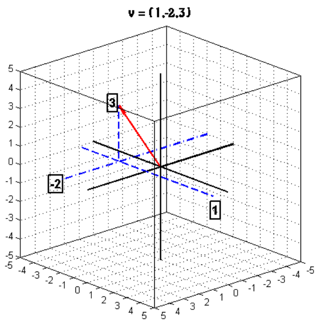
Algebra y Geometría Analítica Lineal

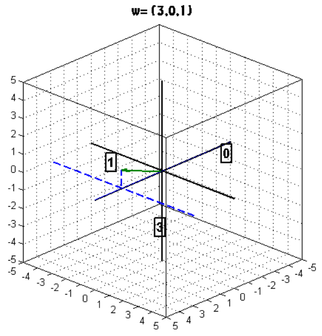
Práctica Resuelta – Guía UTN

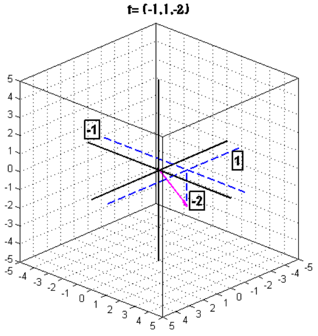
VECTORES EN $\mathbb{R}^3$

1-

-a







-b

$$2\left(\vec{v}-\frac{1}{3}\vec{w}\right)+3\vec{t}\ \rightarrow Vector\ x\ Escalar\ k\vec{u}=(ku_1,ku_2,ku_3)$$

$$2\left(\vec{v}-\left(1,0,\frac{1}{3}\right)\right)+3\vec{t}\rightarrow Resta\ \vec{u}-\vec{v}=(u_1-v_1,u_2-v_2,u_3-v_3)$$

$$2\left(0,-2,\frac{8}{3}\right)+(-3,3,-6)=\left(0,-4,\frac{16}{3}\right)+(-3,3,-6)=$$

$$\left(-3,-1,-\frac{2}{3}\right)$$

-c

$$sea\ \vec{v}=(v_1,v_2,v_3)\Rightarrow ||\vec{v}||=\sqrt{v_1^2+v_2^2+v_3^2}$$

$$\left||\vec{v}+\vec{t}|\right|-||\vec{w}||=||0,-1,1||-\sqrt{10}=$$

$$\sqrt{2}-\sqrt{10}$$

-d

$$\vec{v}=\alpha\vec{w}+\beta\vec{t}\begin{cases}1=3\alpha-\beta\\ -2=\beta\\ 3=\alpha-2\beta\end{cases}\Rightarrow Absurdo$$

2-

a-

$$sea\ A,B\ dos\ Puntos\ en\ el\ Espacio \Rightarrow Dist(A,B) = \left\|\vec{B}-\vec{A}\right\|$$

$$\left\|(2,\sqrt{2},0)-(3,1,-2)\right\| = \left\|(-1,\sqrt{2}-1,2)\right\| = \sqrt{8-2\sqrt{2}}$$

b-

$$sea\ A,B\ dos\ Puntos\ en\ el\ Espacio \Rightarrow Punto_m(A,B) = \frac{\vec{A}+\vec{B}}{2}$$

$$\frac{\vec{A}+\vec{C}}{2} = \frac{\left(7,1-\sqrt{2},-\frac{3}{2}\right)}{2} = \left(\frac{7}{2},\frac{1-\sqrt{2}}{2},-\frac{3}{4}\right)$$

c-

$$sea\ \vec{v}\in R^3 \Rightarrow Versor\ Asociado\ \breve{v} = \frac{\vec{v}}{\|\vec{v}\|}$$

$$\overrightarrow{BC} = \vec{C}-\vec{B} = \left(2,-2\sqrt{2},\frac{1}{2}\right) \Rightarrow \overrightarrow{CB} = \left(-2,2\sqrt{2},-\frac{1}{2}\right) \Rightarrow \breve{CB} = \frac{\left(-2,2\sqrt{2},-\frac{1}{2}\right)}{\sqrt{\frac{49}{4}}} = \left(-\frac{4}{7},\frac{4\sqrt{2}}{7},-\frac{1}{7}\right)$$

$$\overrightarrow{CB} = 7\breve{CB} = \left(-4,4\sqrt{2},-1\right)$$

3-

a-

$$\|t\vec{a}\| = |t|\|\vec{a}\| = \sqrt{5} \Rightarrow |t| = \frac{\sqrt{5}}{\|a\|} = 1$$

b-

$$\left\|\vec{B}-\vec{A}\right\| = \|1-t,1+t,-1\| = \sqrt{1-2t+t^2+t^2+2t+1+1} = 2 \Rightarrow$$

$$2t^2+3 = 4 \Rightarrow t = \pm\sqrt{\frac{1}{2}}$$

c-

$$si\ \vec{x}\ es\ unitario \Rightarrow \|\vec{x}\| = 1 = \|t(2,1,-2)\| = |t|\|2,1,-2\| \Rightarrow |t| = \frac{1}{3}$$

4-

a-

$$sea\ \vec{a},\vec{b}\in R^3 \Rightarrow \vec{a}*\vec{b} = a_1b_1+a_2b_2+a_3b_3$$

$$\begin{cases}\vec{a} = (2,0,-1)\\ \vec{b} = (1,1,-1)\end{cases} \Rightarrow \vec{a}*\vec{b} = 2x1+0*1+1 = 3$$

$$sea\ \vec{a},\vec{b}\in R^3 \Rightarrow \widehat{ab} = ArCos\left(\frac{\vec{a}*\vec{b}}{\|a\|\|b\|}\right)$$

$$\widehat{ab} = ArCos\left(\frac{3}{\sqrt{5x3}}\right) = 0.685\ Rad$$

b-

$$\begin{cases}\vec{a} = (3,-2,1)\\ \vec{b} = (-1,-1,0)\end{cases} \Rightarrow \vec{a}*\vec{b} = -3+2+0 = -1$$

$$\widehat{ab} = ArCos\left(\frac{-1}{\sqrt{14x2}}\right) = 1.761\ Rad$$

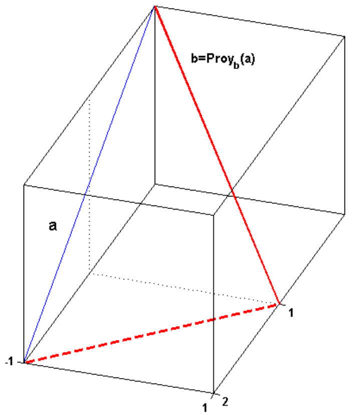
c-

$$\vec{a}*\vec{b} = -2+\frac{2}{3}+\frac{4}{3} = 0$$

$$\widehat{ab} = ArCos(0) = \frac{\pi}{2}Rad$$

5-

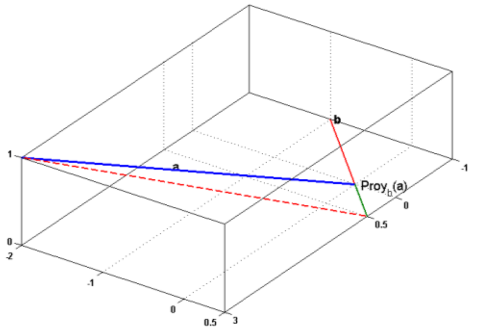
a-



$$sea\ \vec{a},\vec{b}\in R^3 \Rightarrow Proy_{\vec{b}}(\vec{a}) = \frac{\vec{a}*\vec{b}}{\|b\|^2}\vec{b}$$

$$Proy_{\vec{b}}(\vec{a}) = \frac{\vec{a}*\vec{b}}{\left\|\vec{b}\right\|^2}\vec{b} = \frac{3}{3}\vec{b} = \vec{b}$$

b-



$$Proy_{\vec b}(\vec a) = \frac{\vec a * \vec b}{\left\|\vec b\right\|^2}\vec b = -\frac{1}{2}\vec b = \boxed{\left(\frac{1}{2}, \frac{1}{2}, 0\right)}$$

c-

Son perpendiculares: $Proy_{\vec b}(\vec a) = \frac{\vec a * \vec b}{\left\|\vec b\right\|^2}\vec b = \vec 0$

6-

$$\left\|Proy_{\vec y}(\vec x)\right\| = \frac{\vec x * \vec y}{\|\vec y\|} = \frac{2k - 2 + k + 4}{\sqrt{k^2 - 2k + 1 + 1 + 4}} = \frac{3k + 2}{\sqrt{k^2 - 2k + 6}} = 2 \Rightarrow$$

$$\frac{(3k + 2)^2}{k^2 - 2k + 6} = 4 \; Elevamos\ al\ Cuadrado$$

$$9k^2 + 12k + 4 = 4k^2 - 8k + 24$$

$$5k^2 + 20k - 20 = k^2 + 4k - 4$$

$$\boxed{k = -2 \pm 2\sqrt{2}}$$

7-

$$\vec b = \vec v + \vec u \begin{cases} \vec v \parallel \vec a \Rightarrow \vec v = \alpha \vec a \\ \vec u \perp \vec a \end{cases} \Rightarrow$$

$$(4, -6, 5) = \alpha(1, 3, -2) + \vec u \; Multiplicamos\ por\ \vec a$$

$$4 - 18 - 10 = 14\alpha \implies \alpha = -\frac{12}{7} \Rightarrow$$

$$\boxed{\begin{cases} \vec v = \left(-\frac{12}{7}, -\frac{36}{7}, \frac{24}{7}\right) \\ \vec u = \left(\frac{40}{7}, -\frac{6}{7}, \frac{11}{7}\right) \end{cases}}$$

8-

$$Si\ \left\|\vec b\right\| = \sqrt{2}\ y\ Ang(\vec a, \vec b) = \frac{3}{4}\pi \Rightarrow \vec a * \vec b = \|\vec a\| * \left\|\vec b\right\| * \cos\widehat{ab} = -\|\vec a\|$$

$$si\ 4\vec a + 2\vec b \perp \vec a \Rightarrow (4\vec a + 2\vec b)\vec a = 0 \; Distribuimos$$

$$4\|\vec a\|^2 + 2\vec a * \vec b = 4\|\vec a\|^2 - 2\|\vec a\| = 0 \; Factor\ común\ \|\vec a\| \Rightarrow$$

$$4\|a\|\left(\|a\| - \frac{1}{2}\right)$$

$$\boxed{\|\vec a\| = \frac{1}{2}}$$

9-

a-

$$\boxed{Sea\ \vec a, \vec b \in R^3 \Rightarrow \vec a x \vec b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}}$$

$$\vec a x \vec b = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 1 & 1 & -1 \end{vmatrix} = (1.1 - 1.2)i + -(1.-1 - 1.-1)j + (1.-1 - 1.2)k = \boxed{(-1, 0, -1)}$$

b-

$$\vec a x \vec c = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 1 & -1 & 4 \end{vmatrix} = \boxed{(7, -5, -3)}$$

c-

$$\boxed{Sea\ \vec a, \vec b, \vec c \in R^3 \Rightarrow \vec a(\vec b x \vec c) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}}$$

$$\vec a(\vec b x \vec c) = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 4 \end{vmatrix} = 3 - (10) - (-2) = \boxed{-5}$$

d-

$$\vec a x(\vec b x \vec c) = \vec a x \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 1 & -1 & 4 \end{vmatrix} = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 3 & -5 & -2 \end{vmatrix} = \boxed{(-9, -1, -11)}$$

e-

$$\vec a x(\vec a x \vec c) = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 7 & -5 & -3 \end{vmatrix} = \boxed{(-11, -4, -19)}$$

10-

$$si\ \vec{c} \perp \vec{v}\ y\ \vec{c} \perp \vec{u}\ \Rightarrow \vec{v}\,x\vec{u} = \vec{c} = \begin{vmatrix} i & j & k \\ 2 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = (1,-3,5)$$

$$si\ \vec{t} \parallel \vec{c}\ y\ \left\|\vec{t}\right\| = 4 \Rightarrow$$

$$\vec{t} = k\vec{c}\ Aplicamos\ norma$$

$$4 = |k|\sqrt{35} \Rightarrow k = \pm\frac{4}{\sqrt{35}}$$

$$\vec{t} = \pm\frac{4}{\sqrt{35}}(1,-3,5)$$

11-

a-

$$Sea\ \vec{a},\vec{b} \in R^3 \Rightarrow \left\|\vec{a}x\vec{b}\right\| = \|\vec{a}\| * \left\|\vec{b}\right\| * Sin(\widehat{ab}) = Area\ Paralelogramo$$

$$\left\|\vec{a}x\vec{b}\right\| = \sqrt{3}$$

b-

$$Sea\ \vec{a},\vec{b} \in R^3 \Rightarrow \vec{a} * \vec{b} = \|\vec{a}\| * \left\|\vec{b}\right\| * Cos(\widehat{ab})$$

$$\|\vec{a}\| * \left\|\vec{b}\right\| = \frac{\vec{a} * \vec{b}}{Cos(\widehat{ab})} = \frac{\left\|\vec{a}x\vec{b}\right\|}{Sin(\widehat{ab})} \Rightarrow \widehat{ab} = ArcTg\left(\frac{\left\|\vec{a}x\vec{b}\right\|}{\vec{a} * \vec{b}}\right) = -\frac{\pi}{6}$$

12-

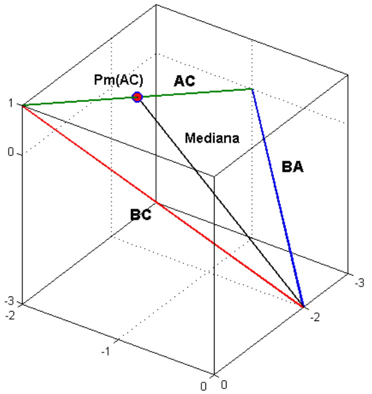
a-

$$Sea \begin{cases} \left\|\overrightarrow{AB}\right\| = \left\|\vec{B} - \vec{A}\right\| = \sqrt{11} \\ \left\|\overrightarrow{BC}\right\| = \left\|C - \vec{B}\right\| = \sqrt{24} \Rightarrow Per = \left\|\overrightarrow{AB}\right\| + \left\|\overrightarrow{BC}\right\| + \left\|\overrightarrow{CA}\right\| = 2\sqrt{11} + \sqrt{24} \\ \left\|\overrightarrow{CA}\right\| = \left\|\vec{A} - \vec{C}\right\| = \sqrt{11} \end{cases}$$

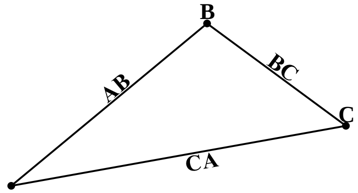
b-

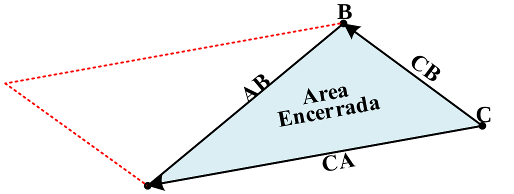
“Nos está pidiendo calcular la distancia del punto medio de AC al vértice opuesto B”

$$Sea \left\{ \left\|\overrightarrow{P_M(\overrightarrow{AC})\vec{B}}\right\| = \left|\left|(-2;0;-3) - \frac{1}{2}\left((-3;-1;0) + (0;-2;1)\right)\right)\right| = \frac{\sqrt{59}}{2} \right.$$



c-





$$Sea \begin{cases} \overrightarrow{CB} = \vec{B} - \vec{C} = (-2,2,-4) \\ \overrightarrow{CA} = \vec{A} - \vec{C} = (-3,1,-1) \end{cases} \Rightarrow$$

$$Area\ Tri = \frac{Area\ Paralelogramo}{2} = \frac{\left\|\overrightarrow{CB}x\overrightarrow{CA}\right\|}{2} = \frac{\left\| \begin{vmatrix} i & j & k \\ -2 & 2 & -4 \\ -3 & 1 & -1 \end{vmatrix} \right\|}{2} = \frac{\|(2,10.4)\|}{2} = \sqrt{30}$$

13-

a-

$$Sea\ \vec{a},\vec{b},\vec{c}\ Antiparelos \in R^3 \Rightarrow \vec{a}\left(\vec{b}x\vec{c}\right) = Volumen\ del\ Paralelipedo$$

$$Area\ Paralelípedo = \left\| \begin{vmatrix} 3 & 1 & 2 \\ 1 & x & 3 \\ 2 & -1 & 0 \end{vmatrix} \right\| = |9 + 6 + 2(-1 - 2x)| = |13 - 4x| = 3$$

$$x = 4\ o\ x = \frac{5}{2}$$

b-

$$Area\ Paralelípedo = 0 = |13 - 4x| \Rightarrow \frac{13}{4} = x$$

14-

$$Si\ \vec{u}\ es\ Coplanar\ con\ \vec{v}\ y\ \vec{w} \Rightarrow \vec{u}(\vec{v}\ x\ \vec{w}) = 0$$

$$\begin{vmatrix} x & y & 0 \\ 3 & 0 & -3 \\ 1 & 2 & -1 \end{vmatrix} = 6x = 0 \Rightarrow x = 0\ ;\ y \in R$$

$$SolVector = \left\{\frac{\vec{u} \in R^3}{\vec{u} = (0,y,0)}\right\} \Rightarrow SolVersor = \{(0,1,0),(0,-1,0)\}$$

15-

a-

$$(\vec{u} + 2\vec{v}) * (\vec{u} - 2\vec{v}) = \|\vec{u}\|^2 - 4\|\vec{v}\|^2 = 0 \Rightarrow \|\vec{u}\| = 2\|\vec{v}\|\ \ V$$

b-

No hay manera de que sean Paralelos y Perpendiculares a la vez *F*

c-

$$Proy_{\vec{v}}(\vec{u} - \vec{v}) = \frac{\vec{v} * (\vec{u} - \vec{v})}{\|\vec{v}\|^2}\vec{v} = \frac{\vec{v} * \vec{u} - \|\vec{v}\|^2}{\|v\|^2}\vec{v} = -\vec{v}\ \Rightarrow V$$

d-

$$si\ \vec{u} \parallel \vec{v} \Rightarrow \vec{u} = k\vec{v} \Rightarrow \begin{cases} 1 = ka \\ 2 = k * 0 \\ -1 = -ka \end{cases} Absurdo \Rightarrow F$$

e-

Falso podrían ser coplanares y ninguno igual a Cero F

f-

$$si\ son\ todos\ Coplanares \begin{cases} \vec{u}\ x\ \vec{v}\ \perp P_\pi \\ \vec{w}\ x\ \vec{t} \perp P_\pi \end{cases} \Rightarrow \vec{u}\ x\ \vec{v} \parallel\ \vec{w}\ x\ \vec{t} \Rightarrow (\vec{u}\ x\ \vec{v}) \parallel (\vec{w}\ x\ \vec{t}) = 0 \Rightarrow V$$

16-

a-

$$Sea\ \vec{n} = (a,b,c) \perp P_\pi\ y\ P_0 \in P_\pi \Rightarrow Ecuaciongeneral: ax + by + cz + d = 0$$

$$2x - 4y + z + d = 0\ como\ (-1,3,-2) \in P_\pi \Rightarrow d = 2 + 12 + 2 = 16 \Rightarrow$$
$$P_\pi:\ 2x - 4y + z + 16 = 0\ EG$$

b-

$$P_\pi \perp \overline{AB} \therefore \vec{n}_\pi = \overrightarrow{AB} \Rightarrow P_\pi: -3x + 2y - z + d = 0\ como\ P_M(\overline{AB}) \in P_\pi \Rightarrow d = \frac{9}{2} + 6 + \frac{3}{2}$$
$$P_\pi: -3x + 2y - z + 12 = 0\ EG$$

c-

$$\begin{cases} \vec{v} = \overrightarrow{AB} \\ \vec{u} = \overrightarrow{AC} \end{cases} \Rightarrow \vec{v} x \vec{u} = \begin{vmatrix} i & j & k \\ -3 & 2 & -2 \\ 0 & 3 & -6 \end{vmatrix} = (-6,-18,-9) = \vec{n} \Rightarrow P_\pi: 2x + 6y + 3z + d = 0$$
$$como\ (-1,3,-2) \in P_\pi \Rightarrow d = 2 - 18 + 6 \Rightarrow$$
$$P_\pi: 2x + 6y + 3z - 10 = 0$$

d-

$$si\ P_\pi \subseteq Eje\ z \Rightarrow \vec{n} = (a,b,0)\ y\ \vec{0} \in P_\pi \Rightarrow P_\pi: ax + by = 0$$
$$(2,-1,3) \in P_\pi \Rightarrow 2a - b = 0 \Rightarrow b = 2a \Rightarrow$$
$$P_\pi: x + 2y = 0$$

e-

$$si\ P_\pi \parallel Eje\ x \Rightarrow P_\pi \parallel (1,0,0)\ y\ P_\pi \parallel \overrightarrow{AB} \Rightarrow \breve{x}\ x\ \overrightarrow{AB} = \vec{n} \Rightarrow$$
$$\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & -4 & 3 \end{vmatrix} = (0,-3,-4) \Longrightarrow P_\pi: -3y - 4z + d = 0$$
$$como\ (1,-1,1) \in P_\pi \Rightarrow d = -3 + 4 = 1 \Rightarrow$$
$$P_\pi: -3y - 4z + 1 = 0$$

f-

$$\vec{v} \parallel P_\pi\ y\ \vec{u} \parallel P_\pi = \begin{vmatrix} i & j & k \\ 3 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix} = (-3,2,-7) = \vec{n} \Rightarrow P_\pi: -3x + 2y - 7z + d = 0$$
$$como\ (1-,3,2) \in P_\pi \Rightarrow d = 3 + 6 + 14 \Rightarrow$$
$$P_\pi: -3x + 2y - 7z + 23 = 0$$

17-

a-

$$Sea\ \vec{n} \perp P_\pi, y\ A = (x_0, y_{0_2}, z_0) Un\ Punto \Rightarrow Dist(P_\pi, A) = \frac{|ax_0 + by_0 + cz_0 + d|}{||\vec{n}||}$$

$$Dist(3x + y + 4z - 1 = 0, (3,-2,-1)) = \frac{|9 - 2 - 4 - 1|}{\sqrt{26}} = \frac{2}{\sqrt{26}}$$

b-

$$Dist(P_\pi, A) = Dist(P_\beta, A)\ y\ \ P_\pi \parallel P_\beta$$
$$\frac{|ax_0 + by_0 + cz_0 + d|}{||\vec{n}||} = \frac{|ax_0 + by_0 + cz_0 + t|}{||\vec{n}||}$$
$$|ax_0 + by_0 + cz_0 + d| = |ax_0 + by_0 + cz_0 + t|$$
$$7 = |3 + t|$$
$$P_\beta: 3x - y + 2z - 10 = 0\ y\ 3x - y + 2z + 4 = 0$$

c-

$$Dist(P_\pi, \vec{0}) = \frac{14}{\sqrt{45 + k^2}} = 2 \Rightarrow 196 = 180 + 4k^2 \Rightarrow$$
$$|k| = 2$$

18-

$$Sea\ \vec{n}_\pi \perp P_\pi, y\ \vec{n}_\beta \perp P_\beta \Rightarrow Ang(P_\pi, P_\beta) = ArcCos\left(\frac{\vec{n}_\pi * \vec{n}_\beta}{||\vec{n}_\pi||\ ||\vec{n}_\beta||}\right)$$

$$Cos\left(\frac{\pi}{3}\right) = \frac{2h + 1}{3 * \sqrt{h^2 + 1}} \Rightarrow 9h^2 + 9 = 16h^2 + 16h + 4 \Rightarrow 7h^2 + 16h - 5 = 0$$
$$\Rightarrow h = -\frac{8}{7} + \frac{3}{7}\sqrt{11}$$

19-

a-

$$Como\ \vec{0} \in Haz \Rightarrow -\alpha + 3\beta = 0 \Rightarrow 3\beta = \alpha \Rightarrow$$
$$3x - 6y + 3z - 3 + x - z + 3 = 0 \Rightarrow Reemplazo$$
$$P_\pi: 2x - 3y + z = 0$$

b-

$$Si\ P_\pi \parallel Eje\ Z\ \Rightarrow \vec{n} = (a,b,0) \Rightarrow \alpha - \beta = 0 \Rightarrow Reemplazo$$
$$P_\pi: x - y + 1 = 0$$

c-

$$Como\ (0,-2,0) \in Haz \Rightarrow 3\alpha + 3\beta = 0 \Rightarrow \beta = -\alpha \Rightarrow Reemplazo$$
$$P_\pi: y - z + 2 = 0$$

d-

$$\begin{cases} \alpha + \beta = 2 \\ -2\alpha = -1 \\ \alpha - \beta = 1 \end{cases} \Rightarrow \begin{cases} \alpha = \frac{3}{2} \\ \beta\ da\ dos\ Valores \end{cases} Absurdo\ \Rightarrow \nexists P_\pi$$

20-

Hayamos π_1:

$$Como\ \pi_1 \subseteq Eje\ y \Rightarrow \pi_1: ax + cz = 0\ y$$
$$como\ \left(1{,}0,\sqrt{2}\right) \in \pi_1 \Rightarrow a + \sqrt{2}c = 0 \Rightarrow$$
$$\pi_1: -\sqrt{2}x + z = 0$$

Hayamos π_2:

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{vmatrix} = (-1,-1{,}1) \Rightarrow \pi_2: -x - y + z + d = 0$$
$$como\ (1{,}0{,}0) \in \pi_2 \Rightarrow$$
$$\pi_2 = -x - y + z + 1 = 0$$

Usamos la Relación de Distancia:

$$Dist(\pi_1, A) = Dist(\pi_2, A)\ con\ A = (0{,}0, z_0)$$
$$\frac{|z_0|}{\sqrt{3}} = \frac{|z_0 + 1|}{\sqrt{3}} \Rightarrow z_0 = -\frac{1}{2}$$

21-

a-

$$Sea\ \vec{u} \parallel R, y\ A\ un\ Punto\ de\ la\ misma \Rightarrow R: \vec{X} = A + \lambda\vec{u}\ ; \lambda \in R$$
$$R\begin{cases} x = 1 - 2\lambda \\ y = -1 + 3\lambda \\ z = 3 + 4\lambda \end{cases}$$

b-

$$X: (1,-3{,}1) + \lambda\big((1,-3{,}1) - (1{,}3,-4)\big)$$
$$R\begin{cases} x = 1 \\ y = -3 - 6\lambda \\ z = 1 + 5\lambda \end{cases}$$

c-

$$X: (3{,}2{,}1) + \lambda(0{,}1{,}0)$$
$$R\begin{cases} x = 3 \\ y = 2 + \lambda \\ z = 1 \end{cases}$$

d-

$$X: (0{,}0{,}0) + \lambda(1{,}1{,}1)$$
$$z = y = x$$

e-

$$X: (0{,}0{,}0) + \lambda\vec{n}$$
$$x = -y = z$$

22-

Planteamos el Sistema

$$\begin{cases} x - y - z = -1 \\ x - 2y - 3z = 2 \end{cases} = \begin{cases} x - y - z = -1 \\ -y - 2z = 3 \end{cases} \Rightarrow Resto \Rightarrow x + z = -4 \Rightarrow$$
$$R\begin{cases} z = -4 - x \\ y = 5 + 2x \\ x = x \end{cases}$$

23-

Para plano Perpendicular a PL_{xy}:

$$Tiene\ la\ forma\ ax + by + d = 0$$
$$Necesitamos\ al\ Menos\ 2\ Ecuaciones \begin{cases} (2,-2{,}3) \in \pi \Rightarrow 2a - 2b + d = 0 \\ (3{,}0{,}2) \in \pi \Rightarrow 3a + d = 0 \end{cases} Restamos \Rightarrow$$
$$a = -2b \Rightarrow -2x + y + d = 0\ Como\ (2,-2{,}3) \in PL_{xy} \Rightarrow d = 6$$
$$PL_{xy}: -2x + y = -6$$

Para plano Perpendicular a PL_{zy}:

$$Tiene\ la\ forma\ by + cz + d = 0$$
$$Necesitamos\ al\ Menos\ 2\ Ecuaciones \begin{cases} (2,-2{,}3) \in \pi \Rightarrow -2b + 3c + d = 0 \\ (3{,}0{,}2) \in R \Rightarrow 2c + d = 0 \end{cases} Restamos \Rightarrow$$
$$c = 2b \Rightarrow y + 2z + d = 0\ Como\ (2,-2{,}3) \in PL_{zy} \Rightarrow d = -4$$
$$PL_{xy}: y + 2z = 4$$

Para plano Perpendicular a PL_{xz}:

$$Tiene\ la\ forma\ ax + cz + d = 0$$
$$Necesitamos\ al\ Menos\ 2\ Ecuaciones \begin{cases} (2,-2{,}3) \in \pi \Rightarrow 2a + 3c + d = 0 \\ (3{,}0{,}2) \in \pi \Rightarrow 3a + 2c + d = 0 \end{cases} Restamos \Rightarrow$$
$$a = c \Rightarrow x + z + d = 0\ Como\ (2,-2{,}3) \in PL_{zy} \Rightarrow d = -5$$
$$PL_{xy}: y + z = 5$$

24-

Hallemos la Recta determinada por:

$$\begin{cases} x - y - z - 8 = 0 \\ 3x - y - 4 = 0 \end{cases} \Rightarrow Restamos \Rightarrow -2x - z - 4 = 0$$
$$Recta\begin{cases} z = -2x - 4 \\ y = 3x - 4 \\ x = x \end{cases} \Rightarrow \begin{cases} (0,-4,-4) \in Recta \\ (1,-1,-6) \in Recta \end{cases}$$

Para formar el Plano nos Alcanza con Tres Puntos no alineados y sabemos que:

$$\begin{cases} A = (0,-4,-4) \in Plano \\ B = (1,-1,-6) \in Plano \\ C = (2{,}1,-3) \in Plano \end{cases} \Rightarrow \vec{n} = \overrightarrow{AB}\ x\ \overrightarrow{AC} \Rightarrow \begin{cases} \overrightarrow{AB} = (13 - 2) \\ \overrightarrow{AC} = (2{,}5{,}1) \end{cases} \Rightarrow \vec{n} =$$

$$\overrightarrow{AB}\,x\,\overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ 2 & 5 & 1 \end{vmatrix} = (13,-5,-1) \Rightarrow P_\pi: 13x-5y-z+d=0 \; Como\; A \in Plano \Rightarrow$$

$$d = -20-4 = -24 \Rightarrow$$

$$P_\pi: 13x-5y-z-24=0$$

25-

$$L \cap P_\pi = \begin{cases} x = 1+2\lambda \\ y = 3\lambda \\ z = -1+4\lambda \\ 2x-2y+3z+1=0 \end{cases} \Rightarrow$$

$$2(1+2\lambda)-2(3\lambda)+3(-1+4\lambda)+1=0$$
$$2+4\lambda-6\lambda-3+12\lambda+1=0 \Rightarrow \lambda=0$$

$$L \cap P_\pi = \{(1,0,-1)\}$$

26-

Hallamos r_2:

$$r_2\begin{cases} x = 1+\lambda \\ y = 2+\lambda \\ z=-5 \end{cases} \Rightarrow r_1 \cap r_2 \begin{cases} 1+\lambda = -t \\ 2+\lambda = 6 \\ -5 = t \end{cases} \Rightarrow \lambda = 4$$

$$r_1 \cap r_2 = \{(5,6,-5)\}$$

27-

Hallamos r:

$$\begin{cases} x-y+z=0 \\ 2x-y+z=2 \end{cases} \Rightarrow Restamos \Rightarrow x=2$$

$$Recta\begin{cases} x=2 \\ y=\lambda \\ z=\lambda-2 \end{cases}$$

Hallamos s:

$$S: \bar{X} = (3,2,4)+\tau(3-k,2,4-k) \Rightarrow$$

$$r \cap s \begin{cases} 2 = 3+\tau(3-k) \\ \lambda = 2+2\tau \\ \lambda-2 = 4+\tau(4-k) \end{cases} = \begin{cases} -1 = 3\tau-\tau k \\ -4 = 2\tau - \tau k \end{cases} Resto \Rightarrow 3=\tau$$

Como vemos, para todo k, el parámetro TAU va a ser siempre uno, es decir que da un punto siempre. Sin embargo, si tau toma ese valor, k ya no siempre se cancela, luego para TAU=3:

$$\begin{cases} -1 = 9-3k \\ -4 = 6-3k \end{cases} si\; k \neq \frac{10}{3}\; SI$$

28-

$$\begin{cases} r_1 \parallel P_\pi \\ r_2 \| P_\pi \end{cases} \Rightarrow \begin{cases} \vec{u} \parallel P_\pi \\ \vec{v} \| P_\pi \end{cases} \Rightarrow \vec{u}\,x\,\vec{v} = \vec{N}_\pi \; Siendo \begin{cases} \vec{u} = (-2,-2,3) \\ \vec{v} = (1,2,3) \end{cases} \Rightarrow$$

$$\vec{N}_\pi = \begin{vmatrix} i & j & k \\ -2 & -2 & 3 \\ 1 & 2 & 3 \end{vmatrix} = (-12,9,-2) \Rightarrow P_\pi: -12x+9y-2z+d=0 \; como\; (3,4,0) \in P_\pi \Rightarrow$$

$$d=0 \Rightarrow P_\pi: -12x+9y-2z=0$$

29-

$$\beta \cap r: (2+2\lambda)-(1-\lambda)+2(-1+\lambda)-4=0 \Rightarrow \lambda=1 \Rightarrow \; Siendo \begin{cases} \vec{u} = (-2,-2,3) \\ \vec{v} = (1,2,3) \end{cases} \Rightarrow$$

$$\beta \cap r = (4,0,0)$$

$$\begin{cases} r_\pi \subseteq \beta \Rightarrow \vec{u} \perp \vec{N}_\beta \\ r_\pi \perp r \end{cases} \Rightarrow \vec{u} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = (1,3,1) \Rightarrow$$

$$r_\pi \begin{cases} x = 4+t \\ y = 3t \quad t \in \mathbb{R} \\ z = t \end{cases}$$

30-

$$Sea\; \vec{n}_\pi \perp P_\pi, y\; \vec{u} \parallel L \Rightarrow Ang(P_\pi, L) = ArcSen\left(\frac{\vec{n}_\pi * \vec{u}}{||\vec{n}_\pi||\,||\vec{u}||}\right)$$

$$\begin{cases} L \perp \vec{n}_{P_\theta} \\ L \perp \vec{n}_{P_\tau} \end{cases} \Rightarrow \vec{u} = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 1 & 2 & 3 \end{vmatrix} = (-8,-2,4) \parallel (4,1,-2) \Rightarrow$$

$$\phi = ArcSen\frac{(4,1,-2)*(3,-7,8)}{\sqrt{21}\sqrt{122}} = ArcSen\frac{-11}{\sqrt{21}\sqrt{122}} = 0.21Rad = \frac{0{,}21Rad*180}{\pi} = 12{,}55º$$

31-

a-

$$Sea\;\; \vec{u} \parallel L\,; P_0 \in L; A \Rightarrow Dist(L,A) = \frac{\left|\left|\vec{u}\,x\,\overrightarrow{P_0A}\right|\right|}{||\vec{u}||}$$

$$Dist(L,A) = \frac{\left\|\begin{vmatrix} i & j & k \\ 3 & 0 & -4 \\ 3 & 2 & -1 \end{vmatrix}\right\|}{5} = \frac{||(8,-9,6)||}{5} = \frac{\sqrt{181}}{5}$$

b-

$$Sea \begin{cases} \vec{u} \parallel L_1 \,; P_1 \in L_1 \\ \vec{v} \parallel L_2 \,;\, P_2 \in L_2 \end{cases} \Rightarrow Dist(L_1, L_2) = \frac{\left|\overrightarrow{P_1P_2}(\vec{u}\ x\ \vec{v})\right|}{\left|\left|\vec{u}\ x\ \vec{v}\right|\right|}$$

$$Dist(L_1, L_2) = \frac{\begin{vmatrix} 3 & -4 & 4 \\ 2 & 1 & 1 \\ -3 & 1 & 2 \end{vmatrix}}{\left\|\begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ -3 & 1 & 2 \end{vmatrix}\right\|} = \frac{|3(2-1)+4(4+3)+4(2+3)|}{\left||1,-7,5|\right|} = \boxed{\frac{51}{5\sqrt{3}}}$$

32-

a-

$$P_{xz} \perp (0,10)\,; \vec{0} \in P_{xz} \Rightarrow y = 0 \Rightarrow \begin{cases} z = -4-1 \\ x = 6-2 \end{cases} \Longrightarrow P_{xz} \cap L = (4,0,-5)$$

b-

$$Sea \begin{cases} \vec{u} \parallel L_1 \,; P_1 \in L_1 \\ \vec{v} \parallel L_2 \,;\, P_2 \in L_2 \end{cases} \Rightarrow Dist(L_1, L_2) = \frac{\left|\overrightarrow{P_1P_2}(\vec{u}\ x\ \vec{v})\right|}{\left|\left|\vec{u}\ x\ \vec{v}\right|\right|}$$

$$Dist(P_\pi, P_{xz} \cap L) = \frac{|ax_0 + by_0 + cz_0 + d|}{\left||a,b,c|\right|} = \frac{|4-5|}{\sqrt{11}} = \boxed{\frac{1}{\sqrt{11}}}$$

33-

a-

$$Proy_{P_\pi}(A) = L \cap\ P_\pi \,/\, \vec{L} = \vec{n}_\pi \lambda + \vec{A}$$

$$L \begin{cases} x = 3\lambda + 3 \\ y = -2\lambda - 1 \; \lambda \in \mathbb{R} \\ z = 4\lambda + 2 \end{cases} \Rightarrow L \cap P_\pi : 3(3\lambda+3) - 2(-2\lambda-1) + 4(4\lambda+2) - 3 = 0 \Rightarrow \lambda = -\frac{16}{29}$$

$$Proy_{P_\pi}(A) = \boxed{\left(\frac{39}{29}, \frac{3}{29}, -\frac{6}{29}\right)}$$

b-

$$Proy_{P_\pi}(L) = P_L \cap\ P_\pi \,/\, \vec{n}_L = \vec{u}\ x\ \vec{n}_\pi\ \wedge P_L \in P_L$$

$$\vec{n}_L = \vec{u}\ x\ \vec{n}_\pi = \begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ 3 & -2 & 4 \end{vmatrix} = (8,2,-5) \Rightarrow$$

$$P_{L:}\, 8x + 2y - 5z + d = 0 \; como \; (2,1,2) \in P_{\setminus L} \Rightarrow d = -8$$

$$Proy_{P_\pi}(L) = P_L \cap\ P_\pi = \begin{cases} P_\pi : 3x - 2y + 4z - 3 = 0 \\ P_L : 8x + 2y - 5z - 8 = 0 \end{cases} \Rightarrow 11x - 11 = z \Rightarrow$$

$$3x - 2y + 4(11x - 11) - 3 = 0$$

$$47x - 47 = 2y$$

$$\frac{47}{2}x - \frac{47}{2} = y \Rightarrow$$

$$\boxed{L = \begin{cases} x = 2\lambda \\ y = 47\lambda - \dfrac{47}{2} \; \lambda \in \mathbb{R} \\ z = 22\lambda - 11 \end{cases}}$$

34-

$$M = P_\pi \cap Eje_x \Rightarrow \begin{cases} \pi : x + 2y - z - 2 = 0 \\ M : (a,0,0) \end{cases} \Rightarrow M = (2,0,0) \Rightarrow$$

$$Dist(M, L) = \frac{\left|\left|\overrightarrow{P_0M}\ x\ \vec{u}\right|\right|}{\left||\vec{u}|\right|} = \frac{\left\|\begin{vmatrix} i & j & i \\ -1 & -k & -2 \\ -1 & -1 & 0 \end{vmatrix}\right\|}{\sqrt{2}} = \frac{\left|\left|(-2,2,1-k)\right|\right|}{\sqrt{2}} = \sqrt{6} \Rightarrow$$

$$\sqrt{k^2 - 2k + 9} = \sqrt{12}$$

$$k^2 - 2k - 3 = 0 \Rightarrow$$

$$\boxed{k = -1 \wedge k = 3}$$

35-

a-

$$Si \begin{cases} t_1 \subseteq \beta \Rightarrow \vec{u}_1 \perp \vec{n}_\beta \\ t_2 \parallel \beta \Rightarrow \vec{u}_2 \perp \vec{n}_\beta \end{cases} \Rightarrow n_\beta = \vec{u}_1\ x\ \vec{u}_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ 1 & 0 & -1 \end{vmatrix} = (-1,3,-1) \; y \; como \; P_1 \in \beta \Rightarrow$$

$$-1 + 3 + d = 0 \Rightarrow d = -2 \Rightarrow$$

$$\boxed{\beta : -x + 3y - z - 2 = 0}$$

b-

$$Proy_\pi(t_2) = P_\pi \cap \left(P_\beta \supseteq t_2\right) \Rightarrow$$

$$\begin{cases} \vec{n}_\pi \parallel \pi \\ \vec{u} \parallel \pi \end{cases} \Rightarrow \vec{n}_\beta = \vec{n}_\pi x\ \vec{u} = \begin{vmatrix} i & j & k \\ 1 & 3 & -1 \\ 1 & 0 & -1 \end{vmatrix} = (-3,0,-3) \parallel (1,0,1) \; como \; t_2 \subseteq P_\beta : P_2 \in P_\beta \Rightarrow$$

$$d = 0 \Rightarrow$$

$$L = P_\pi \cap P_\beta = \begin{cases} \beta : x + z = 0 \Rightarrow z = -x \\ \pi : x + 3y - z + 3 = 0 \end{cases} \Rightarrow 2x + 3y + 3 = 0 \Rightarrow$$

$$\boxed{L \begin{cases} x = \lambda \\ y = -\dfrac{2}{3}\lambda - 1 \\ z = -\lambda \end{cases}}$$