# 6.Stochastic Gradient Descent From Scratch

October 17, 2021

Linear Regression using Stochastic Gradient Descent

[50]:
```
# Import necessary package
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

### 0.0.1 Step 1: Load the dataset

[51]:
```
# Load the dataset into pandas dataframe
dataset = pd.read_csv("E:\\MY LECTURES\\DATA SCIENCE\\3.
↪Programs\\dataset\\Advertising.csv")
# Change this location based on the location of dataset in your machine
```

[52]:
```
# Display the first five records
dataset.head()
```

[52]:

| | TV | radio | newspaper | sales |
|---|---|---|---|---|
| 0 | 230.1 | 37.8 | 69.2 | 22.1 |
| 1 | 44.5 | 39.3 | 45.1 | 10.4 |
| 2 | 17.2 | 45.9 | 69.3 | 9.3 |
| 3 | 151.5 | 41.3 | 58.5 | 18.5 |
| 4 | 180.8 | 10.8 | 58.4 | 12.9 |

Advertising data comprises four features: TV, radio, newspaper, and sales. It explains the budget (in 1000$) spent on different mass media and the net outcome for every week.

sales for a product ( output/dependent/target variable ).

advertising budget for TV, radio, and newspaper media ( input/independent/target variable ).

Planning to perform regression on TV budget (X) as input and sales (Y) as output.

[53]:
```
# Dataset shape (number of rows and columns)
dataset.shape
```

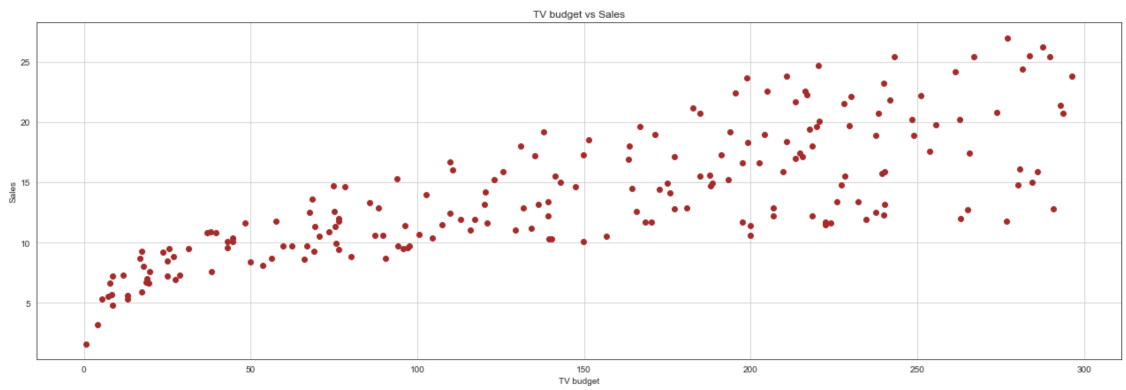
[53]: (200, 4)

Row <=> record, tuple, instance, sample, observation, object, case, entity Column <=> attribute, variable, field, feature, characteristic, dimension

### 0.0.2 Step 2. EDA

**Bivariate analysis**

[54]:
```python
# Scatter plot
sns.set_style(style='white')
fig = plt.figure(figsize=(22,7))
plt.scatter(dataset.TV,dataset.sales,color="brown")
plt.grid(b=None)
plt.xlabel("TV budget")
plt.ylabel("Sales")
plt.title("TV budget vs Sales")
plt.show()
```




### 0.0.3 Step 3. Pre-process and extract the features

[55]:
```python
# Normalize the data (converting features of different scale into values␣
↪between 0 and 1)
from sklearn import preprocessing
x = dataset.values                                  # returns a numpy array
min_max_scaler = preprocessing.MinMaxScaler()
x_scaled = min_max_scaler.fit_transform(x)
df = pd.DataFrame(x_scaled)
df.head()
```

[55]:

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0.775786 | 0.762097 | 0.605981 | 0.807087 |
| 1 | 0.148123 | 0.792339 | 0.394019 | 0.346457 |
| 2 | 0.055800 | 0.925403 | 0.606860 | 0.303150 |

```
3  0.509976  0.832661  0.511873  0.665354
4  0.609063  0.217742  0.510994  0.444882
```

### 0.0.4 Step 4. Split the data for training and testing

[56]:
```
# Splitting dataset into training and testing set
from sklearn.model_selection import train_test_split
# x_train, x_test, y_train, y_test = train_test_split(X, Y, test_size = 0.2,␣
↪random_state = 0)
# The above returns data as numpy array

# The following returns records in random as pandas.core.frame.DataFrame
train, test = train_test_split(df, test_size=0.2)
x_train = test[0].values     # TV
y_train = test[3].values     # sale
x_test = test[0].values      # TV
y_test = test[3].values      # sale
```

### 0.0.5 Step 5: Training phase (bulding the model) using Gradient Descent

**Parameter initialization**

[57]:
```
# np.random.seed(13)
# number of iterations (epochs)
epoch = 1000
# learning rate
learn_rate = 0.001
# batch_size
print("Number of records in traninig set : ",len(x_train),". This is the␣
↪maximum batch size.")
batch_size = 5
```

```
Number of records in traninig set :  40 . This is the maximum batch size.
```

Note: Batch size should be between 1 to number of records (n) in x_train. If batch size is

1 - stochastic gradient descent

2 to at least n-1 - mini batch stochastic gradient descent

n - batch gradient descent (simple gradient decent)

**Objective, Derivative, Loss (error/cost) function**

[58]:
```
# Prediction function
def predict(m, b, x_train):
    return m * x_train + b
```

```python
[59]: # Partial derivative of SSE(m,b) with respect to m
def deriv_m(x_train, y_train, y_predicted):
    return -2 * (x_train * (y_train - y_predicted)).sum()

# Partial derivative of SSE(m,b) with respect to m
def deriv_b(y_train, y_predicted):
    return  -2 * (y_train - y_predicted).sum()
```

```python
[60]: # SSE (cost/loss/error) calculation
def cost_fun(y_train,y_predicted):
    error = (y_train - y_predicted)**2
    SSE = error.sum()
    return SSE
```

**Gradient descent algorithm for 2 parameters**

```python
[61]: # Gradient descent algorithm
def gradient_descent():
    # track all solutions
    solutions_m, solutions_b, cost = list(), list(), list()

    # generate an initial point for m and b
    curr_soln_m = 0
    curr_soln_b = 1

    # run the gradient descent
    for i in range(epoch):

        # Forming the batch
        train = df.sample(batch_size)
        x_train = train[0].values      # TV
        y_train = train[3].values      # sale

        # prediction
        y_predicted = predict(curr_soln_m, curr_soln_b, x_train)

        # gradient calculation
        gradient_m = deriv_m(x_train, y_train, y_predicted)
        gradient_b = deriv_b(y_train, y_predicted)

        # step size calculation
        step_size_m = learn_rate * gradient_m
        step_size_b = learn_rate * gradient_b

        # solution update
        curr_soln_m = curr_soln_m - step_size_m
        curr_soln_b = curr_soln_b - step_size_b
```

```
        # SSE (error/cost/loss) calculation
        SSE = cost_fun(y_train, y_predicted)

        # store the solution
        solutions_m.append(curr_soln_m)
        solutions_b.append(curr_soln_b)
        cost.append(SSE)

        # report progress
        # print('>epoch %d => m %.5f b %.5f cost %.3f ' % (i, curr_soln_m,␣
↪curr_soln_b,SSE))

    return [solutions_m, solutions_b, cost, y_predicted]
```

[62]:
```
# perform the gradient descent search
solutions_m, solutions_b, cost, y_train_pred = gradient_descent()
```

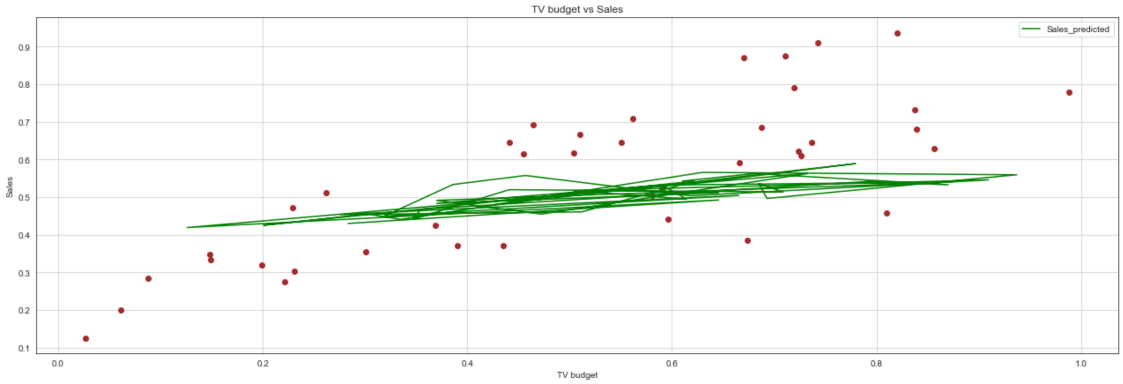
You can observe the fluctuation in the cost (not converging). It is due to selecting records in random.

[63]:
```
m = solutions_m[epoch-1]
b = solutions_b[epoch-1]
print("y = m x + b ==>  y = ",round(m,2)," x + ",round(b,2))
```

```
y = m x + b ==>  y =  0.18  x +  0.41
```

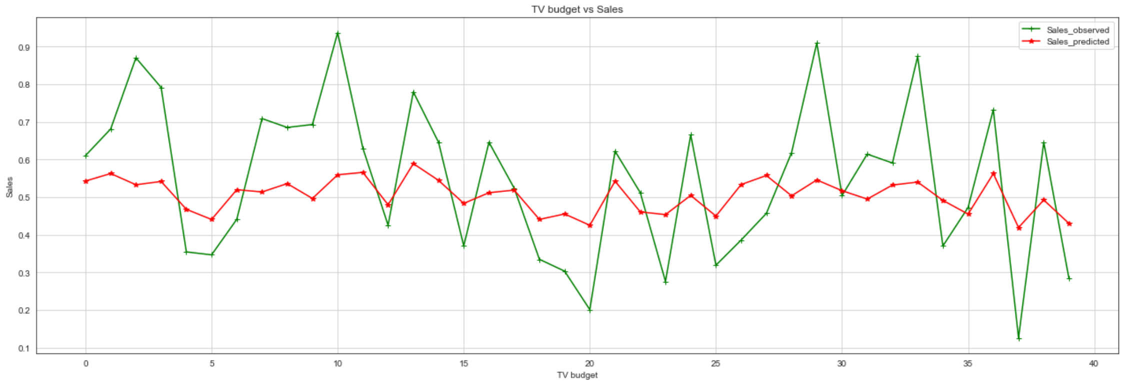
**Visualizing the model**

[64]:
```
sns.set_style(style='white')
fig = plt.figure(figsize=(22,7))
plt.scatter(x_train,y_train,color="brown")
y_train_pred = predict(m, b, x_train)
plt.grid(b=None)
plt.plot(y_train,y_train_pred,"g",label="Sales_predicted")
plt.xlabel("TV budget")
plt.ylabel("Sales")
plt.title("TV budget vs Sales")
plt.legend()
plt.show()
```




**Plotting observed sale (x) and predicted sale (y) for training set**

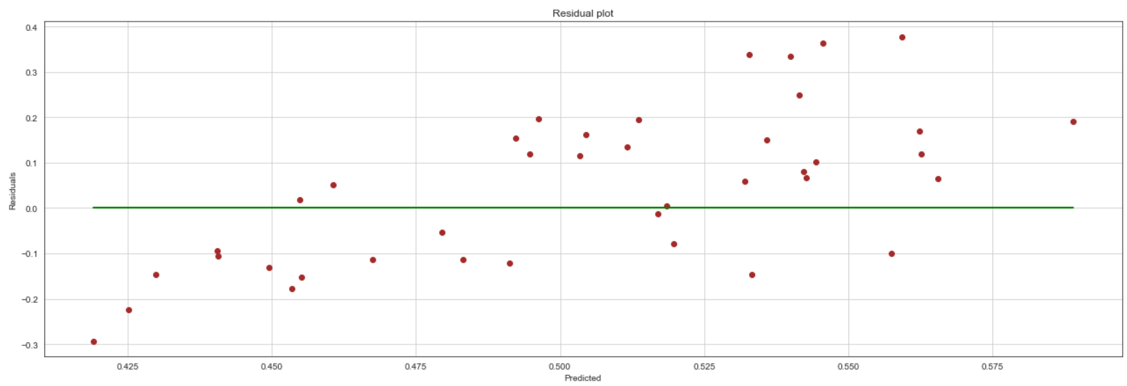
[65]:
```
x = np.arange(len(y_train_pred))
fig = plt.figure(figsize=(22,7))
plt.plot(x,y_train,"g-+",label="Sales_observed")
plt.plot(x,y_train_pred,"r-*",label="Sales_predicted")
plt.grid(b=None)
plt.xlabel("TV budget")
plt.ylabel("Sales")
plt.title("TV budget vs Sales")
plt.legend()
plt.show()
```




**Residual (Error) plot** If the model has done good predictions, then the datapoints must be near around to horizontal line.

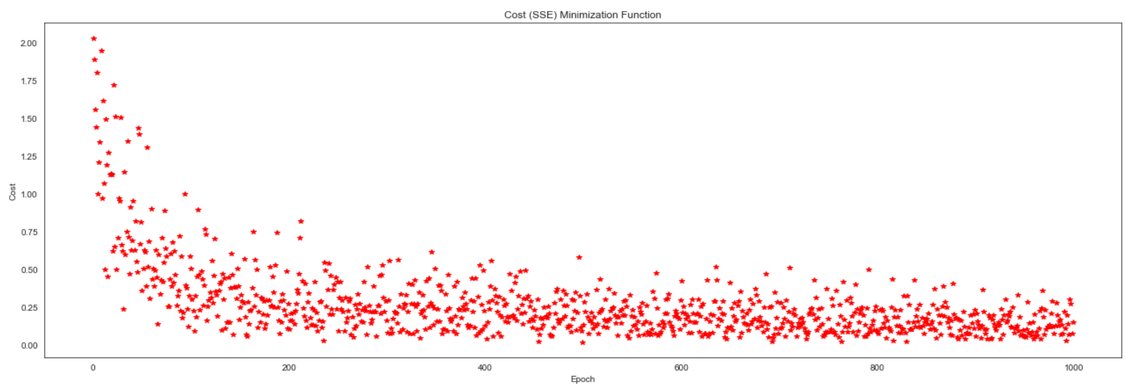
[66]:
```
sns.set_style(style='white')
fig = plt.figure(figsize=(22,7))
residuals = y_train-y_train_pred
```

```
zeros = y_train-y_train
plt.scatter(y_train_pred,residuals,color="brown")
plt.grid(b=None)
plt.plot(y_train_pred,zeros,"g")
plt.xlabel("Predicted")
plt.ylabel("Residuals")
plt.title("Residual plot")
plt.show()
```



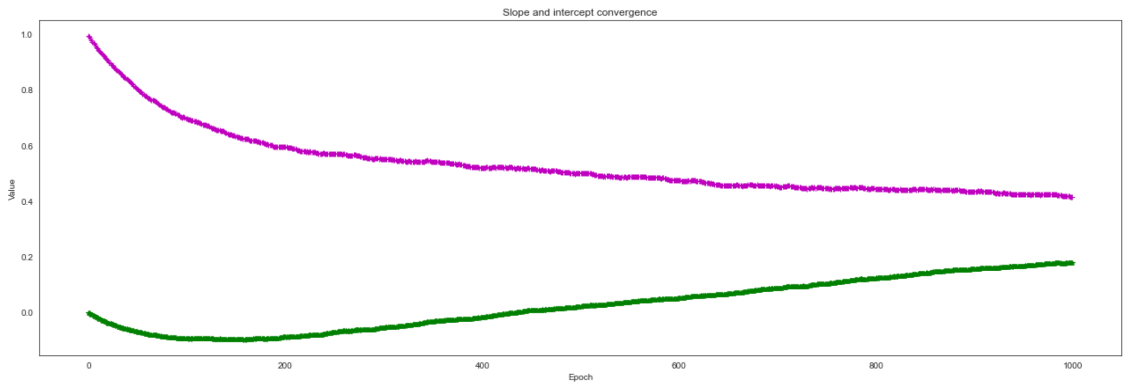

**Plotting SSE minimization**

[67]:
```
x = np.arange(epoch)
fig = plt.figure(figsize=(22,7))
plt.plot(x,cost,"r*")
plt.xlabel("Epoch")
plt.ylabel("Cost")
plt.title("Cost (SSE) Minimization Function")
plt.show()
```



**Plotting slope (m) and intercept (b) convergence**

[68]:
```
x = np.arange(epoch)
fig = plt.figure(figsize=(22,7))
plt.plot(x,solutions_m,"g*")
plt.plot(x,solutions_b,"m+-")
plt.xlabel("Epoch")
plt.ylabel("Value")
plt.title("Slope and intercept convergence")
plt.show()
```




### 0.0.6 Different error calculations to asses the model for training set

**1. Sum of Squared Error (SSE)**

$$SSE(m,b) = \sum_{i=1}^{n}(y_i - \hat{y})^2 = \sum_{i=1}^{n}(y_i - (m * x_i + b))^2 \tag{1}$$

[69]:
```
sum = 0
n = len(x_train)
for i in range (0,n):
  diff = y_train[i] - y_train_pred[i]
  squ_diff = diff**2
  sum = sum + squ_diff
Train_SSE = np.round(sum,2)
print("Sum of Squared Error (SSE) :",Train_SSE)
```

```
Sum of Squared Error (SSE) : 1.2
```

**2. Mean Squared Error (MSE)**

$$MSE(m,b) = \frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{n} = \frac{\sum_{i=1}^{n}(y_i - (m * x_i + b))^2}{n} \tag{2}$$

[70]:
```
Train_MSE = np.round(Train_SSE/n,2)
print("Mean Squared Error (MSE) :",Train_MSE)
```

```
Mean Squared Error (MSE) : 0.03
```

#### 3. Root Mean Squared Error (RMSE)

$$RMSE(m,b) = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{n}} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - (m*x_i + b))^2}{n}} \tag{3}$$

[71]:
```
Train_RMSE = np.round(np.sqrt(Train_MSE),2)
print("Root Mean Squared Error (RMSE) :",Train_RMSE)
```

```
Root Mean Squared Error (RMSE) : 0.17
```

#### 4. Mean Absolute Error (MAE)

$$MAE(m,b) = \frac{\sum_{i=1}^{n}|(y_i - \hat{y})|}{n} \tag{4}$$

[72]:
```
sum = 0
n = len(x_train)
for i in range (0,n):
  diff = y_train[i] - y_train_pred[i]
  sum = sum + np.abs(diff)
Train_MAE = np.round(sum/n,2)
print("Mean Absolute Error (MAE) :",Train_MAE)
```

```
Mean Absolute Error (MAE) : 0.15
```

#### 5. Mean Absolute Percentage Error (MAPE)

$$MAPE(m,b) = \frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{(y_i - \hat{y})}{y_i}\right| = \frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{(y_i - (m*x_i + b))}{y_i}\right| \tag{5}$$

[73]:
```
sum = 0
n = len(x_train)
for i in range (0,n):
    if y_train[i] == 0:
        continue
    else:
        diff = (y_train[i] - y_train_pred[i])/y_train[i]
        sum = sum + np.abs(diff)
Train_MAPE = np.round(sum/n*100,2)
print("Mean Absolute Percentage Error (MAPE) :",Train_MAPE)
```

```
Mean Absolute Percentage Error (MAPE) : 32.8
```

### 0.0.7 Calculating R-Squred value (goodness of model) using SSE

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{6}$$

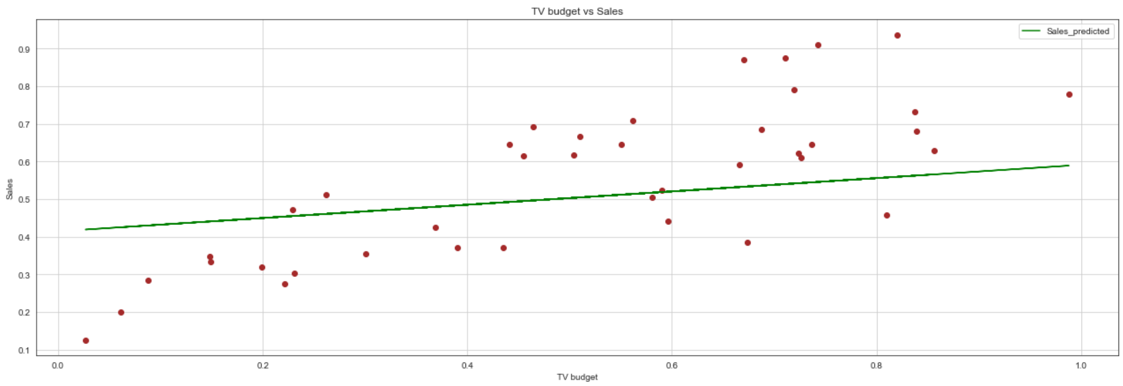
[74]:
```
from sklearn.metrics import r2_score
out = r2_score(y_train,y_train_pred)
Train_RS = np.round(out,2)*100
print("R-Squred value (goodness of model) for training set :",Train_RS,"%")
```

```
R-Squred value (goodness of model) for training set : 25.0 %
```

### 0.0.8 Step 6: Testing phase

[75]:
```
# Predicting values for test input set
y_test_pred = predict(m, b, x_test)
```

[76]:
```
sns.set_style(style='white')
fig = plt.figure(figsize=(22,7))
plt.scatter(x_test,y_test,color="brown")
plt.grid(b=None)
plt.plot(x_test,y_test_pred,"g",label="Sales_predicted")
plt.xlabel("TV budget")
plt.ylabel("Sales")
plt.title("TV budget vs Sales")
plt.legend()
plt.show()
```



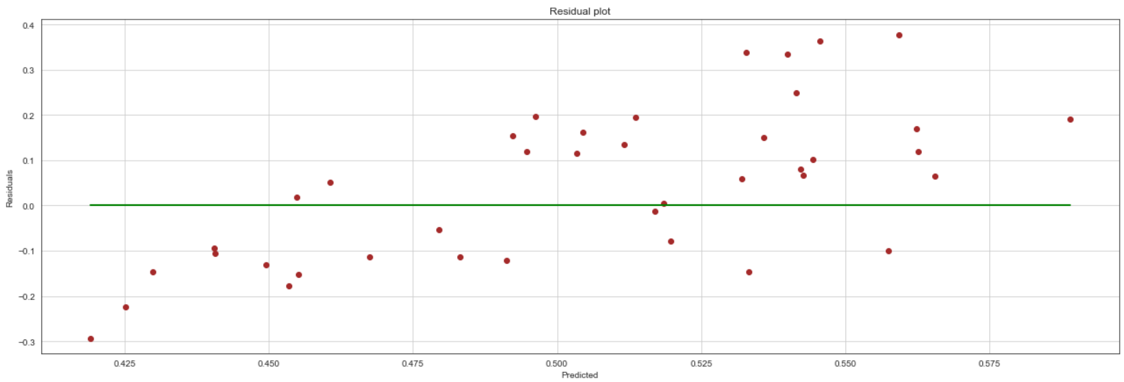

#### Plotting observed sale (x) and predicted sale (y) for test set

[77]:
```
x = np.arange(len(y_test_pred))
fig = plt.figure(figsize=(22,7))
```

```
plt.plot(x,y_test,"g-+",label="Sales_observed")
plt.plot(x,y_test_pred,"r-*",label="Sales_predicted")
plt.grid(b=None)
plt.xlabel("TV budget")
plt.ylabel("Sales")
plt.title("TV budget vs Sales")
plt.legend()
plt.show()
```


**Residual (Error) plot** If the model has done good predictions, then the datapoints must be near around to horizontal line.

[78]:
```
sns.set_style(style='white')
fig = plt.figure(figsize=(22,7))
residuals = y_test-y_test_pred
zeros = y_test-y_test
plt.scatter(y_test_pred,residuals,color="brown")
plt.grid(b=None)
plt.plot(y_test_pred,zeros,"g")
plt.xlabel("Predicted")
plt.ylabel("Residuals")
plt.title("Residual plot")
plt.show()
```




**Storing the outcome in a file**

[79]:
```
# Store the predicted value for sales in new column
dataset.rename(columns={'sales': 'observed_sales'}, inplace=True)
sales_data = dataset.iloc[:,0].values.reshape(-1, 1)
predicted_values = predict(m,b,sales_data)
dataset['predicted_sales'] = predicted_values
dataset.head()
```

[79]:

| | TV | radio | newspaper | observed_sales | predicted_sales |
|---|---|---|---|---|---|
| 0 | 230.1 | 37.8 | 69.2 | 22.1 | 41.075297 |
| 1 | 44.5 | 39.3 | 45.1 | 10.4 | 8.277922 |
| 2 | 17.2 | 45.9 | 69.3 | 9.3 | 3.453740 |
| 3 | 151.5 | 41.3 | 58.5 | 18.5 | 27.185891 |
| 4 | 180.8 | 10.8 | 58.4 | 12.9 | 32.363494 |

[80]:
```
# Write the above output input into new csv
# dataset.to_csv("Gradient Descenet for Linear Regression output.csv")
```

### 0.0.9 Different error calculations to asses the model for the test set

**1. Sum of Squared Error (SSE)**

$$SSE(m,b) = \sum_{i=1}^{n}(y_i - \hat{y})^2 = \sum_{i=1}^{n}(y_i - (m * x_i + b))^2 \tag{7}$$

[81]:
```
sum = 0
n = len(x_test)
for i in range (0,n):
  diff = y_test[i] - y_test_pred[i]
  squ_diff = diff**2
  sum = sum + squ_diff
```

```
Test_SSE = np.round(sum,2)
print("Sum of Squared Error (SSE) :",Test_SSE)
```

```
Sum of Squared Error (SSE) : 1.2
```

#### 2. Mean Squared Error (MSE)

$$MSE(m,b) = \frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{n} = \frac{\sum_{i=1}^{n}(y_i - (m * x_i + b))^2}{n} \tag{8}$$

[82]:
```
Test_MSE = np.round(Train_SSE/n,2)
print("Mean Squared Error (MSE) :",Test_MSE)
```

```
Mean Squared Error (MSE) : 0.03
```

#### 3. Root Mean Squared Error (RMSE)

$$RMSE(m,b) = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{n}} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - (m * x_i + b))^2}{n}} \tag{9}$$

[83]:
```
Test_RMSE = np.round(np.sqrt(Test_MSE),2)
print("Root Mean Squared Error (RMSE) :",Test_RMSE)
```

```
Root Mean Squared Error (RMSE) : 0.17
```

#### 4. Mean Absolute Error (MAE)

$$MAE(m,b) = \frac{\sum_{i=1}^{n}|(y_i - \hat{y})|}{n} \tag{10}$$

[84]:
```
sum = 0
n = len(x_test)
for i in range (0,n):
  diff = y_test[i] - y_test_pred[i]
  sum = sum + np.abs(diff)
Test_MAE = np.round(sum/n,2)
print("Mean Absolute Error (MAE) :",Test_MAE)
```

```
Mean Absolute Error (MAE) : 0.15
```

#### 5. Mean Absolute Percentage Error (MAPE)

$$MAPE(m,b) = \frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{(y_i - \hat{y})}{y_i}\right| = \frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{(y_i - (m * x_i + b))}{y_i}\right| \tag{11}$$

[85]:
```
sum = 0
n = len(x_test)
for i in range (0,n):
    if y_test[i] == 0:
        continue
    else:
        diff = (y_test[i] - y_test_pred[i])/y_test[i]
        sum = sum + np.abs(diff)
Test_MAPE = np.round(sum/n*100,2)
print("Mean Absolute Percentage Error (MAPE) :",Test_MAPE)
```

```
Mean Absolute Percentage Error (MAPE) : 32.8
```

### 0.0.10 Calculating R-Squred value (goodness of model) using SSE

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{12}$$

[86]:
```
from sklearn.metrics import r2_score
out = r2_score(y_test,y_test_pred)
Test_RS = np.round(out,2)*100
print("R-Squred value (goodness of model) for testing set :",Test_RS,"%")
```

```
R-Squred value (goodness of model) for testing set : 25.0 %
```

### 0.0.11 Underfitting and overfitting observation

[87]:
```
print("Error \t From training phase       From testing phase ")
print("===================================================")
print("SSE   \t\t",Train_SSE,"\t\t\t", Test_SSE)
print("MSE   \t\t",Train_MSE,"\t\t\t", Test_MSE)
print("RMSE   \t\t",Train_RMSE,"\t\t\t", Test_RMSE)
print("MAE   \t\t",Train_MAE,"\t\t\t", Test_MAE)
print("RS   \t\t",Train_RS,"\t\t\t", Test_RS)
```

```
Error    From training phase       From testing phase
===================================================
SSE               1.2                        1.2
MSE               0.03                       0.03
RMSE              0.17                       0.17
MAE               0.15                       0.15
RS                25.0                       25.0
```