4.Gradient Descent for Math

October 17, 2021

Simple Gradient Descent for a Math Equation

[1]:
```
# Import necessary libraries
import numpy as np
import matplotlib.pyplot as plt
```

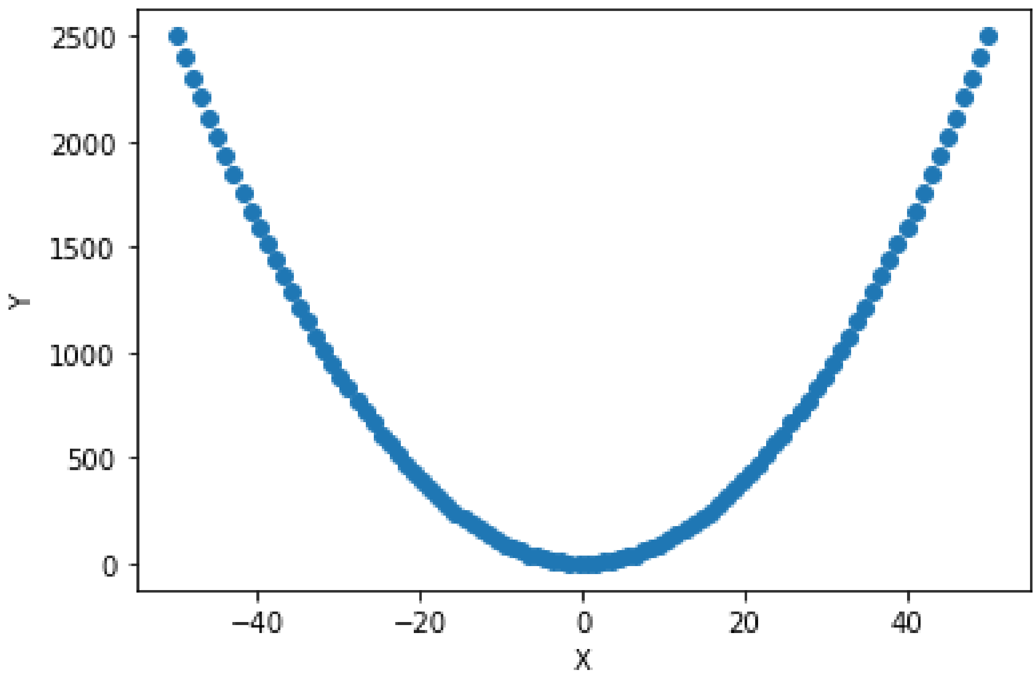
0.0.1 1. Gradient descent to find an optimal value for single variable in a function

$$f(x) = x^2 \tag{1}$$

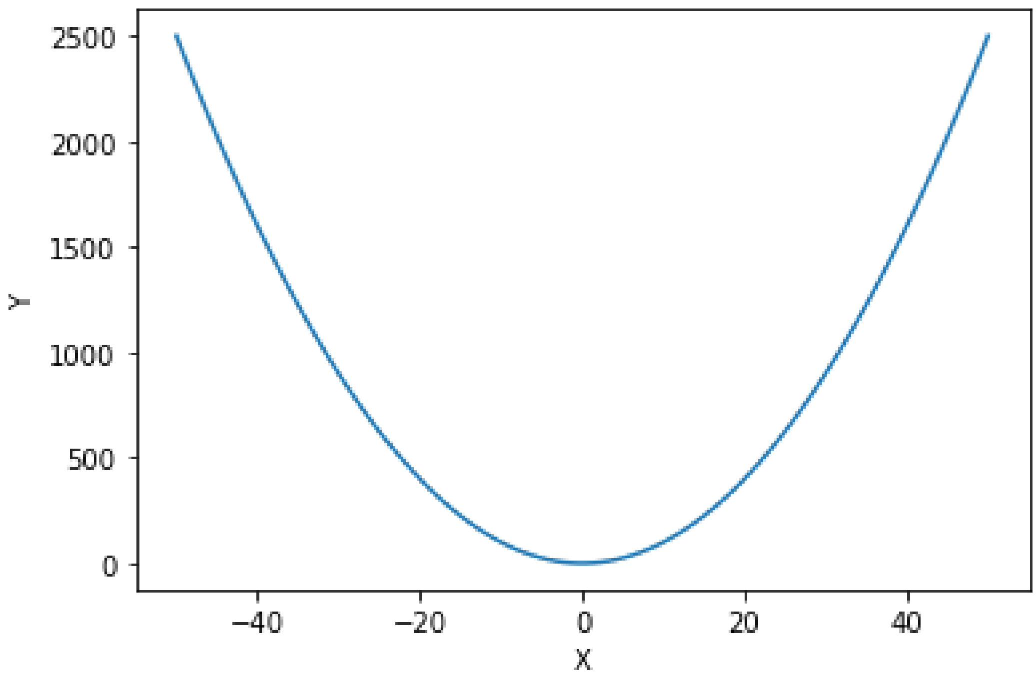
Dataset

[2]:
```
# Dataset
x = np.linspace(-50,50,100)  # generating 100 x values
y = x*x                      # finding y for each x by squaring it
```

[3]:
```
# Scatter plot
plt.scatter(x,y)
plt.xlabel("X")
plt.ylabel("Y")
plt.show()
```



[4]:
```
# Line plot: data points can be connected with a line that can help us to␣
↪approximate y value for x that is not in the 100 values
plt.plot(x,y)
plt.xlabel("X")
plt.ylabel("Y")
plt.show()
```



Objective is to find the x value between -50 to 50 that provides least value for the function

$$f(x) = x^2 \tag{2}$$

Parameter initialization

[5]:
```
# np.random.seed(13)
# range for input x
bounds = np.asarray([[-50, 50]])
# number of iterations (epochs)
epoch = 1000
# learning rate
learn_rate = 0.01
```

Objective and Derivative function

[6]:
```
# Objective function
def objective(x):
    return x**2

# Derivative of objective function
def derivative(x):
    return 2 * x
```

Gradient descent algorithm for 1 parameter

[7]:
```
# Gradient descent algorithm
def gradient_descent():
    # track all solutions
    solutions, scores = list(), list()

    # generate an initial point for x between the values given within bounds
    curr_soln = bounds[:, 0] + np.random.rand(len(bounds)) * (bounds[:, 1] -␣
↪bounds[:, 0])

    # run the gradient descent
    for i in range(epoch):

        # solution evaluation
        solution_eval = objective(curr_soln)

        # gradient calculation
        gradient = derivative(curr_soln)

        # step size calculation
        step_size = learn_rate * gradient

        # solution update
        curr_soln = curr_soln - step_size

        # store the solution
        solutions.append(curr_soln)
        scores.append(solution_eval)

        # report the progress
        # print('epoch >%d => f(%s) = %.5f' % (i, curr_soln, solution_eval))

    return [solutions, scores]
```

Training

[8]:
```
# perform the gradient descent search
solutions, scores = gradient_descent()
```

Solution

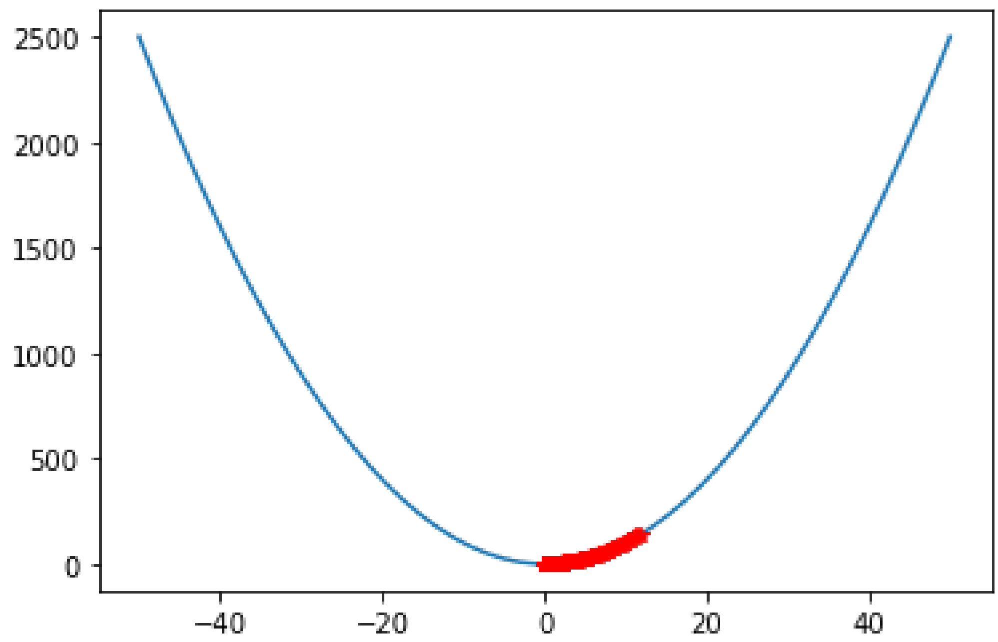
[9]:
```
solutions[epoch-1]
```

[9]:
```
array([2.0154804e-08])
```

Convergence plot

[10]:
```
# sample input range uniformly at 0.1 increments
inputs = np.linspace(bounds[0,0], bounds[0,1], 50)
# compute targets
results = objective(inputs)

# create a line plot of input vs result
plt.plot(inputs, results)
# plot the solutions found
plt.plot(solutions, scores, 'r*')
plt.show()
```



0.0.2 2. Gradient descent to find optimal values for 2 variables in a function

$$f(m,b) = b^2 + m^2 \tag{3}$$

Dataset

[11]:
```
m = np.linspace(-1,1,100)    # creating 100 m values
b = np.linspace(-1,1,100)    # creating 100 b values
```

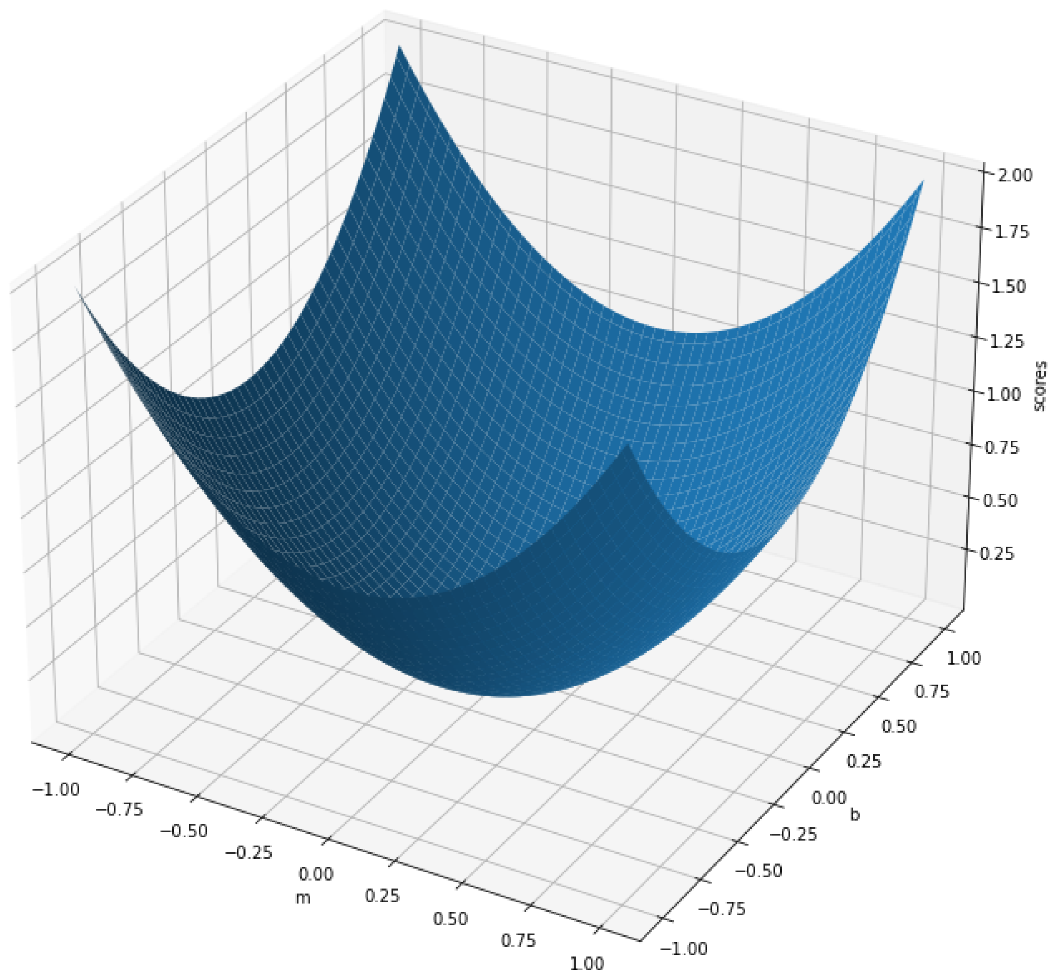
[12]:
```
def objective(m,b):
    return m**2 + b**2
```

[13]:
```
from mpl_toolkits.mplot3d import Axes3D
```

```python
# plotting 3D
fig = plt.figure(figsize=(30,12))
ax = plt.axes(projection = '3d')

axis_m, axis_b = np.meshgrid(m, b)
axis_scores = objective(axis_m,axis_b)
ax.plot_surface(axis_m, axis_b,axis_scores)

ax.set_xlabel('m')
ax.set_ylabel('b')
ax.set_zlabel('scores')
plt.show()
```



Objective is to find optimal values for m and b between -50 to 50 that provides least value for the function

$$f(m, b) = b^2 + m^2 \tag{4}$$

Parameter initialization

[14]:
```
# np.random.seed(13)
# define range for input
bounds = np.asarray([[-50, 50]])
# number of iterations (epochs)
epoch = 100
# learning rate
learn_rate = 0.01
```

Objective and Derivative function

[15]:
```
# Objective function
def objective(m,b):
    return m**2 + b**2
```

[16]:
```
# Partial derivative of f(m,b) with respect to m
def deriv_m(m):
    return 2*m

# Partial derivative of f(m,b) with respect to m
def deriv_b(b):
    return 2*b
```

Gradient descent algorithm for 2 parameters

[17]:
```
# Gradient descent algorithm
def gradient_descent():
    # track all solutions
    solutions_m, solutions_b, scores = list(), list(), list()

    # generate an initial point for x between the values given within bounds
    curr_soln_m = bounds[:, 0] + np.random.rand(len(bounds)) * (bounds[:, 1] -␣
↪bounds[:, 0])
    curr_soln_b = bounds[:, 0] + np.random.rand(len(bounds)) * (bounds[:, 1] -␣
↪bounds[:, 0])

    # run the gradient descent
    for i in range(epoch):

        # solution evaluation
        solution_eval = objective(curr_soln_m, curr_soln_b)

        # gradient calculation
```

```
        gradient_m = deriv_m(curr_soln_m)
        gradient_b = deriv_b(curr_soln_b)

        # step size calculation
        step_size_m = learn_rate * gradient_m
        step_size_b = learn_rate * gradient_b

        # solution update
        curr_soln_m = curr_soln_m - step_size_m
        curr_soln_b = curr_soln_b - step_size_b

        # store the solution
        solutions_m.append(curr_soln_m)
        solutions_b.append(curr_soln_b)
        scores.append(solution_eval)

        # report the progress
        # print('>epoch %d => f(%.5f  %.5f) = %.5f' % (i, curr_soln_m,␣
↪curr_soln_b, solution_eval))

    return [solutions_m, solutions_b, scores]
```

Training

[18]:
```
# perform the gradient descent search
solutions_m, solutions_b, scores = gradient_descent()
```

Soution

[19]:
```
print("Solution for m = ",solutions_m[epoch-1])
print("Solution for b = ",solutions_b[epoch-1])
```

```
Solution for m =  [1.3228589]
Solution for b =  [-2.95775758]
```

Convergence plot

[20]:
```
from mpl_toolkits.mplot3d import Axes3D

# setting up x and y axis tick value range
bounds_m = np.asarray([[min(solutions_m), max(solutions_m)]])
bounds_b = np.asarray([[min(solutions_b), max(solutions_b)]])

# generating tick values
axis_m = np.linspace(bounds_m[0,0], bounds_m[0,1], 50)
axis_b = np.linspace(bounds_b[0,0], bounds_b[0,1], 50)
```

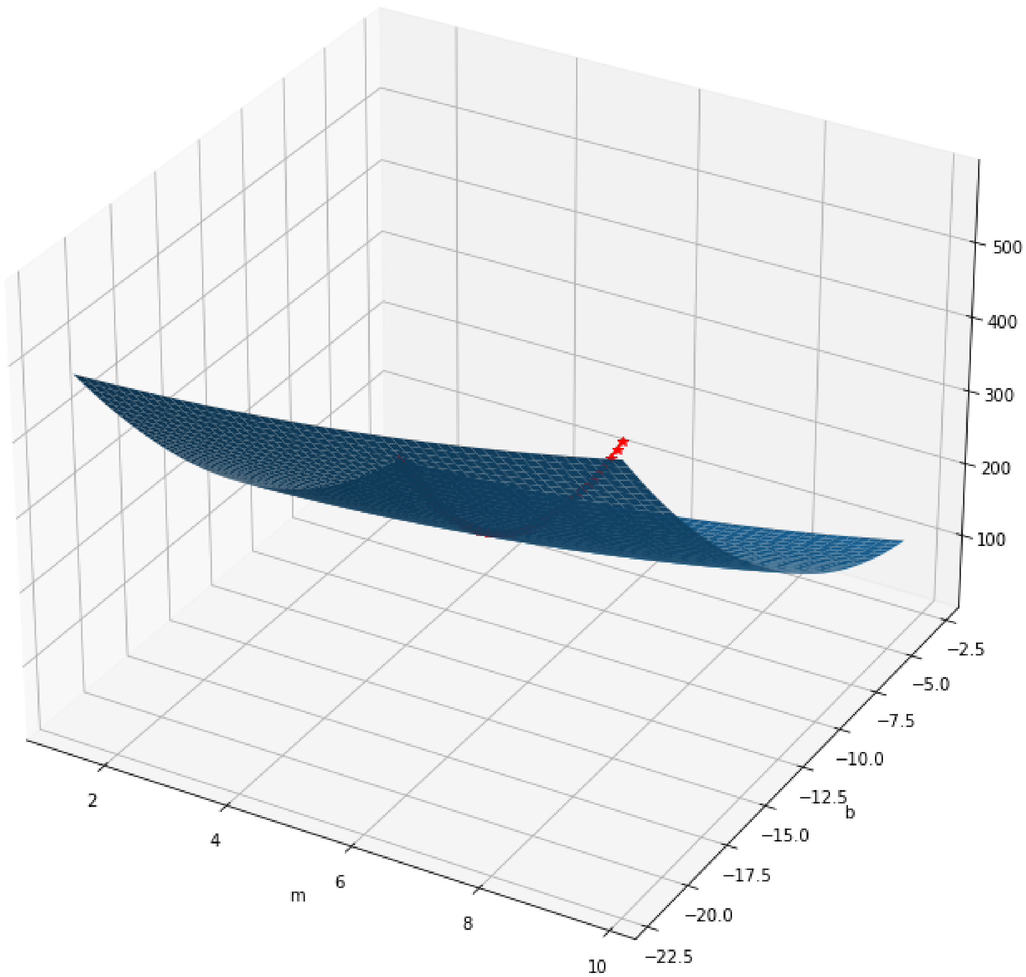
```
# to plot line in 3D with our outcome values
solutions_m = np.array(solutions_m).flatten()
solutions_b = np.array(solutions_b).flatten()
scores = np.array(scores).flatten()

# plotting 3D
fig = plt.figure(figsize=(30,12))
ax = plt.axes(projection = '3d')

axis_m, axis_b = np.meshgrid(axis_m, axis_b)
axis_scores = objective(axis_m,axis_b)

ax.plot_surface(axis_m, axis_b,axis_scores)
ax.plot3D(solutions_m,solutions_b,scores,'r*-')

ax.set_xlabel('m')
ax.set_ylabel('b')
ax.set_zlabel('scores')
plt.show()
```



0.0.3 3. Gradient descent to find optimal value for b and m in SSE

Objective is to find the optimal values for m and b that causes least SSE SSE is

$$SSE(m,b) = \sum_{i=1}^{n}(y_i - (m * x_i + b))^2 \tag{5}$$

Partial derivative of m is

$$\frac{\partial}{\partial m}SSE(m,b) = -2 * \sum_{i=1}^{n} x_i(y_i - (m * x_i + b)) \tag{6}$$

Partial derivative of b is

$$\frac{\partial}{\partial b}SSE(m,b) = -2 * \sum_{i=1}^{n}(y_i - (m * x_i + b)) \tag{7}$$

Dataset

[21]:
```
# Dataset
x = np.array([1,2,3,4,5])
y = np.array([5,7,9,11,13])
```

Parameter initialization

[22]:
```
# np.random.seed(13)
# total iterations (epoch)
epoch = 1000
# learning rate
learn_rate = 0.001
```

Objective, Derivative, Loss (error/cost) function

[23]:
```
# Prediction function
def predict(m, b, x):
    return m*x+b
```

[24]:
```
# Partial derivative of SSE(m,b) with respect to m
def deriv_m(x, y, y_predicted):
    return -2*(x*(y-y_predicted)).sum()

# Partial derivative of SSE(m,b) with respect to m
def deriv_b(x, y, y_predicted):
    return  -2*(y-y_predicted).sum()
```

[25]:
```
# SSE (cost/loss/error) calculation
def cost_fun(y,y_predicted):
    error = (y-y_predicted)**2
    SSE = error.sum()
    return SSE
```

Gradient descent algorithm for 2 parameters

[26]:
```
# Gradient descent algorithm
def gradient_descent():
    # track all solutions
    solutions_m, solutions_b, cost = list(), list(), list()

    # generate an initial point for x between the values given within bounds
```

```
    curr_soln_m = 1
    curr_soln_b = 0

    # run the gradient descent
    for i in range(epoch):

        # prediction
        y_predicted = predict(curr_soln_m, curr_soln_b, x)

        # gradient calculation
        gradient_m = deriv_m(x, y, y_predicted)
        gradient_b = deriv_b(x, y, y_predicted)

        # step size calculation
        step_size_m = learn_rate * gradient_m
        step_size_b = learn_rate * gradient_b

        # solution update
        curr_soln_m = curr_soln_m - step_size_m
        curr_soln_b = curr_soln_b - step_size_b

        # SSE (error/cost/loss) calculation
        SSE = cost_fun(y,y_predicted)

        # store the solution
        solutions_m.append(curr_soln_m)
        solutions_b.append(curr_soln_b)
        cost.append(SSE)

        # report the progress
        # print('>epoch %d => m %.5f b %.5f cost %.3f ' % (i, curr_soln_m,␣
↪curr_soln_b,SSE))

    return [solutions_m, solutions_b, cost]
```

Training

[27]:
```
# perform the gradient descent search
solutions_m, solutions_b, cost = gradient_descent()
```

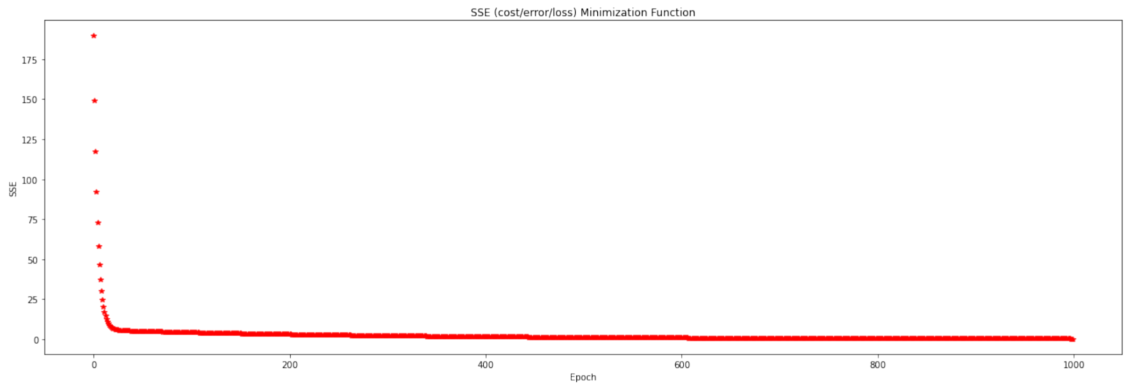
Solution

[28]:
```
m = solutions_m[epoch-1]
b = solutions_b[epoch-1]
print("y = m x + b ==>  y = ",round(m,2)," x + ",round(b,2))
```

```
y = m x + b ==>  y =  2.13  x +  2.53
```

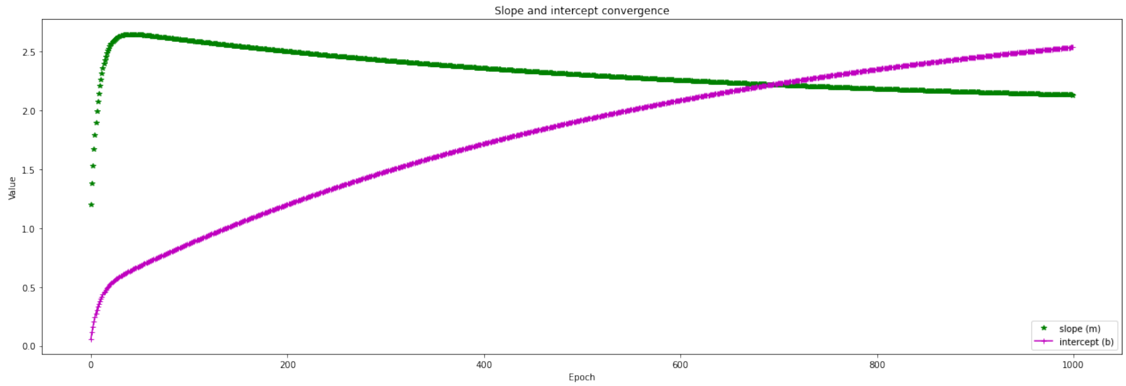
SSE (cost/error/loss) convergence graph

[29]:
```
# Plotting SSE function minimization
x = np.arange(epoch)
fig = plt.figure(figsize=(22,7))
plt.plot(x,cost,"r*")
plt.xlabel("Epoch")
plt.ylabel("SSE")
plt.title("SSE (cost/error/loss) Minimization Function")
plt.show()
```



Slope(m) and intercept(b) convergence graph

[30]:
```
# Plotting slope (m) and intercept (b) convergence
x = np.arange(epoch)
fig = plt.figure(figsize=(22,7))
plt.plot(x,solutions_m,"g*",label="slope (m)")
plt.plot(x,solutions_b,"m+-",label="intercept (b)")
plt.xlabel("Epoch")
plt.ylabel("Value")
plt.title("Slope and intercept convergence")
plt.legend(loc="best")
plt.show()
```


Slope and intercept convergence
Value
Epoch
0.0
0.5
1.0
1.5
2.0
2.5
0
200
400
600
800
1000
slope (m)
intercept (b)