TUTORIAL-2 → DAA

- Riya Rastogi
- 20022525
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- 33.

1.) void fun(int n)

```
{
  int j=1; i=0;
while(i<n);
   i=i+j;
   J++;
 }.
```

Time complexity → O(sqrt n).

1st time = $i = 1$

2nd time = $i = 3$ $(i = 1+2)$.

3rd time $i = 6$ $(i = 1+2+3)$.

⋮

nth time = $i = \frac{i(i+1)}{2} = x^2 < n$

$x = \text{sqrt}(n)$.

2.) Let $T(0) = 1$.

* fib(n) = fib(n-1) + fib(n-1)

```
fib(n):
    if n<=1
      return 1
   return fib(n-1) + fib(n-2).
```

Time complexity :-

$T(n) = T(n-1) + T(n-2) + c$

$= 2T(n-2) + c.$

$T(n-2) = 2*(T2(n-2-2)+c)+c.$

$= 2*(2T(n-2)+c)+c$

$= 4T(n-2)+3c.$

$T(n-4) = 2*(4T(n-2)+3c)+c$

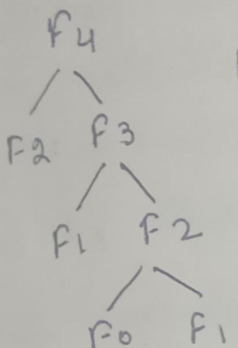
$= 8T(n-3)+7c$

$= 2^k \times T(n-k) + (2^k-1)c$

$n - \cancel{2}k = 0 \Rightarrow n = k \Rightarrow k = n$

$$T(n) = 2^n * T(0) + (2^n - 1)c$$
$$= 2^n * 1 + 2^n c - c$$
$$= 2^n(1 + c) - c$$
$$= 2^n.$$
$$= O(2^n).$$

Space Complexity:- Space is proportional to the maximum depth of the recursion tree.



Hence the space complexity of Fibonacci recursive is $O(N)$.

③ Merge Sort - $n \log n$.

for time complexity:- n^3

We can use three nested loops

```
for (int i = 0; i < n; i++)
{ for (int j = 0; j < n; j++)
  { for (int k = 0; k < n; k++)
    {
       some O(1) expressions
    }
  }
}.
```

→ for time complexity - $\log(\log n)$.

```
for (int i = 2; i < n; i = power(i, j))
}
   // some O(1) expression
}.
```

where K is constant.

for time complexity $n \log n$

```
int fun ( int n)
{
  for(i=1; i<=n; i++)
  {
    for(j=1; j<=n; j+=i)
    { some O(1) expression
    }
  }.
```

Q:- 4.

$T(n) = 2T(n/2) + cn^2$

using master's method

$T(n) = aT(n/b) + fn.$

$a \geq 1, b \geq 1, c = \log_b^a$

$c = \log_2^2 = 1$

$f(n) > n^c$

$T(n) = \Theta(f(n))$.

$\Rightarrow \Theta(n^2)$.

Q:- 5

for $i = 1 \to j = 1, 2, 3, 4 - - - - n$ (run for n times)

for $i = 2 \to j = 1, 3, 5 - - - - -$ (run for $n/2$ times).

for $i = 3 \to j = 1, 4, 7 - - - -$ (run for $n/3$ times).

$T(n) = n + n/2 + n/3 + n/4 + - - - .$

$n(1 + 1/2 + 1/3 + 1/4 + - - -).$

$n \int_1^n 1/x \Rightarrow n \int_1^n dx/x \Rightarrow \log x]_1^n$

$T.C = n \log n$

Q:- 6.

for first iteration $i = 2$

second iteration $i = 2^k$

third iteration $i = (2^k)^k = 2^{k^2}$

nth iteration $i = 2^k$ loop ends at $2^i = n$

apply $\log n = \log 2^{k^i} = k^i = \log n \Rightarrow i = \log_c(\log n)$.

Q:- 7.

99 to 1 in Quick sort

when pivot is either from front or end always

so $T(n) = T(99n/100) + T(n/100) + O(n)$

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$T(n)$

$T\left(\frac{99n}{100}\right)$ $T(n)/_{100}$ $\to n$

$T(1/100) \to$

$T\left(\frac{(99)^2}{100^2} \times n\right)$ $T(99n/100)^2$. $T(99n/100^2)$

$\to n$

$$\frac{n}{(99/100)^k} = 1.$$

$n = (99/100)^k$

$\log n = k \log 99/100$

$K = \log_{\frac{100}{99}} n$.

$\therefore TC = n * \log_{100/99}(n)$.

Ques:- 8

a.) $100 < \log\log(n) < \log^2 n < \log n < \log n!$
$< n < n\log n < n^2 < 2^n < 4^n < 2^{(2^n)} < n!$

b.) $1 < \log(\log n) < \sqrt{\log n} < \log n < \log 2n < 2(\log n) < n <$
$n(\log n) < 2n < 4n < \log(n!) < n^2 < n! < 2^{2^n}$

c.) $96 < \log_8 n < \log_2 n < 5n < n(\log_6 n) < n(\log_2 n) < \log(n!) < 8n^2 <$
$7n^2 < n! < 8^{2n}$.