Finding the most likely sequence (viterbi Algorithm)

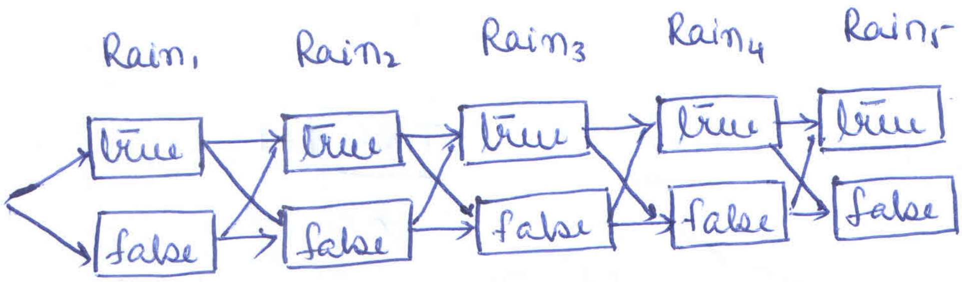
Example from book (chapter 15)

For five days the umbrella sequence is given as

[true, true, false, true, true]

what is the weather sequence most

Possible state sequences of $Rain_t$ can be viewed as paths ~~as~~ through a graph of possible states at each time step as shown below:



There are 2^5 possible weather sequences we could pick.

To find the most likely sequence we must consider the joint probabilities over all the time steps.

$$\max_{x_1 \ldots x_t} P(x_1, \ldots, x_t, X_{t+1} \mid e_{1:t+1})$$

$$= \alpha \, P(e_{t+1} \mid X_{t+1}) \max_{x_t} \left(P(X_{t+1} \mid x_t) \max_{x_1 \ldots x_{t-1}} P(x_1, \ldots, x_{t-1} \mid e_{1:t}) \right)$$

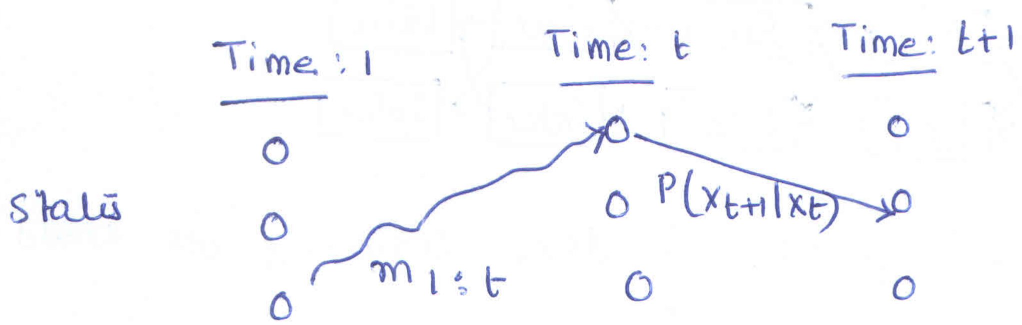
Let $m_{1:t} = \max_{x_1 : x_{t-1}} P(x_1 \ldots x_{t-1}, X_t \mid e_{1:t})$ be the probabili of most likely path to reach state x_t

$$\therefore \quad m_{1:t+1} = \alpha\, P(e_{t+1} \mid x_{t+1}) \max_{x_t} \left(P(x_{t+1} \mid x_t)\, m_{1:t} \right)$$

we need to find $m_{1:5}$

likelihood of any path

= transition probabilities along the path $\times$ the probabilities of the given observations at each state.



(1) $\boxed{m_{1:1}}$ $\quad (m_{1:0+1},\ t=0)$

$$= \alpha\, P(e_1 \mid x_1) \max_{x_0} \left(P(x_1 \mid x_0)\, m_{1:0} \right)$$

$$= \alpha\, P(u_1 \mid R_1) \max_{r_0} \left(P(R_1 \mid x_0)\, m_{1:0} \right)$$

$$= \alpha \langle \underbrace{0.9}_{P(u_1|r_1)} \ \underbrace{0.2}_{P(u_1|\sim r_1)} \rangle \max_{r_0} \left(\langle \underbrace{0.7}_{P(r_1|r_0)} \ \underbrace{0.3}_{P(\sim r_1|r_0)} \rangle \times \underbrace{0.5}_{P(r_0)},\ \langle \underbrace{0.3}_{P(r_1|\sim r_0)} \ \underbrace{0.7}_{P(\sim r_1|\sim r_0)} \rangle\, \underbrace{0.5}_{P(\sim r_0)} \right)$$

compare

$$= \alpha\langle 0.9, 0.2\rangle \max_{r_0} \Big(\underbrace{\langle 0.35 \quad 0.15\rangle}_{r_0 = \text{true}} \quad \underbrace{\langle 0.15 \quad 0.35\rangle}_{r_0 = \text{false}} \Big)$$

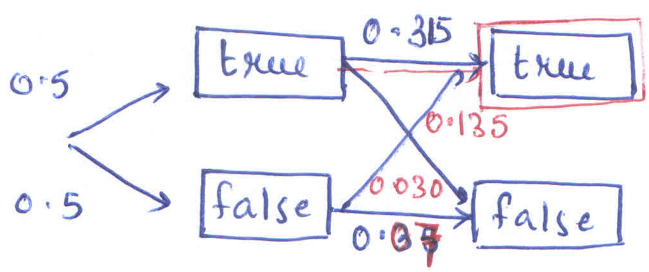
$$= \alpha \langle 0.9 \quad 0.2\rangle \langle 0.35 \quad 0.35\rangle$$

$P(r_1|r_0)P(r_0)$ $\quad$ $P(\sim r_1|\sim r_0)P(\sim r_0)$

$$= \alpha\langle 0.315 \quad 0.07\rangle$$ — not normalized

$$= \langle 0.818 \quad 0.1818\rangle$$

$Rain_0$ $\quad$ $Rain_1$



you can solve separately for R1, ie.

R_1 = true

$R_1 = r_1$

$$P(u_1|r_1) \max_{r_0} \left(P(r_1|r_0)P(r_0)\right)$$

$$P(u_1|r_1) \max_{r_0} \left(P(r_1|r_0)P(r_0), P(r_1|\sim r_0)P(\sim r_0)\right)$$

R_1 = false

$R_1 = \sim r_1$

$$P(u_1|\sim r_1) \max_{r_0} \left(P(\sim r_1|r_0)P(r_0), P(\sim r_1|\sim r_0)P(\sim r_0)\right)$$

Exercise: find $m_{1:2}$, $m_{1:3}$, $m_{1:4}$ and finally $m_{1:5}$