9.Poynomial Regression (SK Learn)

October 26, 2021

Polynomial Regression (Non-Linear fitting)

Let us work on a real dataset with Sci-Kit (SK) Learn

It is not non-linear regression

I will explain with linear, poly-order 2, poly-order 3, poly-order 20, poly-order 40. I am not going to split dataset into training and testing set. I will show you the implementation with whole dataset fitting with the model, SSE, and R_sqaure. You can split dataset into training and testing set and practice with the code.

[103]:
```
# Import necessary package
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import warnings
warnings.filterwarnings("ignore")
%matplotlib inline
```

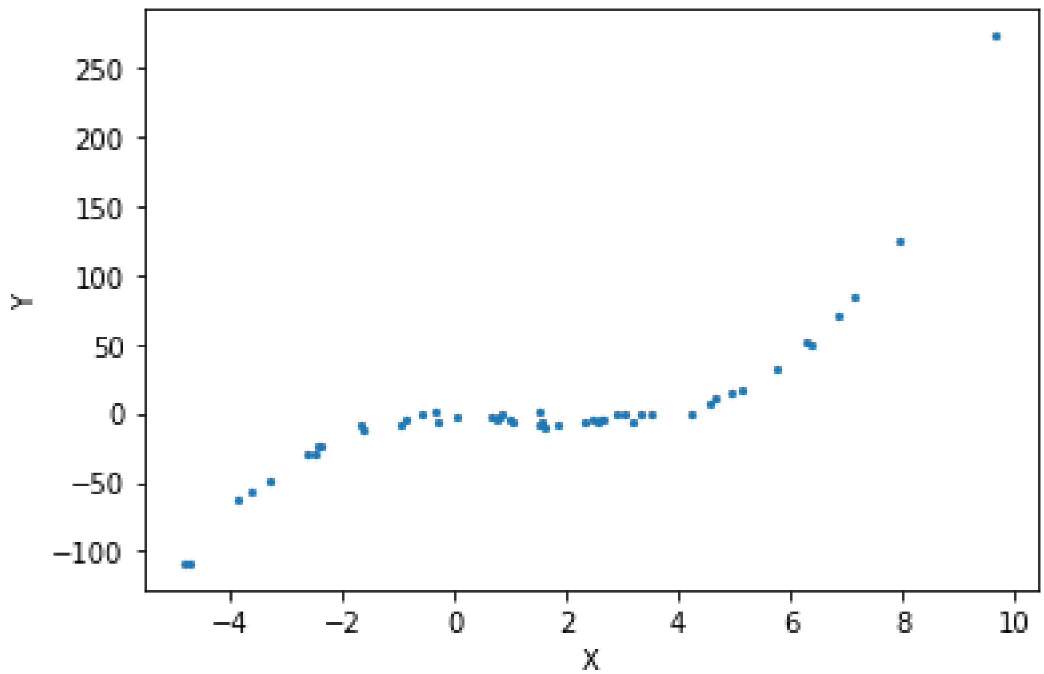
0.0.1 1. Linear Regression

Dataset

[104]:
```
# Let us work with sample data
import numpy as np
import matplotlib.pyplot as plt

np.random.seed(0)
x = 2 - 3 * np.random.normal(0, 1, 50)
y = x - 2 * (x ** 2) + 0.5 * (x ** 3) + np.random.normal(-3, 3, 50)
```

Data visualization

[105]:
```
plt.scatter(x,y, s=5)
plt.xlabel("X")
plt.ylabel("Y")
plt.show()
```



Fitting with simple regression using sklearn

[106]:
```
# Fitting and plotting the predicted value
from sklearn.linear_model import LinearRegression
x = x.reshape(-1,1)

# Fit the linear model
model = LinearRegression()
model.fit(x, y)
```

[106]: LinearRegression()

Predict the output for training input values using the models

[107]:
```
y_pred = model.predict(x)
```

[108]:
```
model.coef_
```

[108]: array([12.91501245])

[109]:
```
model.intercept_
```

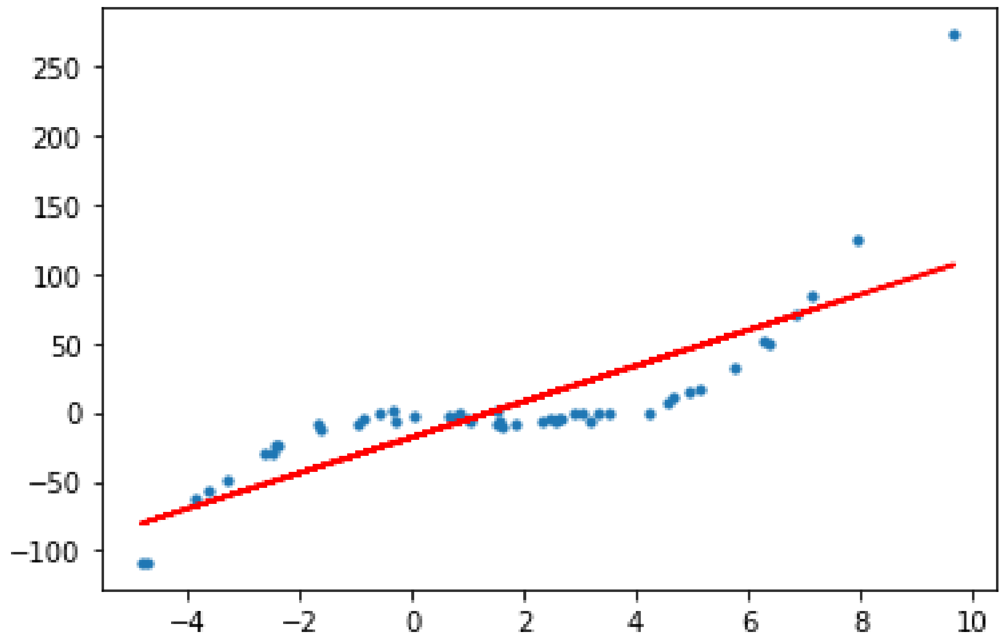
[109]: -18.161443351033988

[110]:
```
print("y = mx + b =>    y = ", model.coef_ ,"x +",model.intercept_)
```

```
y = mx + b =>    y =  [12.91501245] x + -18.161443351033988
```

Visualizting the prediction performance

[111]:
```
# Plotting
plt.scatter(x, y, s=10)
plt.plot(x, y_pred, color='r')
plt.show()
```



Calculating SSE

[112]:
```
# SSE
sum = 0
n = len(x)
for i in range (0,n):
  diff = y[i] - y_pred[i]
  squ_diff = diff**2
  sum = sum + squ_diff
SSE_Linear = round(sum,2)
print("Sum of Squared Error (SSE) for Linear model:",SSE_Linear )
```

```
Sum of Squared Error (SSE) for Linear model: 50092.13
```

Calculating R__2

[113]:
```python
# Calculating R-Squred value (goodness of model) using SSE
from sklearn.metrics import r2_score
out = r2_score(y,y_pred)
RS_Linear = round(out,2)*100
print("R-Squred value for Linear model :",RS_Linear,"%")
```

```
R-Squred value for Linear model : 65.0 %
```

This linear model did not fit well. So, let us try with complicate models (polynomial)

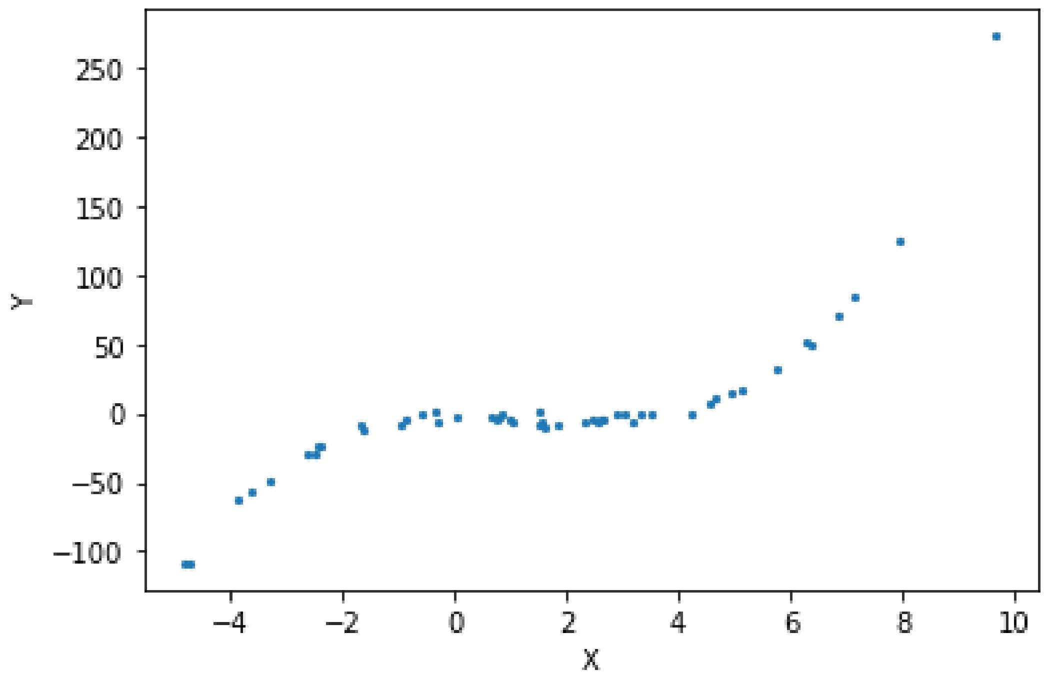
0.0.2 2. Polynomial regression (degree 2)

Dataset

[114]:
```python
# Let us work with sample data
import numpy as np
import matplotlib.pyplot as plt

np.random.seed(0)
x = 2 - 3 * np.random.normal(0, 1, 50)
y = x - 2 * (x ** 2) + 0.5 * (x ** 3) + np.random.normal(-3, 3, 50)
```

Data visualization

[115]:
```python
plt.scatter(x,y, s=5)
plt.xlabel("X")
plt.ylabel("Y")
plt.show()
```



Fitting with polynomial order 2 using sklearn

[116]:
```
import operator
from sklearn.preprocessing import PolynomialFeatures

# To convert the original features into their higher order terms
x = x.reshape(-1,1)
polynomial_features= PolynomialFeatures(degree=2)
x_poly = polynomial_features.fit_transform(x)

# Fit the polynomial model with degree 2
model = LinearRegression()
model.fit(x_poly, y)
```

[116]:
```
LinearRegression()
```

[117]:
```
parameters = model.coef_
parameters
```

[117]:
```
array([0.        , 8.88674291, 1.11862126])
```

[118]:
```
print("a x_2 + b x + c => ", round(parameters[2],2)," x_2␣
↪+",round(parameters[1],2)," x + ",round(parameters[0],2))
```

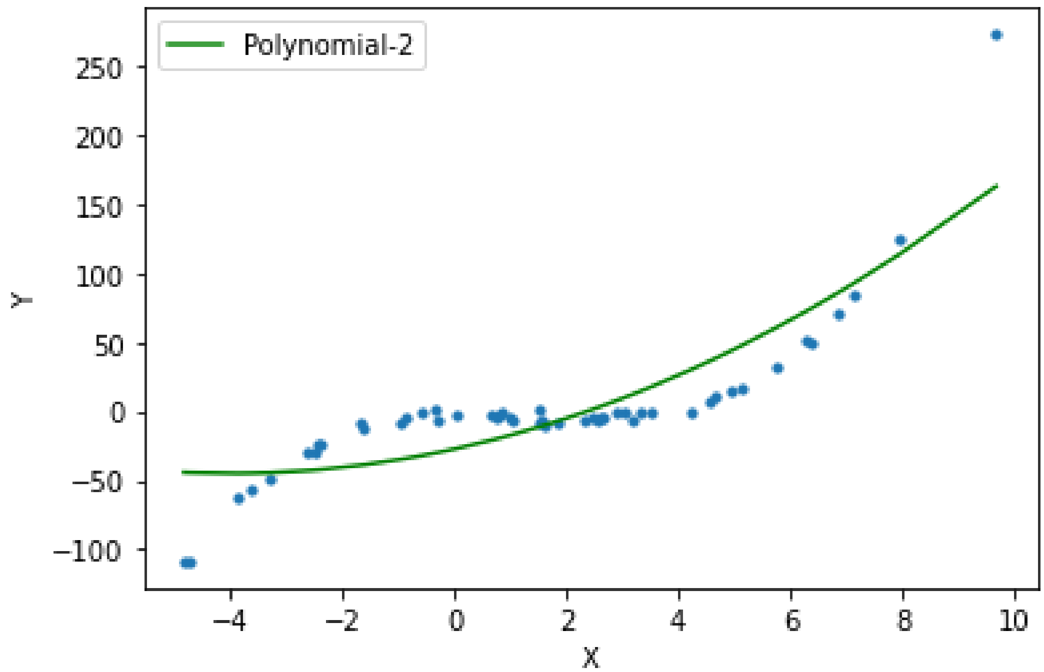
```
a x_2 + b x + c =>  1.12  x_2 + 8.89  x +  0.0
```

Predict the output for training input values using the models

[119]:
```
y_poly_2_pred = model.predict(x_poly)
y_poly_2_pred_temp = y_poly_2_pred
```

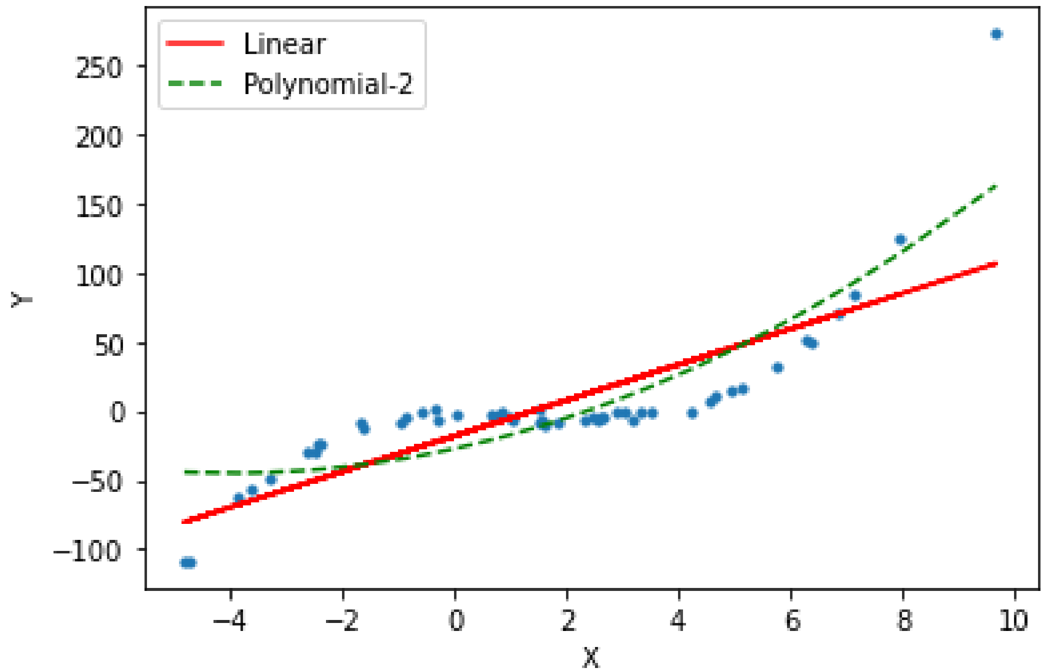
Visualizting the prediction performance

[120]:
```
# Plotting polynomial degree 2 model
plt.scatter(x, y, s=10)

# sort the values of x before plotting for polynomial line
sort_axis = operator.itemgetter(0)
sorted_zip = sorted(zip(x,y_poly_2_pred), key=sort_axis)
mod_x2, y_poly_2_pred = zip(*sorted_zip)
plt.plot(mod_x2, y_poly_2_pred, color="green" ,label="Polynomial-2")

plt.xlabel("X")
plt.ylabel("Y")
plt.legend()
plt.show()
```



[121]:
```
# Comparison with linear model
plt.scatter(x, y, s=10)
plt.plot(x, y_pred, color='r',label="Linear")
```

```
plt.plot(mod_x2, y_poly_2_pred, "g--",label="Polynomial-2")

plt.xlabel("X")
plt.ylabel("Y")
plt.legend()
plt.show()
```



Calculating SSE

[122]:
```
# SSE
sum = 0
n = len(x)
for i in range (0,n):
  diff = y[i] - y_poly_2_pred_temp[i]
  squ_diff = diff**2
  sum = sum + squ_diff
SSE_Poly_2 = round(sum,2)
print("Sum of Squared Error (SSE) for Polynomial degree 2 model:",SSE_Poly_2)
```

```
Sum of Squared Error (SSE) for Polynomial degree 2 model: 37989.36
```

Calculating R_2

[123]:
```
# Calculating R-Squred value (goodness of model) using SSE
from sklearn.metrics import r2_score
out = r2_score(y,y_poly_2_pred_temp)
RS_Poly_2 = round(out,2)*100
print("R-Squred value for Linear model :",RS_Poly_2,"%")
```

```
R-Squred value for Linear model : 74.0 %
```

Still, the polynomial with degree 2 did not fit the shape of the data points fully

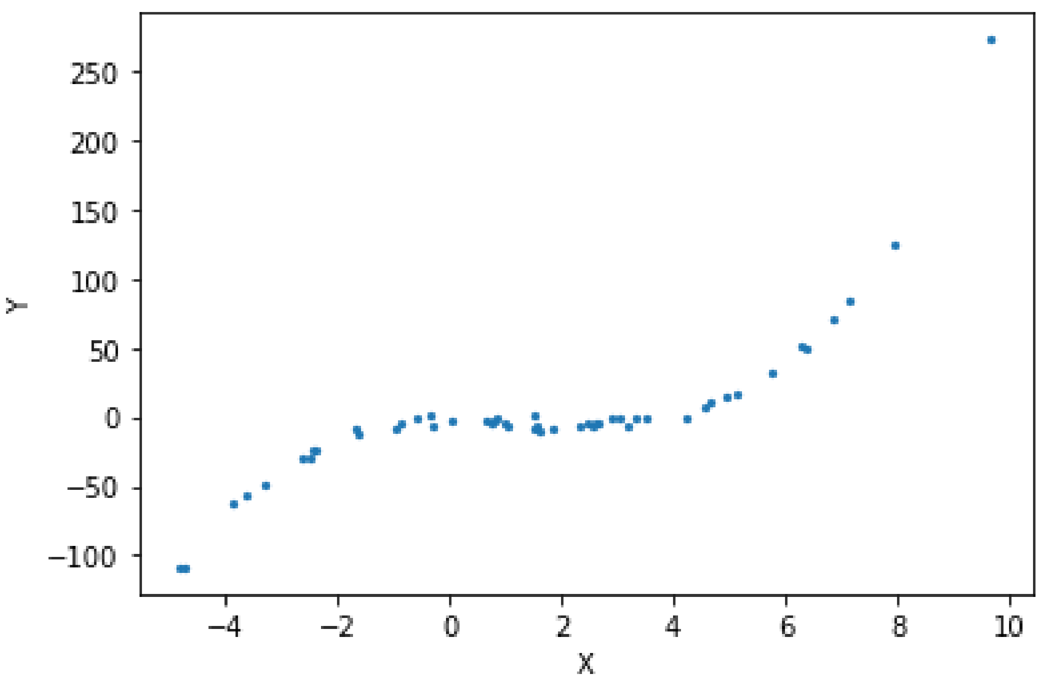
0.0.3 3. Polynomial regression (degree 3)

Dataset

[124]:
```
# Let us work with sample data
import numpy as np
import matplotlib.pyplot as plt

np.random.seed(0)
x = 2 - 3 * np.random.normal(0, 1, 50)
y = x - 2 * (x ** 2) + 0.5 * (x ** 3) + np.random.normal(-3, 3, 50)
```

Data visualization

[125]:
```
plt.scatter(x,y, s=5)
plt.xlabel("X")
plt.ylabel("Y")
plt.show()
```



Fitting with polynomial order 3 using sklearn

[126]:
```
import operator
from sklearn.preprocessing import PolynomialFeatures

# To convert the original features into their higher order terms
x = x.reshape(-1,1)
polynomial_features= PolynomialFeatures(degree=3)
x_poly = polynomial_features.fit_transform(x)

# Fit the polynomial model with degree 3
model = LinearRegression()
model.fit(x_poly, y)
```

[126]:
```
LinearRegression()
```

[127]:
```
parameters = model.coef_
parameters
```

[127]:
```
array([ 0.        ,  1.03253842, -2.02525278,  0.50272631])
```

[128]:
```
print("a x_2 + b x + c => ", round(parameters[3],3)," x_3 +",␣
 ↪round(parameters[3],2)," x_2 +",round(parameters[1],2)," x +␣
 ↪",round(parameters[0],2))
```

```
a x_2 + b x + c =>  0.503  x_3 + 0.5  x_2 + 1.03  x +  0.0
```

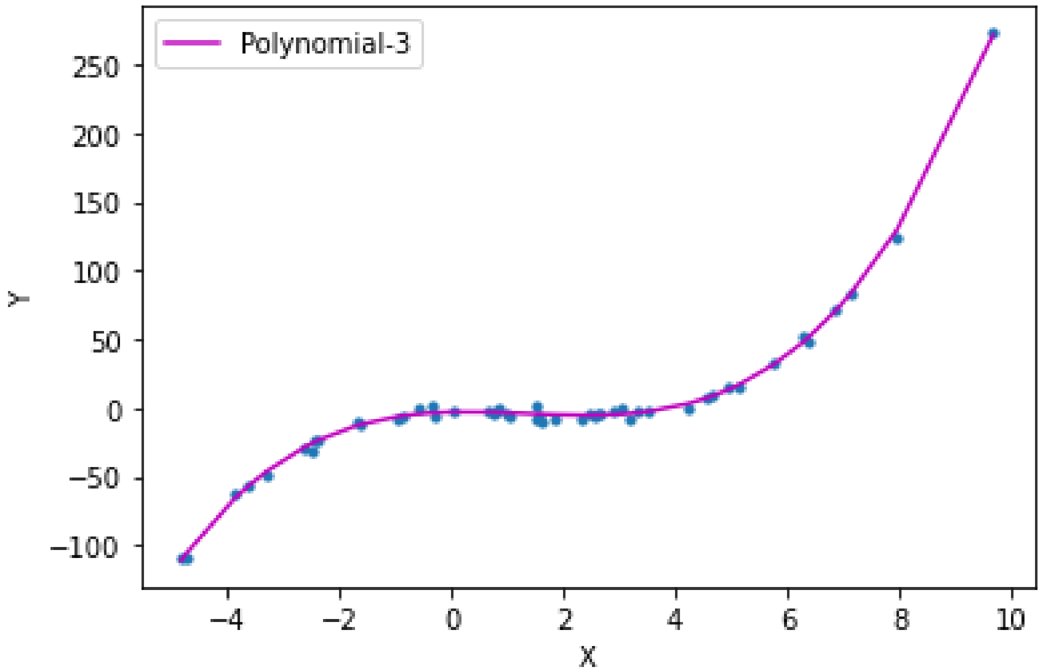
Predict the output for training input values using the model

[129]:
```
y_poly_3_pred = model.predict(x_poly)
y_poly_3_pred_temp = y_poly_3_pred
```

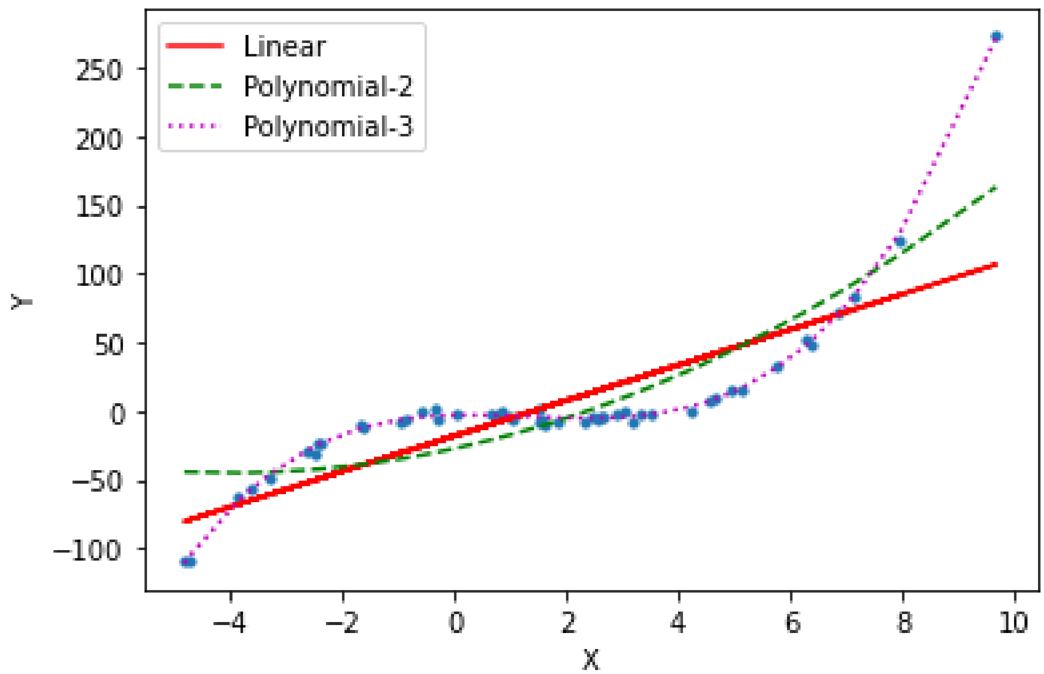
Visualizting the prediction performance¶

[130]:
```
# Plotting polynomial degree 3 model
plt.scatter(x, y, s=10)

# sort the values of x before plotting for polynomial line
sort_axis = operator.itemgetter(0)
sorted_zip = sorted(zip(x,y_poly_3_pred), key=sort_axis)
mod_x3, y_poly_3_pred = zip(*sorted_zip)
plt.plot(mod_x3, y_poly_3_pred, color='m',label="Polynomial-3")

plt.xlabel("X")
```

```
plt.ylabel("Y")
plt.legend()
plt.show()
```



[131]:
```
# Plotting linear, polynomial degree 2, and degree 3 models
plt.scatter(x, y, s=10)
plt.plot(x, y_pred, color='r',label="Linear")

plt.plot(mod_x2, y_poly_2_pred, "g--",label="Polynomial-2")
plt.plot(mod_x3, y_poly_3_pred, "m:",label="Polynomial-3")

plt.xlabel("X")
plt.ylabel("Y")
plt.legend()
plt.show()
```



Calculating SSE

[132]:
```
# SSE
sum = 0
n = len(x)
for i in range (0,n):
  diff = y[i] - y_poly_3_pred_temp[i]
  squ_diff = diff**2
  sum = sum + squ_diff
SSE_Poly_3 = round(sum,2)
print("Sum of Squared Error (SSE) for Polynomial degree 3 model:",SSE_Poly_3)
```

```
Sum of Squared Error (SSE) for Polynomial degree 3 model: 335.37
```

Calculating R_2

[133]:
```
# Calculating R-Squred value (goodness of model) using SSE
from sklearn.metrics import r2_score
out = r2_score(y, y_poly_3_pred_temp)
RS_Poly_3 = round(out,2)*100
print("R-Squred value for Linear model :",RS_Poly_3,"%")
```

```
R-Squred value for Linear model : 100.0 %
```

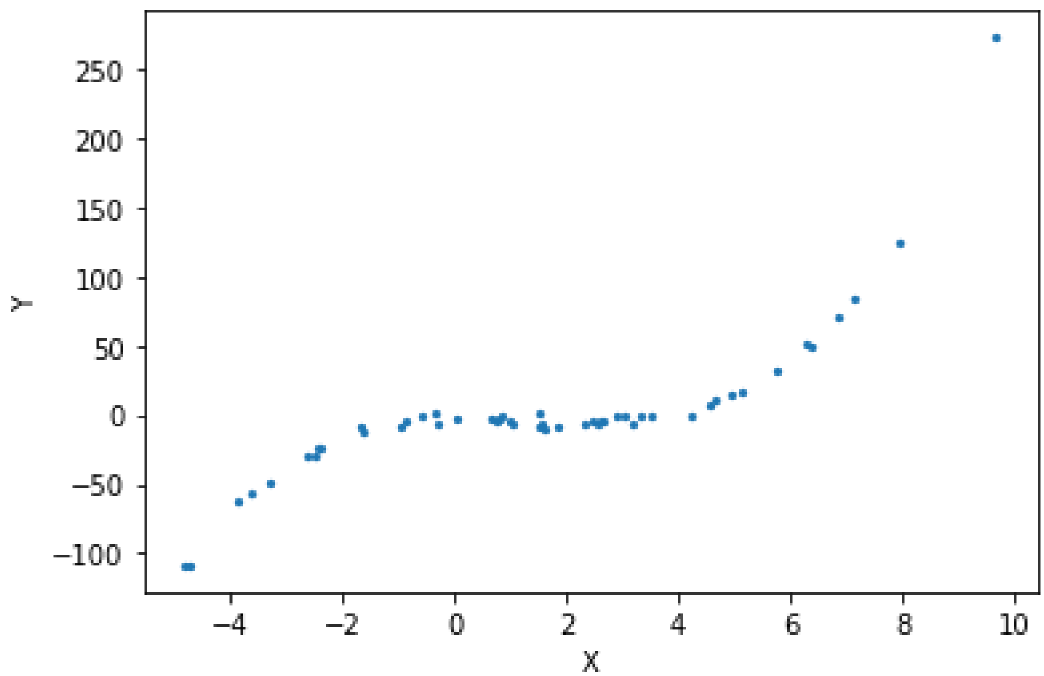
0.0.4 4. Polynomial regression (degree 20)

Dataset

[134]:
```
# Let us work with sample data
import numpy as np
import matplotlib.pyplot as plt

np.random.seed(0)
x = 2 - 3 * np.random.normal(0, 1, 50)
y = x - 2 * (x ** 2) + 0.5 * (x ** 3) + np.random.normal(-3, 3, 50)
```

Data visualization

[135]:
```
plt.scatter(x,y, s=5)
plt.xlabel("X")
plt.ylabel("Y")
plt.show()
```



Fitting with polynomial order 20 using sklearn

[136]:
```
import operator
from sklearn.preprocessing import PolynomialFeatures

# To convert the original features into their higher order terms
x = x.reshape(-1,1)
```

```python
polynomial_features= PolynomialFeatures(degree=20)
x_poly = polynomial_features.fit_transform(x)

# Fit the polynomial model with degree 20
model = LinearRegression()
model.fit(x_poly, y)
```

[136]:
```
LinearRegression()
```

[137]:
```python
parameters = model.coef_
parameters
```

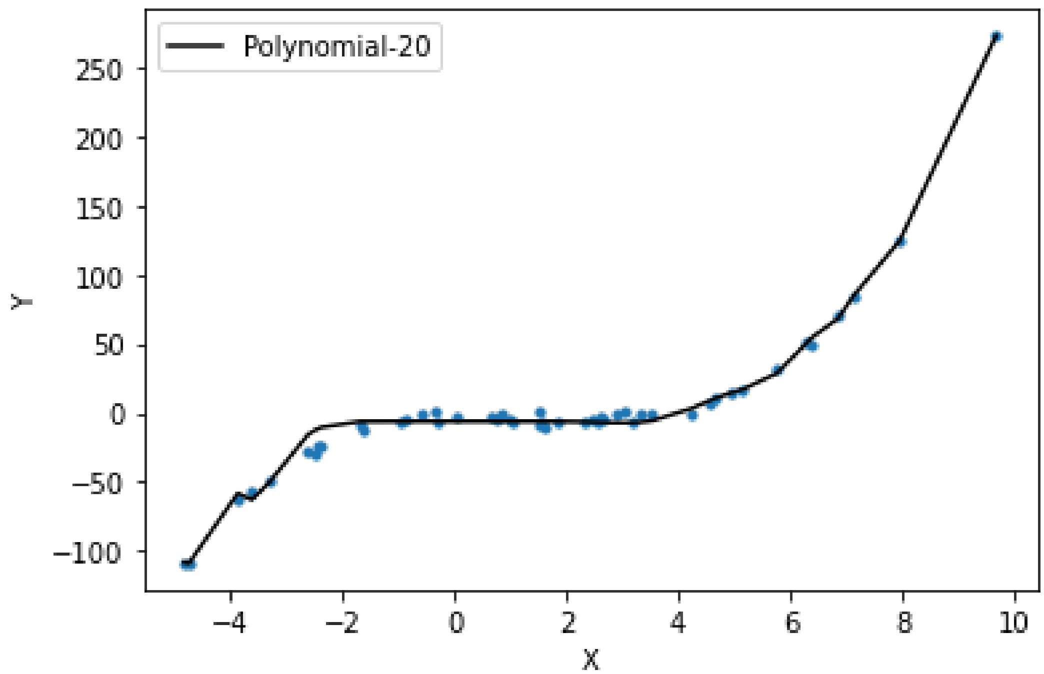
[137]:
```
array([ 0.00000000e+00, -5.80638670e-06, -2.56142132e-07,  9.42812966e-08,
       -2.73251778e-06, -3.34809132e-07, -2.43337476e-05, -7.54489766e-06,
       -1.71053188e-04, -2.84858587e-05, -7.09124271e-04,  3.68238868e-04,
        4.14974138e-05, -4.81533446e-05,  4.22600515e-06,  1.82032258e-06,
       -3.64019229e-07, -3.75353258e-09,  6.52750301e-09, -6.24426594e-10,
        1.90018733e-11])
```

Predict the output for training input values using the model

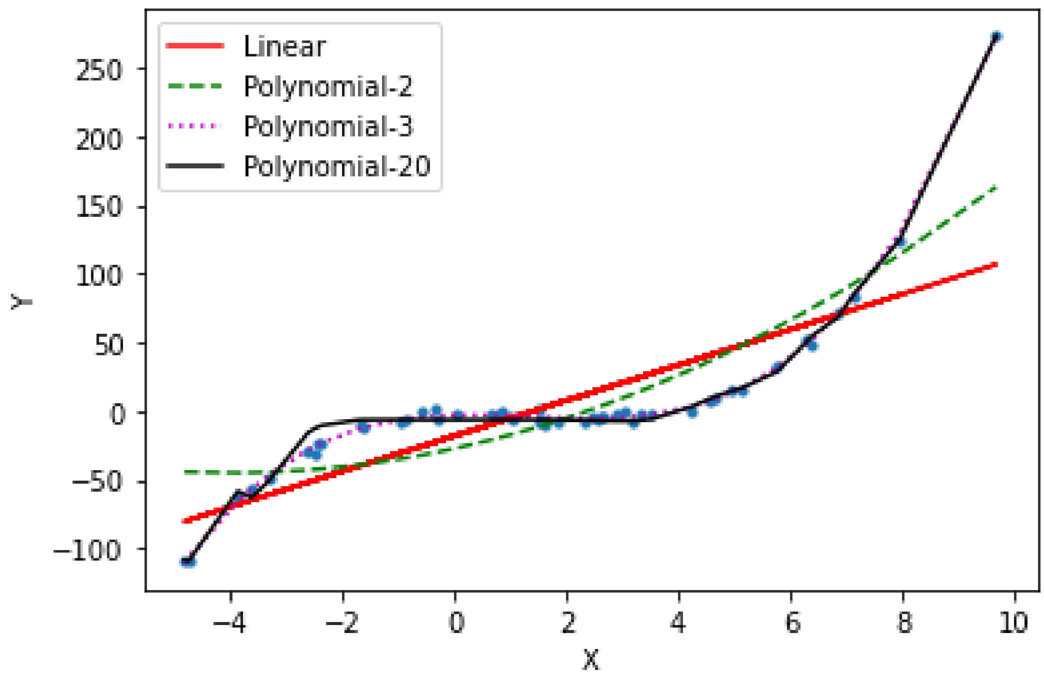
[138]:
```python
y_poly_20_pred = model.predict(x_poly)
y_poly_20_pred_temp = y_poly_20_pred
```

Visualizting the prediction performance

[139]:
```python
# Plotting polynomial degree 3 model
plt.scatter(x, y, s=10)

# sort the values of x before plotting for polynomial line
sort_axis = operator.itemgetter(0)
sorted_zip = sorted(zip(x,y_poly_20_pred), key=sort_axis)
mod_x20, y_poly_20_pred = zip(*sorted_zip)
plt.plot(mod_x20, y_poly_20_pred, color='k',label="Polynomial-20")

plt.xlabel("X")
plt.ylabel("Y")
plt.legend()
plt.show()
```



[140]:
```
# Plotting linear, polynomial degree 2, and degree 3 models
plt.scatter(x, y, s=10)
plt.plot(x, y_pred, color='r',label="Linear")

plt.plot(mod_x2, y_poly_2_pred, "g--",label="Polynomial-2")
plt.plot(mod_x3, y_poly_3_pred, "m:",label="Polynomial-3")
plt.plot(mod_x20, y_poly_20_pred, "k-",label="Polynomial-20")

plt.xlabel("X")
plt.ylabel("Y")
plt.legend()
plt.show()
```



Calculating SSE

[141]:
```
# SSE
sum = 0
n = len(x)
for i in range (0,n):
  diff = y[i] - y_poly_20_pred_temp[i]
  squ_diff = diff**2
  sum = sum + squ_diff
SSE_Poly_20 = round(sum,2)
print("Sum of Squared Error (SSE) for Polynomial degree 3 model:",SSE_Poly_20)
```

```
Sum of Squared Error (SSE) for Polynomial degree 3 model: 1412.95
```

Calculating R_2

[142]:
```
# Calculating R-Squred value (goodness of model) using SSE
from sklearn.metrics import r2_score
out = r2_score(y, y_poly_20_pred_temp)
RS_Poly_20 = round(out,2)*100
print("R-Squred value for Linear model :",RS_Poly_20,"%")
```

```
R-Squred value for Linear model : 99.0 %
```

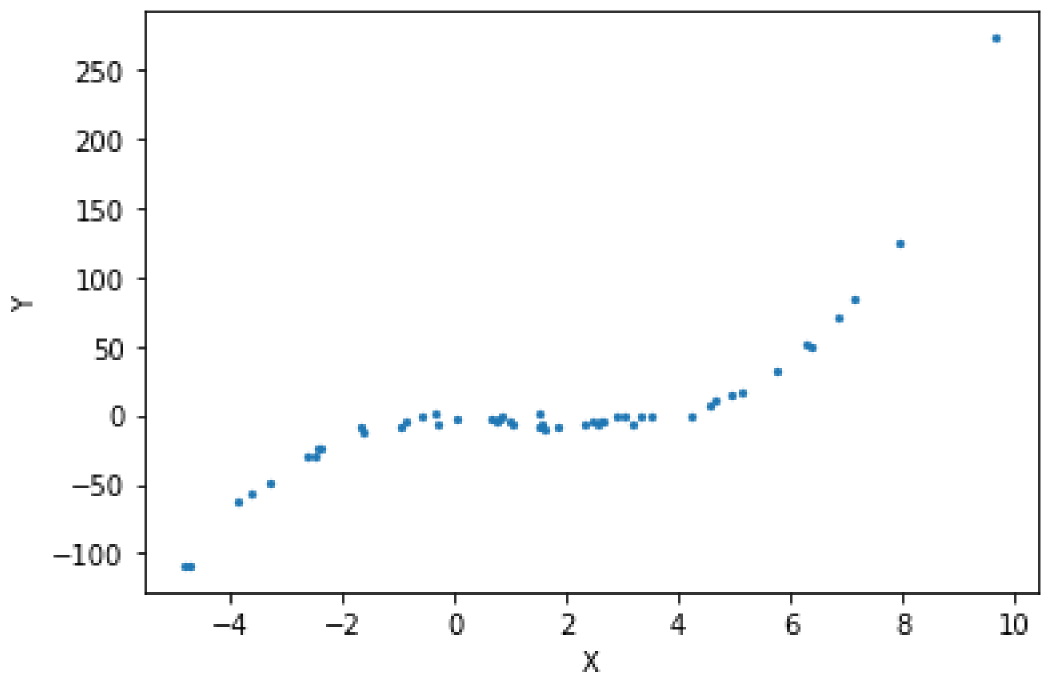
0.0.5 5. Polynomial regression (degree 40)

Dataset

[143]:
```
# Let us work with sample data
import numpy as np
import matplotlib.pyplot as plt

np.random.seed(0)
x = 2 - 3 * np.random.normal(0, 1, 50)
y = x - 2 * (x ** 2) + 0.5 * (x ** 3) + np.random.normal(-3, 3, 50)
```

Data visualization

[144]:
```
plt.scatter(x,y, s=5)
plt.xlabel("X")
plt.ylabel("Y")
plt.show()
```



Fitting with polynomial order 40 using sklearn

[145]:
```
import operator
from sklearn.preprocessing import PolynomialFeatures

# To convert the original features into their higher order terms
```

```
x = x.reshape(-1,1)
polynomial_features= PolynomialFeatures(degree=40)
x_poly = polynomial_features.fit_transform(x)

# Fit the polynomial model with degree 40
model = LinearRegression()
model.fit(x_poly, y)
```

[145]:
```
LinearRegression()
```

[146]:
```
parameters = model.coef_
parameters
```

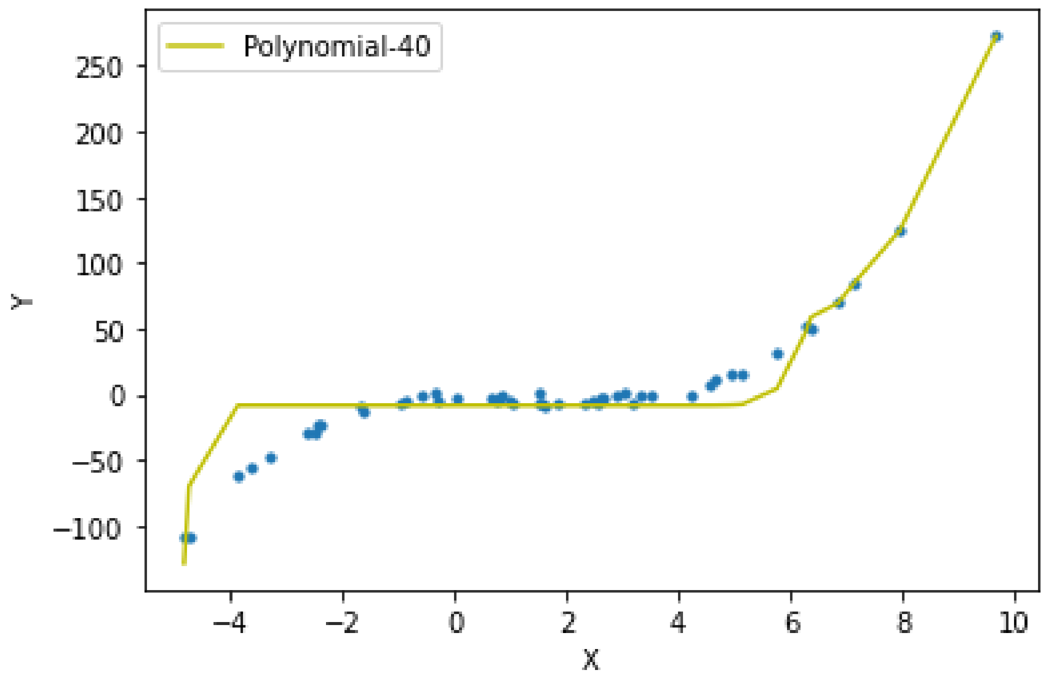
[146]:
```
array([ 1.15292548e-26,  4.88777058e-25,  3.85945961e-24,  3.54435807e-25,
       -4.08928460e-25,  1.65475277e-25,  8.57366583e-25,  1.08844480e-24,
        7.92905187e-25, -5.93122928e-24,  9.49131214e-24, -2.29010482e-23,
        4.24999279e-24,  1.10235548e-23,  7.10928913e-24,  3.65610125e-24,
       -8.35446452e-25,  1.73072959e-34,  9.13323554e-34,  4.55545820e-33,
        2.36155789e-32,  1.16598682e-31,  5.90240318e-31,  2.85966493e-30,
        1.40245942e-29,  6.59030802e-29,  3.09434467e-28,  1.38700206e-27,
        6.11562612e-27,  2.54621085e-26,  1.01825258e-25,  3.74879402e-25,
        1.26240557e-24,  3.66433533e-24,  8.26556952e-24,  1.19088838e-23,
       -9.47979847e-24,  2.50985511e-24, -3.17860822e-25,  1.97256739e-26,
       -4.83579243e-28])
```

Predict the output for training input values using the model

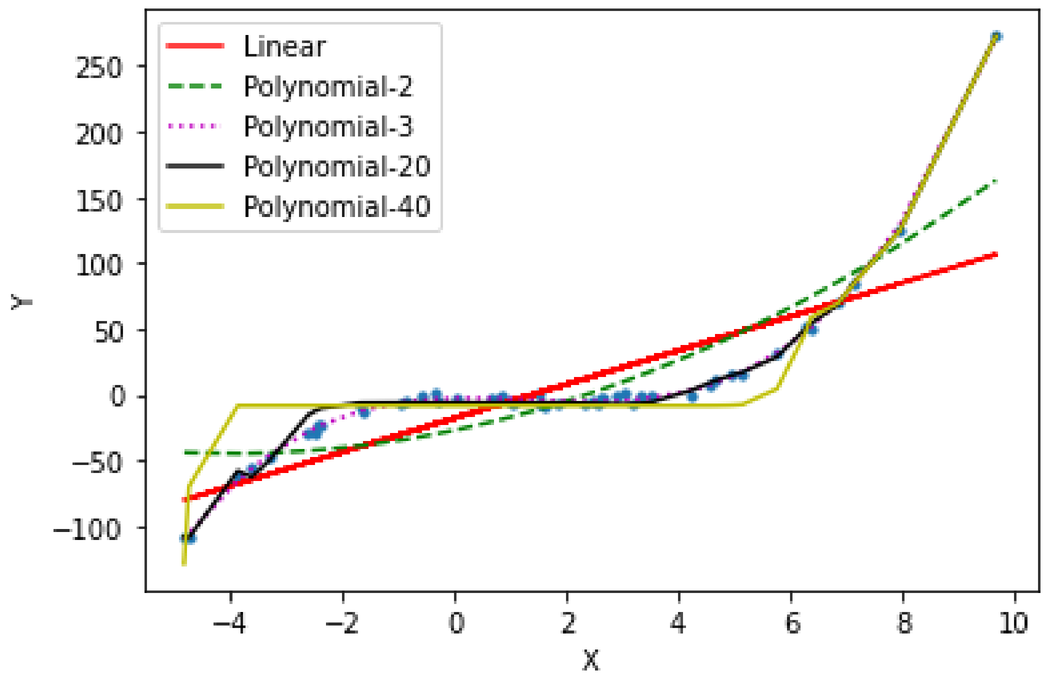
[147]:
```
y_poly_40_pred = model.predict(x_poly)
y_poly_40_pred_temp = y_poly_40_pred
```

Visualizting the prediction performance

[148]:
```
# Plotting polynomial degree 3 model
plt.scatter(x, y, s=10)

# sort the values of x before plotting for polynomial line
sort_axis = operator.itemgetter(0)
sorted_zip = sorted(zip(x,y_poly_40_pred), key=sort_axis)
mod_x40, y_poly_40_pred = zip(*sorted_zip)
plt.plot(mod_x40, y_poly_40_pred, color='y',label="Polynomial-40")

plt.xlabel("X")
plt.ylabel("Y")
plt.legend()
plt.show()
```



[149]:
```
# Plotting linear, polynomial degree 2, and degree 3 models
plt.scatter(x, y, s=10)
plt.plot(x, y_pred, color='r',label="Linear")

plt.plot(mod_x2, y_poly_2_pred, "g--",label="Polynomial-2")
plt.plot(mod_x3, y_poly_3_pred, "m:",label="Polynomial-3")
plt.plot(mod_x20, y_poly_20_pred, "k-",label="Polynomial-20")
plt.plot(mod_x40, y_poly_40_pred, color='y',label="Polynomial-40")

plt.xlabel("X")
plt.ylabel("Y")
plt.legend()
plt.show()
```



Calculating SSE

[150]:
```
# SSE
sum = 0
n = len(x)
for i in range (0,n):
  diff = y[i] - y_poly_40_pred_temp[i]
  squ_diff = diff**2
  sum = sum + squ_diff
SSE_Poly_40 = round(sum,2)
print("Sum of Squared Error (SSE) for Polynomial degree 3 model:",SSE_Poly_40)
```

```
Sum of Squared Error (SSE) for Polynomial degree 3 model: 13431.68
```

Calculating R_2

[151]:
```
# Calculating R-Squred value (goodness of model) using SSE
from sklearn.metrics import r2_score
out = r2_score(y, y_poly_40_pred_temp)
RS_Poly_40 = round(out,2)*100
print("R-Squred value for Linear model :",RS_Poly_40,"%")
```

```
R-Squred value for Linear model : 91.0 %
```

0.0.6 Overall performance

[152]:
```
print("Model   \t SSE              RS ")
print("======================================")
print("Linear   \t",SSE_Linear ,"\t", RS_Linear )
print("Poly_2   \t",SSE_Poly_2,"\t", RS_Poly_2)
print("Poly_3   \t",SSE_Poly_3,"\t", RS_Poly_3)
print("Poly_20  \t",SSE_Poly_20,"\t", RS_Poly_20)
print("Poly_40  \t",SSE_Poly_40,"\t", RS_Poly_40)
```

```
Model            SSE              RS
======================================
Linear           50092.13        65.0
Poly_2           37989.36        74.0
Poly_3           335.37          100.0
Poly_20          1412.95         99.0
Poly_40          13431.68        91.0
```