Homework 2

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1. *Show that the $\leq_p$relation is a transitive relation on languages. That is, show that if L1 $\leq_p$L2 and L2 $\leq_p$ L3, then L1 $\leq_p$L3.*
   Let F1 and F2 be polynomial-reduction algorithm that computes reduction of function f1 and f2.
   f1:{0,1}* -> {0,1}*
   f2:{0,1}* -> {0,1}*
   such that
   $x \in L1 \Leftrightarrow f1(x) \in L2$
   $x \ \in L2 \Leftrightarrow f2(x) \in L3$
   then define composition function f3 of f1 and f2, then L3 is a polynomial-time computable function: {0,1}* -> {0,1}*, and
   $x \in L1 \Leftrightarrow f3(x) \in L3$ holds, therefore $L1 \ \leq_p L3$.
2. To prove that chromatic number of a graph *G* is no less than size of the maximal clique of *G* lets prove it by providing it's contradictory assumption wrong. From the problem that is provided, we know that it's true for Graph *G* with clique size *k*, you must use *k* colors to color all of them. Let's make an assumption that you need less than *k* color and try to figure out if it's possible to do it.
   Using number of colors less than clique size would results in at least two vertices having same color. But the definition of a clique in the problem states that it is a complete subset, which means every vertex is connected to every other vertex. Then in this case these two or more vertices that shares the same colors must be adjacent, which isn't good. So based on that we can state that since the contradictory assumption does not stand, the original statement must be true.
3. *Collaborative Problem: Prove that Efficient Recruiting is NP-Complete*
   Prove that the Efficient Recruiting Problem is NP, ER ∈ NP **:** Suppose that we have a set *u*, that has *n* elements; and *m* subsets of *u*, which must satisfy the condition that union of *m* subset is the same as set of n elements. Now let's check if there is *k* of these *m* subsets whose union equals to *u*. That can be used to check if there is a satisfied counselor for each sport, and time complexity for this is: *O(n) + O(m)*, which is polynomial.

   Prove that the Efficient Recruiting Problem in NP-Complete: Now to prove this problem NP-complete the easiest would be to reduce it from the set cover problem. To construct a graph out of Efficient Recruiting Problem, for each subset create a counselor and assign a sport of expertise to it; going through counselors, and then for each counselor going through all the sports will take *O(mn)* time.
   We want to prove that if we find a set of *k* counselors that covers all sports, we will have a group of sets cover all the capabilities and vice versa.
   Since there is *k* counselors covers all sports and each one is a subset and sport == capabilities, there should be set of *k* subsets covering all capabilities. Conversely, if there is *k* subset covering

all capabilities and each subset == counselor and sport == capabilities, there should be *k* counselors covering all the sports.
From here, because the set cover problem $\leq_p$ Efficient Recruiting Problem, it is NP-Complete.

4. *Collaborative Problem: Prove that DS is NP-Complete*
   Prove DS problem is NP, DS ∈ NP**:** To find such customer/customers, we will have to go through each product and see if there is a product that has been purchased by multiple customers and if so, remove all these customers from the original customer name list. This will result to list of the customers. To do this will take polynomial time complexity *O(mn).*

   Prove DS problem is NP-Complete: From here, we can think of the customer as vertex and products as edges, if two vertex sharing same edges, it means these two customers bought the same product.

   From IS to DS, we need to prove that if we can find a diverse set of size *k* in the above graph (customer = vertex, product = edge), construction takes *O(mn),* we can find an independent set of size *k* in the given graph.

   Our original problem's statement provided that vertex don't share edges. So we need to prove the converse. To prove we can find an independent set of size *k* in the graphs, there is a diverse subset of size *k* in the given array. If there is an IS in the graph of size *k*, it means there is at least *k* vertices with no common edge, meaning there is *k* customers that don't buy the same product, and no common product, meaning it's a diverse subset of size *k*. Thus IS $\leq_p$ DS. Therefore, DS is NP-complete problem.