به‌نام آفریننده بیت‌ها

دانشکده‌ی برق و کامپیوتر

دانشگاه صنعتی اصفهان

نیم‌سال دوم ۱۴۰۱ - ۱۴۰۲

سیگنال‌ها و سیستم‌ها

تمرین پنجم

نام:

دانیال خراسانی‌زاده

شماره دانشجویی:

۹۹۲۲۳۹۳

استاد درس:

دکتر نقش

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1.

$$\mathfrak{Ev}(x(t)) \xleftrightarrow{\mathfrak{FT}} \mathfrak{Re}(X(j\omega))$$

2.

$$\mathfrak{Ev}(x(t)) = \frac{x(t)+x(-t)}{2} \xrightarrow{x(-t)=0, t\geq 0} x(t) = 2\mathfrak{Ev}(x(t))u(t)$$

3.

$$\mathfrak{FT}^{-1}\{\mathfrak{Re}(X(j\omega))\} = \mathfrak{Ev}(x(t)) = |t|e^{-|t|} \xrightarrow{2} x(t) = 2|t|e^{-|t|}u(t) \xrightarrow{t\geq 0} x(t) = 2te^{-t}u(t)$$

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$$Y(j\omega) = \mathfrak{FT}\{y(t)\} = \mathfrak{FT}\{x(t)*h(t)\} = \mathfrak{FT}\{x(t)\}\mathfrak{FT}\{h(t)\} = X(j\omega)H(j\omega)$$

$$G(j\omega) = \mathfrak{FT}\{g(t)\} = \mathfrak{FT}\{x(3t)*h(3t)\} = \mathfrak{FT}\{x(3t)\}\mathfrak{FT}\{h(3t)\} = \frac{X(\frac{j\omega}{3})}{3}\frac{H(\frac{j\omega}{3})}{3} = \frac{X(\frac{j\omega}{3})H(\frac{j\omega}{3})}{9}$$

$$G(j\omega) = \frac{Y(\frac{j\omega}{3})}{9} \xleftrightarrow{\mathfrak{FT}} g(t) = \frac{y(3t)}{3}$$

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$$e^{-3|t|}sin(2t) = e^{-3t}sin(2t)u(t) + e^{3t}sin(2t)u(-t) = x_1(t) + x_2(t) = x_1(t) - x_1(-t)$$

$$X_1(j\omega) = \mathfrak{FT}\{e^{-3t}sin(2t)u(t)\} = \frac{1}{2\pi}\mathfrak{FT}\{e^{-3t}u(t)\} * \mathfrak{FT}\{sin(2t)\} = \frac{1}{2\pi}(\frac{1}{(3+j\omega)} * (\frac{\pi}{j}[\delta(\omega-2) - \delta(\omega+2)]))$$

$$= \frac{1}{2j}(\frac{1}{(3+j\omega)} * [\delta(\omega-2) - \delta(\omega+2)]) = \frac{1}{2j}(\frac{1}{(3+j(\omega-2))} - \frac{1}{(3+j(\omega+2))})$$

$$X(j\omega) = X_1(j\omega) + X_2(j\omega) = X_1(j\omega) - X_1(-j\omega)$$

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\begin{aligned}
X(j\omega) &= \int_{-\infty}^{\infty} \sum_{k=0}^{\infty} a^k \delta(t-kT) e^{-j\omega t}\, dt \\
&= \sum_{k=0}^{\infty} a^k \int_{-\infty}^{\infty} \delta(t-kT) e^{-j\omega t}\, dt \\
&= \sum_{k=0}^{\infty} a^k e^{-j\omega kT} = \sum_{k=0}^{\infty} (ae^{-j\omega T})^k = \frac{1}{1-ae^{-j\omega T}}
\end{aligned}
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$$X(j\omega) = \mathfrak{FT}\{\frac{sin(\pi t)}{\pi t}\frac{sin(2\pi(t-1))}{\pi(t-1)}\} = \frac{1}{2\pi}(\mathfrak{FT}\{\frac{sin(\pi t)}{\pi t}\} * \mathfrak{FT}\{\frac{sin(2\pi(t-1))}{\pi(t-1)}\})$$

$$\mathfrak{FT}\{\frac{sin(\pi t)}{\pi t}\} = \begin{cases} 1 & |\omega| < \pi \\ 0 & |\omega| > \pi \end{cases}$$

$$\mathfrak{FT}\{\frac{sin(2\pi(t-1))}{\pi(t-1)}\} = e^{j\omega}\mathfrak{FT}\{\frac{sin(2\pi t)}{\pi t}\}$$

$$= \begin{cases} e^{-j\omega} & |\omega| < 2\pi \\ 0 & |\omega| > 2\pi \end{cases}$$

$$X(j\omega) = \begin{cases} e^{-j\omega} & |\omega| < 2\pi \\ \frac{(3\pi+\omega)e^{-j\omega}}{2\pi} & -3\pi < \omega < -\pi \\ \frac{(3\pi-\omega)e^{-j\omega}}{2\pi} & \pi < \omega < 3\pi \\ 0 & otherwise \end{cases}$$

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$$G(j\omega) = \mathfrak{FT}\{g(t)\} = \mathfrak{FT}\{x(t)cos(t)\} = \frac{1}{2\pi}(\mathfrak{FT}\{x(t)\} * \mathfrak{FT}\{cos(t)\}) = \frac{1}{2\pi} * (X(j\omega) * \pi[\delta(\omega-1)+\delta(\omega+1)])$$

$$= \frac{X(j(\omega-1)) + X(j(\omega+1))}{2} = \begin{cases} 1 & |\omega| \leq 2 \\ 0 & otherwise \end{cases}$$

$$\Rightarrow X(j\omega) = \begin{cases} 2 & |\omega| \leq 1 \\ 0 & otherwise \end{cases}$$

$$x(t) = \frac{2sin(t)}{\pi t}$$

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$$G(j\omega) = \mathfrak{FT}\{g(t)\} = \mathfrak{FT}\{x(t)cos(\frac{2}{3}t)\} = \frac{1}{2\pi}(\mathfrak{FT}\{x(t)\} * \mathfrak{FT}\{cos(\frac{2}{3}t)\}) = \frac{1}{2\pi} * (X(j\omega) * \pi[\delta(\omega - \frac{2}{3}) + \delta(\omega + \frac{2}{3})])$$
$$= \frac{X(j(\omega - \frac{2}{3})) + X(j(\omega + \frac{2}{3}))}{2} = \begin{cases} 1 & |\omega| \leq 2 \\ 0 & otherwise \end{cases}$$

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$$\mathfrak{FT}^{-1}\{A(j\omega)\} = e^{2j\pi t}\mathfrak{FT}^{-1}\{\frac{2sin(3\omega)}{\omega}\} = \begin{cases} e^{2j\pi t} & |t| < 3 \\ 0 & otherwise \end{cases}$$

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$$\mathfrak{FT}^{-1}\{B(j\omega)\} = \mathfrak{FT}^{-1}\{cos(4\omega + \frac{\pi}{3})\}$$
$$= \frac{1}{2}[\mathfrak{FT}^{-1}\{e^{j(4\omega + \frac{\pi}{3})}\} + \mathfrak{FT}^{-1}\{e^{-j(4\omega + \frac{\pi}{3})}\}]$$
$$= \frac{1}{2}[e^{j\frac{\pi}{3}}\mathfrak{FT}^{-1}\{e^{j4\omega}\} + e^{-j\frac{\pi}{3}}\mathfrak{FT}^{-1}\{e^{-j4\omega}\}]$$
$$= \frac{1}{2}[e^{j\frac{\pi}{3}}\delta(t + 4) + e^{-j\frac{\pi}{3}}\delta(t - 4)]$$

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$$\mathfrak{FT}^{-1}\{C(j\omega)\} = 2\mathfrak{FT}^{-1}\{[\delta(\omega - 1) - \delta(\omega + 1)]\} + 3\mathfrak{FT}^{-1}\{[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]\}$$
$$= \frac{2j}{\pi}sin(t) + \frac{3}{\pi}cos(2\pi t)$$

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$$H(j\omega) = e^{-j\omega}\mathfrak{FT}\{\frac{sin(4t)}{\pi t}\} = \begin{cases} e^{-j\omega} & |\omega| < 4 \\ 0 & otherwise \end{cases}$$

$$X_1(j\omega) = e^{j\frac{\pi}{2}}\pi\delta(\omega - 6) + e^{-j\frac{\pi}{2}}\pi\delta(\omega + 6)$$

$$Y_1(j\omega) = H(j\omega)X_1(j\omega) = 0 \Rightarrow y_1(t) = 0$$

$$X_2(j\omega) = \frac{\pi}{j}\sum_{k=0}^{\infty}(\frac{1}{2})^k(\delta(\omega - 3k) - \delta(\omega + 3k))$$

$$Y_2(j\omega) = H(j\omega)X_2(j\omega) = \frac{\pi}{j}(\frac{1}{2})(\delta(\omega - 3) - \delta(\omega + 3))e^{-j\omega}$$

$$y_2(t) = \frac{1}{2}sin(3t - 1)$$

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$$H_1(j\omega) = \frac{j\omega}{2}\mathfrak{FT}\{\frac{sin(\omega_c t)}{\pi t}\} = \begin{cases} \frac{j\omega}{2} & |\omega| < \omega_c \\ 0 & otherwise \end{cases}$$

$$H_3(j\omega) = \mathfrak{FT}\{\frac{sin(3\omega_c t)}{\pi t}\} = \begin{cases} 1 & |\omega| < 3\omega_c \\ 0 & otherwise \end{cases}$$

$$H_4(j\omega) = \mathfrak{FT}\{u(t)\} = \frac{1}{j\omega} + \pi\delta(\omega)$$

$$h(t) = (h_1(t) + (h_1(t) * h_2(t))) * h_3(t) * h_4(t) \to H(j\omega) = (H_1(j\omega) + H_1(j\omega)H_2(j\omega))H_3(j\omega)H_4(j\omega)$$

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$$(j\omega)^2Y(j\omega) + 6(j\omega)Y(j\omega) + 8Y(j\omega) = 2X(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{(j\omega)^2 + 6(j\omega) + 8} = \frac{1}{(j\omega + 2)} - \frac{1}{(j\omega + 4)}$$

$$h(t) = \mathfrak{FT}^{-1}\{H(j\omega)\} = e^{-2t}u(t) - e^{-4t}u(t)$$

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$$y(t) = \mathfrak{FT}^{-1}\{X(j\omega)H(j\omega)\}$$
$$x(t) = te^{-2t}u(t) \overset{\mathfrak{FT}}{\longleftrightarrow} X(j\omega) = \frac{1}{(2+j\omega)^2}$$
$$Y(j\omega) = X(j\omega)H(j\omega) = \frac{2}{(j\omega)^2 + 6(j\omega) + 8}\frac{1}{(2+j\omega)^2}$$
$$= \frac{1}{4}\frac{1}{2+j\omega} - \frac{1}{2}\frac{1}{(2+j\omega)^2} + \frac{1}{(2+j\omega)^3} - \frac{1}{4}\frac{1}{4+j\omega}$$
$$y(t) = \frac{1}{4}e^{-2t}u(t) - \frac{1}{2}te^{-2t}u(t) + t^2e^{-2t}u(t) - \frac{1}{4}e^{-4t}u(t)$$