

Aufgabe 1

Hier nur die Zwischenergebnisse:

$$(a) (g \circ f)(x, y, z) = xy + y^2 + 2xz + 2yz$$

$$f'(x, y, z) = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}, g'(t, u) = (u, t)$$

$$g'(f(x, y, z)) = (y + 2z, x + y), (g \circ f)'(x, y, z) = (y + 2z, x + 2y + 2z, 2x + 2y)$$

$$(b) (g \circ f)(x, y, z) = (y + x + z + xyz, xyz^3)$$

$$f'(x, y, z) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2z \\ yz & xz & xy \end{pmatrix}, g'(x, y, z, w) = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & w & z \end{pmatrix}$$

$$g'(f(x, y, z)) = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & xyz & z^2 \end{pmatrix}, (g \circ f)'(x, y, z) = \begin{pmatrix} 1 + yz & 1 + xz & 1 + xy \\ yz^3 & xz^3 & 3xyz^2 \end{pmatrix}$$

$$(c) (g \circ f)(x, y, z) = (e^x, e^{2x}z \sin y, \sin e^x, \log(z \sin y) + e^x)$$

$$f'(x, y, z) = \begin{pmatrix} e^x & 0 & 0 \\ 0 & z \cos y & \sin y \end{pmatrix}, g'(t, u) = \begin{pmatrix} 1 & 0 \\ 2tu & t^2 \\ \cos t & 0 \\ 1 & \frac{1}{u} \end{pmatrix}$$

$$g'(f(x, y, z)) = \begin{pmatrix} 1 & 0 \\ 2ze^x \sin y & e^{2x} \\ \cos e^x & 0 \\ 1 & \frac{1}{z \sin y} \end{pmatrix}, (g \circ f)'(x, y, z) = \begin{pmatrix} e^x & 0 & 0 \\ 2ze^{2x} \sin y & ze^{2x} \cos y & e^{2x} \sin y \\ e^x \cos e^x & 0 & 0 \\ e^x & \frac{\cos y}{\sin y} & \frac{1}{z} \end{pmatrix}$$

$$(d) (g \circ f)(x, y) = (2xy^2, \sin xy^2)$$

$$f'(x, y) = (y^2, 2xy), g'(t) = \begin{pmatrix} 2 \\ \cos t \end{pmatrix}$$

$$g'(f(x, y)) = \begin{pmatrix} 2 \\ \cos xy^2 \end{pmatrix}, (g \circ f)'(x, y) = \begin{pmatrix} 2y^2 & 4xy \\ x^2 \cos xy^2 & 2xy \cos xy^2 \end{pmatrix}$$

$$(e) (f \circ g)(t) = 2t \sin^2 t$$

$$f'(g(t)) = (\sin^2 t, 4t \sin t), (f \circ g)'(t) = 2 \sin^2 t + 4t \sin t \cos t$$