

Solⁿ :

given examples

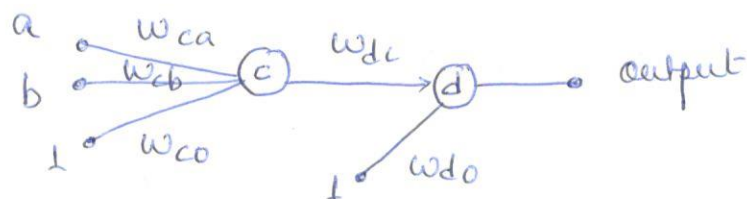
learning rate $\eta = 0.3$

momentum $\alpha = 0.9$

Initial values of weights

$$w_{ca} = w_{cb} = w_{co} = w_{dc} = w_{do} = 0.1$$

a	b	d
1	0	1
0	1	0



$$net_1 = \sum_{i=0}^n w_i x_i$$

Forward phase

$$= w_0 x_0 + w_1 x_1 + w_2 x_2$$

considering

$$a=1, b=0 \rightarrow$$

$$= w_{co} + w_{ca} a + w_{cb} b = 0.2$$

$$\sigma(net_1) = \frac{1}{1 + \exp^{-net_1}} = 0.5498$$

$$net_2 = w_{dc} c + w_{do} = 0.1550$$

$$\sigma(net_2) = \frac{1}{1 + \exp^{-net_2}} = 0.5387$$

Backward phase

weight update for output unit weights (without momentum)

$$\Delta w_{ji} = \eta (d_i - o_i) o_i (1 - o_i) x_i$$

$$\Delta w_{ji} = \eta \delta_j x_{ji}$$

weight update with momentum

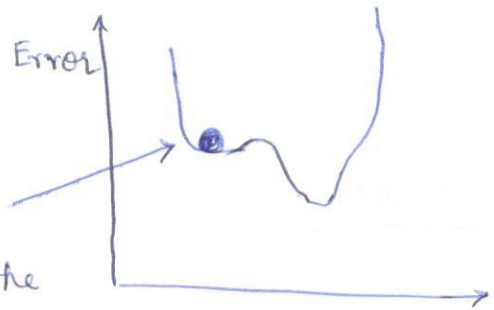
$$\Delta w_{ji}(n) = \eta \delta_j x_{ji} + \alpha \Delta w_{ji}(n-1)$$

(n-1)th iteration

$0 \leq \alpha < 1$: constant : momentum

- * The momentum helps gradually increasing the step size of search in regions where gradient is 0 and in turn speeding the convergence.

analogy to a ball rolling down the error surface. Momentum helps to keep the ball rolling through flat surfaces (local minima). Without momentum ball would stop in such



$$\begin{aligned} \Delta w_{dc} &= 0.3 \times 1 (1 - 0.5387) (1 - 0.5387) \times 0.5498 \\ &= 0.0351 \end{aligned}$$

$$\begin{aligned} \Delta w_{do} &= 0.3 \times 1 (1 - 0.5387) (1 - 0.5387) \times 1 \\ &= 0.0638 \end{aligned}$$

with momentum

(2)

weight update for hidden unit-

$$\Delta w_{ji} = \eta \delta_j x_{ji}$$

$$\delta_j = o_j(1-o_j) \sum_{k \in \text{Downstream}(j)} \delta_k w_{kj}$$

$$\delta_k = o_j(1-o_j)(d_j - o_j)$$

as k is the output node.

$$\delta_c = 0.5498(1-0.5498) \times$$

$$[\delta_d w_{dc} + \delta_o w_{oc}]$$

$$= 0.00011$$

$$\Delta w_{ca} = 0.3 \times 0.00011 \times 1 = 0.00003$$

$$\Delta w_{cb} = 0.3 \times 0.00011 \times 0 = 0$$

$$\Delta w_{co} = 0.3 \times 0.00011 \times 1 = 0.00003$$

Similarly weights with momentum can be determined.

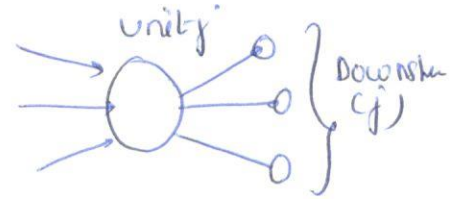
finally update the weights.

$$w_{ji} \leftarrow w_{ji} + \Delta w_{ji}$$

$$\Delta w_{dc} = 0.0351 + 0.1 = 0.1351$$

$$\Delta w_{do} = 0.1 + 0.0638 = 0.1638$$

$$\Delta w_{ca} = 0.1 + 0.00003 = 0.10003$$



Downstream (j):

Set of units whose immediate inputs include the output of unit j .