease the amount of work you have to do. Why can't simple machines change the amount of work that you do? Recall that in closed systems the total amount of energy is conserved. A machine cannot increase the amount of energy you put into it. So, why is a simple machine useful? Although it cannot change the amount of work you do, a simple machine can change the amount of force you must apply to an object, and the distance over which you apply the force. In most cases, a simple machine is used to reduce the amount of force you must exert to do work. The down side is that you must exert the force over a greater distance, because the product of force and distance, $\mathbf{f}d$, (which equals work) does not change.

Let's examine how this works in practice. In Figure 9.7(a), the worker uses a type of **lever** to exert a small force over a large distance, while the pry bar pulls up on the nail with a large force over a small distance. Figure 9.7(b) shows the how a lever works mathematically. The effort force, applied at $\mathbf{F_e}$, lifts the load (the resistance force) which is pushing down at $\mathbf{F}_r$. The triangular pivot is called the fulcrum; the part of the lever between the fulcrum and $\mathbf{F}_e$ is the effort arm, L_e; and the part to the left is the resistance arm, L_r. The **mechanical advantage** is a number that tells us how many times a simple machine multiplies the effort force. The **ideal mechanical advantage**, IMA, is the mechanical advantage of a perfect machine with no loss of useful work caused by friction between moving parts. The equation for IMA is shown in Figure 9.7(b).

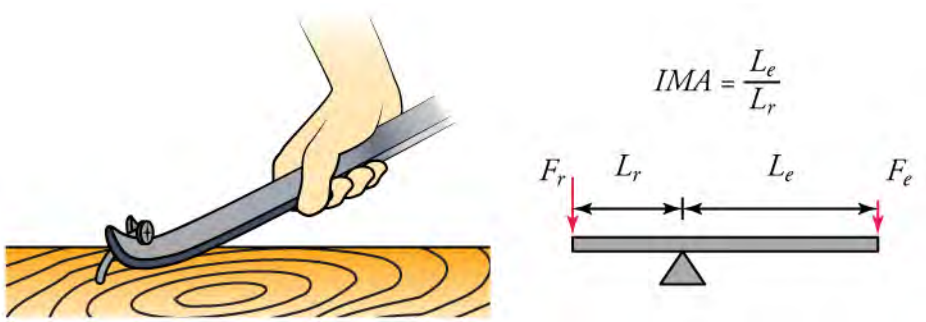

Figure 9.7 (a) A pry bar is a type of lever. (b) The ideal mechanical advantage equals the length of the effort arm divided by the length of the resistance arm of a lever.

In general, the IMA = the resistance force, $\mathbf{F}_r$, divided by the effort force, $\mathbf{F}_e$. IMA also equals the distance over which the effort is applied, d_e, divided by the distance the load travels, d_r.

$$IMA = \frac{\mathbf{F}_r}{\mathbf{F}_e} = \frac{d_e}{d_r}$$

Getting back to conservation of energy, for any simple machine, the work put into the machine, W_i, equals the work the machine puts out, W_o. Combining this with the information in the paragraphs above, we can write

$$W_i = W_o$$
$$\mathbf{F}_e d_e = \mathbf{F}_r d_r$$
$$\text{If } \mathbf{F}_e < \mathbf{F}_r, \text{ then } d_e > d_r.$$

The equations show how a simple machine can output the same amount of work while reducing the amount of effort force by increasing the distance over which the effort force is applied.

WATCH PHYSICS

Introduction to Mechanical Advantage

This video shows how to calculate the *IMA* of a lever by three different methods: (1) from effort force and resistance force; (2) from the lengths of the lever arms, and; (3) from the distance over which the force is applied and the distance the load moves.

Click to view content (https://www.youtube.com/embed/pfzJ-z5Ij48)

GRASP CHECK

Two children of different weights are riding a seesaw. How do they position themselves with respect to the pivot point (the fulcrum) so that they are balanced?

a. The heavier child sits closer to the fulcrum.
b. The heavier child sits farther from the fulcrum.
c. Both children sit at equal distance from the fulcrum.
d. Since both have different weights, they will never be in balance.

Some levers exert a large force to a short effort arm. This results in a smaller force acting over a greater distance at the end of the resistance arm. Examples of this type of lever are baseball bats, hammers, and golf clubs. In another type of lever, the fulcrum is at the end of the lever and the load is in the middle, as in the design of a wheelbarrow.

The simple machine shown in Figure 9.8 is called a **wheel and axle.** It is actually a form of lever. The difference is that the effort arm can rotate in a complete circle around the fulcrum, which is the center of the axle. Force applied to the outside of the wheel causes a greater force to be applied to the rope that is wrapped around the axle. As shown in the figure, the ideal mechanical advantage is calculated by dividing the radius of the wheel by the radius of the axle. Any crank-operated device is an example of a wheel and axle.

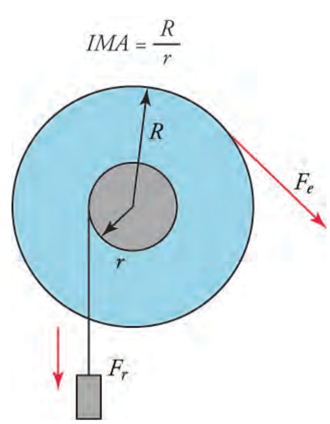

Figure 9.8 Force applied to a wheel exerts a force on its axle.

An **inclined plane** and a **wedge** are two forms of the same simple machine. A wedge is simply two inclined planes back to back. Figure 9.9 shows the simple formulas for calculating the *IMA*s of these machines. All sloping, paved surfaces for walking or driving are inclined planes. Knives and axe heads are examples of wedges.

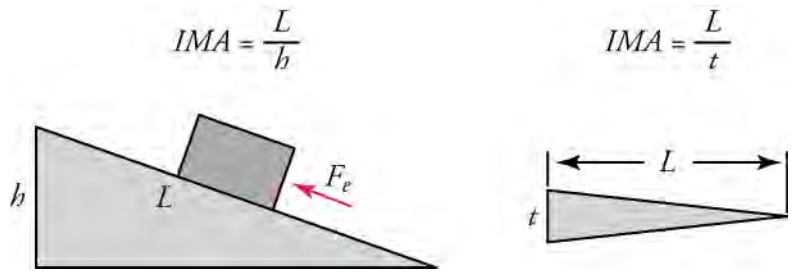

Figure 9.9 An inclined plane is shown on the left, and a wedge is shown on the right.

The **screw** shown in Figure 9.10 is actually a lever attached to a circular inclined plane. Wood screws (of course) are also examples of screws. The lever part of these screws is a screw driver. In the formula for *IMA*, the distance between screw threads is called *pitch* and has the symbol *P*.

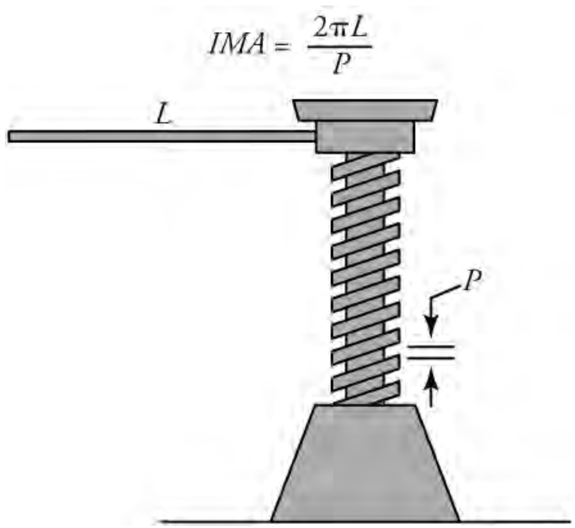

Figure 9.10 The screw shown here is used to lift very heavy objects, like the corner of a car or a house a short distance.

Figure 9.11 shows three different **pulley** systems. Of all simple machines, mechanical advantage is easiest to calculate for pulleys. Simply count the number of ropes supporting the load. That is the *IMA*. Once again we have to exert force over a longer distance to multiply force. To raise a load 1 meter with a pulley system you have to pull *N* meters of rope. Pulley systems are often used to raise flags and window blinds and are part of the mechanism of construction cranes.

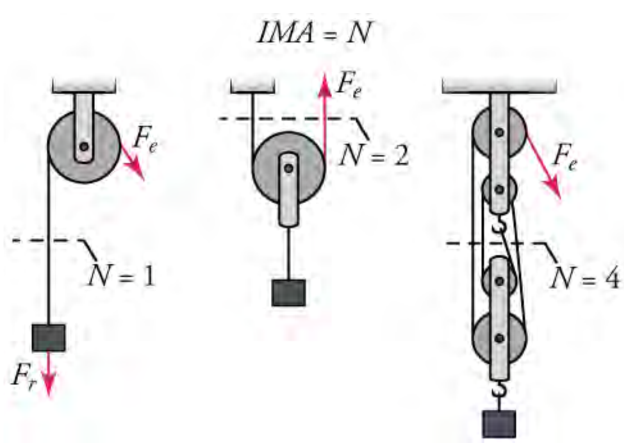

Figure 9.11 Three pulley systems are shown here.

WATCH PHYSICS

Mechanical Advantage of Inclined Planes and Pulleys

The first part of this video shows how to calculate the *IMA* of pulley systems. The last part shows how to calculate the *IMA* of an inclined plane.

Click to view content (https://www.khanacademy.org/embed_video?v=vSsK7Rfa3yA)

GRASP CHECK

How could you use a pulley system to lift a light load to great height?

a. Reduce the radius of the pulley.
b. Increase the number of pulleys.
c. Decrease the number of ropes supporting the load.
d. Increase the number of ropes supporting the load.

A **complex machine** is a combination of two or more simple machines. The wire cutters in Figure 9.12 combine two levers and two wedges. Bicycles include wheel and axles, levers, screws, and pulleys. Cars and other vehicles are combinations of many machines.

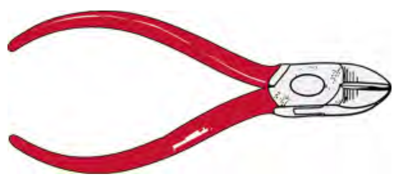

Figure 9.12 Wire cutters are a common complex machine.

Calculating Mechanical Advantage and Efficiency of Simple Machines

In general, the *IMA* = the resistance force, $\mathbf{F}_r$, divided by the effort force, $\mathbf{F}_e$. *IMA* also equals the distance over which the effort is applied, d_e, divided by the distance the load travels, d_r.

$$IMA = \frac{\mathbf{F}_r}{\mathbf{F}_e} = \frac{d_e}{d_r}$$

Refer back to the discussions of each simple machine for the specific equations for the *IMA* for each type of machine.

No simple or complex machines have the actual mechanical advantages calculated by the *IMA* equations. In real life, some of the applied work always ends up as wasted heat due to friction between moving parts. Both the **input work** (W_i) and **output work** (W_o) are the result of a force, **F**, acting over a distance, d.

$$W_i = \mathbf{F}_i d_i \text{and} W_o = \mathbf{F}_o d_o$$

The **efficiency output** of a machine is simply the output work divided by the input work, and is usually multiplied by 100 so that it is expressed as a percent.

$$\% \text{ efficiency} = \frac{W_o}{W_i} \times 100$$

Look back at the pictures of the simple machines and think about which would have the highest efficiency. Efficiency is related to friction, and friction depends on the smoothness of surfaces and on the area of the surfaces in contact. How would lubrication affect the efficiency of a simple machine?

WORKED EXAMPLE

Efficiency of a Lever

The input force of 11 N acting on the effort arm of a lever moves 0.4 m, which lifts a 40 N weight resting on the resistance arm a distance of 0.1 m. What is the efficiency of the machine?

Strategy

State the equation for efficiency of a simple machine, $\%\text{ efficiency} = \frac{W_o}{W_i} \times 100$, and calculate W_o and W_i. Both work values are the product *Fd*.

Solution

$W_i = \mathbf{F}_i d_i = (11)(0.4) = 4.4\text{ J}$ and $W_o = \mathbf{F}_o d_o = (40)(0.1) = 4.0\text{ J}$, then $\%\text{ efficiency} = \frac{W_o}{W_i} \times 100 = \frac{4.0}{4.4} \times 100 = 91\%$

Discussion

Efficiency in real machines will always be less than 100 percent because of work that is converted to unavailable heat by friction and air resistance. W_o and W_i can always be calculated as a force multiplied by a distance, although these quantities are not always as obvious as they are in the case of a lever.

Practice Problems

10. What is the IMA of an inclined plane that is 5 m long and 2 m high?

a. 0.4
b. 2.5
c. 0.4 m
d. 2.5 m

11. If a pulley system can lift a 200N load with an effort force of 52 N and has an efficiency of almost 100 percent, how many ropes are supporting the load?

a. 1 rope is required because the actual mechanical advantage is 0.26.
b. 1 rope is required because the actual mechanical advantage is 3.80.
c. 4 ropes are required because the actual mechanical advantage is 0.26.
d. 4 ropes are required because the actual mechanical advantage is 3.80.

Check Your Understanding

12. True or false—The efficiency of a simple machine is always less than 100 percent because some small fraction of the input work is always converted to heat energy due to friction.

a. True
b. False

13. The circular handle of a faucet is attached to a rod that opens and closes a valve when the handle is turned. If the rod has a diameter of 1 cm and the IMA of the machine is 6, what is the radius of the handle?

A. 0.08 cm
B. 0.17 cm
C. 3.0 cm
D. 6.0 cm

KEY TERMS

complex machine a machine that combines two or more simple machines
efficiency output work divided by input work
energy the ability to do work
gravitational potential energy energy acquired by doing work against gravity
ideal mechanical advantage the mechanical advantage of an idealized machine that loses no energy to friction
inclined plane a simple machine consisting of a slope
input work effort force multiplied by the distance over which it is applied
joule the metric unit for work and energy; equal to 1 newton meter (N•m)
kinetic energy energy of motion
law of conservation of energy states that energy is neither created nor destroyed
lever a simple machine consisting of a rigid arm that pivots on a fulcrum
mechanical advantage the number of times the input force is multiplied
mechanical energy kinetic or potential energy
output work output force multiplied by the distance over which it acts
potential energy stored energy
power the rate at which work is done
pulley a simple machine consisting of a rope that passes over one or more grooved wheels
screw a simple machine consisting of a spiral inclined plane
simple machine a machine that makes work easier by changing the amount or direction of force required to move an object
watt the metric unit of power; equivalent to joules per second
wedge a simple machine consisting of two back-to-back inclined planes
wheel and axle a simple machine consisting of a rod fixed to the center of a wheel
work force multiplied by distance
work–energy theorem states that the net work done on a system equals the change in kinetic energy

SECTION SUMMARY

9.1 Work, Power, and the Work–Energy Theorem

- Doing work on a system or object changes its energy.
- The work–energy theorem states that an amount of work that changes the velocity of an object is equal to the change in kinetic energy of that object.The work–energy theorem states that an amount of work that changes the velocity of an object is equal to the change in kinetic energy of that object.
- Power is the rate at which work is done.

9.2 Mechanical Energy and Conservation of Energy

- Mechanical energy may be either kinetic (energy of motion) or potential (stored energy).
- Doing work on an object or system changes its energy.
- Total energy in a closed, isolated system is constant.

9.3 Simple Machines

- The six types of simple machines make work easier by changing the **f***d* term so that force is reduced at the expense of increased distance.
- The ratio of output force to input force is a machine's mechanical advantage.
- Combinations of two or more simple machines are called complex machines.
- The ratio of output work to input work is a machine's efficiency.

KEY EQUATIONS

9.1 Work, Power, and the Work–Energy Theorem

equation for work	$W = \mathbf{f}d$
force	$\mathbf{f} = w = m\mathbf{g}$
work equivalencies	$W = PE_e = \mathbf{f}m\mathbf{g}$
kinetic energy	$KE = \frac{1}{2}m\mathbf{v}^2$
work–energy theorem	$W = \Delta\text{KE} = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2$
power	$P = \frac{W}{t}$