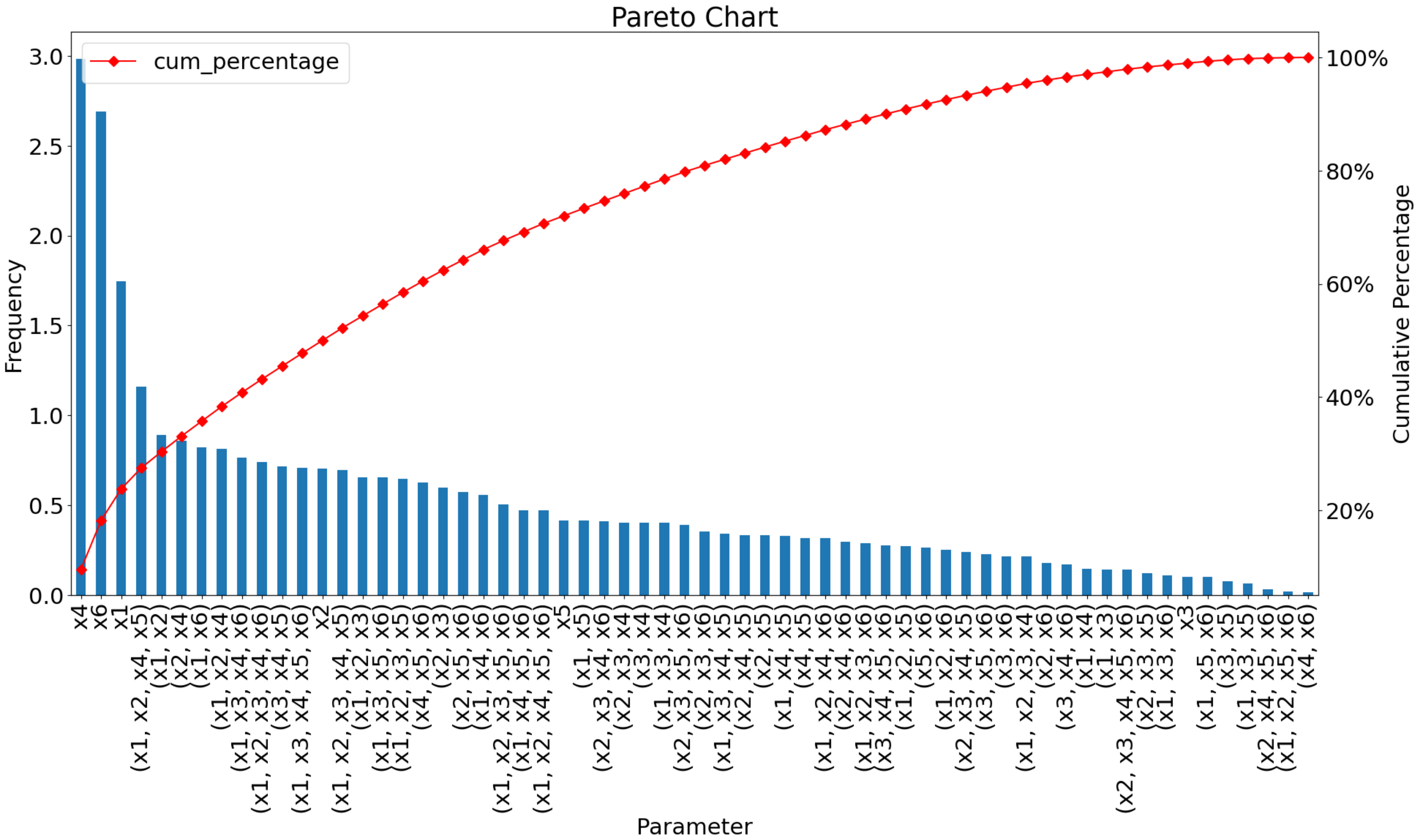
Full, reduced and fractional factorial 2^6 design

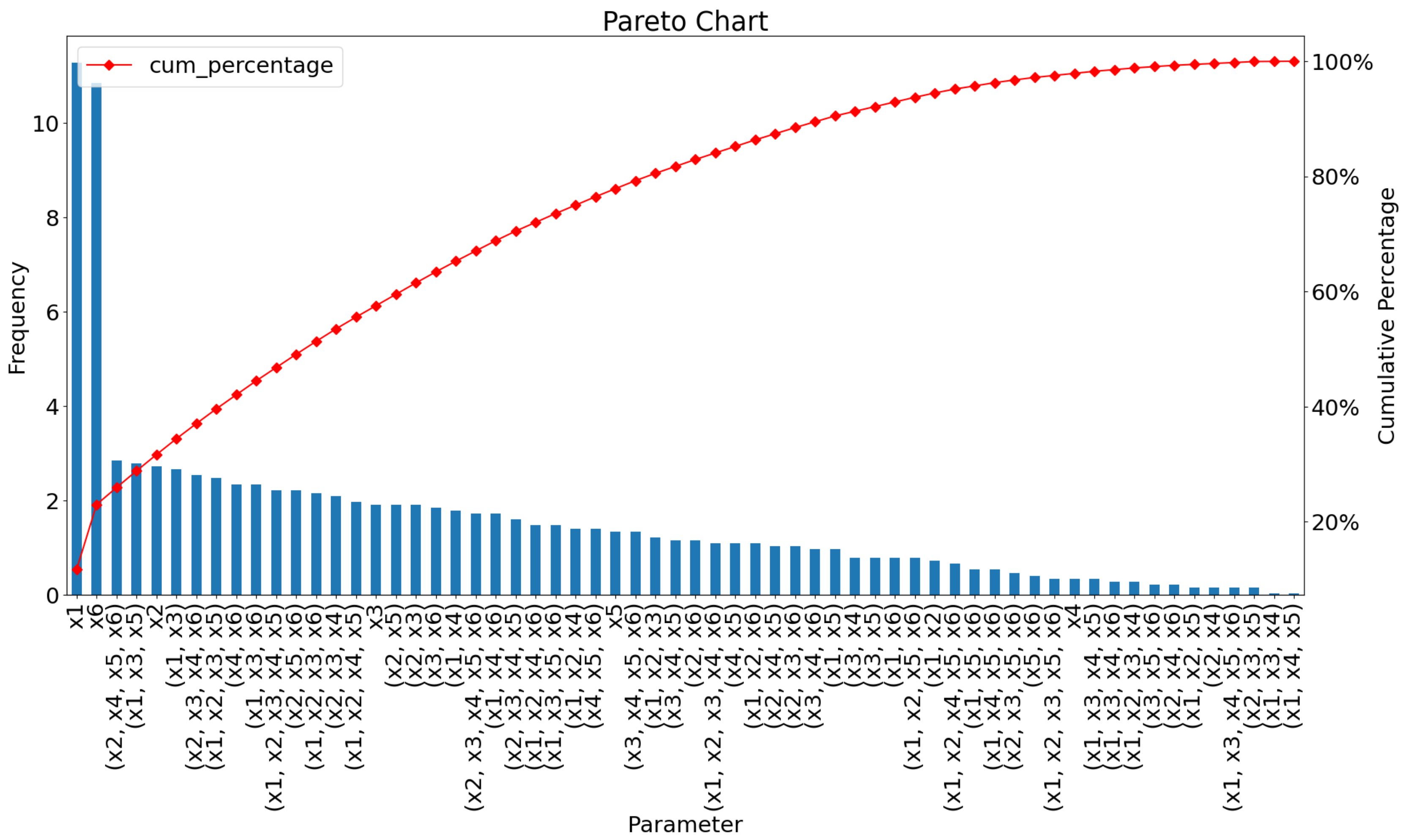
Building the full factorial design

- See jupyter notebook 2-6factor_data_from_book.ipynb

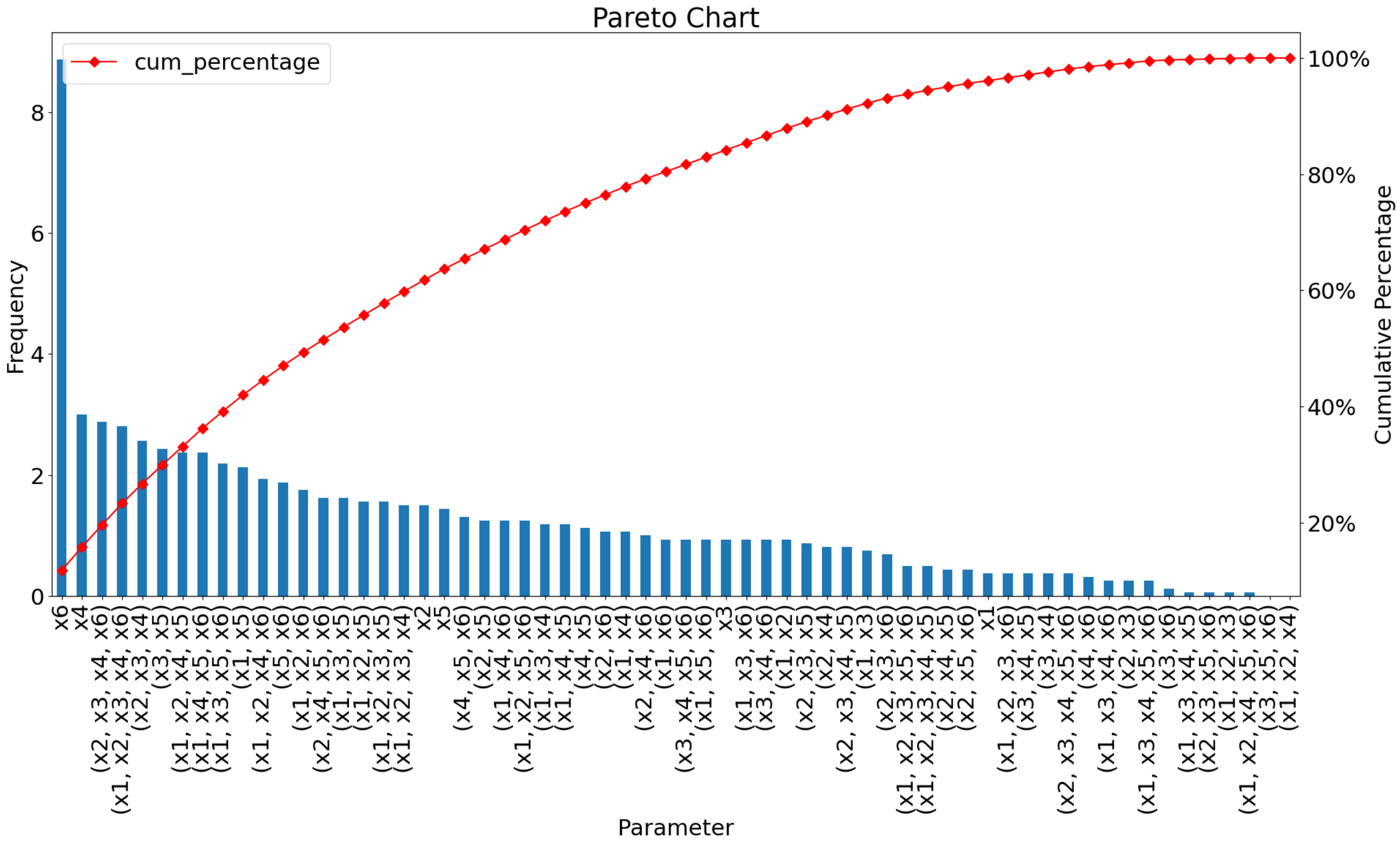
Pareto chart for y1



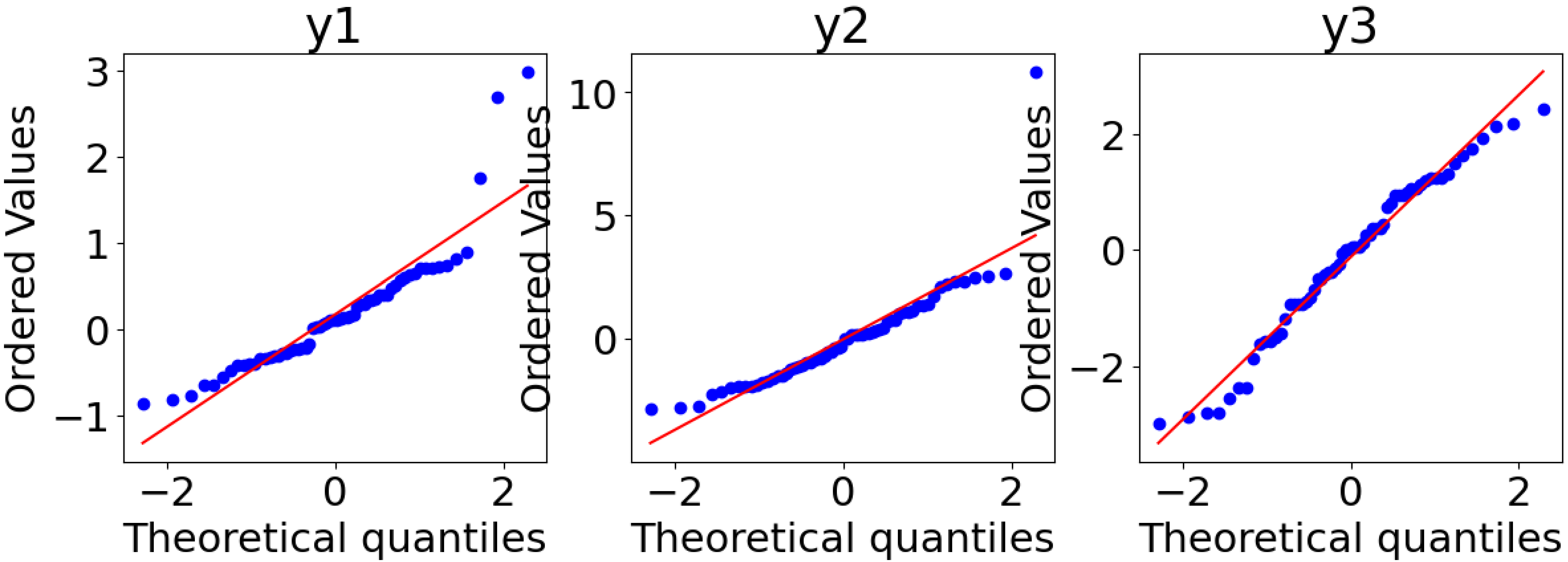
Pareto chart for y1



Pareto chart for y1



Q-Q plots


y1
y2
y3
Ordered Values
Ordered Values
Ordered Values
Theoretical quantiles
Theoretical quantiles
Theoretical quantiles

Reducing the model

- Looks like x1, x4, x6 is needed to make a good model…
- This means that we can reduce the model to only include these variables and use the rest to characterize the error introduced by our approximate model.

- This is done in the jupyterbook

2-6factor-more_analysis_from_book.ipynb

Fractional factorial design

- Number of experiments grow fast with increasing number of factors
- As seen in the full factorial design, cooperative effects often small.
- As we just saw, the three coefficient model gave almost the same result as the full model.

- The solution to this is to use fractional factorial design!

Design Matrix

- Interactions are products of each coded value, for example if:

$$x_1 = -1$$
$$x_2 = +1$$
$$x_3 = +1$$

- then two-variable interaction effects can be computed as:

$$x_{12} = -1 \times +1 = -1$$
$$x_{13} = -1 \times +1 = -1$$
$$x_{23} = +1 \times +1 = +1$$

- And three variable effects:

$$x_{123} = -1 \times -1 \times +1 = +1$$

Thus, any interaction can be added to the design matrix….

Half factorial

- Assume that we can pick any interaction and assume that it is unimportant…for example $x_1x_2x_3x_4$
- This means that for any two groups of experiments where one has

$$x_1x_2x_3x_4 = +1$$

And the other has

$$x_1x_2x_3x_4 = -1$$

One of those two groups can be thrown out…

- The first time we throw one interaction out, the number of experiments is cut in half!!! … 1/2p fractional → $2^{(n-p)}$ experiments

What is the cost?

- Our assumption is that changing $x_1x_2x_3x_4$ from high to low has no effect on y
- This modifies the information we get about higher-order interaction effects. For example, if we assumed $x_1x_2x_3x_4$ = +1, this also changes fith and sixth order interactions:

$$(x_1x_2x_3x_4) = (+1)(x_1x_2x_3x_4)\,x_5 = (+1)x_1x_2x_3x_4x_5 = x_5$$

- i.e., the fifth-order interaction effect $x_1x_2x_3x_4x_5$ has been aliased with the main effect x_5
- Since any factor squared is +1, we can also derive other relations

$$(x_1x_2x_3x_4) = (+1)(x_1x_2x_3x_4)\,x_1 = (+1)x_1^2x_2x_3x_4 = x_1$$

I – the generator

- The sequence of variables selected as the interaction effect to be used as the experimental design basis is called the generator

$I = (x_1 x_2 x_3 x_4)$, which is set to -1 or +1

1/4 – fractional design

- Two identities are used I_1 and I_2

Example: $I_1 = (x_1 x_2 x_3 x_4)$ and $I_2 = (x_4 x_5 x_6)$

- What information do we loose?

$$(x_4 x_5 x_6) = (+1)$$
$$(x_4 x_5 x_6) x_1 = (+1)$$
$$x_1 x_2 x_3 x_4 x_5 = x_5$$

- We can use this information to design experiments to cover particular interaction effects we know to be important and ignore other we don't expect to be significant