CHAPTER 10
Special Relativity

Figure 10.1 Special relativity explains why travel to other star systems, such as these in the Orion Nebula, is unlikely using our current level of technology. (s58y, Flickr)

Chapter Outline

INTRODUCTION Have you ever dreamed of traveling to other planets in faraway star systems? The trip might seem possible by traveling fast enough, but you will read in this chapter why it is not. In 1905, Albert Einstein developed the theory of **special relativity**. Einstein developed the theory to help explain inconsistencies between the equations describing electromagnetism and Newtonian mechanics, and to explain why the ether did not exist. This theory explains the limit on an object's speed among other implications.

Relativity is the study of how different observers moving with respect to one another measure the same events. Galileo and Newton developed the first correct version of classical relativity. Einstein developed the modern theory of relativity. Modern relativity is divided into two parts. Special relativity deals with observers moving at constant velocity. **General relativity** deals with observers moving at constant acceleration. Einstein's theories of relativity made revolutionary predictions. Most importantly, his predictions have been verified by experiments.

In this chapter, you learn how experiments and puzzling contradictions in existing theories led to the development of the theory of special relativity. You will also learn the simple postulates on which the theory was based; a postulate is a statement that is assumed to be true for the purposes of reasoning in a scientific or mathematic argument.

10.1 Postulates of Special Relativity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the experiments and scientific problems that led Albert Einstein to develop the special theory of relativity
- Understand the postulates on which the special theory of relativity was based

Section Key Terms

ether	frame of reference	inertial reference frame
general relativity	postulate	relativity
simultaneity	special relativity	

Scientific Experiments and Problems

Relativity is not new. Way back around the year 1600, Galileo explained that motion is relative. Wherever you happen to be, it seems like you are at a fixed point and that everything moves with respect to you. Everyone else feels the same way. Motion is always measured with respect to a fixed point. This is called establishing a **frame of reference**. But the choice of the point is arbitrary, and all frames of reference are equally valid. A passenger in a moving car is not moving with respect to the driver, but they are both moving from the point of view of a person on the sidewalk waiting for a bus. They are moving even faster as seen by a person in a car coming toward them. It is all relative.

TIPS FOR SUCCESS

A frame of reference is not a complicated concept. It is just something you decide is a fixed point or group of connected points. It is completely up to you. For example, when you look up at celestial objects in the sky, you choose the earth as your frame of reference, and the sun, moon, etc., seem to move across the sky.

Light is involved in the discussion of relativity because theories related to electromagnetism are inconsistent with Galileo's and Newton's explanation of relativity. The true nature of light was a hot topic of discussion and controversy in the late 19th century. At the time, it was not generally believed that light could travel across empty space. It was known to travel as waves, and all other types of energy that propagated as waves needed to travel though a material medium. It was believed that space was filled with an invisible medium that light waves traveled through. This imaginary (as it turned out) material was called the **ether** (also spelled aether). It was thought that everything moved through this mysterious fluid. In other words, ether was the one fixed frame of reference. The Michelson–Morley experiment proved it was not.

In 1887, Albert Michelson and Edward Morley designed the interferometer shown in Figure 10.2 to measure the speed of Earth through the ether. A light beam is split into two perpendicular paths and then recombined. Recombining the waves produces an inference pattern, with a bright fringe at the locations where the two waves arrive in phase; that is, with the crests of both waves arriving together and the troughs arriving together. A dark fringe appears where the crest of one wave coincides with a trough of the other, so that the two cancel. If Earth is traveling through the ether as it orbits the sun, the peaks in one arm would take longer than in the other to reach the same location. The places where the two waves arrive in phase would change, and the interference pattern would shift. But, using the interferometer, there was no shift seen! This result led to two conclusions: that there is no ether and that the speed of light is the same regardless of the relative motion of source and observer. The Michelson–Morley investigation has been called the most famous failed experiment in history.

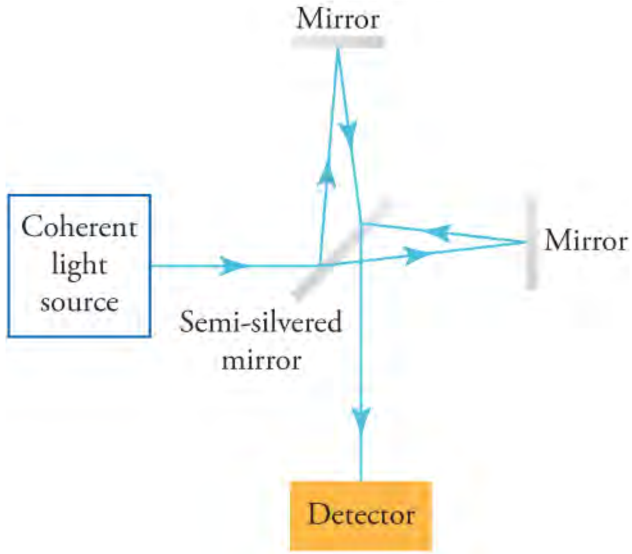

Figure 10.2 This is a diagram of the instrument used in the Michelson–Morley experiment.

To see what Michelson and Morley expected to find when they measured the speed of light in two directions, watch this animation (http://openstax.org/l/28MMexperiment) . In the video, two people swimming in a lake are represented as an analogy to light beams leaving Earth as it moves through the ether (if there were any ether). The swimmers swim away from and back to a platform that is moving through the water. The swimmers swim in different directions with respect to the motion of the platform. Even though they swim equal distances at the same speed, the motion of the platform causes them to arrive at different times.

Einstein's Postulates

The results described above left physicists with some puzzling and unsettling questions such as, why doesn't light emitted by a fast-moving object travel faster than light from a street lamp? A radical new theory was needed, and Albert Einstein, shown in Figure 10.3, was about to become everyone's favorite genius. Einstein began with two simple **postulates** based on the two things we have discussed so far in this chapter.

1. The laws of physics are the same in all inertial reference frames.
2. The speed of light is the same in all inertial reference frames and is not affected by the speed of its source.

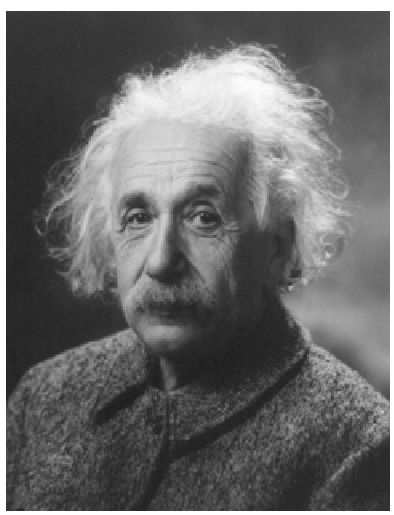

Figure 10.3 Albert Einstein (1879–1955) developed modern relativity and also made fundamental contributions to the foundations of quantum mechanics. (The Library of Congress)

The speed of light is given the symbol *c* and is equal to exactly 299,792,458 m/s. This is the speed of light in vacuum; that is, in the absence of air. For most purposes, we round this number off to 3.00×10^8 m/s The term **inertial reference frame** simply refers to a frame of reference where all objects follow Newton's first law of motion: Objects at rest remain at rest, and objects in motion remain in motion at a constant velocity in a straight line, unless acted upon by an external force. The inside of a car moving along a road at constant velocity and the inside of a stationary house are inertial reference frames.

WATCH PHYSICS

The Speed of Light

This lecture on light summarizes the most important facts about the speed of light. If you are interested, you can watch the whole video, but the parts relevant to this chapter are found between 3:25 and 5:10, which you find by running your cursor along the bottom of the video.

Click to view content (https://www.youtube.com/embed/rLNM8zI4Q_M)

GRASP CHECK

An airliner traveling at 200 m/s emits light from the front of the plane. Which statement describes the speed of the light?

a. It travels at a speed of $c + 200$ m/s.
b. It travels at a speed of $c - 200$ m/s.
c. It travels at a speed c, like all light.
d. It travels at a speed slightly less than c.

Snap Lab

Measure the Speed of Light

In this experiment, you will measure the speed of light using a microwave oven and a slice of bread. The waves generated by a microwave oven are not part of the visible spectrum, but they are still electromagnetic radiation, so they travel at the speed of light. If we know the wavelength, λ, and frequency, f, of a wave, we can calculate its speed, v, using the equation $v = \lambda f$. You can measure the wavelength. You will find the frequency on a label on the back of a microwave oven. The wave in a microwave is a standing wave with areas of high and low intensity. The high intensity sections are one-half wavelength apart.

- High temperature: Very hot temperatures are encountered in this lab. These can cause burns.

- a microwave oven
- one slice of plain white bread
- a centimeter ruler
- a calculator

1. Work with a partner.
2. Turn off the revolving feature of the microwave oven or remove the wheels under the microwave dish that make it turn. It is important that the dish does not turn.
3. Place the slice of bread on the dish, set the microwave on high, close the door, run the microwave for about 15 seconds.
4. A row of brown or black marks should appear on the bread. Stop the microwave as soon as they appear. Measure the distance between two adjacent burn marks and multiply the result by 2. This is the wavelength.
5. The frequency of the waves is written on the back of the microwave. Look for something like "2,450 MHz." Hz is the unit hertz, which means *per second*. The M represents mega, which stands for million, so multiply the number by 10^6.
6. Express the wavelength in meters and multiply it times the frequency. If you did everything correctly, you will get a number very close to the speed of light. Do not eat the bread. It is a general laboratory safety rule never to eat anything in the lab.

GRASP CHECK

How does your measured value of the speed of light compare to the accepted value (% error)?

a. The measured value of speed will be equal to *c*.
b. The measured value of speed will be slightly less than *c*.
c. The measured value of speed will be slightly greater than *c*.
d. The measured value of speed will depend on the frequency of the microwave.

Einstein's postulates were carefully chosen, and they both seemed very likely to be true. Einstein proceeded despite realizing that these two ideas taken together and applied to extreme conditions led to results that contradict Newtonian mechanics. He just took the ball and ran with it.

In the traditional view, velocities are additive. If you are running at 3 m/s and you throw a ball forward at a speed of 10 m/s, the ball should have a net speed of 13 m/s. However, according to relativity theory, the speed of a moving light source is not added to the speed of the emitted light.

In addition, Einstein's theory shows that if you were moving forward relative to Earth at nearly c (the speed of light) and could throw a ball forward at c, an observer at rest on the earth would not see the ball moving at nearly twice the speed of light. The observer would see it moving at a speed that is still less than c. This result conforms to both of Einstein's postulates: The speed of light has a fixed maximum and neither reference frame is privileged.

Consider how we measure elapsed time. If we use a stopwatch, for example, how do we know when to start and stop the watch? One method is to use the arrival of light from the event, such as observing a light turn green to start a drag race. The timing will be more accurate if some sort of electronic detection is used, avoiding human reaction times and other complications.

Now suppose we use this method to measure the time interval between two flashes of light produced by flash lamps on a moving train. (See Figure 10.4)

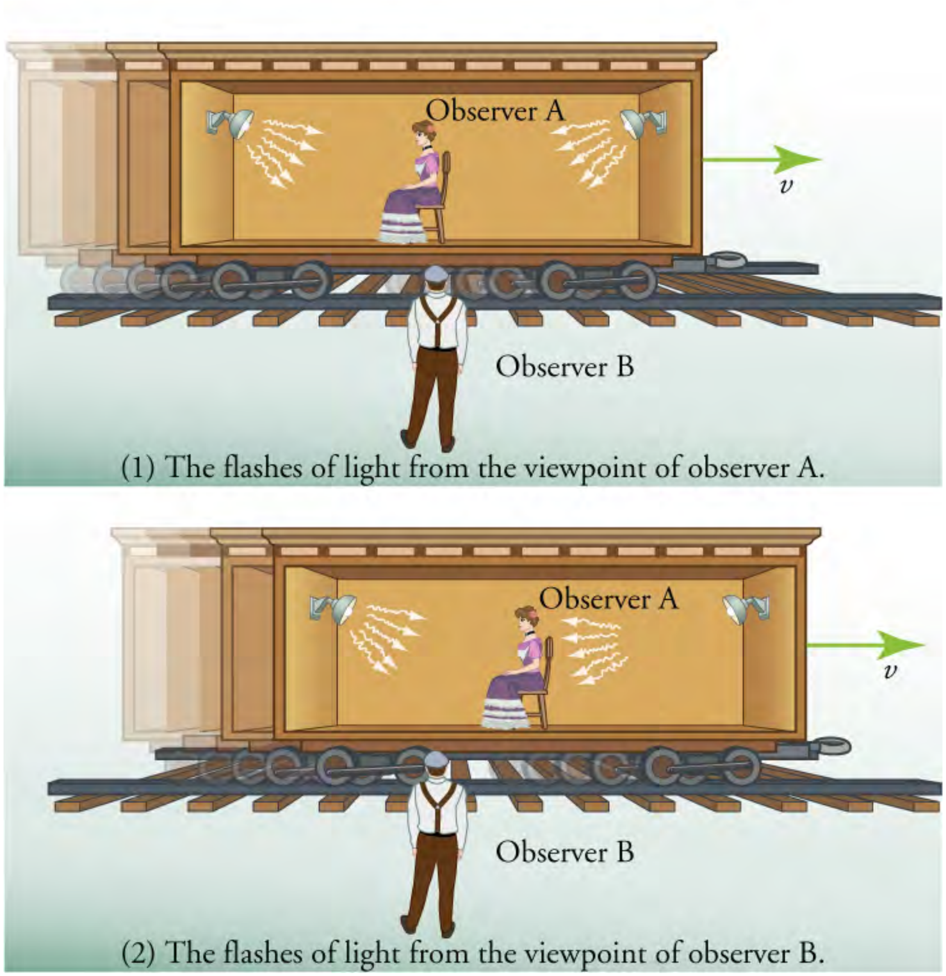

Figure 10.4 Light arriving to observer A as seen by two different observers.

A woman (observer A) is seated in the center of a rail car, with two flash lamps at opposite sides equidistant from her. Multiple light rays that are emitted from the flash lamps move towards observer A, as shown with arrows. A velocity vector arrow for the rail car is shown towards the right. A man (observer B) standing on the platform is facing the woman and also observes the flashes of light.

Observer A moves with the lamps on the rail car as the rail car moves towards the right of observer B. Observer B receives the light flashes simultaneously, and sees the bulbs as both having flashed at the same time. However, he sees observer A receive the flash from the right first. Because the pulse from the right reaches her first, in her frame of reference she sees the bulbs as not having flashed simultaneously. Here, a relative velocity between observers affects whether two events at well-separated locations are observed to be simultaneous. **Simultaneity**, or whether different events occur at the same instant, depends on the frame of reference of the observer. Remember that velocity equals distance divided by time, so $t = d/\mathbf{v}$. If velocity appears to be different, then duration of time appears to be different.

This illustrates the power of clear thinking. We might have guessed incorrectly that, if light is emitted simultaneously, then two observers halfway between the sources would see the flashes simultaneously. But careful analysis shows this not to be the case. Einstein was brilliant at this type of thought experiment (in German, *Gedankenexperiment*). He very carefully considered how an observation is made and disregarded what might seem obvious. The validity of thought experiments, of course, is determined by actual observation. The genius of Einstein is evidenced by the fact that experiments have repeatedly confirmed his theory of relativity. No experiments after that of Michelson and Morley were able to detect any ether medium. We will describe later how experiments also confirmed other predictions of **special relativity**, such as the distance between two objects and the time interval of two events being different for two observers moving with respect to each other.

In summary: Two events are defined to be simultaneous if an observer measures them as occurring at the same time (such as by receiving light from the events). Two events are not necessarily simultaneous to all observers.

The discrepancies between Newtonian mechanics and relativity theory illustrate an important point about how science advances. Einstein's theory did not replace Newton's but rather extended it. It is not unusual that a new theory must be developed to account for new information. In most cases, the new theory is built on the foundation of older theory. It is rare that old theories are completely replaced.

In this chapter, you will learn about the theory of special relativity, but, as mentioned in the introduction, Einstein developed two relativity theories: special and general. Table 10.1 summarizes the differences between the two theories.

Special Relativity	General Relativity
Published in 1905	Final form published in 1916
A theory of space-time	A theory of gravity
Applies to observers moving at constant speed	Applies to observers that are accelerating
Most useful in the field of nuclear physics	Most useful in the field of astrophysics
Accepted quickly and put to practical use by nuclear physicists and quantum chemists	Largely ignored until 1960 when new mathematical techniques made the theory more accessible and astronomers found some important applications
Also note that the theory of general relativity includes the theory of special relativity.	

Table 10.1 Comparing Special Relativity and General Relativity

WORKED EXAMPLE

Calculating the Time it Takes Light to Travel a Given Distance

The sun is 1.50×10^8 km from Earth. How long does it take light to travel from the sun to Earth in minutes and seconds?

Strategy

Identify knowns.

$$\begin{aligned} \text{Distance} &= 1.50 \times 10^8 \text{ km} \\ \text{Speed} &= 3.00 \times 10^8 \text{ km} \end{aligned} \tag{10.1}$$

Identify unknowns.

Time

Find the equation that relates knowns and unknowns.

$$v = \frac{d}{t};\quad t = \frac{d}{v} \tag{10.2}$$

Be sure to use consistent units.

Solution

$$t = \frac{d}{v} = \frac{(1.50 \times 10^8 km) \times \frac{10^3 \text{m}}{\text{km}}}{3.00 \times 10^8 \frac{\text{m}}{\text{s}}} = 5.00 \times 10^2 s$$

$$500 \text{ s} = 8 \text{ min and } 20 \text{ s}$$

Discussion

The answer is written as 5.00×10^2 rather than 500 in order to show that there are three significant figures. When astronomers witness an event on the sun, such as a sunspot, it actually happened minutes earlier. Compare 8 light *minutes* to the distance to stars, which are light *years* away. Any events on other stars happened years ago.

Practice Problems

1. Light travels through 1.00 m of water in 4.42×10^{-9} s. What is the speed of light in water?
   a. 4.42×10^{-9} m/s
   b. 4.42×10^{9} m/s
   c. 2.26×10^{8} m/s
   d. 226×10^{8} m/s

2. An astronaut on the moon receives a message from mission control on Earth. The signal is sent by a form of electromagnetic radiation and takes 1.28 s to travel the distance between Earth and the moon. What is the distance from Earth to the moon?
   a. 2.34×10^{5} km
   b. 2.34×10^{8} km
   c. 3.84×10^{5} km
   d. 3.84×10^{8} km

Check Your Understanding

3. Explain what is meant by a frame of reference.
   a. A frame of reference is a graph plotted between distance and time.
   b. A frame of reference is a graph plotted between speed and time.
   c. A frame of reference is the velocity of an object through empty space without regard to its surroundings.
   d. A frame of reference is an arbitrarily fixed point with respect to which motion of other points is measured.

4. Two people swim away from a raft that is floating downstream. One swims upstream and returns, and the other swims across the current and back. If this scenario represents the Michelson–Morley experiment, what do (i) the water, (ii) the swimmers, and (iii) the raft represent?
   a. the ether rays of light Earth
   b. rays of light the ether Earth
   c. the ether Earth rays of light
   d. Earth rays of light the ether

5. If Michelson and Morley had observed the interference pattern shift in their interferometer, what would that have indicated?
   a. The speed of light is the same in all frames of reference.
   b. The speed of light depends on the motion relative to the ether.
   c. The speed of light changes upon reflection from a surface.
   d. The speed of light in vacuum is less than 3.00×10^{8} m/s.

6. If you designate a point as being fixed and use that point to measure the motion of surrounding objects, what is the point called?
   a. An origin
   b. A frame of reference
   c. A moving frame
   d. A coordinate system

10.2 Consequences of Special Relativity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the relativistic effects seen in time dilation, length contraction, and conservation of relativistic momentum
- Explain and perform calculations involving mass-energy equivalence

Section Key Terms

binding energy	length contraction	mass defect	time dilation

proper length	relativistic	relativistic momentum
relativistic energy	relativistic factor	rest mass

Relativistic Effects on Time, Distance, and Momentum

Consideration of the measurement of elapsed time and simultaneity leads to an important relativistic effect. **Time dilation** is the phenomenon of time passing more slowly for an observer who is moving relative to another observer.

For example, suppose an astronaut measures the time it takes for light to travel from the light source, cross her ship, bounce off a mirror, and return. (See Figure 10.5.) How does the elapsed time the astronaut measures compare with the elapsed time measured for the same event by a person on the earth? Asking this question (another thought experiment) produces a profound result. We find that the elapsed time for a process depends on who is measuring it. In this case, the time measured by the astronaut is smaller than the time measured by the earth bound observer. The passage of time is different for the two observers because the distance the light travels in the astronaut's frame is smaller than in the earth bound frame. Light travels at the same speed in each frame, and so it will take longer to travel the greater distance in the earth bound frame.

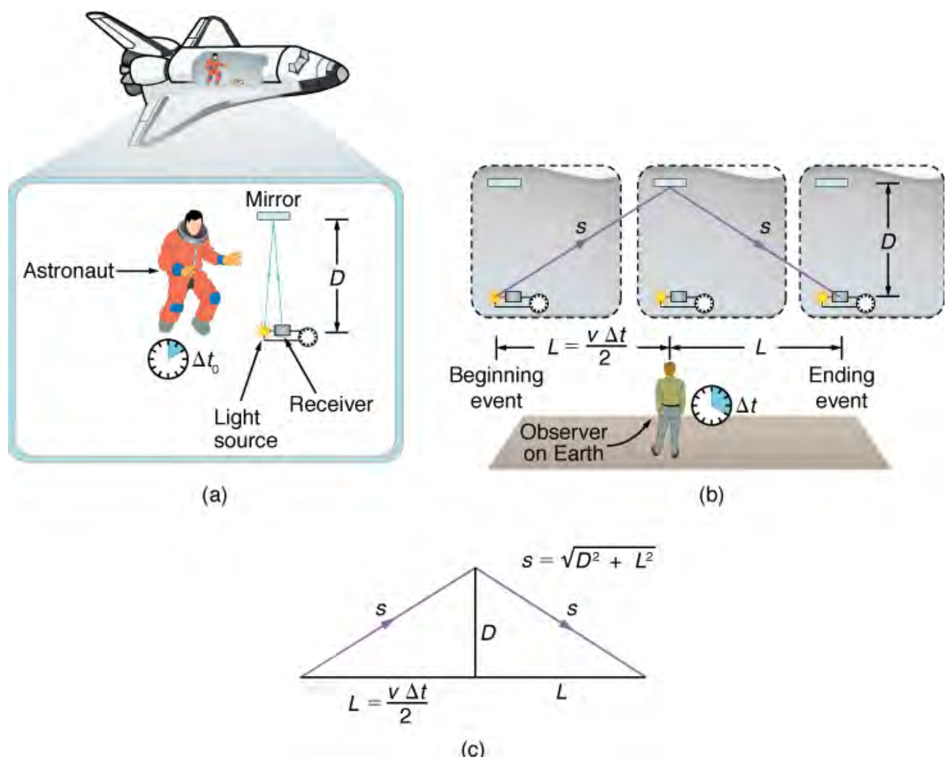

Figure 10.5 (a) An astronaut measures the time Δt_0 for light to cross her ship using an electronic timer. Light travels a distance $2D$ in the astronaut's frame. (b) A person on the earth sees the light follow the longer path $2s$ and take a longer time Δt.

The relationship between Δt and Δt_0 is given by

$$\Delta t = \gamma \Delta t_0,$$

where γ is the **relativistic factor** given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

and v and c are the speeds of the moving observer and light, respectively.

TIPS FOR SUCCESS

Try putting some values for *v* into the expression for the relativistic factor (γ). Observe at which speeds this factor will make a difference and when γ is so close to 1 that it can be ignored. Try 225 m/s, the speed of an airliner; 2.98×10^4 m/s, the speed of Earth in its orbit; and 2.990×10^8 m/s, the speed of a particle in an accelerator.

Notice that when the velocity *v* is small compared to the speed of light *c*, then *v/c* becomes small, and γ becomes close to 1. When this happens, time measurements are the same in both frames of reference. **Relativistic** effects, meaning those that have to do with special relativity, usually become significant when speeds become comparable to the speed of light. This is seen to be the case for time dilation.

You may have seen science fiction movies in which space travelers return to Earth after a long trip to find that the planet and everyone on it has aged much more than they have. This type of scenario is a based on a thought experiment, known as the twin paradox, which imagines a pair of twins, one of whom goes on a trip into space while the other stays home. When the space traveler returns, she finds her twin has aged much more than she. This happens because the traveling twin has been in two frames of reference, one leaving Earth and one returning.

Time dilation has been confirmed by comparing the time recorded by an atomic clock sent into orbit to the time recorded by a clock that remained on Earth. GPS satellites must also be adjusted to compensate for time dilation in order to give accurate positioning.

Have you ever driven on a road, like that shown in Figure 10.6, that seems like it goes on forever? If you look ahead, you might say you have about 10 km left to go. Another traveler might say the road ahead looks like it is about 15 km long. If you both measured the road, however, you would agree. Traveling at everyday speeds, the distance you both measure would be the same. You will read in this section, however, that this is not true at relativistic speeds. Close to the speed of light, distances measured are not the same when measured by different observers moving with respect to one other.

Figure 10.6 People might describe distances differently, but at relativistic speeds, the distances really are different. (Corey Leopold, Flickr)

One thing all observers agree upon is their relative speed. When one observer is traveling away from another, they both see the other receding at the same speed, regardless of whose frame of reference is chosen. Remember that speed equals distance divided by time: $v = d/t$. If the observers experience a difference in elapsed time, they must also observe a difference in distance traversed. This is because the ratio *d/t* must be the same for both observers.

The shortening of distance experienced by an observer moving with respect to the points whose distance apart is measured is called **length contraction**. **Proper length**, L_0, is the distance between two points measured in the reference frame where the observer and the points are at rest. The observer in motion with respect to the points measures *L*. These two lengths are related by the equation

$$L = \frac{L_0}{\gamma}.$$

Because γ is the same expression used in the time dilation equation above, the equation becomes

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}.$$

To see how length contraction is seen by a moving observer, go to this simulation (http://openstax.org/l/28simultaneity) . Here you can also see that simultaneity, time dilation, and length contraction are interrelated phenomena.

This link is to a simulation that illustrates the relativity of simultaneous events.

In classical physics, momentum is a simple product of mass and velocity. When special relativity is taken into account, objects that have mass have a speed limit. What effect do you think mass and velocity have on the momentum of objects moving at relativistic speeds; i.e., speeds close to the speed of light?

Momentum is one of the most important concepts in physics. The broadest form of Newton's second law is stated in terms of momentum. Momentum is conserved in classical mechanics whenever the net external force on a system is zero. This makes momentum conservation a fundamental tool for analyzing collisions. We will see that momentum has the same importance in modern physics. **Relativistic momentum** is conserved, and much of what we know about subatomic structure comes from the analysis of collisions of accelerator-produced relativistic particles.

One of the postulates of special relativity states that the laws of physics are the same in all inertial frames. Does the law of conservation of momentum survive this requirement at high velocities? The answer is yes, provided that the momentum is defined as follows.

Relativistic momentum, $\mathbf{p}$, is classical momentum multiplied by the relativistic factor γ.

$$\mathbf{p} = \gamma m\mathbf{u}, \tag{10.3}$$

where m is the **rest mass** of the object (that is, the mass measured at rest, without any γ factor involved), $\mathbf{u}$ is its velocity relative to an observer, and γ, as before, is the relativistic factor. We use the mass of the object as measured at rest because we cannot determine its mass while it is moving.

Note that we use $\mathbf{u}$ for velocity here to distinguish it from relative velocity $\mathbf{v}$ between observers. Only one observer is being considered here. With $\mathbf{p}$ defined in this way, $\mathbf{p}_{\text{tot}}$ is conserved whenever the net external force is zero, just as in classical physics. Again we see that the relativistic quantity becomes virtually the same as the classical at low velocities. That is, relativistic momentum $\gamma m\mathbf{u}$ becomes the classical $m\mathbf{u}$ at low velocities, because γ is very nearly equal to 1 at low velocities.

Relativistic momentum has the same intuitive feel as classical momentum. It is greatest for large masses moving at high velocities. Because of the factor γ, however, relativistic momentum behaves differently from classical momentum by approaching infinity as $\mathbf{u}$ approaches c. (See Figure 10.7.) This is another indication that an object with mass cannot reach the speed of light. If it did, its momentum would become infinite, which is an unreasonable value.

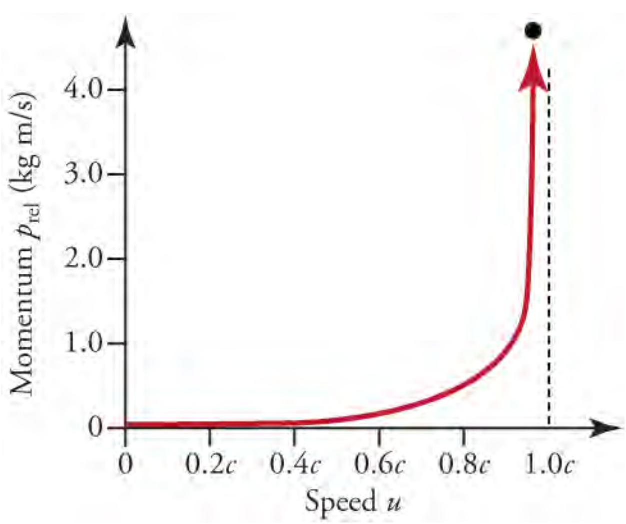

Figure 10.7 Relativistic momentum approaches infinity as the velocity of an object approaches the speed of light.

Relativistic momentum is defined in such a way that the conservation of momentum will hold in all inertial frames. Whenever the net external force on a system is zero, relativistic momentum is conserved, just as is the case for classical momentum. This has been verified in numerous experiments.

Mass-Energy Equivalence

Let us summarize the calculation of relativistic effects on objects moving at speeds near the speed of light. In each case we will need to calculate the relativistic factor, given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}},$$

where **v** and c are as defined earlier. We use **u** as the velocity of a particle or an object in one frame of reference, and **v** for the velocity of one frame of reference with respect to another.

Time Dilation

Elapsed time on a moving object, Δt_0, as seen by a stationary observer is given by $\Delta t = \gamma \Delta t_0$, where Δt_0 is the time observed on the moving object when it is taken to be the frame or reference.

Length Contraction

Length measured by a person at rest with respect to a moving object, L, is given by

$$L = \frac{L_0}{\gamma},$$

where L_0 is the length measured on the moving object.

Relativistic Momentum

Momentum, $\mathbf{p}$, of an object of mass, m, traveling at relativistic speeds is given by $\mathbf{p} = \gamma m \mathbf{u}$, where **u** is velocity of a moving object as seen by a stationary observer.

Relativistic Energy

The original source of all the energy we use is the conversion of mass into energy. Most of this energy is generated by nuclear reactions in the sun and radiated to Earth in the form of electromagnetic radiation, where it is then transformed into all the forms with which we are familiar. The remaining energy from nuclear reactions is produced in nuclear power plants and in Earth's interior. In each of these cases, the source of the energy is the conversion of a small amount of mass into a large amount of energy. These sources are shown in Figure 10.8.

Figure 10.8 The sun (a) and the Susquehanna Steam Electric Station (b) both convert mass into energy. ((a) NASA/Goddard Space Flight Center, Scientific Visualization Studio; (b) U.S. government)

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Einstein showed that the law of conservation of energy is valid relativistically, if we define energy to include a relativistic factor. The result of his analysis is that a particle or object of mass m moving at velocity **u** has **relativistic energy** given by

$$E = \gamma mc^2.$$

This is the expression for the total energy of an object of mass m at any speed **u** and includes both kinetic and potential energy. Look back at the equation for γ and you will see that it is equal to 1 when **u** is 0; that is, when an object is at rest. Then the rest energy, E_0, is simply

$$E_0 = mc^2.$$

This is the correct form of Einstein's famous equation.

This equation is very useful to nuclear physicists because it can be used to calculate the energy released by a nuclear reaction. This is done simply by subtracting the mass of the products of such a reaction from the mass of the reactants. The difference is the m in $E_0 = mc^2$. Here is a simple example:

A positron is a type of antimatter that is just like an electron, except that it has a positive charge. When a positron and an electron collide, their masses are completely annihilated and converted to energy in the form of gamma rays. Because both particles have a rest mass of 9.11×10^{-31} kg, we multiply the mc^2 term by 2. So the energy of the gamma rays is

$$\begin{aligned} E_0 &= 2(9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^8\ \tfrac{\text{m}}{\text{s}}\right)^2 \\ &= 1.64 \times 10^{-13}\ \tfrac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \\ &= 1.64 \times 10^{-13}\ \text{J} \end{aligned} \tag{10.4}$$

where we have the expression for the joule (J) in terms of its SI base units of kg, m, and s. In general, the nuclei of stable isotopes have less mass then their constituent subatomic particles. The energy equivalent of this difference is called the **binding energy** of the nucleus. This energy is released during the formation of the isotope from its constituent particles because the product is more stable than the reactants. Expressed as mass, it is called the **mass defect**. For example, a helium nucleus is made of two neutrons and two protons and has a mass of 4.0003 atomic mass units (u). The sum of the masses of two protons and two neutrons is 4.0330 u. The mass defect then is 0.0327 u. Converted to kg, the mass defect is 5.0442×10^{-30} kg. Multiplying this mass times c^2 gives a binding energy of 4.540×10^{-12} J. This does not sound like much because it is only one atom. If you were to make one gram of helium out of neutrons and protons, it would release 683,000,000,000 J. By comparison, burning one gram of coal releases about 24 J.

BOUNDLESS PHYSICS

The RHIC Collider

Figure 10.9 shows the Brookhaven National Laboratory in Upton, NY. The circular structure houses a particle accelerator called the RHIC, which stands for Relativistic Heavy Ion Collider. The heavy ions in the name are gold nuclei that have been stripped of their electrons. Streams of ions are accelerated in several stages before entering the big ring seen in the figure. Here, they are accelerated to their final speed, which is about 99.7 percent the speed of light. Such high speeds are called relativistic. All the relativistic phenomena we have been discussing in this chapter are very pronounced in this case. At this speed $\gamma = 12.9$, so that relativistic time dilates by a factor of about 13, and relativistic length contracts by the same factor.

Figure 10.9 Brookhaven National Laboratory. The circular structure houses the RHIC. (energy.gov, Wikimedia Commons)

Two ion beams circle the 2.4-mile long track around the big ring in opposite directions. The paths can then be made to cross, thereby causing ions to collide. The collision event is very short-lived but amazingly intense. The temperatures and pressures produced are greater than those in the hottest suns. At 4 trillion degrees Celsius, this is the hottest material ever created in a laboratory

But what is the point of creating such an extreme event? Under these conditions, the neutrons and protons that make up the gold nuclei are smashed apart into their components, which are called quarks and gluons. The goal is to recreate the conditions that theorists believe existed at the very beginning of the universe. It is thought that, at that time, matter was a sort of soup of quarks and gluons. When things cooled down after the initial bang, these particles condensed to form protons and neutrons.

Some of the results have been surprising and unexpected. It was thought the quark-gluon soup would resemble a gas or plasma. Instead, it behaves more like a liquid. It has been called a *perfect* liquid because it has virtually no viscosity, meaning that it has no resistance to flow.

GRASP CHECK

Calculate the relativistic factor γ, for a particle traveling at 99.7 percent of the speed of light.

a. 0.08
b. 0.71
c. 1.41
d. 12.9

WORKED EXAMPLE

The Speed of Light

One night you are out looking up at the stars and an extraterrestrial spaceship flashes across the sky. The ship is 50 meters long and is travelling at 95 percent of the speed of light. What would the ship's length be when measured from your earthbound frame of reference?

Strategy

List the knowns and unknowns.

Knowns: proper length of the ship, L_O = 50 m; velocity, $\mathbf{v}$, = 0.95c

Unknowns: observed length of the ship accounting for relativistic length contraction, L.

Choose the relevant equation.

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1 - \frac{\mathbf{u}^2}{c^2}}$$

Solution

$$L = 50\text{ m}\sqrt{1 - \frac{(0.95)^2 c^2}{c^2}} = 50\text{ m}\sqrt{1 - (0.95)^2} = 16\ \text{ m}$$

Discussion

Calculations of γ can usually be simplified in this way when v is expressed as a percentage of c because the c^2 terms cancel. Be sure to also square the decimal representing the percentage before subtracting from 1. Note that the aliens will still see the length as L_0 because they are moving with the frame of reference that is the ship.

Practice Problems

7. Calculate the relativistic factor, γ, for an object traveling at 2.00×10^8 m/s.
   a. 0.74
   b. 0.83
   c. 1.2
   d. 1.34

8. The distance between two points, called the proper length, Lo, is 1.00 km. An observer in motion with respect to the frame of

reference of the two points measures 0.800 km, which is L. What is the relative speed of the frame of reference with respect to the observer?

a. 1.80×10^8 m/s
b. 2.34×10^8 m/s
c. 3.84×10^8 m/s
d. 5.00×10^8 m/s

9. Consider the nuclear fission reaction $n + {}^{235}_{92}U \rightarrow {}^{137}_{55}Cs + {}^{97}_{37}Rb + 2n + E$. If a neutron has a rest mass of 1.009u, ${}^{235}_{92}U$ has a rest mass of 235.044u, ${}^{137}_{55}Cs$ has rest mass of 136.907u, and ${}^{97}_{37}Rb$ has a rest mass of 96.937u, what is the value of E in joules?
   a. 1.8×10^{-11} J
   b. 2.9×10^{-11} J
   c. 1.8×10^{-10} J
   d. 2.9×10^{-10} J

Solution

The correct answer is (b). The mass deficit in the reaction is $235.044\ \text{u} - (136.907 + 96.937 + 1.009)\ \text{u}$, or 0.191u. Converting that mass to kg and applying $E = mc^2$ to find the energy equivalent of the mass deficit gives

$$(0.191\ \text{u})\left(1.66 \times 10^{-27}\text{kg/u}\right)\left(3.00 \times 10^{8}\text{m/s}\right)^2 \cong 2.85 \times 10^{-11}\text{J}.$$

10. Consider the nuclear fusion reaction ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}H + E$. If ${}^{2}_{1}H$ has a rest mass of 2.014u, ${}^{3}_{1}H$ has a rest mass of 3.016u, and ${}^{1}_{1}H$ has a rest mass of 1.008u, what is the value of E in joules?
   a. 6×10^{-13} J
   b. 6×10^{-12} J
   c. 6×10^{-11} J
   d. 6×10^{-10} J

Solution

The correct answer is (a). The mass deficit in the reaction is $2\,(2.014\ \text{u}) - (3.016 + 1.008)\ \text{u}$, or 0.004u. Converting that mass to kg and applying $E = mc^2$ to find the energy equivalent of the mass deficit gives

$$(0.004\ \text{u})\left(1.66 \times 10^{-27}\text{kg/u}\right)\left(3.00 \times 10^{8}\text{m/s}\right)^2 \cong 5.98 \times 10^{-13}\text{J}.$$

Check Your Understanding

11. Describe time dilation and state under what conditions it becomes significant.
   a. When the speed of one frame of reference past another reaches the speed of light, a time interval between two events at the same location in one frame appears longer when measured from the second frame.
   b. When the speed of one frame of reference past another becomes comparable to the speed of light, a time interval between two events at the same location in one frame appears longer when measured from the second frame.
   c. When the speed of one frame of reference past another reaches the speed of light, a time interval between two events at the same location in one frame appears shorter when measured from the second frame.
   d. When the speed of one frame of reference past another becomes comparable to the speed of light, a time interval between two events at the same location in one frame appears shorter when measured from the second frame.

12. The equation used to calculate relativistic momentum is $p = \gamma \cdot m \cdot u$. Define the terms to the right of the equal sign and state how m and u are measured.
   a. γ is the relativistic factor, m is the rest mass measured when the object is at rest in the frame of reference, and u is the velocity of the frame.
   b. γ is the relativistic factor, m is the rest mass measured when the object is at rest in the frame of reference, and u is the velocity relative to an observer.
   c. γ is the relativistic factor, m is the relativistic mass $\left(\text{i.e.,} \frac{m}{\sqrt{1-\frac{u^2}{c^2}}}\right)$ measured when the object is moving in the frame of reference, and u is the velocity of the frame.

d. γ is the relativistic factor, m is the relativistic mass $\left(\text{i.e.,} \frac{m}{\sqrt{1-\frac{u^2}{c^2}}}\right)$ measured when the object is moving in the frame of reference, and u is the velocity relative to an observer.

13. Describe length contraction and state when it occurs.

a. When the speed of an object becomes the speed of light, its length appears to shorten when viewed by a stationary observer.
b. When the speed of an object approaches the speed of light, its length appears to shorten when viewed by a stationary observer.
c. When the speed of an object becomes the speed of light, its length appears to increase when viewed by a stationary observer.
d. When the speed of an object approaches the speed of light, its length appears to increase when viewed by a stationary observer.

KEY TERMS

binding energy the energy equivalent of the difference between the mass of a nucleus and the masses of its nucleons

ether scientists once believed there was a medium that carried light waves; eventually, experiments proved that ether does not exist

frame of reference the point or collection of points arbitrarily chosen, which motion is measured in relation to

general relativity the theory proposed to explain gravity and acceleration

inertial reference frame a frame of reference where all objects follow Newton's first law of motion

length contraction the shortening of an object as seen by an observer who is moving relative to the frame of reference of the object

mass defect the difference between the mass of a nucleus and the masses of its nucleons

postulate a statement that is assumed to be true for the purposes of reasoning in a scientific or mathematic argument

proper length the length of an object within its own frame of reference, as opposed to the length observed by an observer moving relative to that frame of reference

relativistic having to do with modern relativity, such as the effects that become significant only when an object is moving close enough to the speed of light for γ to be significantly greater than 1

relativistic energy the total energy of a moving object or particle $E = \gamma mc^2$, which includes both its rest energy mc^2 and its kinetic energy

relativistic factor $\gamma = \frac{1}{\sqrt{1-\frac{\mathbf{u}^2}{c^2}}}$, where **u** is the velocity of a moving object and c is the speed of light

relativistic momentum $\mathbf{p} = \gamma m\mathbf{u}$, where γ is the relativistic factor, m is rest mass of an object, and **u** is the velocity relative to an observer

relativity the explanation of how objects move relative to one another

rest mass the mass of an object that is motionless with respect to its frame of reference

simultaneity the property of events that occur at the same time

special relativity the theory proposed to explain the consequences of requiring the speed of light and the laws of physics to be the same in all inertial frames

time dilation the contraction of time as seen by an observer in a frame of reference that is moving relative to the observer

SECTION SUMMARY

10.1 Postulates of Special Relativity

- One postulate of special relativity theory is that the laws of physics are the same in all inertial frames of reference.
- The other postulate is that the speed of light in a vacuum is the same in all inertial frames.
- Einstein showed that simultaneity, or lack of it, depends on the frame of reference of the observer.

10.2 Consequences of Special Relativity

- Time dilates, length contracts, and momentum increases as an object approaches the speed of light.
- Energy and mass are interchangeable, according to the relationship $E = mc2$. The laws of conservation of mass and energy are combined into the law of conservation of mass-energy.

KEY EQUATIONS

10.1 Postulates of Special Relativity

speed of light	$v = \lambda f$
constant value for the speed of light	$c = 3.00 \times 10^8\,\text{m/s}$

10.2 Consequences of Special Relativity

elapsed time	$\Delta t = \gamma \Delta t_0$
relativistic factor	$\gamma = \frac{1}{\sqrt{1-\frac{\mathbf{u}^2}{c^2}}}$
length contraction	$L = \frac{L_0}{\gamma}$
relativistic momentum	$\mathbf{p} = \gamma m\mathbf{u}$