Day.... Month.... Year.... ۹۸۲۴۴۷۵۳ Subject. HW2

۱ـ الف)

$$w(t) = \int_{-\infty}^{+\infty} e^{-\tau}u(\tau)\left(\delta(t-\tau) - e^{-2(t-\tau)}u(t-\tau)\right) d\tau$$

$$= \left[\int_{-\infty}^{+\infty} e^{-\tau}u(\tau)\delta(t-\tau)\,d\tau\right] - \left[\int_{-\infty}^{+\infty} e^{-\tau}u(\tau)e^{-2(t-\tau)}u(t-\tau)\right]$$

$$= \int_{0}^{+\infty} e^{-\tau}\delta(t-\tau)\,d\tau - \int_{0}^{t} e^{+\tau}e^{-2t}\,d\tau =$$

$$e^{-t}u(t) - e^{-2t}\int_{0}^{t} e^{\tau}d\tau = e^{-t}u(t) - \left(e^{-t} + e^{-2t}\right)u(t) =$$

$$e^{-2t}u(t)$$

ب-ا

$$z(t) = x(t) * h_2(t) =$$

$$= \int_{-\infty}^{+\infty} e^{-\tau} u(\tau) \left(\delta(t-\tau) - e^{t-\tau} u(\tau - t) \right) d\tau$$

$$= \int_{0}^{+\infty} e^{-\tau} \delta(t-\tau) d\tau - u(t) \int_{t}^{+\infty} e^{t-2\tau} d\tau - u(-t) \int_{0}^{+\infty} e^{t-2\tau} d\tau$$

$$e^{-t} u(t) - \frac{e^{-t} u(t)}{2} - \frac{e^{t} u(-t)}{2} =$$

$$\frac{e^{-t} u(t)}{2} - \frac{e^{t} u(-t)}{2}$$

$h(t) = h_1(t) * h_2(t) =$ (ج-۱

$$\int_{-\infty}^{+\infty} \left(\delta(\tau) - e^{-2\tau} u(\tau)\right)\left(\delta(t-\tau) - e^{t-\tau} u(t-\tau)\, d\tau\right) =$$

$$\int_{-\infty}^{+\infty} \delta(\tau)\delta(t-\tau)\, d\tau - \int_{-\infty}^{+\infty} e^{t-\tau} \delta(\tau)\, u(t-\tau)\, d\tau \dots$$

$$-\int_{-\infty}^{+\infty} e^{-2\tau} u(\tau)\, \delta(t-\tau)\, d\tau + \int_{-\infty}^{+\infty} e^{-2\tau} e^{t-\tau}\, u(\tau)\, u(t-\tau)\, d\tau$$

$$= \delta(t) - e^{-2t} u(t) - e^{t} u(-t) \int_{t}^{+\infty} e^{-\tau} \delta(\tau)\, d\tau \dots$$

$$+ u(t) \int_{t}^{+\infty} e^{t-3\tau}\, d\tau + u(-t) \int_{0}^{+\infty} e^{t-3\tau}\, d\tau =$$

$$\delta(t) - e^{-2t} u(t) - e^{t} u(-t) + \frac{e^{-2t} u(t)}{3} + \frac{e^{t} u(-t)}{3} =$$

$$\delta(t) - \frac{2}{3}\left(e^{-2t} u(t) + e^{t} u(-t)\right)$$

$$y_1(t) = w(t) * h_2(t) = \qquad ()-1$$

$$\int_{-\infty}^{+\infty} \left(e^{-2\tau} u(\tau)\right)\left(\delta(t-\tau) - e^{t-\tau} u(\tau-t)\right) d\tau =$$

$$\int_{-\infty}^{+\infty} e^{-2\tau} u(\tau)\, \delta(t-\tau) - \int_{-\infty}^{+\infty} e^{t-2\tau} u(\tau)\, u(\tau-t)\, d\tau =$$

$$e^{-2t} u(t) - u(t)\int_{t}^{+\infty} e^{t-2\tau} d\tau - u(-t)\int_{0}^{+\infty} e^{t-2\tau} d\tau =$$

$$e^{-2t} u(t) - \frac{e^{-2t} u(t)}{2} - \frac{e^{t} u(-t)}{2} =$$

$$\frac{2e^{-2t} u(t)}{2} - \frac{e^{t} u(-t)}{2}.$$

$$y_2(t) = x(t) * h_1(t) =$$ ۱- ص)

$$\frac{1}{2}\int_{-\infty}^{+\infty}\left(e^{-\tau}u(\tau) - e^{\tau}u(-\tau)\right)\left(\delta(t-\tau) - e^{-2(t-\tau)}u(t-\tau)\right)d\tau =$$

$$\frac{e^{-t}u(t)}{2} - \frac{e^{t}u(-t)}{2} - \frac{1}{2}\int_{-\infty}^{+\infty} e^{-2t+\tau}u(\tau)u(t-\tau)\,d\tau \ldots$$

$$+\frac{1}{2}\int_{-\infty}^{+\infty} e^{3\tau-2t}u(-\tau)u(t-\tau)\,d\tau =$$

$$\frac{e^{-t}u(t)}{2} - \frac{e^{t}u(-t)}{2} - \frac{u(t)}{2}\int_{0}^{t} e^{-2t+\tau}d\tau + \frac{u(t)}{2}\int_{-\infty}^{0} e^{3\tau-2t}d\tau +$$

$$\frac{u(-t)}{2}\int_{-\infty}^{t} e^{3\tau-2t}d\tau =$$

$$\cancel{\frac{e^{-t}u(t)}{2}} - \frac{e^{t}u(-t)}{2} + \frac{e^{-2t}u(t)}{2} - \cancel{\frac{e^{-t}u(t)}{2}} + \frac{e^{-2t}u(t)}{6} + \frac{e^{t}u(-t)}{6}$$

$$= \frac{-e^{t}u(-t)}{3} + \frac{2e^{-2t}u(t)}{3}$$

$$y_3(t) = x(t) * h(t) =$$ (9-1

$$\int_{-\infty}^{+\infty} e^{-\tau} u(\tau)\left(\delta(t-\tau) - \frac{2}{3}\left(e^{-2(t-\tau)} u(t-\tau) + e^{t-\tau} u(\tau-t)\right)\right) d\tau$$

$$= e^{-t}u(t) - \frac{2}{3}\int_{-\infty}^{+\infty} e^{\tau-2t} u(\tau)u(t-\tau)\,d\tau \cdots$$

$$-\frac{2}{3}\int_{-\infty}^{+\infty} e^{t-2\tau} u(\tau)u(\tau-t)\,d\tau =$$

$$e^{-t}u(t) - \frac{2}{3}\left(u(t)\int_0^t e^{\tau-2t}\,d\tau + u(t)\int_t^{+\infty} e^{t-2\tau}\,d\tau + u(-t)\int_0^{+\infty} e^{t-2\tau}\,d\tau\right)$$

$$= e^{-t}u(t) - \frac{2}{3}\left(e^{-t}u(t) - e^{-2t}u(t) + \frac{e^{-t}u(t)}{2} + \frac{e^{t}u(-t)}{2}\right) =$$

$$\frac{+2e^{-2t}u(t)}{3} - \frac{e^{t}u(-t)}{3}$$

بله، با توجه به خصوصیات کانولوشن این تساوی برقرار است

۲ـ الف)

$$y[n] = h[n] * x[n] =$$

$$\sum_{k=-\infty}^{+\infty} h[k]\, x[n-k] = \sum_{k=-\infty}^{+\infty} h[k]\, u[n-k-3] =$$

$$\sum_{k=0}^{3} u[n-k-3] + \sum_{k=4}^{6} -2\, u[n-k-3] =$$

$$u[n-3] + u[n-4] + u[n-5] + u[n-6] - 2u[n-7] - 2[n-8] - 2[n-9]$$

$$y[n] = \begin{cases} 0 & n < 3 \\ n-2 & 3 \leq n \leq 6 \\ -2n+16 & 7 \leq n \leq 9 \\ -2 & n \geq 10 \end{cases}$$

$$y[n] = x[n] * h[n] =$$

$$\sum_{k=-\infty}^{+\infty} a^{-k} u[k]\, h[n-k] =$$

$$\sum_{k=-\infty}^{+\infty} a^{-k} u[k] \left(u[n-k] - 2u[n-k-2] + 2u[n-k-7] \right) =$$

$$\sum_{k=0}^{n} a^{-k} - 2\sum_{k=0}^{n-2} a^{-k} + 2\sum_{k=0}^{n-7} a^{-k} =$$

$$\left(\frac{a-a^{-n}}{a-1}\right) u[n] - 2\left(\frac{a-a^{2-n}}{a-1}\right) u[n-2] + 2\left(\frac{a-a^{7-n}}{k=0}\right) a[n-7]$$

$$y[n] = h[n] * x[n] = \sum_{-\infty}^{+\infty} h[k]\, x[n-k] =$$ —۵

$$\sum_{-\infty}^{+\infty} \left[\frac{1}{۲}^{k} \cos \frac{\pi k}{۲} \right] u[k]\, A \sin \frac{\pi (n-k)}{۲} \Rightarrow$$

$$y[1] = A \sum_{k=0}^{\infty} \frac{1}{۲}^{k} \cos \frac{\pi k}{۲} \quad \sin \frac{\pi - \pi k}{۲} =$$

$$A \sum_{k=0}^{\infty} \cos\left(\frac{k\pi}{۲}\right)^{۲} \frac{1}{۲}^{k} = A \sum_{k=0}^{\infty} \frac{1}{۲}^{۲k} = A \times \frac{1}{1-\frac{1}{۴}} \Rightarrow$$

$$\frac{۴}{۳} A = ۴ \Rightarrow \boxed{A = ۳}$$