CHAPTER 2
Motion in One Dimension

Figure 2.1 Shanghai Maglev. At this rate, a train traveling from Boston to Washington, DC, a distance of 439 miles, could make the trip in under an hour and a half. Presently, the fastest train on this route takes over six hours to cover this distance. (Alex Needham, Public Domain)

Chapter Outline

INTRODUCTION Unless you have flown in an airplane, you have probably never traveled faster than 150 mph. Can you imagine traveling in a train like the one shown in Figure 2.1 that goes over 300 mph? Despite the high speed, the people riding in this train may not notice that they are moving at all unless they look out the window! This is because motion, even motion at 300 mph, is relative to the observer.

In this chapter, you will learn why it is important to identify a reference frame in order to clearly describe motion. For now, the motion you describe will be one-dimensional. Within this context, you will learn the difference between distance and displacement as well as the difference between speed and velocity. Then you will look at some graphing and problem-solving techniques.

2.1 Relative Motion, Distance, and Displacement

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe motion in different reference frames
- Define distance and displacement, and distinguish between the two
- Solve problems involving distance and displacement

Section Key Terms

displacement	distance	kinematics	magnitude
position	reference frame	scalar	vector

Defining Motion

Our study of physics opens with **kinematics**—the study of motion without considering its causes. Objects are in motion everywhere you look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. Even in inanimate objects, atoms are always moving.

How do you know something is moving? The location of an object at any particular time is its **position**. More precisely, you need to specify its position relative to a convenient **reference frame**. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. In other cases, we use reference frames that are not stationary but are in motion relative to Earth. To describe the position of a person in an airplane, for example, we use the airplane, not Earth, as the reference frame. (See Figure 2.2.) Thus, you can only know how fast and in what direction an object's position is changing against a background of something else that is either not moving or moving with a known speed and direction. The reference frame is the coordinate system from which the positions of objects are described.

Figure 2.2 Are clouds a useful reference frame for airplane passengers? Why or why not? (Paul Brennan, Public Domain)

Your classroom can be used as a reference frame. In the classroom, the walls are not moving. Your motion as you walk to the door, can be measured against the stationary background of the classroom walls. You can also tell if other things in the classroom are moving, such as your classmates entering the classroom or a book falling off a desk. You can also tell in what direction something is moving in the classroom. You might say, “The teacher is moving toward the door.” Your reference frame allows you to determine not only that something is moving but also the direction of motion.

You could also serve as a reference frame for others’ movement. If you remained seated as your classmates left the room, you would measure their movement away from your stationary location. If you and your classmates left the room together, then your perspective of their motion would be change. You, as the reference frame, would be moving in the same direction as your other moving classmates. As you will learn in the **Snap Lab**, your description of motion can be quite different when viewed from different reference frames.

Snap Lab

Looking at Motion from Two Reference Frames

In this activity you will look at motion from two reference frames. Which reference frame is correct?

- Choose an open location with lots of space to spread out so there is less chance of tripping or falling due to a collision and/or loose basketballs.
- 1 basketball

Procedure

1. Work with a partner. Stand a couple of meters away from your partner. Have your partner turn to the side so that you are looking at your partner's profile. Have your partner begin bouncing the basketball while standing in place. Describe the motion of the ball.
2. Next, have your partner again bounce the ball, but this time your partner should walk forward with the bouncing ball. You will remain stationary. Describe the ball's motion.
3. Again have your partner walk forward with the bouncing ball. This time, you should move alongside your partner while continuing to view your partner's profile. Describe the ball's motion.
4. Switch places with your partner, and repeat Steps 1–3.

GRASP CHECK

How do the different reference frames affect how you describe the motion of the ball?

a. The motion of the ball is independent of the reference frame and is same for different reference frames.
b. The motion of the ball is independent of the reference frame and is different for different reference frames.
c. The motion of the ball is dependent on the reference frame and is same for different reference frames.
d. The motion of the ball is dependent on the reference frames and is different for different reference frames.

LINKS TO PHYSICS

History: Galileo's Ship

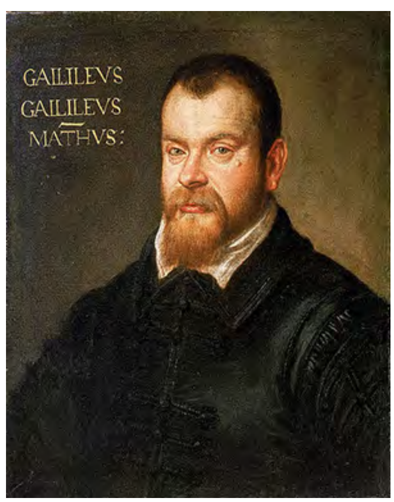


Figure 2.3 Galileo Galilei (1564–1642) studied motion and developed the concept of a reference frame. (Domenico Tintoretto)

The idea that a description of motion depends on the reference frame of the observer has been known for hundreds of years. The 17th-century astronomer Galileo Galilei (Figure 2.3) was one of the first scientists to explore this idea. Galileo suggested the following thought experiment: Imagine a windowless ship moving at a constant speed and direction along a perfectly calm sea. Is there a way that a person inside the ship can determine whether the ship is moving? You can extend this thought experiment by also imagining a person standing on the shore. How can a person on the shore determine whether the ship is moving?

Galileo came to an amazing conclusion. Only by looking at each other can a person in the ship or a person on shore describe the motion of one relative to the other. In addition, their descriptions of motion would be identical. A person inside the ship would describe the person on the land as moving past the ship. The person on shore would describe the ship and the person inside it as moving past. Galileo realized that observers moving at a constant speed and direction relative to each other describe motion in the same way. Galileo had discovered that a description of motion is only meaningful if you specify a reference frame.

GRASP CHECK

Imagine standing on a platform watching a train pass by. According to Galileo's conclusions, how would your description of motion and the description of motion by a person riding on the train compare?

a. I would see the train as moving past me, and a person on the train would see me as stationary.
b. I would see the train as moving past me, and a person on the train would see me as moving past the train.
c. I would see the train as stationary, and a person on the train would see me as moving past the train.
d. I would see the train as stationary, and a person on the train would also see me as stationary.

Distance vs. Displacement

As we study the motion of objects, we must first be able to describe the object's position. Before your parent drives you to school, the car is sitting in your driveway. Your driveway is the starting position for the car. When you reach your high school, the car has changed position. Its new position is your school.

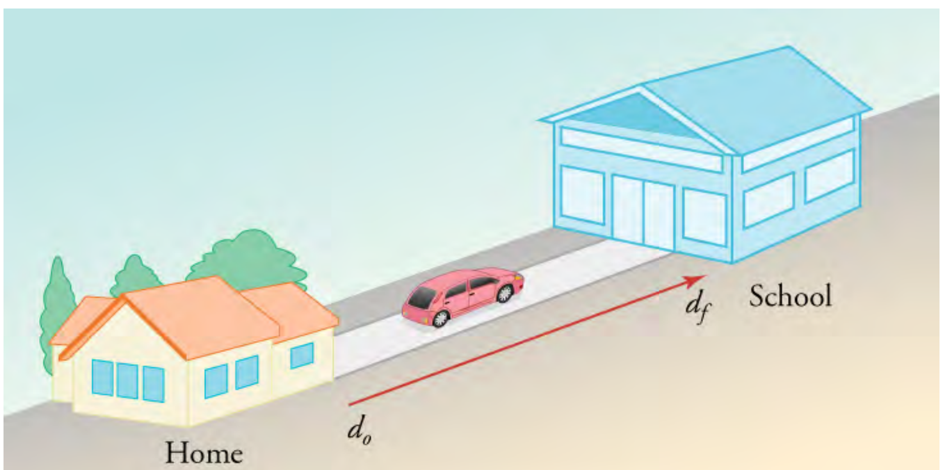


Figure 2.4 Your total change in position is measured from your house to your school.

Physicists use variables to represent terms. We will use **d** to represent car's position. We will use a subscript to differentiate between the initial position, $\mathbf{d}_0$, and the final position, $\mathbf{d}_f$. In addition, vectors, which we will discuss later, will be in bold or will have an arrow above the variable. Scalars will be italicized.

TIPS FOR SUCCESS

In some books, **x** or **s** is used instead of **d** to describe position. In $\mathbf{d}_0$, said *d naught*, the subscript 0 stands for *initial*. When we begin to talk about two-dimensional motion, sometimes other subscripts will be used to describe horizontal position, $\mathbf{d}_x$, or vertical position, $\mathbf{d}_y$. So, you might see references to $\mathbf{d}_{0x}$ and $\mathbf{d}_{fy}$.

Now imagine driving from your house to a friend's house located several kilometers away. How far would you drive? The **distance** an object moves is the length of the path between its initial position and its final position. The distance you drive to your friend's house depends on your path. As shown in Figure 2.5, distance is different from the length of a straight line between two points. The distance you drive to your friend's house is probably longer than the straight line between the two houses.

Figure 2.5 A short line separates the starting and ending points of this motion, but the distance along the path of motion is considerably longer.

We often want to be more precise when we talk about position. The description of an object's motion often includes more than just the distance it moves. For instance, if it is a five kilometer drive to school, the distance traveled is 5 kilometers. After dropping you off at school and driving back home, your parent will have traveled a total distance of 10 kilometers. The car and your parent will end up in the same starting position in space. The net change in position of an object is its **displacement**, or $\Delta \mathbf{d}$. The Greek letter delta, Δ, means *change in*.

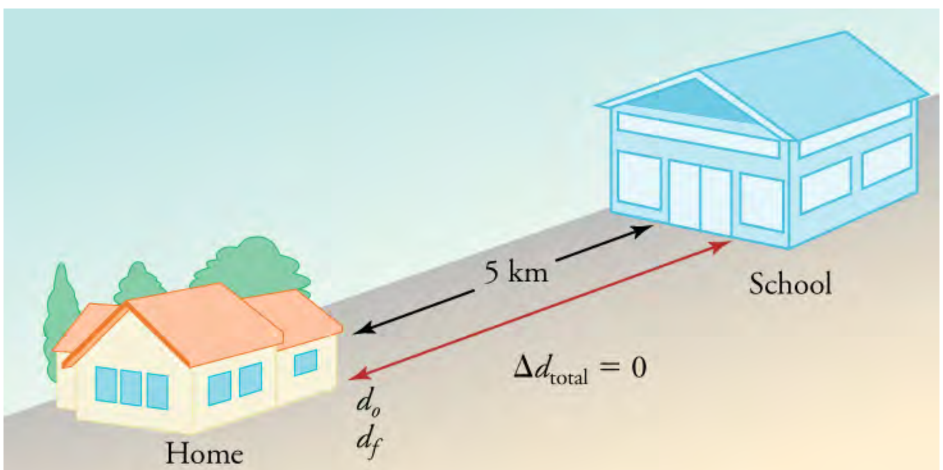


Figure 2.6 The total distance that your car travels is 10 km, but the total displacement is 0.

Snap Lab

Distance vs. Displacement

In this activity you will compare distance and displacement. Which term is more useful when making measurements?

- 1 recorded song available on a portable device
- 1 tape measure
- 3 pieces of masking tape
- A room (like a gym) with a wall that is large and clear enough for all pairs of students to walk back and forth without running into each other.

Procedure

1. One student from each pair should stand with their back to the longest wall in the classroom. Students should stand at least 0.5 meters away from each other. Mark this starting point with a piece of masking tape.
2. The second student from each pair should stand facing their partner, about two to three meters away. Mark this point

with a second piece of masking tape.
3. Student pairs line up at the starting point along the wall.
4. The teacher turns on the music. Each pair walks back and forth from the wall to the second marked point until the music stops playing. Keep count of the number of times you walk across the floor.
5. When the music stops, mark your ending position with the third piece of masking tape.
6. Measure from your starting, initial position to your ending, final position.
7. Measure the length of your path from the starting position to the second marked position. Multiply this measurement by the total number of times you walked across the floor. Then add this number to your measurement from step 6.
8. Compare the two measurements from steps 6 and 7.

GRASP CHECK

1. Which measurement is your total distance traveled?
2. Which measurement is your displacement?
3. When might you want to use one over the other?

a. Measurement of the total length of your path from the starting position to the final position gives the distance traveled, and the measurement from your initial position to your final position is the displacement. Use distance to describe the total path between starting and ending points,and use displacement to describe the shortest path between starting and ending points.
b. Measurement of the total length of your path from the starting position to the final position is distance traveled, and the measurement from your initial position to your final position is displacement. Use distance to describe the shortest path between starting and ending points, and use displacement to describe the total path between starting and ending points.
c. Measurement from your initial position to your final position is distance traveled, and the measurement of the total length of your path from the starting position to the final position is displacement. Use distance to describe the total path between starting and ending points, and use displacement to describe the shortest path between starting and ending points.
d. Measurement from your initial position to your final position is distance traveled, and the measurement of the total length of your path from the starting position to the final position is displacement. Use distance to describe the shortest path between starting and ending points, and use displacement to describe the total path between starting and ending points.

If you are describing only your drive to school, then the distance traveled and the displacement are the same—5 kilometers. When you are describing the entire round trip, distance and displacement are different. When you describe distance, you only include the **magnitude**, the size or amount, of the distance traveled. However, when you describe the displacement, you take into account both the magnitude of the change in position and the direction of movement.

In our previous example, the car travels a total of 10 kilometers, but it drives five of those kilometers forward toward school and five of those kilometers back in the opposite direction. If we ascribe the forward direction a positive (+) and the opposite direction a negative (–), then the two quantities will cancel each other out when added together.

A quantity, such as distance, that has magnitude (i.e., how big or how much) but does not take into account direction is called a **scalar**. A quantity, such as displacement, that has both magnitude and direction is called a **vector**.

WATCH PHYSICS

Vectors & Scalars

This video (http://openstax.org/l/28vectorscalar) introduces and differentiates between vectors and scalars. It also introduces quantities that we will be working with during the study of kinematics.

Click to view content (https://www.khanacademy.org/embed_video?v=ihNZlp7iUHE)

GRASP CHECK

How does this video (https://www.khanacademy.org/science/ap-physics-1/ap-one-dimensional-motion/ap-physics-foundations/v/introduction-to-vectors-and-scalars) help you understand the difference between distance and displacement? Describe the differences between vectors and scalars using physical quantities as examples.

a. It explains that distance is a vector and direction is important, whereas displacement is a scalar and it has no direction attached to it.
b. It explains that distance is a scalar and direction is important, whereas displacement is a vector and it has no direction attached to it.
c. It explains that distance is a scalar and it has no direction attached to it, whereas displacement is a vector and direction is important.
d. It explains that both distance and displacement are scalar and no directions are attached to them.

Displacement Problems

Hopefully you now understand the conceptual difference between distance and displacement. Understanding concepts is half the battle in physics. The other half is math. A stumbling block to new physics students is trying to wade through the math of physics while also trying to understand the associated concepts. This struggle may lead to misconceptions and answers that make no sense. Once the concept is mastered, the math is far less confusing.

So let's review and see if we can make sense of displacement in terms of numbers and equations. You can calculate an object's displacement by subtracting its original position, $\mathbf{d}_o$, from its final position $\mathbf{d}_f$. In math terms that means

$$\Delta\mathbf{d} = \mathbf{d}_f - \mathbf{d}_0.$$

If the final position is the same as the initial position, then $\Delta\mathbf{d} = 0$.

To assign numbers and/or direction to these quantities, we need to define an axis with a positive and a negative direction. We also need to define an origin, or *O*. In Figure 2.6, the axis is in a straight line with home at zero and school in the positive direction. If we left home and drove the opposite way from school, motion would have been in the negative direction. We would have assigned it a negative value. In the round-trip drive, $\mathbf{d}_f$ and $\mathbf{d}_o$ were both at zero kilometers. In the one way trip to school, $\mathbf{d}_f$ was at 5 kilometers and $\mathbf{d}_o$ was at zero km. So, $\Delta\mathbf{d}$ was 5 kilometers.

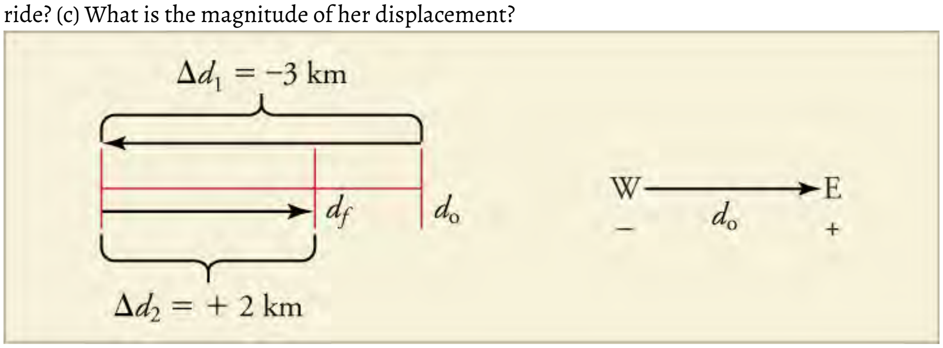
TIPS FOR SUCCESS

You may place your origin wherever you would like. You have to make sure that you calculate all distances consistently from your zero and you define one direction as positive and the other as negative. Therefore, it makes sense to choose the easiest axis, direction, and zero. In the example above, we took home to be zero because it allowed us to avoid having to interpret a solution with a negative sign.

WORKED EXAMPLE

Calculating Distance and Displacement

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?



Strategy

To solve this problem, we need to find the difference between the final position and the initial position while taking care to note the direction on the axis. The final position is the sum of the two displacements, $\Delta\mathbf{d}_1$ and $\Delta\mathbf{d}_2$.

Solution

a. Displacement: The rider's displacement is $\Delta\mathbf{d} = \mathbf{d}_f - \mathbf{d}_0 = -1 \text{ km}$.
b. Distance: The distance traveled is 3 km + 2 km = 5 km.
c. The magnitude of the displacement is 1 km.

Discussion

The displacement is negative because we chose east to be positive and west to be negative. We could also have described the displacement as 1 km west. When calculating displacement, the direction mattered, but when calculating distance, the direction did not matter. The problem would work the same way if the problem were in the north–south or *y*-direction.

TIPS FOR SUCCESS

Physicists like to use standard units so it is easier to compare notes. The standard units for calculations are called *SI* units (International System of Units). SI units are based on the metric system. The SI unit for displacement is the meter (m), but sometimes you will see a problem with kilometers, miles, feet, or other units of length. If one unit in a problem is an SI unit and another is not, you will need to convert all of your quantities to the same system before you can carry out the calculation.

Practice Problems

1. On an axis in which moving from right to left is positive, what is the displacement and distance of a student who walks 32 m to the right and then 17 m to the left?
   a. Displacement is -15 m and distance is -49 *m*.
   b. Displacement is -15 m and distance is 49 m.
   c. Displacement is 15 m and distance is -49 m.
   d. Displacement is 15 m and distance is 49 m.

2. Tiana jogs 1.5 km along a straight path and then turns and jogs 2.4 km in the opposite direction. She then turns back and jogs 0.7 km in the original direction. Let Tiana's original direction be the positive direction. What are the displacement and distance she jogged?
   a. Displacement is 4.6 km,and distance is -0.2 km.
   b. Displacement is -0.2 km, and distance is 4.6 km.
   c. Displacement is 4.6 km, and distance is +0.2 km.
   d. Displacement is +0.2 km, and distance is 4.6 km.

WORK IN PHYSICS

Mars Probe Explosion

Figure 2.7 The Mars Climate Orbiter disaster illustrates the importance of using the correct calculations in physics. (NASA)

Physicists make calculations all the time, but they do not always get the right answers. In 1998, NASA, the National Aeronautics and Space Administration, launched the Mars Climate Orbiter, shown in Figure 2.7, a $125-million-dollar satellite designed to monitor the Martian atmosphere. It was supposed to orbit the planet and take readings from a safe distance. The American scientists made calculations in English units (feet, inches, pounds, etc.) and forgot to convert their answers to the standard metric SI units. This was a very costly mistake. Instead of orbiting the planet as planned, the Mars Climate Orbiter ended up flying into the Martian atmosphere. The probe disintegrated. It was one of the biggest embarrassments in NASA's history.

GRASP CHECK

In 1999 the Mars Climate Orbiter crashed because calculation were performed in English units instead of SI units. At one point the orbiter was just 187,000 feet above the surface, which was too close to stay in orbit. What was the height of the orbiter at this time in kilometers? (Assume 1 meter equals 3.281 feet.)

a. 16 km
b. 18 km
c. 57 km
d. 614 km

Check Your Understanding

3. What does it mean when motion is described as relative?

a. It means that motion of any object is described relative to the motion of Earth.
b. It means that motion of any object is described relative to the motion of any other object.
c. It means that motion is independent of the frame of reference.
d. It means that motion depends on the frame of reference selected.

4. If you and a friend are standing side-by-side watching a soccer game, would you both view the motion from the same reference frame?

a. Yes, we would both view the motion from the same reference point because both of us are at rest in Earth's frame of reference.
b. Yes, we would both view the motion from the same reference point because both of us are observing the motion from two points on the same straight line.
c. No, we would both view the motion from different reference points because motion is viewed from two different points; the reference frames are similar but not the same.
d. No, we would both view the motion from different reference points because response times may be different; so, the motion observed by both of us would be different.

5. What is the difference between distance and displacement?

a. Distance has both magnitude and direction, while displacement has magnitude but no direction.
b. Distance has magnitude but no direction, while displacement has both magnitude and direction.
c. Distance has magnitude but no direction, while displacement has only direction.
d. There is no difference. Both distance and displacement have magnitude and direction.

6. Which situation correctly identifies a race car's distance traveled and the magnitude of displacement during a one-lap car race?

a. The perimeter of the race track is the distance, and the shortest distance between the start line and the finish line is the magnitude of displacement.
b. The perimeter of the race track is the magnitude of displacement, and the shortest distance between the start and finish line is the distance.
c. The perimeter of the race track is both the distance and magnitude of displacement.
d. The shortest distance between the start line and the finish line is both the distance and magnitude of displacement.

7. Why is it important to specify a reference frame when describing motion?

a. Because Earth is continuously in motion; an object at rest on Earth will be in motion when viewed from outer space.
b. Because the position of a moving object can be defined only when there is a fixed reference frame.

c. Because motion is a relative term; it appears differently when viewed from different reference frames.
d. Because motion is always described in Earth's frame of reference; if another frame is used, it has to be specified with each situation.

2.2 Speed and Velocity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Calculate the average speed of an object
- Relate displacement and average velocity

Section Key Terms

average speed	average velocity	instantaneous speed
instantaneous velocity	speed	velocity

Speed

There is more to motion than distance and displacement. Questions such as, "How long does a foot race take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts. In this section we will look at time, speed, and velocity to expand our understanding of motion.

A description of how fast or slow an object moves is its speed. **Speed** is the rate at which an object changes its location. Like distance, speed is a scalar because it has a magnitude but not a direction. Because speed is a rate, it depends on the time interval of motion. You can calculate the elapsed time or the change in time, Δt, of motion as the difference between the ending time and the beginning time

$$\Delta t = t_{\mathrm{f}} - t_0.$$

The SI unit of time is the second (s), and the SI unit of speed is meters per second (m/s), but sometimes kilometers per hour (km/h), miles per hour (mph) or other units of speed are used.

When you describe an object's speed, you often describe the average over a time period. **Average speed**, v_{avg}, is the distance traveled divided by the time during which the motion occurs.

$$v_{\mathrm{avg}} = \frac{\text{distance}}{\text{time}}$$

You can, of course, rearrange the equation to solve for either distance or time

$$\text{time} = \frac{\text{distance}}{v_{\mathrm{avg}}}.$$

$$\text{distance} = v_{\mathrm{avg}} \times \text{time}$$

Suppose, for example, a car travels 150 kilometers in 3.2 hours. Its average speed for the trip is

$$\begin{aligned} v_{\mathrm{avg}} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{150 \text{ km}}{3.2 \text{ h}} \\ &= 47 \text{ km/h}. \end{aligned}$$

A car's speed would likely increase and decrease many times over a 3.2 hour trip. Its speed at a specific instant in time, however, is its **instantaneous speed**. A car's speedometer describes its instantaneous speed.

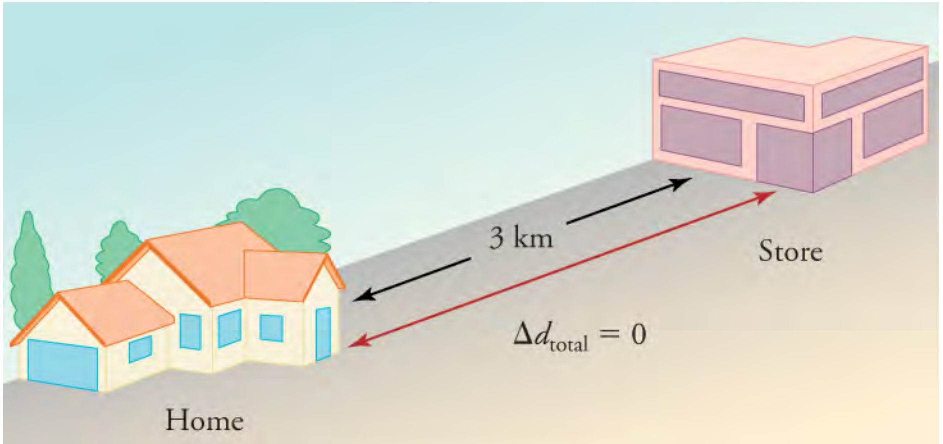


Figure 2.8 During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, because there was no net change in position.

WORKED EXAMPLE

Calculating Average Speed

A marble rolls 5.2 m in 1.8 s. What was the marble's average speed?

Strategy

We know the distance the marble travels, 5.2 m, and the time interval, 1.8 s. We can use these values in the average speed equation.

Solution

$$v_{\text{avg}} = \frac{\text{distance}}{\text{time}} = \frac{5.2\ \text{m}}{1.8\ \text{s}} = 2.9\ \text{m/s}$$

Discussion

Average speed is a scalar, so we do not include direction in the answer. We can check the reasonableness of the answer by estimating: 5 meters divided by 2 seconds is 2.5 m/s. Since 2.5 m/s is close to 2.9 m/s, the answer is reasonable. This is about the speed of a brisk walk, so it also makes sense.

Practice Problems

8. A pitcher throws a baseball from the pitcher's mound to home plate in 0.46 s. The distance is 18.4 m. What was the average speed of the baseball?
   a. 40 m/s
   b. - 40 m/s
   c. 0.03 m/s
   d. 8.5 m/s

9. Cassie walked to her friend's house with an average speed of 1.40 m/s. The distance between the houses is 205 m. How long did the trip take her?
   a. 146 s
   b. 0.01 s
   c. 2.50 min
   d. 287 s

Velocity

The vector version of speed is velocity. **Velocity** describes the speed and direction of an object. As with speed, it is useful to describe either the average velocity over a time period or the velocity at a specific moment. **Average velocity** is displacement divided by the time over which the displacement occurs.

$$v_{avg} = \frac{\text{distance}}{\text{time}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{\mathbf{d}_f - \mathbf{d}_0}{t_f - t_0}$$

Velocity, like speed, has SI units of meters per second (m/s), but because it is a vector, you must also include a direction. Furthermore, the variable **v** for velocity is bold because it is a vector, which is in contrast to the variable *v* for speed which is italicized because it is a scalar quantity.

TIPS FOR SUCCESS

It is important to keep in mind that the average speed is not the same thing as the average velocity without its direction. Like we saw with displacement and distance in the last section, changes in direction over a time interval have a bigger effect on speed and velocity.

Suppose a passenger moved toward the back of a plane with an average velocity of –4 m/s. We cannot tell from the average velocity whether the passenger stopped momentarily or backed up before he got to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals such as those shown in Figure 2.9. If you consider infinitesimally small intervals, you can define **instantaneous velocity**, which is the velocity at a specific instant in time. Instantaneous velocity and average velocity are the same if the velocity is constant.

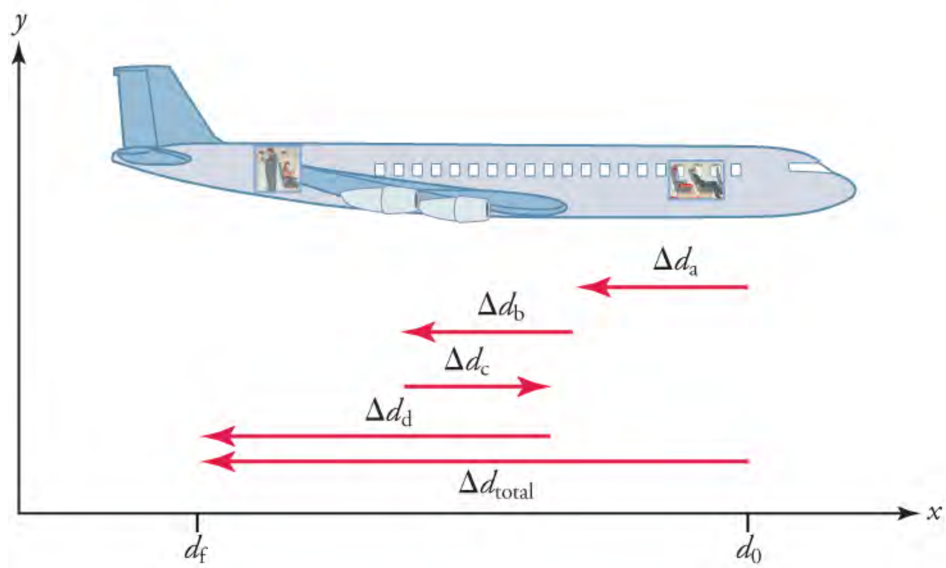


Figure 2.9 The diagram shows a more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

Earlier, you have read that distance traveled can be different than the magnitude of displacement. In the same way, speed can be different than the magnitude of velocity. For example, you drive to a store and return home in half an hour. If your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero because your displacement for the round trip is zero.

WATCH PHYSICS

Calculating Average Velocity or Speed

This video (http://openstax.org/l/28avgvelocity) reviews vectors and scalars and describes how to calculate average velocity and average speed when you know displacement and change in time. The video also reviews how to convert km/h to m/s.

Click to view content (https://www.khanacademy.org/embed_video?v=MAS6mBRZZXA)

GRASP CHECK

Which of the following fully describes a vector and a scalar quantity and correctly provides an example of each?

a. A scalar quantity is fully described by its magnitude, while a vector needs both magnitude and direction to fully describe it. Displacement is an example of a scalar quantity and time is an example of a vector quantity.
b. A scalar quantity is fully described by its magnitude, while a vector needs both magnitude and direction to fully describe it. Time is an example of a scalar quantity and displacement is an example of a vector quantity.
c. A scalar quantity is fully described by its magnitude and direction, while a vector needs only magnitude to fully describe it. Displacement is an example of a scalar quantity and time is an example of a vector quantity.
d. A scalar quantity is fully described by its magnitude and direction, while a vector needs only magnitude to fully describe it. Time is an example of a scalar quantity and displacement is an example of a vector quantity.

WORKED EXAMPLE

Calculating Average Velocity

A student has a displacement of 304 m north in 180 s. What was the student's average velocity?

Strategy

We know that the displacement is 304 m north and the time is 180 s. We can use the formula for average velocity to solve the problem.

Solution

$$\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{304\ \text{m}}{180\ \text{s}} = 1.7\ \text{m/s north} \tag{2.1}$$

Discussion

Since average velocity is a vector quantity, you must include direction as well as magnitude in the answer. Notice, however, that the direction can be omitted until the end to avoid cluttering the problem. Pay attention to the significant figures in the problem. The distance 304 m has three significant figures, but the time interval 180 s has only two, so the quotient should have only two significant figures.

TIPS FOR SUCCESS

Note the way scalars and vectors are represented. In this book d represents distance and displacement. Similarly, v represents speed, and v represents velocity. A variable that is not bold indicates a scalar quantity, and a bold variable indicates a vector quantity. Vectors are sometimes represented by small arrows above the variable.

WORKED EXAMPLE

Solving for Displacement when Average Velocity and Time are Known

Layla jogs with an average velocity of 2.4 m/s east. What is her displacement after 46 seconds?

Strategy

We know that Layla's average velocity is 2.4 m/s east, and the time interval is 46 seconds. We can rearrange the average velocity formula to solve for the displacement.

Solution

$$\begin{aligned} \mathbf{v}_{\text{avg}} &= \frac{\Delta \mathbf{d}}{\Delta t} \\ \Delta \mathbf{d} &= \mathbf{v}_{avg} \Delta t \\ &= (2.4\ \text{m/s})(46\ \text{s}) \\ &= 1.1 \times 10^2\ \text{m east} \end{aligned} \tag{2.2}$$

Discussion

The answer is about 110 m east, which is a reasonable displacement for slightly less than a minute of jogging. A calculator shows the answer as 110.4 m. We chose to write the answer using scientific notation because we wanted to make it clear that we only used two significant figures.

TIPS FOR SUCCESS

Dimensional analysis is a good way to determine whether you solved a problem correctly. Write the calculation using only units to be sure they match on opposite sides of the equal mark. In the worked example, you have
m = (m/s)(s). Since seconds is in the denominator for the average velocity and in the numerator for the time, the unit cancels out leaving only m and, of course, m = m.

WORKED EXAMPLE

Solving for Time when Displacement and Average Velocity are Known

Phillip walks along a straight path from his house to his school. How long will it take him to get to school if he walks 428 m west with an average velocity of 1.7 m/s west?

Strategy

We know that Phillip's displacement is 428 m west, and his average velocity is 1.7 m/s west. We can calculate the time required for the trip by rearranging the average velocity equation.

Solution

$$\begin{aligned} \mathbf{v}_{\text{avg}} &= \frac{\Delta \mathbf{d}}{\Delta t} \\ \Delta t &= \frac{\Delta \mathbf{d}}{\mathbf{v}_{\text{avg}}} \\ &= \frac{428 \;\; \text{m}}{1.7 \;\; \text{m/s}} \\ &= 2.5 \times 10^2 \text{ s} \end{aligned} \tag{2.3}$$

Discussion

Here again we had to use scientific notation because the answer could only have two significant figures. Since time is a scalar, the answer includes only a magnitude and not a direction.

Practice Problems

10. A trucker drives along a straight highway for 0.25 h with a displacement of 16 km south. What is the trucker's average velocity?
    a. 4 km/h north
    b. 4 km/h south
    c. 64 km/h north
    d. 64 km/h south

11. A bird flies with an average velocity of 7.5 m/s east from one branch to another in 2.4 s. It then pauses before flying with an average velocity of 6.8 m/s east for 3.5 s to another branch. What is the bird's total displacement from its starting point?
    a. 42 m west
    b. 6 m west
    c. 6 m east
    d. 42 m east

Virtual Physics

The Walking Man

In this simulation you will put your cursor on the man and move him first in one direction and then in the opposite direction. Keep the *Introduction* tab active. You can use the *Charts* tab after you learn about graphing motion later in this chapter. Carefully watch the sign of the numbers in the position and velocity boxes. Ignore the acceleration box for now. See if you can make the man's position positive while the velocity is negative. Then see if you can do the opposite.

[Click to view content (https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/)](https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/)

GRASP CHECK

Which situation correctly describes when the moving man's position was negative but his velocity was positive?

a. Man moving toward 0 from left of 0
b. Man moving toward 0 from right of 0
c. Man moving away from 0 from left of 0
d. Man moving away from 0 from right of 0

Check Your Understanding

12. Two runners travel along the same straight path. They start at the same time, and they end at the same time, but at the halfway mark, they have different instantaneous velocities. Is it possible for them to have the same average velocity for the trip?

a. Yes, because average velocity depends on the net or total displacement.
b. Yes, because average velocity depends on the total distance traveled.
c. No, because the velocities of both runners must remain the exactly same throughout the journey.
d. No, because the instantaneous velocities of the runners must remain same midway but can be different elsewhere.

13. If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity, and under what circumstances are these two quantities the same?

a. Average speed. Both are the same when the car is traveling at a constant speed and changing direction.
b. Average speed. Both are the same when the speed is constant and the car does not change its direction.
c. Magnitude of average velocity. Both are same when the car is traveling at a constant speed.
d. Magnitude of average velocity. Both are same when the car does not change its direction.

14. Is it possible for average velocity to be negative?

a. Yes, in cases when the net displacement is negative.
b. Yes, if the body keeps changing its direction during motion.
c. No, average velocity describes only magnitude and not the direction of motion.
d. No, average velocity describes only the magnitude in the positive direction of motion.

2.3 Position vs. Time Graphs

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the meaning of slope in position vs. time graphs
- Solve problems using position vs. time graphs

Section Key Terms

dependent variable   independent variable   tangent

Graphing Position as a Function of Time

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information, they also reveal relationships between physical quantities. In this section, we will investigate kinematics by analyzing graphs of position over time.

Graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against each other, the horizontal axis is usually considered the **independent variable**, and the vertical axis is the **dependent variable**. In algebra, you would have referred to the horizontal axis as the x-axis and the vertical axis as the y-axis. As in Figure 2.10, a straight-line graph has the general form $y = mx + b$.

Here m is the slope, defined as the rise divided by the run (as seen in the figure) of the straight line. The letter b is the y-intercept which is the point at which the line crosses the vertical, y-axis. In terms of a physical situation in the real world, these quantities will take on a specific significance, as we will see below. (Figure 2.10.)

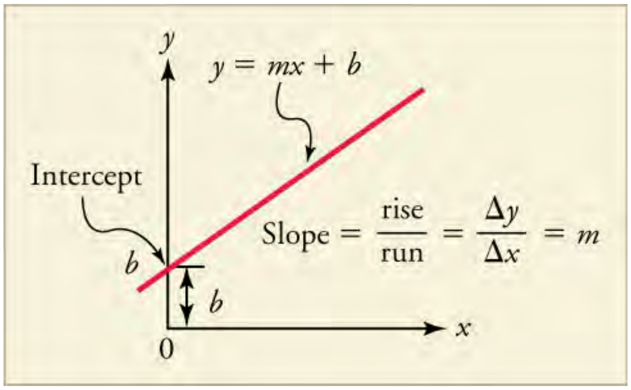


Figure 2.10 The diagram shows a straight-line graph. The equation for the straight line is y equals $mx + b$.

In physics, time is usually the independent variable. Other quantities, such as displacement, are said to depend upon it. A graph of position versus time, therefore, would have position on the vertical axis (dependent variable) and time on the horizontal axis (independent variable). In this case, to what would the slope and y-intercept refer? Let's look back at our original example when studying distance and displacement.

The drive to school was 5 km from home. Let's assume it took 10 minutes to make the drive and that your parent was driving at a constant velocity the whole time. The position versus time graph for this section of the trip would look like that shown in Figure 2.11.

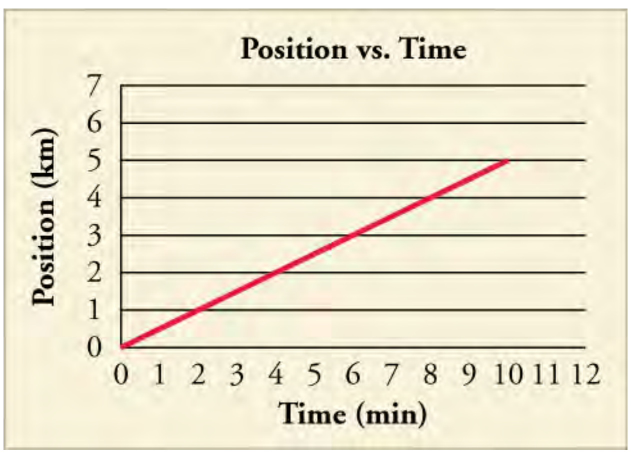


Figure 2.11 A graph of position versus time for the drive to school is shown. What would the graph look like if we added the return trip?

As we said before, $\mathbf{d}_0 = 0$ because we call home our O and start calculating from there. In Figure 2.11, the line starts at $\mathbf{d} = 0$, as well. This is the b in our equation for a straight line. Our initial position in a position versus time graph is always the place where the graph crosses the x-axis at $t = 0$. What is the slope? The *rise* is the change in position, (i.e., displacement) and the *run* is the change in time. This relationship can also be written

$$\frac{\Delta \mathbf{d}}{\Delta t}. \tag{2.4}$$

This relationship was how we defined average velocity. Therefore, the slope in a $\mathbf{d}$ versus t graph, is the average velocity.

TIPS FOR SUCCESS

Sometimes, as is the case where we graph both the trip to school and the return trip, the behavior of the graph looks different during different time intervals. If the graph looks like a series of straight lines, then you can calculate the average velocity for each time interval by looking at the slope. If you then want to calculate the average velocity for the entire trip, you can do a weighted average.

Let's look at another example. Figure 2.12 shows a graph of position versus time for a jet-powered car on a very flat dry lake bed in Nevada.

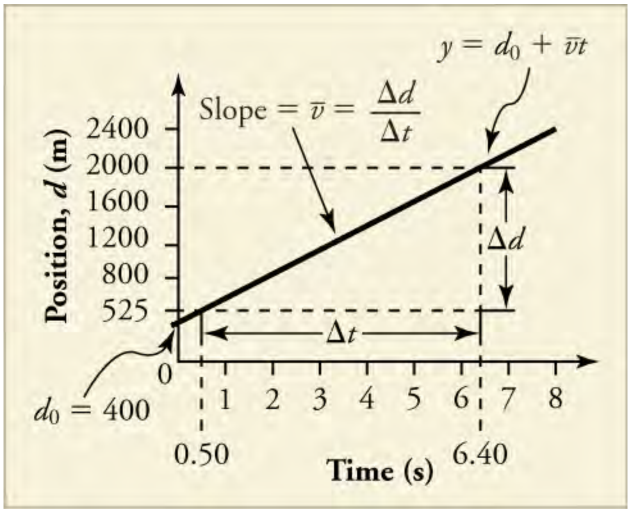


Figure 2.12 The diagram shows a graph of position versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph in Figure 2.12 is average velocity, $\mathbf{v}_{avg}$ and the intercept is displacement at time zero—that is, $\mathbf{d}_0$. Substituting these symbols into $y = mx + b$ gives

$$\mathbf{d} = \mathbf{v}t + \mathbf{d}_0 \tag{2.5}$$

or

$$\mathbf{d} = \mathbf{d}_0 + \mathbf{v}t. \tag{2.6}$$

Thus a graph of position versus time gives a general relationship among displacement, velocity, and time, as well as giving detailed numerical information about a specific situation. From the figure we can see that the car has a position of 400 m at $t = 0$ s, 650 m at $t = 1.0$ s, and so on. And we can learn about the object's velocity, as well.

Snap Lab

Graphing Motion

In this activity, you will release a ball down a ramp and graph the ball's displacement vs. time.

- Choose an open location with lots of space to spread out so there is less chance for tripping or falling due to rolling balls.

- 1 ball
- 1 board
- 2 or 3 books
- 1 stopwatch
- 1 tape measure
- 6 pieces of masking tape
- 1 piece of graph paper
- 1 pencil

Procedure

1. Build a ramp by placing one end of the board on top of the stack of books. Adjust location, as necessary, until there is no obstacle along the straight line path from the bottom of the ramp until at least the next 3 m.
2. Mark distances of 0.5 m, 1.0 m, 1.5 m, 2.0 m, 2.5 m, and 3.0 m from the bottom of the ramp. Write the distances on the tape.

3. Have one person take the role of the experimenter. This person will release the ball from the top of the ramp. If the ball does not reach the 3.0 m mark, then increase the incline of the ramp by adding another book. Repeat this Step as necessary.
4. Have the experimenter release the ball. Have a second person, the timer, begin timing the trial once the ball reaches the bottom of the ramp and stop the timing once the ball reaches 0.5 m. Have a third person, the recorder, record the time in a data table.
5. Repeat Step 4, stopping the times at the distances of 1.0 m, 1.5 m, 2.0 m, 2.5 m, and 3.0 m from the bottom of the ramp.
6. Use your measurements of time and the displacement to make a position vs. time graph of the ball's motion.
7. Repeat Steps 4 through 6, with different people taking on the roles of experimenter, timer, and recorder. Do you get the same measurement values regardless of who releases the ball, measures the time, or records the result? Discuss possible causes of discrepancies, if any.

GRASP CHECK

True or False: The average speed of the ball will be less than the average velocity of the ball.

a. True
b. False

Solving Problems Using Position vs. Time Graphs

So how do we use graphs to solve for things we want to know like velocity?

WORKED EXAMPLE

Using Position–Time Graph to Calculate Average Velocity: Jet Car

Find the average velocity of the car whose position is graphed in Figure 1.13.

Strategy

The slope of a graph of d vs. t is average velocity, since slope equals rise over run.

$$\text{slope} = \frac{\Delta \mathbf{d}}{\Delta t} = \mathbf{v} \tag{2.7}$$

Since the slope is constant here, any two points on the graph can be used to find the slope.

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)
2. Substitute the **d** and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$\begin{aligned} \mathbf{v} &= \frac{\Delta \mathbf{d}}{\Delta t} \\ &= \frac{2000\ \text{m} - 525\ \text{m}}{6.4\ \text{s} - 0.50\ \text{s}}, \\ &= 250\ \text{m/s} \end{aligned} \tag{2.8}$$

Discussion

This is an impressively high land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 27 m/s or 96 km/h, but considerably shy of the record of 343 m/s or 1,234 km/h, set in 1997.

But what if the graph of the position is more complicated than a straight line? What if the object speeds up or turns around and goes backward? Can we figure out anything about its velocity from a graph of that kind of motion? Let's take another look at the jet-powered car. The graph in Figure 2.13 shows its motion as it is getting up to speed after starting at rest. Time starts at zero for this motion (as if measured with a stopwatch), and the displacement and velocity are initially 200 m and 15 m/s, respectively.

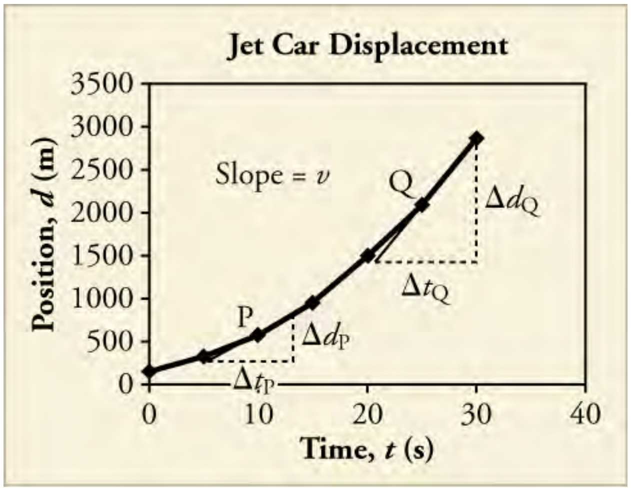


Figure 2.13 The diagram shows a graph of the position of a jet-powered car during the time span when it is speeding up. The slope of a distance versus time graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

Figure 2.14 A U.S. Air Force jet car speeds down a track. (Matt Trostle, Flickr)

The graph of position versus time in Figure 2.13 is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a position-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line **tangent** to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in Figure 2.13. The average velocity is the net displacement divided by the time traveled.

WORKED EXAMPLE

Using Position–Time Graph to Calculate Average Velocity: Jet Car, Take Two

Calculate the instantaneous velocity of the jet car at a time of 25 s by finding the slope of the tangent line at point Q in Figure 2.13.

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point.

Solution

1. Find the tangent line to the curve at $t = 25\ \text{s}$.
2. Determine the endpoints of the tangent. These correspond to a position of 1,300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, **v**.

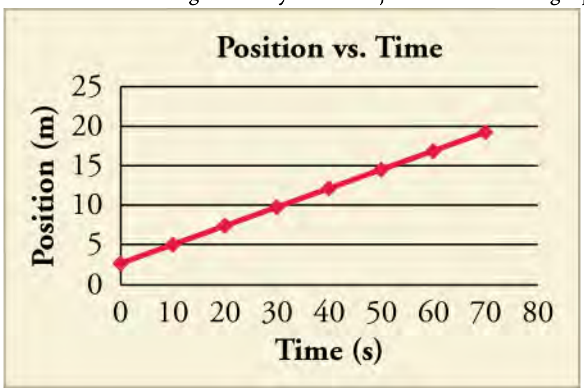
$$\begin{aligned}\text{slope} &= v_Q = \frac{\Delta d_Q}{\Delta t_Q} \\ &= \frac{(3120-1300)\ \text{m}}{(32-19)\ \text{s}} \\ &= \frac{1820\ \ \text{m}}{13\ \ \text{s}} \\ &= 140\ \ \text{m/s}\end{aligned} \tag{2.9}$$

Discussion

The entire graph of **v** versus *t* can be obtained in this fashion.

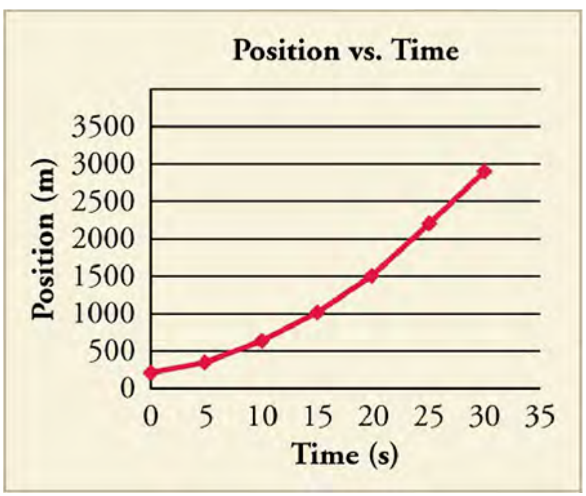
Practice Problems

15. Calculate the average velocity of the object shown in the graph below over the whole time interval.



a. 0.25 m/s
b. 0.31 m/s
c. 3.2 m/s
d. 4.00 m/s

16. True or False: By taking the slope of the curve in the graph you can verify that the velocity of the jet car is $115\ \text{m/s}$ at $t = 20\ \text{s}$.



a. True
b. False

Check Your Understanding

17. Which of the following information about motion can be determined by looking at a position vs. time graph that is a straight line?

a. frame of reference
b. average acceleration
c. velocity
d. direction of force applied

18. True or False: The position vs time graph of an object that is speeding up is a straight line.
a. True
b. False

2.4 Velocity vs. Time Graphs

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the meaning of slope and area in velocity vs. time graphs
- Solve problems using velocity vs. time graphs

Section Key Terms

acceleration

Graphing Velocity as a Function of Time

Earlier, we examined graphs of position versus time. Now, we are going to build on that information as we look at graphs of velocity vs. time. Velocity is the rate of change of displacement. **Acceleration** is the rate of change of velocity; we will discuss acceleration more in another chapter. These concepts are all very interrelated.

Virtual Physics

Maze Game

In this simulation you will use a vector diagram to manipulate a ball into a certain location without hitting a wall. You can manipulate the ball directly with position or by changing its velocity. Explore how these factors change the motion. If you would like, you can put it on the *a* setting, as well. This is acceleration, which measures the rate of change of velocity. We will explore acceleration in more detail later, but it might be interesting to take a look at it here.

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GRASP CHECK

Click to view content (https://archive.cnx.org/specials/30e37034-2fbd-11e5-83a2-03be60006ece/maze-game/#sim-maze-game)

a. The ball can be easily manipulated with displacement because the arena is a position space.
b. The ball can be easily manipulated with velocity because the arena is a position space.
c. The ball can be easily manipulated with displacement because the arena is a velocity space.
d. The ball can be easily manipulated with velocity because the arena is a velocity space.

What can we learn about motion by looking at velocity vs. time graphs? Let's return to our drive to school, and look at a graph of position versus time as shown in Figure 2.15.

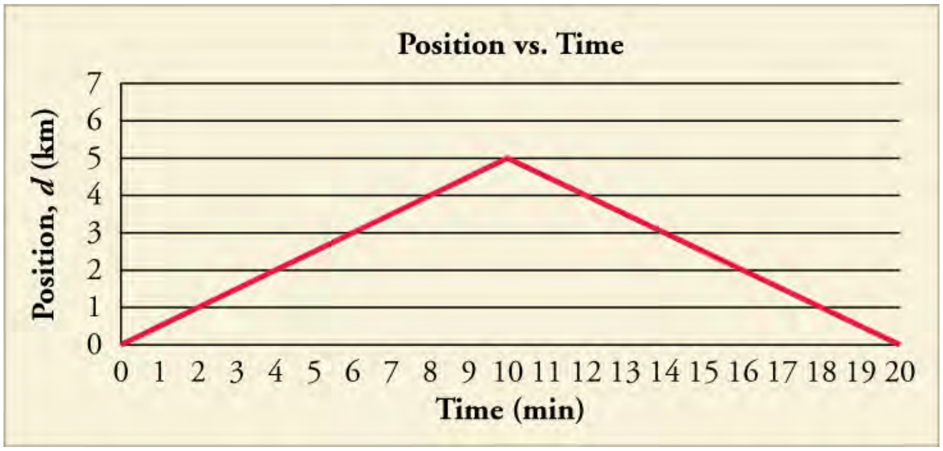

Figure 2.15 A graph of position versus time for the drive to and from school is shown.

We assumed for our original calculation that your parent drove with a constant velocity to and from school. We now know that the car could not have gone from rest to a constant velocity without speeding up. So the actual graph would be curved on either end, but let's make the same approximation as we did then, anyway.

TIPS FOR SUCCESS

It is common in physics, especially at the early learning stages, for certain things to be *neglected*, as we see here. This is because it makes the concept clearer or the calculation easier. Practicing physicists use these kinds of short-cuts, as well. It works out because usually the thing being *neglected* is small enough that it does not significantly affect the answer. In the earlier example, the amount of time it takes the car to speed up and reach its cruising velocity is very small compared to the total time traveled.

Looking at this graph, and given what we learned, we can see that there are two distinct periods to the car's motion—the way to school and the way back. The average velocity for the drive to school is 0.5 km/minute. We can see that the average velocity for the drive back is –0.5 km/minute. If we plot the data showing velocity versus time, we get another graph (Figure 2.16):

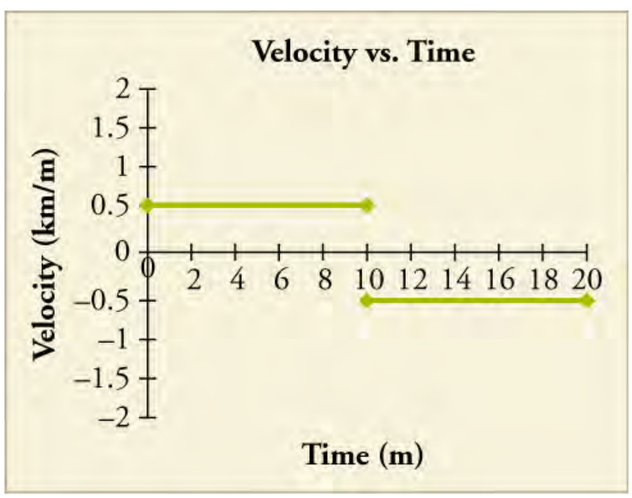

Figure 2.16 Graph of velocity versus time for the drive to and from school.

We can learn a few things. First, we can derive a **v** versus *t* graph from a **d** versus *t* graph. Second, if we have a straight-line position–time graph that is positively or negatively sloped, it will yield a horizontal velocity graph. There are a few other interesting things to note. Just as we could use a position vs. time graph to determine velocity, we can use a velocity vs. time graph to determine position. We know that $\mathbf{v} = \mathbf{d}/t$. If we use a little algebra to re-arrange the equation, we see that $\mathbf{d} = \mathbf{v} \times t$. In Figure 2.16, we have velocity on the *y*-axis and time along the *x*-axis. Let's take just the first half of the motion. We get 0.5 km/minute × 10 minutes. The units for *minutes* cancel each other, and we get 5 km, which is the displacement for the trip to school. If we calculate the same for the return trip, we get –5 km. If we add them together, we see that the net displacement for the whole trip is 0 km, which it should be because we started and ended at the same place.

TIPS FOR SUCCESS

You can treat units just like you treat numbers, so a km/km=1 (or, we say, it cancels out). This is good because it can tell us whether or not we have calculated everything with the correct units. For instance, if we end up with m × s for velocity instead of m/s, we know that something has gone wrong, and we need to check our math. This process is called dimensional analysis, and it is one of the best ways to check if your math makes sense in physics.

The area under a velocity curve represents the displacement. The velocity curve also tells us whether the car is speeding up. In our earlier example, we stated that the velocity was constant. So, the car is not speeding up. Graphically, you can see that the slope of these two lines is 0. This slope tells us that the car is not speeding up, or accelerating. We will do more with this information in a later chapter. For now, just remember that the area under the graph and the slope are the two important parts of the graph. Just like we could define a linear equation for the motion in a position vs. time graph, we can also define one for a velocity vs. time graph. As we said, the slope equals the acceleration, **a**. And in this graph, the *y*-intercept is $\mathbf{v}_0$. Thus, $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$.

But what if the velocity is not constant? Let's look back at our jet-car example. At the beginning of the motion, as the car is speeding up, we saw that its position is a curve, as shown in Figure 2.17.

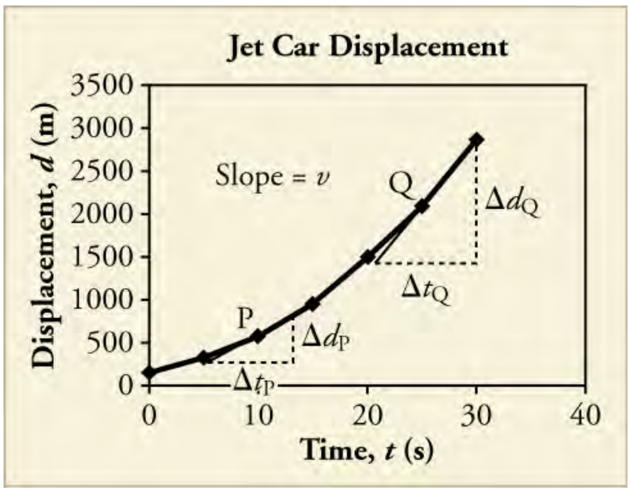

Figure 2.17 A graph is shown of the position of a jet-powered car during the time span when it is speeding up. The slope of a d vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

You do not have to do this, but you could, theoretically, take the instantaneous velocity at each point on this graph. If you did, you would get Figure 2.18, which is just a straight line with a positive slope.

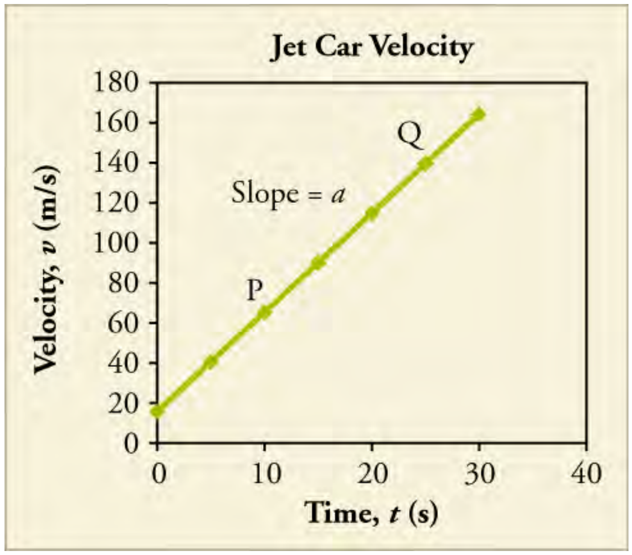

Figure 2.18 The graph shows the velocity of a jet-powered car during the time span when it is speeding up.

Again, if we take the slope of the velocity vs. time graph, we get the acceleration, the rate of change of the velocity. And, if we take the area under the slope, we get back to the displacement.

Solving Problems using Velocity–Time Graphs

Most velocity vs. time graphs will be straight lines. When this is the case, our calculations are fairly simple.

WORKED EXAMPLE

Using Velocity Graph to Calculate Some Stuff: Jet Car

Use this figure to (a) find the displacement of the jet car over the time shown (b) calculate the rate of change (acceleration) of the velocity. (c) give the instantaneous velocity at 5 s, and (d) calculate the average velocity over the interval shown.

Strategy

a. The displacement is given by finding the area under the line in the velocity vs. time graph.
b. The acceleration is given by finding the slope of the velocity graph.
c. The instantaneous velocity can just be read off of the graph.
d. To find the average velocity, recall that $\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{\mathbf{d}_f - \mathbf{d}_0}{t_f - t_0}$

Solution

a. 1. Analyze the shape of the area to be calculated. In this case, the area is made up of a rectangle between 0 and 20 m/s stretching to 30 s. The area of a rectangle is length × width. Therefore, the area of this piece is 600 m.
   2. Above that is a triangle whose base is 30 s and height is 140 m/s. The area of a triangle is 0.5 × length × width. The area of this piece, therefore, is 2,100 m.
   3. Add them together to get a net displacement of 2,700 m.

b. 1. Take two points on the velocity line. Say, $t = 5$ s and $t = 25$ s. At $t = 5$ s, the value of $\mathbf{v} = 40$ m/s. At $t = 25$ s, $\mathbf{v} = 140$ m/s.
   2. Find the slope.
$$\begin{aligned} \mathbf{a} &= \frac{\Delta \mathbf{v}}{\Delta t} \\ &= \frac{100 \text{ m/s}}{20 \text{ s}} \\ &= 5 \text{ m/s}^2 \end{aligned}$$

c. The instantaneous velocity at $t = 5$ s, as we found in part (b) is just 40 m/s.

d. 1. Find the net displacement, which we found in part (a) was 2,700 m.
   2. Find the total time which for this case is 30 s.
   3. Divide 2,700 m/30 s = 90 m/s.

Discussion

The average velocity we calculated here makes sense if we look at the graph. 100m/s falls about halfway across the graph and since it is a straight line, we would expect about half the velocity to be above and half below.

TIPS FOR SUCCESS

You can have negative position, velocity, and acceleration on a graph that describes the way the object is moving. You should never see a graph with negative time on an axis. Why?

Most of the velocity vs. time graphs we will look at will be simple to interpret. Occasionally, we will look at curved graphs of velocity vs. time. More often, these curved graphs occur when something is speeding up, often from rest. Let's look back at a more realistic velocity vs. time graph of the jet car's motion that takes this *speeding up* stage into account.

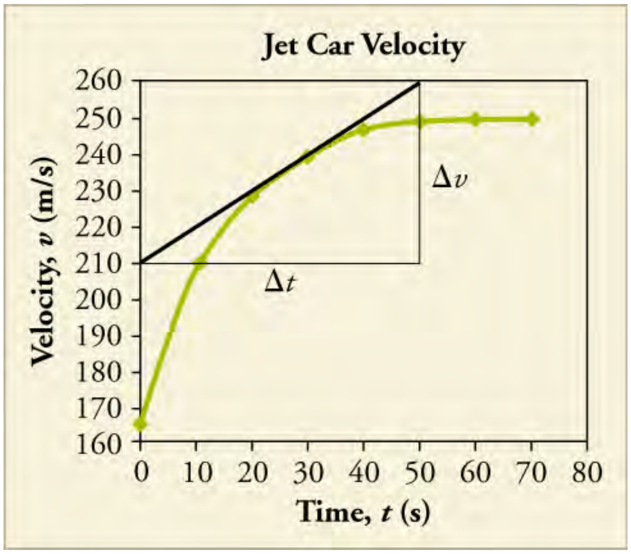

Figure 2.19 The graph shows a more accurate graph of the velocity of a jet-powered car during the time span when it is speeding up.

WORKED EXAMPLE

Using Curvy Velocity Graph to Calculate Some Stuff: jet car, Take Two

Use Figure 2.19 to (a) find the approximate displacement of the jet car over the time shown, (b) calculate the instantaneous acceleration at $t = 30$ s, (c) find the instantaneous velocity at 30 s, and (d) calculate the approximate average velocity over the interval shown.

Strategy

a. Because this graph is an undefined curve, we have to estimate shapes over smaller intervals in order to find the areas.
b. Like when we were working with a curved displacement graph, we will need to take a tangent line at the instant we are interested and use that to calculate the instantaneous acceleration.
c. The instantaneous velocity can still be read off of the graph.
d. We will find the average velocity the same way we did in the previous example.

Solution

a. 1. This problem is more complicated than the last example. To get a good estimate, we should probably break the curve into four sections. $0 \rightarrow 10$ s, $10 \rightarrow 20$ s, $20 \rightarrow 40$ s, and $40 \rightarrow 70$ s.
   2. Calculate the bottom rectangle (common to all pieces). $165\text{ m/s} \times 70\text{ s} = 11{,}550\text{ m}$.
   3. Estimate a triangle at the top, and calculate the area for each section. Section 1 = 225 m; section 2 = 100 m + 450 m = 550 m; section 3 = 150 m + 1,300 m = 1,450 m; section 4 = 2,550 m.
   4. Add them together to get a net displacement of 16,325 m.

b. Using the tangent line given, we find that the slope is 1 m/s^2.

c. The instantaneous velocity at $t = 30$ s, is 240 m/s.
d. 1. Find the net displacement, which we found in part (a), was 16,325 m.
   2. Find the total time, which for this case is 70 s.
   3. Divide $\frac{16{,}325 \text{ m}}{70 \text{ s}} \sim 233 \text{ m/s}$

Discussion

This is a much more complicated process than the first problem. If we were to use these estimates to come up with the average velocity over just the first 30 s we would get about 191 m/s. By approximating that curve with a line, we get an average velocity of 202.5 m/s. Depending on our purposes and how precise an answer we need, sometimes calling a curve a straight line is a worthwhile approximation.

Practice Problems

19.

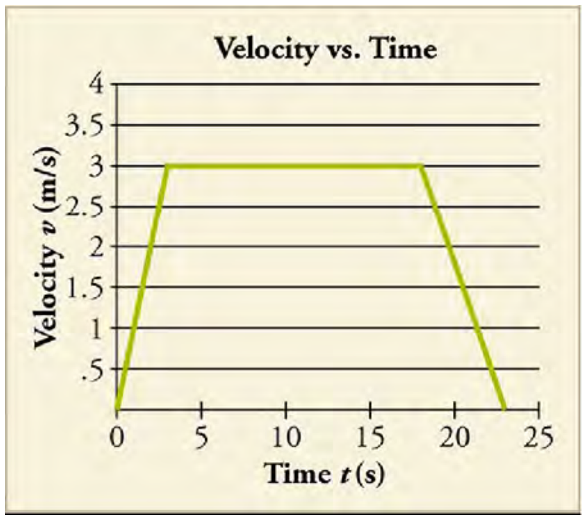

Figure 2.20

Consider the velocity vs. time graph shown below of a person in an elevator. Suppose the elevator is initially at rest. It then speeds up for 3 seconds, maintains that velocity for 15 seconds, then slows down for 5 seconds until it stops. Find the instantaneous velocity at $t = 10$ s and $t = 23$ s.

a. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 0 m/s and 0 m/s.
b. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 0 m/s and 3 m/s.
c. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 3 m/s and 0 m/s.
d. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 3 m/s and 1.5 m/s.

20.

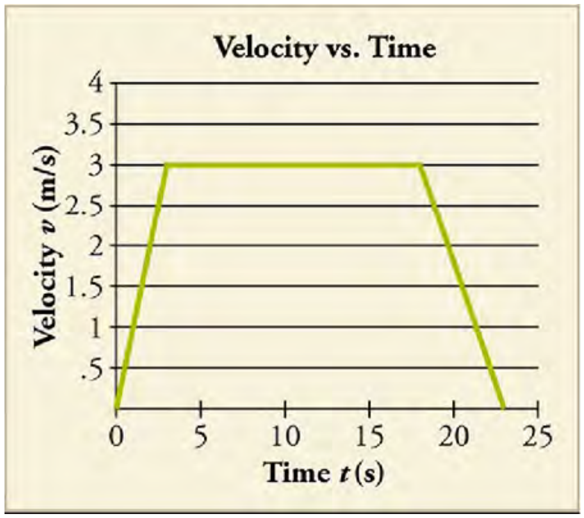

Figure 2.21

Calculate the net displacement and the average velocity of the elevator over the time interval shown.

a. Net displacement is 45 m and average velocity is 2.10 m/s.
b. Net displacement is 45 m and average velocity is 2.28 m/s.
c. Net displacement is 57 m and average velocity is 2.66 m/s.
d. Net displacement is 57 m and average velocity is 2.48 m/s.

Snap Lab

Graphing Motion, Take Two

In this activity, you will graph a moving ball's velocity vs. time.

- your graph from the earlier Graphing Motion Snap Lab!
- 1 piece of graph paper
- 1 pencil

Procedure

1. Take your graph from the earlier Graphing Motion Snap Lab! and use it to create a graph of velocity vs. time.
2. Use your graph to calculate the displacement.

GRASP CHECK

Describe the graph and explain what it means in terms of velocity and acceleration.

a. The graph shows a horizontal line indicating that the ball moved with a constant velocity, that is, it was not accelerating.
b. The graph shows a horizontal line indicating that the ball moved with a constant velocity, that is, it was accelerating.
c. The graph shows a horizontal line indicating that the ball moved with a variable velocity, that is, it was not accelerating.
d. The graph shows a horizontal line indicating that the ball moved with a variable velocity, that is, it was accelerating.

Check Your Understanding

21. What information could you obtain by looking at a velocity vs. time graph?

a. acceleration
b. direction of motion
c. reference frame of the motion

d. shortest path

22. How would you use a position vs. time graph to construct a velocity vs. time graph and vice versa?

a. Slope of position vs. time curve is used to construct velocity vs. time curve, and slope of velocity vs. time curve is used to construct position vs. time curve.
b. Slope of position vs. time curve is used to construct velocity vs. time curve, and area of velocity vs. time curve is used to construct position vs. time curve.
c. Area of position vs. time curve is used to construct velocity vs. time curve, and slope of velocity vs. time curve is used to construct position vs. time curve.
d. Area of position/time curve is used to construct velocity vs. time curve, and area of velocity vs. time curve is used to construct position vs. time curve.

KEY TERMS

acceleration the rate at which velocity changes
average speed distance traveled divided by time during which motion occurs
average velocity displacement divided by time over which displacement occurs
dependent variable the variable that changes as the independent variable changes
displacement the change in position of an object against a fixed axis
distance the length of the path actually traveled between an initial and a final position
independent variable the variable, usually along the horizontal axis of a graph, that does not change based on human or experimental action; in physics this is usually time
instantaneous speed speed at a specific instant in time
instantaneous velocity velocity at a specific instant in time
kinematics the study of motion without considering its causes
magnitude size or amount
position the location of an object at any particular time
reference frame a coordinate system from which the positions of objects are described
scalar a quantity that has magnitude but no direction
speed rate at which an object changes its location
tangent a line that touches another at exactly one point
vector a quantity that has both magnitude and direction
velocity the speed and direction of an object

SECTION SUMMARY

2.1 Relative Motion, Distance, and Displacement

- A description of motion depends on the reference frame from which it is described.
- The distance an object moves is the length of the path along which it moves.
- Displacement is the difference in the initial and final positions of an object.

2.2 Speed and Velocity

- Average speed is a scalar quantity that describes distance traveled divided by the time during which the motion occurs.
- Velocity is a vector quantity that describes the speed and direction of an object.
- Average velocity is displacement over the time period during which the displacement occurs. If the velocity is constant, then average velocity and instantaneous velocity are the same.

2.3 Position vs. Time Graphs

- Graphs can be used to analyze motion.
- The slope of a position vs. time graph is the velocity.
- For a straight line graph of position, the slope is the average velocity.
- To obtain the instantaneous velocity at a given moment for a curved graph, find the tangent line at that point and take its slope.

2.4 Velocity vs. Time Graphs

- The slope of a velocity vs. time graph is the acceleration.
- The area under a velocity vs. time curve is the displacement.
- Average velocity can be found in a velocity vs. time graph by taking the weighted average of all the velocities.

KEY EQUATIONS

2.1 Relative Motion, Distance, and Displacement

Displacement	$\Delta \mathbf{d} = \mathbf{d_f} - \mathbf{d_0}$

2.2 Speed and Velocity

Average speed	$v_{\text{avg}} = \frac{\textbf{distance}}{\textbf{time}}$
Average velocity	$\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{\mathbf{d}_\text{f} - \mathbf{d}_0}{t_\text{f} - t_0}$

2.3 Position vs. Time Graphs

Displacement	$\mathbf{d} = \mathbf{d}_0 + \mathbf{v}t$.

2.4 Velocity vs. Time Graphs

Velocity	$\mathbf{v} = \mathbf{v}_0 + at$
Acceleration	$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$