3.Multiple Linear Regression

October 23, 2021

Multiple (Multi-variate) Linear Regression

[53]:
```
# Import necessary package
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

0.0.1 Step 1: Load the dataset

[54]:
```
# Load the dataset into pandas dataframe
df=pd.read_csv("E:\\MY LECTURES\\DATA SCIENCE\\3.Programs\\dataset\\Advertising.
↪csv")
# Change this location based on the location of dataset in your machine
```

[55]:
```
# Display the first five records
df.head()
```

[55]:

	TV	radio	newspaper	sales
0	230.1	37.8	69.2	22.1
1	44.5	39.3	45.1	10.4
2	17.2	45.9	69.3	9.3
3	151.5	41.3	58.5	18.5
4	180.8	10.8	58.4	12.9

Advertising data comprises four features: TV, radio, newspaper, and sales. It explains the budget (in 1000$) spent on different mass media and the net outcome for every week.

sales for a product (output/dependent/target variable).

advertising budget for TV, radio, and newspaper media (input/independent/target variable).

Planning to perform regression on TV budget (X1), Radio budget (X2), Newspaper budget (X3) as input and sales (Y) as output.

[56]:
```
# Dataset shape (number of rows and columns)
df.shape
```

[56]: (200, 4)

Row <=> record, tuple, instance, sample, observation, object, case, entity Column <=> attribute, variable, field, feature, characteristic, dimension

0.0.2 Step 2: Apply EDA

Univariate analysis

[57]:
```
# Statistics summary
df["TV"].describe()
```

[57]:
```
count    200.000000
mean     147.042500
std       85.854236
min        0.700000
25%       74.375000
50%      149.750000
75%      218.825000
max      296.400000
Name: TV, dtype: float64
```

[58]:
```
df["radio"].describe()
```

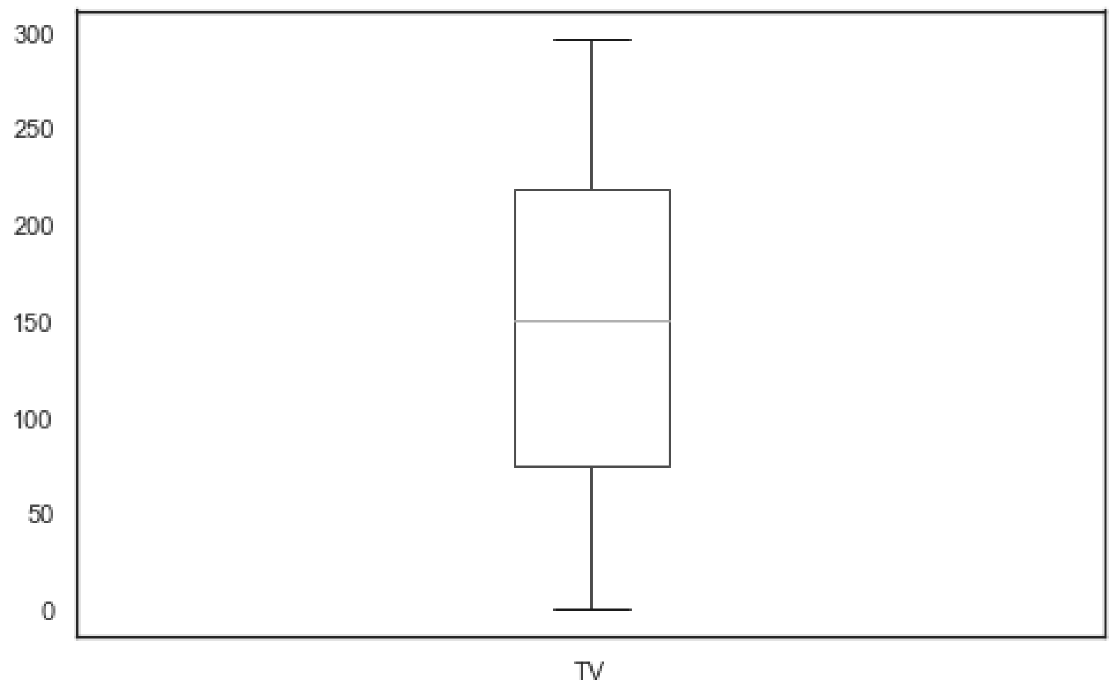
[58]:
```
count    200.000000
mean      23.264000
std       14.846809
min        0.000000
25%        9.975000
50%       22.900000
75%       36.525000
max       49.600000
Name: radio, dtype: float64
```

[59]:
```
df["newspaper"].describe()
```

[59]:
```
count    200.000000
mean      30.554000
std       21.778621
min        0.300000
25%       12.750000
50%       25.750000
75%       45.100000
max      114.000000
Name: newspaper, dtype: float64
```

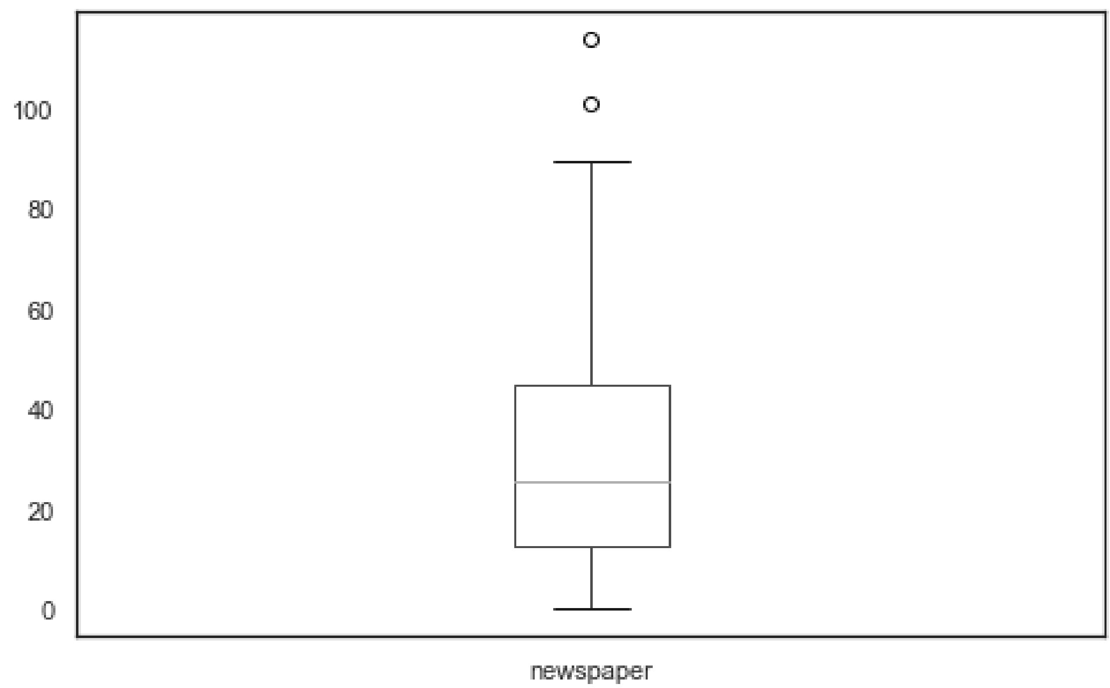
[60]:
```
# Univariate Analysis using Boxplot
sns.set_style(style='white')
df.boxplot(column =['TV'], grid = False,figsize=(8,5))
```

[60]: <AxesSubplot:>



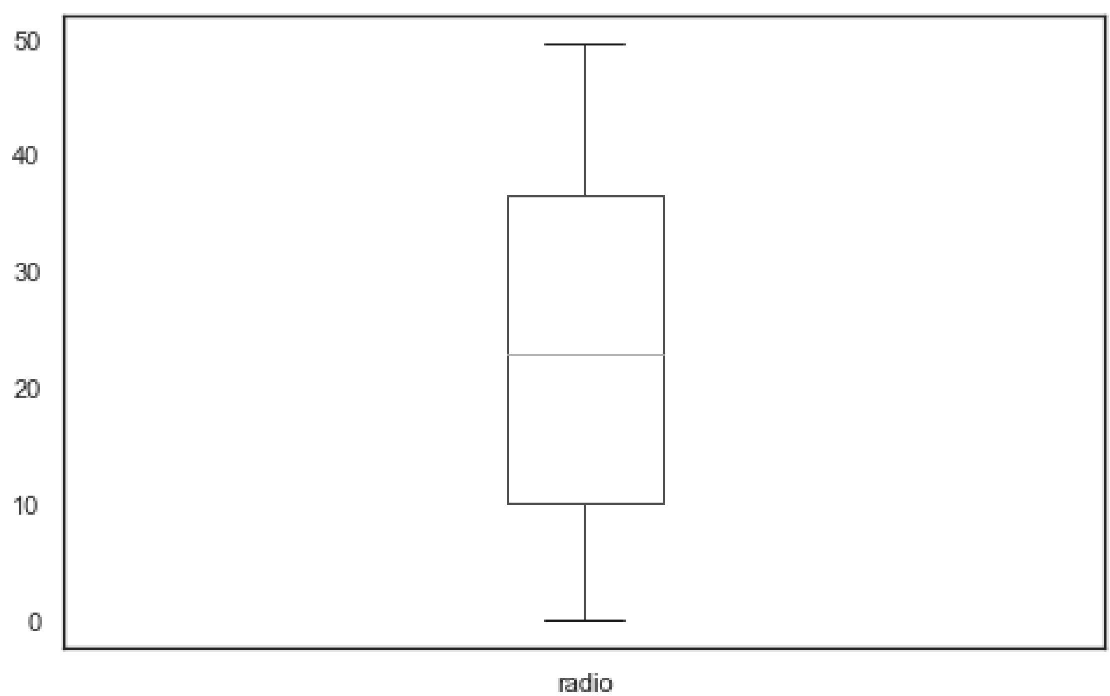
[61]:
```
df.boxplot(column =['newspaper'], grid = False,figsize=(8,5))
```

[61]: <AxesSubplot:>



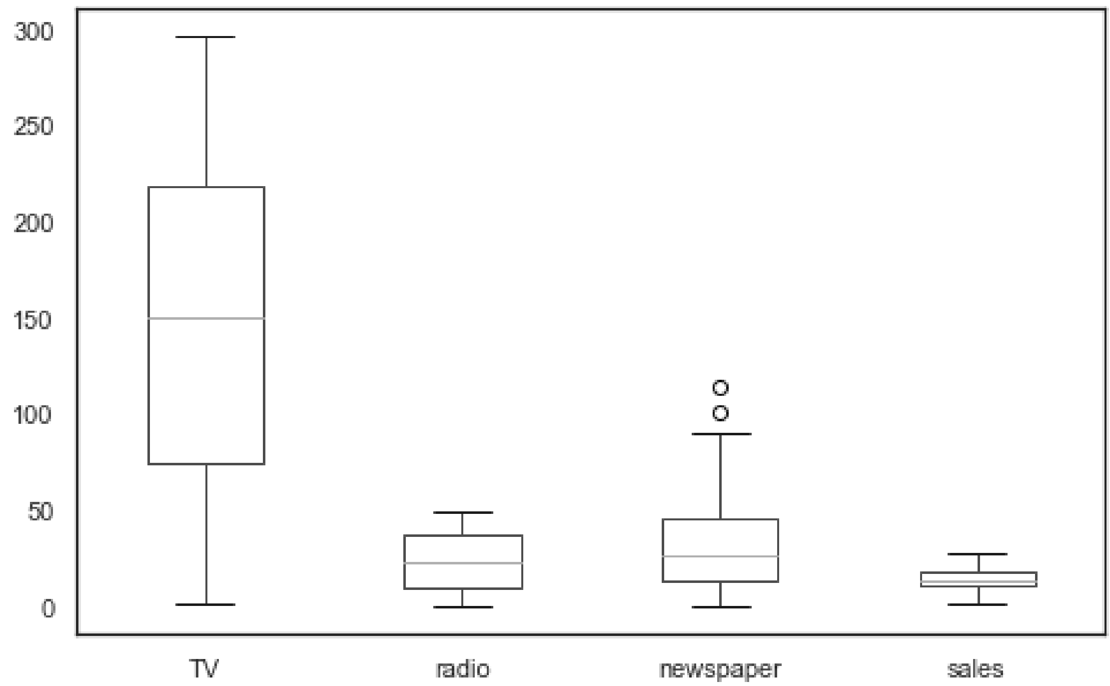

```
[62]: df.boxplot(column =['radio'], grid = False,figsize=(8,5))
```

```
[62]: <AxesSubplot:>
```



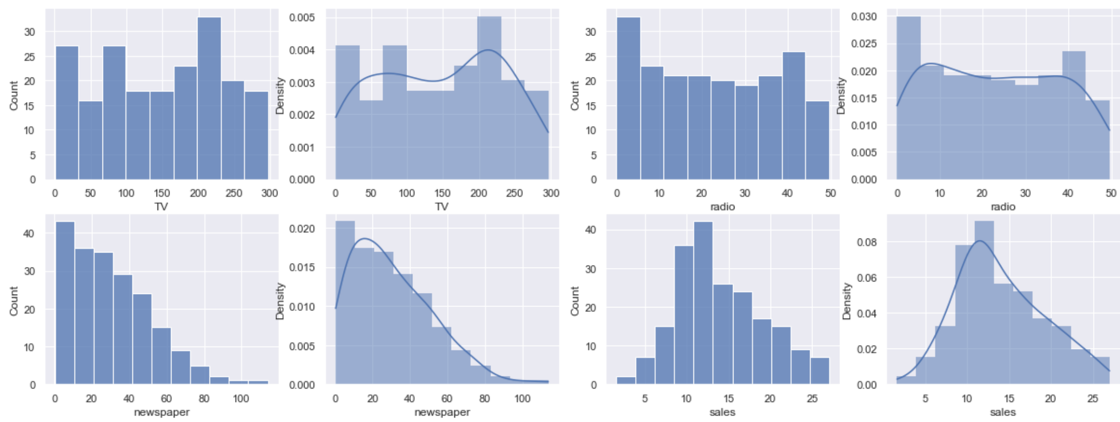
[63]:
```
df.boxplot(grid = False,figsize=(8,5))
```

[63]: <AxesSubplot:>



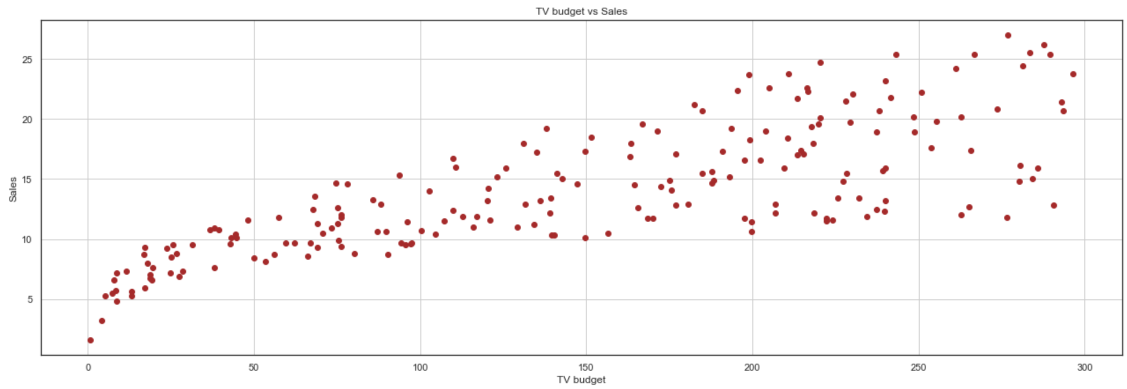
[64]:
```
# Distribution plot to find skewness
from pylab import *
sns.set(rc={"figure.figsize": (20, 15)});

subplot(4,4,1)
ax = sns.histplot(df["TV"])

subplot(4,4,2)
ax = sns.histplot(df["TV"], kde=True, stat="density", linewidth=0)

subplot(4,4,3)
ax = sns.histplot(df["radio"])

subplot(4,4,4)
ax = sns.histplot(df["radio"], kde=True, stat="density", linewidth=0)

subplot(4,4,5)
ax = sns.histplot(df["newspaper"])
```

```
subplot(4,4,6)
ax = sns.histplot(df["newspaper"], kde=True, stat="density", linewidth=0)

subplot(4,4,7)
ax = sns.histplot(df["sales"])

subplot(4,4,8)
ax = sns.histplot(df["sales"], kde=True, stat="density", linewidth=0)

plt.show()
```

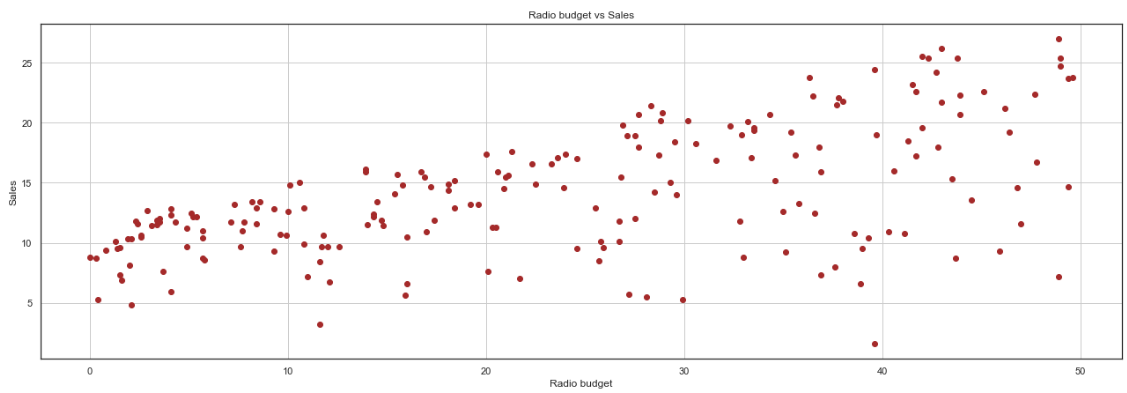


Bivariate analysis

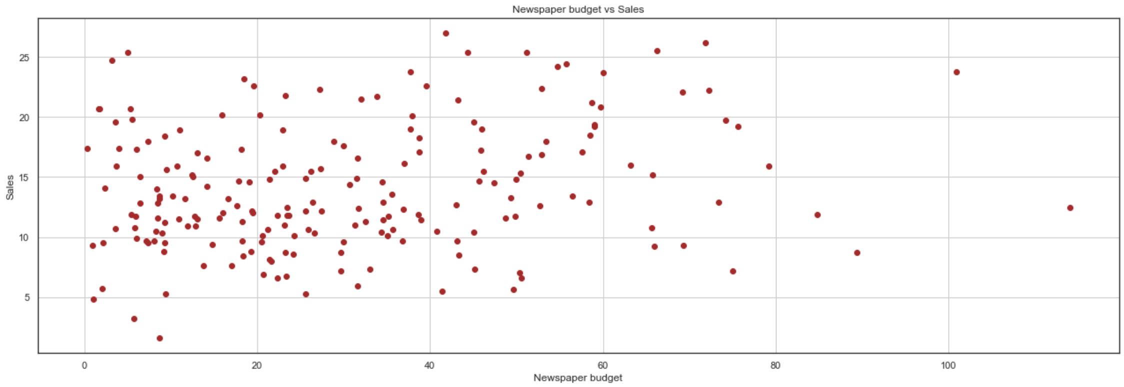
[65]:
```
# Scatter plot
sns.set_style(style='white')
fig = plt.figure(figsize=(22,7))
plt.scatter(df["TV"],df["sales"],color="brown")
plt.grid(b=None)
plt.xlabel("TV budget")
plt.ylabel("Sales")
plt.title("TV budget vs Sales")
plt.show()
```



[66]:
```
sns.set_style(style='white')
fig = plt.figure(figsize=(22,7))
plt.scatter(df["radio"],df["sales"],color="brown")
plt.grid(b=None)
plt.xlabel("Radio budget")
plt.ylabel("Sales")
plt.title("Radio budget vs Sales")
plt.show()
```



[67]:
```
sns.set_style(style='white')
fig = plt.figure(figsize=(22,7))
plt.scatter(df["newspaper"],df["sales"],color="brown")
plt.grid(b=None)
plt.xlabel("Newspaper budget")
plt.ylabel("Sales")
plt.title("Newspaper budget vs Sales")
plt.show()
```



[68]:
```
# Correlation
df.corr()
# Spearman's rho
# df.corr(method='spearman')
# Kendall's tau
# df.corr(method='kendall')
```

[68]:

	TV	radio	newspaper	sales
TV	1.000000	0.054809	0.056648	0.782224
radio	0.054809	1.000000	0.354104	0.576223
newspaper	0.056648	0.354104	1.000000	0.228299
sales	0.782224	0.576223	0.228299	1.000000

0.0.3 Step 3. Pre-process and extract the features

[69]:
```
# Load TV, and radio, and newspaper X1, X2, X3 as feature vector and sales into␣
↪Y variable
X = df.iloc[:,0:3].values     # TV, radio, newspaper
Y = df.iloc[:,3].values       # Sales
```

[70]:
```
X[:10,:]
```

[70]:
```
array([[230.1,  37.8,  69.2],
       [ 44.5,  39.3,  45.1],
       [ 17.2,  45.9,  69.3],
       [151.5,  41.3,  58.5],
       [180.8,  10.8,  58.4],
       [  8.7,  48.9,  75. ],
       [ 57.5,  32.8,  23.5],
       [120.2,  19.6,  11.6],
       [  8.6,   2.1,   1. ],
       [199.8,   2.6,  21.2]])
```

[71]:
```
Y[:10]
```

[71]:
```
array([22.1, 10.4,  9.3, 18.5, 12.9,  7.2, 11.8, 13.2,  4.8, 10.6])
```

input feature independent feature or predictor feature. Here, X1 (TV), X2 (Radio), X3 (Newspaper) are the input features. output feature dependent feature or response feature or target feature. Here, Y (sales) is the output feature.

0.0.4 Step 4. Split the data for training and testing

[72]:
```
# Splitting dataset into training and testing set
from sklearn.model_selection import train_test_split
x_train, x_test, y_train, y_test = train_test_split(X, Y, test_size = 0.2,␣
↪random_state = 0)
```

[73]:
```
x_train[:5,:]
```

[73]:
```
array([[ 36.9,  38.6,  65.6],
       [ 31.5,  24.6,   2.2],
       [142.9,  29.3,  12.6],
       [209.6,  20.6,  10.7],
       [215.4,  23.6,  57.6]])
```

[74]:
```
x_test[:5,:]
```

[74]:
```
array([[ 69.2,  20.5,  18.3],
       [ 50. ,  11.6,  18.4],
       [ 90.4,   0.3,  23.2],
       [289.7,  42.3,  51.2],
       [170.2,   7.8,  35.2]])
```

[75]:
```
y_train[:5]
```

[75]:
```
array([10.8,  9.5, 15. , 15.9, 17.1])
```

[76]:
```
y_test[:5]
```

[76]:
```
array([11.3,  8.4,  8.7, 25.4, 11.7])
```

0.0.5 Step 5: Training phase (bulding the model)

[77]:
```
# Fitting line on multiple dimensions on the training set
from sklearn import linear_model
model = linear_model.LinearRegression(normalize=True)
model.fit(x_train, y_train)
```

[77]: LinearRegression(normalize=True)

[78]:
```
m1, m2, m3 = model.coef_
```

[79]:
```
b = model.intercept_
```

[80]:
```
print("The multi-linear model is :")
print('\t\t\t\t Y = m1 X1 + m2 X2 + m3 X3 + b \n')
print('\t\t\t\t Y = ',round(m1,3),'X1 + ',round(m2,3),'X2 + ',round(m3,3),'X3 +␣
↪',round(b,3))
```

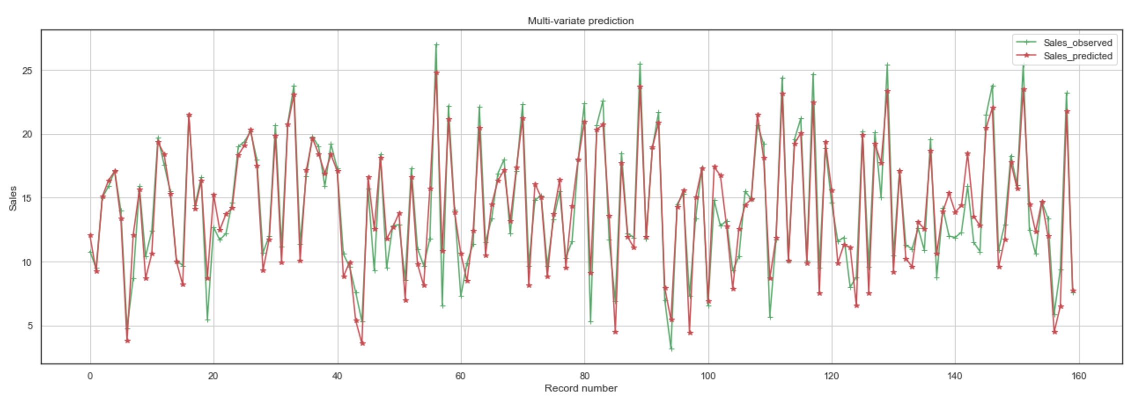
```
The multi-linear model is :
                                 Y = m1 X1 + m2 X2 + m3 X3 + b

                                 Y =  0.045 X1 +  0.196 X2 +  -0.003 X3 +  2.995
```

[81]:
```
# Predicting the output for training input
y_train_pred = model.predict(x_train)
```

Visualizing the model It involves 4 dimensions, so imagine yourself

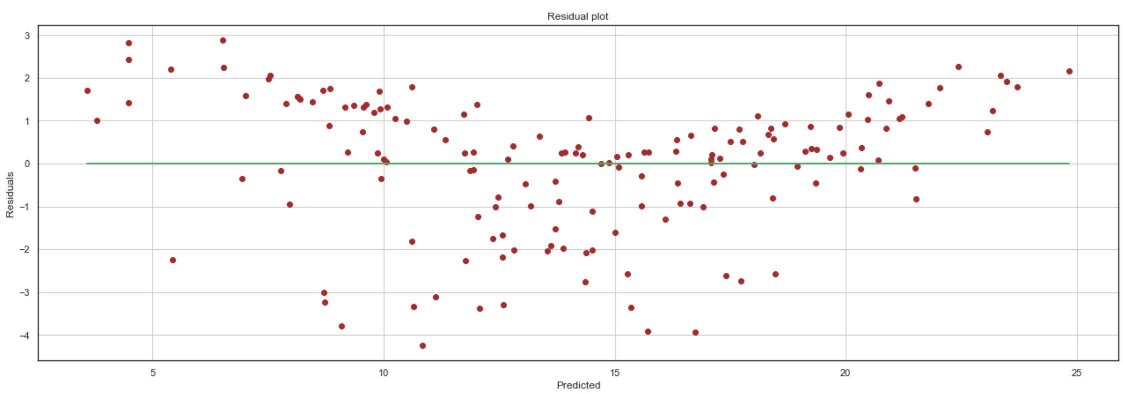
Plotting observed sale (x) and predicted sale (y) for training set

[82]:
```
x = np.arange(len(y_train_pred))
fig = plt.figure(figsize=(22,7))
plt.plot(x,y_train,"g-+",label="Sales_observed")
plt.plot(x,y_train_pred,"r-*",label="Sales_predicted")
plt.grid(b=None)
plt.xlabel("Record number")
plt.ylabel("Sales")
plt.title("Multi-variate prediction")
plt.legend()
plt.show()
```



Residual (Error) plot If the model has done good predictions, then the datapoints must be near around to horizontal line.

[83]:
```
sns.set_style(style='white')
fig = plt.figure(figsize=(22,7))
residuals = y_train-y_train_pred
zeros = y_train-y_train
plt.scatter(y_train_pred,residuals,color="brown")
plt.grid(b=None)
plt.plot(y_train_pred,zeros,"g")
plt.xlabel("Predicted")
plt.ylabel("Residuals")
plt.title("Residual plot")
plt.show()
```



0.0.6 Different error calculations to asses the model for training set

1. Sum of Squared Error (SSE)

$$SSE(m,b) = \sum_{i=1}^{n}(y_i - \hat{y})^2 = \sum_{i=1}^{n}(y_i - (m * x_i + b))^2 \tag{1}$$

[84]:
```
sum = 0
n = len(x_train)
for i in range (0,n):
  diff = y_train[i] - y_train_pred[i]
  squ_diff = diff**2
  sum = sum + squ_diff
Train_SSE = np.round(sum,2)
```

```python
print("Sum of Squared Error (SSE) :",Train_SSE)
```

```
Sum of Squared Error (SSE) : 385.09
```

2. Mean Squared Error (MSE)

$$MSE(m,b) = \frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{n} = \frac{\sum_{i=1}^{n}(y_i - (m * x_i + b))^2}{n} \tag{2}$$

[85]:
```python
Train_MSE = np.round(Train_SSE/n,2)
print("Mean Squared Error (MSE) :",Train_MSE)
```

```
Mean Squared Error (MSE) : 2.41
```

3. Root Mean Squared Error (RMSE)

$$RMSE(m,b) = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{n}} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - (m * x_i + b))^2}{n}} \tag{3}$$

[86]:
```python
Train_RMSE = np.round(np.sqrt(Train_MSE),2)
print("Root Mean Squared Error (RMSE) :",Train_RMSE)
```

```
Root Mean Squared Error (RMSE) : 1.55
```

4. Mean Absolute Error (MAE)

$$MAE(m,b) = \frac{\sum_{i=1}^{n}|(y_i - \hat{y})|}{n} \tag{4}$$

[87]:
```python
sum = 0
n = len(x_train)
for i in range (0,n):
  diff = y_train[i] - y_train_pred[i]
  sum = sum + np.abs(diff)
Train_MAE = np.round(sum/n,2)
print("Mean Absolute Error (MAE) :",Train_MAE)
```

```
Mean Absolute Error (MAE) : 1.21
```

5. Mean Absolute Percentage Error (MAPE)

$$MAPE(m,b) = \frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{(y_i - \hat{y})}{y_i}\right| = \frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{(y_i - (m * x_i + b))}{y_i}\right| \tag{5}$$

[88]:
```
sum = 0
n = len(x_train)
for i in range (0,n):
  diff = (y_train[i] - y_train_pred[i])/y_train[i]
  sum = sum + np.abs(diff)
Train_MAPE = np.round(sum/n*100,2)
print("Mean Absolute Percentage Error (MAPE) :",Train_MAPE)
```

```
Mean Absolute Percentage Error (MAPE) : 11.43
```

0.0.7 Calculating R-Squred value (goodness of model) using SSE

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{6}$$

[89]:
```
from sklearn.metrics import r2_score
out = r2_score(y_train,y_train_pred)
Train_RS = np.round(out,2)*100
print("R-Squred value (goodness of model) for training set :",Train_RS,"%")
```

```
R-Squred value (goodness of model) for training set : 91.0 %
```

0.0.8 Calculating Adjusted R-Squred value (goodness of model) using SSE

$$R^2 = 1 - (1 - R^2)\frac{(n-1)}{n-p-1} \tag{7}$$

[90]:
```
out = 1 - (1-Train_RS)*(len(y_train)-1)/(len(y_train)-X.shape[1]-1)
Train_Adj_RS = round(out,2)
print("Adjusted R-Squred value (goodness of model) for training set :
 ↪",Train_Adj_RS,"%")
```

```
Adjusted R-Squred value (goodness of model) for training set : 92.73 %
```

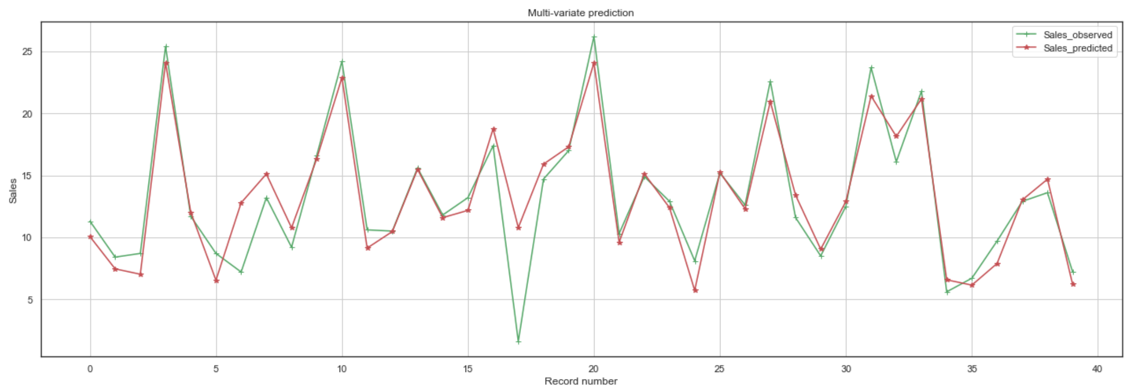
0.0.9 Step 6: Testing phase

[91]:
```
# Predicting values for test input set
y_test_pred = model.predict(x_test)
```

Visualizing the model It involves 4 dimensions, so imagine yourself

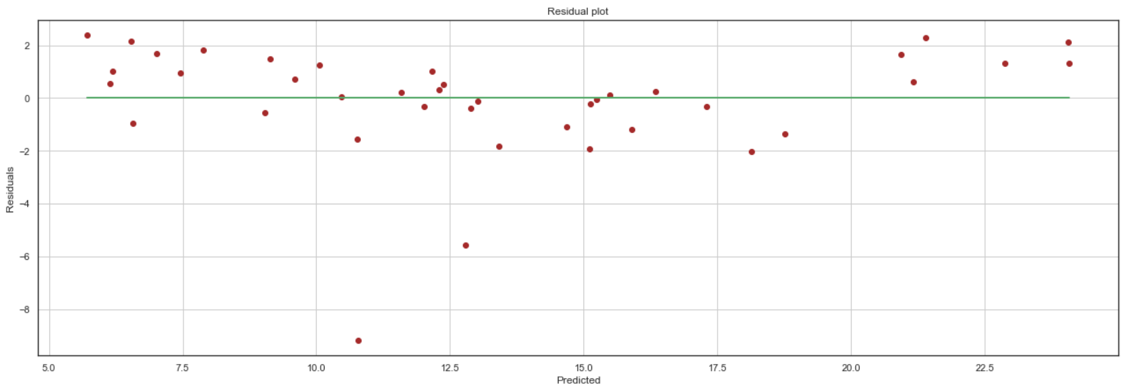
Plotting observed sale (x) and predicted sale (y) for test set

[92]:
```
x = np.arange(len(y_test_pred))
fig = plt.figure(figsize=(22,7))
plt.plot(x,y_test,"g-+",label="Sales_observed")
plt.plot(x,y_test_pred,"r-*",label="Sales_predicted")
plt.grid(b=None)
plt.xlabel("Record number")
plt.ylabel("Sales")
plt.title("Multi-variate prediction")
plt.legend()
plt.show()
```



Residual (Error) plot If the model has done good predictions, then the datapoints must be near around to horizontal line.

[93]:
```
sns.set_style(style='white')
fig = plt.figure(figsize=(22,7))
residuals = y_test-y_test_pred
zeros = y_test-y_test
plt.scatter(y_test_pred,residuals,color="brown")
plt.grid(b=None)
plt.plot(y_test_pred,zeros,"g")
plt.xlabel("Predicted")
plt.ylabel("Residuals")
plt.title("Residual plot")
plt.show()
```



Storing the outcome in a file

[94]:
```
# Store the predicted value for sales in new column
df.rename(columns={'sales': 'observed_sales'}, inplace=True)
sales_data = df.iloc[:,0:3]
predicted_values = model.predict(sales_data)
df['predicted_sales'] = predicted_values
df.head()
```

[94]:

	TV	radio	newspaper	observed_sales	predicted_sales
0	230.1	37.8	69.2	22.1	20.488787
1	44.5	39.3	45.1	10.4	12.575771
2	17.2	45.9	69.3	9.3	12.588197
3	151.5	41.3	58.5	18.5	17.701984
4	180.8	10.8	58.4	12.9	13.015414

[95]:
```
# Write the above output input into new csv
# df.to_csv("Multi Linear Regression Output.csv")
```

0.0.10 Different error calculations to asses the model

1. Sum of Squared Error (SSE)

$$SSE(m,b) = \sum_{i=1}^{n}(y_i - \hat{y})^2 = \sum_{i=1}^{n}(y_i - (m * x_i + b))^2 \tag{8}$$

[96]:
```
sum = 0
n = len(x_test)
for i in range (0,n):
  diff = y_test[i] - y_test_pred[i]
  squ_diff = diff**2
  sum = sum + squ_diff
```

```
Test_SSE = np.round(sum,2)
print("Sum of Squared Error (SSE) :",Test_SSE)
```

```
Sum of Squared Error (SSE) : 176.08
```

2. Mean Squared Error (MSE)

$$MSE(m,b) = \frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{n} = \frac{\sum_{i=1}^{n}(y_i - (m*x_i + b))^2}{n} \tag{9}$$

[97]:
```
Test_MSE = np.round(Train_SSE/n,2)
print("Mean Squared Error (MSE) :",Test_MSE)
```

```
Mean Squared Error (MSE) : 9.63
```

3. Root Mean Squared Error (RMSE)

$$RMSE(m,b) = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{n}} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - (m*x_i + b))^2}{n}} \tag{10}$$

[98]:
```
Test_RMSE = np.round(np.sqrt(Test_MSE),2)
print("Root Mean Squared Error (RMSE) :",Test_RMSE)
```

```
Root Mean Squared Error (RMSE) : 3.1
```

4. Mean Absolute Error (MAE)

$$MAE(m,b) = \frac{\sum_{i=1}^{n}|(y_i - \hat{y})|}{n} \tag{11}$$

[99]:
```
sum = 0
n = len(x_test)
for i in range (0,n):
  diff = y_test[i] - y_test_pred[i]
  sum = sum + np.abs(diff)
Test_MAE = np.round(sum/n,2)
print("Mean Absolute Error (MAE) :",Test_MAE)
```

```
Mean Absolute Error (MAE) : 1.36
```

5. Mean Absolute Percentage Error (MAPE)

$$MAPE(m,b) = \frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{(y_i - \hat{y})}{y_i}\right| = \frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{(y_i - (m*x_i + b))}{y_i}\right| \tag{12}$$

```
[100]: sum = 0
n = len(x_test)
for i in range (0,n):
  diff = (y_test[i] - y_test_pred[i])/y_test[i]
  sum = sum + np.abs(diff)
Test_MAPE = np.round(sum/n*100,2)
print("Mean Absolute Percentage Error (MAPE) :",Test_MAPE)
```

```
Mean Absolute Percentage Error (MAPE) : 24.61
```

0.0.11 Calculating R-Squred value (goodness of model) using SSE

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{13}$$

```
[101]: from sklearn.metrics import r2_score
out = r2_score(y_test,y_test_pred)
Test_RS = np.round(out,2)*100
print("R-Squred value (goodness of model) for testing set :",Test_RS,"%")
```

```
R-Squred value (goodness of model) for testing set : 86.0 %
```

0.0.12 Calculating Adjusted R-Squred value (goodness of model) using SSE

$$R^2 = 1 - (1 - R^2)\frac{(n-1)}{n-p-1} \tag{14}$$

```
[102]: out = 1 - (1-Test_RS)*(len(y_test)-1)/(len(y_test)-X.shape[1]-1)
Test_Adj_RS = round(out,2)
print("Adjusted R-Squred value (goodness of model) for testing set :
 ↪",Train_Adj_RS,"%")
```

```
Adjusted R-Squred value (goodness of model) for testing set : 92.73 %
```

0.0.13 Underfitting and overfitting observation

```
[103]: print("Error \t From training phase       From testing phase ")
print("=======================================================")
print("SSE   \t\t",Train_SSE,"\t\t", Test_SSE)
print("MSE   \t\t",Train_MSE,"\t\t\t", Test_MSE)
print("RMSE   \t\t",Train_RMSE,"\t\t\t", Test_RMSE)
print("MAE   \t\t",Train_MAE,"\t\t\t", Test_MAE)
print("RS   \t\t",Train_RS,"\t\t\t", Test_RS)
print("ARS   \t\t",Train_Adj_RS,"\t\t\t", Test_Adj_RS)
```

Error	From training phase	From testing phase
SSE	385.09	176.08
MSE	2.41	9.63
RMSE	1.55	3.1
MAE	1.21	1.36
RS	91.0	86.0
ARS	92.73	93.08