Epidemiology model (SIR model)

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- ▸ The Epidemiological models are mathematical models used to better understand and predict how diseases spread and affect populations

The SIR Model: A Basic Framework

The model divides the population into three categories:

- ▸ (S): Susceptible People: Individuals capable of becoming infected but not yet being infected.

- ▸ (I): Infected People: These are individuals who have been infected and are capable of infecting susceptible individuals.

- ▸ (R): Recovered People: These are people who have been recovered from the infection.
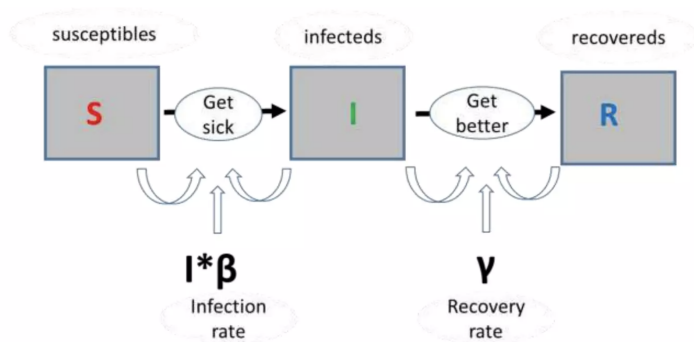
SIR Model

Figure: kermack & mckendrick, 1927

Ordinary differential equations (ODEs) to describe the changes in the number of individuals in each catagory:

$$\frac{dS}{dt} = -\frac{\beta IS}{N}$$
$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I,$$
$$\frac{dR}{dt} = \gamma I$$

S: Suspected group

I: Infected group

R: Recovered group

β: Transmission rate or mixing rate of population

γ: Recovery rate (depends on medical facilities, medicine)

- Closed System: Ignoring birth and deaths
  $S(t) + I(t) + R(t) = N$
- Reinfection is not considered
- All individuals have the same chances of getting infected.
- the population is considered to be well mixed

4th order Runge-Kutta method

$$k_1 = f_s\left(t_i, I_i, S_i\right) = -\frac{\beta S_i I_i}{N}$$

$$l_1 = f_I\left(t_i, I_i, S_i\right) = \frac{\beta S_i I_i}{N} - \gamma I_i$$

$$m_1 = f_R\left(t_i, I_i\right) = \gamma I_i$$

$$k_2 = -\frac{\beta\left(S_i + \frac{hk_1}{2}\right)\left(I_i + \frac{hl_1}{2}\right)}{N}$$

$$l_2 = \frac{\beta\left(S_i + \frac{hk_1}{2}\right)\left(I_i + \frac{hl_1}{2}\right)}{N} - \gamma\left(I_i + \frac{hl_1}{2}\right)$$

$$m_2 = \gamma\left(I_i + \frac{hl_1}{2}\right)$$

4th order Runge-Kutta method

$$k_3 = -\frac{\beta \left(S_i + \frac{hk_2}{2}\right)\left(I_i + \frac{hl_2}{2}\right)}{N}$$

$$l_3 = \frac{\beta \left(S_i + \frac{hk_2}{2}\right)\left(I_i + \frac{hl_2}{2}\right)}{N} - \gamma \left(I_i + \frac{hl_2}{2}\right)$$

$$m_3 = \gamma \left(I_i + \frac{hl_2}{2}\right)$$

$$k_4 = -\frac{\beta \left(S_i + hk_3\right)\left(I_i + hl_3\right)}{N}$$

$$l_4 = \frac{\beta \left(S_i + hk_3\right)\left(I_i + hl_3\right)}{N} - \gamma \left(I_i + hl_3\right)$$

$$m_4 = \gamma \left(I_i + hl_3\right)$$

4th order Runge-Kutta method

▸ Thus, the numerical solution for the SIR Model :

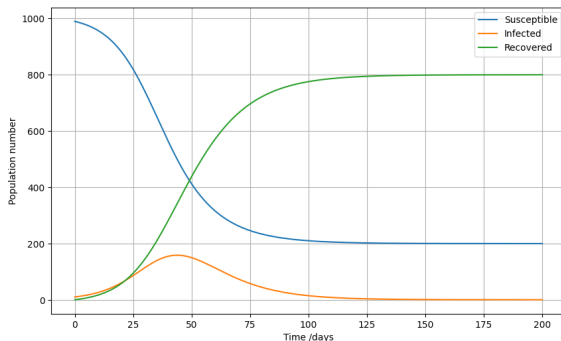$$S_{i+1} = S_i + \frac{1}{6}h\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$I_{i+1} = I_i + \frac{1}{6}h\left(l_1 + 2l_2 + 2l_3 + l_4\right)$$

$$R_{i+1} = R_i + \frac{1}{6}h\left(m_1 + 2m_2 + 2m_3 + m_4\right)$$

where h is stepsizes

Modeling Covid 19 with SIR

▸ for population number N=1000, $I_0 = 10$, $R_0 = 0$ and $\beta = 0.2$, $\gamma = 0.1 days^{-1}$ for the period of 200 days

▸ As we can see, the transformation between the categories is similar to what is seen from the ordinary differential equations.

- The ratio β/γ is often used to calculate the basic reproduction number
- which represents the average number of secondary infections produced by a typical case of an infection in a population where everyone is susceptible.

  If a disease has an $R_0 = 3$ for example, so on average, a person who has this sickness will pass it on to three other people.

$$R_0 = \begin{cases} > 1, & \text{the disease will spread in the population} \\ < 1, & \text{the disease will decay out in the long run.} \end{cases}$$

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▸ Let's consider the case where we have great medical care systems boosting recovery rate of infected individuals γ , let $\gamma = 0.6$ and $\beta = 0.5$, where $R_0 = 0.8$
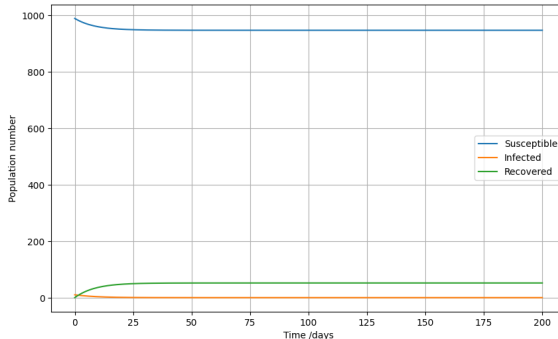
▸ because the recovery rate is dominant over transmission rate, the disease will decay out soon.

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▸ Lets consider a case where there is no serious lockdowns or social distancing rules applied, $\beta = 0.6$ and $\gamma = 0.1$ where $R_0 = 6$
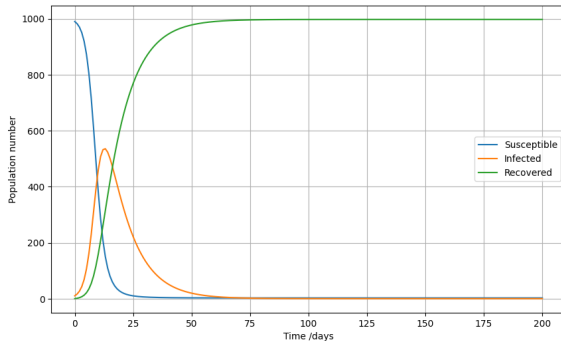
▸ As higher peak for infected individuals can be seen as the contact rate increases.
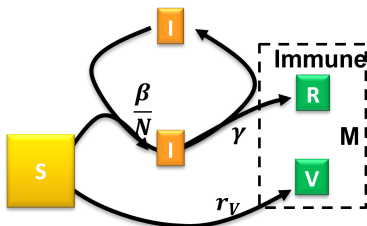
Plotting for different values of β and γ

▸ Plotting for different values of β by fixing $\gamma = 0.1$

SIRV Model
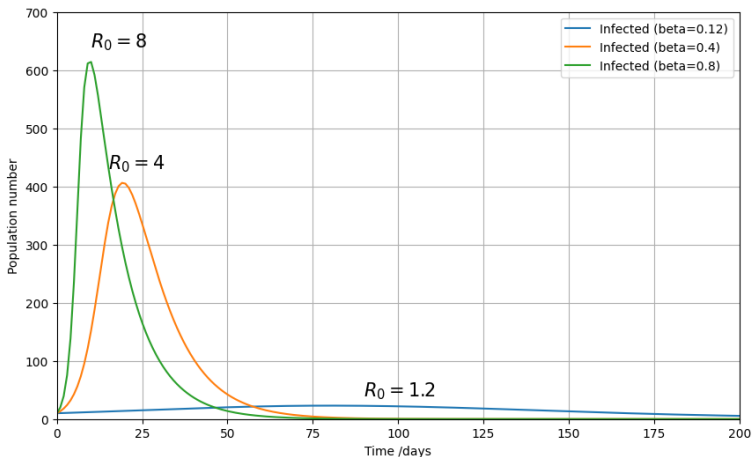
▸ The Susceptible-Infected-Recovered-Vaccinated model is an extended SIR model that accounts for vaccination of the susceptible population.
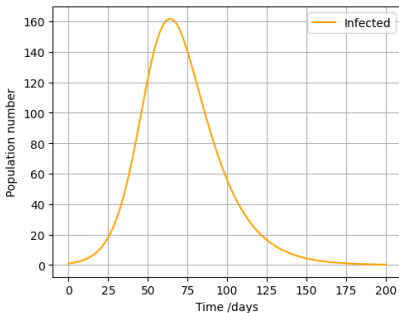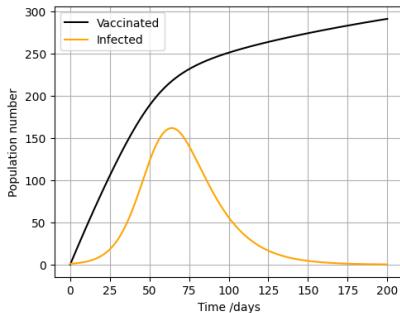
$$\frac{dS}{dt} = -\frac{\beta(t)IS}{N} - v(t)S$$
$$\frac{dI}{dt} = \frac{\beta(t)IS}{N} - \gamma(t)I$$
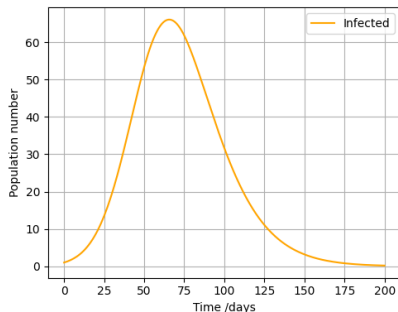$$\frac{dR}{dt} = \gamma(t)I$$
$$\frac{dV}{dt} = v(t)S$$

where v is called vaccination rate, and V is the vaccinated group of population

plotting for SIRV model

▸ In the case of low vaccination rate $\nu = 0.0045$

for a lower vaccination rate, more people get infected, around 164.

plotting for SIRV model

▸ In the case of higher vaccination rate $\nu = 0.01$
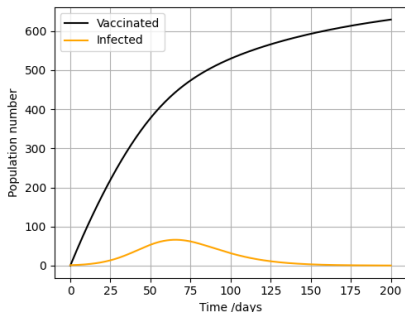
for a higher vaccination rate, number of infected people decreases to around 66.