CSC 8980 Deep Learning

Assignment 1

Due Date: 08/31/2018 (by 11:59 pm)

Name: ______________________________ Campus ID: ____________________

Background Knowledge Test

Problem 1. (1 point) We are machine learners with a slight gambling problem (very different from gamblers with a machine learning problem!). Our friend, Bob, is proposing the following payout on the roll of a dice:

$$payout = \begin{cases} \$1 & x = 1 \\ -\$1/4 & x \neq 1 \end{cases}$$

Where $x \in \{1,2,3,4,5,6\}$ is the outcome of the roll, (+) means payout to us and (-) means payout to Bob. Is this a good bet? Are we expected to make money?

Solution:

$$E_x[f(x)] = \sum_x p(x)f(x) = \frac{1}{6} \times 1 - \frac{5}{6} \times \frac{1}{4} = \frac{1}{6} - \frac{5}{24} = -\frac{1}{24}$$

Since the expected payout to us is negative, this is not a good bet for us and we are expected to loss money.

Problem 2. (1 point) X is a continuous random variable with the probability density function

$$p(x) = \begin{cases} 4x & 0 \leq x \leq 1/2 \\ -4x + 4 & 1/2 \leq x \leq 1 \end{cases}$$

What is the equation for the corresponding cumulative density function (cdf) $C(x)$?
[Hint: Recall that CDF is defined as $C(x) = P(X \leq x)$.]

Solution:

$$C(x) = P_r(X \leq x)$$
$$= \int_0^x 4x\,dx = 2x^2 \ \ when\ 0 \leq x \leq \frac{1}{2}$$

$$C(x) = P_r(X \leq x) = \int_0^{\frac{1}{2}} 4x\,dx + \int_{\frac{1}{2}}^{x} (-4x + 4)dx$$
$$= (-2x^2 + 4x)|_{1/2}^{x} + \frac{1}{2}$$

$$= -2x^2 + 4x - 1 \ \ when \frac{1}{2} \leq x \leq 1$$

Problem 3. (1 point) Recall that the variance of a random variable is defined as $Var[X] = E[(X-\mu)^2]$, where $\mu = E[X]$. Use the properties of expectation to show that we can rewrite the variance of a random variable X as

$$Var[X] = E[X^2] - (E[X])^2$$

Solution:

$$\begin{aligned} Var[x] &= E[(x-\mu)^2] \\ &= E[x^2 + \mu^2 - 2\mu x] \\ &= E[x^2] + \mu^2 - 2\mu E[x] \\ &= E[x^2] - (E[X])^2 \end{aligned}$$

Problem 4. (1 point) A random variable x in standard Gaussian distribution has following probability density

$$p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Evaluate following integral

$$\int_{-\infty}^{\infty} p(x)(ax^2 + bx + c)dx$$

[Hint: This is not a calculus question!]

Solution:

$$\begin{aligned} \int_{-\infty}^{\infty} p(x)(ax^2 + bx + c)dx &= E[ax^2 + bx + c] \\ &= aE[x^2] + bE[x] + c \\ &= a(Var[x] + \mu^2) + b\mu + c \qquad (According\ to\ Problem\ 3) \\ &= a(1+0) + b \times 0 + c \qquad \big(Given\ x \sim N(0,1)\big) \\ &= a + c \end{aligned}$$

Problem 5. (3 points) Consider the following function of $\boldsymbol{x} = (x_1, x_2, x_3, x_4, x_5, x_6)$:

$$f(x) = \sigma\left(\log\left(5\left(\max\{x_1, x_2\}.\frac{x_3}{x_4} - (x_5 + x_6)\right)\right) + \frac{1}{2}\right)$$

where σ is the sigmoid function

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Evaluate $f(\boldsymbol{x})$ at $\hat{\boldsymbol{x}} = (5, -1, 6, 12, 7, -5)$. Then, compute the gradient $\nabla_x f(\boldsymbol{x})$ and evaluate it at the same point.

Solution:

First evaluate $f(\hat{x}) = \sigma\left(\log\left(5\left(\max\{5, -1\}.\frac{6}{12} - (7 - 5)\right)\right) + \frac{1}{2}\right)$

$$= \sigma\left(\log\left(\frac{5}{2}\right) + \frac{1}{2}\right) = 0.805$$

Next let's compute the gradient

$$\text{Define } y = \max\{x_1, x_2\} \times \frac{x_3}{x_4} - (x_5 + x_6)$$

$$z = 5y$$

$$u = \log(z)$$

$$v = u + \frac{1}{2}$$

$$f = \sigma(v)$$

By using Chain rule:

$$\frac{\partial f(x)}{\partial x_i} = \frac{\partial f}{\partial v}\frac{\partial v}{\partial u}\frac{\partial u}{\partial z}\frac{\partial z}{\partial y}\frac{\partial y}{\partial x_i}$$

$$= \sigma(v)\sigma(-v)\frac{1}{z}\, 5\, \frac{\partial y}{\partial x_i}$$

$$Given\ x = (5, -1, 6, 12, 7, -5)$$

$$y = 5 \times \frac{1}{2} - 2 = 0.5$$

$$z = 2.5$$

$$u = log 2.5$$

$$v = log 2.5 + 0.5$$

$$f = 0.805$$

$$\frac{\partial f}{\partial x_1} = \sigma(v)\sigma(-v)\frac{5}{z}\frac{\partial y}{\partial x_1} = 2\sigma(v)\sigma(-v) \times \frac{x_3}{x_4} = 0.157$$

$$\frac{\partial f}{\partial x_2} = 0$$

$$\frac{\partial f}{\partial x_3} = 2\sigma(v)\sigma(-v) \times \frac{x_1}{x_4} = 0.131$$

$$\frac{\partial f}{\partial x_4} = 2\sigma(v)\sigma(-v) \times -\frac{\mathrm{x}_1\mathrm{x}_3}{\mathrm{x}_4^2} = -0.065$$

$$\frac{\partial f}{\partial x_5} = 2\sigma(v)\sigma(-v) \times (-1) = -0.314$$

$$\frac{\partial f}{\partial x_6} = 2\sigma(v)\sigma(-v) \times (-1) = -0.314$$

In summary,

$$\nabla_{x} f(\boldsymbol{x})|_{\hat{x}} = [0.157, \quad 0, \quad 0.131, \quad -0.065, \quad -0.314, \quad -0.314]$$

Problem 6. (3 points) Set up your Python programming environment by following the instruction below.

Setting up a Python programming environment for deep learning can be challenging. Depending on the system and privilege you have, it can be as easy as running a few scripts or completely a headache. In this class, we will use Python 2.7 and Numpy for programming assignments. (Note Python 3.5+ isn't fully tested and may or may not work with the assignment scripts.) In generally, you can set up your environment in Linux and Windows. Linux is preferred in case you have sudo privilege to install some needed packages. If this is the case, you may just need to run a few scripts and complete this task. Otherwise, you may set up your environment in Windows, but it's a little bit more involved.

0. Download assignment1.zip from iCollege hw section

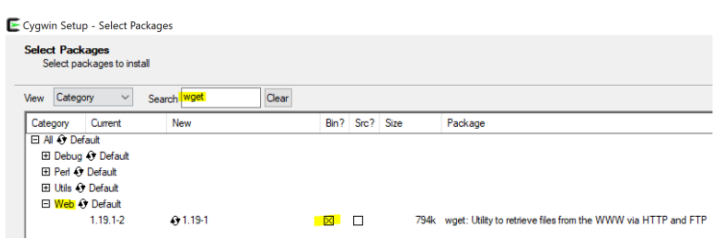
Linux (Preferred, but you may need sudo):

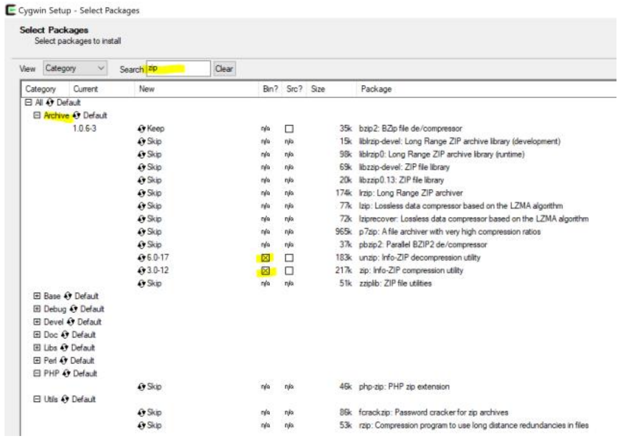
1. Unzip assignment1.zip
2. cd assignment1
3. sudo pip install -r requirements.txt
4. cd assignment1/lib/datasets
5. source get_datasets.sh
6. cd assignment1
7. jupyter notebook (and save your results)
8. If your notebook runs without problem, run ". collectSubmission.sh" to collect your results and send your zip file along with your completed problem set to iCollege hw1 dropbox.

Windows:

- Install Cygwin https://cygwin.com/install.html

  Select packages: wget, zip and unzip





1. Run cygwin64 Terminal
2. Unzip assignment1.zip

- Install Anaconda https://www.anaconda.com/download/ (Python 2.7)

1. Run Anaconda Prompt as Adminstrator
2. cd assignment1
3. sudo pip install -r requirements.txt
4. cd assignment1/lib/datasets
5. source `get_datasets.sh`
6. cd assignment1
7. jupyter notebook (and save your results)
8. If your notebook runs without problem, run “. collectSubmission.sh” to collect your results and send your zip file along with your completed problem set to iCollege hw1 dropbox.