

2.Linear Regression From Scratch

October 17, 2021

Simple Linear Regression without Sci-Kit Learn

```
[1]: # Import necessary package
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

0.0.1 Step 1: Load the dataset

```
[2]: # Load the dataset into pandas dataframe
df = pd.read_csv("E:\\MY LECTURES\\DATA SCIENCE\\3.
↳Programs\\dataset\\Advertising.csv")
# Change this location based on the location of dataset in your machine
```

```
[3]: # Display the first five records
df.head()
```

```
[3]:
```

	TV	radio	newspaper	sales
0	230.1	37.8	69.2	22.1
1	44.5	39.3	45.1	10.4
2	17.2	45.9	69.3	9.3
3	151.5	41.3	58.5	18.5
4	180.8	10.8	58.4	12.9

Advertising data comprises four features: TV, radio, newspaper, and sales. It explains the budget (in 1000\$) spent on different mass media and the net outcome for every week.

sales for a product (output/dependent/target variable).

advertising budget for TV, radio, and newspaper media (input/independent/target variable).

Planning to perform regression on TV budget (X) as input and sales (Y) as output.

```
[4]: # Load TV into X and sales into Y variable
X = df.iloc[:,0].values          # Budget spent on TV
Y = df.iloc[:,3].values          # Sales
```

```
[5]: def disp_data(feature1,feature2):
    print('Displaying first 10 records')
    print('-----')
    print('TV budget (X)', '|', 'Sales (Y)')
    print('-----')
    count = 0
    for x,y in zip(feature1,feature2):
        if count == 10:
            break
        else:
            print(x, '      ', y)
            count = count + 1
disp_data(X,Y)
```

Displaying first 10 records

```
-----
TV budget (X) | Sales (Y)
-----
230.1          22.1
44.5           10.4
17.2           9.3
151.5          18.5
180.8          12.9
8.7            7.2
57.5           11.8
120.2          13.2
8.6            4.8
199.8          10.6
```

Row <=> record, tuple, instance, sample, observation, object, case, entity Column <=> attribute, variable, field, feature, characteristic, dimension input feature <=> independent feature or predictor feature. Here, X1 (TV) is the input feature. output feature <=> dependent feature or response feature or target feature. Here, Y (sales) is the output feature.

0.0.2 Step 2: Split the data for training and testing

```
[6]: # Dataset shape (number of rows and columns)
df.shape
```

```
[6]: (200, 4)
```

```
[7]: # Splitting dataset into training set (80%) and testing set (20%)
x_train = X[0:160]
x_test = X[160:]
y_train = Y[0:160]
y_test = Y[160:]
```

```
[8]: print("Training data")
      print("=====")
      disp_data(x_train,y_train)
```

Training data

=====

Displaying first 10 records

TV budget (X) | Sales (Y)

230.1	22.1
44.5	10.4
17.2	9.3
151.5	18.5
180.8	12.9
8.7	7.2
57.5	11.8
120.2	13.2
8.6	4.8
199.8	10.6

```
[9]: print("Testing data")
      print("=====")
      disp_data(x_test,y_test)
```

Testing data

=====

Displaying first 10 records

TV budget (X) | Sales (Y)

172.5	14.4
85.7	13.3
188.4	14.9
163.5	18.0
117.2	11.9
234.5	11.9
17.9	8.0
206.8	12.2
215.4	17.1
284.3	15.0

0.0.3 Step 3: Training phase (bulding a model)

Variance function

```
[10]: def variance(values, mean):
        var = 0
        for val in values:
            var = var + (val-mean)**2
        return var
```

Co-variance function

```
[11]: def covariance(X_val, X_val_mean, Y_val , Y_val_mean):
        covariance = 0
        for r in range(len(X_val)):
            covariance = covariance + (X_val[r] - X_val_mean) * (Y_val[r] -
↪Y_val_mean)
        return covariance
```

The linear model $Y = m X + b$

```
[12]: x_train_mean = np.mean(x_train)
        y_train_mean = np.mean(y_train)
        x_train_var = variance(x_train,x_train_mean)
        x_train_y_test_cov = covariance(x_train, x_train_mean, y_train, y_train_mean)
```

$$m = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad (1)$$

```
[13]: m = x_train_y_test_cov / x_train_var
        b = y_train_mean - m * x_train_mean
```

```
[14]: print('Y = ',round(m,3),'X + ',round(b,3))
```

Y = 0.049 X + 7.069

Predicting value for training set using the model above

```
[15]: # Predicting the Training set results
        y_train_pred = np.zeros(len(x_train)).reshape(len(x_train),1)
        for x in range(len(x_train)):
            y_train_pred[x] = m * x_train[x] + b
```

Visualizing the model

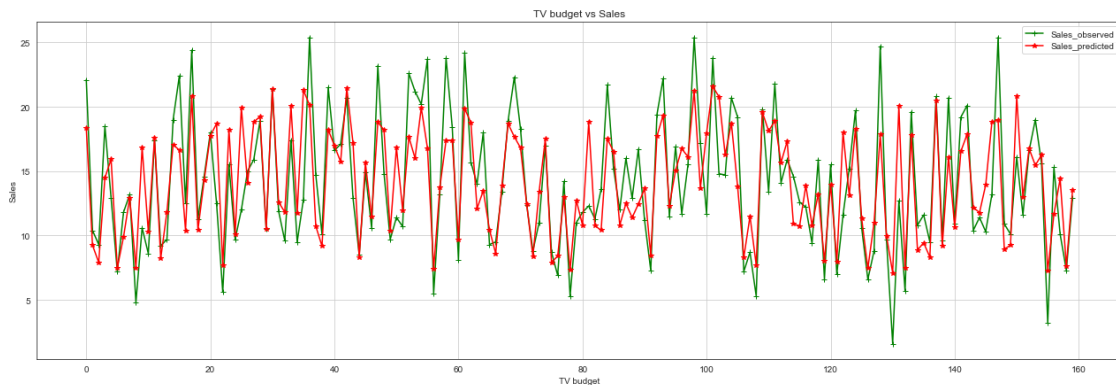
```
[16]: sns.set_style(style='white')
        fig = plt.figure(figsize=(22,7))
        plt.scatter(x_train,y_train,color="brown")
        plt.grid(b=None)
        plt.plot(x_train,y_train_pred,"g",label="Sales_predicted")
```

```
plt.xlabel("TV budget")
plt.ylabel("Sales")
plt.title("TV budget vs Sales")
plt.legend()
plt.show()
```



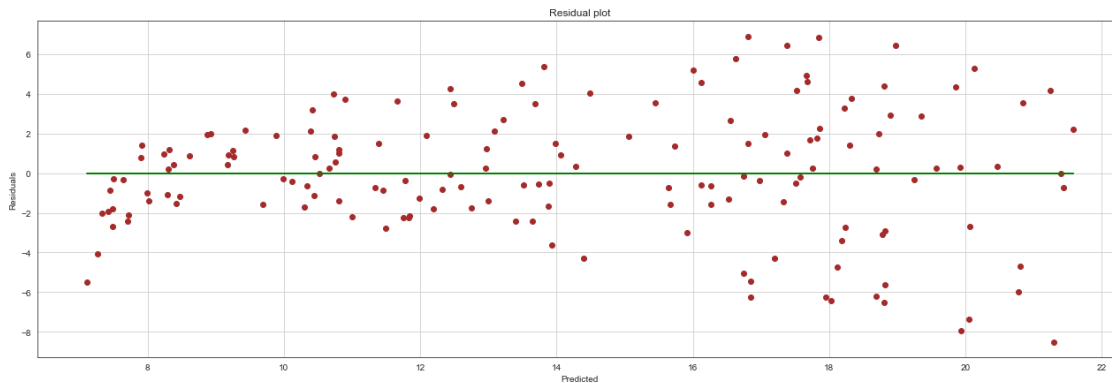
Plotting observed sale (x) and predicted sale (y) for training set

```
[17]: # Predicting the Test set results
x = np.arange(len(y_train_pred))
fig = plt.figure(figsize=(22,7))
plt.plot(x,y_train,"g-+",label="Sales_observed")
plt.plot(x,y_train_pred,"r-*",label="Sales_predicted")
plt.grid(b=None)
plt.xlabel("TV budget")
plt.ylabel("Sales")
plt.title("TV budget vs Sales")
plt.legend()
plt.show()
```



Residual (Error) plot If the model has done good predictions, then the datapoints must be near around to horizontal line.

```
[18]: sns.set_style(style='white')
fig = plt.figure(figsize=(22,7))
residuals = y_train.reshape((-1, 1))-y_train_pred
zeros = y_train-y_train
plt.scatter(y_train_pred,residuals,color="brown")
plt.grid(b=None)
plt.plot(y_train_pred,zeros,"g")
plt.xlabel("Predicted")
plt.ylabel("Residuals")
plt.title("Residual plot")
plt.show()
```



0.0.4 Different error calculations to asses the model for training set

1. Sum of Squared Error (SSE)

$$SSE(m, b) = \sum_{i=1}^n (y_i - \hat{y})^2 = \sum_{i=1}^n (y_i - (m * x_i + b))^2 \quad (2)$$

```
[19]: sum = 0
n = len(x_train)
for i in range (0,n):
    diff = y_train[i] - y_train_pred[i]
    squ_diff = diff**2
    sum = sum + squ_diff
Train_SSE = np.round(sum,2)
print("Sum of Squared Error (SSE) :",Train_SSE)
```

Sum of Squared Error (SSE) : [1551.95]

2. Mean Squared Error (MSE)

$$MSE(m, b) = \frac{\sum_{i=1}^n (y_i - \hat{y})^2}{n} = \frac{\sum_{i=1}^n (y_i - (m * x_i + b))^2}{n} \quad (3)$$

```
[20]: Train_MSE = np.round(Train_SSE/n,2)
print("Mean Squared Error (MSE) :",Train_MSE)
```

Mean Squared Error (MSE) : [9.7]

3. Root Mean Squared Error (RMSE)

$$RMSE(m, b) = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y})^2}{n}} = \sqrt{\frac{\sum_{i=1}^n (y_i - (m * x_i + b))^2}{n}} \quad (4)$$

```
[21]: Train_RMSE = np.round(np.sqrt(Train_MSE),2)
print("Root Mean Squared Error (RMSE) :",Train_RMSE)
```

Root Mean Squared Error (RMSE) : [3.11]

4. Mean Absolute Error (MAE)

$$MAE(m, b) = \frac{\sum_{i=1}^n |(y_i - \hat{y})|}{n} \quad (5)$$

```
[22]: sum = 0
n = len(x_train)
for i in range (0,n):
    diff = y_train[i] - y_train_pred[i]
    sum = sum + np.abs(diff)
Train_MAE = np.round(sum/n,2)
print("Mean Absolute Error (MAE) :",Train_MAE)
```

Mean Absolute Error (MAE) : [2.43]

5. Mean Absolute Percentage Error (MAPE)

$$MAPE(m, b) = \frac{100\%}{n} \sum_{i=1}^n \left| \frac{(y_i - \hat{y})}{y_i} \right| = \frac{100\%}{n} \sum_{i=1}^n \left| \frac{(y_i - (m * x_i + b))}{y_i} \right| \quad (6)$$

```
[23]: sum = 0
n = len(x_train)
for i in range (0,n):
    diff = (y_train[i] - y_train_pred[i])/y_train[i]
    sum = sum + np.abs(diff)
```

```
Train_MAPE = np.round(sum/n*100,2)
print("Mean Absolute Percentage Error (MAPE) :",Train_MAPE)
```

Mean Absolute Percentage Error (MAPE) : [20.43]

0.0.5 Calculating R-Squared value (goodness of model) using SSE

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y})^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (7)$$

```
[24]: # SSE variance for mean model
y_train_mean = np.mean(y_train)
SSE_mean = variance(y_train,y_train_mean)
# SSE variance for fit model
y_train_pred_mean = np.mean(y_train_pred)
SSE_fit = variance(y_train_pred,y_train_pred_mean)
# calculating R_square
R = 1 - (SSE_fit/SSE_mean)
Train_RS = np.round(R,2)*100
print("R-Squared value (goodness of model) for training set :",Train_RS,"%")
```

R-Squared value (goodness of model) for training set : [36.] %

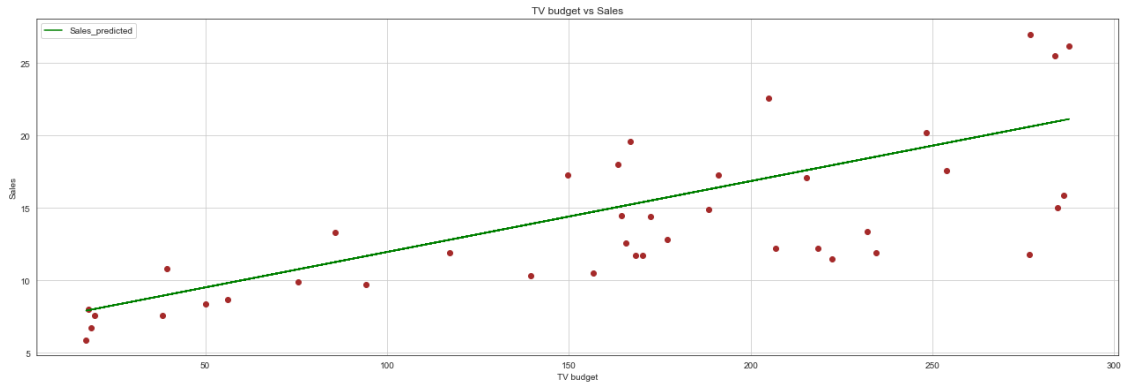
0.0.6 Step 6: Testing phase

Predicting value for training set using the model above

```
[25]: # Predicting values for test input set
y_test_pred = np.zeros(len(x_test)).reshape(len(x_test),1)
for x in range(len(x_test)):
    y_test_pred[x] = m * x_test[x] + b
```

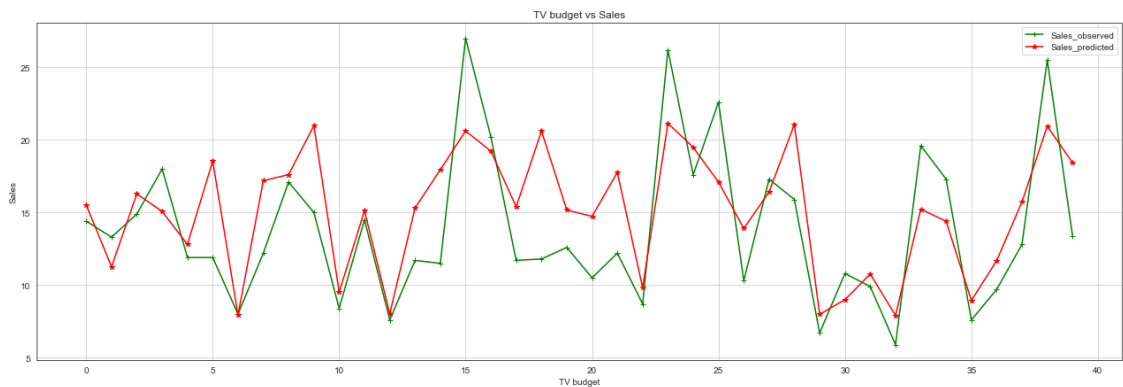
Visualizing the model

```
[26]: # Plotting the predicted values
sns.set_style(style='white')
fig = plt.figure(figsize=(22,7))
plt.scatter(x_test,y_test,color="brown")
plt.grid(b=None)
plt.plot(x_test,y_test_pred,"g",label="Sales_predicted")
plt.xlabel("TV budget")
plt.ylabel("Sales")
plt.title("TV budget vs Sales")
plt.legend()
plt.show()
```

Plotting observed sale (x) and predicted sale (y) for training set

```
[27]: x = np.arange(len(y_test_pred))
fig = plt.figure(figsize=(22,7))
plt.plot(x,y_test,"g-+",label="Sales_observed")
plt.plot(x,y_test_pred,"r-*",label="Sales_predicted")
plt.grid(b=None)
plt.xlabel("TV budget")
plt.ylabel("Sales")
plt.title("TV budget vs Sales")
plt.legend()
plt.show()
```



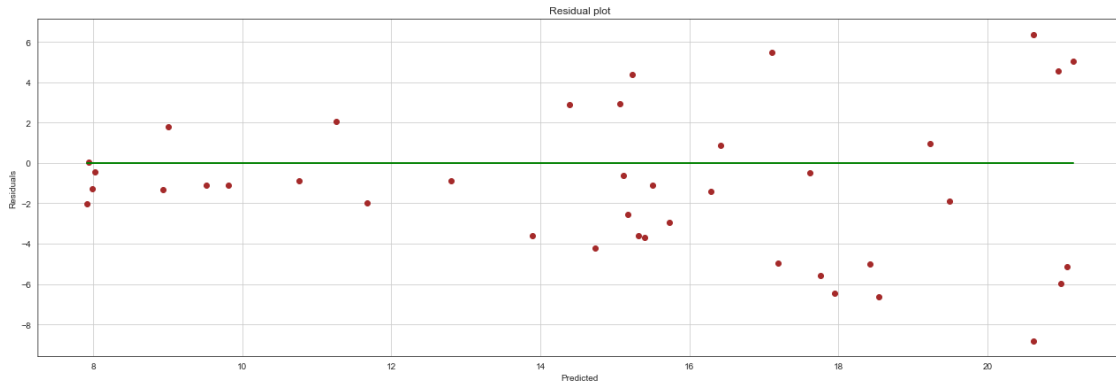
Residual (Error) plot If the model has done good predictions, then the datapoints must be near around to horizontal line.

```
[28]: sns.set_style(style='white')
fig = plt.figure(figsize=(22,7))
residuals = y_test.reshape((-1, 1))-y_test_pred
```

```

zeros = y_test-y_test
plt.scatter(y_test_pred,residuals,color="brown")
plt.grid(b=None)
plt.plot(y_test_pred,zeros,"g")
plt.xlabel("Predicted")
plt.ylabel("Residuals")
plt.title("Residual plot")
plt.show()

```



0.0.7 Different error calculations to asses the model for the test set

1. Sum of Squared Error (SSE)

$$SSE(m, b) = \sum_{i=1}^n (y_i - \hat{y})^2 = \sum_{i=1}^n (y_i - (m * x_i + b))^2 \quad (8)$$

```

[29]: sum = 0
n = len(x_test)
for i in range (0,n):
    diff = y_test[i] - y_test_pred[i]
    squ_diff = diff**2
    sum = sum + squ_diff
Test_SSE = np.round(sum,2)
print("Sum of Squared Error (SSE) :",Test_SSE)

```

Sum of Squared Error (SSE) : [565.15]

2. Mean Squared Error (MSE)

$$MSE(m, b) = \frac{\sum_{i=1}^n (y_i - \hat{y})^2}{n} = \frac{\sum_{i=1}^n (y_i - (m * x_i + b))^2}{n} \quad (9)$$

```
[30]: Test_MSE = np.round(Train_SSE/n,2)
print("Mean Squared Error (MSE) :",Test_MSE)
```

Mean Squared Error (MSE) : [38.8]

3. Root Mean Squared Error (RMSE)

$$RMSE(m,b) = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y})^2}{n}} = \sqrt{\frac{\sum_{i=1}^n (y_i - (m * x_i + b))^2}{n}} \quad (10)$$

```
[31]: Test_RMSE = np.round(np.sqrt(Test_MSE),2)
print("Root Mean Squared Error (RMSE) :",Test_RMSE)
```

Root Mean Squared Error (RMSE) : [6.23]

4. Mean Absolute Error (MAE)

$$MAE(m,b) = \frac{\sum_{i=1}^n |(y_i - \hat{y})|}{n} \quad (11)$$

```
[32]: sum = 0
n = len(x_test)
for i in range (0,n):
    diff = y_test[i] - y_test_pred[i]
    sum = sum + np.abs(diff)
Test_MAE = np.round(sum/n,2)
print("Mean Absolute Error (MAE) :",Test_MAE)
```

Mean Absolute Error (MAE) : [3.08]

5. Mean Absolute Percentage Error (MAPE)

$$MAPE(m,b) = \frac{100\%}{n} \sum_{i=1}^n \left| \frac{(y_i - \hat{y})}{y_i} \right| = \frac{100\%}{n} \sum_{i=1}^n \left| \frac{(y_i - (m * x_i + b))}{y_i} \right| \quad (12)$$

```
[33]: sum = 0
n = len(x_test)
for i in range (0,n):
    diff = (y_test[i] - y_test_pred[i])/y_test[i]
    sum = sum + np.abs(diff)
Test_MAPE = np.round(sum/n*100,2)
print("Mean Absolute Percentage Error (MAPE) :",Test_MAPE)
```

Mean Absolute Percentage Error (MAPE) : [23.03]

0.0.8 Calculating R-Squared value (goodness of model) using SSE

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y})^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (13)$$

```
[34]: # SSE variance for mean model
y_test_mean = np.mean(y_test)
SSE_mean = variance(y_test,y_test_mean)
# SSE variance for fit model
y_test_pred_mean = np.mean(y_test_pred)
SSE_fit = variance(y_test_pred,y_test_pred_mean)
# calculating R_square
R = 1 - (SSE_fit/SSE_mean)
Test_RS = np.round(R,2)*100
print("R-Squared value (goodness of model) for testing set :",Test_RS,"%")
```

R-Squared value (goodness of model) for testing set : [36.] %

0.0.9 Underfitting and overfitting observation

```
[35]: print("Error \t Train \t Test")
print("=====")
print("SSE \t",Train_SSE,"\t", Test_SSE)
print("MSE \t",Train_MSE,"\t\t", Test_MSE)
print("RMSE \t",Train_RMSE,"\t", Test_RMSE)
print("MAE \t",Train_MAE,"\t", Test_MAE)
print("RS \t",Train_RS,"\t\t", Test_RS)
```

Error	Train	Test
=====		
SSE	[1551.95]	[565.15]
MSE	[9.7]	[38.8]
RMSE	[3.11]	[6.23]
MAE	[2.43]	[3.08]
RS	[36.]	[36.]

Repeat this process by changing the values of parameters m and b until you get least SSE

Painful process, so we use Sci-Kit (SK) Learn library.