1

Premises (KB):

P

$V \vee T$

$\neg P \vee U$

$R \vee \neg Q$

$V \Rightarrow W \equiv \neg V \vee W$

$P \Rightarrow Q \equiv \neg P \vee Q$

$S \Rightarrow (U \vee T) \equiv \neg S \vee U \vee T$

$(P \wedge R) \Rightarrow S \equiv \neg P \vee \neg R \vee S$

Conclusion: S

Proposition: $KB \vDash S$

We show that $KB \wedge \neg S$ is unsatisfiable

Resolving:

$\neg P \vee \neg R \vee S , \neg S \Rightarrow \neg P \vee \neg R$

$\neg P \vee \neg R , P \Rightarrow \neg R$

$R \vee \neg Q , \neg P \vee Q \Rightarrow R \vee \neg P$

$\neg R , R \vee \neg P \Rightarrow \neg P$

$\neg P, P \Rightarrow$

$KB \wedge \neg S$ is unsatisfiable therefore $KB \vDash S$ is a tautology

2

Premises:

$P \Rightarrow R$

$R \Rightarrow S$

$T \lor \neg S$

$\neg T \lor U$

$\neg U$

Conclusion:

$\neg P$

Proof:

step		reason
1	$\neg U$	premise
2	$\neg T \lor U$	premise
3	$\neg T$	Disjunctive syllogism 1 and 2
4	$T \lor \neg S$	premise
5	$\neg S$	Disjunctive syllogism 3 and 4
6	$R \Rightarrow S$	premise
7	$\neg R$	Modus tollens 5 and 6
8	$P \Rightarrow R$	premise
9	$\neg P$	Modus tollens 7 and 8

3

$\neg K(X) \lor M(X) \equiv \boldsymbol{K(X) \Rightarrow M(X)}$

$\neg \boldsymbol{K(X)} \Rightarrow \neg \boldsymbol{Q(X)}$

$\boldsymbol{L(X)} \land \neg \boldsymbol{Q(X)} \Rightarrow \boldsymbol{N(X)}$

$\neg(N(X) \land M(Y) \land \neg P(X,Y)) \equiv \boldsymbol{N(X)} \land \boldsymbol{M(Y)} \Rightarrow \boldsymbol{P(X,Y)}$

$\neg \boldsymbol{Q(Ali)}$

$\boldsymbol{K(Amir)}$

$\boldsymbol{L(Ali)}$

$\neg Q(Ali) \land L(Ali) \Rightarrow \boldsymbol{N(Ali)}$

$K(Amir) \Rightarrow \boldsymbol{M(Amir)}$

$N(Ali) \land M(Amir) \Rightarrow \boldsymbol{P(Ali, Amir)}$

Conclusion: $\boldsymbol{K(Amir)} \Rightarrow \boldsymbol{P(Ali, Amir)}$

4

inferred = { } ۴

count = { }

queue = { Shiny(Sat), Healthy(Amin), Swim(Ali, Fri) }

P ← PoP(queue) : Shiny(Sat)

inferred ← ∪ Shiny(Sat)

count [C_1(Sat)] = 0 ⇒ queue ← ∪ Nice weather (Sat)

P ← PoP(queue) : Healthy (Amin)

inferred ← ∪ Healthy (Amin)

count [C_2(Amin, x)] = 1

P ← PoP(queue) : Swim (Ali, Fri)

inferred ← ∪ Swim (Ali, Fri)

count [C_3(Ali, Fri)] = 0 ⇒ queue ← Healthy (Ali)

P ← PoP(queue) : Nice weather (Sat)

inferred ← U Nice weather (Sat)

Count [C_2 (y, Sat)] = 1

Count [C_2 (Amin, Sat)] = 0 ⇒ queue ← Swim (Amin, Sat)

P ← PoP(queue) : Healthy (Ali) ✓

inferred ← U Healthy (Ali)

Count [C_r (Ali, x)] = 1

Count [C_r (Ali, Sat)] = 0 ⇒ queue ← Swim (Ali, Sat)

P ← PoP (queue) : Swim (Amin, Sat)

inferred ← U Swim (Amin, Sat)

P ← PoP (queue) : Swim (Ali, Sat)

inferred ← U Swim (Ali, Sat)

5

$\neg C \;\Rightarrow\; \neg E \equiv C \;\vee\; \neg E$

$D \;\Rightarrow\; \neg A \equiv \neg D \;\vee\; \neg A$

$B \;\Leftarrow\!\!\Rightarrow\; C \equiv (B \;\Rightarrow\; C) \wedge (C \;\Rightarrow\; B) \equiv (\neg B \;\vee\; C) \wedge (\neg C \;\vee\; B)$

$A \;\oplus\; B \equiv (A \vee B) \wedge (\neg A \vee \neg B)$

$C \;\Rightarrow\; \neg D \equiv \neg C \;\vee\; \neg D$

$D \;\vee\; \mathrm{E}$

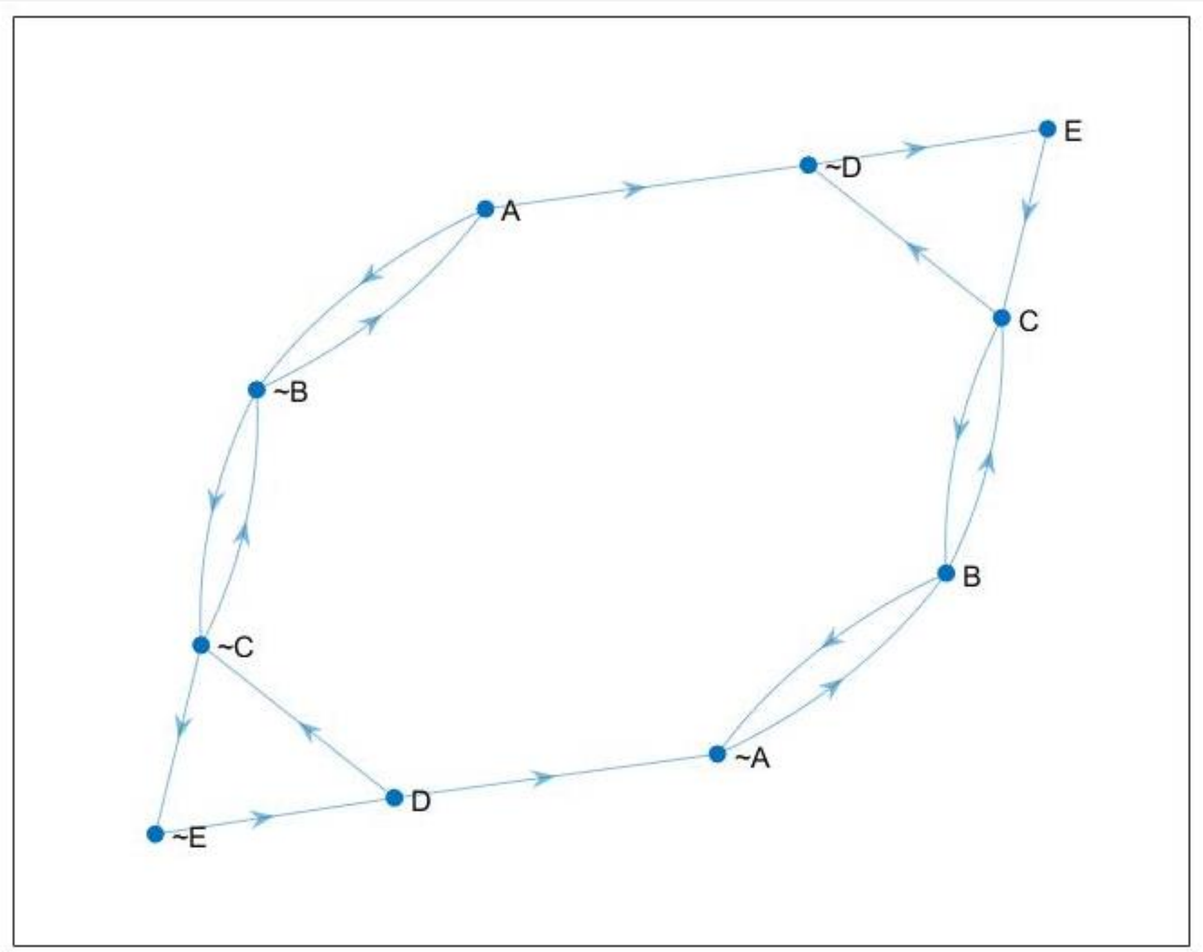
5.1

2CNF:

$(C \vee \neg E) \wedge (\neg D \vee \neg A) \wedge (\neg B \vee C) \wedge (\neg C \vee B) \wedge$

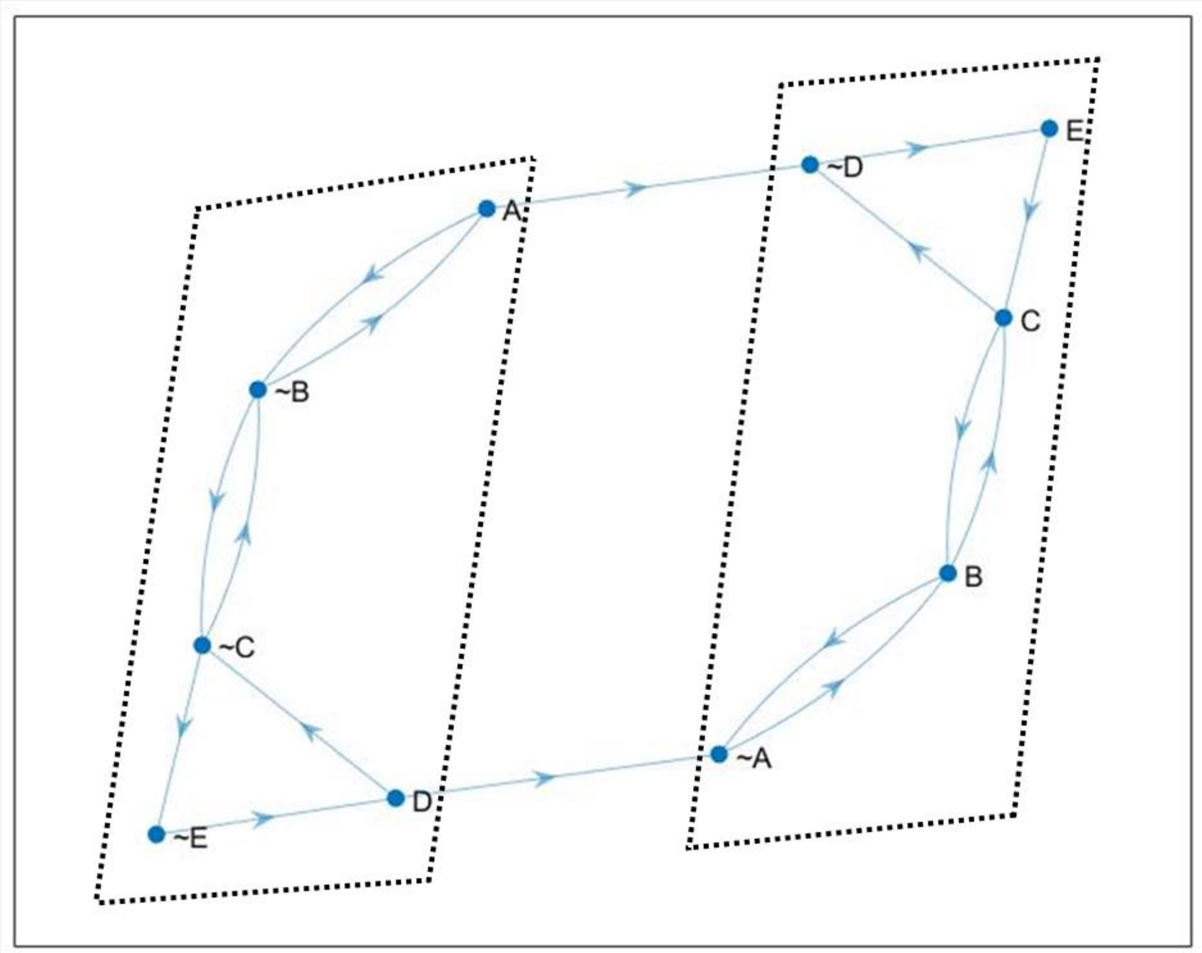
$(A \vee B) \wedge (\neg A \vee \neg B) \wedge (\neg C \vee \neg D) \wedge (D \vee \mathrm{E})$

5.2



5.3

As you can see in the figure below, no strong component contains both x_i and $\neg x_i$



5.4

1	$(C \vee \neg E)$	P
2	$(\neg D \vee \neg A)$	P
3	$(\neg B \vee C)$	P
4	$(\neg C \vee B)$	P
5	$(A \vee B)$	P
6	$(\neg A \vee \neg B)$	P
7	$(\neg C \vee \neg D)$	P
8	$(D \vee E)$	P
9	$(\neg E \vee B)$	1,4
10	$(\neg D \vee \neg E)$	1,7
11	$(C \vee D)$	1,8
12	$(\neg D \vee B)$	2,5
13	$(E \vee \neg A)$	2,8
14	$(C \vee \neg A)$	2,11
15	$(A \vee C)$	3,5
16	$(\neg B \vee \neg D)$	3,7
17	$(\neg D \vee C)$	3,12
18	$(\neg C \vee \neg A)$	4,6
19	$(D \vee B)$	4,11
20	$(\neg A \vee B)$	4,14
21	$(E \vee B)$	5,13
22	$(C \vee B)$	5,14
23	$(A \vee \neg D)$	5,16
24	B	5,20
25	$(\neg A \vee \neg E)$	6,9
26	$(\neg A \vee B)$	6,19
27	$\neg A$	6,20
28	$(\neg C \vee E)$	7,8
29	$\neg D$	7,17
30	$(\neg B \vee E)$	8,16
31	$(A \vee E)$	8,23
32	$(D \vee \neg A)$	8,25
33	E	8,29
34	$(\neg D \vee E)$	2,31
35	C	15,27

6

$Smoke$	$Smoke \Rightarrow Smoke$
F	T
T	T

Tautology and satisfiable

$Smoke$	$Fire$	$Smoke \Rightarrow Fire$	$\neg Smoke \Rightarrow \neg Fire$	$(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	T
T	T	T	T	T

Non-Tautology but satisfiable

$Smoke$	$Fire$	$\neg Fire$	$Fire \vee \neg Fire$	$Smoke \vee Fire \vee \neg Fire$
F	F	T	T	T
F	T	F	T	T
T	F	T	T	T
T	T	F	T	T

Tautology and satisfiable

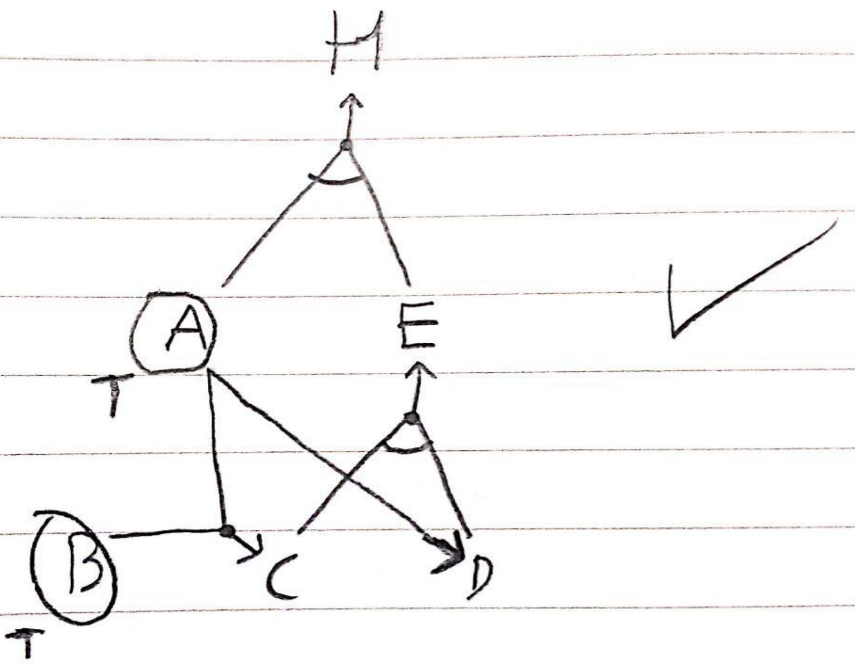
$Fire$	$Smoke$	$Heat$	$Smoke \Rightarrow Fire$	$Smoke \wedge Heat$	$(Smoke \wedge Heat) \Rightarrow Fire$	$(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \wedge Heat) \Rightarrow Fire)$
F	F	F	T	F	T	T
F	F	T	T	F	T	T
F	T	F	F	F	T	T
F	T	T	F	T	F	T
T	F	F	T	F	T	T
T	F	T	T	F	T	T
T	T	F	T	F	T	T
T	T	T	T	T	T	T

Tautology and satisfiable

Big	$Dumb$	$Big \vee Dumb$	$Big \Rightarrow Dumb$	$Big \vee Dumb \vee (Big \Rightarrow Dumb)$
F	F	F	T	T
F	T	T	T	T
T	F	T	F	T
T	T	T	T	T

Tautology and satisfiable

7



$$\frac{A \land B \land F}{\therefore\ A \land B} \qquad \frac{A \land B}{\therefore\ A} \qquad \frac{A \land B}{\therefore\ B}$$