Algebra y Geometría Analítica Lineal

Guía Resuelta-UTN

SISTEMAS DE ECUACIONES LINEALES

1-

a-

$$\begin{pmatrix}2 & 1 & -4\\4 & 1 & -6\\2 & -2 & 2\end{pmatrix}=\begin{pmatrix}2 & 1 & -4\\0 & -2 & 4\\0 & -6 & 12\end{pmatrix}=\begin{pmatrix}2 & 1 & -4\\0 & -2 & 4\\0 & 0 & 0\end{pmatrix}\ 2\ FIL\ LI \Rightarrow \mathrm{Rg(A)}=2$$

b-

$$\begin{pmatrix}2 & 1 & -2 & -1\\4 & 4 & -3 & 1\\2 & 7 & 1 & 8\end{pmatrix}=\begin{pmatrix}2 & 1 & -2 & -1\\0 & 4 & 2 & 6\\0 & 12 & 6 & 18\end{pmatrix}\ 2\ FIL\ LI \Rightarrow \mathbf{Rg(B)=2}$$

c-

$$\begin{pmatrix}2 & -3 & 4\\-1 & 2 & 1\\0 & 1 & 6\\1 & -2 & 1\end{pmatrix}=\begin{pmatrix}2 & -3 & 4\\0 & 1 & 6\\0 & 1 & 6\\0 & -1 & -2\end{pmatrix}=\ \mathbf{3\ FIL\ LI \Rightarrow Rg(C)=3}$$

2-

a-

$$det\begin{pmatrix}1 & 3\\2 & 5\end{pmatrix}=1*5-2*3=-1\Rightarrow \exists A^{-1}=\frac{Adj(A)}{Det(A)}=\frac{Cof^T(A)}{Det(A)}=\frac{\begin{pmatrix}5 & -3\\-2 & 1\end{pmatrix}}{-1}=\begin{pmatrix}\mathbf{-5} & \mathbf{3}\\\mathbf{2} & \mathbf{-1}\end{pmatrix}$$

b-

$$det\begin{pmatrix}2 & 1 & -4\\4 & 1 & -6\\2 & -2 & 2\end{pmatrix}=2(2-12)-(8+12)-4(-8-2)=0\Rightarrow \nexists \boldsymbol{B^{-1}}$$

c-

$$det\begin{pmatrix}1 & 2 & -1\\1 & 3 & -3\\2 & 2 & 4\end{pmatrix}=18-20+4=2\Rightarrow C^{-1}=\frac{\begin{pmatrix}18 & -10 & -4\\-10 & 6 & 2\\-3 & 2 & 1\end{pmatrix}^T}{2}=\begin{pmatrix}\mathbf{9} & \mathbf{-5} & -\frac{\mathbf{3}}{\mathbf{2}}\\\mathbf{-5} & \mathbf{3} & \mathbf{1}\\\mathbf{-2} & \mathbf{1} & \frac{\mathbf{1}}{\mathbf{2}}\end{pmatrix}$$

3-

a-

$$\begin{pmatrix}1 & 2 & 0\\ 3 & 2h+1 & 1\\ -1 & -5 & h\end{pmatrix} = \begin{pmatrix}1 & 2 & 0\\ 0 & 2h-5 & 1\\ 0 & -3 & h\end{pmatrix} = \begin{pmatrix}1 & 2 & 0\\ 0 & 2h-5 & 1\\ 0 & 0 & \mathbf{2h^2-5h+3=0}\end{pmatrix} \Rightarrow$$

$$\mathbf{h = 1 \vee h = \frac{3}{2}}$$

b-

$$Det(A) \neq 0 \Rightarrow \exists \mathbf{A^{-1}}$$

4 -

$$AX = B_i \Rightarrow X = A^{-1}B_i \Rightarrow Det(A) = 2 \Rightarrow A^{-1} = \frac{\begin{pmatrix}6 & -2\\ -2 & 1\end{pmatrix}}{2} = \begin{pmatrix}\mathbf{3} & \mathbf{-1}\\ \mathbf{-1} & \mathbf{\frac{1}{2}}\end{pmatrix}$$

$X_1 = \begin{pmatrix}3 & -1\\ -1 & \frac{1}{2}\end{pmatrix} * \begin{pmatrix}3\\ 5\end{pmatrix}$ $\mathbf{X_1} = \begin{pmatrix}\mathbf{4}\\ \mathbf{-\frac{1}{2}}\end{pmatrix}$	$X_2 = \begin{pmatrix}3 & -1\\ -1 & \frac{1}{2}\end{pmatrix} * \begin{pmatrix}-1\\ 2\end{pmatrix}$ $\mathbf{X_2} = \begin{pmatrix}\mathbf{-5}\\ \mathbf{2}\end{pmatrix}$	$X_3 = \begin{pmatrix}3 & -1\\ -1 & \frac{1}{2}\end{pmatrix} * \begin{pmatrix}2\\ 0\end{pmatrix}$ $\boldsymbol{X_3} = \begin{pmatrix}\mathbf{6}\\ \mathbf{-2}\end{pmatrix}$

5-

a-

$$\begin{pmatrix}1 & 2 & -3 & | & 9\\ 2 & -1 & 1 & | & 0\\ 4 & -1 & 1 & | & 4\end{pmatrix} = \begin{pmatrix}1 & 2 & -3 & | & 9\\ 0 & -5 & 7 & | & -18\\ 0 & -9 & 13 & | & -32\end{pmatrix}\begin{pmatrix}1 & 2 & -3 & | & 9\\ 0 & -5 & 7 & | & -18\\ 0 & 0 & -2 & | & -2\end{pmatrix} \Rightarrow$$

$$\mathbf{Rg(A) = Rg(A') = 3 = n \ \Rightarrow SCD\ Sol = \{(2,5,1)\}}$$

b-

$$\begin{pmatrix}1 & 2 & 1 & | & 3\\ 2 & 1 & 1 & | & 2\\ -1 & 1 & 0 & | & 3\end{pmatrix} = \begin{pmatrix}1 & 2 & 1 & | & 3\\ 0 & -3 & -1 & | & -4\\ 0 & 3 & 1 & | & 6\end{pmatrix} = \begin{pmatrix}1 & 2 & -3 & | & 9\\ 0 & -3 & -1 & | & -4\\ 0 & 0 & 0 & | & -6\end{pmatrix} \Rightarrow \begin{cases}\mathbf{Rg(A) = 2}\\ \mathbf{Rg(A') = 3}\end{cases}$$

$$\Rightarrow \mathbf{SI}$$

c-

$$\begin{pmatrix}1 & 2 & -1 & | & 3\\ 2 & 3 & 1 & | & 1\end{pmatrix} = \begin{pmatrix}1 & 2 & -1 & | & 3\\ 0 & -1 & 3 & | & -5\end{pmatrix} \Rightarrow \begin{cases}\boldsymbol{Rg(A) = 2}\\ \boldsymbol{Rg(A') = 2}\end{cases} \boldsymbol{2 < n \Rightarrow SCI}$$

$$\begin{cases}-y + 3z = -5 \Rightarrow y = 5 + 3z\\ x + 2y - z = 3\end{cases} \Rightarrow x + 10 + 6z - z = 3$$

$$\mathbf{\Rightarrow Sol = \{x \in \mathbb{R}^3 / x = -7 - 5z\ \wedge y = 5 + 3z\ ; z \in \mathbb{R}\}}$$

d-

$$\begin{pmatrix}-1 & -1 & 2 & 1 & | & 1\\ -2 & -1 & 3 & 2 & | & 3\\ 1 & -1 & 0 & -1 & | & -3\end{pmatrix} = \begin{pmatrix}-1 & -1 & 2 & 1 & | & 1\\ 0 & -1 & 1 & 0 & | & -1\\ 0 & 2 & -2 & 0 & | & 2\end{pmatrix} =$$

$$\begin{pmatrix}-1 & -1 & 2 & 1 & | & 1\\ 0 & -1 & 1 & 0 & | & -1\\ 0 & 0 & 0 & 0 & | & 0\end{pmatrix} \Rightarrow \boldsymbol{Rg(A) = Rg(A') = 2 < n \Rightarrow SCI}$$

$$\begin{cases}-x - y + 2z + w = 1\\ -y + z = -1 \Rightarrow y = z + 1\end{cases} \Rightarrow -x - z - 1 + 2z + w = 1$$

$$\boldsymbol{\Rightarrow Sol = \{x \in \mathbb{R}^3 / y = z + 1\ ; x = z + w - 2\ ; z, w \in \mathbb{R}\}}$$

6-

a-

$$\begin{pmatrix}1 & 2 & -2\\ -2 & 1 & 4\\ -1 & 3 & 2\end{pmatrix} = \begin{pmatrix}1 & 2 & -2\\ 0 & 5 & 0\\ 0 & 5 & 0\end{pmatrix} \Rightarrow \begin{cases}x - 2z = 0\\ 5y = 0\end{cases} \Rightarrow$$

$$\mathbf{Sol = Gen\{(2,0,1)\} \Rightarrow Dim(Sol) = 1}$$

Recta intersección de los Planos A y B.

b-

$$\begin{pmatrix}1 & 2 & -2\\ -2 & 1 & 4\\ -1 & 3 & 2\end{pmatrix}^T = \begin{pmatrix}1 & -2 & -1\\ 2 & 1 & 3\\ -2 & 4 & 2\end{pmatrix} = \begin{pmatrix}1 & -2 & -1\\ 0 & 5 & 5\\ 0 & 0 & 0\end{pmatrix} \Rightarrow$$

$$Base\big(Col(A)\big) = \left\{\begin{pmatrix}1\\ -2\\ -1\end{pmatrix} + \begin{pmatrix}0\\ 2\\ 2\end{pmatrix}; \begin{pmatrix}0\\ 1\\ 1\end{pmatrix}\right\} = \left\{\begin{pmatrix}\mathbf{1}\\ \mathbf{0}\\ \mathbf{1}\end{pmatrix}; \begin{pmatrix}\mathbf{0}\\ \mathbf{1}\\ \mathbf{1}\end{pmatrix}\right\}$$

7-

a-

$$\begin{pmatrix}1 & -k & 0\\ 0 & 1 & k\\ 1 & 2 & -1\end{pmatrix} = \begin{pmatrix}1 & -k & 0\\ 0 & 1 & k\\ 0 & 2+k & -1\end{pmatrix} = \begin{pmatrix}1 & -k & 0\\ 0 & 1 & k\\ 0 & 0 & -1-2k-k^2\end{pmatrix} \Rightarrow (k+1)^2 = 0 \Rightarrow$$

$$\mathbf{k = -1}$$

b-

$$\begin{cases}x = -y\\ y = z\end{cases} \Rightarrow sol = \mathbf{Gen\{(-1,1,1)\} \Rightarrow Dim(Sol) = 1}$$

8-

a-

$$\begin{pmatrix} -2 & 2 & -2 \\ k-1 & 1-k & k^2-1 \\ -1 & 1 & -1 \end{pmatrix} = \underset{k=-1}{\begin{pmatrix} -2 & 2 & -2 \\ 0 & 0 & -2k^2+2k \\ 0 & 0 & 0 \end{pmatrix}} \Rightarrow \mathbf{SCI}\ \forall \mathbf{k} \in \mathbb{R}$$

b-

¡¡¡Ojo!!! Está hablando de la BASE SOLUCION no de la Matriz

Si el rango de la Matriz fuera dos, la DIMENSION SOLUCION sería 1. De manera que nuestra Matriz la necesitamos de RANGO 1 para que la DIMENSION SOLUCION sea 2

$$-2k^2+2k = 2k(1-k) = 0 \Rightarrow \mathbf{k=1} \vee \mathbf{k=0}$$

c-

$$-2k^2+2k = 2k(1-k) \neq 0 \Rightarrow \boldsymbol{k} \in \mathbb{R} - \{\mathbf{0}, \mathbf{1}\}$$

9-

a-

$$\begin{pmatrix} 2 & 1 & 1 & | & 2 \\ 1 & a & 2 & | & -b \\ 1 & -2 & 3 & | & -4 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 & | & 2 \\ 0 & 2a-1 & 3 & | & -2b-2 \\ 0 & -5 & 5 & | & -10 \end{pmatrix} =$$

$$\begin{pmatrix} 2 & 1 & 1 & | & 2 \\ 0 & 2a-1 & 3 & | & -2b-2 \\ 0 & 0 & 10a+10 & | & -20a-10b \end{pmatrix}$$

$$\begin{cases} \mathbf{SCD\ a \neq -1} \\ \mathbf{SI\ a = -1 \wedge b \neq 2} \\ \mathbf{SCI\ a = -1 \wedge b = 2} \end{cases}$$

b-

$$\begin{pmatrix} 2 & 1 & 1 & | & 2 \\ 0 & -3 & 3 & | & -6 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \Rightarrow \begin{cases} S_H = Gen\{(-1,1,1)\} \\ S_P = (-1,3,1) \end{cases}$$

$$\Rightarrow \mathbf{Sol} = \{\mathbf{x} \in \mathbb{R}^3 / \mathbf{x} = \boldsymbol{\lambda}(-\mathbf{1}, \mathbf{1}, \mathbf{1}) + (-\mathbf{1}, \mathbf{3}, \mathbf{1})\ ; \boldsymbol{\lambda} \in \mathbb{R}\}$$

10-

a-

$$\begin{cases} AX_1 = B \\ AX_2 = B \end{cases} \Rightarrow A(X_1 - X_2) = N \Rightarrow Sol_H = \left\{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}\right\} = \left\{\begin{pmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{4} \end{pmatrix}\right\}$$

b-

$$X_3 = \lambda \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} \mathbf{3} \\ \mathbf{4} \\ \mathbf{11} \end{pmatrix} \mathbf{con}\ \boldsymbol{\lambda} = \mathbf{2}$$

11-

a-

$$\begin{pmatrix} 1 & 2 & 1 & | & -1 \\ -1 & -2 & -1 & | & 1 \\ 1 & 2 & 1 & | & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 & | & -1 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \Rightarrow \begin{cases} Sol_H = Gen\{(1,0,-1)(0,1,-2)\} \\ Sol_P = (1,0,-2) \end{cases}$$

$$\mathbf{Sol} = (\mathbf{1}, \mathbf{0}, -\mathbf{2}) + \mathbf{a}(\mathbf{1}, \mathbf{0}, -\mathbf{1}) + \mathbf{b}(\mathbf{0}, \mathbf{1}, -\mathbf{2})$$

b-

$$\begin{cases} (1,0,-1)(x,y,z) = 0 \\ (0,1,-2)(x,y,z) = 0 \end{cases} \Rightarrow \begin{cases} \mathbf{x - z = 0} \\ \mathbf{y - 2z = 0} \end{cases}$$