

2025 CFA®

Exam Prep

SchweserNotes™

Fixed Income and Derivatives

Level I Book 3

KAPLAN  **SCHWESER**

Book 3: Fixed Income and Derivatives

SchweserNotes™ 2025

Level I CFA®



SCHWESERNOTES™ 2025 LEVEL I CFA® BOOK 3: FIXED INCOME AND DERIVATIVES

©2024 Kaplan, Inc. All rights reserved.

Published in 2024 by Kaplan, Inc.

ISBN: 978-1-0788-4564-9

These materials may not be copied without written permission from the author. The unauthorized duplication of these notes is a violation of global copyright laws and the CFA Institute Code of Ethics. Your assistance in pursuing potential violators of this law is greatly appreciated.

Required CFA Institute Disclaimer: “Kaplan Schweser is a CFA Institute Prep Provider. Only CFA Institute Prep Providers are permitted to make use of CFA Institute copyrighted materials which are the building blocks of the exam. We are also required to create / use updated materials every year and this is validated by CFA Institute. Our products and services substantially cover the relevant curriculum and exam and this is validated by CFA Institute. In our advertising, any statement about the numbers of questions in our products and services relates to unique, original, proprietary questions. CFA Institute Prep Providers are forbidden from including CFA Institute official mock exam questions or any questions other than the end of reading questions within their products and services.

CFA Institute does not endorse, promote, review or warrant the accuracy or quality of the product and services offered by Kaplan Schweser. CFA Institute®, CFA® and “Chartered Financial Analyst®” are trademarks owned by CFA Institute.”

Certain materials contained within this text are the copyrighted property of CFA Institute. The following is the copyright disclosure for these materials: “© Copyright CFA Institute”.

Disclaimer: The Schweser study tools should be used in conjunction with the original readings as set forth by CFA Institute. The information contained in these study tools covers topics contained in the readings referenced by CFA Institute and is believed to be accurate. However, their accuracy cannot be guaranteed nor is any warranty conveyed as to your ultimate exam success. The authors of the referenced readings have not endorsed or sponsored these study tools.

CONTENTS

Learning Outcome Statements (LOS)

FIXED INCOME

READING 47

Fixed-Income Instrument Features

Module 47.1: Fixed-Income Instrument Features

Key Concepts

Answer Key for Module Quizzes

READING 48

Fixed-Income Cash Flows and Types

Module 48.1: Fixed-Income Cash Flows and Types

Key Concepts

Answer Key for Module Quizzes

READING 49

Fixed-Income Issuance and Trading

Module 49.1: Fixed-Income Issuance and Trading

Key Concepts

Answer Key for Module Quizzes

READING 50

Fixed-Income Markets for Corporate Issuers

Module 50.1: Fixed-Income Markets for Corporate Issuers

Key Concepts

Answer Key for Module Quizzes

READING 51

Fixed-Income Markets for Government Issuers

Module 51.1: Fixed-Income Markets for Government Issuers

Key Concepts

Answer Key for Module Quizzes

READING 52

Fixed-Income Bond Valuation: Prices and Yields

Module 52.1: Fixed-Income Bond Valuation: Prices and Yields

Key Concepts

Answer Key for Module Quizzes

READING 53

Yield and Yield Spread Measures for Fixed-Rate Bonds

Module 53.1: Yield and Yield Spread Measures for Fixed-Rate Bonds

Key Concepts

Answer Key for Module Quizzes

READING 54

Yield and Yield Spread Measures for Floating-Rate Instruments

Module 54.1: Yield and Yield Spread Measures for Floating-Rate Instruments

Key Concepts

Answer Key for Module Quizzes

READING 55

The Term Structure of Interest Rates: Spot, Par, and Forward Curves

Module 55.1: The Term Structure of Interest Rates: Spot, Par, and Forward
Curves

Key Concepts

Answer Key for Module Quizzes

READING 56

Interest Rate Risk and Return

Module 56.1: Interest Rate Risk and Return

Key Concepts

Answer Key for Module Quizzes

READING 57

Yield-Based Bond Duration Measures and Properties

Module 57.1: Yield-Based Bond Duration Measures and Properties

Key Concepts

Answer Key for Module Quizzes

READING 58

Yield-Based Bond Convexity and Portfolio Properties

Module 58.1: Yield-Based Bond Convexity and Portfolio Properties

Key Concepts

Answer Key for Module Quizzes

READING 59

Curve-Based and Empirical Fixed-Income Risk Measures

Module 59.1: Curve-Based and Empirical Fixed-Income Risk Measures

Key Concepts

Answer Key for Module Quizzes

READING 60

Credit Risk

Module 60.1: Credit Risk

Key Concepts

Answer Key for Module Quizzes

READING 61

Credit Analysis for Government Issuers

Module 61.1: Credit Analysis for Government Issuers

Key Concepts

Answer Key for Module Quizzes

READING 62

Credit Analysis for Corporate Issuers

Module 62.1: Credit Analysis for Corporate Issuers

Key Concepts

Answer Key for Module Quizzes

READING 63

Fixed-Income Securitization

Module 63.1: Fixed-Income Securitization

Key Concepts

Answer Key for Module Quizzes

READING 64

Asset-Backed Security (ABS) Instrument and Market Features

Module 64.1: Asset-Backed Security (ABS) Instrument and Market Features

Key Concepts

Answer Key for Module Quizzes

READING 65

Mortgage-Backed Security (MBS) Instrument and Market Features

Module 65.1: Mortgage-Backed Security (MBS) Instrument and Market Features

Key Concepts

Answer Key for Module Quizzes

Topic Quiz: Fixed Income

DERIVATIVES

READING 66

Derivative Instrument and Derivative Market Features

Module 66.1: Derivatives Markets

Key Concepts

Answer Key for Module Quizzes

READING 67

Forward Commitment and Contingent Claim Features and Instruments

Module 67.1: Forwards and Futures

Module 67.2: Swaps and Options

Key Concepts

Answer Key for Module Quizzes

READING 68

Derivative Benefits, Risks, and Issuer and Investor Uses

Module 68.1: Uses, Benefits, and Risks of Derivatives

Key Concepts

Answer Key for Module Quizzes

READING 69

Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

Module 69.1: Arbitrage, Replication, and Carrying Costs

Key Concepts

Answer Key for Module Quizzes

READING 70

Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

Module 70.1: Forward Contract Valuation

Key Concepts

Answer Key for Module Quizzes

READING 71

Pricing and Valuation of Futures Contracts

Module 71.1: Futures Valuation

Key Concepts

Answer Key for Module Quizzes

READING 72

Pricing and Valuation of Interest Rates and Other Swaps

Module 72.1: Swap Valuation

Key Concepts

Answer Key for Module Quizzes

READING 73

Pricing and Valuation of Options

Module 73.1: Option Valuation

Key Concepts

Answer Key for Module Quizzes

READING 74

Option Replication Using Put–Call Parity

Module 74.1: Put-Call Parity

Key Concepts

Answer Key for Module Quizzes

READING 75

Valuing a Derivative Using a One-Period Binomial Model

Module 75.1: Binomial Model for Option Values

Key Concepts

Answer Key for Module Quizzes

Topic Quiz: Derivatives

Appendix

Formulas

Index

Learning Outcome Statements (LOS)

47. Fixed-Income Instrument Features

The candidate should be able to:

- a. describe the features of a fixed-income security.
- b. describe the contents of a bond indenture and contrast affirmative and negative covenants.

48. Fixed-Income Cash Flows and Types

The candidate should be able to:

- a. describe common cash flow structures of fixed-income instruments and contrast cash flow contingency provisions that benefit issuers and investors.
- b. describe how legal, regulatory, and tax considerations affect the issuance and trading of fixed-income securities.

49. Fixed-Income Issuance and Trading

The candidate should be able to:

- a. describe fixed-income market segments and their issuer and investor participants.
- b. describe types of fixed-income indexes.
- c. compare primary and secondary fixed-income markets to equity markets.

50. Fixed-Income Markets for Corporate Issuers

The candidate should be able to:

- a. compare short-term funding alternatives available to corporations and financial institutions.
- b. describe repurchase agreements (repos), their uses, and their benefits and risks.
- c. contrast the long-term funding of investment-grade versus high-yield corporate issuers.

51. Fixed-Income Markets for Government Issuers

The candidate should be able to:

- a. describe funding choices by sovereign and non-sovereign governments, quasi-government entities, and supranational agencies.
- b. contrast the issuance and trading of government and corporate fixed-income instruments.

52. Fixed-Income Bond Valuation: Prices and Yields

The candidate should be able to:

- a. calculate a bond's price given a yield-to-maturity on or between coupon dates.
- b. identify the relationships among a bond's price, coupon rate, maturity, and yield-to-maturity.
- c. describe matrix pricing.

53. Yield and Yield Spread Measures for Fixed-Rate Bonds

The candidate should be able to:

- a. calculate annual yield on a bond for varying compounding periods in a year.
- b. compare, calculate, and interpret yield and yield spread measures for fixed-rate bonds.

54. Yield and Yield Spread Measures for Floating-Rate Instruments

The candidate should be able to:

- a. calculate and interpret yield spread measures for floating-rate instruments.
- b. calculate and interpret yield measures for money market instruments.

55. The Term Structure of Interest Rates: Spot, Par, and Forward Curves

The candidate should be able to:

- a. define spot rates and the spot curve, and calculate the price of a bond using spot rates.
- b. define par and forward rates, and calculate par rates, forward rates from spot rates, spot rates from forward rates, and the price of a bond using forward rates.
- c. compare the spot curve, par curve, and forward curve.

56. Interest Rate Risk and Return

The candidate should be able to:

- a. calculate and interpret the sources of return from investing in a fixed-rate bond.
- b. describe the relationships among a bond's holding period return, its Macaulay duration, and the investment horizon.
- c. define, calculate, and interpret Macaulay duration.

57. Yield-Based Bond Duration Measures and Properties

The candidate should be able to:

- a. define, calculate, and interpret modified duration, money duration, and the price value of a basis point (PVBP).
- b. explain how a bond's maturity, coupon, and yield level affect its interest rate risk.

58. Yield-Based Bond Convexity and Portfolio Properties

The candidate should be able to:

- a. calculate and interpret convexity and describe the convexity adjustment.
- b. calculate the percentage price change of a bond for a specified change in yield, given the bond's duration and convexity.
- c. calculate portfolio duration and convexity and explain the limitations of these measures.

59. Curve-Based and Empirical Fixed-Income Risk Measures

The candidate should be able to:

- a. explain why effective duration and effective convexity are the most appropriate measures of interest rate risk for bonds with embedded options.
- b. calculate the percentage price change of a bond for a specified change in benchmark yield, given the bond's effective duration and convexity.
- c. define key rate duration and describe its use to measure price sensitivity of fixed-income instruments to benchmark yield curve changes.
- d. describe the difference between empirical duration and analytical duration.

60. Credit Risk

The candidate should be able to:

- a. describe credit risk and its components, probability of default and loss given default.
- b. describe the uses of ratings from credit rating agencies and their limitations.
- c. describe macroeconomic, market, and issuer-specific factors that influence the level and volatility of yield spreads.

61. Credit Analysis for Government Issuers

The candidate should be able to:

- a. explain special considerations when evaluating the credit of sovereign and non-sovereign government debt issuers and issues.

62. Credit Analysis for Corporate Issuers

The candidate should be able to:

- a. describe the qualitative and quantitative factors used to evaluate a corporate borrower's creditworthiness.
- b. calculate and interpret financial ratios used in credit analysis.
- c. describe the seniority rankings of debt, secured versus unsecured debt and the priority of claims in bankruptcy, and their impact on credit ratings.

63. Fixed-Income Securitization

The candidate should be able to:

- a. explain benefits of securitization for issuers, investors, economies, and financial markets.
- b. describe securitization, including the parties and the roles they play.

64. Asset-Backed Security (ABS) Instrument and Market Features

The candidate should be able to:

- a. describe characteristics and risks of covered bonds and how they differ from other asset-backed securities.
- b. describe typical credit enhancement structures used in securitizations.
- c. describe types and characteristics of non-mortgage asset-backed securities, including the cash flows and risks of each type.
- d. describe collateralized debt obligations, including their cash flows and risks.

65. Mortgage-Backed Security (MBS) Instrument and Market Features

The candidate should be able to:

- a. define prepayment risk and describe time tranching structures in securitizations and their purpose.
- b. describe fundamental features of residential mortgage loans that are securitized.
- c. describe types and characteristics of residential mortgage-backed securities, including mortgage pass-through securities and collateralized mortgage obligations, and explain the cash flows and risks for each type.
- d. describe characteristics and risks of commercial mortgage-backed securities.

66. Derivative Instrument and Derivative Market Features

The candidate should be able to:

- a. define a derivative and describe basic features of a derivative instrument.
- b. describe the basic features of derivative markets, and contrast over-the-counter and exchange-traded derivative markets.

67. Forward Commitment and Contingent Claim Features and Instruments

The candidate should be able to:

- a. define forward contracts, futures contracts, swaps, options (calls and puts), and credit derivatives and compare their basic characteristics.
- b. determine the value at expiration and profit from a long or a short position in a call or put option.
- c. contrast forward commitments with contingent claims.

68. Derivative Benefits, Risks, and Issuer and Investor Uses

The candidate should be able to:

- a. describe benefits and risks of derivative instruments.
- b. compare the use of derivatives among issuers and investors.

69. Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

The candidate should be able to:

- a. explain how the concepts of arbitrage and replication are used in pricing derivatives.
- b. explain the difference between the spot and expected future price of an underlying and the cost of carry associated with holding the underlying asset.

70. Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

The candidate should be able to:

- a. explain how the value and price of a forward contract are determined at initiation, during the life of the contract, and at expiration.
- b. explain how forward rates are determined for interest rate forward contracts and describe the uses of these forward rates.

71. Pricing and Valuation of Futures Contracts

The candidate should be able to:

- a. compare the value and price of forward and futures contracts.
- b. explain why forward and futures prices differ.

72. Pricing and Valuation of Interest Rates and Other Swaps

The candidate should be able to:

- a. describe how swap contracts are similar to but different from a series of forward contracts.
- b. contrast the value and price of swaps.

73. Pricing and Valuation of Options

The candidate should be able to:

- a. explain the exercise value, moneyness, and time value of an option.
- b. contrast the use of arbitrage and replication concepts in pricing forward commitments and contingent claims.
- c. identify the factors that determine the value of an option and describe how each factor affects the value of an option.

74. Option Replication Using Put–Call Parity

The candidate should be able to:

- a. explain put–call parity for European options.
- b. explain put–call *forward* parity for European options.

75. Valuing a Derivative Using a One-Period Binomial Model

The candidate should be able to:

- a. explain how to value a derivative using a one-period binomial model.
- b. describe the concept of risk neutrality in derivatives pricing.

READING 47

FIXED-INCOME INSTRUMENT FEATURES

MODULE 47.1: FIXED-INCOME INSTRUMENT FEATURES



Video covering this content is available online.

LOS 47.a: Describe the features of a fixed-income security.

Major types of fixed-income instruments include loans, which are private (nontradable) agreements between a borrower and lender, and **bonds (or fixed-income securities)**, which are standardized, tradable securities representing a debt investment.

Investors in **bonds** are lending capital (referred to as principal, par, or face value) to the issuer of the bond. The issuer of the bond promises to repay this principal amount plus interest, typically in the form of a regular periodic **coupon** that is stated as a percentage of par. The capital raised is usually used to finance the long-term investments of the bond issuer. For a corporate issuer, loans and bonds are classified as long-term liabilities in the balance sheet.

Key features that are specified in a fixed-income security include the following:

- **Issuer.** Major issuers of bonds are sovereign national governments and corporations. Other issuers include local governments, supranational entities (e.g., the International Monetary Fund), quasi-government entities sponsored by the government (e.g., national railways), and special purpose entities, which are corporations set up to purchase financial assets and issue **asset-backed securities**, which are bonds backed by the cash flows from those assets.
- **Maturity.** The maturity date of a bond is the date on which the final cash flow is to be paid. Once a bond has been issued, the time remaining until maturity is referred to as the tenor of a bond. Bonds with original maturities (their tenor when they were first issued) of one year or less are referred to as **money market securities**. Bonds with original maturities of more than one year are referred to as **capital market securities**. Bonds that have no stated maturity date are called **perpetual bonds**.
- **Principal** (par or face value). The par value of a bond is the principal amount that will be repaid. Repayment of principal typically occurs at maturity, but debt instruments may specify that principal is paid back gradually over the life of the instrument, such as with a mortgage loan.

- *Coupon rate and frequency.* The **coupon rate** on a bond is the **annual percentage of its par value that will be paid to bondholders**. Some bonds make coupon interest payments annually, while others make semiannual, quarterly, or monthly payments. A \$1,000 par value semiannual-pay bond with a fixed 5% coupon would pay 2.5% of \$1,000, or \$25, every six months.
 - Some bonds **pay coupons** based on a **variable market rate of interest** at the date of coupon payment. These bonds are called **floating-rate notes (FRNs)** or **floaters**. The **variable market rate of interest** is called the **market reference rate (MRR)**, and an FRN promises to pay the variable reference rate plus a fixed margin. This added margin is typically expressed in **basis points**, which are hundredths of 1%.
 - Some **bonds pay no interest before maturity** and are called **zero-coupon bonds** or **pure discount bonds**. *Pure discount* refers to the fact that these bonds are sold at a discount to their par value, and the interest is all paid at maturity when bondholders receive the par value. A 10-year, \$1,000, zero-coupon bond yielding 7% would sell for a bit more than \$500 initially and pay \$1,000 at maturity. (In our reading on Fixed-Income Bond Valuation we will show how to calculate the exact price.)
- *Seniority.* In the event of bankruptcy or liquidation of an issuer, debt investors' claims on the issuer's assets rank above those of equity investors, making debt *senior* to equity in the capital structure of the issuer. However, not all debt claims rank equally. **Senior debt** ranks higher than **junior debt** (also called **subordinated debt**), making senior debt a less risky investment from a credit risk perspective.
- *Contingency provisions.* A bond may have an **embedded option**, such as a call option, put option, or the right to convert the debt into equity. We will describe these options in later readings.

Yield Measures

Given a bond's price and its expected cash flows, we can calculate the expected return from investing in the bond, referred to as the bond's **yield**. For a fixed-coupon bond, when prices fall, the bond offers a higher yield, and when prices rise, the bond offers a lower yield. As such, prices and yields are inversely related. We will perform yield calculations in later readings.

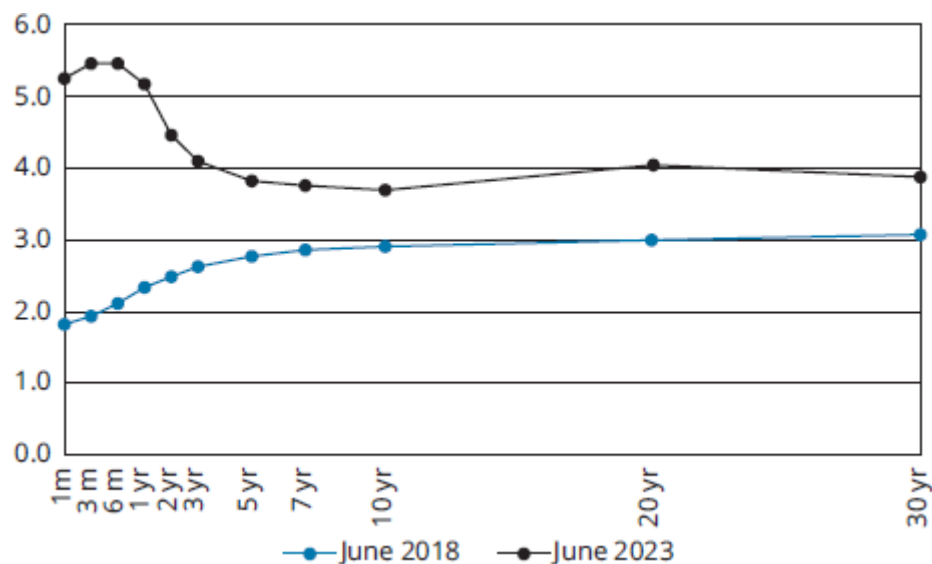


PROFESSOR'S NOTE

The inverse price/yield relationship for fixed-coupon bonds is a crucial concept that runs through the whole fixed income topic. If a bond with fixed cash flows is to offer a higher return (yield), the only way this is possible is through investors paying a lower price for the bond today. Hence, increasing bond yields imply decreasing bond prices, and decreasing bond yields imply increasing bond prices.

For a given issuer, we will likely find that bonds of different maturity will offer different yields. A graphical plot of these yields versus maturity is referred to as a **yield curve**. An example of yield curves for U.S. Treasury bonds is displayed in Figure 47.1.

Figure 47.1: U.S. Treasury Yield Curve



An upward-sloping yield curve (i.e., higher expected returns for longer-dated maturities), as U.S. Treasuries exhibited in mid-2018, is referred to as a normal yield curve because this is the shape most frequently observed. A normal yield curve reflects investor demand for higher returns for longer-dated maturities due to higher levels of uncertainty (i.e., risk) over longer time frames. A downward-sloping yield curve, as U.S. Treasuries exhibited in mid-2023, is less common and is referred to as an **inverted yield curve**.

Government bonds are often deemed to be of the lowest credit risk (highest credit quality) in a particular market due to the fact the bonds are backed by the tax-raising powers of the government. A government bond yield curve is commonly used as a benchmark to assess the extra returns (called spreads) offered by more risky issuers, such as corporations. For example, if a 5-year corporate bond were yielding 6% and 5-year government bonds were yielding 5%, then the spread offered by the corporate bond is $6\% - 5\% = 1\%$. We will discuss credit spreads in more detail in later readings.

LOS 47.b: Describe the contents of a bond indenture and contrast affirmative and negative covenants.

The legal contract between the bond issuer (borrower) and bondholders (lenders) is called the **bond indenture**. The indenture defines obligations of, and restrictions on, the borrower, including the sources of repayment, and it forms the basis for all future interactions between the bondholder and the issuer.

Sources of Repayment

The source of the cash flows required to be paid by the bond issuer depends on the nature of the issuer and type of bond issue.

Sovereign (national government) bonds are repaid from taxes on economic activity and, in some cases, the ability of a government to create new currency. This tends to result in sovereign debt being perceived as the lowest credit risk in a particular region.

Local government bonds are repaid from local government taxes or revenue from operational infrastructure, such as toll roads.

The sources of repayment for a corporate bond depend on the type of bond issue. A **secured bond** is repaid from the operating cash flow of the company, with the added security of a legal claim (called a **lien** or **pledge**) on specific assets of the company (referred to as **collateral**) in the event of issuer default. This contrasts with an **unsecured bond**, which, having no such claim, is repaid only from the operating cash flow of the issuing company.

For an asset-backed security (ABS), financial assets held by the special purpose entity that has issued the ABS provide the cash flows promised to the ABS investors. We will discuss these in more detail in later readings.

Bond Covenants

While debt investments do not provide voting rights in the same way as an equity investment, certain legal rules known as **covenants** can be written into the bond indenture.

Affirmative covenants specify requirements the issuer must fulfill. These may require the issuer to provide timely financial reports to bondholders, specify the use of proceeds from the bond issue, or specify a bondholder's right to redeem at a premium to par if the issuer is acquired in a merger or corporate takeover.

Two examples of affirmative covenants are **cross-default** and **pari passu** provisions. A cross-default clause states that if the issuer defaults on any other debt obligation, the issuer will also be considered in default on this bond. A pari passu clause states that the bond will have the same priority of claims as the issuer's other senior debt issues.

Negative covenants place restrictions on the issuer. These can include restrictions on:

- entering into asset sales and leaseback agreements;
- pledges of collateral (the company cannot use the same assets to back several debt issues simultaneously);
- issuance of debt that ranks more senior than existing debt (referred to as a **negative pledge clause**); and
- additional borrowings, share repurchases, or dividend payments. These actions can be subject to an **incurrence test** relating to the financial ratios of the company—for example, they can only be carried out if debt/EBITDA is below a specified threshold.

Negative covenants protect the interests of bondholders and prevent the issuing firm from taking actions that would increase the risk of default. However, covenants must not be so restrictive that they prevent the firm from taking advantage of opportunities or responding appropriately to changing business circumstances.



MODULE QUIZ 47.1

1. A fixed-coupon bond will pay a coupon equal to its:
 - A. yield multiplied by price.
 - B. stated coupon rate multiplied by price.

- C. stated coupon rate multiplied by face value.
2. When fixed-coupon bond prices fall:
- A. their yields rise.
 - B. their yields fall.
 - C. their coupon rates fall.
3. A bond's indenture:
- A. contains its covenants.
 - B. is only required in the event of a lien on collateral.
 - C. relates only to its interest and principal payments.
4. A clause in a bond indenture that requires the borrower to perform a certain action is *most accurately* described as a(n):
- A. trust deed.
 - B. negative covenant.
 - C. affirmative covenant.

KEY CONCEPTS

LOS 47.a

Basic features of a fixed income security include the issuer, maturity date, par value, coupon rate, coupon frequency, seniority, and contingency provisions.

- Issuers include corporations, governments, quasi-government entities, supranational entities and special purpose entities set up to issue asset-backed securities.
- Bonds with original maturities of one year or less are money market securities. Bonds with original maturities of more than one year are capital market securities. Bonds with no stated maturity are perpetual bonds.
- Par value is the principal amount that will be repaid to bondholders, usually at maturity.
- Coupon rate is the percentage of par value that is paid annually as interest. Coupon frequency may be annual, semiannual, quarterly, or monthly. Zero-coupon bonds pay no coupon interest and are pure discount securities.
- Senior debt ranks above junior (subordinated) debt should an issuer file for bankruptcy or undergo liquidation. Junior bonds with lower credit quality must offer investors higher yields to compensate for the greater probability of default.
- Contingency provisions are rights to take actions in response to some potential future event, such as the right for the issuer to call the bond back earlier than maturity.

The return earned from investing in a bond is referred to as the bond's yield. For a fixed coupon bond, there is an inverse relationship between the price and the yield (return) of the instrument. A plot of yield versus maturity for a certain issuer or class of bond is referred to as a yield curve.

The source of repayment for sovereign bonds is the country's taxing authority. For non-sovereign government bonds, the sources may be taxing authority or revenues from a

project. Corporate bonds are repaid with funds from the firm's operations. Securitized bonds are repaid with cash flows from a pool of financial assets.

Bonds are secured if they are backed by specific collateral or unsecured if they represent an overall claim against the issuer's cash flows and assets.

LOS 47.b

A bond indenture is a contract between a bond issuer and the bondholders which defines the bond's features and the issuer's obligations. An indenture specifies the entity issuing the bond, the source of funds for repayment, assets pledged as collateral, credit enhancements, and any covenants with which the issuer must comply.

Affirmative covenants specify actions an issuer must take, negative covenants specify restrictions on the issuer.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 47.1

1. **C** A fixed-coupon bond has a stated coupon rate that is applied to the bond's face (principal or par) value. The yield of the bond is the return earned through paying the bond's price today and holding the bond to maturity. (LOS 47.a)
2. **A** For fixed-coupon bonds, prices and yields have an inverse relationship. If the price of the bond is falling, then the return (yield) from buying the bond at the lower price is rising. (LOS 47.a)
3. **A** An indenture is the contract between the company and its bondholders and contains the bond's covenants. (LOS 47.b)
4. **C** Affirmative covenants require the borrower to perform certain actions. Negative covenants restrict the borrower from performing certain actions. Trust deed is another name for a bond indenture. (LOS 47.b)

READING 48

FIXED-INCOME CASH FLOWS AND TYPES

MODULE 48.1: FIXED-INCOME CASH FLOWS AND TYPES



Video covering this content is available online.

LOS 48.a: Describe common cash flow structures of fixed-income instruments and contrast cash flow contingency provisions that benefit issuers and investors.

A typical bond has a **bullet structure**, where principal (par value) is paid back in a single payment at maturity. Periodic payments across the life of the bond (referred to as the bond's **coupons**) are purely interest payments.

Consider a \$1,000 par value 5-year bond with an annual coupon rate of 5%, issued at par. With a bullet structure, the bond's promised payments at the end of each year would be as follows.

Year	1	2	3	4	5
PMT	\$50	\$50	\$50	\$50	\$1,050
Principal remaining	\$1,000	\$1,000	\$1,000	\$1,000	\$0

A loan structure in which the periodic payments include both interest and some repayment of principal (the amount borrowed) is called an **amortizing loan**. If a bond (loan) is **fully amortizing**, this means the principal is fully paid off when the last periodic payment is made. Typically, automobile loans and home loans are fully amortizing loans. If the 5-year, 5% bond in the previous table had a fully amortizing structure rather than a bullet structure, the payments and remaining principal balance at each year-end would be as follows (final payment reflects rounding of previous payments).

Year	1	2	3	4	5
PMT	\$230.97	\$230.97	\$230.97	\$230.97	\$230.98
Principal remaining	\$819.03	\$629.01	\$429.49	\$219.99	\$0

This constant PMT can be calculated using a financial calculator:

$$N = 5; I/Y = 5; PV = 1,000; FV = 0; CPT \rightarrow PMT = -230.97$$

Note that the constant yearly payment of \$230.97, here, is partly interest and partly principal loan repayment. For example, in the first year, the interest component is $0.05 \times \$1,000 = \50 ; hence, the principal component is $\$230.97 - \$50 = \$180.97$. The opening principal balance for the second year is, therefore, $\$1,000 - \$180.97 = \$819.03$. In subsequent years, the interest component of the \$230.97 will decrease and the proportion relating to principal repayment will increase.

A bond can also be structured to be **partially amortizing** so that there is a repayment of some principal at maturity (referred to as a **balloon payment**). Unlike a bullet structure, the final payment includes just the remaining unamortized principal amount rather than the full principal amount. In the following table, the final payment includes \$200 to repay the remaining principal outstanding.

Year	1	2	3	4	5
PMT	\$194.78	\$194.78	\$194.78	\$194.78	\$394.78
Principal remaining	\$855.22	\$703.20	\$543.58	\$375.98	\$0

This constant PMT can be calculated using a financial calculator:

$$N = 5; I/Y = 5; PV = 1,000; FV = -200; CPT \rightarrow PMT = -194.78$$

Other types of amortization schedules include **sinking fund provisions** for bonds and **waterfall structures** for asset-backed securities (ABSs) and mortgage-backed securities (MBSs).

Sinking fund provisions provide for the repayment of principal through a series of payments over the life of a bond issue. For example, a 20-year issue with a face amount of \$300 million may require that the bond trustee redeems \$20 million of the principal from investors selected at random every year beginning in the sixth year.

Sinking fund provisions offer both advantages and disadvantages to bondholders. On the plus side, bonds with a sinking fund provision have less credit risk because the periodic redemptions reduce the total amount of principal to be repaid at maturity. The presence of a sinking fund, however, can be a disadvantage to bondholders when interest rates fall due to **reinvestment risk**, which is the possibility of receiving cash flows early and only being able to reinvest them at lower yields.

Waterfall structures are used to establish principal repayments to holders of ABSs and MBSs. These structured products can be split into *tranches* of varying seniority. A common waterfall structure is for junior tranches not to receive any principal payment from the collateral pool until all senior tranches have been fully repaid. Interest payments would still be made to all tranches.

There are several coupon structures besides a fixed-coupon structure. We summarize the most important ones here.

Variable Interest Debt

Some bonds pay periodic interest that depends on the prevailing market rate of interest at the time future coupon payments are made. These bonds are called **floating-rate notes (FRNs)** or **floaters**. The variable market rate of interest is called the **market**

reference rate (MRR), and an FRN promises to pay the MRR plus some fixed margin (called a **credit spread**). This added margin is typically expressed in **basis points**, which are hundredths of 1%. A 120 basis point margin is equivalent to 1.2%.

Most floaters pay quarterly coupons and are based on a quarterly (90-day) reference rate. As an example, consider an FRN that pays a quarterly interest rate of MRR plus 0.75% (75 basis points). If the annualized MRR in the current quarter is 2.3%, the bond will pay $(2.3\% + 0.75\%)/4 = 0.7625\%$ of its par value at the end of the quarter.

Other Coupon Structures

Step-up coupon bonds are structured so that the coupon rate increases over time according to a predetermined schedule, providing protection to investors against interest rates rising over the life of the bond.

Coupon changes could also be linked to future potential events. For example, loans to borrowers of lower credit quality (called **leveraged loans**) often have a coupon that increases if the credit quality of the issuer decreases further. For example, the term sheet of a leveraged loan might specify that the coupon to be paid is $\text{MRR} + 2.5\%$; however, should the issuer's debt/EBITDA ratio rise above 3, then the coupon paid will increase to $\text{MRR} + 3\%$. A similar provision is often included in a **credit-linked note**, whereby the coupon rate increases if the credit rating of the issuer deteriorates (or decreases if the credit rating improves).

A **payment-in-kind (PIK) bond** allows the issuer to make the coupon payments by increasing the principal amount of the outstanding bonds, essentially paying bond interest with more bonds. Firms that issue PIK bonds typically do so because they anticipate that firm cash flows may be less than required to service the debt, often because of high levels of debt financing (leverage). These bonds typically have higher yields because of the lower perceived credit quality implied by expected cash flow shortfalls, or simply because of the high leverage of the issuing firm.

More recently, **green bonds** have been issued whereby the coupon paid increases if certain environmental goals (for example CO₂ emissions reduction) are not met by the issuer over a specified time frame.

An **index-linked bond** has coupon payments or a principal value that is based on a specified published index. **Inflation-linked bonds** (also called **linkers**) are the most common type of index-linked bonds, which increase their cash flows in line with a specified inflation index, such as the Consumer Price Index (CPI) in the United States, to protect the real value of the cash flows promised to investors.

The different structures of inflation-indexed bonds include the following:

- **Interest-indexed bonds.** The coupon rate is adjusted for inflation, while the principal value remains unchanged. This means the principal value of the debt is not inflation-protected.
- **Capital-indexed bonds.** This is the most common structure. An example is U.S. **Treasury Inflation-Protected Securities (TIPS)**. The coupon rate remains constant, but the principal value is increased by the rate of inflation, or decreased by deflation.

In the case of deflation, TIPS investors receive the maximum of inflation-adjusted principal or the unindexed par amount at maturity.

To better understand the structure of capital-indexed bonds, consider a bond with a par value of \$1,000 at issuance, a 3% annual coupon rate paid semiannually, and a provision that the principal value will be adjusted for inflation (or deflation). If, six months after issuance, the reported inflation has been 1% over the period, the principal value of the bonds is increased by 1% from \$1,000 to \$1,010, and the six-month coupon is calculated as 1.5% of the adjusted principal value of \$1,010 (i.e., $\$1,010 \times 1.5\% = \15.15).

With this structure, we can view the coupon rate of 3% as a real rate of interest. Unexpected inflation will not decrease the purchasing power of the coupon interest payments, and the principal value paid at maturity will have approximately the same purchasing power as the \$1,000 par value did at bond issuance. Thus, investors are fully protected against inflation over the life of the bond.

Zero-coupon bonds are the simplest form of fixed-income instrument, offering only a single payment of par at maturity. These bonds are popular with investors that wish to minimize reinvestment risk. With no periodic coupon, zero-coupon bonds must trade below par to offer investors a positive return.

With a **deferred coupon bond**, regular coupon payments do not begin until a specified time after issuance. These bonds may be appropriate financing for issuers with a low credit rating or with a large project that will not be completed and generating revenue for some period after bond issuance. Zero-coupon bonds can be considered the most extreme type of deferred coupon bond—and, like zero-coupon bonds, deferred coupon bonds often trade below par to provide investors with the yields they demand.

Fixed-Income Contingency Provisions

A **contingency provision** in a contract describes an action that may be taken if an event (the contingency) actually occurs. Contingency provisions in bond indentures are referred to as **embedded options**—embedded in the sense that they are an integral part of the bond contract and are not a separate security. Some embedded options are exercisable at the option of the issuer of the bond, so they are valuable to the issuer; others are exercisable at the option of the purchaser of the bond, so they have value to the bondholder.

We will discuss three types of bonds with embedded options here: callable bonds, puttable bonds, and convertible bonds. Bonds that do not have contingency provisions are referred to as *straight bonds* or *option-free* bonds.

Callable Bonds

A **callable bond** gives the *issuer* the right, but not the obligation, to redeem (through buying bonds back from investors before maturity) all or part of a bond issue at a predetermined fixed price (known as a call price).

As an example of a call provision, consider a 6% 20-year bond issued at par on June 1, 2022, for which the indenture includes the following *call schedule*:

- The bonds can be redeemed by the issuer at 102% of par after June 1, 2027.
- The bonds can be redeemed by the issuer at 101% of par after June 1, 2030.
- The bonds can be redeemed by the issuer at 100% of par after June 1, 2032.

For the 5-year period from the issue date until June 2027, the bond is not callable; hence, we say the bond has five years of **call protection**.

June 1, 2027, is referred to as the *first call date* (the start of the **call period** where the bond can be called by the issuer) and the *call price* is 102 (102% of par value) between that date and June 2030. The call price declines to 101 (101% of par) after June 1, 2030. After, June 1, 2032, the bond is callable at par.

A call option has value to the issuer because it gives the issuer the right to redeem the bond early and issue a new bond (borrow) if the market yield on the bond declines. This could occur either because interest rates in general have decreased, or because the credit quality of the bond has increased (default risk has decreased).

Consider a situation where the market yield on the 6% 20-year bond has declined from 6% at issuance to 4% on June 1, 2027 (the first call date). If the bond did not have a call option, it would trade at approximately \$1,224. With a call price of 102, the issuer can redeem the bonds at \$1,020 each and borrow that amount at the current market yield of 4%, reducing the annual interest payment from \$60 per bond to \$40.80.



PROFESSOR'S NOTE

This is analogous to refinancing a home mortgage when mortgage rates fall to reduce the monthly payments.

The issuer holding the call option creates **call risk** for the bondholder. Call risk relates to the fact that bondholders face an uncertain redemption date. For a bond that is in its call period, the call price will put an upper limit on the value of the bond in the market. Due to call risk, bondholders will demand a higher yield and will pay a lower price for a callable bond than they would for an otherwise equivalent straight bond. The difference in price between a callable bond and an otherwise identical noncallable bond is equal to the value of the call option to the issuer.

Putable Bonds

A **putable bond** gives the *bondholder* the right to sell the bond back to the issuing company at a prespecified price, typically par. Bondholders are likely to exercise such a put option when the price of the bond is less than the put price because interest rates have risen or the credit quality of the issuer has fallen.

Unlike a callable bond, the embedded option for a putable bond has value to the bondholder because the choice of whether to exercise the option is the bondholder's. For this reason, a putable bond will sell at a higher price (offer a lower yield) than an otherwise equivalent straight bond. The difference in price between an otherwise identical straight bond and a putable bond is equal to the value of the put option to the bondholder.

Convertible Bonds

Convertible bonds give bondholders the option to exchange the bond for a specific number of shares of the issuing corporation's common stock. This gives bondholders the opportunity to profit from increases in the value of the common shares. Because the conversion option is valuable to bondholders, convertible bonds can be issued with higher prices (and, therefore, lower yields, which is an advantage to the issuer) compared to otherwise identical straight bonds.

Some terms related to convertible bonds are as follows:

- **Conversion price.** This is the par amount per share at which the bond may be converted to common stock.
- **Conversion ratio.** This is equal to the par value of the bond divided by the conversion price. If a bond with a \$1,000 par value has a conversion price of \$40, its conversion ratio is $1,000 / 40 = 25$ shares per bond.
- **Conversion value.** This is the market value of the shares that would be received upon conversion. A bond with a conversion ratio of 25 shares when the current market price of a common share is \$50 would have a conversion value of $25 \times \$50 = \$1,250$.



PROFESSOR'S NOTE

Valuation of convertible bonds is addressed in the Level II CFA curriculum.

Warrants

An alternative way to give bondholders an opportunity for additional returns when the firm's common shares increase in value is to attach **warrants** to straight bonds when they are issued. Warrants give their holders the right to buy the firm's common shares at a fixed price over a given period. As an example, warrants that give their holders the right to buy shares for \$40 will have value if the common shares increase in value above \$40 before the warrants expire. Warrants can be detached from the bond issue and traded on securities exchanges.

Contingent Convertible Bonds

Contingent convertible bonds (referred to as *CoCos*) are bonds that convert from debt to common equity automatically if a specific event occurs. This type of bond has been issued by some European banks. Banks must maintain specific levels of equity financing. If a bank's equity falls below the required level, they must somehow raise more equity financing to comply with regulations. CoCos are often structured so that if the bank's equity capital falls below a given level, they are automatically converted to common stock. This has the effect of decreasing the bank's debt liabilities and increasing its equity capital at the same time, which helps the bank to meet its minimum equity requirement.

LOS 48.b: Describe how legal, regulatory, and tax considerations affect the issuance and trading of fixed-income securities.

Bonds are subject to different legal and regulatory requirements that depend on where they are issued and traded. Bonds of issuers domiciled in the same country as the market in which the bonds are issued and traded are referred to as **domestic bonds**. Bonds of issuers from countries other than the market in which the bond trades are referred to as **foreign bonds**. For example, a U.K. company raising U.S. dollars to expand their U.S. operations by issuing bonds that trade in the United States is a foreign bond issuance.

Eurobonds are issued outside the jurisdiction of any one country and can be issued in any currency. They are subject to less regulation than domestic bonds in most jurisdictions and were initially introduced to avoid U.S. regulations. Eurobonds should not be confused with bonds denominated in euros or thought to originate in Europe, although they can be both. Eurobonds got the “euro” name because they were first introduced in Europe, and most are still traded by firms in European capitals. A bond issued by a Chinese firm that is denominated in yen and traded in markets outside Japan would fit the definition of a Eurobond (it would be referred to as a Euroyen bond). Eurobonds that trade in at least one domestic bond market and in the Eurobond market are referred to as **global bonds**.

Eurobonds are referred to by the currency they are denominated in. Eurodollar bonds are denominated in U.S. dollars, and Euroyen bonds are denominated in yen. At one time, most Eurobonds were issued in **bearer bond** form. Ownership of bearer bonds was not officially recorded by the issuer; hence, ownership was evidenced simply by possessing the bonds. Today Eurobonds, like most other bonds, are issued as **registered bonds** with a record of ownership.

Foreign bonds, global bonds, and Eurobonds that involve more than one country are often collectively referred to as **international bonds** to distinguish them from domestic bonds, which involve only a single country.

While domestic and international bonds are subject to varying laws and regulations in different jurisdictions and have different conventions relating to coupon frequency and calculation methods, the factor that is likely to best describe differences in yields across different markets is the currency of the bond. This is because the interest rates that determine the bond’s yield will be driven by the market interest rates of the country in whose currency the bond is denominated.

Sukuk bonds are Sharia-compliant bonds with specific restrictions on the payment of interest and use of the proceeds of the bond issue to comply with Islamic law. The periodic payments on these bonds are considered to be cash flows from rent on underlying assets.

Taxation of Bond Income

Most often, the interest income paid to bondholders is taxed as ordinary income at the same rate as wage and salary income. The interest income from bonds issued by

municipal governments in the United States, however, is most often exempt from national income tax and often from state income tax in the state of issue.

When a bondholder sells a coupon bond before maturity, it may be at a gain or a loss relative to its purchase price. Such gains and losses are considered capital gains income (rather than ordinary taxable income). Capital gains are often taxed at a lower rate than ordinary income. Capital gains on the sale of an asset that has been owned for more than some minimum amount of time may be classified as long-term capital gains and taxed at an even lower rate.

Zero-coupon bonds and other bonds sold at significant discounts to par when issued are termed **original issue discount (OID) bonds**. Because the gains over an OID bond's tenor as its price moves toward par value are really interest income, these bonds can generate a tax liability even when no cash interest payment has been made. In many tax jurisdictions, a portion of the discount from par at issuance is treated as taxable interest income each year.

Some tax jurisdictions provide a symmetric treatment for bonds issued at a premium to par, allowing part of the premium to be used to reduce the taxable portion of coupon interest payments.



MODULE QUIZ 48.1

1. Compared to a fully amortizing loan, an equivalent loan with a balloon payment will *most likely* have:
☒ A. lower regular periodic payments and a higher final payment amount.
☐ B. higher regular periodic payments and a lower final payment amount.
☐ C. lower regular periodic payments and a lower final payment amount.
2. With which of the following features of a corporate bond issue does an investor *most likely* face the risk of redemption before maturity?
☐ A. Floating-rate notes.
☒ B. Sinking fund.
☐ C. Term maturity structure.
3. A 10-year bond pays no interest for three years, then pays \$229.25, followed by payments of \$35 semiannually for seven years, and an additional \$1,000 at maturity. This bond is *most likely* a:
☐ A. step-up bond.
☐ B. zero-coupon bond.
☒ C. deferred coupon bond.
4. Which of the following *most accurately* describes the maximum price for a currently callable bond?
☐ A. Its par value.
☒ B. The call price.
☐ C. The present value of its par value.
5. An investor buys a pure-discount bond, holds it to maturity, and receives its par value. For tax purposes, the increase in the bond's value is *most likely* to be treated as:
☐ A. a capital gain.
☒ B. interest income.

KEY CONCEPTS

LOS 48.a

A bond with a bullet structure pays coupon interest periodically and repays the entire principal value at maturity, along with the final coupon interest payment.

A bond with an amortizing structure repays part of its principal at each payment date. A fully amortizing structure makes equal payments throughout the bond's life. A partially amortizing structure has a balloon payment at maturity, which repays the remaining principal as a lump sum.

A sinking fund provision requires the issuer to retire a portion of a bond issue at specified times during the bond's life.

Floating-rate notes have coupon rates that adjust based on a variable market reference rate. Other coupon structures include step-up coupon notes, credit-linked coupon bonds, payment-in-kind bonds, deferred coupon bonds, and index-linked bonds.

Callable bonds allow the issuer to redeem bonds at a specified call price.

Putable bonds allow the bondholder to sell bonds back to the issuer at a specified put price.

Convertible bonds allow the bondholder to exchange bonds for a specified number of shares of the issuer's common stock.

Embedded options benefit the party who has the right to exercise them. Embedded call options benefit the issuer, while embedded put and conversion options benefit the bondholder.

LOS 48.b

Legal and regulatory matters that affect fixed-income securities vary by the places where they are issued and traded, and the location of the issuing entities.

Domestic bonds trade in the issuer's home country and currency. Foreign bonds are from foreign issuers but denominated in the currency of the country where they trade. Eurobonds are issued outside the jurisdiction of any single country and can be issued in any currency. Global bonds are traded in the Eurobond market and at least one domestic market.

Interest income is typically taxed at the same rate as ordinary income, while gains or losses from selling a bond are taxed at the capital gains tax rate. However, the increase in value toward par of original issue discount bonds is considered interest income. In the United States, interest income from municipal bonds is usually tax exempt at the national level and in the issuer's state.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 48.1

1. **A** A balloon payment in a loan schedule is a partial payment of principal that is made at the end of the loan's life. Compared to an otherwise equivalent fully amortizing loan, the existence of the balloon payment will lead to lower periodic payments over the life of the loan because the borrower has to repay less principal before maturity. There will be a higher final payment, however, at maturity. (LOS 48.a)
2. **B** With a sinking fund, the issuer must redeem part of the issue before maturity, but the specific bonds to be redeemed are not known. Floating-rate notes have an unknown future coupon because it relates to a variable market reference rate; however, they have a known maturity date. In an issue with a term maturity structure, all the bonds are scheduled to mature on the same date. (LOS 48.a)
3. **C** This pattern describes a deferred-coupon bond. The first payment of \$229.25 is the value of the accrued coupon payments for the first three years. (LOS 48.a)
4. **B** If the price of the bond increases above the call price stipulated in the bond indenture, it will benefit the issuer to call the bond. Theoretically, the price of a currently callable bond should never rise above its call price. (LOS 48.a)
5. **B** Tax authorities typically treat the increase in value of a pure-discount bond toward par as interest income to the bondholder. In many jurisdictions, this interest income is taxed periodically during the life of the bond, even though the bondholder does not receive any cash until maturity. (LOS 48.b)

READING 49

FIXED-INCOME ISSUANCE AND TRADING

MODULE 49.1: FIXED-INCOME ISSUANCE AND TRADING



Video covering this content is available online.

LOS 49.a: Describe fixed-income market segments and their issuer and investor participants.

Global bond markets are primarily segmented by the type of issuer (or *sector*), credit quality, and time to maturity. Other classifications used include currency, the issuer's geography, and environmental, social, and governance (ESG) features.

- *Type of issuer.* Major classifications of bond issuers are governments (sovereign and non-sovereign), corporates, and special purpose entities issuing asset-backed securities (ABSs).
- *Credit quality.* Standard & Poor's (S&P) and Moody's are major examples of **credit rating agencies** that provide credit ratings on bonds. Ratings from AAA down to BBB- (for S&P) and Aaa through Baa3 (Moody's) are considered **investment-grade bonds**. Bonds BB+ or lower (Ba1 or lower for Moody's) are termed **high-yield bonds** (speculative, or "junk" bonds).
- *Original maturities.* Fixed-income markets are usually segmented into short-term investments with original maturities of less than 1 year (known as money market securities), intermediate-term securities with original maturities of 1 to 10 years, and long-term securities with original maturities over 10 years.

The credit/maturity spectrum for issuers is summarized in Figure 49.1.

Figure 49.1: Issuer Credit/Maturity Spectrum

	Years to Maturity →		
	< 1y Short-Term	1y-10y Intermediate-Term	> 10y Long-Term
"Default Risk Free"	Treasury bills	Treasury notes	Treasury bonds
Investment Grade	Repo Commercial Paper	Unsecured Corporate bonds	Unsecured Corporate bonds
	ABCP	ABS	MBS
High Yield		Secured Corporate bonds	
		Leveraged loans	
Credit Quality ↓			

Note: ABCP is asset-backed commercial paper.

Source: Reproduced from Level I CFA Curriculum learning module, "Fixed-Income Issuance and Trading," with permission from CFA Institute.



PROFESSOR'S NOTE

Repos here refers to sale and repurchase agreements, which we will describe in a later reading. For now, it is enough to know that repos are short-term secured borrowing agreements that can be used for short-term financing. Asset-backed commercial paper is a form of short-term asset-backed securities.

Secured corporate bonds appear in the "high-yield" category because we are referring to *new* issues. Companies with less reliable operating cash flows will have to offer security to investors when issuing debt. The high-yield sector also includes the bonds of previously investment-grade issuers that have been downgraded by credit rating agencies due to deteriorating credit quality (such bonds are known as **fallen angels**).

The type of bond that a corporate issuer chooses to issue is generally driven by the access the issuer has to capital markets and by the intended use of the proceeds of the issue.

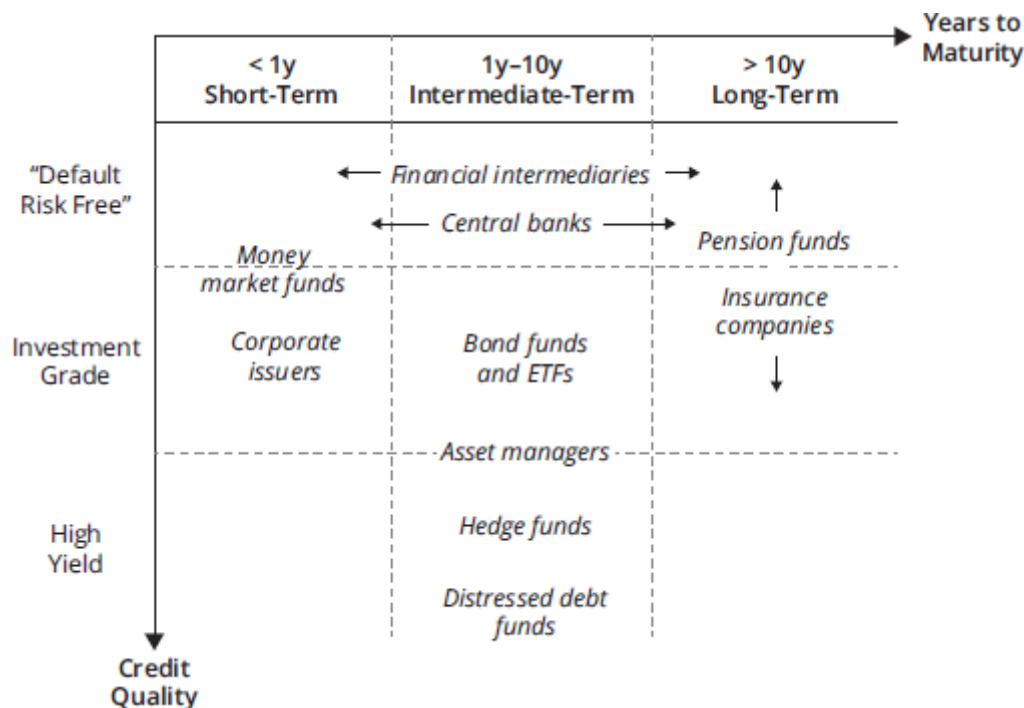
A well-established, investment-grade company could choose to issue commercial paper to fund short-term working capital requirements, intermediate-term debt to fund medium-term investments and permanent working capital, and long-term debt to fund capital investment in fixed assets. The short- and medium-term issues could be arranged for a fee with a **syndicate** of banks offering **credit facilities**, allowing the issuer to issue securities when required for their business operations. However, a riskier company with less stable operating cash flows is likely to have limited access to secured short-term financing and leveraged loans.

Investor Positioning in the Credit/Maturity Spectrum

Where in the credit/maturity spectrum investors choose to invest will depend on their desired interest rate and credit risk exposure, and the maturity of any obligations they need to meet with the cash flows from the bonds.

Common positioning for different types of fixed-income investors is summarized in Figure 49.2.

Figure 49.2: Investor Positioning in Credit/Maturity Spectrum



Source: Reproduced from Level I CFA Curriculum learning module, "Fixed-Income Issuance and Trading," with permission from CFA Institute.

Key points here are the following:

- Pension funds and insurance companies invest in long-term, investment-grade securities to match their long-term liabilities (paying pensions and claims on insurance policies). These institutions are often prohibited by regulations from owning high-yield securities.
- Corporations seek to earn returns on excess liquidity by investing in commercial paper, repos, and ABCP.
- Central banks use intermediate-term Treasury notes as a monetary policy tool to increase or decrease the monetary reserves of commercial banks.
- Bond funds and ETFs will position according to their stated mandate, usually in investment-grade intermediate securities excluding Treasuries. Asset managers seeking higher returns would invest in riskier high-yield intermediate securities, alongside hedge funds and, at the lowest end of the credit spectrum, distressed debt funds (discussed later in this reading).

- Financial intermediaries (banks) use Treasuries across the whole maturity spectrum to manage interest rate and liquidity risks.

LOS 49.b: Describe types of fixed-income indexes.

Fixed-income indexes differ from equity indexes in the following three ways:

1. Corporate bond issuers can, and often do, have many different bonds outstanding, while they will likely have no more than two or three different classes of shares. This leads to fixed-income indexes having many more constituents than equity indexes. Consequently, bond tracker funds employ sampling techniques rather than purchasing all the constituents of a fixed-income index to keep transaction complexity reasonable.
2. Bonds maturing and being issued more frequently cause a higher frequency of removal and replacement of constituents (called turnover) in fixed-income indexes versus in equity indexes.
3. Bonds are issued across multiple sectors. Usually, governments are the largest issuer of bonds; hence, broad bond indexes have large weights in sovereign bonds. Changes in debt issuance trends in terms of maturity and credit quality affect the weights of fixed-income indexes over time.

Bond indexes that contain a broad selection of bonds are called **aggregate indexes**. An example of an aggregate index is the Bloomberg Barclays Aggregate Index, which includes bonds from all sectors across 28 currencies. The index excludes high-yield and unrated issues and bonds that do not meet minimum size for inclusion.

Indexes can have a narrower focus on geography, credit quality, sector, or maturity. For example, the JP Morgan Emerging Markets Bond Index Plus contains U.S. dollar-denominated emerging market sovereign debt with a rating of Baa1/BBB+ or below, with minimum size and maturity limits.

Indexes can also incorporate ESG factors in their construction. For example, the Bloomberg Barclays MSCI Euro Corporate Sustainable SRI Index has a minimum credit rating of Baa3/BBB- and a minimum ESG rating of BBB. The index also screens out certain business sectors deemed incompatible with sustainable investing (e.g., alcohol or generation of thermal coal).

An index chosen to act as a benchmark for a bond fund should match the exposure of the fund in terms of sector focus, credit quality, and maturity.

LOS 49.c: Compare primary and secondary fixed-income markets to equity markets.

Primary Markets

Sales of newly issued bonds are referred to as **primary market** transactions, whereby the issuer sells new securities to investors and receives new capital in return. Newly issued bonds can be registered with securities regulators for sale to the public (a *public*

offering) or sold only to selected investors (a *private placement*). Both are usually carried out through financial intermediaries (i.e., investment banks). An issuer that is offering its first-ever bond is referred to as a **debut issuer** and is typically a growing and maturing firm that is replacing bank loans in its capital structure with the proceeds from the bond issue.

Bonds can be sold through an **underwritten offering** or a **best-efforts offering**. In an underwritten offering, the bond issue price is guaranteed by the financial intermediaries conducting the bond sale to investors. Issues that are not underwritten are said to be conducted on a best-efforts basis. An issue price is not guaranteed by the intermediaries, but they will charge a commission for placing the bonds with investors at the best price possible.

In a **shelf registration**, a bond issue is registered with securities regulators in its aggregate value with a master prospectus. The bonds can then be issued over time when the issuer needs to raise funds.

Debut issues usually require weeks of “roadshows” by underwriters prior to the issue date to introduce investors to the debut issuer. Subsequent **repeat issues** of fixed-income securities usually take much less time. For an investment-grade frequent issuer with a fully underwritten shelf registration, the time between the transaction being agreed and being completed and allocated to investors may be a matter of hours, because bond funds and other investors are likely already familiar with the issuer and regulatory registrations have already been made. Issuance is likely to take significantly longer for a smaller high-yield issue with more detailed covenants and collateral provisions, made on a best-efforts basis.

Some bonds, especially government bonds, are sold through a public auction, a process we will describe in our reading on Fixed-Income Markets for Government Issuers.

Secondary Markets

Secondary markets refer to the trading of previously issued bonds among investors.

While some electronic trading platforms and exchange-based trading exist, the majority of trading in the secondary market remains in the dealer, or over-the-counter (OTC), market.

Dealers post quotes comprising *bid* (purchase) prices and *ask* or *offer* (selling) prices for various bond issues. The difference between the bid and ask prices is the dealer’s spread. The spread varies across individual bonds according to their liquidity, ranging from a fraction of a basis point for liquid, recently issued (“on-the-run”), developed market sovereign bonds and high-credit-quality corporate frequent issuers, to 10–20 basis points or more for less liquid, smaller, or older (“seasoned”) corporate issues.

Distressed debt is a name given to the bonds of issuers that are in, or expected to file for, bankruptcy. A distressed debt investor might buy the debt from other institutions that are prohibited from owning securities with low credit ratings, and aims to profit from the issuer’s fortunes reversing, higher-than-expected recovery rates in liquidation, or value-enhancing restructuring of the issuer. For an otherwise infrequently traded issue, entering a distressed situation may temporarily increase its trading activity.



MODULE QUIZ 49.1

1. Funds required by a corporation to finance investment in seasonal working capital are *most likely* raised through issuing:
A. secured bonds.
B. Treasury notes.
☒ C. commercial paper.
2. Compared to equity indexes, aggregate fixed-income indexes are *most likely* to have a lower:
A. turnover.
☒ B. weight in the corporate sector.
C. number of constituents.
3. In which type of primary market transaction does an investment bank sell bonds on a commission basis?
A. Single-price auction.
☒ B. Best-efforts offering.
C. Underwritten offering.
4. Secondary market bond transactions *most likely* take place:
A. in dealer markets.
☒ B. in brokered markets.
C. on organized exchanges.
5. Sovereign bonds are described as “on the run” when they:
☒ A. are the most recent issue in a specific maturity.
B. have increased substantially in price since they were issued.
C. receive greater-than-expected demand from auction bidders.

KEY CONCEPTS

LOS 49.a

Global bond markets can be classified by the following:

- *Type of issuer.* Sovereign, corporate, and special purpose entities issuing ABSs.
- *Credit quality.* Investment-grade (Baa3/BBB– and above), non-investment grade (Ba1/BB+ and below).
- *Original maturity.* Short term/money market (one year or less), intermediate term (one year to 10 years), long term (more than 10 years).

Well-established investment-grade corporations may issue commercial paper to fund short-term working capital requirements; intermediate-term debt to fund medium-term investments and permanent working capital; and long-term debt to fund capital investment in fixed assets.

Where in the credit/maturity spectrum investors choose to invest will depend on their desired interest rate and credit risk exposure and the maturity of any obligations they need to meet.

- Pension funds and insurance companies: long-term investment grade
- Corporations: short-term investment grade (excluding Treasury bills)

- Central banks: intermediate-term Treasury notes used to conduct monetary policy
- Bond funds and ETFs: intermediate investment-grade (excluding Treasury notes)
- Asset managers seeking higher returns: high-yield intermediate securities, distressed debt
- Financial intermediaries (banks): Treasuries across the maturity spectrum

LOS 49.b

Relative to equity indexes, fixed-income indexes have more constituents, higher turnover, and weights more affected by issuer sector and changes in borrowing trends over time.

Broad bond indexes that include all relevant bonds are referred to as aggregate indexes. Narrower indexes focus on sector, credit quality, geography, or ESG considerations.

LOS 49.c

Bonds may be issued in the primary market through a public offering or a private placement. Bond issues that are underwritten have a price guaranteed by financial intermediaries; those on a best-efforts basis have no such guarantee. Public offerings of government debt commonly take place through auctions. Debut issues are likely to be more time consuming and costly than subsequent repeat issues.

In secondary markets, while some bonds trade on electronic platforms or exchanges, most are traded in OTC dealer markets. Spreads between bid and ask prices are narrower for liquid issues and wider for less liquid issues.

Distressed debt refers to bonds of issuers that are in, or expected to file for, bankruptcy.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 49.1

1. **C** Corporations are most likely to fund short-term seasonal investment in working capital by issuing short-term commercial paper because its maturity (less than a year) will match the required investment period in working capital. Intermediate-term corporate bonds are usually issued to invest in longer-term working capital requirements and medium-term fixed capital investments. A corporation cannot issue Treasury notes, which are intermediate-term securities issued by governments. (LOS 49.a)
2. **B** Aggregate fixed-income indexes include constituents from all sectors. Hence, fixed-income indexes will have higher weights allocated to sovereign issuers and lower weights to corporate issuers than equity indexes, which are based purely on corporate issuers of equity. A fixed-income index is expected to have higher turnover (due to maturing bonds and frequent re-issues) and a higher number of constituents (due to a single issuer typically having more bond issues in existence than equity issues). (LOS 49.b)

3. **B** In a best-efforts offering, the investment bank or banks do not underwrite (i.e., purchase all of) a bond issue, but rather sell the bonds on a commission basis. Bonds sold by auction are offered directly to buyers by the issuer, typically a government. (LOS 49.c)
4. **A** The secondary market for bonds is primarily a dealer market in which dealers post bid and ask prices. (LOS 49.c)
5. **A** Sovereign bonds are described as “on the run” when they represent the most recent issue in a specific maturity. (LOS 49.c)

READING 50

FIXED-INCOME MARKETS FOR CORPORATE ISSUERS

MODULE 50.1: FIXED-INCOME MARKETS FOR CORPORATE ISSUERS



Video covering this content is available online.

LOS 50.a: Compare short-term funding alternatives available to corporations and financial institutions.

Short-Term Funding for Nonfinancial Corporations

A nonfinancial company usually raises external funds for investment in short-term assets (cash, short-term investments, accounts receivable, and inventory) through financial intermediaries that provide either loan financing or security-based financing.

External Loan Financing

External loan financing, or **bank lines of credit**, refers to agreements between borrowers and banks to draw down funds as required. These primarily consist of three types, which are uncommitted, committed, or revolving lines of credit:

1. *Uncommitted line of credit.* A bank extends an offer of credit for a principal amount (credit line), usually charging a floating market reference rate (MRR) plus a fixed credit spread on funds drawn down. The credit is “uncommitted” in the sense that the bank may refuse to lend if circumstances change. As a result, this is a less reliable source of funds than the other two types. However, it is likely to be a flexible agreement with no fees outside of interest charges. These agreements can be offered unsecured if the borrower maintains stable cash balances with the bank.
2. *Committed (regular) line of credit.* A bank commits to an offer of credit for a specific time period, providing a more reliable source of funding for borrowers than an uncommitted line of credit. Banks charge a commitment fee, typically about 50 basis points, on either the full or unused amount over the commitment period. Regulators require banks to hold a higher level of reserves to cover potential defaults on committed lines versus uncommitted lines. Banks can mitigate these default risks by offering commitments for less than one year or by acting in syndicate with other banks to provide such agreements. Although banks commit to extend the line of credit for the stated maturity of the agreement, they may withdraw the agreement at maturity should credit conditions worsen, leading to **renewal risk** for borrowers.

3. *Revolving (operating) line of credit.* An even more reliable source of short-term financing than a committed line of credit, “revolvers” are typically for a longer term, sometimes years (with potential medium-term loan facilities). Banks typically place restrictive covenants on borrowers under such agreements. Fees and rates are similar to a committed line of credit.

Companies with weaker credit ratings typically have to pledge assets as collateral for bank borrowings. **Secured (asset-backed) loans** are backed by collateral (e.g., fixed assets, receivables, or inventory). Companies can assign receivables to lenders to act as collateral for loans. **Factoring** refers to the actual transfer of credit granting and collection of receivables to a lender (“factor”) at a discount from their face value. The size of the discount, which represents the interest rate on the loan from the factor, depends on the creditworthiness of the firm’s customers, and on collection costs.

External Security-Based Financing

Large corporations with high credit ratings can reduce their funding costs by issuing short-term unsecured debt securities, referred to as **commercial paper (CP)**. For these firms, the interest cost of CP is less than the interest on a bank loan. With maturity of typically less than three months, CP is issued by firms to fund working capital and as a temporary source of funds before issuing longer-term debt. Debt that is temporary until permanent financing can be secured is referred to as **bridge financing**.

CP is often reissued, or “rolled over,” when it matures. The risk that a company will not be able to sell new CP to replace maturing paper is termed **rollover risk**. To manage this risk, borrowers maintain **backup lines of credit** with banks. These are sometimes referred to as liquidity enhancement or backup liquidity lines, whereby lenders agree to provide funds to make repayments on CP when it matures, if needed.

Similar to U.S. T-bills, CP in the United States is typically issued as a pure discount security, making a single payment equal to the face value at maturity. A smaller, less liquid international market in CP also exists, referred to as **Eurocommercial paper (ECP)**.

Short-Term Funding for Financial Institutions

Commercial and retail deposits are a major short-term funding source for banks.

- *Checking accounts* (“demand deposits”) provide transactions services and immediate availability of funds but typically pay no interest.
- *Operational deposits* are made by larger customers who require cash management, custody, and clearing services.
- *Savings deposits* have a stated term and interest rate. These may take the form of an interest-bearing **certificate of deposit (CD)**, which pays interest at a specified maturity of less than a year. *Nonnegotiable CDs* cannot be sold before maturity, and early withdrawal of funds incurs a penalty. *Negotiable CDs* can be sold in the open market before maturity as a means of early withdrawal of funds. At the wholesale (institutional investor) level, negotiable CDs are an important funding source for banks. They trade in domestic bond markets as well as in the Eurobond market.

Funds that are loaned by one bank to another are referred to as **interbank funds**. Banks lend to each other for periods of one day to a year, on either a secured or unsecured basis, at an interest rate based on a market reference rate (MRR) that varies across markets. The most common type of secured interbank borrowing and lending is carried out through repurchase agreements (repos), which we discuss later in this reading.



PROFESSOR'S NOTE

We saw this interbank MRR when we defined the coupon paid by FRNs as being equal to a variable MRR plus a fixed margin. In a stable banking system, the short-term interbank rate is likely to be of very low credit risk. This MRR can be used as a starting point for other interest payments that need to be made on riskier loans (e.g., the coupon on FRNs issued by corporations). An example of an MRR is the Secured Overnight Funding Rate (SOFR) in the United States.

Another source of short-term funding for banks is to borrow excess reserves from other banks in the **central bank funds market**. Banks in most countries must maintain a portion of their funds as reserves on deposit with the central bank. Banks with excess reserves lend them to banks that need funds at the **central bank funds rate**, which is strongly influenced by the central bank's open market operations and by the availability of short-term funds. The central bank may act as the lender of last resort ("discount window lending") to banks struggling to access liquidity. This will likely be made at a higher rate than the central bank funds rate and may bring extra scrutiny and restrictions on the activities of the borrower.

Financial institutions issue more commercial paper than nonfinancial companies. Financial institutions often sponsor **asset-backed commercial paper (ABCP)**, a type of short-term asset-backed security. The ABCP creation process is as follows:

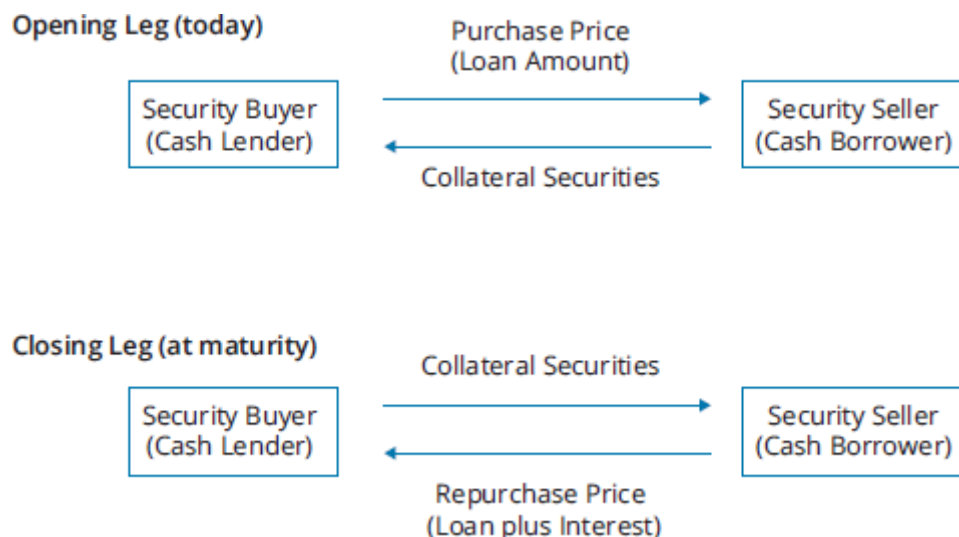
1. A financial institution transfers collateral (usually existing short-term loans made by the bank) to a separate legal entity called a special purpose entity (SPE), in return for cash. The SPE is set up as an off-balance-sheet vehicle with respect to the financial institution.
2. The SPE sells ABCP to investors, who accept the risk and return of the collateral backing the ABCP. The sponsoring financial institution provides a backup credit liquidity line.

LOS 50.b: Describe repurchase agreements (repos), their uses, and their benefits and risks.

A **repurchase agreement (repo)** is an arrangement by which one party sells a security to a counterparty with a commitment to buy it back at a later date at a prespecified higher price. The original purchase price is effectively a loan by the security buyer to the security seller, with the security as collateral. The difference between the repurchase price and the original purchase price accounts for the interest paid to the security buyer. The annualized interest rate implied by the difference between the two prices is called the **repo rate**.

The basic structure of a repo is displayed in Figure 50.1.

Figure 50.1: Repurchase Agreement



To protect the lender against a potential decrease in the value of the securities posted as collateral, the borrower typically must post extra collateral above the loan amount (the purchase price) by an amount known as the **initial margin**. In practice this means the loan amount will be a discount to the value of the securities.

As an example, consider a firm that wishes to borrow by entering into a repo agreement to sell a bond today with a market value of \$1 million and repurchase it 90 days later (the **repurchase date**). The lender requires a repo rate of 2% and initial margin of 103%.

The initial purchase price (the initial loan amount) is calculated as the market value of the securities posted as collateral divided by the initial margin:

$$\begin{aligned} \text{purchase price (loan amount)} &= \frac{\text{market value of securities}}{\text{initial margin}} = \frac{\$1,000,000}{1.03} \\ &= \$970,874 \end{aligned}$$

The repurchase price after 90 days is calculated as the purchase price (loan amount) multiplied by one plus the de-annualized repo rate. Assuming a 360-day count convention, this is calculated as follows for our example:

$$\text{repurchase price} = \$970,874 \times [1 + (0.02 \times (90 / 360))] = \$975,728$$

The discount applied to the market value of collateral to get the purchase price is referred to as a **haircut**. In this example, the haircut is calculated as follows:

$$\text{haircut} = \frac{(\$1,000,000 - \$970,874)}{\$1,000,000} = 2.91\%$$

$$\text{The haircut can also be calculated quickly as } 1 - \frac{1}{\text{initial margin}}.$$

The loan value increases during its life at the repo rate. Should the market value of the collateral fall below this value times the initial margin, the lender will ask the borrower for more collateral, known as **variation margin**.

For example, given the repo details just listed, assume that after 30 days the market value of the bond has fallen to \$990,000:

$$\begin{aligned}\text{adjusted loan amount after 30 days} &= \$970,874 \times [1 + (0.02 \times (30 / 360))] \\ &= \$972,492\end{aligned}$$

$\$972,492 \times 1.03 = \$1,001,667$. Because the collateral value is less than this, the borrower must provide variation margin.

$$\begin{aligned}\text{variation margin} &= (\text{initial margin} \times \text{adjusted purchase price}) - \text{market value of collateral} \\ &= \$1,001,667 - \$990,000 = \$11,667\end{aligned}$$

In this case, the security buyer/lender will ask the security seller/borrower to post an extra \$11,667 of securities as collateral. If the variation margin is negative, then the loan is overcollateralized, and the borrower can request a release of collateral equal in value to the variation margin amount.

A repurchase agreement for one day is called an **overnight repo**, and an agreement covering a longer period is called a **term repo**. Due to the short-term collateralized nature of a repo and the fact that collateral is usually high-quality, liquid, sovereign bonds, interest rates are customarily less than the rate on bank loans or other short-term borrowing.

Although the transactions underlying the repo are described as a “sale” and “repurchase” of securities, the seller/borrower retains rights to the benefits of holding the bond over the repo term. The collateral used in the repo may involve a specific security or a general type of security (e.g., Treasury bonds of a certain range of maturities), in which case it is known as a **general collateral repo**. The details of the contractual terms of the repo are contained in the **master repurchase agreement** between the counterparties.

Repo Applications

The main uses of repurchase agreements are as follows:

- Financial institutions enter repos as security sellers/borrowers to finance positions in securities held in their trading activities.
- Banks and institutional investors, such as mutual funds and pension funds, enter repos as security buyers/lenders to earn the repo rate on excess short-term funds.
- Central banks may use repos to enact monetary policy, buying securities/lending to increase the money supply and selling securities/borrowing to decrease the money supply.
- Short sellers, such as hedge funds, can use repo agreements to borrow securities that they intend to short sell, speculating that the security value will decrease. This is executed through the following trades:
 1. Buy securities/lend in a repo.
 2. Short sell the securities in the open market.
 3. Buy back the securities in the open market later (before the repo term ending).
 4. Deliver the securities back to the repo counterparty at the maturity of the repo.

The hedge fund earns the repo rate from being the lender in the repo, and gains if the security value decreases over the repo term. When the motivation to enter a repo is to borrow a security in this way, the participant is said to be entering a **reverse repo**.

A short seller engaging in the type of transaction just described will need to specify the security they wish to borrow (known as a special trade). If the specified security is difficult to borrow, the hedge fund will be willing to accept a lower repo rate than that earned on general collateral, or may even be willing to accept a negative repo rate for extremely-hard-to-source collateral.

Factors Affecting the Repo Rate

The repo rate is usually:

- Higher, when interest rates for alternative sources of short-term (money market) funds are higher
- Lower, the higher the credit quality of the collateral security
- Higher, the longer the repo term (when longer-term rates are generally higher than short-term rates)
- Lower, when the collateral security is in high demand or low supply
- Higher, if the repo is undercollateralized, or if collateral is specified but not actually delivered to the lender

Repo Risks

While a repo agreement is a safer form of lending than most sources of short-term funds, it remains a source of debt financing to the borrower, and overuse can lead to financial distress or insolvency. Like other forms of collateralized borrowing, the major risks include the following:

- Default risk, that the lender of the security (borrower of cash) fails to make the repurchase payment at the end of the repo
- Collateral risk, relating to the value that can be generated for collateral in event of default
- Margining risk, relating to the timely and accurate calculation and payment of margin
- Legal risk, that the contracts cannot be legally enforced
- Netting and settlement risk, relating to the ability to net off payments across different contracts with the same nondefaulting counterparty, and the ability to settle the cash and collateral transactions underlying the repo

Many of the risks just listed can be mitigated through using **tri-party repos**, which employ a third-party intermediary (usually a custodian bank or clearinghouse) as an agent to arrange and administer repo transactions. While this does not reduce credit risk, it does likely improve cost efficiencies with respect to access to collateral and counterparties, and the valuation and safekeeping of assets. A repo agreement that is struck directly between two parties without a third party is referred to as a **bilateral repo**.

LOS 50.c: Contrast the long-term funding of investment-grade versus high-yield corporate issuers.

Recall that in a normal yield curve environment, bond yields are higher for longer-dated maturities, reflecting higher risk-free rates and credit spreads over longer time frames. In this environment, both investment-grade and high-yield corporate issuers must offer higher yields on longer-maturity bond issues. This difference in yields across maturity is likely to be greater, however, for high yield issuers due to the higher spreads offered on high yield debt. Companies that choose to issue shorter-dated bonds to reduce the yield they pay on their debt assume rollover risk.

Other major differences between issuances of investment grade debt versus high yield debt include the following:

- Default risk, and loss given default, are primary concerns for high yield investors due to the lower credit quality of the issuers. For investment grade investors, the primary credit-related concern is the chance of a ratings downgrade and the probability of future default increasing, rather than an imminent default.
- Credit spreads are likely to be a smaller proportion of yield for investment grade issues, where yields are largely tied to benchmark rates (e.g., sovereign debt).
- Investment grade issuers usually face only a few restrictive covenants on debt issues, typically limiting liens and sale and leaseback arrangements on the core operating assets that are expected to generate the operating cash flows required to repay the debt. High yield issuers are likely to face a larger number of restrictive covenants relating to debt-based ratios, issuance of additional debt, and distributions of capital to equity investors (e.g., limits on dividend payments). High yield issuers are also likely to need to provide collateral as security for bond issues.
- Investment grade issues are somewhat standardized (similar across different issues) and typically issued across multiple maturities, which reduces rollover risk for the issuer. This choice of maturity allows investment grade issuers to take advantage of changes in market conditions. High yield issues are likely to have more specific covenants and liens, making issues less standardized. High yield issuers also typically have less flexibility with regard to maturity, with bonds usually being issued with maturity of 10 years or less. With less flexibility and less standardization, high yield issuers are less able to take advantage of opportunities to refinance debt when borrowing costs fall.
- High yield issuers are more likely to structure their debt so that it can be repaid earlier if their credit quality improves. This is done either by taking leveraged loans with prepayment options or by issuing callable debt.
- Due to the higher uncertainty in the cash flows used to repay high yield debt, returns are likely to be more equity-like than the returns of investment grade debt.



MODULE QUIZ 50.1

1. Restrictive covenants are *most likely* to be placed on borrowers under a:
 - A. revolving line of credit.
 - B. factoring arrangement.

- C. committed line of credit.
2. A borrower pledges \$100 million of securities as collateral for an overnight repo with a repo rate of 4% and an initial margin of 101%. The purchase price of the repo is *closest* to:
- A. \$99,000,000.
 - B. \$99,009,901.
 - C. \$101,000,000.
3. A borrower pledges \$100 million of securities as collateral for an overnight repo with a repo rate of 4% and an initial margin of 101%. Assuming 360 days in a year, the interest paid under the repo is *closest* to:
- A. \$10,891.
 - B. \$11,001.
 - C. \$11,111.
4. A borrower pledges \$100 million of securities as collateral for an overnight repo with a repo rate of 4% and an initial margin of 101%. The haircut on collateral is *closest* to:
- A. 0.01%.
 - B. 0.99%.
 - C. 1.01%.
5. Relative to a long-term high-yield bond issue, an investment-grade bond issue is *most likely* to have a:
- A. longer maturity.
 - B. greater number of covenants.
 - C. higher proportion of its yield related to credit spreads.

KEY CONCEPTS

LOS 50.a

Forms of short-term funding for nonfinancial corporations include lines of credit (uncommitted, committed, and revolving in order of increasing reliability and cost), secured loans, factoring arrangements, and commercial paper issuance.

A major form of short-term funding for financial institutions is deposits from customers. Savings deposits can take the form of certificates of deposit, which can be nonnegotiable (no early withdrawal without penalty), or negotiable (can be sold to a third party). Other sources of short-term financing for financial institutions include interbank funds (including repos), central bank funds, and commercial paper (including asset-backed commercial paper).

LOS 50.b

A repurchase agreement is a collateralized loan whereby a borrower sells a security to a counterparty with a commitment to buy it back later at a higher price. The annualized percentage difference between the two prices represents the repo rate.

Borrowers under a repo must post collateral, which must maintain a value greater than the initial margin or the borrower must provide variation margin.

The main uses of repos are for borrowers to finance positions in securities, for lenders to earn the repo rate on excess liquidity, for central banks to engage in monetary policy,

and for short sellers to temporarily borrow securities (as the lending party in the repo).

The repo rate will be higher for longer-term repos, lower quality collateral, or when the repo is undercollateralized. The repo rate will be lower for a specific security that is difficult to source.

Repo risks include default risk, collateral risk, margining risk, legal risk, and netting and settlement risk. The use of a tri-party repo can help mitigate these risks.

LOS 50.c

Relative to investment-grade debt, high-yield debt is likely to have a higher proportion of its yield related to credit spread, a greater number of covenants, shorter maturity, and more equity-like returns, and is likelier to have call or prepayment provisions.

Probability of default risk, and loss given default, are a greater concern for high-yield investors than investment-grade investors, whose primary concern is the prospect of credit downgrades.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 50.1

1. **A** Restrictive covenants are most likely to be placed on borrowers using revolving lines of credit because these agreements require banks to commit funds over the longest terms. A factoring arrangement involves selling accounts receivable to a third party. (LOS 50.a)
2. **B** Repo purchase price (loan amount) = market value of securities / initial margin
$$= \$100,000,000 / 1.01 = \$99,009,901$$

(LOS 50.b)
3. **B** Repo purchase price (loan amount) = market value of securities / initial margin
$$= \$100,000,000 / 1.01 = \$99,009,901$$

Repo interest = $\$99,009,901 \times 0.04 \times (1 / 360) = \$11,001.10$
(LOS 50.b)
4. **B** $1 - 1 / 1.01 = 0.0099$, or 0.99%.
(LOS 50.b)
5. **A** An investment grade bond is likely to have a longer maturity than a high yield bond because high yield issues tend to be restricted to 10-year maturity or less, while investment grade issuers can issue bonds of any maturity. Investment-grade bond issues typically have fewer restrictive covenants than high yield issues. The proportion of yield that is credit spread is likely to be lower for investment grade issues, where yield is primarily composed of benchmark rates. (LOS 50.c)

READING 51

FIXED-INCOME MARKETS FOR GOVERNMENT ISSUERS

MODULE 51.1: FIXED-INCOME MARKETS FOR GOVERNMENT ISSUERS



Video covering this content is available online.

LOS 51.a: Describe funding choices by sovereign and non-sovereign governments, quasi-government entities, and supranational agencies.

Sovereign Debt

National governments issue bonds to raise funds for spending on public goods and services and investment in public infrastructure. These sovereign issues are backed by the power to collect taxes and therefore typically carry the highest credit rating in their domestic market. Sovereign issuers are usually the largest debt issuers in their domestic market.

Public sector issuers such as governments prepare financial reports, but the accounting standards that apply differ from those in the private sector. Generally, public sector accounting standards are based more on cash transactions and less on accruals (e.g., depreciation, unfunded liabilities). When assessing the financial statements of a government, an analyst should consider the issuer to have an “economic balance sheet” that includes implied assets (e.g., expected future tax revenues) and implied liabilities (e.g., promised future expenditures) in addition to the financial assets and liabilities reported in the public accounts.

A key distinction in sovereign debt markets is the difference between **developed market** versus **emerging market** issuers.

Developed market sovereign issuers have stable, diversified economies with consistent and transparent fiscal policy. Debt is denominated in a **reserve currency** (i.e., a major currency held as reserves by central banks across the world and widely used in international trade, such as the U.S. dollar or the Chinese renminbi).

Emerging market sovereign issuers typically have faster growing, less stable, and more concentrated economies and, consequently, less stable tax revenues, which are sometimes tied to a dominant industry or commodity. Emerging market debt is often

raised to fund investment in economic growth and can be **domestic debt** or **external debt**.

- Domestic debt is issued in the nation's home currency and is held by domestic investors. That currency might not be freely convertible into other currencies due to illiquidity or restrictions on capital flows.
- External debt is debt owed to foreign creditors and may be denominated in the government's home currency or a foreign reserve currency. When external debt is denominated in a reserve currency, foreign investors avoid the direct currency risk of the issuer's currency weakening, but the investor still faces indirect currency risk related to the emerging market government generating enough flows of the reserve currency to make repayments on its external debt.

A government's **debt management policy** sets out the amount and type of securities the government intends to issue. As we discussed in the Economics topic area, fiscal policy is often used as a tool to manage the business cycle. A government that believes the economy is below full employment may tax less and spend more to stimulate economic activity. This may require the government to issue new debt. Analysts who wish to forecast a government's debt issuance needs should focus on its fiscal policy, as well as how cyclical and inflation-sensitive its revenues and spending are. Analysts should also be aware of debt features such as floating rates or inflation indexing, as well as whether the government guarantees any non-sovereign debt.

Recall from the Economics topic area the condition of Ricardian equivalence, which suggests taxpayers expect that future taxes will have to repay any debt the government issues. This would imply that a government should be indifferent among the possible maturities of its debt, but only under all of the following assumptions about taxpayer behavior:

- They will increase savings when they expect higher future taxes.
- They have rational expectations, expecting tax decreases in the present to be offset by tax increases in the future.
- They can borrow and lend in capital markets that have no transactions costs.
- They can and will pass tax savings on to future generations.

Because these assumptions do not hold in practice, governments must manage how much of their debt is short-term or longer-term. Governments typically issue securities across maturities to maintain a stable split between long-term and short-term debt over time.

Different maturity sectors for sovereign debt have some benefits for market participants. For example, many investors view short-term sovereign debt as safe and highly liquid, and it may function as an alternative to bank deposits. This liquidity benefit to investors likely causes short-term government debt yields to be lower than they would be otherwise. While governments have a great deal of flexibility as to how much short-term debt they issue, relying too heavily on it creates rollover risk.

Having liquid markets in longer maturities of government debt also has benefits. Market participants use government debt yields as benchmarks against which to

$$\text{bond yield} = \text{coupon} / \text{market price}$$

measure the credit risk of non-government debt. Asset managers and financial institutions often rely on government debt in their interest rate risk management, and use it as collateral in transactions such as repurchase agreements. Central banks buy and sell government debt of various maturities to conduct monetary policy.

Nonsovereign Government Debt

Nonsovereign government bonds are issued by states, provinces, counties, and entities created to fund and provide services (e.g., for the construction of hospitals, airports, and other municipal services).

Agency bonds or **quasi-government bonds** are issued by entities that national governments create for specific purposes, such as financing infrastructure investment or providing mortgage financing. An example of such an agency is the Government National Mortgage Association (Ginnie Mae) in the United States, which securitizes and guarantees mortgage loans to facilitate home ownership. Ginnie Mae issues debt securities to finance its operations, the repayment sources for which are guarantor fees from their business alongside U.S. government backing. When they are backed by the sovereign entity, agency bonds typically have yields and credit ratings closely aligned with those of the government.

Local and regional government authorities may issue debt raised for general public spending backed by local tax raising powers (referred to as **general obligation bonds** or GO bonds), or debt issued to fund a specific project (called **revenue bonds**) where the source of repayment is fees from the use of the infrastructure funded by the bond issue (e.g., a toll road or bridge).

Supranational bonds are issued by international institutions such as the World Bank, the IMF, and the Asian Development Bank, which have been set up by multiple sovereign governments to promote economic cooperation, trade, or economic growth. Bonds issued by supranational agencies typically have high credit quality and some issues are highly liquid.

LOS 51.b: Contrast the issuance and trading of government and corporate fixed-income instruments.

While corporate issuers of debt raise debt finance as required, sovereign issuers use regular public auctions to issue government debt securities.

Buyers can make **competitive bids** or **noncompetitive bids** at government debt auctions. Competitive bids are used to set the price of the debt issue, while noncompetitive bids are guaranteed to have their allocation met at the price determined by the competitive bids. The auction is conducted by first allocating bonds to noncompetitive bids. Then, competitive bids are ranked in order of highest price (lowest yield). Bonds are allocated to competitive bids starting with the highest price and moving through the auction order book until the offering amount is met. The yield of the successful competitive bid with the lowest price is referred to as the **cut-off yield**. Under a **single-price auction**, all investors pay the price associated with this

cut-off yield, regardless of the yield they actually bid. Under a **multiple-price auction**, successful competitive bidders actually pay the price that they bid.

A government issuer that wishes to **minimize yield volatility** would likely choose to **conduct single-price auctions** because all the bonds will be **issued at a single yield**. This decrease in yield volatility may increase the chance of a successful auction, distribute bonds more broadly among investors, and result in a lower cost of funds for the issuer. Because successful competitive bidders in a multiple-price auction pay what they actually bid, it is likely that bids will be close together and large in size.

A sovereign issuer typically designates certain financial institutions as **primary dealers** that are required to make competitive bids in auctions, submit bids in auctions on behalf of third parties, and act as counterparty to the central bank when it buys and sells securities to carry out monetary policy.

Once issued, sovereign debt typically trades in quote-driven OTC dealer markets in a similar fashion to corporate bonds. Trading is most active, and prices most informative, for the most recently issued government securities of a particular maturity. These issues are referred to as **on-the-run** bonds and their yields are typically used to represent default-risk-free “benchmark” yields when constructing yield curves.

Investors in government securities may have “noneconomic” objectives. For example, government bonds are often used by central banks to conduct monetary policy; foreign governments purchase sovereign bonds of other nations as reserves; and some financial institutions are required to hold government bonds to comply with regulations. The presence of such investors decreases the yields of sovereign bonds relative to those of non-sovereign issuers.



MODULE QUIZ 51.1

1. Bonds issued by the World Bank are *best* described as:
A. quasi-government bonds.
B. global bonds.
C. **supranational** bonds.
2. A foreign investor who invests in the USD-denominated external debt of an emerging market government has currency risk that is *best* described as:
A. direct.
B. **indirect**.
C. **hedged**.
3. Investors are guaranteed a bond allocation in a Treasury bond auction when:
A. **they submit a noncompetitive bid**.
B. the auction is a single-price auction.
C. they submit a competitive bid in a multiple-price auction.

KEY CONCEPTS

LOS 51.a

Sovereign issuers are usually the highest credit quality and largest issuers of debt in any given bond market.

Developed market sovereign issuers have stable, diversified economies with consistent and transparent fiscal policy, with debt denominated in a major reserve currency. Emerging market sovereign issuers typically have faster growing, less stable, and more concentrated economies and, consequently, less stable tax revenues, sometimes tied to a dominant industry or commodity. Emerging market debt can be domestic (issued in domestic currency and held by domestic investors) or external (owed to foreign creditors).

Ricardian equivalence theory states that governments should be indifferent about raising taxes today or issuing debt of any maturity. The high rollover risk of short-term debt and the need for predictable fiscal policy means that, in practice, governments tend to issue debt evenly across the maturity spectrum.

Benefits of having a broad range of maturities of government securities in existence include the identification of benchmark government yield curves; the ability of investors to hedge interest rate risk; the ability to pledge the securities as collateral for repos; and the ability of the central bank to use government bonds to execute monetary policy.

Agency or quasi-government bonds are issued by entities created by national governments for specific purposes such as financing infrastructure investment or providing mortgage financing.

Local and regional government authorities may issue debt for general public spending backed by local tax raising powers (general obligation bonds) or to fund a specific project (revenue bonds).

Supranational bonds are issued by international institutions that promote economic cooperation, trade, or economic growth.

LOS 51.b

Sovereign issuers use regular public auctions to issue government debt securities.

Auctions bids can be either competitive or noncompetitive. Competitive bids are used to set the price of the debt issue, while noncompetitive bids are guaranteed to have their allocation met at the auction price.

In a single-price auction, all investors pay the price associated with the cut-off yield. In a multiple-price auction, successful competitive bidders actually pay the price that they bid.

Primary dealers are financial institutions required to make competitive bids in auctions, submit bids in auctions on behalf of third parties, and act as counterparties to the central bank.

Once issued, sovereign debt typically trades in quote-driven OTC dealer markets. Trading is most active and prices most informative for on-the-run bonds.

Module Quiz 51.1

1. **C** Bonds issued by the World Bank, which is a multilateral agency operating globally, are termed supranational bonds. (LOS 51.a)
2. **B** While the U.S. investor will not have the direct currency exposure of holding foreign-denominated debt, they still face indirect currency exposure from the emerging market government having to raise USD through international transactions to repay their external debt. (LOS 51.a)
3. **A** Investors who make noncompetitive bids in government bond auctions are guaranteed to have their allocations met at a price determined by the competitive bids. Competitive bidders are not guaranteed allocations in either single-price or multiple-price auctions. (LOS 51.b)

READING 52

FIXED-INCOME BOND VALUATION: PRICES AND YIELDS

MODULE 52.1: FIXED-INCOME BOND VALUATION: PRICES AND YIELDS



Video covering
this content is
available online.

LOS 52.a: Calculate a bond's price given a yield-to-maturity on or between coupon dates.

Calculating the Value of an Annual Coupon Bond

The value of a coupon bond can be calculated by summing the present values of all of the bond's promised cash flows. The market discount rate appropriate for discounting a bond's cash flows is called the bond's **yield to maturity (YTM)**. If we know a bond's YTM, we can calculate its value, and if we know its value (market price), we can calculate its YTM.

Consider a newly issued 5-year, 10% coupon, annual-pay bond. For \$100 of par value, the coupon payments will be \$10 at the end of each year, and the \$100 par value will be paid at the end of Year 5. First, let's value this bond assuming that the appropriate discount rate (also called "yield") is 10%. The present value of the bond's cash flows discounted at 10% is as follows:

$$\frac{10}{(1.10)^1} + \frac{10}{(1.10)^2} + \frac{10}{(1.10)^3} + \frac{10}{(1.10)^4} + \frac{110}{(1.10)^5}$$

The calculator solution is as follows:

$N = 5; PMT = 10; FV = 100; I/Y = 10; CPT \rightarrow PV = -100$

where:

N = number of years

PMT = the *annual* coupon payment

I/Y = the *annual* discount rate (YTM)

FV = the par value or face value of the bond received at maturity

This calculation shows that when the coupon of a bond is equal to its yield, the bond's price is equal to par.



PROFESSOR'S NOTE

Take note of a couple points here. The discount rate is entered as a whole number in a percentage, 10, not 0.10. The five coupon payments of \$10 each are taken care of in the $N = 5$ and $PMT = 10$ entries. The principal repayment is in the $FV = 100$ entry. Lastly, note that the PV is negative; it will be the opposite sign to the sign of PMT and FV. The calculator is just “thinking” that to receive the payments and future value (to own the bond), you must pay the present value of the bond today (you must buy the bond). That’s why the PV amount is negative; it is a cash outflow to a bond buyer.

Now, let’s value that same bond with a discount rate of 8%:

$$\frac{10}{(1.08)^1} + \frac{10}{(1.08)^2} + \frac{10}{(1.08)^3} + \frac{10}{(1.08)^4} + \frac{110}{(1.08)^5}$$

The calculator solution is as follows:

$$N = 5; PMT = 10; FV = 100; I/Y = 8; CPT \rightarrow PV = -107.99$$

If the market discount rate for this bond were 8%, it would sell at a **premium** of \$7.99 above its par value. When bond yields decrease, the present value of a bond’s payments, its market value, increases. Here, we also see that a bond with a coupon greater than its yield will be trading above par.

If we discount the bond’s cash flows at 12%, this is the present value of the bond:

$$\frac{10}{(1.12)^1} + \frac{10}{(1.12)^2} + \frac{10}{(1.12)^3} + \frac{10}{(1.12)^4} + \frac{110}{(1.12)^5}$$

The calculator solution is as follows:

$$N = 5; PMT = 10; FV = 100; I/Y = 12; CPT \rightarrow PV = -92.79$$

If the market discount rate for this bond were 12%, it would sell at a **discount** of \$100 – \$92.79 = \$7.21 to its par value. When bond yields increase, the present value of a bond’s payments, its market value, decreases. Here, we also see that a bond with a coupon less than its yield will be trading below par.



PROFESSOR'S NOTE

It’s worth noting here that a 2% decrease in YTM increases the bond’s value by more (\$7.99) than a 2% increase in yield decreases the bond’s value (\$7.21). This illustrates that the bond’s price-yield relationship is convex, as we will explain in more detail in a later reading.

Calculating the Value of a Semiannual Coupon Bond

Let’s calculate the value of the same bond with semiannual payments. Rather than \$10 per year, the security will pay \$5 every six months. Assuming an annual YTM of 8%, we need to discount the coupon payments at 4% per period, which results in this present value:

$$\frac{5}{(1.04)^1} + \frac{5}{(1.04)^2} + \frac{5}{(1.04)^3} + \dots + \frac{5}{(1.04)^9} + \frac{105}{(1.04)^{10}}$$

The calculator solution is as follows:

$$N = 10; PMT = 5; FV = 100; I/Y = 4; CPT \rightarrow PV = -108.11$$

The stated annualized YTM is equal to the periodic return of the bond multiplied by the number of periods in the year. In this case, the bond actually earns 4% every six months, hence the stated annualized YTM = $4\% \times 2 = 8\%$.

Calculating Yield to Maturity

Now let's calculate the stated yield of the same bond after changes in market conditions have caused the bond price to move to 105. (This price is stated as a percentage of par quote—in other words, investors will pay 105% of the par value they purchase. The easiest way to interpret this price is as the price of \$100 of par.)

We solve for the semiannual return of the bond using the following calculator inputs:

$$N = 10; PMT = 5; FV = 100; PV = -105; CPT \rightarrow I/Y = 4.37\%$$

This is the true semiannual return of the bond. To quote a YTM, we need to multiply by two to give a stated YTM of $4.37\% \times 2 = 8.74\%$. Also note the negative sign for the PV input: we must respect the direction of cash flows (negative for the cash outflow today, positive for the cash inflow later), or else the calculator will not be able to calculate a rate of return.

To actually earn the YTM over the life of a bond, the investor must hold the bond to maturity, the issuer must make all the promised payments, and the investor must be able to reinvest the periodic cash flows and earn the same YTM.

Accrued Interest, Flat Price, and Full Price

So far we have been calculating bond values on the date a coupon is paid, as the present value of the remaining coupons. For most actual bond trades, the settlement date, which is when cash is exchanged for the bond, will fall between coupon payment dates.

Bond pricing has to account for the fact that the next coupon will be paid to the buyer, but a portion of it (the **accrued interest**) is owed to the seller. The accrued interest since the last payment date can be calculated as the coupon payment times the portion of the coupon period that has passed between the last coupon payment date and the settlement date of the transaction. That is:

$$\text{accrued interest} = \text{coupon payment} \times \frac{\text{days from last coupon to settlement}}{\text{days in coupon period}}$$

Financial markets use a variety of **day count methods**. Two of the most common are the **actual/actual convention**, which uses the actual number of days between coupon payments and the actual number of days between the last coupon date and the settlement date; and the **30/360 convention**, which assumes each month has 30 days and a year has 360 days. Government bonds typically use actual/actual, while corporate bonds typically use 30/360.

EXAMPLE: Accrued interest

An investor buys a 4% annual-pay bond that pays its coupons on May 15. The investor's order settles on August 10. Calculate the accrued interest that is owed to the bond seller, using the 30/360 method and the actual/actual method.

Answer:

The annual coupon payment is $4\% \times \$100 = \4 .

Using the 30/360 method, interest is accrued for $30 - 15 = 15$ days in May; 30 days each in June and July; and 10 days in August, or $15 + 30 + 30 + 10 = 85$ days:

$$\text{accrued interest (30/360 method)} = \frac{85}{360} \times \$4 = \$0.944$$

Using the actual/actual method, interest is accrued for $31 - 15 = 16$ days in May; 30 days in June; 31 days in July; and 10 days in August, or $16 + 30 + 31 + 10 = 87$ days.

$$\text{accrued interest (actual/actual method)} = \frac{87}{365} \times \$4 = \$0.953$$

Bond prices are quoted without accrued interest. This is because, holding yield constant, including accrued interest would make a bond's price appear to increase on each day of a coupon period and drop suddenly on the coupon payment date. A bond's quoted price is known as its **flat price** (or clean price).

A bond's **full price** (also known as its invoice price or dirty price) is the sum of its flat price and the accrued interest. However, we cannot simply calculate a flat price and add accrued interest to it. Instead we must calculate the full price and derive the flat price from it:

$$\boxed{\text{flat price} = \text{full price} - \text{accrued interest}}$$

The method for calculating the full price is as follows: |

Step 1: Calculate the value of the bond on the last coupon date.

Step 2: Compound this value at the YTM per period, over the number of days since the last coupon payment:

$$\text{full price} = \text{PV on last coupon date} \times \left(1 + \frac{\text{YTM}}{\text{periods per year}} \right)^{\frac{\text{days since last coupon}}{\text{days in coupon period}}}$$

Let's work an example for a specific bond.

EXAMPLE: Calculating the full and flat prices of a bond

A 5% bond makes coupon payments on June 15 and December 15, and is trading with a YTM of 4%. The bond is purchased, and will settle on August 21 when there will be four coupons remaining until maturity. Calculate the full price, accrued interest, and the flat price of the bond using actual days.

Answer:

Step 1: Calculate the value of the bond on the last coupon date (coupons are semiannual, so we use $4 / 2 = 2\%$ for the periodic discount rate):

$$N = 4; PMT = 2.5; FV = 100; I/Y = 2; CPT \rightarrow PV = -101.904$$

Step 2: Adjust for the number of days since the last coupon payment:

days between June 15 and December 15 = 183 days

days between June 15 and settlement on August 21 = 67 days

$$\text{full price} = 101.904 \times (1.02)^{67/183} = 102.645$$

Accrued interest on the settlement date of August 21 is as follows:

$$\$2.5 \times (67 / 183) = \$0.915$$

$$\text{Flat price} = 102.645 - 0.915 = 101.73.$$

Note that the flat price is *not* the present value of the bond on its last coupon payment date, $101.73 < 101.904$.

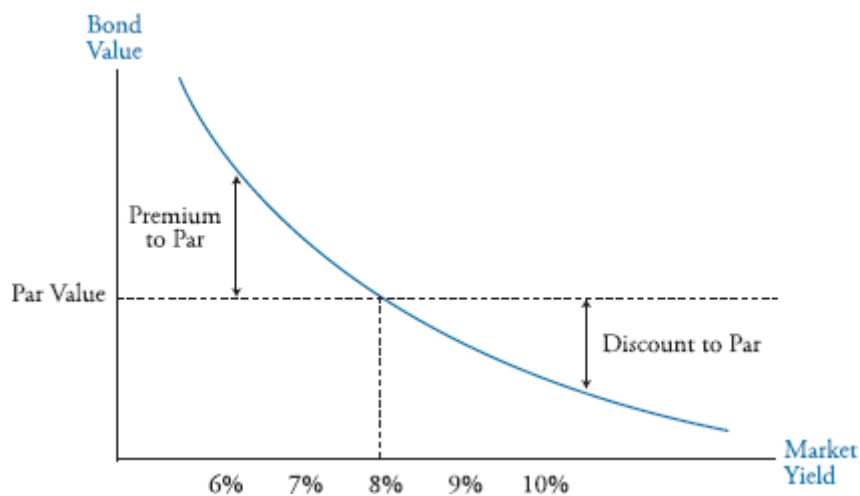
LOS 52.b: Identify the relationships among a bond's price, coupon rate, maturity, and yield-to-maturity.

We can summarize the relationships between price and specific bond features as follows:

1. At a point in time, a decrease (increase) in a bond's YTM will increase (decrease) its price. That is, there is an inverse relationship between yield and price.
2. Other things equal, the price of a bond with a lower coupon rate is more sensitive to a change in yield than is the price of a bond with a higher coupon rate.
3. Other things equal, the price of a bond with a longer maturity is more sensitive to a change in yield than is the price of a bond with a shorter maturity.
4. The percentage decrease in value when the YTM increases by a given amount is smaller than the increase in value when the YTM decreases by the same amount (the price-yield relationship is convex).

Figure 52.1 illustrates the convex relationship between a bond's price and its YTM.

Figure 52.1: Market Yield vs. Bond Value for an 8% Coupon Bond



Relationship Between Price and Maturity

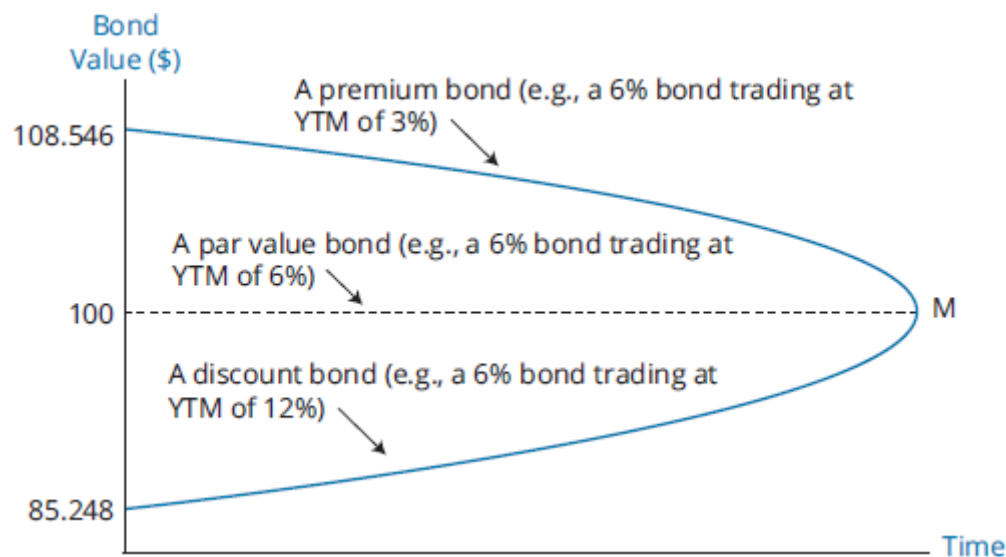
Before maturity, a bond can be selling at a significant discount or premium to par value. However, regardless of its required yield, the price will converge to par value as maturity approaches. Consider a bond with a 3-year life paying 6% semiannual coupons. The bond values corresponding to required yields of 3%, 6%, and 12% as the bond approaches maturity are presented in Figure 52.2.

Figure 52.2: Bond Values and the Passage of Time

Time to Maturity (In Years)	YTM = 3%	YTM = 6%	YTM = 12%
3.0	108.546	100	85.248
2.5	107.174	100	87.363
2.0	105.782	100	89.605
1.5	104.368	100	91.981
1.0	102.934	100	94.500
0.5	101.478	100	97.169
0.0	100.000	100	100.000

The change in value associated with the passage of time for the three bonds represented in Figure 52.2 is presented graphically in Figure 52.3. This convergence to par value (“pull to par”) at maturity is known as the **constant-yield price trajectory** because it shows how the bond’s price would change as time passes if its YTM remained constant.

Figure 52.3: Premium, Par, and Discount Bonds



LOS 52.c: Describe matrix pricing.

Matrix pricing is a method of estimating the required YTM (or price) of bonds that are currently not traded, or infrequently traded. The procedure is to use the YTM of traded bonds that have credit quality very close to that of a nontraded or infrequently traded bond and are similar in maturity and coupon, to estimate the required YTM.

EXAMPLE: Pricing an illiquid bond

Rob Phelps, CFA, is estimating the value of a nontraded 4% annual-pay, A+ rated bond that has three years remaining until maturity. He has obtained the following yields to maturity on similar corporate bonds:

- A+ rated, 2-year annual-pay, YTM = 4.3%
- A+ rated, 5-year annual-pay, YTM = 5.1%
- A+ rated, 5-year annual-pay, YTM = 5.3%

Estimate the value of the nontraded bond.

Answer:

Step 1: Take the average YTM of the 5-year bonds:

$$(5.1 + 5.3) / 2 = 5.2\%$$

Step 2: Interpolate the 3-year YTM based on the 2-year and average 5-year YTM:

$$4.3\% + (5.2\% - 4.3\%) \times [(3 \text{ years} - 2 \text{ years}) / (5 \text{ years} - 2 \text{ years})] = 4.6\%$$

Step 3: Price the nontraded bond with a YTM of 4.6%:

$$N = 3; PMT = 4; FV = 100; I/Y = 4.6; CPT \rightarrow PV = -98.354$$

The estimated value is \$98.354 per \$100 par value.

In Step 2 in the preceding example, we have used simple linear interpolation. Because the maturity of the nontraded bond is three years, we estimate the YTM on the 3-year bond as the yield on the 2-year bond, plus one-third of the difference between the YTM of the 2-year bond and the average YTM of the 5-year bonds. (The difference between the maturities of the 2-year bond and the 3-year bond is one year, and the difference between the maturities of the 2-year and 5-year bonds is three years.)

A variation of matrix pricing used for new bond issues focuses on spreads. The required yield spread to a benchmark for a new issue bond can be estimated by observing spreads on existing similar securities, as demonstrated in the following example.

EXAMPLE: Estimating the spread for a new 6-year, A rated bond issue

Consider the following market yields:

- 4-year, U.S. Treasury bond, YTM 1.48%
- 5-year, A rated corporate bond, YTM 2.64%
- 6-year, U.S. Treasury bond, YTM 2.15%

Estimate the required yield spread on a newly issued 6-year, A rated corporate bond.

Answer:

We will use the existing 5-year, A rated corporate bond to estimate the required yield spread of the issuer by comparing the YTM of the 5-year corporate bond to the interpolated 5-year Treasury bond YTM.

Interpolated 5-year Treasury bond YTM:

$$= 1.48\% + (2.15\% - 1.48\%) \times [(5 \text{ years} - 4 \text{ years}) / (6 \text{ years} - 4 \text{ years})] = 1.815\%$$

Note: Because the target maturity of the existing bond (5 years) is midway between the two Treasury bond maturities (4 and 6 years), we could have simply averaged the two Treasury bond yields here $(1.48\% + 2.15\%) / 2 = 1.815\%$.

The yield spread on existing 5-year corporate debt is $2.64\% - 1.815\% = 0.825\%$.

We will apply this yield spread to the new 6-year corporate debt issue:

$$\text{YTM for the new 6-year corporate bond} = 2.15\% + 0.825\% = 2.975\%$$



MODULE QUIZ 52.1

1. A 20-year bond has a 10% annual-pay coupon. What is the price of the bond if it has a yield to maturity of 15%?
A. 68.514.
☒ B. 68.703.
C. 82.839.
2. An analyst observes a 5-year, 10% semiannual-pay bond. The face amount is £1,000. The analyst believes that the yield to maturity on a semiannual bond basis should be 15%. Based on this yield estimate, the value of this bond is:
☒ A. £828.40.
B. £1,189.53.

- C. £1,193.04.
3. An analyst observes a 20-year, 8% option-free bond with semiannual coupons. The required yield to maturity on a semiannual bond basis was 8%, but suddenly it decreased to 7.25%. As a result, the price of this bond:
- ☒ A. increased.
 - ☐ B. decreased.
 - ☐ C. stayed the same.
4. A \$1,000 par, 5% coupon, 20-year annual-pay bond has a YTM of 6.5%. If the YTM remains unchanged, how much will the bond value increase over the next three years?
- ☐ A. \$13.62.
 - ☐ B. \$13.78.
 - ☐ C. \$13.96.
5. An investor paid a full price of 105.904 for \$1 million face value of a bond issue. The purchase was between coupon dates, and accrued interest was 2.354. What is each bond's flat price?
- ☐ A. 100.000.
 - ☒ B. 103.550.
 - ☐ C. 108.258.
6. Cathy Moran, CFA, is estimating a value for an infrequently traded bond with six years to maturity, an annual coupon of 7%, and a single-B credit rating. Moran obtains yields to maturity for more liquid bonds with the same credit rating:
- 5% coupon, eight years to maturity, yielding 7.20%.
 - 6.5% coupon, five years to maturity, yielding 6.40%.
- The infrequently traded bond is *most likely* trading at:
- ☐ A. par value.
 - ☐ B. a discount to par value.
 - ☒ C. a premium to par value.

KEY CONCEPTS

LOS 52.a

The price of a bond is the present value of its future cash flows, discounted at the bond's yield to maturity.

For an annual coupon bond with N years to maturity:

$$\text{price} = \frac{\text{coupon}}{(1 + \text{YTM})} + \frac{\text{coupon}}{(1 + \text{YTM})^2} + \dots + \frac{\text{coupon} + \text{principal}}{(1 + \text{YTM})^N}$$

For a semiannual coupon bond with N years to maturity:

$$\text{price} = \frac{\text{coupon}}{\left(1 + \frac{\text{YTM}}{2}\right)} + \frac{\text{coupon}}{\left(1 + \frac{\text{YTM}}{2}\right)^2} + \dots + \frac{\text{coupon} + \text{principal}}{\left(1 + \frac{\text{YTM}}{2}\right)^{N \times 2}}$$

The full price of a bond includes interest accrued between coupon dates. The flat price of a bond is the full price minus accrued interest.

Accrued interest for a bond transaction is calculated as the coupon payment times the portion of the coupon period from the previous payment date to the settlement date.

Methods for determining the period of accrued interest include actual days or 30-day months and 360-day years.

LOS 52.b

A bond's price and YTM are inversely related. An increase in YTM decreases the price, and a decrease in YTM increases the price.

Bond prices are convex with respect to yield movements, which means price increases when yields fall are greater in magnitude than the fall in prices caused by an equivalent yield rise.

A bond will be priced at a discount to par value if its coupon rate is less than its YTM (deficient coupon), and at a premium to par value if its coupon rate is greater than its YTM (excessive coupon).

Prices are more sensitive to changes in YTM for bonds with lower coupon rates and longer maturities, and less sensitive to changes in YTM for bonds with higher coupon rates and shorter maturities.

A bond's price moves toward par value as time passes and maturity approaches.

LOS 52.c

Matrix pricing is a method used to estimate the yield to maturity for bonds that are not traded or infrequently traded. The yield is estimated based on the yields of traded bonds with the same credit quality. If these traded bonds have different maturities than the bond being valued, linear interpolation is used to estimate the subject bond's yield.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 52.1

1. **B** $N = 20$; $I/Y = 15$; $FV = 100$; $PMT = 10$; and $CPT \rightarrow PV = -68.703$ (LOS 52.a)
2. **A** $N = 10$; $I/Y = 7.5$; $FV = 1,000$; $PMT = 50$; and $CPT \rightarrow PV = -\$828.40$ (LOS 52.a)
3. **A** The price-yield relationship is inverse. If the required yield decreases, the bond's price will increase, and vice versa. (LOS 52.b)
4. **A** With 20 years to maturity, the value of the bond with an annual-pay yield of 6.5% is $N = 20$; $PMT = 50$; $FV = 1,000$; $I/Y = 6.5$; and $CPT \rightarrow PV = -834.72$. With $N = 17$, $CPT \rightarrow PV = -848.34$, so the value will increase \$13.62. (LOS 52.a, 52.b)
5. **B** The full price includes accrued interest, while the flat price does not. Therefore, the flat (or clean) price is $105.904 - 2.354 = 103.550$. (LOS 52.b)
6. **C** Using linear interpolation, the yield on a bond with six years to maturity should be $6.40\% + (1 / 3)(7.20\% - 6.40\%) = 6.67\%$. A bond with a 7% coupon and a yield of 6.67% is at a premium to par value. (LOS 52.c)

READING 53

YIELD AND YIELD SPREAD MEASURES FOR FIXED-RATE BONDS

MODULE 53.1: YIELD AND YIELD SPREAD MEASURES FOR FIXED-RATE BONDS



Video covering this content is available online.

LOS 53.a: Calculate annual yield on a bond for varying compounding periods in a year.

Given a bond's price in the market, we can say that the yield to maturity (YTM) is the discount rate that makes the present value of a bond's cash flows equal to its price.

For a 5-year, annual-pay 7% bond that is priced in the market at 102.078, the YTM will satisfy the following equation:

$$\frac{7}{(1 + \text{YTM})^1} + \frac{7}{(1 + \text{YTM})^2} + \frac{7}{(1 + \text{YTM})^3} + \frac{7}{(1 + \text{YTM})^4} + \frac{107}{(1 + \text{YTM})^5} = 102.078$$

We can calculate the YTM (discount rate) that satisfies this equality as follows:

$N = 5$; $\text{PMT} = 7$; $\text{FV} = 100$; $\text{PV} = -102.078$; $\text{CPT} \rightarrow \text{I/Y} = 6.5\%$

By convention, the YTM on a semiannual coupon bond is stated as two times the semiannual discount rate. For a 5-year, semiannual-pay 7% coupon bond, we can calculate the semiannual discount rate as $\text{YTM} / 2$ and then double it to get the YTM expressed as an annualized yield:

$$\frac{3.5}{\left(1 + \frac{\text{YTM}}{2}\right)^1} + \frac{3.5}{\left(1 + \frac{\text{YTM}}{2}\right)^2} + \frac{3.5}{\left(1 + \frac{\text{YTM}}{2}\right)^3} + \dots + \frac{3.5}{\left(1 + \frac{\text{YTM}}{2}\right)^9} + \frac{103.5}{\left(1 + \frac{\text{YTM}}{2}\right)^{10}} = 102.078$$

$N = 10$; $\text{PMT} = 3.5$; $\text{FV} = 100$; $\text{PV} = -102.078$; $\text{CPT} \rightarrow \text{I/Y} = 3.253\%$

The quoted annualized YTM is $3.253 \times 2 = 6.506\%$.

Yield Measures for Fixed-Rate Bonds

The number of bond coupon payments per year is referred to as the **periodicity** of a bond. A bond with a periodicity of 2 will have its YTM quoted on a **semiannual bond basis**. For a given coupon rate, the greater the periodicity, the more compounding periods, and the greater the effective annual yield (which reflects compounding).

In general, the effective annual yield for a bond with its YTM stated for a periodicity of n (n compounding periods per year) is as follows:

$$\text{annual yield} = \left(1 + \frac{\text{YTM}}{n}\right)^n - 1$$

EXAMPLE: Effective annual yields

What is the effective annual yield for a bond with a stated YTM of 10%:

1. When the periodicity of the bond is 2 (pays semiannually)?
2. When the periodicity of the bond is 4 (pays quarterly)?

Answer:

1. $\text{annual yield} = \left(1 + \frac{0.10}{2}\right)^2 - 1 = 1.05^2 - 1 = 0.1025 = 10.25\%$

2. $\text{annual yield} = \left(1 + \frac{0.10}{4}\right)^4 - 1 = 1.025^4 - 1 = 0.1038 = 10.38\%$

It may be necessary to adjust the quoted yield on a bond to make it comparable with the yield on a bond with a different periodicity. This is illustrated in the following example.

EXAMPLE: Adjusting yields for periodicity

An Atlas Corporation bond is quoted with a YTM of 4% on a semiannual bond basis. What yields should be used to compare it with a quarterly-pay bond and an annual-pay bond?

Answer:

The first thing to note is that a YTM of 4% on a semiannual bond basis means a periodic return of 2% per 6-month period.

To compare this with the yield on an annual-pay bond, which is an effective annual yield, we need to calculate the effective annual yield on the semiannual coupon bond, which is $1.02^2 - 1 = 4.04\%$.

For the quoted annual YTM on the quarterly-pay bond, we need to calculate the effective quarterly yield and multiply by four. The quarterly yield (yield per quarter) that is equivalent to a yield of 2% per six months is $1.02^{1/2} - 1 = 0.995\%$. The quoted annual rate for the equivalent yield on a quarterly bond basis is $4 \times 0.995\% = 3.98\%$.

Note that we have shown that the effective annual yields are the same for the following:

- An annual coupon bond with a yield of 4.04% on an annual basis (periodicity of one)
- A semiannual coupon bond with a yield of 4.0% on a semiannual basis (periodicity of two)
- A quarterly coupon bond with a yield of 3.98% on quarterly basis (periodicity of four)

Bond yields calculated using the stated coupon payment dates are referred to as following the **street convention**. When coupon dates fall on weekends and holidays, coupon payments will actually be made the next business day. The yield calculated using these actual coupon payment dates is referred to as the **true yield**. Because coupon payments will be made later when holidays and weekends are taken into account, true yields are usually slightly lower than street convention yields, if only by a few basis points.

Current yield (also called **income yield** or **running yield**) looks at just one source of return, which is *a bond's annual interest income*—it does not consider capital gains or losses or reinvestment income. The formula for the current yield is as follows:

$$\text{current yield} = \frac{\text{annual cash coupon payment}}{\text{bond price}}$$

EXAMPLE: Computing current yield

Consider a 20-year, \$1,000 par value, 6% semiannual-pay bond that is currently trading at a flat price of \$802.07. Calculate the current yield.

Answer:

The annual cash coupon payments total:

$$\text{annual cash coupon payment} = \text{par value} \times \text{stated coupon rate} = \$1,000 \times 0.06 = \$60$$

Because the bond is trading at \$802.07, the current yield is:

$$\text{current yield} = \frac{60}{802.07} = 0.0748, \text{ or } 7.48\%$$

Note that current yield is based on *annual* coupon interest so that it is the same for a semiannual-pay and annual-pay bond with the same coupon rate and price.

A bond's **simple yield** takes a discount or premium into account by assuming that any discount or premium declines evenly over the remaining years to maturity. The sum of the annual coupon payment plus (minus) the straight-line amortization of a discount (premium) is divided by the flat price to get the simple yield.

EXAMPLE: Computing simple yield

A 3-year, 8% coupon, semiannual-pay bond is priced at 90.165. Calculate the simple yield.

Answer:

The discount from par value is $100 - 90.165 = 9.835$. Annual straight-line amortization of the discount is $9.835 / 3 = 3.278$:

$$\text{simple yield} = \frac{8 + 3.278}{90.165} = 12.51\%$$

For a callable bond, an investor's yield will depend on whether and when the bond is called. The **yield to call** can be calculated for each possible call date and price. The lowest of YTM and the various yields to call is termed the **yield to worst**. The following example illustrates these calculations.

EXAMPLE: Yield to call and yield to worst

Consider a 5-year, semiannual-pay 6% bond trading at 102 on January 1, 20X4. The bond is callable according to the following schedule:

- Callable at 102 on or after January 1, 20X7
- Callable at 101 on or after January 1, 20X8

Calculate the bond's YTM, yield to first call, yield to second call, and yield to worst.

Answer:

The yield to maturity on the bond is calculated as:

$$N = 10; PMT = 3; FV = 100; PV = -102; CPT \rightarrow I/Y = 2.768\% \\ 2 \times 2.768 = 5.54\% = YTM$$

To calculate the *yield to first call*, we calculate the YTM using the number of semiannual periods until the first call date in 20X7 (6) for N and the call price (102) for FV :

$$N = 6; PMT = 3; FV = 102; PV = -102; CPT \rightarrow I/Y = 2.941\% \\ 2 \times 2.941 = 5.88\% = \text{yield to first call}$$

To calculate the *yield to second call*, we calculate the YTM using the number of semiannual periods until the second call date in 20X8 (8) for N and the call price (101) for FV :

$$N = 8; PMT = 3; FV = 101; PV = -102; CPT \rightarrow I/Y = 2.830\% \\ 2 \times 2.830 = 5.66\% = \text{yield to second call}$$

The lowest yield, 5.54%, is realized if the bond is held to maturity and not called, so the *yield to worst* is 5.54%.

A callable bond can be viewed as an equivalent straight (option-free) bond combined with a *short* call option position (because the right to call the bond lies with the issuer, not the investor):

$$\text{callable bond value} = \text{straight bond value} - \text{call option value}$$

Stated differently, we can view the value of an equivalent straight bond (known as the **option-adjusted price**) as the value of the callable bond *plus* the value of the call

option embedded in the callable bond (which could be derived from an option pricing model).

This option-adjusted price can then be used to calculate an **option-adjusted yield**, which represents the yield that the bond would be offering if it were not callable. Because the existence of the call option decreases the bond price and increases an investor's required yield, "removing" the option in this manner will cause the option-adjusted yield to be *lower* than the yield of the callable bond. The option-adjusted yield is useful because it can be used to compare the yields of bonds with various embedded options to each other and to similar option-free bonds on a consistent basis.



PROFESSOR'S NOTE

Take care to understand that option-adjusted prices and yields remove the impact of the option from a bond with an embedded option. It is a common mistake when first encountering these measures to incorrectly think they are adjusting to incorporate the impact of the option, when in fact they are doing the opposite. When you see "option-adjusted," think "option removed" or "option taken away."

LOS 53.b: Compare, calculate, and interpret yield and yield spread measures for fixed-rate bonds.

A **yield spread**, or **benchmark spread**, is the difference between the yields of a bond and a benchmark security. For example, if a 5-year corporate bond has a yield of 6.25% and its benchmark, the 5-year Treasury note, has a yield of 3.50%, the corporate bond has a benchmark spread of $625 - 350 = 275$ basis points.

For fixed-coupon bonds, on-the-run government bond yields for the same or nearest maturity are frequently used as benchmarks because they are the most actively traded bonds, and therefore give the most useful price and yield information. A yield spread in basis points over a government bond is also known as a **G-spread**. If a benchmark government bond with exactly the same maturity as the riskier bond does not exist, interpolation should be used to estimate the appropriate maturity benchmark yield.

EXAMPLE: G-spread

A 3-year, 8% coupon, semiannual-pay bond is priced at 103.165, and 1-year and 4-year U.S. Treasury yields are 3% and 5%, respectively. Calculate the G-spread of the bond.

Answer:

The YTM of the bond is calculated as:

$$N = 6; PMT = 4; FV = 100; PV = -103.165; CPT \rightarrow I/Y = 3.408\%$$

$$\text{Quoted YTM} = 2 \times 3.408\% = 6.82\%$$

The interpolated 3-year government bond yield is:

$$3\% + [(3 - 1) / (4 - 1)] \times (5\% - 3\%) = 4.33\%$$

Then, the G-spread = $6.82\% - 4.33\% = 2.49\%$, or 249 basis points.

An alternative to using government bond yields as benchmarks is to use rates for interest rate swaps in the same currency and with the same tenor as a bond. Yield spreads relative to swap rates are known as **interpolated spreads** or **I-spreads** and represent the extra return of a bond in excess of the interbank market reference rates (MRRs) used in swap contracts. I-spreads are frequently stated for bonds denominated in euros.

Yield spreads are useful for analyzing the factors that affect a bond's yield. If a corporate bond's yield increases from 6.25% to 6.50%, this may have been caused by factors that affect all bond yields (macroeconomic factors) or by firm-specific or industry-specific (microeconomic) factors. If a bond's yield increases but its spread remains the same, the yield on its benchmark must have also increased, which suggests macroeconomic factors caused bond yields in general to increase. However, if the yield spread increases, this suggests the increase in the bond's yield was caused by microeconomic factors, such as credit risk of the issuer increasing or the issue's liquidity deteriorating.



PROFESSOR'S NOTE

Recall from Quantitative Methods that an interest rate is composed of the real risk-free rate, the expected inflation rate, and a risk premium. We can think of macroeconomic factors as those that affect the real risk-free rate and expected inflation (which make up the benchmark yield), and microeconomic factors as those that affect credit and liquidity risk premiums (which make up the yield spread). Differences in taxation of the returns from bonds of different issuers can also affect yield spreads.

Zero-Volatility and Option-Adjusted Spreads

The G-spread and I-spread are based on the difference between the yields of a specific bond and a benchmark. Recall that the YTM of a bond is the *single* discount rate that sets the present value of the cash flows of the bond equal to its market price.

We can observe from the prices of zero-coupon bonds that different individual cash flows occurring at different maturities can have different yields. Yields earned by individual cash flows at different maturities are referred to as **spot rates**. The single YTM of a coupon-paying bond represents a weighted average of the different spot rates offered by the individual cash flows of the bond.



PROFESSOR'S NOTE

Spot rates are formally introduced and discussed later in our reading on The Term Structure of Interest Rates. For now, it is enough to know that calculating spreads over benchmark spot rates is a more precise way of calculating spreads than basing the measure on YTMs because it better captures how rates vary over different maturities (referred to as the term structure of rates).

A method for deriving a bond's yield spread to a benchmark spot yield curve that accounts for the shape of the yield curve is to add an equal amount to each benchmark spot rate and value the bond with those rates. When we find an amount which, when added to the benchmark spot rates, produces a value equal to the market price of the bond, we have the appropriate yield curve spread. A yield spread calculated this way is known as a **zero-volatility spread** or **Z-spread**.

EXAMPLE: Zero-volatility spread

The 1-, 2-, and 3-year spot rates on Treasuries are 4%, 8.167%, and 12.377%, respectively. Consider a 3-year, 9% annual coupon corporate bond trading at 89.464. The YTM is 13.50%, and the YTM of a 3-year Treasury is 12%. Calculate the G-spread and the Z-spread of the corporate bond.

Answer:

The G-spread is $YTM_{\text{bond}} - YTM_{\text{Treasury}} = 13.50 - 12.00 = 1.50\%$.

To compute the Z-spread, set the present value of the bond's cash flows equal to today's market price. Discount each cash flow at the appropriate zero-coupon bond spot rate *plus* a fixed-spread ZS. Solve for ZS in the following equation, and you have the Z-spread:

$$89.464 = \frac{9}{(1.04 + ZS)^1} + \frac{9}{(1.08167 + ZS)^2} + \frac{109}{(1.12377 + ZS)^3}$$

$\Rightarrow ZS = 1.67\%$, or 167 basis points

Note that this spread is found by trial and error. In other words, pick a number "ZS," plug it into the right-hand side of the equation, and see if the result equals 89.464. If the right-hand side equals the left, then you have found the Z-spread. If not, adjust ZS in the appropriate direction and recalculate.

An **option-adjusted spread (OAS)** is used for bonds with embedded options. Loosely speaking, the OAS takes the option yield component out of the Z-spread measure; the OAS is the spread to the government spot rate curve that the bond would have if it were option free. This is similar to the option-adjusted yields discussed earlier, the only difference being OAS respects the term structure of rates instead of being based on YTM.

If we calculate an **OAS for a callable bond, it will be less than the bond's Z-spread**. The difference is the extra yield required to compensate bondholders for the call option. That extra yield is referred to as the option value. Thus, we can write the following:

$$\text{option value} = \text{Z-spread} - \text{OAS}$$

$$\text{OAS} = \text{Z-spread} - \text{option value}$$

For example, if a callable bond has a Z-spread of 180 bp and the value of the call option is 60 bp, the bond's OAS is $180 - 60 = 120$ bp.

The interpretation here is that investors are demanding the Z-spread (180 bp) for the credit, liquidity, taxation, and optionality risks of the callable bond. When the

component of the spread relating to optionality is *removed*, we are left with the OAS (120 bp) that rewards investors for facing credit, liquidity, and taxation risks.



MODULE QUIZ 53.1

1. Based on semiannual compounding, what is the YTM of a 15-year, zero-coupon, \$1,000 par value bond that is currently trading at \$331.40?
A. 3.750%.
B. 5.151%.
☒ C. 7.500%.
2. An analyst observes a Widget & Co. 7.125%, 4-year, semiannual-pay bond trading at 102.347. The bond is callable at 101 in two years. The bond's yield to call is *closest* to:
A. 3.2%.
☒ B. 6.3%.
C. 9.4%.
3. Holding the effective annual yield constant, if the periodicity of a bond is increased, its stated YTM will:
☒ A. decrease.
B. stay the same.
C. increase.
4. A corporate bond is quoted at a spread of +235 basis points over an interpolated 12-year U.S. Treasury bond yield. This spread is a(n):
☒ A. G-spread.
B. I-spread.
C. Z-spread.
5. For a callable bond, relative to its option-adjusted spread, its Z-spread is *most likely* to be:
A. lower.
B. the same.
☒ C. higher.

KEY CONCEPTS

LOS 53.a

The effective yield of a bond depends on its periodicity, or frequency of coupon payments. For an annual-pay bond, the effective yield is equal to the yield to maturity (YTM). For bonds with greater periodicity, the effective yield is greater than the YTM.

A YTM quoted on a semiannual bond basis is two times the semiannual discount rate.

Bond yields that follow street convention use the stated coupon payment dates. A true yield accounts for coupon payments that are delayed by weekends or holidays and may be slightly lower than a street convention yield.

Current yield is the ratio of a bond's annual coupon payments to its price. Simple yield adjusts current yield by using straight-line amortization of any discount or premium.

For a callable bond, a yield to call may be calculated using each of its call dates and prices. The lowest of these yields or its YTM is a callable bond's yield to worst.

LOS 53.b

A yield spread or benchmark spread is the difference between a bond's yield and a benchmark yield or yield curve. If the benchmark is a government bond yield, the spread is known as a government spread or G-spread. If the benchmark is a swap rate, the spread is known as an interpolated spread or I-spread.

A zero-volatility spread or Z-spread is the percentage spread that must be added to each spot rate on the benchmark yield curve to make the present value of a bond's cash flows equal to its price.

An option-adjusted spread (OAS) is used for bonds with embedded options and represents the spread the bond would offer if it had no embedded options. For a callable bond, the OAS is equal to the Z-spread minus the call option value in basis points.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 53.1

1. **C** $N = 30$; $FV = 1,000$; $PMT = 0$; $PV = -331.40$; $CPT \rightarrow I/Y = 3.750 \times 2 = 7.500\%$

$$\text{Alternatively, } \left[\left(\frac{1,000}{331.4} \right)^{\frac{1}{30}} - 1 \right] \times 2 = 7.5\%.$$

(LOS 53.a)

2. **B** $N = 4$; $FV = 101$; $PMT = 3.5625$; $PV = -102.347$; $CPT \rightarrow I/Y = 3.167 \times 2 = 6.334\%$

(LOS 53.a)

3. **A** Due to their increased compounding frequency, bonds with higher periodicity will have a lower stated YTM for a certain level of effective annual yield (EAY). For example, for an EAY of 5% and periodicity of 2, the stated YTM must solve $[1 + (YTM / 2)]^2 = 1.05$. Hence, $YTM = 2 (1.05^{1/2} - 1) = 4.94\%$. With periodicity of 4, the stated YTM must solve $[1 + (YTM / 4)]^4 = 1.05$. Hence, $YTM = 4 (1.05^{1/4} - 1) = 4.91\%$. (LOS 53.a)

4. **A** G-spreads are quoted relative to an actual or interpolated government bond yield. I-spreads are quoted relative to swap rates. Z-spreads are calculated based on the shape of the benchmark yield curve. (LOS 53.b)

5. **C** A callable bond will offer a higher yield than an equivalent straight bond because the investor faces the call risk of the option. Hence, the Z-spread, which includes the impact of the option, will be higher than the OAS, which has removed the impact of the option. (LOS 53.b)

READING 54

YIELD AND YIELD SPREAD MEASURES FOR FLOATING-RATE INSTRUMENTS

MODULE 54.1: YIELD AND YIELD SPREAD MEASURES FOR FLOATING-RATE INSTRUMENTS



Video covering this content is available online.

LOS 54.a: Calculate and interpret yield spread measures for floating-rate instruments.

Floating-Rate Note Yields

The values of floating-rate notes (FRNs) are more stable than those of fixed-rate debt of similar maturity because the coupon rate is reset periodically based on a variable market reference rate (MRR). Recall that the coupon rate on an FRN consists of a relatively risk-free (usually interbank) MRR plus a fixed margin based on the credit risk of the issuer (at the time of issuance) relative to the credit risk of the MRR. The coupon rate for the next period is set using the current MRR for the reset period, and the payment at the end of the period is based on this rate. For this reason, we say that interest is paid *in arrears*.

If an FRN is issued by a company that has more (less) credit risk than the financial institutions from which the MRR is derived, a margin is added to (subtracted from) the MRR. The liquidity of an FRN and its tax treatment can also affect the margin.

The fixed margin above the MRR actually paid in the coupon is referred to as the **quoted margin (QM)**. The margin required to price the FRN at par is called the **required margin** or the **discount margin (DM)**.

FRNs are usually issued at par with the QM equal to the DM at issuance. If the credit quality of an FRN remains unchanged after issuance, the QM will remain equal to the DM and the FRN will trade at par on its coupon reset dates.

If the credit quality of the issuer decreases after issuance of the FRN, investors will demand a higher DM in compensation for increased credit risk. This will cause the DM to be greater than the fixed QM, and the FRN will trade at a discount to its par value.

Similarly, if the issuer's credit quality improves during an FRN's life, the DM will be less than the fixed QM, and the FRN will sell at a premium to its par value.



PROFESSOR'S NOTE

This is analogous to the relationship between the coupon and yield for fixed-coupon instruments. If investors demand more yield from a fixed-coupon bond than the regular coupon, then the coupon is said to be deficient, and the bond trades at a discount to par. With an FRN, the bond pays a coupon of $MRR + QM$ and investors demand a yield of $MRR + DM$. When the $MRR + DM$ is greater than $MRR + QM$, the QM is deficient and the FRN will trade below par.

A simplified way of calculating the value of an FRN on a reset date is to use the current MRR plus the QM to estimate the future cash flows for the FRN, and discount these future cash flows at the $MRR + DM$. More complex models produce better estimates of value.

EXAMPLE: Valuation of an FRN

A \$100,000 FRN with a semiannual coupon pays a 180-day MRR plus a quoted margin of 120 basis points. On a reset date with five years remaining to maturity, the 180-day MRR is quoted as 3.0% (annualized), and the discount margin (based on the issuer's current credit rating) is 1.5% (annualized). Estimate the value of the FRN.

Answer:

The current annualized coupon rate on the note is $3.0\% + 1.2\% = 4.2\%$, so the next semiannual coupon payment will be $4.2\% / 2 = 2.1\%$ of face value.

The required return in the market ($MRR + \text{discount margin}$) as an effective 180-day discount rate is $4.5\% / 2 = 2.25\%$.

Using a face value of 100%, 10 coupon payments of 2.1%, and a discount rate per period of 2.25%, we can calculate the present value of the FRN as:

$$N = 10; I/Y = 2.25; FV = 100; PMT = 2.1; CPT @ PV = -98.67$$

By this method, we can estimate current value of the note as 98.67% of its face value, or \$98,670.

LOS 54.b: Calculate and interpret yield measures for money market instruments.

For **money market securities** (debt securities maturing in a year or less), yields are quoted using various conventions. Some yield quotes are **based on a 360-day year**, while others are based on a 365-day year. Some yield quotes are add-on yields, and others are discount yields. Add-on yields are simply the interest to be earned on the amount paid or deposited today. Discount yields are annualized current discounts from the face values of money market securities received at maturity.

Bank CDs, repos, and market reference rates are typically quoted as annualized add-on rates. U.S. Treasury bills (T-bills) and commercial paper are quoted as their annualized discount from face value, based on a 360-day year.

The relation between a security's yield quoted as an annualized add-on yield based on a 365-day year and its holding period yield (HPY) is as follows:

$$\text{quoted add-on yield} = \text{HPY} \times 365 / \text{days to maturity}$$

Consider a 100-day bank CD with an add-on yield (annualized) of 1.5%, based on a 365-day year. We can calculate the HPY of the CD as the quoted yield of $1.5\% \times 100 / 365 = 0.41\%$. The purchase of a \$1,000 CD would provide a payment of \$1,004.10 in 100 days.

The relation between a quoted discount and the actual unannualized discount based on a 360-day year is as follows:

$$\text{quoted discount yield} = \text{actual discount on the security} \times 360 / \text{days to maturity}$$

Consider a 180-day U.S. T-bill quoted at a 2.2% (annualized) discount yield based on a 360-day year. The actual discount from face value on the T-bill is $180 / 360 \times 2.2\% = 1.1\%$. A \$1,000 T-bill would be priced at $(1 - 0.011) \times 1,000 = \989 . The HPY of the T-bill is $1,000 / 989 - 1 = 1.11\%$, slightly higher than its discount from face value of 1.1%.

An analyst should be able to convert the yield of a security calculated on one basis to its yield on another basis. Such adjustments allow us to compare the yields of two money market securities for which quoted yields are calculated differently. The following provides some examples of converting a yield to one based on a different convention.

EXAMPLE: Money market yields

1. A \$1,000 90-day T-bill is priced with an annualized discount of 1.2%. Calculate its market price and its annualized add-on yield based on a 365-day year.
2. A \$1 million negotiable CD with 120 days to maturity is quoted with an add-on yield of 1.4% based on a 365-day year. Calculate the payment at maturity for this CD and its bond equivalent yield.
3. A bank deposit for 100 days is quoted with an add-on yield of 1.5% based on a 360-day year. Calculate the bond equivalent yield and the yield on a semiannual bond basis.

Answer:

1. The discount from face value is $1.2\% \times 90 / 360 \times 1,000 = \3 , so the current price is $1,000 - 3 = \$997$.
The equivalent add-on yield for 90 days is $3 / 997 = 0.3009\%$. The annualized add-on yield based on a 365-day year is $365 / 90 \times 0.3009\% = 1.2203\%$. This add-on yield based on a 365-day year is referred to as the *bond equivalent yield* for a money market security.
2. The add-on interest for the 120-day period is $120 / 365 \times 1.4\% = 0.4603\%$.
At maturity, the CD will pay $\$1 \text{ million} \times (1 + 0.004603) = \$1,004,603$.
The quoted yield on the CD of 1.4% is already the bond equivalent yield by definition because it is an add-on yield annualized based on a 365-day year.
3. Because the yield of 1.5% is an annualized yield calculated based on a 360-day year, the bond equivalent yield, which is based on a 365-day year, is:

$$(365 / 360) \times 1.5 \% = 1.5208\%$$

We may want to compare the yield on a money market security to the YTM of a semiannual-pay bond. The method is to convert the money market security's holding period return to an effective semiannual yield, and then double it.

Because the quoted yield of 1.5% is calculated as the add-on yield for 100 days times $360 / 100$, the 100-day holding period return is $1.5\% \times 100 / 360 = 0.4167\%$. The effective annual yield is $1.004167^{365/100} - 1 = 1.5294\%$, the equivalent semiannual yield is $1.015294^{1/2} - 1 = 0.7618\%$, and the annual yield on a semiannual bond basis is $2 \times 0.7618\% = 1.5236\%$.

Because the periodicity of the money market security, $365 / 100$, is greater than the periodicity of 2 for a semiannual-pay bond, the simple annual rate for the money market security, 1.5%, is less than the yield on a semiannual bond basis.



MODULE QUIZ 54.1

1. A floating-rate note has a quoted margin of +50 basis points and a required margin of +75 basis points. On its next reset date, the price of the note will be:
 - A. equal to par value.
 - ☒ B. less than par value.
 - C. greater than par value.
2. Which of the following money market yields is a bond equivalent yield?
 - ☒ A. Add-on yield based on a 365-day year.
 - B. Discount yield based on a 360-day year.
 - C. Discount yield based on a 365-day year.
3. Which of the following money market instruments has the highest bond equivalent yield?
 - ☒ A. A 90-day Treasury bill quoted with a discount of 1% on a 360-day basis.
 - B. A 183-day commercial paper quoted with a discount of 1% on a 365-day basis.
 - C. A 91-day certificate of deposit offering an add-on rate of 1% on a 365-day basis.

KEY CONCEPTS

LOS 54.a

Floating-rate notes (FRNs) pay a coupon equal to a fixed quoted margin relative over a market reference rate (MRR). The required margin (or discount margin) on an FRN is the extra return over MRR demanded by investors due to credit and liquidity risk of the issuer. At issuance, FRNs usually have a quoted margin equal to the discount margin; hence, the FRN is issued at par value.

When credit conditions deteriorate and the discount margin rises above the quoted margin, the FRN will trade below par.

When credit conditions improve and the discount margin falls below the quoted margin, the FRN will trade above par.

LOS 54.b

For money market instruments, yields may be quoted on a discount basis or an add-on basis, and they may use 360-day or 365-day years. A bond equivalent yield is an add-on yield based on a 365-day year.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 54.1

1. **B** If the required margin is greater than the quoted margin, the credit quality of the issue has decreased, and the price on the reset date will be less than par value. (LOS 54.a)
2. **A** An add-on yield based on a 365-day year is a bond equivalent yield. (LOS 54.b)
3. **A** The bond equivalent yield is defined as the add-on yield quoted on a 365-day basis.

The 90-day Treasury is trading at a $1\% \times 90 / 360 = 0.25\%$ discount to par. Hence, it is trading at a price of 99.75. The HPR is, therefore, $100 / 99.75 - 1 = 0.2506\%$, and the bond equivalent yield is $0.2506\% \times 365 / 90 = 1.016\%$.

The commercial paper is trading at a $1\% \times 183 / 365 = 0.5014\%$ discount to par. Hence, it is trading at a price of 99.4986. The HPR is, therefore, $100 / 99.4986 - 1 = 0.5039\%$, and the bond equivalent yield is $0.5039\% \times 365 / 183 = 1.005\%$.

The quoted return of the certificate of deposit is already in bond equivalent form; hence, no translation is needed here. (LOS 54.b)

READING 55

THE TERM STRUCTURE OF INTEREST RATES: SPOT, PAR, AND FORWARD CURVES

MODULE 55.1: THE TERM STRUCTURE OF INTEREST RATES: SPOT, PAR, AND FORWARD CURVES



Video covering this content is available online.

LOS 55.a: Define spot rates and the spot curve, and calculate the price of a bond using spot rates.

The yield to maturity of a bond is calculated as if the discount rate for every bond cash flow is the same. In reality, discount rates vary according to the time period in which a future cash flow is made. These discount rates for a single payment to be received in the future are called **spot rates** and can be observed by calculating the discount rates for **zero-coupon bonds** (hence, spot rates are sometimes referred to as *zero-coupon rates* or simply *zero rates*).

To price a bond with spot rates, we sum the present values of the bond's payments, each discounted at the spot rate for the number of periods before it will be paid. The general equation for calculating a bond's value using spot rates (S_i) is as follows:

$$\frac{CPN_1}{1 + S_1} + \frac{CPN_2}{(1 + S_2)^2} + \dots + \frac{CPN_N + FV_N}{(1 + S_N)^N} = PV$$

EXAMPLE: Valuing a bond using spot rates

Given the following spot rates, calculate the value of a 3-year, 5% annual coupon bond.

Spot rates:

1-year: 3%

2-year: 4%

3-year: 5%

Answer:

$$\frac{5}{(1.03)^1} + \frac{5}{(1.04)^2} + \frac{105}{(1.05)^3} = 100.180$$

This price, calculated using spot rates, is sometimes called the *no-arbitrage price* of a bond because if a bond is priced differently, there will be a profit opportunity from arbitrage among bonds.

Because the bond value is slightly greater than its par value, we know its YTM is slightly less than its coupon rate of 5%. Using the price of 100.180, we can calculate the YTM for this bond as follows:

$$N = 3; PMT = 5; FV = 100; PV = -100.180; CPT \rightarrow I/Y = 4.93\%$$



PROFESSOR'S NOTE

It's useful to think of a spot rate as a return offered by a single cash flow occurring at a certain time in the future. A weighted average of these spot rates gives us an idea of what the yield is likely to be. For the bond in the previous example, we have the first coupon returning 3%, the second coupon returning 4%, and the third coupon and par payment returning 5%. Given that most of the cash flows occur at Time 3 when the par payment is made, the average is going to be heavily weighted to 5%. This means we can easily see that the yield must be just below 5%.

The **spot curve** displays spot rates versus maturity for a particular type of bond or issuer (e.g., U.S. Treasury government spot rates). We will construct and use spot curves later in this reading.

LOS 55.b: Define par and forward rates, and calculate par rates, forward rates from spot rates, spot rates from forward rates, and the price of a bond using forward rates.

Par Yields

Par yields reflect the coupon rate that a hypothetical bond at each maturity would need to have to be priced at par, given a specific spot curve. Alternatively, they can be viewed as the YTM of a hypothetical par bond at each maturity.

Consider a 3-year annual-pay bond and spot rates for one, two, and three years of S_1 , S_2 , and S_3 . The following equation can be used to calculate the coupon rate, PMT, necessary for the bond to be trading at par:

$$\frac{PMT}{1 + S_1} + \frac{PMT}{(1 + S_2)^2} + \frac{PMT + 100}{(1 + S_3)^3} = 100$$

With spot rates of 1%, 2%, and 3%, a 3-year annual par bond will have a payment that will satisfy the following:

$$\frac{PMT}{1.01} + \frac{PMT}{(1.02)^2} + \frac{PMT + 100}{(1.03)^3} = 100$$

In this case, the payment is 2.96 and the par bond coupon rate is 2.96%.



PROFESSOR'S NOTE

If this type of calculation appears on the exam, the best tactic is to plug in the middle answer choice and see if it gives a value of 100. If it results in a value less than 100, the larger choice must be correct. If it produces a value greater than 100, the smaller choice must be correct. Algebraically solving for PMT can be done, but it takes much more time than simply trying the coupon rates in the answers.

Forward Rates

A **forward rate** is a borrowing/lending rate for a loan to be made at some future date. The notation used must identify both the start and length of the lending/borrowing period. The most common convention (and the one used on the Level I CFA exam) is to denote a forward period using two numbers, each followed by a letter indicating the compounding period (*y* for years, *m* for months). The first number represents the future period in which the loan begins, and the second denotes the length of the loan. Hence, $2y1y$ is the rate for a 1-year loan to be made two years from now; $3y2y$ is the 2-year forward rate three years from now; $1y1y$ is the rate for a 1-year loan one year from now; and so on.

When linking forward rates to spot rates—say, for example, over the next three years—a key no-arbitrage idea is that *borrowing for three years at the 3-year spot rate, or borrowing for one-year periods in three successive years, should have the same cost.*

This relation is illustrated as $(1 + S_3)^3 = (1 + S_1)(1 + 1y1y)(1 + 2y1y)$.

Thus, $S_3 = [(1 + S_1)(1 + 1y1y)(1 + 2y1y)]^{1/3} - 1$, which is a basic geometric mean periodic return.

More generally, the spot rate earned between now and a future maturity must be equal to the compounded forward rates that apply to each period out to that maturity.

EXAMPLE: Computing spot rates from forward rates

If the current 1-year spot rate is 2%, the 1-year forward rate one year from today ($1y1y$) is 3%, and the 1-year forward rate two years from today ($2y1y$) is 4%, what is the 3-year spot rate?

Answer:

$$S_3 = [(1.02)(1.03)(1.04)]^{1/3} - 1 = 2.997\%$$

This can be interpreted to mean that a dollar compounded at 2.997% for three years would produce the same ending value as a dollar that earns compound interest of 2% the first year, 3% the next year, and 4% for the third year.



PROFESSOR'S NOTE

You can get a very good approximation of the 3-year spot rate with the simple average of the forward rates. In the previous example, we calculated 2.997%, and the simple average of the three annual rates is as follows:

$$\frac{2 + 3 + 4}{3} = 3\%$$

We can use the same relationships we use to calculate spot rates from forward rates to calculate forward rates from spot rates.

This is our basic relation between forward rates and spot rates (for two periods):

$$(1 + S_2)^2 = (1 + S_1)(1 + 1y1y)$$

This again tells us that an investment has the same expected yield (borrowing has the same expected cost) whether we invest (borrow) for two periods at the 2-period spot rate, S_2 , or for one period at the current 1-year rate, S_1 , and for the next period at the forward rate, $1y1y$. Given two of these rates, we can solve for the other.

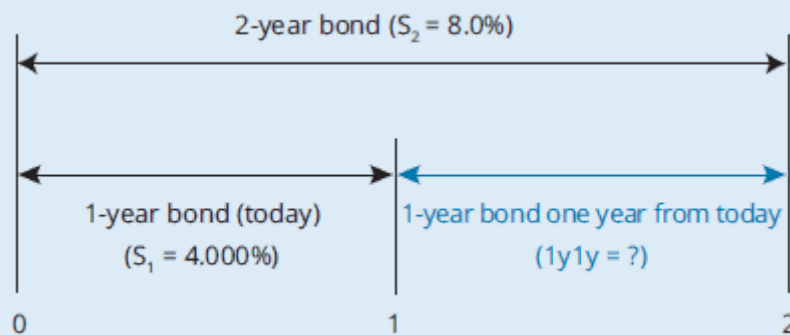
EXAMPLE: Computing a forward rate from spot rates

The 2-period spot rate, S_2 , is 8%, and the 1-period spot rate, S_1 , is 4%. Calculate the forward rate for one period, one period from now, $1y1y$.

Answer:

The following figure illustrates the problem.

Finding a Forward Rate



From our original equality, $(1 + S_2)^2 = (1 + S_1)(1 + 1y1y)$, we can get:

$$\frac{(1 + S_2)^2}{(1 + S_1)} = (1 + 1y1y)$$

Or, because we know that both choices have the same payoff in two years:

$$(1.08)^2 = (1.04)(1 + 1y1y)$$

$$(1 + 1y1y) = \frac{(1.08)^2}{(1.04)}$$

$$1y1y = \frac{(1.08)^2}{(1.04)} - 1 = \frac{1.1664}{1.04} - 1 = 12.154\%$$

In other words, investors are willing to accept 4.0% on the 1-year bond today (when they could get 8.0% on the 2-year bond today) only because they can get 12.154% on

a 1-year bond one year from today. This future rate that can be locked in today is a forward rate.

Similarly, we can back other forward rates out of the spot rates. We know that:

$$(1 + S_3)^3 = (1 + S_1)(1 + 1y1y)(1 + 2y1y)$$

And that:

$$(1 + S_2)^2 = (1 + S_1)(1 + 1y1y), \text{ so we can write} \\ (1 + S_3)^3 = (1 + S_2)^2(1 + 2y1y).$$

This last equation says that investing for three years at the 3-year spot rate should produce the same ending value as investing for two years at the 2-year spot rate—and then for a third year at 2y1y, the 1-year forward rate, two years from now.

Solving for the forward rate, 2y1y, we get:

$$\frac{(1 + S_3)^3}{(1 + S_2)^2} - 1 = 2y1y$$

EXAMPLE: Forward rates from spot rates

Let's extend the previous example to three periods. The current 1-year spot rate is 4.0%, the current 2-year spot rate is 8.0%, and the current 3-year spot rate is 12.0%. Calculate the 1-year forward rate two years from now.

Answer:

We know the following relation must hold:

$$(1 + S_3)^3 = (1 + S_2)^2(1 + 2y1y)$$

Substituting values for S_3 and S_2 , we have:

$$(1.12)^3 = (1.08)^2 \times (1 + 2y1y)$$

This is so that the 1-year forward rate two years from now is:

$$2y1y = \frac{(1.12)^3}{(1.08)^2} - 1 = 20.45\%$$

We can check our results by calculating:

$$S_3 = [(1.04)(1.12154)(1.2045)]^{1/3} - 1 = 12.00\%$$

This may all seem a bit complicated, but the basic relation, that borrowing for successive periods at 1-period rates should have the same cost as borrowing at multiperiod spot rates, can be summed up as follows:

$$(1 + S_2)^2 = (1 + S_1)(1 + 1y1y) \text{ for two periods, and} \\ (1 + S_3)^3 = (1 + S_2)^2(1 + 2y1y) \text{ for three periods}$$



PROFESSOR'S NOTE

Simple averages also give decent approximations for calculating forward rates from spot rates (particularly when rates are small). In the preceding example, we had spot rates of 4% for one year and 8% for two years. Two years at 8% is 16%, so if the first-year rate is 4%, the second-year forward rate is close to $16 - 4 = 12\%$ (actual is 12.154). Given a 2-year spot rate of 8% and a 3-year spot rate of 12%, we could approximate the 1-year forward rate from Time 2 to Time 3 as $(3 \times 12) - (2 \times 8) = 20$. That may be close enough (actual is 20.45) to answer a multiple-choice question—and, in any case, it serves as a good check to make sure the exact rate you calculate is reasonable.

We can also calculate implied forward rates for loans for more than one period. Given spot rates (1-year = 5%, 2-year = 6%, 3-year = 7%, and 4-year = 8%), we can calculate 2y2y.

The implied forward rate on a 2-year loan two years from now, 2y2y, is as follows:

$$\left[\frac{(1 + S_4)^4}{(1 + S_2)^2} \right]^{1/2} - 1 = \left(\frac{1.08^4}{1.06^2} \right)^{1/2} - 1 = 10.04\%$$



PROFESSOR'S NOTE

The approximation works for multiperiod forward rates as well.

The difference between four years at 8% (= 32%) and two years at 6% (= 12%) is 20%. Because that difference is for two years, we divide by two to get an annual rate of 10%, $\frac{(4 \times 8 - 6 \times 2)}{2} = 10$, which is very close to the exact solution of 10.04%.

EXAMPLE: Computing a bond value using forward rates

The current 1-year rate, S_1 , is 4%, the 1-year forward rate for lending from time = 1 to time = 2 is 1y1y = 5%, and the 1-year forward rate for lending from time = 2 to time = 3 is 2y1y = 6%. Value a 3-year annual-pay bond with a 5% coupon and a par value of \$1,000.

Answer:

$$\begin{aligned} \text{bond value} &= \frac{50}{1+S_1} + \frac{50}{(1+S_1)(1+1y1y)} + \frac{1,050}{(1+S_1)(1+1y1y)(1+2y1y)} \\ &= \frac{50}{1.04} + \frac{50}{(1.04)(1.05)} + \frac{1,050}{(1.04)(1.05)(1.06)} = \$1,000.98 \end{aligned}$$



PROFESSOR'S NOTE

If you think this looks a little like valuing a bond using spot rates, as we did for arbitrage-free valuation, you are correct. The discount factors are equivalent to spot rate discount factors.

If we have a semiannual coupon bond, the calculation methods are the same, but we would use the semiannual discount rate rather than the annualized rate—and the number of periods would be the number of semiannual periods.

LOS 55.c: Compare the spot curve, par curve, and forward curve.

The **spot rate yield curve** (spot curve) for U.S. Treasury bonds, also referred to as the **zero curve** or the **strip curve** (because zero-coupon U.S. Treasury bonds are also called stripped Treasuries), is a plot of spot rates versus maturity.

Usually, a spot curve is upward sloping, with higher spot rates for longer maturities (referred to as a normal yield curve), reflecting investor demand for higher returns over longer time frames. When spot rates are lower for longer-dated maturities, the spot curve is downward-sloping and is said to be inverted. Spot rates are usually quoted on a semiannual bond basis, so they are directly comparable to YTM's quoted for coupon government bonds.

A **yield curve for coupon bonds** shows the YTM's for a similar type of actively traded coupon bonds at various maturities (e.g., U.S. Treasury bonds). Yields are calculated for several available maturities, and yields for bonds with maturities between these are estimated by linear interpolation. Yields are usually expressed on a semiannual bond basis.

A practical issue with constructing yield curves directly from traded coupon bond prices is that distortions in yields can occur due to illiquidity and taxation differences between interest income and gains and losses from buying bonds at a discount or premium. To avoid the illiquidity issue, "on-the-run" (most recently issued) bond yields are used; however, there may not be enough on-the-run securities to construct a meaningful curve, and the ones that do exist may trade at a premium or discount to par, causing taxation distortions.

To avoid these practical issues when constructing a coupon bond yield curve, a **par bond yield curve**, or *par curve*, can be constructed from spot curves. As we discussed earlier, par yields are hypothetical yields of bonds that would trade at par for a specific maturity. A par yield curve displays the yields of par bonds versus maturity.

A **forward yield curve** shows forward rates for bonds or money market securities for annual periods in the future. Typically, the forward curve would show the yields of 1-year securities for each future year, quoted on a semiannual bond basis.

As we have seen, the spot rate for a given maturity is a geometric average of the forward rates that apply to each period between now and that maturity. Therefore, when the forward curve is upward sloping, the spot curve is also upward sloping, but less so. We also saw earlier how a par yield at a certain maturity is a weighted average of the spot rates that apply to the individual cash flows of the bond (most heavily weighted toward the longest-dated spot rate when par payment takes place). Hence, par yields will also be upward sloping, and very close to spot rates (but slightly below them) in a normal (upward-sloping) forward curve environment.



PROFESSOR'S NOTE

It might be helpful, once again, to approximate using simple averages to get a clear picture. Assume the 1-year spot rate is 1% and the 1y1y forward rate is 3%. Note that we have rising periodic rates here (the forward curve is rising

with maturity). The annualized 2-year spot is approximated as $(1\% + 3\%) / 2 = 2\%$, so we can see the spot curve is also rising with maturity, but at a slower pace than forward rates. The yield of a 1-year annual coupon bond would be 1% because there would only be one cash flow at maturity for this bond, and it would therefore earn the 1-year spot rate. The 2-year yield would be a weighted average of the 1-year spot of 1% and the 2-year spot of 2%, but mostly 2% because that is where most of the 2-year bond's cash flows occur. So, we can see that par yields, too, are rising with maturity—and are close to, but slightly below, spot rates. The key takeaway here is that forward rates drive spot rates, which in turn drive par yields.

Likewise, when the forward curve is downward sloping, the spot curve will also be downward sloping, but less so; and par yields will also be downward sloping, and close to spot rates (but slightly above them). One potential explanation for an inverted yield curve is that interest rates are expected to decrease.

When forward rates are constant, it means that all future periodic rates are the same. This means that spot rates to all maturities will be the same—and therefore, bond yields will be the same for all maturities. We describe this as a flat yield curve environment.



MODULE QUIZ 55.1

1. If spot rates are 3.2% for one year, 3.4% for two years, and 3.5% for three years, the price of a \$100,000 face value, 3-year, annual-pay bond with a coupon rate of 4% is *closest* to:
 - A. \$101,420.
 - ☒ B. \$101,790.
 - C. \$108,230.
2. A market rate of discount for a single payment to be made in the future is a:
 - ☒ A. spot rate.
 - B. simple yield.
 - C. forward rate.
3. Which of the following yield curves is least likely to consist of observed yields in the market?
 - A. Forward yield curve.
 - B. ~~Par~~ bond yield curve.
 - ☒ C. Coupon bond yield curve.
4. The 4-year spot rate is 9.45%, and the 3-year spot rate is 9.85%. What is the 1-year forward rate three years from today?
 - ☒ A. 8.258%.
 - B. 9.850%.
 - C. 11.059%.
5. Given the following spot and forward rates:
 - Current 1-year spot rate is 5.5%.
 - 1-year forward rate one year from today is 7.63%.
 - 1-year forward rate two years from today is 12.18%.
 - 1-year forward rate three years from today is 15.5%.

The value of a 4-year, 10% annual-pay, \$1,000 par value bond is *closest* to:

- A. \$870.
- B. \$996.
- ☒ C. \$1,009.

6. The 1-year spot rate is 1% and the 1y1y rate is 3%. Which of the following statements is *most accurate*?

- ☒ A. The 2-year spot is just below 2%, and the 2-year par yield is just below the 2-year spot rate.
- B. The 2-year spot is just below 2%, and the 2-year par yield is just above the 2-year spot rate.
- C. The 2-year spot is just below 4%, and the 2-year par yield is just below the 2-year spot rate.

KEY CONCEPTS

LOS 55.a

Spot rates are market discount rates for single payments to be made in the future.

The no-arbitrage price of a bond is calculated using no-arbitrage spot rates as follows:

$$\text{no-arbitrage price} = \frac{\text{coupon}}{(1 + S_1)} + \frac{\text{coupon}}{(1 + S_2)^2} + \dots + \frac{\text{coupon} + \text{principal}}{(1 + S_N)^N}$$

LOS 55.b

Spot curves can be used to derive the par yields of hypothetical bonds trading at par across different maturities. It is useful to interpret the par yield of a bond as a weighted average of the spot rates applying to each individual cash flow of the bond.

Forward rates are current lending/borrowing rates for short-term loans to be made in future periods.

A spot rate for a maturity of N periods is the geometric mean of forward rates over the N periods. The same relation can be used to solve for a forward rate given spot rates for two different periods.

To value a bond using forward rates, discount the cash flows at Periods 1 to N by the product of one plus each forward rate for Periods 1 to N , and sum them.

This is for a 3-year annual-pay bond:

$$\text{price} = \frac{\text{coupon}}{(1 + S_1)} + \frac{\text{coupon}}{(1 + S_1)(1 + 1y1y)} + \frac{\text{coupon} + \text{principal}}{(1 + S_1)(1 + 1y1y)(1 + 2y1y)}$$

LOS 55.c

The spot curve plots spot rates versus maturity. It can be derived from the prices of instruments offering single payments in the future, such as zero-coupon bonds or stripped Treasury bonds.

The par curve shows the coupon rates for bonds of various maturities that would result in bond prices equal to their par values.

A forward curve is a yield curve composed of forward rates, such as 1-year rates, available at each year over a future period.

In an upward-sloping (normal) yield curve environment, forward rates will be higher than spot rates, which will be higher than par yields. In a downward-sloping (inverted) yield curve environment, forward rates will be lower than spot rates, which will be lower than par yields. In a flat yield curve environment, forward rates will be equal to spot rates, which will be equal to par yields across all maturities.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 55.1

$$1. \text{ A bond value} = \frac{4,000}{1.032} + \frac{4,000}{(1.034)^2} + \frac{104,000}{(1.035)^3} = \$101,419.28$$

(LOS 55.a)

2. **A** A spot rate is a discount rate for a single future payment. Simple yield is a measure of a bond's yield that accounts for coupon interest and assumes straight-line amortization of a discount or premium. A forward rate is an interest rate for a future period, such as a 3-month rate six months from today. (LOS 55.a)

3. **B** Par bond yield curves are based on the theoretical yields that would cause bonds at each maturity to be priced at par. Coupon bond yields and forward interest rates can be observed directly from market transactions. (LOS 55.b)

$$4. \text{ A } (1.0945)^4 = (1.0985)^3 \times (1 + 3y1y)$$

$$3y1y = \frac{(1.0945)^4}{(1.0985)^3} - 1 = 8.258\%$$

$$\text{approximate forward rate} = 4(9.45\%) - 3(9.85\%) = 8.25\%$$

(LOS 55.b)

$$5. \text{ C bond value} = \frac{100}{1.055} + \frac{100}{(1.055)(1.0763)} + \frac{100}{(1.055)(1.0763)(1.1218)} + \frac{1,100}{(1.055)(1.0763)(1.1218)(1.155)} = 1,009.03$$

(LOS 55.b)

6. **A** The 2-year spot rate must reflect the periodic rates of 1% in the first period and 3% in the second period—hence, the 2-year spot will be approximately 2%. Given that forward rates are rising with maturity, spot rates will be rising with maturity at a slower pace, and par yields will also be rising, but slightly below spot rates. So, the correct answer is that the 2-year spot is just below 2%, and the 2-year par yield is just below the 2-year spot rate. (LOS 55.c)

READING 56

INTEREST RATE RISK AND RETURN

MODULE 56.1: INTEREST RATE RISK AND RETURN



Video covering
this content is
available online.

LOS 56.a: Calculate and interpret the sources of return from investing in a fixed-rate bond.

There are **three sources of returns** from investing in a fixed-rate bond:

1. Coupon and principal payments
2. Interest earned on coupon payments that are reinvested over the investor's holding period for the bond
3. Any capital gain or loss if the bond is sold before maturity

We will assume that a bond makes all of its promised coupon and principal payments on time (i.e., we are not addressing credit risk). Additionally, we assume that the *interest rate earned on reinvested coupon payments is the same as the prevailing yield to maturity (YTM) on the bond.*

Given the assumptions just listed, we may draw five key results:

1. An investor who holds a fixed-rate bond to maturity will earn an annualized rate of return equal to the YTM of the bond when purchased, if the YTM of the bond (and hence the reinvestment rate) does not change over the life of the bond.
2. An investor who sells a bond before maturity will earn a rate of return equal to the YTM at purchase if the YTM has not changed since purchase.
3. If the market YTM for the bond, our assumed reinvestment rate, increases (decreases) after the bond is purchased but before the first coupon date, an investor who holds the bond to maturity will earn a realized return that is higher (lower) than the original YTM of the bond when purchased.
4. If the market YTM for the bond, our assumed reinvestment rate, *increases* after the bond is purchased but before the first coupon date, a bond investor will earn a rate of return that is lower than the YTM at bond purchase if the bond is held for a *short* period.
5. If the market YTM for the bond, our assumed reinvestment rate, *decreases* after the bond is purchased but before the first coupon date, a bond investor will earn a rate of

return that is lower than the YTM at bond purchase if the bond is held for a *long* period.

We will present mathematical examples to demonstrate each of these results as well as some intuition as to why these results must hold.

We call the time that the bond will be held the investor's **investment horizon**, which may be shorter than the bond's maturity. A bond investor's **horizon yield** is the compound annual return earned from the bond over the investment horizon. It is calculated by comparing the purchase price of the bond to the end value derived from holding the bond, which includes coupons, interest earned on reinvested coupons, and the sale price (or principal payment amount, if the bond is held to maturity).

Unchanged YTM, Bond Held to Maturity

We will illustrate this calculation (and the first result listed earlier) with a 6% annual-pay 3-year bond purchased at a YTM of 7% and held to maturity.

With an annual YTM of 7%, the bond's purchase price is 97.376:

$$N = 3; I/Y = 7; PMT = 6; FV = 100; CPT \rightarrow PV = -97.376$$

At maturity, the investor will have received coupon income and reinvestment income equal to the future value of an annuity of three \$6 coupon payments calculated with an interest rate equal to the bond's YTM. This is the amount:

$$6(1.07)^2 + 6(1.07) + 6 = \$19.289$$

$$N = 3; I/Y = 7; PV = 0; PMT = 6; CPT \rightarrow FV = -19.289$$

We can easily calculate the amount earned from reinvestment of the coupons as follows:

$$19.289 - 3(6) = \$1.289$$

Adding the principal payment at maturity of \$100 to \$19.289, we can calculate the investor's rate of return over the 3-year holding period as $\left(\frac{\$119.289}{\$97.376}\right)^{1/3} - 1 = 7\%$ and demonstrate that \$97.376 invested at a compound annual rate of 7% would return \$119.289 after three years.

With this example, we have demonstrated our first result: for a fixed-rate bond that does not default and has a reinvestment rate equal to the YTM, an investor who holds the bond until maturity will earn a rate of return equal to the YTM at purchase.

Unchanged YTM, Bond Sold Before Maturity

Now let's examine the second result—that an investor who sells a bond before maturity will earn a rate of return equal to the YTM *as long as the YTM has not changed since purchase*. If the YTM of the bond remains unchanged, the value of a bond will move toward the par value of the bond by the maturity date. At dates between the purchase and the maturity, the value of a bond at the same YTM as when it was

purchased is its **carrying value**, and it reflects the amortization of the discount or premium since the bond was purchased.



PROFESSOR'S NOTE

Carrying value is a price along a bond's constant-yield price trajectory (in other words, it is the value of the bond at a certain time after purchase, assuming the original yield of the bond has not changed). It is referred to as carrying value because it is the value that is often shown on the balance sheet in financial reporting when a bond is held to maturity. This is not the same as the market value of the bond if its yield *has* changed.

Capital gains or losses at the time a bond is sold are measured relative to this carrying value, as illustrated in the following example.

EXAMPLE: Capital gain or loss on a bond

An investor purchases a 20-year bond with a 5% semiannual coupon and a yield to maturity of 6%. Five years later, the investor sells the bond for a price of 91.40. Determine whether the investor realizes a capital gain or loss, and calculate its amount.

Answer:

Any capital gain or loss is based on the bond's carrying value at the time of sale, when it has 15 years (30 semiannual periods) to maturity. The carrying value is calculated using the bond's YTM at the time the investor purchased it:

$$N = 30; I/Y = 3; PMT = 2.5; FV = 100; CPT \rightarrow PV = -90.20$$

Because the selling price of 91.40 is greater than the carrying value of 90.20, the investor realizes a capital gain of $91.40 - 90.20 = 1.20$ per 100 of face value.

Bonds held to maturity have no capital gain or loss. Bonds sold before maturity at the same YTM as at purchase will also have no capital gain or loss. Using the 6% 3-year bond from our earlier examples, we can demonstrate this for an investor with a two-year holding period (investment horizon).

After the bond is purchased at a YTM of 7% (for 97.376), we have the following.

Price at sale (at end of Year 2, YTM = 7%):

$$106 / 1.07 = 99.065$$

$$N = 1; I/Y = 7; FV = 100; PMT = 6; CPT \rightarrow PV = -99.065$$

which is the carrying value of the bond.

Coupon interest and reinvestment income for two years:

$$6(1.07) + 6 = \$12.420$$

$$N = 2; I/Y = 7; PV = 0; PMT = 6; CPT \rightarrow FV = -12.420$$

Investor's annual compound rate of return over the two-year holding period:

$$\left(\frac{12.420 + 99.065}{97.376}\right)^{1/2} - 1 = 7\%$$

This demonstrates the second key result we listed: that for a bond investor with a horizon less than the bond's term to maturity, the annual return will be equal to the YTM at purchase if the bond is sold at that YTM and all coupons are reinvested at the original YTM.

Changed YTM, Bond Held to Maturity

Next let's examine our third result—that if rates rise (fall) before the first coupon date, an investor who holds a bond to maturity will earn a rate of return greater (less) than the YTM at purchase.

Based on our previous result that an investor who holds a bond to maturity will earn a rate of return equal to the YTM at purchase if the reinvestment rate is also equal to the YTM at purchase, the intuition of the third result is straightforward. If the YTM, which is also the reinvestment rate for the bond, increases (decreases) after purchase, the return from coupon payments and reinvestment income will increase (decrease) as a result, which will increase (decrease) the investor's rate of return on the bond above (below) its YTM at purchase. The following calculations demonstrate these results for the 3-year 6% bond in our previous examples.

For a 3-year 6% bond purchased at 97.376 (YTM of 7%), first assume that the YTM and reinvestment rate increases to 8% after purchase but before the first coupon payment date. The bond's annualized holding period return is calculated as follows.

Coupons and reinvestment interest:

$$6(1.08)^2 + 6(1.08) + 6 = \$19.478$$

$$N = 3; I/Y = 8; PV = 0; PMT = 6; CPT \rightarrow FV = -19.478$$

Investor's annual compound holding period return:

$$\left(\frac{119.478}{97.376}\right)^{1/3} - 1 = 7.06\%$$

which is greater than the 7% YTM at purchase.

If the YTM decreases to 6% after purchase but before the first coupon date, we have the following.

Coupons and reinvestment interest:

$$6(1.06)^2 + 6(1.06) + 6 = \$19.102$$

$$N = 3; I/Y = 6; PV = 0; PMT = 6; CPT \rightarrow FV = -19.102$$

Investor's annual compound holding period return:

$$\left(\frac{119.102}{97.376}\right)^{1/3} - 1 = 6.94\%$$

which is less than the 7% YTM at purchase.

Note that in both cases, the investor's rate of return is between the YTM at purchase and the assumed reinvestment rate (the new YTM).

Changed YTM, Bond Sold Before Maturity

We now turn our attention to the fourth and fifth results concerning the effects of the length of an investor's holding period on the rate of return for a bond that experiences an increase or decrease in its YTM before the first coupon date.

Consider a 3-year 6% bond purchased at 97.376 by an investor with a one-year investment horizon. If the YTM increases from 7% to 8% after purchase and the bond is sold after one year, the rate of return can be calculated as follows.

Bond price just after first coupon has been paid with YTM = 8%:

$$N = 2; I/Y = 8; FV = 100; PMT = 6; CPT \rightarrow PV = -96.433$$

There is no reinvestment income and only one coupon of \$6 received, so the holding period rate of return is simply:

$$\left(\frac{6 + 96.433}{97.376} \right) - 1 = 5.19\%$$

which is less than the YTM at purchase.

If the YTM decreases to 6% after purchase and the bond is sold at the end of one year, the investor's rate of return can be calculated as follows.

Bond price just after first coupon has been paid with YTM = 6%:

$$N = 2; I/Y = 6; FV = 100; PMT = 6; CPT \rightarrow PV = -100.00$$

And the holding period rate of return is simply:

$$\left(\frac{6 + 100.00}{97.376} \right) - 1 = 8.86\%$$

which is greater than the YTM at purchase.

The intuition of this result is based on the idea of a tradeoff between **price risk** (the uncertainty about a bond's price due to uncertainty about the prevailing market YTM at a time of sale) and **reinvestment risk** (uncertainty about the total of coupon payments and reinvestment income on those payments due to the uncertainty about future reinvestment rates).

Previously, we showed that for a bond held to maturity, the investor's rate of return increased with an increase in the bond's YTM and decreased with a decrease in the bond's YTM. For an investor who intends to hold a bond to maturity, there is no price risk as we have defined it (because they do not intend to sell the bond). Assuming no default, the bond's value at maturity is its par value regardless of interest rate changes, so that the investor has only reinvestment risk. Her realized return will increase when interest earned on reinvested cash flows increases, and decrease when the reinvestment rate decreases.

For an investor with a short investment horizon, because they intend to sell the bond before maturity, price risk increases and reinvestment risk decreases. For the investor

with a one-year investment horizon, there was no reinvestment risk because the bond was sold before any interest on coupon payments was earned. The investor had only price risk, so an increase in yield decreased the rate of return over the one-year holding period because the sale price was lower. Conversely, a decrease in yield increased the one-year holding period return to more than the YTM at purchase because the sale price was higher.

To summarize:

- Short investment horizon: price risk > reinvestment risk
- Long investment horizon: reinvestment risk > price risk

LOS 56.b: Describe the relationships among a bond's holding period return, its Macaulay duration, and the investment horizon.

LOS 56.c: Define, calculate, and interpret Macaulay duration.

Is there an investment horizon where price risk and reinvestment risk are in balance?
There is, as demonstrated by the following example.

EXAMPLE: Investment horizon yields

Consider a 5-year, 11% annual coupon bond priced at 86.59 to yield 15% to maturity. Calculate the horizon yield for an investment horizon of 4 years, assuming that the YTM does the following:

- a) Falls to 14% before the first coupon date
- b) Rises to 16% before the first coupon date

Answer:

- a) **Falls to 14% before the first coupon date**

Sale after 4 years

Bond price:

$$N = 1; PMT = 11; FV = 100; I/Y = 14; CPT \rightarrow PV = 97.368$$

Coupons and interest on reinvested coupons:

$$N = 4; PMT = 11; PV = 0; I/Y = 14; CPT \rightarrow FV = 54.133$$

Horizon return:

$$[(97.368 + 54.133) / 86.59]^{1/4} - 1 = 15.0\%$$

- b) **Rises to 16% before the first coupon date**

Sale after 4 years

Bond price:

$$N = 1; PMT = 11; FV = 100; I/Y = 16; CPT \rightarrow PV = 95.690$$

Coupons and interest on reinvested coupons:

$$N = 4; PMT = 11; PV = 0; I/Y = 16; CPT \rightarrow FV = 55.731$$

Horizon return:

$$[(95.690 + 55.731) / 86.59]^{1/4} - 1 = 15.0\%$$

For an investment horizon of 4 years, the horizon return is equal to the original YTM of 15%, regardless of changes in the YTM of the bond.

This example shows that for a particular fixed-coupon bond, we can find an investment horizon that is neither short enough to face price risk nor long enough to face reinvestment risk. For this bond it was an investment horizon of four years. At this horizon, if the YTM decreases, losses on reinvestment income are just balanced by a gain in price. If the YTM increases, gains in reinvestment income are just offset by a loss in price.

How can we easily find this “sweet spot” investment horizon? We can determine the average time until the receipt of the cash flows of the bond, referred to as its **Macaulay duration**.

A bond’s (annual) Macaulay duration is calculated as the weighted average of the number of years until each of the bond’s promised cash flows is to be paid, where the weights are the present values of each cash flow as a percentage of the bond’s full value.

Consider the bond in the example. The present values of each of the bond’s promised payments, discounted at the bond’s YTM of 15%, and their weights in the calculation of Macaulay duration, are shown in the following table.

$C_1 = 11$	$PV_1 = 11 / 1.15 = 9.565$	$W_1 = 9.565 / 86.59 = 0.1105$
$C_2 = 11$	$PV_2 = 11 / 1.15^2 = 8.318$	$W_2 = 8.318 / 86.59 = 0.0961$
$C_3 = 11$	$PV_3 = 11 / 1.15^3 = 7.233$	$W_3 = 7.233 / 86.59 = 0.0835$
$C_4 = 11$	$PV_4 = 11 / 1.15^4 = 6.289$	$W_4 = 6.289 / 86.59 = 0.0726$
$C_5 = 111$	$PV_5 = 111 / 1.15^5 = 55.187$	$W_5 = 55.187 / 86.59 = 0.6373$
	86.59	1.0000

The present values of all the promised cash flows sum to 86.59 (the full price of the bond) and the weights sum to 1.

Now that we have the weights, and because we know the time until each promised payment is to be made, we can calculate the Macaulay duration for this bond:

$$0.1105(1) + 0.0961(2) + 0.0835(3) + 0.0726(4) + 0.6373(5) = 4.03 \text{ years}$$

Our interpretation of this Macaulay duration is that, as demonstrated in the example, if we have an investment horizon of four years, then we still earn the original YTM of the bond out to this horizon—even if the YTM of the bond immediately changes after purchasing the bond.

The Macaulay duration of a semiannual-pay bond can be calculated in the same way: as a weighted average of the number of *semiannual periods* until the cash flows are to be received. In this case, the result is the number of semiannual periods rather than years (to express this in annualized terms, divide the number of semiannual periods by two).

The difference between a bond’s Macaulay duration and the bondholder’s investment horizon is referred to as a **duration gap**. A positive duration gap (Macaulay duration greater than the investment horizon) exposes the investor to price risk from increasing

interest rates. A negative duration gap (Macaulay duration less than the investment horizon) exposes the investor to reinvestment risk from decreasing interest rates.



MODULE QUIZ 56.1

1. The largest component of returns for a 7-year zero-coupon bond yielding 8% and held to maturity is:
A. capital gains.
☒ B. interest income.
C. reinvestment income.
2. An investor buys a 10-year bond with a 6.5% annual coupon and a YTM of 6%. Before the first coupon payment is made, the YTM for the bond decreases to 5.5%. Assuming coupon payments are reinvested at the YTM, the investor's return when the bond is held to maturity is:
☒ A. less than 6.0%.
B. equal to 6.0%.
C. greater than 6.0%.
3. Assuming coupon interest is reinvested at a bond's YTM, what is the interest portion of an 18-year, \$1,000 par, 5% annual coupon bond's return if it is purchased at par and held to maturity?
A. \$576.95.
☒ B. \$1,406.62.
C. \$1,476.95.
4. An investor buys a 15-year, £800,000, zero-coupon bond with an annual YTM of 7.3%. If she sells the bond after three years for £346,333, she will have:
☒ A. a capital gain.
B. a capital loss.
C. neither a capital gain nor a capital loss.
5. An investor with an investment horizon of six years buys a bond with a Macaulay duration of seven years. This investment has:
A. no duration gap.
B. a positive duration gap.
C. a negative duration gap.
6. The Macaulay duration (in years) of a 2-year semiannual-pay 7% coupon bond yielding 5% is *closest* to:
A. 0.38.
B. 1.90.
C. 3.81.

KEY CONCEPTS

LOS 56.a

Sources of return from a bond investment include the following:

- Coupon and principal payments
- Reinvestment of coupon payments
- Capital gain or loss if bond is sold before maturity

Changes in yield to maturity (YTM) produce price risk (uncertainty about a bond's price) and reinvestment risk (uncertainty about income from reinvesting coupon payments). An increase (a decrease) in YTM decreases (increases) a bond's price but increases (decreases) its reinvestment income.

LOS 56.b

Over a short investment horizon, a change in YTM affects price more than it affects reinvestment income.

Over a long investment horizon, a change in YTM affects reinvestment income more than it affects price.

The Macaulay duration may be interpreted as the investment horizon for which a bond's price risk and reinvestment risk offset each other:

$$\text{duration gap} = \text{Macaulay duration} - \text{investment horizon}$$

LOS 56.c

Macaulay duration is calculated as the weighted average of the number of years until each of the bond's promised cash flows is to be paid, where the weights are the present values of each cash flow as a percentage of the bond's full value.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 56.1

- B** The increase in value of a zero-coupon bond over its life is interest income. A zero-coupon bond has no reinvestment risk over its life. A bond held to maturity has no capital gain or loss. (LOS 56.a)
- A** The investment horizon is maturity, which means that the investor faces reinvestment risk (on average, the cash flows of the bond are received before maturity) and zero price risk. The decrease in the YTM to 5.5% will decrease the reinvestment income over the life of the bond so that the investor will earn less than 6%, the YTM at purchase. (LOS 56.a)
- B** The interest portion of a bond's return is the sum of the coupon payments and interest earned from reinvesting coupon payments over the holding period:
$$N = 18; PMT = 50; PV = 0; I/Y = 5\%; CPT \rightarrow FV = -1,406.62$$

(LOS 56.a)
- A** The price of the bond after three years that will generate neither a capital gain nor a capital loss is the price if the YTM remains at 7.3%. After three years, the present value of the bond is $800,000 / 1.073^{12} = 343,473.57$, so she will have a capital gain relative to the bond's carrying value. (LOS 56.a)
- B** Duration gap is the Macaulay duration minus the investment horizon. This bond has a Macaulay duration greater than six years, and the investment has a positive duration gap. (LOS 56.b)

6. B

$C_1 = 3.5$	$PV_1 = 3.5 / 1.025$	$= 3.415$	$W_1 = 3.415 / 103.762$	$= 0.0329$
$C_2 = 3.5$	$PV_2 = 3.5 / 1.025^2$	$= 3.331$	$W_2 = 3.331 / 103.762$	$= 0.0321$
$C_3 = 3.5$	$PV_3 = 3.5 / 1.025^3$	$= 3.250$	$W_3 = 3.250 / 103.762$	$= 0.0313$
$C_4 = 103.5$	$PV_4 = 103.5 / 1.025^4$	$= \underline{93.766}$	$W_4 = 93.766 / 103.762$	$= \underline{0.9037}$
		103.762		1.0000

The Macaulay duration is, therefore, calculated as weighted average time as follows:

$$0.0329(1) + 0.0321(2) + 0.0313(3) + 0.9037(4) = 3.806$$

Then, the annualized Macaulay duration is $3.806 / 2 = 1.90$ years. (LOS 56.c)

READING 57

YIELD-BASED BOND DURATION MEASURES AND PROPERTIES

MODULE 57.1: YIELD-BASED BOND DURATION MEASURES AND PROPERTIES



Video covering this content is available online.

LOS 57.a: Define, calculate, and interpret modified duration, money duration, and the price value of a basis point (PVBP).

Modified duration (ModDur) is calculated as Macaulay duration (MacDur) divided by one plus the bond's periodic rate of return (YTM divided by periodicity).

For an annual-pay bond, this is the general form of ModDur:

$$\text{ModDur} = \text{MacDur} / (1 + \text{YTM})$$

For a semiannual-pay bond with a YTM quoted on a semiannual bond basis, this is the form:

$$\text{ModDur}_{\text{SEMI}} = \text{MacDur}_{\text{SEMI}} / (1 + \text{YTM} / 2)$$

EXAMPLE: Modified duration

A 5-year, 11% annual coupon bond priced at 86.59 to yield 15% to maturity has a Macaulay duration of 4.03. Calculate the modified duration of this bond.

Answer:

Because it is an annual coupon bond (periodicity = 1), its modified duration can be calculated as follows:

$$\text{ModDur} = 4.03 / 1.15 = 3.50$$

Modified duration provides an estimate for the percentage change in a bond's price given a 1% change in YTM:

$$\text{approximate percentage change in bond price} = -\text{ModDur} \times \Delta\text{YTM}$$

Based on a ModDur of 3.50, in response to an 0.5% increase in YTM the price of the bond should fall by approximately $3.50 \times 0.5\% = 1.75\%$. The resulting price estimate of

$86.59 \times (1 - 0.0175) = 85.075$ is close to the value of the bond calculated directly using a YTM of 15.5%, which is 85.092:

$N = 5; I/Y = 15.5; FV = 100; PMT = 11; CPT \rightarrow PV = -85.092$

For a semiannual-pay bond, $\text{ModDur}_{\text{SEMI}}$ can be annualized (from semiannual periods to annual periods) by dividing by two, and then used as the approximate change in price for a 1% change in a bond's YTM.

Approximate Modified Duration

We can approximate modified duration directly using bond values for an increase in YTM and for a decrease in YTM of the same size.

The calculation of approximate ModDur is based on a given change in YTM. V_- is the price of the bond if YTM is *decreased* by ΔYTM , and V_+ is the price of the bond if the YTM is *increased* by ΔYTM . Approximate ModDur is given by the following:

$$\text{approximate ModDur} = \frac{V_- - V_+}{2V_0 \Delta\text{YTM}}$$

The formula uses the average of the magnitudes of the price increase and the price decrease, which is why $V_- - V_+$ in the numerator is divided by two in the denominator.

V_0 , the current price of the bond, is in the denominator to convert this average price change to a percentage, and the ΔYTM term is in the denominator to scale the duration measure to a 1% change in yield by convention. Note that the ΔYTM term in the denominator must be entered as a decimal (rather than in a whole percentage) to properly scale the duration estimate.

EXAMPLE: Calculating approximate modified duration

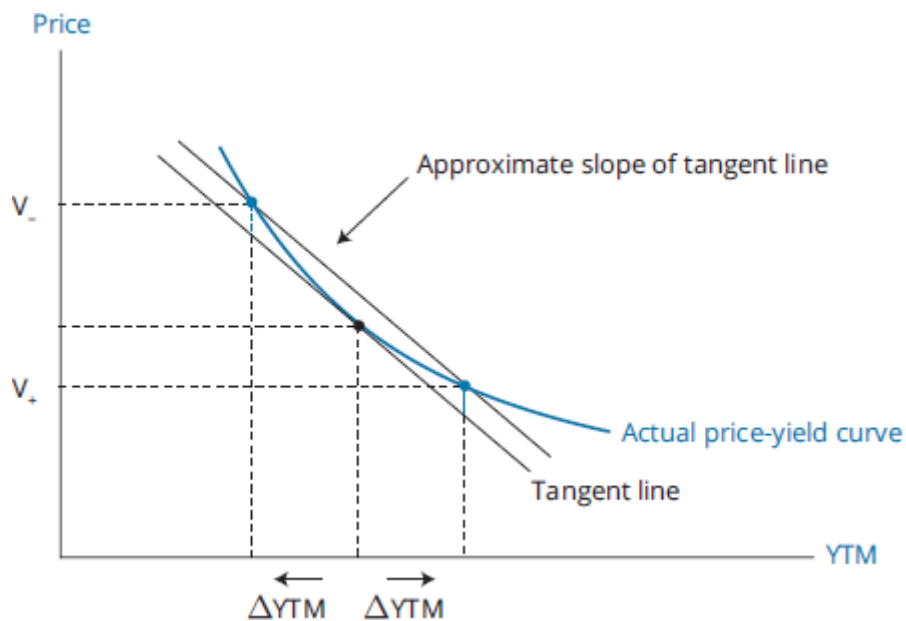
Consider a 5-year, 11% annual coupon bond priced at 86.59 to yield 15% to maturity. If its YTM increases by 50 basis points, its price will decrease to 85.092. If its YTM decreases by 50 basis points, its price will increase to 88.127. Calculate the approximate ModDur.

Answer:

The approximate ModDur is $\frac{88.127 - 85.092}{2 \times 86.59 \times 0.005} = 3.505$ and the

Modified duration is a *linear estimate* of the relation between a bond's price and YTM, whereas the actual relation is *convex*—not a straight line. This means that the modified duration measure provides good estimates of bond prices for small changes in yield, but increasingly poor estimates for larger changes in yield as the effect of the curvature of the price-yield curve is more pronounced. We illustrate this in Figure 57.1.

Figure 57.1: Approximate ModDur



Money Duration

The **money duration** of a bond position (also called dollar duration) is expressed in currency units:

$$\text{money duration} = \text{annual ModDur} \times \text{full price of bond position}$$

Multiplying the money duration of a bond by a given change in YTM (as a decimal) will provide an estimate for the change in bond value for that change in YTM.

EXAMPLE: Money duration

1. Calculate the money duration on a coupon date of a \$2 million par value bond that has a ModDur of 7.42 and a full price of 101.32, expressed for the whole bond and per \$100 of face value.
2. What will be the impact on the value of the bond of a 25 basis point increase in its YTM?

Answer:

1. The money duration for the bond is ModDur times the full value of the bond:

$$7.42 \times \$2,000,000 \times 101.32\% = \$15,035,888$$

The money duration per \$100 of par value is:

$$7.42 \times 101.32 = \$751.79$$

$$\text{Or, } \$15,035,888 / (\$2,000,000 / \$100) = \$751.79.$$

2. $\$15,035,888 \times 0.0025 = \$37,589.72$

The bond value decreases by \$37,589.72.

The **price value of a basis point (PVBP)** is the money change in the full price of a bond when its YTM changes by one basis point, or 0.01%. We can calculate the PVBP

directly for a bond by calculating the average of the decrease in the full value of a bond when its YTM increases by one basis point, and the increase in the full value of the bond when its YTM decreases by one basis point.

EXAMPLE: Calculating the PVBP

A newly issued, 20-year, 6% annual-pay straight bond is priced at 101.39. Calculate the PVBP for this bond, assuming it has a par value of \$1 million.

Answer:

First, we need to find the YTM of the bond:

$$N = 20; PV = -101.39; PMT = 6; FV = 100; CPT \rightarrow I/Y = 5.88$$

Now, we need the values for the bond with YTM of 5.89 and 5.87:

$$I/Y = 5.89; CPT \rightarrow PV = -101.273 (V_+)$$

$$I/Y = 5.87; CPT \rightarrow PV = -101.507 (V_-)$$

$$PVBP \text{ (per \$100 of par value)} = (101.507 - 101.273) / 2 = 0.117$$

For the \$1 million par value bond, each 1 basis point change in the YTM will change the bond's price by $0.117 \times \$1 \text{ million} \times 0.01 = \$1,170$.

LOS 57.b: Explain how a bond's maturity, coupon, and yield level affect its interest rate risk.

Other things equal, *bonds with longer maturity* will (usually) have higher interest rate risk. The present values of payments made further in the future are more sensitive to changes in the discount rate used to calculate present value than are the present values of payments made sooner.

We must say *usually* because there are instances where an increase in a discount coupon bond's maturity will decrease its Macaulay duration. For a discount bond, duration first increases with longer maturity and then decreases over a range of relatively long maturities until it approaches the duration of a perpetuity, which is $(1 + YTM) / YTM$.

Between coupon dates, if the YTM of a coupon bond remains constant, its Macaulay duration decreases smoothly with the passage of time, and then goes back up slightly at each coupon payment date as the time to the next coupon resets to a full coupon period.

Other things equal, a *higher coupon rate* on a bond will decrease its interest rate risk. For a given maturity and YTM, the duration of a zero-coupon bond will be greater than that of a coupon bond. Increasing the coupon rate means more of a bond's value will be from payments received sooner, so that the value of the bond will be less sensitive to changes in yield.

For floating-rate notes (FRNs) where coupons are periodically reset to a market reference rate (MRR), when interest rates rise, the coupon will also rise, limiting the

price risk of the bond. Macaulay duration for an FRN can be calculated as the time to the next coupon reset date at the end of the current coupon period.

Other things equal, an *increase (decrease) in a bond's YTM* will decrease (increase) its *interest rate risk*. To understand this, we can look to the convexity of the price-yield curve and use its slope as our proxy for interest rate risk. At lower yields, the price-yield curve has a steeper slope, indicating that price is more sensitive to a given change in yield.



MODULE QUIZ 57.1

1. A 14% annual-pay coupon bond has six years to maturity. The bond is currently trading at par. Using a 25 basis point change in yield, the approximate modified duration of the bond is *closest* to:
A. 0.392.
☒ B. 3.888.
C. 3.970.
2. The current price of a \$1,000, 7-year, 5.5% semiannual coupon bond is \$1,029.23. The bond's price value of a basis point is *closest* to:
A. \$0.05.
☒ B. \$0.60.
C. \$5.74.
3. The modified duration of a bond is 7.87. The approximate percentage change in price using duration only for a yield decrease of 110 basis points is *closest* to:
A. -8.657%.
B. +7.155%.
C. +8.657%.
4. All else equal, which of the following bonds is likely to have the highest price risk?
A. A 10-year maturity semiannual-pay floating-rate note.
☒ B. A 2-year zero-coupon bond.
C. A 2-year 10% semiannual-pay bond.

KEY CONCEPTS

LOS 57.a

Modified duration is a linear estimate of the percentage change in a bond's price that would result from a 1% change in its YTM:

$$\text{ModDur} = \text{MacDur} / (1 + \text{periodic return of bond})$$

For an expected change in yield, ΔYTM , the expected change in the bond's price is given by the following:

$$\text{approximate percentage change in bond price} = -\text{ModDur} \times \Delta\text{YTM}$$

Modified duration can be approximated by repricing the bond at different yields:

$$\text{approximate modified duration} = \frac{V_- - V_+}{2V_0 \Delta\text{YTM}}$$

Money duration is stated in currency units and is sometimes expressed per 100 of bond value:

$$\text{money duration} = \text{annual ModDur} \times \text{full price of bond position}$$

The price value of a basis point is the change in the value of a bond, expressed in currency units, for a change in YTM of one basis point:

$$\text{PVBP} = [(V_- - V_+) / 2] \times \text{par value} \times 0.01$$

PVBP can also be calculated as money duration \times 0.0001.

LOS 57.b

Holding other factors constant, the following are true:

- Duration increases when maturity increases.
- Duration decreases when the coupon rate increases.
- Duration decreases when YTM increases.
- Duration decreases as time passes, but increases slightly on coupon dates.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 57.1

$$\begin{aligned} V_- &= 100.979 \\ N &= 6; \text{PMT} = 14.00; \text{FV} = 100; \text{I/Y} = 13.75; \text{CPT} \rightarrow \text{PV} = -100.979 \\ V_+ &= 99.035 \\ \text{1. B } \text{I/Y} &= 14.25; \text{CPT} \rightarrow \text{PV} = -99.035 \\ V_0 &= 100.000 \\ \Delta y &= 0.0025 \\ \text{So, approximate ModDur} &= \frac{V_- - V_+}{2V_0 \Delta \text{YTM}} = \frac{100.979 - 99.035}{2(100)(0.0025)} = 3.888. \end{aligned}$$

(LOS 57.a)

2. B PVBP = initial price – price if yield is changed by 1 basis point.

First, we need to calculate the yield so we can calculate the price of the bond with a 1 basis point change in yield. Using a financial calculator, PV = -1,029.23; FV = 1,000; PMT = 27.5 = $(0.055 \times 1,000) / 2$; N = 14 = 2×7 years; and CPT \rightarrow I/Y = 2.49998, multiplied by 2 = 4.99995, or 5.00%.

Next, compute the price of the bond at a yield of 5.00% + 0.01%, or 5.01%. Using the calculator: FV = 1,000; PMT = 27.5; N = 14; I/Y = 2.505 (5.01 / 2); CPT \rightarrow PV = \$1,028.63.

Next, compute the price of the bond at a yield of 5.00% – 0.01%, or 4.99%. Using the calculator: FV = 1,000; PMT = 27.5; N = 14; I/Y = 2.495 (4.99 / 2); CPT \rightarrow PV = \$1,029.82.

Finally, PVBP = $(\$1,029.82 - \$1,028.63) / 2 = \$0.595$.

(LOS 57.a)

3. C $-7.87 \times (-1.10\%) = 8.657\%$

(LOS 57.a)

4. **B** The price risk of the FRN is very low because at the next coupon payment date, the coupons will reset to market rates, and the FRN price will reset to par. Lower coupons, all else equal, lead to greater price risk. Therefore the 2-year zero-coupon bond will have more price risk than the 2-year 10% semiannual-pay bond. (LOS 57.b)

READING 58

YIELD-BASED BOND CONVEXITY AND PORTFOLIO PROPERTIES

MODULE 58.1: YIELD-BASED BOND CONVEXITY AND PORTFOLIO PROPERTIES

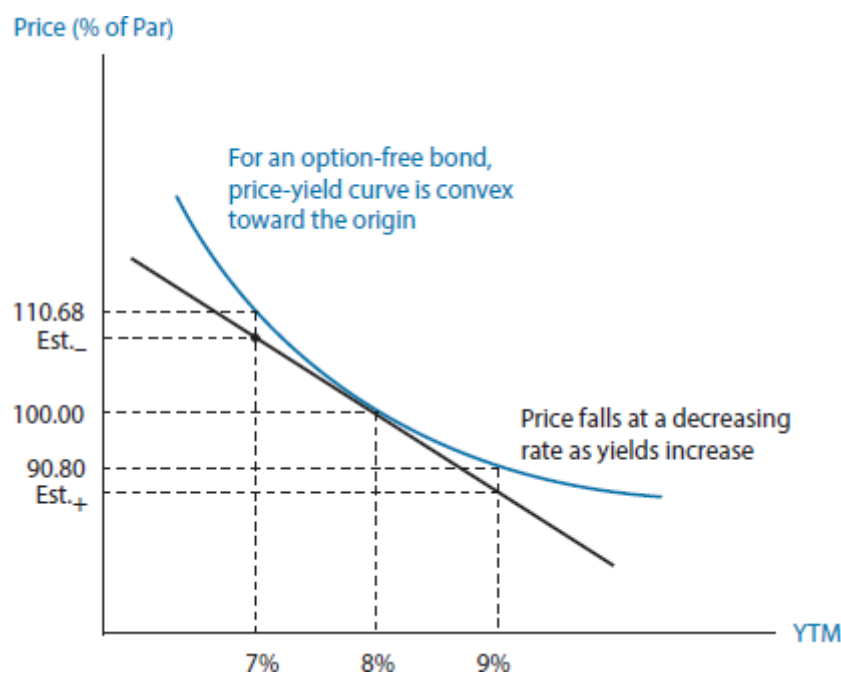


Video covering
this content is
available online.

LOS 58.a: Calculate and interpret convexity and describe the convexity adjustment.

In our reading on Yield-Based Bond Duration Measures and Properties, we showed that modified duration is a linear approximation of the price-yield relationship. Because the true relationship is convex, duration-based estimates of a bond's full price will be increasingly different from actual prices as we increase the changes in yield. This is illustrated in Figure 58.1. Duration-based price estimates for a decrease and for an increase in YTM are shown as Est.₋ and Est.₊.

Figure 58.1: Price-Yield Curve for an Option-Free, 8%, 20-Year Semiannual-Pay Bond



We can improve our estimates of the price impact of a change in yield by introducing a second term. **Convexity** is a measure of the curvature of the price-yield relation. The more curved it is, the greater the convexity adjustment.

One way to calculate convexity is by considering each of a bond's cash flows separately. The convexity of a single cash flow at period t is given by the following:

$$\text{convexity of cash flow at period } t = \frac{t \times (t + 1)}{(1 + r)^2}$$

where r is the periodic yield of the cash flow (YTM/periodicity).

The convexity of a coupon-paying bond can then be calculated as the weighted average convexity of its individual cash flows, using the present value of cash flows as the weights (the same weighting we use to calculate Macaulay duration).

For a 5-year, 11% annual coupon bond priced at 86.59 (a 15% yield to maturity), the convexity of the coupon at Time 1 is $(1 \times 2) / 1.15^2 = 1.512$, and the convexity of the coupon at Time 2 is $(2 \times 3) / 1.15^2 = 4.537$. The following table completes the calculations for all five of the bond's cash flows:

				<u>Convexity</u>
$C_1 = 11$	$PV_1 = 11 / 1.15 = 9.565$	$W_1 = 9.565 / 86.59 = 0.1105$		1.512
$C_2 = 11$	$PV_2 = 11 / 1.15^2 = 8.318$	$W_2 = 8.318 / 86.59 = 0.0961$		4.537
$C_3 = 11$	$PV_3 = 11 / 1.15^3 = 7.233$	$W_3 = 7.233 / 86.59 = 0.0835$		9.074
$C_4 = 11$	$PV_4 = 11 / 1.15^4 = 6.289$	$W_4 = 6.289 / 86.59 = 0.0726$		15.123
$C_5 = 111$	$PV_5 = 111 / 1.15^5 = 55.187$	$W_5 = 55.187 / 86.59 = 0.6373$		22.684
86.59		1.0000		

Using the weights given in the table, we can calculate the convexity for this bond as follows:

$$0.1105(1.512) + 0.0961(4.537) + 0.0835(9.074) + 0.0726(15.123) + 0.6373(22.684) = 16.915$$

For bonds with non-annual coupons, convexity needs to be divided by the number of periods per year *squared* to annualize the measure. For a semiannual coupon bond, the final convexity figure would be annualized by dividing by $2^2 = 4$.

In a similar way to how we approximated modified duration, we can also determine a bond's **approximate convexity** using the following formula:

$$\text{approximate convexity} = \frac{V_- + V_+ - 2V_0}{(\Delta YTM)^2 V_0}$$

where:

V_- = price of the bond if YTM is decreased by ΔYTM

V_+ = price of the bond if the YTM is increased by ΔYTM

V_0 = current price of the bond

EXAMPLE: Calculating approximate convexity

Consider our 5-year, 11% annual coupon bond priced at 86.59138 to yield 15% to maturity. If its YTM increases by 50 basis points, its price will decrease to 85.09217.

If its YTM decreases by 50 basis points, its price will increase to 88.12721. Calculate the approximate convexity of the bond.

Answer:

The approximate convexity is:

$$\frac{88.12721 + 85.09217 - (2 \times 86.59138)}{86.59138 \times 0.005^2} = 16.916$$

This result is nearly the same as the convexity measure we calculated by considering each cash flow separately.

A bond's convexity is increased or decreased by the same bond characteristics that affect duration. A longer maturity, a lower coupon rate, or a lower YTM will all increase convexity, and vice versa. For two bonds with equal duration, the one with cash flows that are more dispersed over time will have more convexity.

LOS 58.b: Calculate the percentage price change of a bond for a specified change in yield, given the bond's duration and convexity.

By taking account of both a bond's duration (first-order effects) and convexity (second-order effects), we can improve an estimate of the effects of a change in yield on a bond's value, especially for larger changes in yield:

$$\text{percent change in full bond price} = -\text{annual modified duration } (\Delta\text{YTM}) + \frac{1}{2} \text{annual convexity } (\Delta\text{YTM})^2$$

EXAMPLE: Estimating price changes with duration and convexity

Consider our 5-year, 11% annual coupon bond priced at 86.59138 to yield 15% to maturity. We have calculated the modified duration to be 3.50 and the convexity of the bond to be 16.9. Estimate the new price of the bond if its yield decreases by 50 basis points.

Answer:

The duration effect is $-3.50 \times -0.005 = 1.75\%$.

The convexity effect is $1/2 \times 16.9 \times (-0.005)^2 = 0.000211 = 0.0211\%$.

The expected change in bond price is $(1.75\% + 0.0211\%) = 1.7711\%$.

The new price of the bond is estimated to be $86.59138 \times 1.017711 = 88.125$.

Analogous to money duration (MoneyDur), the **money convexity** (MoneyCon) of a bond position is expressed in currency units:

$$\text{money convexity} = \text{annual convexity} \times \text{full price of bond position}$$

We can use money duration and money convexity to estimate the change in price of a bond as follows:

$$\begin{aligned} \text{change in full price of bond} = & -(\text{MoneyDur} \times \Delta\text{YTM}) \\ & + \left(\frac{1}{2} \times \text{MoneyCon} \times \Delta\text{YTM}^2 \right) \end{aligned}$$

EXAMPLE: Estimating price changes with duration and convexity

For the bond in our previous examples, calculate the money duration and money convexity of a \$10 million par position in the bond and estimate the new price of the bond for a 50 basis point decrease in yield. Recall that the modified duration of the bond is 3.50 and its convexity is 16.9.

Answer:

The market value of the position is $0.8659138 \times 10,000,000 = \$8,659,138$.

The money duration of the position is $3.50 \times \$8,659,138 = \$30,306,983$.

The money convexity of the bond is $16.9 \times \$8,659,138 = \$146,339,432$.

The duration effect is $-(\$30,306,983 \times -0.005) = \$151,534.92$.

The convexity effect is $\frac{1}{2} \times \$146,339,432 \times (-0.005)^2 = \$1,829.25$.

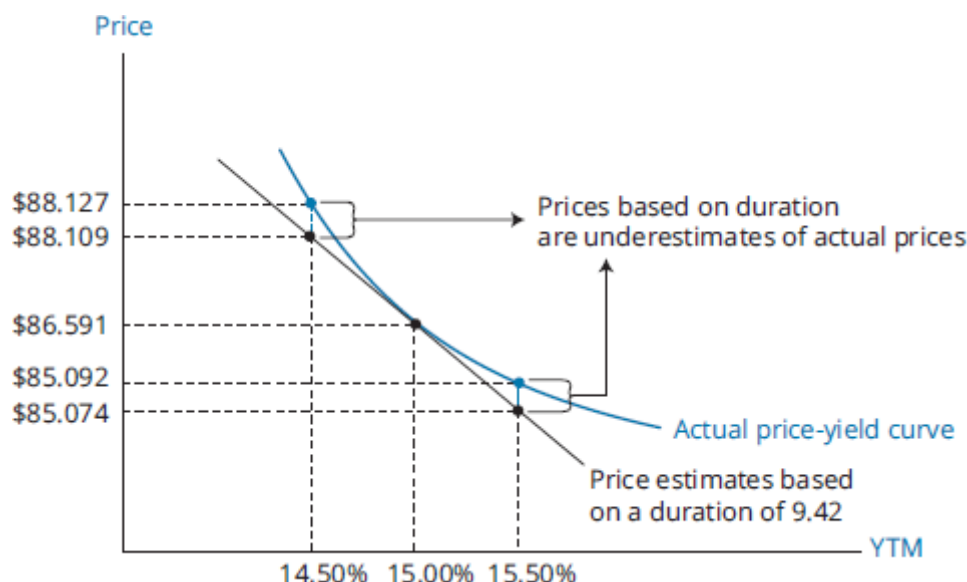
The expected change in bond price is $(\$151,534.92 + \$1,829.25) = \$153,364.17$.

The new value of the bond is estimated to be $\$8,659,138 + \$153,364.17 = \$8,812,502$.

This is consistent with the estimate for new price of 88.125 in the previous example. Both methods are doing the same thing in a slightly different way.

The convexity adjustment to the price change is the same for either an increase or a decrease in yield. As illustrated in Figure 58.2, the duration-only based estimate of the increase in price resulting from a decrease in yield is too low for a bond with positive convexity, and it is improved by a positive adjustment for convexity. The duration-only based estimate of the decrease in price resulting from an increase in yield is larger than the actual decrease, so it is also improved by a positive adjustment for convexity.

Figure 58.2: Duration-Based Price Estimates vs. Actual Bond Prices



LOS 58.c: Calculate portfolio duration and convexity and explain the limitations of these measures.

There are two approaches to estimating the duration and convexity of a portfolio. The first is to base a single duration and convexity calculation on the portfolio's aggregate cash flows across all bonds. The second approach is to calculate the duration and convexity of each bond in the portfolio, then take a weighted average of these based on the weight of the bond in the portfolio.

The first approach is theoretically correct; however, the second approach is typically used in practice because it is often easier to apply. The second approach can be formulated as follows:

$$\text{portfolio duration} = W_1 D_1 + W_2 D_2 + \dots + W_N D_N$$

where:

W_i = full price of bond i divided by the total value of the portfolio

D_i = duration of bond i

N = number of bonds in the portfolio

One limitation of this approach is that it assumes the YTM of every bond (of any maturity) in the portfolio must change by the same amount. Only under this assumption of a **parallel shift** in the yield curve will portfolio duration and convexity calculated with this approach produce the percentage change in portfolio value per 1% change in YTM.

Changes in the yield curve are rarely a simple parallel shift. Changes in shape, such as a steepening or twist, are common.



MODULE QUIZ 58.1

1. The annualized convexity of a 2-year, annual-pay, 10% coupon bond trading at par is *closest* to:
 - A. 1.65.
 - B. 4.66.
 - C. 4.96.
2. A bond is trading at a price of 104.4518. If its yield increases by 10 basis points, its price will decrease to 103.9954. If its yield decreases by 10 basis points, its price will increase to 104.9108. The approximate convexity of this bond is *closest* to:
 - A. 12.45.
 - B. 24.89.
 - C. 49.78.
3. The annualized convexity of a 2-year, semiannual-pay, 10% coupon bond trading at par is *closest* to:
 - A. 4.12.
 - B. 4.66.
 - C. 16.47.
4. A bond has a convexity of 114.6. The convexity effect, if the yield decreases by 110 basis points, is *closest* to:
 - A. -1.673%.
 - B. +0.693%.

- C. +1.673%.
5. Portfolio duration based on weighted average durations of constituents of the portfolio assumes:
- A. yields change uniformly across all maturities.
 - B. the portfolio does not include bonds with embedded options.
 - C. the portfolio's internal rate of return is equal to its cash flow yield.

KEY CONCEPTS

LOS 58.a

Convexity refers to the curvature of a bond's price-yield relationship.

The convexity of a single cash flow at period t is given by the following:

$$\text{convexity of cash flow at period } t = \frac{t \times (t + 1)}{(1 + r)^2}$$

where:

t = period at which the cash flow occurs

r = periodic yield of the bond (YTM/periodicity)

The convexity of a coupon-paying bond is the weighted average convexity of its cash flows.

To annualize convexity for non-annual coupons, divide by periodicity squared.

Convexity can be approximated using the following formula:

$$\text{approximate convexity} = \frac{V_- + V_+ - 2V_0}{(\Delta\text{YTM})^2 V_0}$$

where:

V_- = price of the bond if YTM is decreased by ΔYTM

V_+ = price of the bond if the YTM is increased by ΔYTM

V_0 = current price of the bond

LOS 58.b

Given values for approximate annual modified duration and approximate annual convexity, the percentage change in the full price of a bond can be estimated as follows:

$$\begin{aligned} \%\Delta \text{ full bond price} = & -\text{annual modified duration } (\Delta\text{YTM}) \\ & + \frac{1}{2} \text{ annual convexity } (\Delta\text{YTM})^2 \end{aligned}$$

Money convexity is stated in currency units and is sometimes expressed per 100 of bond value:

$$\text{money convexity} = \text{annual convexity} \times \text{full price of bond position}$$

Using money duration and money convexity to directly estimate the change in price of a bond, this is the equation:

$$\begin{aligned} \text{change in full price of bond} = & -(\text{MoneyDur} \times \Delta\text{YTM}) \\ & + \left(\frac{1}{2} \times \text{MoneyCon} \times \Delta\text{YTM}^2 \right) \end{aligned}$$

LOS 58.c

There are two methods for calculating portfolio duration and convexity:

1. Calculate a single duration and convexity measure based on the aggregate cash flows of the bond portfolio.
2. Calculate the weighted average of durations of bonds in the portfolio. This method is used most often in practice, but it assumes a parallel shift of the yield curve.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 58.1

1. **B** Because the bond is trading at par, its yield is equal to its coupon of 10%. The convexity of the first cash flow at Time 1 is $(1 \times 2) / (1.10)^2 = 1.653$. The convexity of the second cash flow is $(2 \times 3) / (1.10)^2 = 4.959$. The weights for each cash flow time are calculated as follows:

					<u>Convexity</u>
$C_1 = 10$	$PV_1 = 10 / 1.10$	$= 9.091$	$W_1 = 9.091 / 100$	$= 0.09091$	1.653
$C_2 = 110$	$PV_2 = 110 / 1.10^2$	$= \underline{90.909}$	$W_2 = 90.909 / 100$	$= 0.90909$	4.959
		100			

The convexity of the bond is $0.09091(1.653) + 0.90909(4.959) = 4.66$. (LOS 58.a)

2. **B** The approximate convexity of the bond is calculated as follows:

$$\text{approximate convexity} = \frac{V_- + V_+ - 2V_0}{(\Delta YTM)^2 V_0}$$

where:

V_- = price of the bond if YTM is decreased by ΔYTM

V_+ = price of the bond if the YTM is increased by ΔYTM

V_0 = current price of the bond

$$\begin{aligned} \text{In this case, approximate convexity} &= \frac{104.9108 + 103.9954 - 2(104.4518)}{0.001^2 \times 104.4518} \\ &= 24.89. \end{aligned}$$

(LOS 58.a)

3. **A** Because the bond is trading at par, its yield is equal to its coupon of 10%. Note that this is a semiannual coupon bond; hence, it has coupons of \$5 every six months and has a periodic six-month return of 5%. The weights for each cash flow time are calculated as follows:

					<u>Convexity</u>
$C_1 = 5$	$PV_1 = 5 / 1.05$	$= 4.762$	$W_1 = 4.762 / 100$	$= 0.0476$	1.8141
$C_2 = 5$	$PV_2 = 5 / 1.05^2$	$= 4.535$	$W_2 = 4.535 / 100$	$= 0.0454$	5.4422
$C_3 = 5$	$PV_3 = 5 / 1.05^3$	$= 4.319$	$W_3 = 4.319 / 100$	$= 0.0432$	10.8844
$C_4 = 105$	$PV_4 = 105 / 1.05^4$	$= \underline{86.384}$	$W_4 = 86.384 / 100$	$= 0.8638$	18.1406
		100			

The convexity of the first cash flow at Time 1 is $(1 \times 2) / (1.05)^2 = 1.8141$.

The convexity of the second cash flow is $(2 \times 3) / (1.05)^2 = 5.4422$.

The convexity of the third cash flow is $(3 \times 4) / (1.05)^2 = 10.8844$.

The convexity of the fourth cash flow is $(4 \times 5) / (1.05)^2 = 18.1406$.

The convexity of the bond is $0.0476(1.8141) + 0.0454(5.4422) + 0.0432(10.8844) + 0.8638(18.1406) = 16.47$.

The annualized convexity is calculated by dividing convexity by the periodicity of the bond (2) squared.

Hence, annualized convexity = $16.47 / 2^2 = 4.12$.

(LOS 58.a)

4. **B** The convexity effect = $1/2 \times \text{convexity} \times (\Delta\text{YTM})^2 = (0.5)(114.6)(-0.011)^2 = 0.00693 = 0.693\%$.

(LOS 58.b)

5. **A** Portfolio duration is limited as a measure of interest rate risk because it assumes parallel shifts in the yield curve; that is, the discount rate at each maturity changes by the same amount. (LOS 58.c)

READING 59

CURVE-BASED AND EMPIRICAL FIXED-INCOME RISK MEASURES

MODULE 59.1: CURVE-BASED AND EMPIRICAL FIXED-INCOME RISK MEASURES



Video covering
this content is
available online.

LOS 59.a: Explain why effective duration and effective convexity are the most appropriate measures of interest rate risk for bonds with embedded options.

LOS 59.b: Calculate the percentage price change of a bond for a specified change in benchmark yield, given the bond's effective duration and convexity.

So far, all of our duration measures have been calculated using the YTM and prices of straight (option-free) bonds. This is straightforward because both the future cash flows and their timing are known. This is not the case with bonds that have embedded options, such as callable bonds, putable bonds, or a mortgage-backed security (MBS). Embedded options may bring about the early termination of a bond, either at the choice of the investor (for a putable bond) or at the choice of the issuer or underlying borrowers (for callable bonds and MBSs).



PROFESSOR'S NOTE

Recall that a callable bond gives the issuer the right to buy the bond back before maturity. As such, it is equivalent to a straight (option-free) bond and a short call option position. A putable bond gives the investor the right to sell the bond back to the issuer before maturity. As such, it is equivalent to a straight (option-free) bond and a long put option position.

MBSs are similar to callable bonds because mortgage borrowers have the right to prepay their loans. We will explain MBSs in detail in our reading on Mortgage-Backed Security Instrument and Market Features.

The fact that bonds with embedded options have uncertain future cash flows and redemption dates means they do not have a single well-defined yield. The yield of the bond will depend on whether the option embedded in the bond is exercised (recall that we can calculate a yield to maturity and a yield to each call date for a callable bond).

Thus, analyzing interest rate risk for bonds with embedded options is based on shifts in the *benchmark curve* (e.g., government par rates), rather than changes in the bond's own

yield. This measure of price sensitivity is referred to as **effective duration** (EffDur).

Calculating effective duration is the same as calculating approximate modified duration, but we replace the change in YTM with ΔCurve , the change in the benchmark yield curve (used with a bond pricing model to generate V_- and V_+). The formula for calculating effective duration is as follows:

$$\text{effective duration} = \frac{V_- - V_+}{2V_0 \Delta\text{Curve}}$$

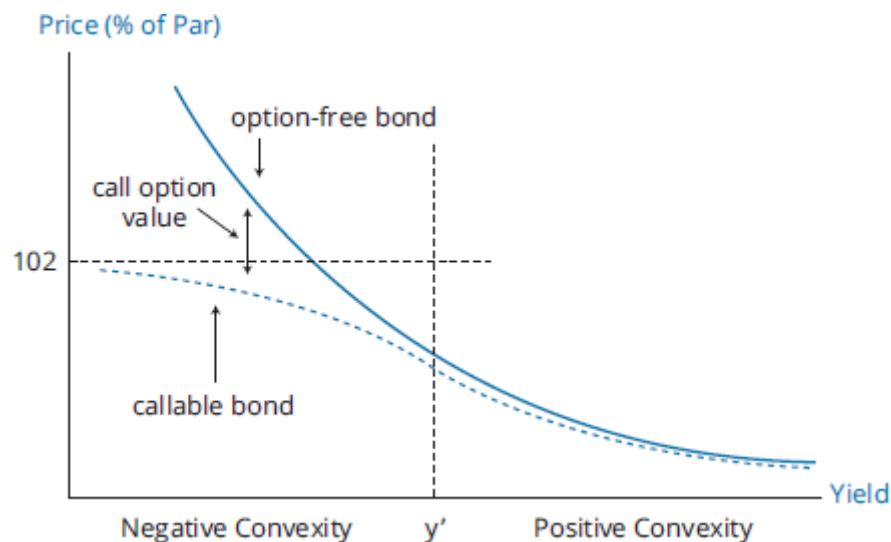
Another difference between effective duration and the methods we have discussed so far is that effective duration separates the effects of changes in benchmark yields from changes in the spread for credit and liquidity risk. Modified duration makes no distinction between changes in the benchmark yield and changes in the spread. Effective duration reflects only the sensitivity of the bond's value to changes in the benchmark yield curve and assumes all else (including spreads) remains the same.

When calculating the convexity of bonds with embedded options, we use an analogous measure, **effective convexity** (EffCon), which is once again based on changes in the benchmark curve, rather than on changes in the bond's YTM:

$$\text{effective convexity} = \frac{V_- + V_+ - 2V_0}{(\Delta\text{Curve})^2 V_0}$$

While the convexity of any option-free bond is positive, a callable bond can exhibit **negative convexity**. This is because at low yields, the call option becomes more valuable, and the call price puts an effective limit on increases in bond value, as shown in Figure 59.1. For a bond with negative convexity, the price increase that results from a decrease in YTM is *smaller* than the price decrease that results from an equal-sized increase in YTM. This means the duration of a callable bond is less than that of an equivalent option-free bond at low yields.

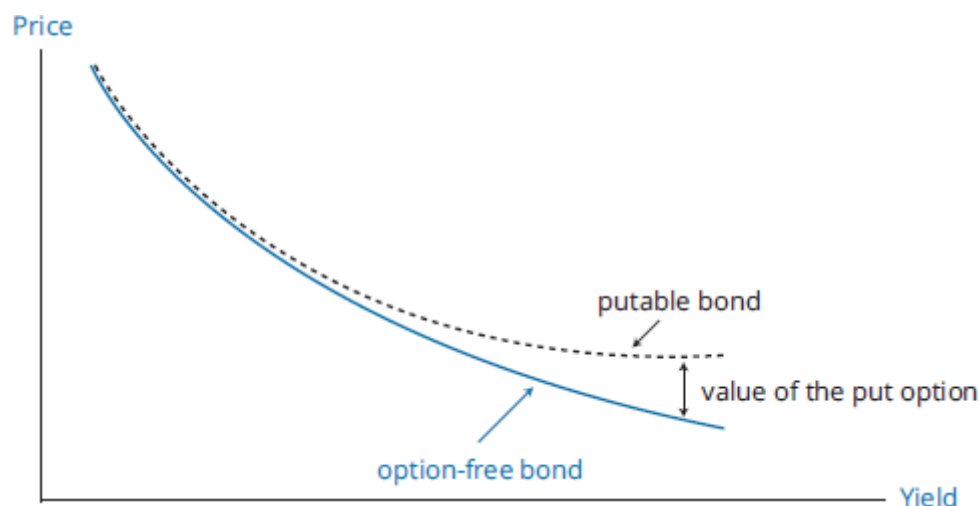
Figure 59.1: Price-Yield Function of Callable vs. Option-Free Bond With a Call Price of 102



A puttable bond always has positive convexity. In Figure 59.2, we illustrate the price-yield relation for a puttable bond. At higher yields, the put becomes more valuable, so

that the value of the puttable bond decreases less than that of an option-free bond as yield increases. This means the duration of a puttable bond is less than that of an equivalent option-free bond at high yields.

Figure 59.2: Comparing the Price-Yield Curves for Option-Free and Puttable Bonds



For an option-free bond, small differences can be observed between modified duration (with respect to ΔYTM) and effective duration (with respect to $\Delta Curve$). This may be surprising, because it seems natural to assume that any shift in government par yield curve would automatically flow through into a similar change in the yields of risky bonds. However, in a nonflat yield curve environment, a shift in the benchmark par yield curve will generate a *nonparallel* shift in the government spot curve. If we assume that credit spreads above government spot rates remain the same, this will mean the change in the risky bond's yield will be slightly different to the original shift in government par yield curves, causing ModDur to be slightly different to EffDur.



PROFESSOR'S NOTE

Recall that bond yields are a weighted average of the spot rates that apply to the individual cash flows of the bond. When we shift the government par yield curve to calculate EffDur, we must also change the government spot rate curve that generates these yields. Not all spot rates have an equal weight in calculating a particular par yield; hence, spot rates change by different amounts (the spot curve undergoes a nonparallel shift). This nonparallel shift in spots means that the yields of risky bonds, assumed to offer a constant spread above government spot rates, may not move exactly in line with the original $\Delta Curve$ shift. This is a fairly technical point, so for the exam just understand that for option-free bonds, ModDur and EffDur are not exactly the same for a given $\Delta Curve$ unless the yield curve is flat.

We can estimate the expected price change for a bond with respect to an expected change in the yield curve using EffDur and EffCon in the same way we use modified duration and convexity with respect to ΔYTM :

$$\text{change in full bond price} = -\text{EffDur} \times (\Delta \text{Curve}) + \frac{1}{2} \times \text{EffCon} \times (\Delta \text{Curve})^2$$

Unlike modified duration and convexity, effective duration and convexity do not necessarily provide better estimates of bond prices for smaller changes in yield. This is because for bonds with embedded options, considerations other than the level of government rates determine whether the option is likely to be exercised (e.g., the level of credit spreads on a corporate bond or the amount of principal outstanding on a mortgage).

LOS 59.c: Define key rate duration and describe its use to measure price sensitivity of fixed-income instruments to benchmark yield curve changes.

Recall that effective duration is an adequate measure of bond price risk only for parallel shifts in the benchmark yield curve. The impact of nonparallel shifts can be measured using a concept known as **key rate duration**. A key rate duration, also known as a **partial duration**, is defined as the sensitivity of the value of a bond or portfolio to changes in the benchmark yield for a *specific* maturity, holding other yields constant. The sum of a bond's key rate durations equals its effective duration.

Key rate duration is particularly useful for measuring **shaping risk**, defined as the effect of a nonparallel shift in the yield curve on a bond portfolio. We can use the key rate duration for each maturity to compute the effect on the portfolio of a yield change at that maturity. The effect on the overall portfolio is the sum of these individual effects.

The key rate duration of a cash flow in a portfolio is the cash flow's modified duration multiplied by its weight in the portfolio. This key rate duration can then be used to assess the impact of a nonparallel shift in the yield curve, as demonstrated in the following example.

EXAMPLE: Key rate duration

A portfolio has equally weighted investments in a 5-year zero-coupon bond yielding 5% and a 10-year bond yielding 6%. Yields are quoted on an annual coupon basis. What would be the performance of the portfolio if 5-year yields increase by 50 basis points and 10-year yields decrease by 25 basis points?

Answer:

Recall that modified duration is equal to Macaulay duration divided by $(1 + \text{periodic yield})$. Also recall that the Macaulay duration of a single cash flow is equal to its maturity.

For the 5-year cash flow: $\text{ModDur} = 5 / (1.05) = 4.762$

The 5-year key rate duration is $\text{ModDur} \times \text{weight in portfolio} = 4.762 \times 0.5 = 2.381$.

The impact of a 50 bp increase in the 5-year yield is, therefore, $-2.381 \times 0.0050 = -0.0119$, or -1.19% .

For the 10-year cash flow: $\text{ModDur} = 10 / (1.06) = 9.434$

The 10-year key rate duration is $\text{ModDur} \times \text{weight in portfolio} = 9.434 \times 0.5 = 4.717$.

The impact of a 25 bp decrease in the 10-year yield is, therefore, $-4.717 \times -0.0025 = 0.0118$, or 1.18%.

Overall, the portfolio value will change by $-1.19\% + 1.18\% = -0.01\%$. The portfolio value is expected to remain roughly unchanged in response to this nonparallel shift in yields.

LOS 59.d: Describe the difference between empirical duration and analytical duration.

The duration measures we have introduced so far, based on mathematical analysis, are often referred to as **analytical durations**. A different approach is to estimate **empirical durations** using the actual observed historical relationship between benchmark yield changes and bond price changes.

When we estimate corporate bond durations based on a shift in the benchmark (government) yield curve, we implicitly assume that the credit spread for the corporate bond remains unchanged (i.e., changes in the benchmark yield curve and a bond's yield spread are uncorrelated). When this assumption is not justified, estimates of empirical duration, based on the actual relationship between changes in the benchmark yield curve and bond values, may be more appropriate.

An example of such a situation is an increase in market uncertainty during which investor demand shifts sharply toward bonds with low credit risk (a "flight to quality"). Yields on government bonds decrease, but credit spreads increase at the same time. As a result, government bond prices increase, but corporate bond prices increase by less or possibly even fall. For a corporate bond portfolio, an estimate of empirical duration that accounts for this effect would be lower (i.e., less price response to a decrease in benchmark yields) than an estimate of analytical duration would indicate. An analytical estimate of the duration of a portfolio consisting primarily of government debt securities, in this case, would still be appropriate, while an empirically derived estimate of duration would be more appropriate for a portfolio comprising corporate bonds (risky credits).



MODULE QUIZ 59.1

1. Effective duration is more appropriate than modified duration for estimating interest rate risk for bonds with embedded options because these bonds:
 - A. tend to have greater credit risk than option-free bonds.
 - B. exhibit high convexity that makes modified duration less accurate.
 - C. have uncertain cash flows that depend on the path of interest rate changes.
2. When an embedded option has significant value, relative to an equivalent option-free bond, the effective duration of a bond with an embedded option will *most likely* be:
 - A. lower.
 - B. the same.
 - C. higher.
3. Which of the following bonds is *most likely* to exhibit negative convexity?
 - A. Callable bonds in a low-yield environment.
 - B. Callable bonds in a high-yield environment.

- C. Puttable bonds in a high-yield environment.
4. A bond portfolio manager who wants to estimate the sensitivity of the portfolio's value to changes in the 5-year yield only should use a(n):
- key rate duration.
 - Macaulay duration.
 - effective duration.
5. Assume that a bond has an effective duration of 10.5 and a convexity of 97.3. Using both of these measures, the estimated percentage change in price for this bond, in response to a decline in the yield curve of 200 basis points, is *closest* to:
- 19.05%.
 - 22.95%.
 - 24.89%.

KEY CONCEPTS

LOS 59.a

Because bonds with embedded options have uncertain cash flows, they do not have a single well-defined yield. Therefore, effective duration and effective convexity must be calculated with respect to shifts in the benchmark curve rather than the bond's yield for bonds with embedded options.

Effective duration is a linear estimate of the percentage change in a bond's price that would result from a 1% change in the benchmark yield curve:

$$\text{effective duration} = \frac{V_- - V_+}{2V_0 \Delta \text{Curve}}$$

$$\text{effective convexity} = \frac{V_- + V_+ - 2V_0}{(\Delta \text{Curve})^2 V_0}$$

Callable bonds and MBS may exhibit negative convexity at low yields.

LOS 59.b

The expected price change for a bond with respect to an expected ΔCurve is estimated as follows:

$$\text{change in full bond price} = -\text{EffDur} \times (\Delta \text{Curve}) + \frac{1}{2} \times \text{EffCon} \times (\Delta \text{Curve})^2$$

LOS 59.c

Key rate duration is a measure of the price sensitivity of a bond or a bond portfolio to a change in yield for a specific maturity while other yields remain the same. Key rate durations of a bond or portfolio can be used to estimate price sensitivity to changes in the shape of the yield curve.

LOS 59.d

Macaulay, modified, and effective duration are examples of analytical duration. Empirical duration is estimated from historical data using models. Empirical duration may be lower than analytical duration in interest rate environments where the

assumptions underlying analytical duration may not hold, such as for credit-risky bonds in a flight-to-quality scenario.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 59.1

1. **C** Because bonds with embedded options have cash flows that are uncertain and depend on future interest rates, effective duration must be used. (LOS 59.a)
2. **A** Embedded options decrease the duration of the bond relative to an equivalent option-free bond when the option has significant value. (LOS 59.b)
3. **A** Negative convexity is exhibited only by callable bonds in a low-yield environment where the call price creates a ceiling on the price of the callable bond, causing the rate of price rises to decrease as yields fall. Putable bonds never exhibit negative convexity. (LOS 59.b)
4. **A** Key rate duration refers to the sensitivity of a bond or portfolio value to a change in one specific maturity yield. (LOS 59.c)
5. **B** Total estimated price change = (duration effect + convexity effect) = $\{-10.5 \times (-0.02)\} + \{1/2 \times 97.3 \times (-0.02)^2\} \times 100 = 21.0\% + 1.95\% = 22.95\%$
(LOS 59.b)

READING 60

CREDIT RISK

MODULE 60.1: CREDIT RISK



Video covering
this content is
available online.

LOS 60.a: Describe credit risk and its components, probability of default and loss given default.

Credit risk is the risk associated with losses to fixed income investors stemming from the failure of a borrower to make payment of interest or principal (referred to as servicing their debt). When a borrower fails to service their debt, they are said to be in **default**.

The key drivers of credit risk are either specific to the borrower (bottom up) or relate to general economic conditions (top down). These are often referred to as the *Cs* of credit analysis.

Bottom-up credit analysis factors are as follows:

- **Capacity.** The borrower's ability to make their debt payments on time.
- **Capital.** Other resources available to the borrower that reduce reliance on debt.
- **Collateral.** The value of assets pledged to provide the lender with security in the event of default.
- **Covenants.** The legal terms and conditions the borrowers and lenders agree to as part of a bond issue.
- **Character.** The borrower's integrity (e.g., management for a corporate bond) and their commitment to make payments under their debt obligations.

Top-down credit analysis factors are as follows:

- **Conditions.** The general economic environment that affects all borrowers' ability to make payments on their debt.
- **Country.** The geopolitical environment, legal system, and political system that apply to the debt.
- **Currency.** Foreign exchange fluctuations and their impact on a borrower's ability to service foreign-denominated debt.

At its core, credit risk stems from the possibility that the borrower's *sources of repayments* will not provide enough cash to service their debt.

The sources of repayment for a debt issuer depend on the nature of the borrower and the specifics of the loan or bond issue. **Secured corporate debt** is backed primarily by the operating cash flows and investments of the business plus cash flows generated from collateral specifically pledged as security for the debt. **Unsecured corporate debt** is only backed by the operating cash flows and investments of the issuer (as well as secondary sources of cash flow such as asset sales, divestitures of subsidiaries, or additional debt/equity issuance). Credit risk for a corporate issuer may come from poor economic and market conditions, increased competition, low profitability, or having excessive debt levels.

Sovereign debt is backed by tax revenue, tariffs, and other fees charged by the government issuer. Secondary sources of cash flow include additional debt issuance and sale of public assets (privatizations). Credit risk for a sovereign issuer may stem from poor economic conditions and political uncertainty, fiscal deficits (tax revenue being less than government spending), and high debt levels relative to the size of the economy.

Credit analysts should distinguish between an issuer being *illiquid*, or unable to raise cash to service debt, and being *insolvent*, where the assets of an issuer fall below the value of its debt. An illiquid issuer may not necessarily be insolvent, but could still default.

When default occurs, clauses written into a bond indenture are important. A **cross-default clause** means that a default on one bond issue causes a default on all issues. A **pari passu clause** means all bonds of a certain type rank equally in the default process. When pari passu and cross-default provisions exist on unsecured debt, a default on one issue implies that all holders of unsecured claims have access to the general assets of the issuer to satisfy their obligations. For secured debtholders, such clauses mean default on any obligation of the issuer will grant access to the general assets of the company and to the assets pledged as collateral for the debt. Only when the value of the pledged assets falls below the amount of pari passu secured debt will a secured bond investor suffer credit losses.

Measuring Credit Risk

Credit risk is measured by assessing the **expected loss** from a debt investment in the event of default:

$$\text{expected loss} = \text{probability of default} \times \text{loss given default}$$

Probability of default is the probability that a borrower fails to pay interest or repay principal when due. The probability is typically expressed on an annualized basis. **Loss given default** is the loss an investor will suffer if the issuer defaults. This can be stated as a monetary amount or as a percentage.

A bond's expected **recovery rate** is the proportion of a claim an investor will recover if the issuer defaults. The proportion an investor will not recover, or one minus the recovery rate, is known as **loss severity**.

A debt investor's **expected exposure** or **exposure at default** is the difference between the amount the investor is owed (principal and accrued interest) and the value

of the collateral available to repay the investor. Loss given default, stated as a percentage, is the product of the expected exposure and the loss severity:

$$\text{LGD}\% = \text{expected exposure} \times (1 - \text{recovery rate})$$



PROFESSOR'S NOTE

Technically, loss given default is defined as a monetary amount, and the loss expressed as a rate is defined as loss severity. However, the Level I curriculum regularly states loss given default as a rate and then uses it as a rate in examples; hence, the definition is slightly loose. Be careful to read what you are given and what you need to calculate. For clarity in our SchweserNotes, when we use loss given default as a rate, we will refer to it as LGD%.

We can use the annualized expected loss (as a percentage) as an estimate of the annualized credit spread over a risk-free benchmark that an investor should demand for facing the credit risk of the investment:

$$\text{credit spread} \approx \text{probability of default} \times \text{LGD}\%$$

If the actual credit spread of the issue is higher than this estimated credit spread, the investor is more than fairly compensated for the credit risk of the investment. If the actual credit spread of a bond is less than this estimated credit spread, investors are not adequately compensated for credit risk and should avoid investing.

EXAMPLE: Expected loss and credit spreads

A bond issuer has a 3% probability of default, and one of its bond issues has a recovery rate of 75%. The bond has a 4% coupon and is currently trading at par. A government security of similar maturity yields 2.5%. Assess whether the credit spread of the bond issue is adequately compensating investors for credit risk.

Answer:

The bond is trading at par, so its coupon of 4% is also its yield. The actual credit spread of the bond is, therefore, $4\% - 2.5\% = 1.5\%$.

The estimated credit spread for the bond is its probability of default times (1 – recovery rate):

$$= 0.03 \times (1 - 0.75)$$

$$= 0.0075, \text{ or } 0.75\%$$

Hence, the bond is providing an actual spread that is double that which is fair, meaning that bond investors are more than adequately compensated for the credit risk of the bond.

To assess the required returns from credit-risky bonds, an analyst will need to estimate the probability of default for the issuer and the loss given default for the bond issue.

Probability of default can be assessed through quantitative metrics relating to capacity to repay. For example, a profitable company with high EBIT margin, a high interest coverage ratio (EBIT/interest), low leverage multiples (e.g., debt/EBITDA), and a high

ratio of cash flow to net debt would be deemed of high credit quality (low probability of default).

Estimates of loss given default depend on whether the bond is secured or unsecured, and the level of seniority of the bond issue in the capital structure of the issuer. More senior, secured debt will have lower losses given default than junior, unsecured debt.

Due to their financial strength, investment grade issuers have lower probability of default than high-yield issuers. However, high-yield issuers often issue secured debt with a secondary source of repayment in the event of default. As a result, high-yield debt can have lower losses given default than unsecured bonds of an investment grade issuer. The greatest risk to the investors in unsecured investment grade debt is not an increase in loss given default, but an increase in the probability of default due to deterioration in the issuer's financial situation.



PROFESSOR'S NOTE

The terms *investment grade* and *high yield* are formally defined in terms of credit ratings, which we will describe next.

LOS 60.b: Describe the uses of ratings from credit rating agencies and their limitations.

Credit rating agencies assign forward-looking ratings to both the issuers of bonds and their debt issues, based on qualitative and quantitative credit risk factors.

Uses of credit ratings include the following:

- Comparing the credit risk of issuers across industries and bond types, and assessing changing credit conditions over time.
- Assessing **credit migration risk**, the risk that a credit rating downgrade will decrease the value of the bonds and potentially trigger other contractual clauses.
- Meeting regulatory, statutory, or contractual requirements.

Figure 60.1 shows ratings scales used by Standard & Poor's, Moody's, and Fitch, three of the major credit rating agencies.

Figure 60.1: Credit Rating Categories

(a) Investment Grade Ratings		(b) Non-Investment Grade Ratings	
Moody's	Standard & Poor's, Fitch	Moody's	Standard & Poor's, Fitch*
Aaa	AAA	Ba1	BB+
Aa1	AA+	Ba2	BB
Aa2	AA	Ba3	BB–
Aa3	AA–	B1	B+
A1	A+	B2	B
A2	A	B3	B–
A3	A–	Caa1	CCC+
Baa1	BBB+	Caa2	CCC
Baa2	BBB	Caa3	CCC–
Baa3	BBB–	Ca	CC
		C	C
		C	D

*Fitch omits the use of +/- symbols for the CCC rating.

Triple A (AAA or Aaa) is the highest rating. Bonds with ratings of Baa3/BBB– or higher are considered **investment grade**. Bonds rated Ba1/BB+ or lower are considered **non-investment grade** and are often called *high-yield bonds* or *junk bonds*.

Bonds in default are rated D by Standard & Poor's and Fitch and are included in Moody's lowest rating category, C.

Relying on ratings from credit rating agencies has some risks:

1. *Credit ratings lag market pricing.* Market prices and credit spreads can change much faster than credit ratings. Additionally, two bonds with the same rating can trade at different yields because credit ratings focus on expected loss, whereas market pricing for distressed debt focuses more on default timing and expected recoveries.
2. *Some risks are difficult to assess.* Risks such as litigation, natural disasters, environmental risks, acquisitions, and equity buybacks using debt are not easily predicted, or captured in credit ratings. Agencies may take different views on the likelihood of such events, leading to **split ratings** where the same debt issue gets assigned different ratings from different agencies.
3. *Rating agencies are not perfect.* Mistakes occur from time to time. Infamously, subprime mortgage securities were assigned much higher ratings than they deserved in the lead-up to the global financial crisis of 2008–2009. Cases of corporate fraud can also lead to companies with high credit ratings suddenly defaulting.

Investors should also do their own due diligence when assessing credit risk and not rely purely on credit ratings. Investors who trade correctly in anticipation of credit rating changes will experience far superior performance to those who trade in reaction to rating changes.

LOS 60.c: Describe macroeconomic, market, and issuer-specific factors that influence the level and volatility of yield spreads.

Credit spread risk is the risk that yield spreads widen due to deteriorating conditions, causing credit-risky bond prices to decrease. This is a primary concern for investment grade bond investors because default is unlikely to occur suddenly. A more realistic concern is that spreads widen and prices fall as credit conditions worsen. Credit spread risk arises from macroeconomic, issuer-specific, and market (trading related) factors.

Macroeconomic Factors

Credit risk changes largely in line with the economic cycle. In times of strong growth and high profits, the probability of default decreases, causing spreads to contract; at times of recession, the probability of default increases, causing spreads to widen.

Credit spreads for high-yield issuers may behave differently than credit spreads for investment grade issuers over a business cycle. Examples of the typical behavior of both are as follows:

- Investment grade issuers have lower yield spreads than high-yield issuers due to their lower expected loss.
- Yield spreads usually increase with maturity because the probability of default increases over longer time frames, giving rise to upward-sloping credit spread curves (plots of credit spread vs. maturity).
 - During economic contractions (recessions), high-yield and investment grade credit curves rise and flatten as the probability of a near-term default increases. The high-yield credit curve may even invert (turn downward sloping) in this stage of the economic cycle.
 - During economic expansions, high-yield and investment grade credit curves fall and steepen as the probability of a near-term default decreases. Credit curves will be lowest and most steep at the peak of the cycle.
- Across issuers, the dispersion of yield spreads for high-yield issuers is higher than for investment grade issuers.
- High-yield spreads tend to fluctuate more than investment grade spreads as economic conditions change. High-yield spreads can widen dramatically in times of crisis as investors sell riskier assets and buy safer ones in a **flight to quality**. Because high-yield issues tend to be less liquid, bid–offer spreads for high-yield debt may widen more than for investment grade debt in times of crisis.



PROFESSOR'S NOTE

Notice that two different kinds of spread are involved here. The yield spread of a bond is the extra return over benchmark risk-free yields. A bid–offer spread is the difference between the prices at which a bond dealer is willing to buy or sell a bond. Pay attention to which spread an exam question is referring, particularly when the component of yield spread relating to liquidity risk is identified through analyzing the size of bid–offer spreads.

It is clear that owning high-yield bonds is riskier than holding investment grade bonds from a credit spread risk perspective because spreads are more volatile for high-yield

issuers. The incentive to do so is the greater yield spread offered by high-yield debt. Other incentives for exposure to this higher credit spread risk include the following:

- *Diversification.* High-yield bond prices have low or even negative correlation with investment grade bonds, so they can diversify a fixed-income portfolio.
- *Capital appreciation.* The larger spread changes for high-yield issues produce larger price gains during economic recoveries compared to investment grade issues.
- *Equity-like returns.* According to some empirical data, high-yield debt offers equity-like returns with lower volatility than equity markets.

Other systematic factors that can drive yield spreads higher include the following:

- Increasing regulations of broker-dealers and market makers in corporate bonds have increased the cost of funding bond positions.
- Funding stresses in markets may increase risk aversion.
- Heavy new issuance of debt into bond markets might not be met by increased demand.

Issuer-Specific Factors

As noted previously, the financial performance of the issuer will have a significant impact on the yield spread level and volatility on their debt. Investors often compare an issuer's yield spread to the average yield spread offered by bonds of a similar credit rating to assess issuer-specific concerns. For an issuer with problems servicing its debt, yield spreads will be wider than the average for the issuer's credit rating.

Market Factors

Market liquidity risk relates to the transaction costs of trading a bond. It can be assessed through analyzing the bid-offer spreads of market makers in a bond. The bid is the price at which investors can sell bonds, while the offer is the price at which investors can buy. A wider bid-offer spread implies higher costs of trading to investors and indicates higher market liquidity risk.

Generally, issuers with more debt outstanding or with higher credit ratings will have more actively traded bonds (and therefore narrower bid-offer spreads) with less market liquidity risk. Market liquidity risk is higher for less actively traded bonds, for issuers with lower credit quality, and for issuers with less debt outstanding in bond markets.

Bid-offer spreads can widen substantially for high-yield issuers during times of financial stress as market liquidity falls dramatically. This can lead to a general increase in risk aversion, which may cause wider bid-offer spreads to spill over to investment grade issuers.

The bid-offer spreads of market makers can be used to isolate the component of the yield spread that is due to liquidity risk. This is done by calculating the yield at the bid and offer prices, and assuming this difference is reflected in the yield spread of the bond

as compensation for liquidity risk. The remaining component of the yield spread is assumed to be related to the credit risk of the issuer.

EXAMPLE: Decomposing yield spread into liquidity and credit spreads

A 10-year bond has an annual coupon of 5% and a bid-offer spread of 99.5/100.5. The benchmark 10-year yield is 3%. Decompose the yield spread into liquidity spread and credit spread.

Answer:

First, we assess the yield spread of the bond based on the midprice quote, which is the average of the bid and offer prices.

$$\text{midprice quote} = (99.5 + 100.5) / 2 = 100$$

At a price of par, a bond's yield is its coupon; hence, the bond is yielding 5% at its midprice quote.

The yield spread of the bond over the benchmark yield is $5\% - 3\% = 2\%$.

Yield at the bid price:

$$N = 10; PV = -99.5; PMT = 5; FV = 100; CPT \rightarrow I/Y = 5.065$$

Yield at the offer price:

$$N = 10; PV = -100.5; PMT = 5; FV = 100; CPT \rightarrow I/Y = 4.935$$

The liquidity spread of the bond is the bid yield minus the offer yield:

$$\text{liquidity spread} = 5.065\% - 4.935\% = 0.13\%$$

The credit component of the yield spread is therefore $2\% - 0.13\% = 1.87\%$.

We have seen that the sensitivity of a bond's price to a change in yield can be estimated using the bond's duration and convexity. We can also use these measures to estimate the price impact of a change in spread, simply by replacing the yield change (ΔYTM) with the spread change ($\Delta Spread$):

$$\text{change in full price of bond} = -\text{ModDur}(\Delta \text{Spread}) + \frac{1}{2} \text{convexity}(\Delta \text{Spread})^2$$



MODULE QUIZ 60.1

1. The two components of expected loss are:
 - A. default risk and yield spread.
 - B. probability of default and loss severity.
 - C. loss severity and yield spread.
2. Expected loss can decrease with an increase in a bond's:
 - A. probability of default.
 - B. loss severity.
 - C. recovery rate.
3. Which of the following factors in credit analysis is *least likely* a top-down factor?
 - A. Capital.

- B. Conditions.
 - C. Currency.
4. Higher credit risk is indicated by a higher:
 - A. cash flow/debt ratio.
 - B. debt/EBITDA ratio.
 - C. EBITDA/interest expense ratio.
 5. Compared to other firms in the same industry, an issuer with a credit rating of AAA is *most likely* to have a lower:
 - A. cash flow/debt ratio.
 - B. operating margin.
 - C. debt/capital ratio.
 6. Credit spreads tend to widen as:
 - A. the credit cycle improves.
 - B. economic conditions worsen.
 - C. broker-dealers become more willing to provide capital.
 7. Compared to shorter-duration bonds, longer-duration bonds:
 - A. have smaller bid-ask spreads.
 - B. are less sensitive to credit spreads.
 - C. have less certainty regarding future creditworthiness.

KEY CONCEPTS

LOS 60.a

Credit risk refers to the possibility that a borrower fails to make the scheduled interest payments or return of principal.

Bottom-up credit analysis factors include capacity, capital, collateral, character, and covenants.

Top-down credit analysis factors include country, conditions, and currency.

Credit risk is measured through expected loss, which is the product of the probability of default and the loss given default.

Expected loss as a percentage is an estimate of the credit spread investors should demand:

$$\text{credit spread} \approx \text{POD} \times \text{LGD}\%$$

Loss given default expressed as a rate is also called loss severity, and equals one minus the recovery rate.

A profitable company with high EBIT margin, high interest coverage ratio, low leverage multiples, and high cash flow to net debt would be deemed of high credit quality with a low probability of default.

More senior, secured debt will have a smaller loss given default than junior, unsecured debt.

LOS 60.b

Credit ratings reflect a debt issuer or debt issue's overall creditworthiness and assess the likelihood that debt will be fully paid back.

Ratings of BBB-/Baa3 and above are classed as investment grade and of lower credit risk. Ratings of BB+/Ba1 and below are deemed to be non-investment grade (high-yield) and of higher credit risk.

Investors should not rely exclusively on credit ratings from rating agencies for these reasons:

- Rating agencies cannot always judge credit risk accurately.
- Firms are subject to risk of unforeseen events that credit ratings do not reflect.
- Market prices of bonds often adjust more rapidly than credit ratings.

LOS 60.c

An issue's yield spread over its benchmark reflects credit risk and liquidity risk.

The level and volatility of yield spreads are affected by the credit and business cycles, availability of capital from broker-dealers, the supply and demand for debt issues, and the financial performance of the bond issuer.

Yield spreads tend to narrow when the credit cycle is improving, the economy is expanding, and financial markets and investor demand for new debt issues are strong. Yield spreads tend to widen when the credit cycle, the economy, and financial markets are weakening, and in periods when the supply of new debt issues is heavy or broker-dealer capital is insufficient for market making.

The liquidity spread can be estimated as the difference between yields of a bond at bid and offer prices. The remainder of the yield spread is considered credit spread.

Given an expected change in spread, duration and convexity can be used to estimate the impact on price:

$$\text{change in full price of bond} = -\text{ModDur}(\Delta\text{Spread}) + \frac{1}{2}\text{convexity}(\Delta\text{Spread})^2$$

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 60.1

1. **B** Expected loss is composed of probability of default and loss severity. Yield spreads reflect the credit risk of a borrower, but they are a reflection of expected loss rather than a component of it. (LOS 60.a)
2. **C** An increase in the recovery rate means that the loss severity has decreased, which decreases expected loss. (LOS 60.a)
3. **A** Top-down credit analysis factors include country, conditions, and currency. Capital is a bottom-up factor relating to the reliance of the issuer on debt and the availability of other sources of funding. (LOS 60.b)

4. **B** A higher debt/EBITDA ratio is sign of higher leverage and higher credit risk. Higher cash flow/debt and EBITDA/interest expense ratios indicate lower credit risk. (LOS 60.a)
5. **C** A low debt/capital ratio is an indicator of low leverage. An issuer rated AAA is likely to have a high operating margin and a high cash flow/debt ratio compared to its industry group. (LOS 60.a)
6. **B** Credit spreads widen as economic conditions worsen. Spreads narrow as the credit cycle improves and as broker-dealers provide more capital to bond markets. (LOS 60.c)
7. **C** Longer-duration bonds usually have longer maturities and carry more uncertainty of future creditworthiness. (LOS 60.c)

READING 61

CREDIT ANALYSIS FOR GOVERNMENT ISSUERS

MODULE 61.1: CREDIT ANALYSIS FOR GOVERNMENT ISSUERS



Video covering
this content is
available online.

LOS 61.a: Explain special considerations when evaluating the credit of sovereign and non-sovereign government debt issuers and issues.

Sovereign Government Debt

The ability of a **sovereign debt** issuer to service its debts comes primarily from its ability to tax economic activity in its jurisdiction. Assessing the credit risk of sovereign issuers requires an analyst to evaluate the factors that provide for stable economic growth with low inflation. These factors can generally be split into five qualitative factors and three quantitative factors.

The following are five qualitative factors in sovereign creditworthiness:

1. *Institutions and policy factors* address whether a government encourages political and economic stability. Economically, this means enforcement of basic legal protections and property rights, a culture of debt repayment, transparency and consistency in data reporting, and policies that encourage business activity. Politically, this means a stable political system and peaceful coexistence with neighboring countries. The willingness of a government to repay debt is also important (unlike with corporate issues) because bondholders usually have no legal recourse if a government refuses to pay its debts (national governments are said to have **sovereign immunity**).
2. *Fiscal flexibility factors* relate to the government's ability to increase tax collection or decrease public spending to ensure debt service payments are made.
3. *Monetary effectiveness factors* relate to the ability of the central bank to vary the money supply and interest rates in a credible manner to encourage stable economic growth. A central bank that is independent from the government is less likely to print money to service government debts, reducing the risk of high inflation and currency weakness.
4. *Economic flexibility factors* relate to growth trends, income per capita, and diversity of sources for economic growth.

5. *External status factors* relate to the standing of a country's currency in international markets. Countries with **reserve currencies**, which are widely held as foreign exchange reserves at central banks across the world, have more ability to issue debt to foreign investors in their domestic currency and maintain larger budget deficits and higher levels of debt. Geopolitical risks relating to international conflict are also a consideration.

The following are three quantitative factors in sovereign creditworthiness:

1. *Fiscal strength* is measured by low debt burden ratios (debt to GDP and debt to revenue) and low interest-to-GDP or interest-to-revenue ratios (which measure debt affordability).
2. *Economic growth and stability* is measured by high real GDP growth, large real economy size, high per-capita GDP, and low volatility of real GDP growth.
3. *External stability* is measured by high foreign exchange reserves to GDP, high foreign exchange reserves to external debt, low long-term external debt to GDP, and low near-term external debt (due in the next 12 months) relative to GDP. Foreign exchange reserves are usually built up by a country that exports more than it imports (has a current account surplus). If exports are concentrated in a specific commodity, sovereign credit risk becomes aligned with the price of that commodity.

Non-Sovereign Government Debt

Issuers of **non-sovereign government debt** include the following:

- *Agencies*. These quasi-government entities are established to carry out a particular government-sponsored role, such as infrastructure development and operation. Backed by law, and with implicit government support, credit ratings for agency debt issues are usually similar to the relevant sovereign debt rating.
- *Government sector banks or financing institutions*. These financial intermediaries are set up with a specific government-sponsored mission, such as issuing green bonds to raise finance for projects designed to mitigate climate change. Similar to agencies, implied government support means that credit ratings are similar to the sovereign entity.
- *Supranational issuers*. Entities such as the World Bank and the Development Bank of Latin America are set up by groups of sovereign governments to carry out projects with varied missions, such as alleviating poverty or encouraging sustainable economic growth. Credit ratings for these issues depend on the implicit support of the governments and global development institutions that sponsor them.
- *Regional governments*. These include provinces, states, and local governments (the debt issues of which are referred to as **municipal bonds** in the United States). Most regional government bonds can be classified as general obligation bonds or revenue bonds. **General obligation (GO) bonds** are unsecured bonds backed by the full faith and credit of the issuing non-sovereign government entity, which is to say they are supported by its taxing power. **Revenue bonds** are issued to finance specific projects, such as airports, toll bridges, hospitals, and power generation facilities.

Unlike sovereigns, regional governments cannot use monetary policy to service their debt and usually must balance their operating budgets. Municipal governments' ability to service their general obligation debt depends ultimately on the local economy (i.e., the tax base). Revenue bonds often have higher credit risk than GO bonds because the project is the sole source of funds to service the debt.

Analysis of revenue bonds uses techniques similar to those for analyzing corporate bonds, with a focus on the cash flows generated by the project and the ability of the issuer to service their debt as measured by the debt-service coverage ratio (revenue after operating costs relative to interest and principal repayments required).



MODULE QUIZ 61.1

1. One key difference between sovereign bonds and municipal bonds is that sovereign issuers:
 - A. can print money.
 - B. have government taxing power.
 - C. are affected by economic conditions.
2. All else equal, a sovereign debt issuer with higher credit quality will *most likely* have a higher:
 - A. debt burden ratio.
 - B. debt affordability ratio.
 - C. foreign exchange reserve ratio.

KEY CONCEPTS

LOS 61.a

The five qualitative factors relevant for assessing sovereign debt are (1) government institutions and policy, (2) fiscal flexibility, (3) monetary effectiveness, (4) economic flexibility, and (5) external status.

The three quantitative factors relevant for assessing sovereign debt are (1) fiscal strength, (2) economic growth and stability, and (3) external stability.

Non-sovereign government debt issuers include agencies, government sector banks, supranational entities, and regional governments. Major types of regional government bonds include general obligation (GO) bonds, backed by tax-raising powers, and revenue bonds, issued to fund a specific project.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 61.1

1. **A** Sovereign entities can print money to repay debt, while municipal borrowers cannot. Both sovereign and municipal entities have taxing powers, and both are affected by economic conditions. (LOS 61.a)

2. **C** A foreign exchange reserve ratio looks at the level of foreign exchange reserves relative to external debt. A high ratio implies higher credit quality on an external stability basis. Debt burden ratios measure the level of sovereign debt relative to GDP or tax revenue. As such, a higher ratio indicates a more indebted nation and therefore lower credit quality due to less fiscal strength. Debt affordability ratios compare interest payments to GDP or revenue. When these ratios are high, this suggests lower credit quality on a fiscal strength basis. (LOS 61.a)

READING 62

CREDIT ANALYSIS FOR CORPORATE ISSUERS

MODULE 62.1: CREDIT ANALYSIS FOR CORPORATE ISSUERS



Video covering this content is available online.

LOS 62.a: Describe the qualitative and quantitative factors used to evaluate a corporate borrower's creditworthiness.

An analyst will use both qualitative and quantitative factors when evaluating the likelihood and impact of default by a corporate bond issuer.

Qualitative Factors

Qualitative factors include an issuer's business model, the degree of competition in its industry, its business risk, and the quality of its corporate governance.

Business model. A corporate issuer with high credit quality will have a business model with stable and predictable cash flows. For longer-term debt issues, a credit analyst should consider both the existing business model and any long-term changes to it that the issuer will need to make to remain competitive. Changes in the business model will increase business risk.

Industry competition. Less intensive competition is favorable for an issuer's credit quality. As with the business model, analysts need to consider any change in the competitive landscape over the long term.

Business risk. High-credit-quality issuers have low risk of unexpected deviations from expected revenues and margins. Business risk can originate from issuer-specific, industry-specific, or external sources.

Corporate governance. An issuer with high credit quality should have sufficient processes in place relating to the fair and legal treatment of debtholders. Specific concerns include debt covenants and accounting policies.

- *Covenants.* For unsecured investment-grade issuers, it is likely that covenants are primarily affirmative, relating to compliance with rules and laws, maintenance of company assets, and paying taxes. Analysts must assess the potential for management to issue additional debt that would dilute the claims of existing debtholders. High-

yield issuers are likely to have negative covenants, restricting the issuer's ability to pay dividends or issue further debt, in addition to affirmative covenants. Here, analysts need to assess the past actions of management for evidence of preferential treatment of equity investors over debt investors, particularly if this led to credit rating downgrades (e.g., a debt-financed stock buyback program).

- *Accounting policies.* While evidence of fraud is an obvious concern, the use of aggressive accounting policies that accelerate revenue recognition, significant use of off-balance-sheet financing, a heavy preference for capitalizing spending rather than immediate expensing, and frequently changing auditors or the chief financial officer are also warning signs that the character of management may hurt the creditworthiness of the issuer.

Quantitative Factors

Quantitative modeling of the future performance of corporate issuers involves estimating future financial statements and cash flows of the issuer to identify key factors driving their probability of default and loss given default, and how these are likely to change over the economic cycle. While investors in unsecured investment-grade debt are likely to be primarily concerned about the probability of default increasing, investors in secured high-yield debt are also concerned about loss given default, due to the higher probability of default for high-yield debt.

Quantitative analysis can be performed on a top-down or bottom-up basis, or both. *Top-down* inputs relate to the macroeconomic cycle, the size of the industry and potential market share, and event risk related to potential external shocks. *Bottom-up* inputs relate to issuer-specific factors driving revenue, costs, balance sheet assets and liabilities, and future cash flows. A *hybrid* approach considers both top-down and bottom-up factors.

All else equal, companies are deemed to be of higher credit quality if the issuer has:

- Strong operating profits and recurring revenues
- Low levels of leverage and less reliance on debt in the capital structure
- High coverage of debt service payments with periodic income
- High levels of liquidity to meet short-term debt payments

LOS 62.b: Calculate and interpret financial ratios used in credit analysis.

Credit analysts calculate ratios to assess the creditworthiness of a company, to find trends over time, and to compare companies to industry averages and peers. Common ratios relate to profitability, coverage, and leverage.

Some key terms that are commonly used in calculating ratios are as follows:

- **Earnings before interest, taxes, depreciation, and amortization (EBITDA).** EBITDA is a commonly used measure that is calculated as operating income plus depreciation and amortization. A drawback to using this measure for credit analysis is that it does not adjust for capital expenditures and changes in working capital,

which are necessary uses of funds for a going concern. Cash needed for these uses is not available to debtholders.

- **Cash flow from operating activities (CFO).** CFO is net cash paid or received in the continuing operations of the business. It is calculated as net income plus noncash charges minus increase in working capital. CFO is disclosed by companies in their cash flow statements.
- **Funds from operations (FFO).** FFO is net income from continuing operations plus depreciation, amortization, deferred taxes, and noncash items. FFO is similar to CFO, except that FFO excludes changes in working capital.
- **Free cash flow (FCF).** FCF is CFO minus fixed asset expenditures plus net interest expense. It represents the discretionary cash flow of a company because it could be paid to providers of financing after all the obligations of the company have been met.
- **Retained cash flow (RCF).** RCF is operating cash flow minus dividends. Analysts may define operating cash as CFO, FFO, or another preferred measure.

Common credit analysis ratios are summarized in Figure 62.1.

Figure 62.1: Financial Ratios for Corporate Credit Analysis

Ratio Type	Ratio Name	Calculation	Indication of Higher Credit Quality
Profitability	EBIT margin	EBIT / revenue	Higher ratio
Coverage	EBIT to interest expense	EBIT / interest expense	Higher ratio
Leverage	Debt to EBITDA	Debt / EBITDA	Lower ratio
Leverage	RCF to net debt	RCF / (debt – cash and marketable securities)	Higher ratio

EXAMPLE: Credit analysis based on ratios

An analyst is assessing the credit quality of York, Inc., and Zale, Inc. Selected financial information appears in the following table.

	York, Inc.	Zale, Inc.
Revenue	\$2,200,000	\$11,000,000
Depreciation and amortization	\$220,000	\$900,000
Earnings before interest and taxes	\$550,000	\$2,250,000
Cash flow from operations	\$300,000	\$850,000
Interest expense	\$40,000	\$160,000
Total debt	\$1,900,000	\$2,700,000
Cash and marketable securities	\$500,000	\$1,000,000
Dividends	\$30,000	\$200,000

Using profitability, coverage, and leverage ratios, explain how the analyst should evaluate the relative creditworthiness of York and Zale.

Answer:

Profitability ratios based on these data are as follows:

EBIT margin = EBIT / revenue:

$$\text{York: } \$550,000 / \$2,200,000 = 25\%$$

$$\text{Zale: } \$2,250,000 / \$11,000,000 = 20.5\%$$

York has higher EBIT margin, so from a profitability perspective, it has higher credit quality than Zale.

Coverage ratios based on these data are as follows:

EBIT / interest:

$$\text{York: } \$550,000 / \$40,000 = 13.8\times$$

$$\text{Zale: } \$2,250,000 / \$160,000 = 14.1\times$$

York has a slightly lower interest coverage ratio than Zale, suggesting slightly lower credit quality. The difference is small, though, and both companies can comfortably meet their interest payments from EBIT.

Leverage ratios based on these data are as follows:

Debt / EBITDA:

$$\text{York: } \$1,900,000 / (\$550,000 + \$220,000) = 2.5\times$$

$$\text{Zale: } \$2,700,000 / (\$2,250,000 + \$900,000) = 0.9\times$$

York is more leveraged than Zale, suggesting a lower credit quality for York.

RCF / net debt:

$$\text{York: } (\$300,000 - \$30,000) / (\$1,900,000 - \$500,000) = 19\%$$

$$\text{Zale: } (\$850,000 - \$200,000) / (\$2,700,000 - \$1,000,000) = 38\%$$

Zale's retained cash flow (using CFO as operating cash) relative to its net debt level is greater than York. This, too, suggests a lower credit quality for York relative to Zale.

Overall, the picture is mixed. Relative to Zale, York is a more profitable company, suggesting a higher credit rating—though it relies more on debt in its capital structure, which suggests a lower credit rating. Both companies report similarly strong coverage ratios.

LOS 62.c: Describe the seniority rankings of debt, secured versus unsecured debt and the priority of claims in bankruptcy, and their impact on credit ratings.

Each different type of bond of a particular issuer is ranked according to a **priority of claims** in the event of a default. A bond's position in the priority of claims to the issuer's assets and cash flows is referred to as its **seniority ranking**.

Debt can be either **secured debt** or **unsecured debt**. Secured debt is backed by collateral, which can be sold to recover funds for bond investors in the event of default

by the issuer. Unsecured debt represents a general claim to the issuer's assets and cash flows. Secured debt has a higher priority of claims than unsecured debt.

Secured debt can be further distinguished as *first lien* (where a specific asset is pledged) or *first mortgage* (where a specific property is pledged), *senior secured (second lien)*, or *junior secured* debt. Unsecured debt is further divided into *senior*, *junior*, and *subordinated* gradations. The highest rank of unsecured debt is senior unsecured. Subordinated debt ranks below other unsecured debt.

The general seniority rankings for debt repayment priority are the following:

1. First lien/mortgage
2. Senior secured (second lien)
3. Junior secured
4. Senior unsecured
5. Senior subordinated
6. Subordinated
7. Junior subordinated

All debt within the same category is said to rank **pari passu**, or have the same priority of claims. All senior secured debtholders, for example, are treated alike in a corporate bankruptcy. Any senior secured claims that are not met by the value of collateral upon which they are secured will automatically rank *pari passu* with senior unsecured claims in the bankruptcy process.

Recovery rates are the highest for debt with the highest priority of claims and decrease with each lower rank of seniority. The lower the seniority ranking of a bond, the higher its credit risk.

In a default or reorganization, senior lenders have claims on the assets before junior lenders and equity holders. A strict priority of claims, however, is not always applied in practice. Although in theory the priority of claims is absolute, in many cases, lower-priority debtholders (and even equity investors) may get paid even if senior debtholders are not paid in full in order to accelerate the bankruptcy process. Bankruptcies can be costly and take a long time to settle, during which the value of a company's assets could deteriorate due to loss of customers and key employees, while legal expenses mount.

Credit rating agencies rate both the issuer (i.e., the company issuing the bonds) and the debt issues, or the bonds themselves. Issuer credit ratings are called **corporate family ratings (CFRs)**, and are typically based on their senior unsecured debt, while issue-specific ratings are called **corporate credit ratings (CCRs)**.

The ratings of a firm's individual bonds can differ from its corporate (issuer) rating. The seniority and covenants (including collateral pledged) of an individual bond issue are the primary determinants of differences between an issuer's rating and the ratings of its individual bond issues. The assignment of individual issue ratings that are higher or lower than that of the issuer is referred to as **notching**.

Another example of a factor that rating agencies consider when notching an issue credit rating is **structural subordination**. In a holding company structure, both the parent company and the subsidiaries may have outstanding debt. A subsidiary's debt covenants may restrict the transfer of cash or assets "upstream" to the parent company before the subsidiary's debt is serviced. In such a case, even though the parent company's bonds are not junior to the subsidiary's bonds, the subsidiary's bonds have a priority claim to the subsidiary's cash flows. Thus, the parent company's bonds are effectively subordinated to the subsidiary's bonds with respect to the subsidiary's cash flows.

Notching is less common for highly rated issuers than for lower-rated issuers. For firms with high overall credit ratings, differences in expected recovery rates among a firm's individual bonds are less important, so their bonds might not be notched at all. For firms with higher probabilities of default (lower ratings), differences in expected recovery rates among a firm's bonds are more significant. For this reason, notching is more likely for issues with lower creditworthiness in general.



MODULE QUIZ 62.1

1. Which of the following scenarios is *most likely* to occur as a result of a corporate issuer moving from issuing unsecured debt to secured debt?
 - A. The issuer is able to access debt markets that were previously unavailable to them.
 - B. The market yield on the debt will increase.
 - C. The issuer needs to include fewer covenants in their bond indentures.
2. The absolute priority of claims in a bankruptcy might be violated because:
 - A. of the pari passu principle.
 - B. creditors negotiate a different outcome.
 - C. available funds must be distributed equally among creditors.
3. Notching is *best* described as a difference between a(n):
 - A. issuer credit rating and an issue credit rating.
 - B. company credit rating and an industry average credit rating.
 - C. investment-grade credit rating and a non-investment-grade credit rating.
4. Higher credit risk is indicated by a higher:
 - A. RCF/net debt ratio.
 - B. debt/EBITDA ratio.
 - C. EBIT/interest expense ratio.

KEY CONCEPTS

LOS 62.a

Qualitative factors that imply a corporate issuer has high creditworthiness include having a stable business model, being in an industry with low levels of competition, having low business risk, and having adequate corporate governance.

Quantitative factors that imply a corporate issuer has a high creditworthiness include strong profitability and recurring revenues, low leverage, high coverage ratios, and high levels of liquidity.

LOS 62.b

Credit analysts use profitability, leverage, and coverage ratios to assess debt issuers' capacity:

- Profitability can be measured by EBIT margin (EBIT / revenue).
- Coverage can be measured by EBIT to interest expense.
- Leverage ratios include debt to EBITDA and retained cash flow to net debt.

LOS 62.c

Corporate debt is ranked by seniority or priority of claims. Secured debt is a direct claim on specific firm assets and has priority over unsecured debt. Secured or unsecured debt may be further ranked as senior or subordinated. Priority of claims may be summarized as follows:

1. First lien/mortgage
2. Senior secured (second lien)
3. Junior secured
4. Senior unsecured
5. Senior subordinated
6. Subordinated
7. Junior subordinated

Issuer credit ratings, or corporate family ratings (CFRs), reflect a debt issuer's overall creditworthiness and typically apply to a firm's senior unsecured debt.

Issue credit ratings, or corporate credit ratings, reflect the credit risk of a specific debt issue. Notching refers to adjusting an issue credit rating upward or downward from the issuer credit rating.

For a specific debt issue, secured collateral implies lower credit risk compared to unsecured debt, and higher seniority implies lower credit risk compared to lower seniority.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 62.1

1. **A** By offering secured debt, the issuer may attract investments from investors that would have otherwise considered the unsecured bond for the issuer to be too risky. It is likely that the security will decrease the cost of borrowing for the issuer. It is also likely that secured debt will have more covenants stating the legal rights that lenders have over the collateral pledged as security for the issue. (LOS 62.a)
2. **B** A negotiated bankruptcy settlement does not always follow the absolute priority of claims. (LOS 62.c)

3. **A** Notching refers to the credit rating agency practice of distinguishing between the credit rating of an issuer (generally for its senior unsecured debt) and the credit rating of particular debt issues from that issuer, which may differ from the issuer rating because of provisions such as seniority. (LOS 62.c)
4. **B** A higher debt/EBITDA ratio is a sign of higher leverage and higher credit risk. Higher RCF/net debt and EBITDA/interest expense ratios indicate lower credit risk. (LOS 62.b)

READING 63

FIXED-INCOME SECURITIZATION

MODULE 63.1: FIXED-INCOME SECURITIZATION



Video covering this content is available online.

LOS 63.a: Explain benefits of securitization for issuers, investors, economies, and financial markets.

The **securitization** process involves the following steps:

Step 1: A bank making loans or a corporation extending credit to customers (referred to as the **originator**) creates a pool of debt-based assets.

Step 2: The pool of assets (referred to as the collateral) is sold to a separate legal entity, referred to as a **special purpose entity (SPE)**.

Step 3: The SPE issues fixed-income securities (referred to as asset-backed securities, or ABSs) supported by the cash flows from the collateral (i.e., the borrowers making repayments on their loans). These ABSs are purchased by investors, such as pension funds or fixed-income funds, that deem the risk/reward of the underlying collateral to be attractive.

Securitization connects the owners of capital (investors) with those that require capital (the borrowers in the collateral pool) and removes the originating bank or corporation from the intermediation process.



PROFESSOR'S NOTE

Care must be taken with the use of the word *issuer*, as the Level I curriculum applies it two different ways. In this LOS, the curriculum uses the term *issuer* to refer to the banks and corporations that act as *originators* in the securitization process in Step 1. In a securitization, the actual issuer of the ABS is the special purpose entity in Step 2. If you are asked about the “advantages of securitization to issuers,” you should assume the question is asking you about the advantages to the originator. However, a general reference to the issuer in a securitization should be assumed to refer to the SPE.

Benefits of securitization to issuers (the originating bank/corporation) are as follows:

- *Increased business activity.* By securitizing loans, banks are able to lend more than they could if they had to finance the loans through their own balance sheet (i.e., by

raising more deposits or other liabilities). When a loan portfolio is securitized, the bank receives the proceeds, which can then be used to make more loans.

- *Improved profitability.* The originating bank or corporation can charge fees for originating the initial transaction that creates the collateral (e.g., making the mortgage loan in the first place) and for selling the collateral to the SPE.
- *Lower capital reserves for banks.* By selling loans to the SPE, an originating bank removes credit risk from its balance sheet, thereby reducing the capital reserves that regulators require the bank to hold. This allows the bank to allocate more funds to profitable activities such as lending or other investments.
- *Improved liquidity.* Banks can use securitization to sell illiquid loan portfolios, thereby operating more efficiently from a risk/return perspective.

Benefits of securitization to investors are as follows:

- *Tailored risk and return.* Securitization allows issuers to create securities with risk/return profiles that align with the needs of investors. As an example, an investor with a long investment horizon can invest in a portfolio of long-term mortgage loans at acceptable levels of risk and earn returns higher than those of long-dated Treasuries.
- *Access.* By investing in ABSs, investors are able to access the collateral pool without having the specialized resources and expertise necessary to provide loan origination and loan servicing functions.
- *Liquidity.* ABSs are liquid securities that can be sold to other investors more easily than the underlying collateral, which allows investors to react more quickly to changes in market conditions than they could if they held the loans directly.

Benefits of securitization to economies and financial markets are as follows:

- *Decreased liquidity risk.* ABSs are more liquid than the underlying collateral. Hence, securitization improves liquidity in financial markets.
- *Improved market efficiency.* The greater liquidity of ABSs than the underlying collateral allows investors to set equilibrium prices, which makes the market more efficient (i.e., more reflective of current investor opinion).
- *Lower financing costs for originators.* Securitization can provide a source of finance for originators that is lower than the cost of issuing debt or equity directly to investors.
- *Lower leverage for originators.* Securitization allows originators to grow their business without having to increase debt on their balance sheets.

The risks to investors in ABSs are twofold:

- The cash flows from the collateral to the ABS investors are uncertain and can vary in timing and size (e.g., mortgage borrowers may prepay their mortgages at a rate that is different to what was expected).
- The credit risk of the collateral is passed through to investors in the ABS. Systemic buildup of credit risk in ABSs can create risks for the financial system, as shown by the financial crisis of 2007–2009.

LOS 63.b: Describe securitization, including the parties and the roles they play.

We can illustrate the basic structure of a securitization transaction with this simplified, fictitious example of Fred Motor Company.

Fred Motor Company (Fred) sells most of its cars on retail sales installment contracts (i.e., auto loans). The customers buy the automobiles, and Fred loans the customers the money for the purchase (i.e., Fred *originates* the loans) secured on the autos and receives principal and interest payments on the loans until they mature. Fred is also the *servicer* of the loans (i.e., collects principal and interest payments, sends out delinquency notices, and repossesses and disposes of autos if customers do not make timely payments) through a subsidiary established by Fred to perform financial services.

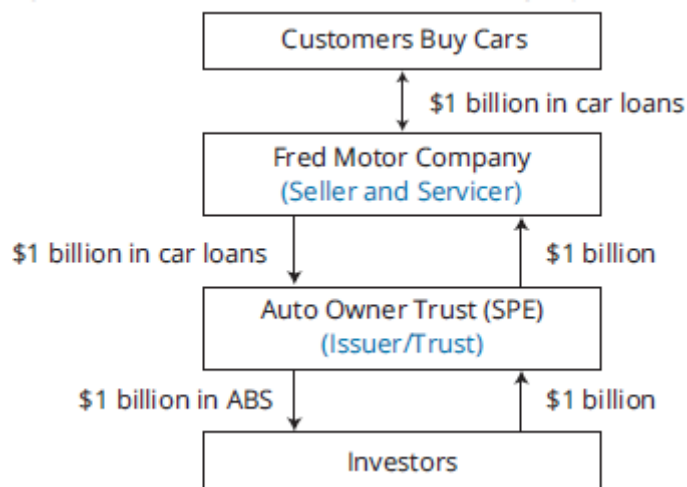
Fred has 50,000 auto loans totaling \$1 billion that it would like to remove from its balance sheet and use the proceeds to make more auto loans. It accomplishes this by selling the loan portfolio to an SPE called Auto Loan Trust for \$1 billion (Fred is called the *seller* or *depositor*). The SPE, which is set up for the specific purpose of buying these auto loans and selling ABSs, is referred to as the *trust* or the *issuer*. The SPE then sells ABSs to investors. The loan portfolio is the collateral supporting the ABS because the cash flows from the loans are the source of the funds to make the promised payments to ABS investors. An SPE is sometimes also called a special purpose company or special purpose vehicle (SPV). The SPE is a separate legal entity from Fred.

Let's review the parties to this transaction and their functions:

- The seller or depositor (Fred) originates the auto loans and sells the portfolio of loans to Auto Loan Trust, the SPE.
- The issuer/trust (Auto Loan Trust) is the SPE that buys the loans from the seller and issues ABSs to investors.
- The servicer (Fred) services the loans.
- In this case, the seller and the servicer are the same entity (Fred Motor Company), but that is not always the case.

The structure of this securitization transaction is illustrated in Figure 63.1.

Figure 63.1: Structure of Fred Motor Company Asset Securitization



Subsequent to the initial transaction, the principal and interest payments on the original loans are allocated to pay servicing fees to the servicer and principal and interest payments to the owners of the ABSs. A trustee is also appointed to oversee the safekeeping of collateral and cash flows due to the ABS investors and provide information to ABS holders (known as a **disinterested trustee** because they have no other interest in the structure).

Because the SPE is a separate legal entity from Fred, a decline in the financial position of Fred does *not* affect the value of the claims of ABS owners to the cash flows from the trust collateral owned by the SPE. This is referred to as the SPE being **bankruptcy remote** from the originator. This also means that the buyers of the ABS have no claim on other assets of Fred—only on the collateral sold to the SPE. If Fred had instead issued corporate bonds to finance more auto loans, the bondholders would be subject to the financial risks of Fred.

Outside of the bond indenture and covenants, important documents involved in the securitization process include the following:

- The purchase agreement, which describes the terms of the purchase of the collateral by the SPE from the seller
- The prospectus, which describes the terms of the securitization with respect to fees made to servicers and other administrators and how cash flows from the collateral are distributed to ABS investors



MODULE QUIZ 63.1

1. Economic benefits of securitization *least likely* include:
 - A. reducing excessive lending by banks.
 - B. reducing funding costs for firms that securitize assets.
 - C. increasing the liquidity of the underlying financial assets.
2. In a securitization, the issuer of asset-backed securities is *best* described as the:
 - A. special purpose entity.
 - B. seller.

KEY CONCEPTS

LOS 63.a

Benefits of securitization to the originators of the collateral include:

- Increased business activity through re-lending the proceeds
- Improved profitability from fees for originating and arranging the sale of collateral
- Lower capital reserves after removing the collateral from their balance sheets
- Increased liquidity of the underlying collateral

Benefits of securitization to ABS investors include:

- Risk and return tailored to the requirements of the investor
- Access to collateral that investors would have difficulty accessing directly
- Greater liquidity than the underlying collateral

Benefits of securitization to economies and financial markets include:

- Decreased liquidity risk, because ABSs can be traded more easily than the underlying collateral
- Improved market efficiency, because increased trading helps markets trade at equilibrium prices
- Lower financing costs and leverage for originators

Risks to ABS investors include uncertain size and timing of cash flows and credit risk of the collateral.

LOS 63.b

Parties to a securitization are a seller of financial assets, an SPE, and a servicer:

- The seller is the firm that is raising funds through the securitization.
- An SPE is an entity independent of the seller. The SPE buys financial assets from the seller and issues ABSs supported by these financial assets.
- The servicer carries out collections and other responsibilities related to the financial assets. The servicer may be the same entity as the seller, but does not have to be.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 63.1

1. **A** Companies that securitize loans they hold as assets receive cash with which they can make additional loans. The primary benefits of securitization to the economy include reducing firms' funding costs and increasing the liquidity of the financial assets that are securitized. (LOS 63.a)

2. **A** ABSs are issued by a special purpose entity (SPE), which is an entity created for that specific purpose. In a securitization, the firm that is securitizing financial assets is described as the seller because it sells the assets to the SPE. The servicer is the entity that deals with collections on the securitized assets. (LOS 63.b)

READING 64

ASSET-BACKED SECURITY (ABS) INSTRUMENT AND MARKET FEATURES

MODULE 64.1: ASSET-BACKED SECURITY (ABS) INSTRUMENT AND MARKET FEATURES



Video covering
this content is
available online.

LOS 64.a: Describe characteristics and risks of covered bonds and how they differ from other asset-backed securities.

Covered bonds are senior debt obligations of financial institutions that are similar to ABSs. However, the underlying assets (the cover pool), while segregated from other assets of the issuer, remain on the balance sheet of the issuing corporation (i.e., no SPE is created). Covered bonds are issued primarily by European, Asian, and Australian banks, and the cover pool typically consists of mortgage loans (though other types of assets can be used).

In the event of issuer default, covered bond investors have the **dual recourse** of claims on both the cover pool and, in contrast to a securitization involving an SPE, claims over other assets of the issuers that have not been pledged as collateral for other debt (referred to as unencumbered assets). To mitigate credit risk faced by covered bond investors, the mortgages that make up the cover pool are subject to upper limits on loan-to-value ratios, and the value of the collateral is usually higher than the face value of covered bonds issued (referred to as overcollateralization). The cover pool is monitored by a third party to ensure adherence to these conditions. These credit enhancements, along with dual recourse, result in covered bonds generally having lower yields than comparable ABSs.

Because the cover pool remains on the balance sheet of the issuer, the issuer will not benefit from any reduction in required capital reserves that would occur under a securitization.

Unlike an ABS, in which the pool of assets is fixed at issuance, a covered bond requires the issuer to replace or augment nonperforming or prepaid assets in the cover pool so that it always provides for the covered bond's promised interest and principal payments. Covered bonds typically are not structured with credit tranching.

Covered bonds may have different provisions in case their issuer defaults. A **hard-bullet covered bond** is in default if the issuer fails to make a scheduled payment, leading to the acceleration of payments to covered bondholders. A **soft-bullet covered bond** may postpone the originally scheduled maturity date by as much as a year, should a payment on the covered bond be missed—effectively postponing default and associated payment acceleration. A **conditional pass-through covered bond** converts to a pass-through bond on the maturity date if any payments remain due, meaning that any payments subsequently recovered on the cover pool are passed through to investors.

LOS 64.b: Describe typical credit enhancement structures used in securitizations.

Internal credit enhancements of ABSs are features of the structure designed to mitigate the credit risk faced by investors due to defaults in the collateral pool. They take three main forms: overcollateralization, excess spread, and subordination (or credit tranching).

Overcollateralization occurs when the value of the collateral exceeds the face value of the ABS. For example, if the value of the collateral is \$600 million and the face value of the ABS issued is \$500 million, then there is \$100 million overcollateralization. The collateral could experience default of up to $100 / 600 = 16.7\%$ of value before investors in the ABS begin to suffer credit losses.

An **excess spread** feature builds up reserves in the ABS structure by earning higher income on the collateral than the coupon promised to ABS investors. This income can then be used to absorb credit losses in the collateral.

With **credit tranching** (also called **subordination**), the ABS is structured with multiple classes of securities (referred to as **tranches**), each with a different priority of claims to the cash flows of the collateral. Tranches ranked as subordinated absorb credit losses first (up to their principal values), thereby providing protection to senior tranches against credit losses. The level of protection for the senior tranche increases with the proportion of subordinated bonds in the structure.

Let's look at an example to illustrate how a senior/subordinated structure redistributes the credit risk compared to a single-class structure. Consider an ABS with the following bond classes:

Tranche Name	Face Value (\$)	Interest Rate
Tranche A senior notes	300,000,000	MRR + 0.5%
Tranche B subordinated notes	80,000,000	MRR + 1.5%
Tranche C subordinated notes	<u>30,000,000</u>	<u>Variable</u>
Total	410,000,000	

Tranche C is first to absorb any losses, because it is the most junior tranche, until losses exceed \$30 million in principal. Being the lowest-ranking tranche, it has a residual claim to any value left over after other, more senior tranches have been paid—and for this reason, it is often referred to as the **equity tranche**. Any losses from default of the underlying assets greater than \$30 million, and up to \$110 million, will be absorbed by

subordinated Tranche B. The senior Tranche A is protected from any credit losses of \$110 million or less, and therefore it will have the highest credit rating and offer the lowest yield of the three bond classes. It is through this credit tranching, and the bankruptcy remote nature of the SPE, that senior tranches of ABSs can receive credit ratings higher than that of the originating company that sells the collateral to the SPE.



PROFESSOR'S NOTE

This subordination structure is not new to us—it's the same idea as the senior/junior debt and equity capital structure of a corporate issuer.

This structure is also called a **waterfall** structure because in liquidation, each subordinated tranche would receive only the “overflow” from the more senior tranche(s) if they are repaid their principal value in full.

As long as a tranche remains outstanding, it will receive its coupon payment. For example, say an investor purchases \$10 million face value of the Tranche B notes. If the MRR is 4%, and if there are no defaults in the underlying collateral, the investor will receive an annual coupon of $\$10,000,000 \times (0.04 + 0.015) = \$550,000$. If defaults in the collateral pool amount to \$50 million, the first \$30 million of losses will be allocated to Tranche C, while the remaining losses of \$20 million will reduce the outstanding face value of the Tranche B note from \$80 million to \$60 million. The coupon earned by the investor in Tranche B notes, in this case, is equal to $\$550,000 \times (60 / 80) = \$412,500$.

LOS 64.c: Describe types and characteristics of non-mortgage asset-backed securities, including the cash flows and risks of each type.

In addition to those backed by mortgages, there are ABSs backed by various types of financial assets including business loans, accounts receivable, or automobile loans. In fact, any asset that generates a future stream of cash flows could be used as collateral for securitization, including music royalties or franchise license payments. Each of these types of ABS has different risk characteristics, and their structures vary to some extent as well. Here, we explain the characteristics of two types: ABSs backed by credit card receivables and ABSs backed by loans for homeowners to install solar panels on their property (referred to as a residential solar ABS).

A key distinction between the debt-based assets acting as collateral for these ABSs is whether it is amortizing or nonamortizing. Recall that an amortizing loan has a principal balance that is scheduled to be paid down over the life of the loan. A nonamortizing loan has no such schedule for repayment of principal.

Credit Card ABS

Credit card receivable-backed securities are ABSs backed by pools of credit card debt owed to banks, retailers, travel and entertainment companies, and other credit card issuers.

The cash flow to a pool of credit card receivables includes finance charges (i.e., interest), membership and late payment fees, and principal repayments. Credit card receivables are nonamortizing loans; however, borrowers can choose to repay principal

at their discretion. Interest rates on credit card receivables can be fixed or floating, typically subject to a cap (i.e., a maximum rate that the credit card lender can charge).

A credit card securitization structure will typically include a **lockout period** or **revolving period** during which ABS investors only receive interest and fees paid on the collateral. If the underlying credit card holders make principal payments during the lockout period, these payments are used to purchase additional credit card receivables, keeping the overall value of the pool relatively constant.



PROFESSOR'S NOTE

In our reading on mortgage-backed securities, we will discuss prepayment risk at length. Prepayment risk occurs when borrowers repay loans at a different speed to that originally anticipated by investors. During the revolving period of a credit card ABS, investors are not subject to prepayment risk because any principal payments in the underlying collateral are used to make additional loans.

Once the lockout period ends, the **amortization period** of the ABS begins, and principal payments made on the underlying collateral are passed through to security holders. Credit card ABSs typically have an early (rapid) amortization provision that provides for earlier amortization of principal when it is necessary to preserve the credit quality of the securities, akin to an acceleration of payment for a debt issuer when credit conditions worsen.

Solar ABSs

Solar ABSs are backed by loans to homeowners wishing to finance the installation of solar energy systems to reduce their energy bills. These loans are offered by specialist finance companies or the financing subsidiaries of solar energy companies.

Solar ABSs are attractive to investors focused on environmental, social, and governance (ESG) factors because, through investing in the solar ABS, they are providing funds for homeowners to switch to a renewable energy source. Solar loans themselves can be secured on the solar energy system itself or on the homeowner's property as a junior mortgage, providing extra security to ABS investors.

Internal credit enhancement methods such as overcollateralization, excess spread, or subordination are common in these structures. Solar loans are usually made to homeowners with good credit scores, who are saving on energy bills through installing the solar energy system; hence, credit risk is likely to be low. However, solar ABSs are a relatively new asset class and are yet to be tested through a full credit cycle.

Many solar ABSs have a **pre-funding period**, which allows the trust to make investments in solar-related loans for a fixed time period after raising funds through issuing the ABS. A pre-funding period allows the ABS structure to invest in a larger, more diversified pool of solar-related investments.

LOS 64.d: Describe collateralized debt obligations, including their cash flows and risks.

A **collateralized debt obligation (CDO)** is a structured security issued by an SPE for which the collateral is a pool of debt obligations. When the collateral securities are corporate and emerging market debt, they are called collateralized bond obligations (CBOs). Collateralized loan obligations (CLOs) are supported by a portfolio of leveraged bank loans. Unlike the ABSs we have discussed, CDOs do not rely solely on payments from a static collateral pool to meet obligations from issuing securities. What sets CDOs apart from regular ABSs is that CDOs have a **collateral manager** who dynamically buys and sells securities in the collateral pool to generate the cash to make the promised payments to investors.



PROFESSOR'S NOTE

Leveraged loans are senior secured bank loans made to companies that have high levels of debt already, or have a poor credit history.

CDOs issue subordinated tranches in a similar fashion to ABSs. In creating a CDO, the structure must be able to offer an attractive return on the lowest-ranked equity tranche, after accounting for the required yields on the more senior CDO debt tranches.

Before the 2007–2009 global financial crisis, CDOs were based on a wide variety of underlying collateral. Since the crisis, CDO structures have become less complex, and more stringent requirements have been placed on the quality of the collateral. Collateral today comprises mostly leveraged loans; hence, the most common form of CDO is the CLO.

Three major types of CLOs are as follows:

1. *Cash flow CLOs*: Payments to CLO investors are generated through cash flows on the underlying collateral.
2. *Market value CLOs*: Payments to CLO investors are generated through trading the market value of the underlying collateral.
3. *Synthetic CLOs*: The collateral pool exposure is generated through credit derivative contracts. In this type of CLO, the CLO trust does not take ownership of the collateral.

The collateral of a CDO is subject to a series of prespecified tests to protect CLO investors from default:

- Coverage of payment obligations to the CLO investors by cash flows from the collateral
- Overcollateralization levels for each tranche—breaches of overcollateralization limits cause cash flows to be redirected to purchase additional collateral or to pay off the senior-most tranches of the CDO
- Diversification in the collateral pool
- Limitations on the amount of CCC rated debt in the collateral pool



MODULE QUIZ 64.1

1. During the lockout period of a credit card ABS:
 - A. no new receivables are added to the pool.
 - B. investors do not receive interest payments.
 - C. investors do not receive principal payments.
2. A debt security that is collateralized by a pool of the sovereign debt of several developing countries is *best* described as a:
 - A. CLO.
 - B. CBO.
 - C. CMO.
3. A covered bond is *most likely* to feature:
 - A. a fixed cover pool.
 - B. recourse to the issuer.
 - C. a special purpose entity.

KEY CONCEPTS

LOS 64.a

Covered bonds are similar to asset-backed securities, but instead of creating an SPE, the collateral (cover pool) remains on the balance sheet of the issuer. Covered bonds give bondholders recourse to the issuer as well as the asset pool, which increases the bonds' credit quality.

LOS 64.b

Internal credit enhancements of ABS include the following:

- *Overcollateralization*. The value of the collateral is greater than the face value of the ABS securities.
- *Excess spread*. Income from the collateral in excess of the payment obligations to ABS investors acts as a reserve to absorb default losses in the collateral pool.
- *Credit tranching*. Credit losses are first absorbed by the tranche with the lowest priority, and after that, by any other subordinated tranches, in order.

LOS 64.c

Credit card ABSs are backed by credit card receivables, which are nonamortizing receivables. Credit card ABSs typically have a lockout/revolving period during which only interest and fees are passed through from the collateral pool to investors, while any principal payments on the receivables are used to purchase additional receivables. Following this period, the ABSs have an amortization period where principal payments are passed through to ABS investors.

Solar ABSs are backed by loans to homeowners to finance installation of domestic solar energy equipment. Solar loans can be secured on the solar energy equipment or the homeowner's residence. Solar ABSs may be attractive to ESG investors.

LOS 64.d

Collateralized debt obligations (CDOs) are structured securities backed by a pool of debt obligations that are managed by a collateral manager. CDOs include collateralized bond obligations (CBOs) backed by corporate and emerging market debt, and collateralized loan obligations (CLOs) backed by leveraged bank loans.

Three major types of CLO are as follows:

- *Cash flow CLOs*. Payments to CLO investors are generated through cash flows on the underlying collateral.
- *Market value CLOs*. Payments to CLO investors are generated through trading the market value of the underlying collateral.
- *Synthetic CLOs*. Collateral pool exposure is generated through credit derivative contracts.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 64.1

1. **C** During the lockout period on a credit card receivables backed ABS, no principal payments are made to investors. (LOS 64.c)
2. **B** A collateralized bond obligation (CBO) is backed by an underlying pool of fixed-income securities, which may include emerging markets debt. Collateralized loan obligations (CLOs) are backed by leveraged loans. Collateralized mortgage obligations (CMOs) are backed by pools of mortgages or mortgage-backed securities. (LOS 64.d)
3. **B** Covered bonds differ from ABSs in that bondholders have recourse to the issuer as well as the cover pool. Covered bonds are not issued through special purpose entities. A covered bond issuer must maintain a dynamic cover pool, replacing any nonperforming or prepaid assets. (LOS 64.a)

READING 65

MORTGAGE-BACKED SECURITY (MBS) INSTRUMENT AND MARKET FEATURES

MODULE 65.1: MORTGAGE-BACKED SECURITY (MBS) INSTRUMENT AND MARKET FEATURES



Video covering this content is available online.

LOS 65.a: Define prepayment risk and describe time tranching structures in securitizations and their purpose.

An important characteristic of investments in mortgage-backed securities (MBSs) is their **prepayment risk**. Prepayments are principal repayments by mortgage borrowers in excess of the scheduled principal repayments for amortizing loans. In a pool of mortgages, some of the borrowers are likely to prepay, typically because they sell their homes or refinance their mortgages.

MBS valuation is based on an assumed prepayment rate for the underlying mortgages. Prepayment risk is the risk that prepayment speeds turn out to be *different to the expectations* of MBS investors when they purchased the MBS.

The risk that prepayments will be slower than expected, leading to MBS investors waiting longer than originally anticipated for their cash flows, is called **extension risk**. The risk that prepayments will be faster than expected, leading to cash flows arriving sooner than expected, is called **contraction risk**.

A key driver of prepayment speeds is the prevailing level of interest rates. When interest rates decrease, borrowers often refinance their mortgages at lower rates. This will cause the original higher-rate mortgages to be repaid early, increasing prepayment speeds and leading to contraction in the average life of an MBS. This is bad for MBS investors for two reasons:

- They receive cash flows sooner than expected in a low-rate environment and face lower reinvestment returns.
- Because the prices of MBS reflect expectations for prepayments in low-rate environments, they will not rise as much in response to decreasing interest rates as other fixed-income instruments that do not have an embedded prepayment option.



PROFESSOR'S NOTE

MBSs are like callable bonds, where the borrower has the right to repay the loan early, which leads to negative convexity for prices at low yields (i.e., prices rise at a slower rate as yields fall).

When interest rates increase, borrowers' prepayment speeds will be slower than expected because refinancing activity will slow, leading to extension in the average life of an MBS. This is also bad for MBS investors because cash flows will be discounted by more periods at a higher discount rate.

One way to reapportion the prepayment risk inherent in the underlying mortgage pool is to use **time tranching** in the MBS structure. An MBS with time tranching will have different bond classes with different maturities. Contraction and extension risk still exist with this structure, but the risks are redistributed to some extent among the tranches. The tranches that mature first offer investors protection against extension risk. Tranches with longer maturities provide relatively more protection against contraction risk. We will describe one method of time tranching, called sequential pay tranching, later in this reading.

LOS 65.b: Describe fundamental features of residential mortgage loans that are securitized.

A **residential mortgage loan** is a loan for which the collateral that underlies the loan is residential real estate. If the borrower defaults on the loan, the lender has a legal claim to the collateral property (referred to as a first lien), whereby they can take possession of the property and sell it to recover losses (a process called foreclosure).



PROFESSOR'S NOTE

MBSs backed by residential mortgages are called residential mortgage-backed securities (RMBSs). These residential mortgages are the focus of this LOS. Commercial mortgages, which relate to investments in properties to generate income rather than to provide a home for the borrower, as well as their related commercial mortgage-backed securities, are discussed later in the reading.

Common features of residential mortgage loans include the following:

- *Prepayment penalties.* A prepayment penalty is an additional payment that must be made by borrowers to compensate lenders if principal is prepaid when interest rates decline. Common in Europe, but rare in the United States, these penalties are designed to reduce prepayment risk for lenders.
- *Recourse/nonrecourse loans.* Nonrecourse loans only have the specified property as collateral. With recourse loans, the lender also has a claim against other assets of the borrower for any amount by which the foreclosure falls short of the payments outstanding on a defaulting loan. When the property value falls below the outstanding mortgage balance (a situation known as *underwater mortgages* or *negative equity*), borrowers with nonrecourse loans are more likely to strategically default, preferring to face foreclosure and a reduction in their credit score rather than continue to make

payments on a loan exceeding the value of the property. In Europe, most residential mortgages are recourse loans. In the United States, the nature of the loans varies by state, but most loans are nonrecourse.

Loan-to-Value and Debt-to-Income Ratios

A key measure of default risk for a mortgage loan is its **loan-to-value ratio (LTV)**, the percentage of the value of the collateral real estate that is loaned to the borrower. The lower the LTV, the higher the borrower's equity in the property. For a lender, loans with lower LTVs are less risky because the borrower has more to lose in the event of default (so is less likely to default). Also, if the property value is high compared to the loan amount, the lender is more likely to recover the amount loaned if the borrower defaults and the lender repossesses and sells the property.

Another key measure is the borrower's **debt-to-income ratio (DTI)**, which measures the monthly debt payments of the individual as a percentage of their monthly pretax gross income. A borrower with a lower DTI is deemed of lower risk to default.

For example, a borrower wishing to take out a \$300,000 mortgage on a \$400,000 property will have an LTV of $\$300,000 / \$400,000 = 75\%$.

If the mortgage has an annual interest rate of 6%, is to be repaid monthly over 25 years, and the borrower's annual pretax gross income is \$80,000, then we can calculate the DTI as follows.

First, calculate the constant monthly payment required to service the mortgage:

$$N = 25 \times 12 = 300; I/Y = 6 / 12 = 0.5; PV = 300,000; FV = 0; CPT \rightarrow PMT \\ = \$1,932.90$$

Second, calculate the monthly pretax gross income of the borrower as $\$80,000 / 12 = \$6,667$.

Then, calculate DTI as $\$1,932.90 / \$6,667 = 29\%$.

In the United States, mortgages made to borrowers with good credit, low LTV, and low DTI are termed **prime loans**. Mortgages to borrowers of lower credit quality, with higher DTI, higher LTV, or that have a lower-priority claim to the collateral in event of default, are termed **subprime loans**.

Agency and Non-Agency RMBSs

In the United States, a distinction is made between *agency* and *non-agency* RMBSs.

Agency RMBSs are either guaranteed by the government or guaranteed by a government-sponsored enterprise (GSE), which are companies created by the government. Because GSEs are technically distinct from the government, the securities guaranteed by GSEs do not carry the full faith and credit of the government—only that of the GSE that provides the guarantee. Mortgages need to meet minimum underwriting standards to qualify as collateral for an agency RMBS.

Non-agency RMBSs are issued by private entities such as banks and have no government or GSE guarantee. Non-agency RMBSs typically include credit

enhancements through external insurance, letters of credit, tranching, and private guarantees. Non-agency RMBSs are often cited as a catalyst for the global financial crisis of 2007–2009, when investment-grade senior tranches backed by subprime mortgages suffered large losses due to significant defaults in the underlying collateral. Issuance of non-agency RMBSs effectively ceased after the credit crisis due to changes in regulation.

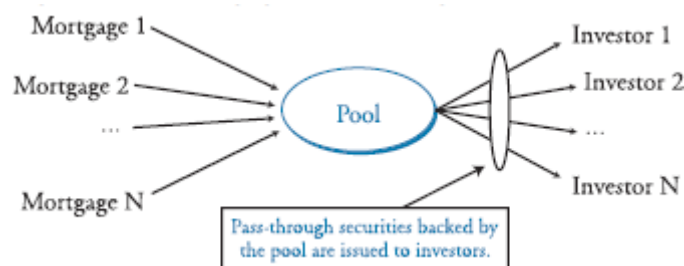
LOS 65.c: Describe types and characteristics of residential mortgage-backed securities, including mortgage pass-through securities and collateralized mortgage obligations, and explain the cash flows and risks for each type.

A **mortgage pass-through security** represents a claim on the cash flows from a pool of mortgages, net of administration fees. Any number of mortgages may be used to form the pool, and any mortgage included in the pool is referred to as a **securitized mortgage**.

The mortgages in the pool typically have different maturities and different mortgage rates. The **weighted average maturity (WAM)** of the pool is equal to the weighted average of the final maturities of all the mortgages in the pool, weighted by each mortgage's outstanding principal balance as a proportion of the total outstanding principal value of all the mortgages in the pool. The **weighted average coupon (WAC)** of the pool is the weighted average of the interest rates of all the mortgages in the pool (weighted in the same way as WAM).

Investors in mortgage pass-through securities receive the monthly cash flows generated by the underlying pool of mortgages, less any servicing and guarantee/insurance fees. The fees account for the fact that **pass-through rates** (i.e., the coupon rate on the MBS, also called its *net interest* or *net coupon*) are less than the WAC of the underlying mortgage pool.

Figure 65.1: Mortgage Pass-Through Cash Flow



EXAMPLE: WAM and WAC

An MBS contains the three mortgages A, B, and C, with details displayed in the following table.

Mortgage	Interest Rate	Beginning Balance (000 USD)	Current Balance (000 USD)	Original Term (Months)	Months to Maturity
A	2.6%	100	90	240	210
B	1.0%	200	72	300	100
C	5.4%	300	247	360	280

Calculate the WAM and the WAC for this MBS.

Answer:

Both the WAM and the WAC are calculated using the current balance of the mortgages as weights. Given that the sum of current balances is $90 + 72 + 247 = 409$, the WAM is calculated as follows:

$$\text{WAM} = 210\left(\frac{90}{409}\right) + 100\left(\frac{72}{409}\right) + 280\left(\frac{247}{409}\right) = 233 \text{ months}$$

The WAC is calculated as follows:

$$\text{WAC} = 2.6\%\left(\frac{90}{409}\right) + 1.0\%\left(\frac{72}{409}\right) + 5.4\%\left(\frac{247}{409}\right) = 4.0\%$$

Collateralized Mortgage Obligations

Collateralized mortgage obligations (CMOs) are securities that are collateralized by pass-through MBSs and pools of mortgages. Each CMO has multiple bond classes (CMO tranches) that have different exposures to prepayment risk. The total prepayment risk of the underlying RMBS is not changed, but is reapportioned among the various CMO tranches.

Institutional investors have different tolerances for prepayment risk. Some are primarily concerned with extension risk, while others may want to minimize exposure to contraction risk. By partitioning and distributing the cash flows generated by RMBSs into different risk packages to better match investor preferences, CMOs increase the potential market for securitized mortgages, and perhaps reduce funding costs as a result.

Common CMO structures include sequential pay tranches, planned amortization class tranches, support tranches, and floating-rate tranches.

In a **sequential pay CMO**, principal payments for the collateral flow to tranches in a prespecified order. Once the first tranche has been fully paid, principal from the underlying collateral will flow to the next tranche in the structure.

As an example, consider a simple CMO with two tranches. Both tranches receive interest payments at a specified coupon rate, but principal payments (both scheduled payments and prepayments) are paid to Tranche 1 (the “short” tranche) until its principal is paid off. Principal payments then flow to Tranche 2 until its principal is paid off.

Contraction and extension risk still exist with this structure, but they have been redistributed to some extent between the two tranches. The short tranche, which

matures first, offers investors relatively more protection against extension risk. The other tranche provides relatively more protection against contraction risk.

Investors with preferences for shorter-term securities will prefer tranches paid back sooner because they will have a lower expected average life than other tranches that ranked lower in the order for receiving principal payments.

Other CMO structures include the following:

- **Z-tranches.** A Z-tranche (also called an *accrual* or *accretion* bond) is a CMO tranche that receives no interest payments during a specified accrual period. Instead, during the accrual period, the tranche accrues interest as extra principal. For example, if a Z-tranche with a coupon rate of 5% has \$100 of principal outstanding at the start of the accrual period, then at the end of the first period, the principal amount of the Z-tranche would be increased to \$105 (rather than Z-tranche holders receiving a \$5 cash interest payment). Once the accrual payment has ended, interest and principal payments on the Z-tranche begin. A Z-tranche is typically the lowest-ranking tranche in a CMO structure.
- **Principal-only (PO) securities.** These securities pay only principal from the collateral pool; hence, they are effectively zero-coupon securities. Decreasing interest rates and increasing prepayment speeds benefit PO tranche holders because their return comes purely from the difference between the price paid and the repayment of principal. The sooner the principal is repaid, the higher the annualized return to PO security investors.
- **Interest-only (IO) securities.** These securities pay only interest from the collateral pool. Decreasing interest rates and increasing prepayment speeds harm IO tranche holders because their return comes purely from coupon payments on outstanding principal. The sooner the principal is repaid, the fewer coupon payments IO security investors receive.
- **Floating-rate tranches.** These tranches pay a coupon that is linked to a variable market reference rate, often subject to a cap and a floor. Tranches can also be structured as inverse floaters, where the coupon paid rises (falls) as interest rates fall (rise).
- **Residual tranches.** Residual tranches of CMOs rank junior to all other tranches, similar to the equity tranche of an ABS.
- **Planned amortization class (PAC) tranches and support tranches.** A PAC tranche is structured to make predictable payments to investors as long as prepayment speeds remain within a certain range. If prepayment speeds increase, the support tranche receives the principal repayments in excess of those specifically allocated to the PAC tranches. Conversely, prepayment speeds decrease, principal repayments to the support tranche are curtailed so the scheduled PAC payments can be made. In this way, both contraction and extension risk are reduced for PAC tranche investors. However, this protection is limited by the range of prepayment speeds under which the support tranche can operate. Should speeds move outside this range, the PAC tranche will not receive payments in line with its planned amortization schedule.

LOS 65.d: Describe characteristics and risks of commercial mortgage-backed securities.

Commercial mortgage-backed securities (CMBSs) are backed by pools of commercial mortgages on income-producing real estate, typically in the form of apartments (multifamily), industrial property (e.g., warehouses), shopping centers, office buildings, health care facilities (e.g., senior housing), and hotels.

CMBSs typically have fewer mortgages in the collateral pool than RMBSs—in some cases, a CMBS can even be backed by a single mortgage on a high-value, well-known property in a major city. Hence, there is less diversification against default risk. CMBSs can be backed by mortgages from an individual lender, or multiple commercial mortgages by one borrower. CMBSs are common in the United States, where CMBS coupons are fixed. CMBS volumes are increasing in Europe, where they typically have floating coupons.

An important difference between residential and commercial MBSs is the obligations of the borrowers of the underlying loans. Residential MBS loans are repaid by the owners or occupiers of the property; commercial MBS loans are repaid by real estate investors who, in turn, rely on tenants (usually businesses) of the property and their customers to provide the cash flow to repay the mortgage loan. The regular income from the collateral of a CMBS is referred to as the weighted average proceeds from the mortgages (WAMP), which plays the same role as WAC for an RMBS.

For these reasons, the analysis of CMBSs focuses on the credit risk of the property and not the credit risk of the borrower. Analysts focus on two key ratios to assess credit risk.

1. The **debt service coverage ratio (DSCR, or DSC ratio)** is a basic cash flow coverage ratio of the amount of cash flow from a commercial property available to make debt service payments, compared to the required debt service cost:

$$\text{debt service coverage ratio} = \frac{\text{net operating income}}{\text{debt service}}$$

Net operating income is calculated as rental income less cash operating expenses and a noncash replacement reserve that represents the depreciation of the property over time. This ratio indicates greater protection to the lender when it is higher.

2. The **loan-to-value ratio (LTV)** compares the loan amount on the property to its current fair market or appraisal value:

$$\text{LTV} = \frac{\text{current mortgage amount}}{\text{current appraised value}}$$

This is the same ratio we discussed in the context of residential mortgages. As was the case there, a higher LTV indicates higher default risk.

Two differences between CMBS and the RMBS we discussed earlier are call protection and balloon maturity provisions.

Call protection is equivalent to prepayment protection (i.e., restrictions on the early return of principal through prepayments). CMBSs provide call protection in two ways:

loan-level call protection provided by the terms of the individual mortgages, and call protection provided by the CMBS structure.

There are several means of creating **loan-level call protection**:

- *Prepayment lockout.* For a specific time period (typically, two to five years), the borrower is prohibited from prepaying the mortgage loan.
- *Prepayment penalty points.* A penalty fee expressed in points may be charged to borrowers who prepay mortgage principal. Each point is 1% of the principal amount prepaid.
- *Defeasance.* The borrower uses payments in excess of scheduled loan payments to purchase a portfolio of government securities that is sufficient to make the remaining scheduled principal and interest payments on the loan. This protects the lender from prepayments (because they continue to expect to receive the same payments as under the original loan schedule), and it allows the borrower to remove the lender's lien on the property, should they wish to sell the property before the end of the mortgage term.

To create **CMBS-level call protection**, loan pools can be subject to sequential pay tranching in a similar fashion to the process used for CMOs.

Commercial mortgages are typically not fully amortized, meaning that at the end of the loan term, some principal remains outstanding that needs to be paid. This is referred to as a **balloon payment**. If the borrower is unable to arrange refinancing to make this payment, the borrower is in default. This possibility is called **balloon risk**. In this case, the lender might extend the term of the loan during a “workout period” under modified terms. Because balloon risk entails extending the term of the loan, it is a form of extension risk for CMBS investors.



MODULE QUIZ 65.1

1. The risk that mortgage prepayments will occur more slowly than expected is *best* characterized as:
 - A. default risk.
 - B. extension risk.
 - C. contraction risk.
2. A sequential pay security is structured with three tranches: A, B, and C. Tranche A receives principal first, followed by Tranche B, and finally Tranche C. An investor concerned about extension risk would *most likely* prefer:
 - A. Tranche A.
 - B. Tranche B.
 - C. Tranche C.
3. For investors in commercial mortgage-backed securities, balloon risk in commercial mortgages results in:
 - A. call risk.
 - B. extension risk.
 - C. contraction risk.

KEY CONCEPTS

LOS 65.a

Prepayment risk refers to uncertainty about the timing of the principal cash flows from an MBS. Contraction risk is the risk that loan principal will be repaid more rapidly than expected, typically when interest rates have decreased. Extension risk is the risk that loan principal will be repaid more slowly than expected, typically when interest rates have increased.

Time tranching can be used to distribute prepayment risk across the tranches of an MBS.

LOS 65.b

The loan-to-value ratio (LTV) indicates the percentage of the value of the real estate collateral that is loaned. Lower LTVs indicate less credit risk.

The debt-to-income ratio (DTI) indicates the size of the monthly debt payments of the borrower relative to their monthly pretax gross income. Lower DTIs indicate lower credit risk.

Agency residential mortgage-backed securities (RMBSs) are guaranteed and issued by the federal government or a government-sponsored enterprise. Mortgages that back agency RMBS must meet certain minimum credit quality standards. Non-agency RMBSs are issued by private companies and may be backed by riskier subprime loans.

LOS 65.c

Key characteristics of RMBS pass-through securities include the pass-through rate (the coupon rate on the RMBS) and the weighted average maturity (WAM) and weighted average coupon (WAC) of the underlying pool of mortgages.

Collateralized mortgage obligations (CMOs) are collateralized by RMBSs or pools of mortgages. CMOs are structured with tranches that have different exposures to prepayment risks.

In a sequential pay CMO, all scheduled principal payments and prepayments are paid to each tranche in sequence until that tranche is paid off. The first tranche to be paid principal has the most contraction risk, and the last tranche to be paid principal has the most extension risk.

A planned amortization class CMO has PAC tranches that receive predictable cash flows as long as the prepayment rate remains within a predetermined range, and support tranches that have more contraction risk and more extension risk than the PAC tranches.

Other types of CMO tranches include Z-tranches, principal-only tranches, interest-only tranches, floating-rate tranches, and residual tranches.

LOS 65.d

Commercial mortgage-backed securities (CMBSs) are backed by mortgages on income-producing real estate properties. Because commercial mortgages are nonrecourse loans, analysis of CMBSs focuses on credit risk of the properties. CMBSs are structured in tranches with credit losses absorbed by the lowest-priority tranches in sequence.

Call (prepayment) protection in CMBSs includes loan-level call protection such as prepayment lockout periods, defeasance, prepayment penalty points, and CMBS-level call protection provided by lower-priority tranches.

CMBS loans are more likely to be partially amortizing than residential loans, leading to balloon risk.

Ratios used to analyze the credit risk of a commercial mortgage include the debt service coverage ratio and the loan-to-value ratio.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 65.1

1. **B** Extension risk is the risk that prepayments will be slower than expected. Contraction risk is the risk that prepayments will be faster than expected. (LOS 65.a)
2. **A** An investor concerned about extension risk would prefer to purchase a tranche that is paid back sooner than other tranches; hence, the investor would prefer Tranche A. (LOS 65.c)
3. **B** Balloon risk is the possibility that a commercial mortgage borrower will not be able to refinance the principal that is due at the maturity date of the mortgage. This results in a default that is typically resolved by extending the term of the loan during a workout period. Thus, balloon risk is a source of extension risk for CMBS investors. (LOS 65.d)

TOPIC QUIZ: FIXED INCOME

You have now finished the Fixed Income topic section. Please log into your Schweser online dashboard and take the Topic Quiz on this section. The Topic Quiz provides immediate feedback on how effective your study has been for this material. Questions are more exam-like than typical Module Quiz or QBank questions; a score of less than 70% indicates that your study likely needs improvement. These tests are best taken timed; allow 1.5 minutes per question.

READING 66

DERIVATIVE INSTRUMENT AND DERIVATIVE MARKET FEATURES

MODULE 66.1: DERIVATIVES MARKETS

LOS 66.a: Define a derivative and describe basic features of a derivative instrument.



Video covering this content is available online.

A **derivative** is a security that *derives* its value from the value of another security or a variable (such as an interest rate or stock index value) at some specific future date. The security or variable that determines the value of a derivative security is referred to as the **underlying** for the derivative. The value of a derivative at a point in time is derived from the value of the underlying (asset or variable) on which the derivative contract is based.

A relatively simple example of a derivative is a **forward contract** that specifies the price at which one party agrees to buy or sell an underlying security at a specified future date. Consider a forward contract to buy 100 shares of Acme at \$30 per share three months from now:

- Acme shares are the **underlying asset** for the forward contract.
- \$30 is the **forward price** in the contract.
- The date of the future transaction, when the shares will be exchanged for cash, is referred to as the **settlement date** (maturity date) of the forward contract.
- 100 shares is the **contract size** of the forward contract.
- The forward price is set so that the forward contract has zero value to both parties at contract initiation; neither party pays at the initiation of the contract.

We can examine three outcomes for the price of Acme shares at settlement:

1. The **spot price**, or the market price of the underlying, is \$30, equal to the forward price of \$30.

Neither party has profits or losses on the forward contract. Ignoring transactions costs, the party selling Acme shares could buy them back at \$30 per share and the party buying the shares could sell them at \$30 per share.

2. The spot price of Acme shares is \$40, greater than the forward price of \$30.

The party buying 100 Acme shares for \$3,000 at settlement can sell those shares at the spot price for \$4,000, realizing a profit of \$1,000 on the forward contract. The party that must deliver the Acme shares delivers shares with a market value of \$4,000 and receives \$3,000, realizing a \$1,000 loss on the forward contract.

3. The spot price of Acme shares is \$25 at settlement, less than the forward price of \$30.

The party buying 100 Acme shares for \$3,000 at settlement can sell those shares at the market price of \$25 to get \$2,500 and realize a loss of \$500 on the forward contract. The party that must deliver the Acme shares delivers shares with a market value of \$2,500 and receives \$3,000, realizing a \$500 gain on the forward contract.

To summarize, the buyer of the shares in a forward contract will have gains when the market price of the shares at settlement is greater than the forward price, and losses when the market price of the shares at settlement is less than the forward price. The party that must deliver the shares in a forward contract will have gains when the market price of the shares at settlement is less than the forward price, and losses when the market price of the shares at settlement is greater than the forward price. The gains of one party equal the losses of the other party at settlement.

We refer to the party that agrees to buy the underlying asset in a forward contract as the buyer of the forward. The buyer of the forward gains when the price of the underlying increases (and loses when it falls), similarly to a long position in the underlying. In this case we say the forward buyer has *long exposure* to the underlying, while the seller of the underlying has *short exposure* to the underlying, gaining when the price of the underlying decreases and losing when the value of the underlying increases.

In practice, a forward contract may be a **deliverable contract**. In our forward contract example, this means that the payment and the shares must be exchanged at the settlement date. A **cash-settled contract** specifies that only the gains and losses from the forward contract are exchanged at settlement. In our example above, with a share price of \$25, cash settlement would require the buyer of Acme shares to pay \$500 to the seller of Acme shares at settlement. Ignoring the transaction cost of buying and selling shares in the market, the gains and losses to the parties in our forward contract example are economically equivalent under the two alternative settlement methods.

We can view a derivatives contract as a way to transfer risk from one party to another. Consider a situation where the share seller in the forward contract owns 100 shares of Acme. She has existing risk because the future price of Acme shares is uncertain; the share price three months from now is a random variable. If she enters the forward contract from our example, she will receive \$3,000 for her shares at settlement, regardless of their market price. This effectively transfers her existing Acme price risk to the buyer of the shares in the forward contract.

When a party to a derivative contract has an existing risk that is transferred to another party, we say that party has **hedged** (offset, reduced) their existing risk. If the risk of the forward contract exactly matches an existing risk, then the forward contract can be

used to fully hedge the existing risk. If a derivative is used to reduce, but not entirely offset, an existing risk, we say the existing risk is **partially hedged**.

If the Acme share buyer in our example has no existing Acme price risk, she clearly increases her risk by entering into the forward contract. In this situation, the Acme share buyer is said to be **speculating** on the future price of Acme shares.

You may have realized our share seller could have achieved her goal of eliminating her Acme price risk by simply selling her shares (a **cash market transaction**). Derivatives have potential advantages over cash market transactions:

- Investors can gain exposure to a risk at low cost, effectively creating a highly leveraged investment in the underlying.
- Transaction costs for a derivatives position may be significantly lower than for the equivalent cash market trade.
- Initiating a derivatives position may have less impact on market prices of the underlying, relative to initiating an equivalent position in the underlying through a cash market transaction.

Underlying Assets and Variables

The underlying for a derivative is most often a stock or bond price, the level of a stock or bond index, or an interest rate. Here, we give examples of different underlying assets and variables for derivative contracts and the nature of the risks they transfer or modify. We present more details about derivatives based on these underlying assets in subsequent readings.

- A bond, for example a forward contract on a 30-year U.S. Treasury bond or other specific bond. The risk involved is the uncertainty about future bond prices.
- An index, for example the S&P 500 Index or the Citi Goldman Sachs Investment Grade Corporate Bond Index. The risk involved is the uncertainty about the future value of the index at a specific date. A portfolio manager can reduce the risk of a portfolio of large U.S. stocks for a period of time by selling a forward on the S&P 500 index. An investor can gain long exposure to a portfolio of high-grade corporate bonds, quickly and at low cost, by buying a forward on the index.
- A currency, for example British pounds (GBP). A U.S. manufacturer that expects a large payment in GBP in six months can offset the uncertainty about the USD value of this payment by selling a forward contract on the expected amount of GBP. A UK manufacturer that must make a large payment in USD in one year can offset the uncertainty about the GBP cost of the payment by buying a forward contract on the USD that is priced in GBP.
- An interest rate, for example the 1-year Treasury bill rate. Similar to derivatives based on bonds, except that a higher interest rate means gains for the buyer of an interest rate forward, whereas higher interest rates mean lower bond prices and losses for the buyer of a bond forward.
- Commodities, which are physical assets including **hard commodities** (typically mined or extracted, such as gold and oil) and **soft commodities** (typically grown, such as cotton, coffee, pork, and cattle). A farmer expecting a cotton crop in four

months can reduce her cotton price risk by selling cotton forward. A utility that will require thousands of gallons of oil over the next year can reduce its oil price risk by buying oil forwards that settle at various times over the coming year.



PROFESSOR'S NOTE

It may help you to remember this common rule for hedging risk with futures, which are similar to forwards: “Do in the futures market what you must do in the future.” A baking company that must buy wheat in the future should buy wheat futures (or forwards) to reduce the effects of the uncertainty about future wheat prices on their profits. A farmer who must sell wheat at harvest time should sell wheat futures (or forwards) to reduce price risk.

- Credit derivatives include credit default swaps (CDS), in which one party makes fixed periodic payments to another party, which will make a payment only if the underlying credit instrument (or portfolio of such credit securities) suffers a loss in value due to a default by the issuer (borrower of funds) of the credit instrument.
- Derivative contracts are also created with the weather (for farmers, energy producers, travel and tourism companies), cryptocurrencies, or longevity (for life insurers or annuity providers) as the underlying asset.

Along with forwards and futures, derivatives types that we will cover in the remainder of our derivatives readings are:

- *Options*: Put options give the buyer the right (but not the obligation) to sell the underlying for a specific price in the future. Call options give the buyer the right (but not the obligation) to buy the underlying for a specific price in the future.
- *Swaps*: In a simple interest rate swap, one party agrees to make periodic payments on a given amount at a fixed interest rate, and the other agrees to make periodic interest payments on the same given amount, but at an interest rate based on a future market reference rate (MRR). The resulting cash flows are equivalent to one party (the fixed-rate payer) borrowing at a fixed rate and using the proceeds to buy a floating-rate bond.

LOS 66.b: Describe the basic features of derivative markets, and contrast over-the-counter and exchange-traded derivative markets.

Exchange-Traded Derivatives

Centralized physical exchanges provide markets for futures contracts, some options contracts, and some other derivative contracts. The largest by volume of trades are the National Securities Exchange (India), the B3 market (Brazil), and the CME Group (the United States).

Exchange-traded derivatives are standardized and backed by a central clearinghouse. The exchange specifies the terms of each of the derivative contracts that will be traded and rules for trading on the exchange.

A **central clearinghouse (CCH)** essentially takes the opposite position to each side of a trade (called **novation**), guaranteeing the payments promised under the contract. The CCH requires deposits from both participants when a trade is initiated, and additional deposits for accounts that decline in value, to support its guarantee and minimize counterparty credit risk.

Exchange members (dealers or market makers) buy and sell derivatives at slightly different prices and primarily earn trading profits from the bid/ask spreads between buy and sell prices, rather than from holding (speculating on) specific derivatives positions, although they may hold such positions from time to time to meet customer needs.

The standardization of contracts allows exchange-traded derivatives to be more liquid and more transparent to market participants, compared to customized derivatives. A market participant who has taken a position in exchange-traded derivatives can easily exit that position by entering into a contract with a position opposite to their existing derivatives. Standardization of contracts reduces trading costs compared to customized derivative contracts.

Standardization also facilitates the clearing and settlement of trades. Clearing refers to executing the trade, recording the participants, and handling the exchange of any required payments. Settlement refers to the exchange of underlying assets or payments of the final amounts due at contract settlement (maturity).

Dealer (OTC) Markets for Derivatives

Forwards, most swaps, and some options are custom instruments created and traded by dealers in a market with no central location. Some dealer markets are quite structured (e.g., the Nasdaq market), while others are not. A dealer market with no central location is referred to as an over-the-counter (OTC) market. OTC markets are largely unregulated and less transparent than exchange markets. In OTC markets with no central clearinghouse, each side of a trade faces counterparty credit risk. Dealers (market makers) make derivatives trades with end users of derivatives and may also trade with each other to reduce their exposures to changes in prices of underlying assets.

OTC derivatives contracts can be customized to fit the needs of an end user regarding contract size, definition of the underlying, settlement date, whether the contract is deliverable of cash settled, and other relevant details. Users trying to gain or hedge a specific risk use OTC derivatives when a standardized derivative contract will not meet their needs (including a desire for privacy).

After the financial crisis of 2008, regulators worldwide instituted a **central clearing mandate** requiring that, for many swap trades, a **central counterparty (CCP)** takes on the counterparty credit risk of both sides of a trade, similar to the role of a central clearinghouse. As an example, multiple dealers record their swap trades on a **swap execution facility (SEF)**. When a dealer makes a swap trade, that information is sent to the SEF and the CCP replaces the trade with two trades, with the CCP as the counterparty to both of them, reducing counterparty risk. The downside of this

structure is that counterparty risks are concentrated rather than distributed among financial intermediaries.

The following offers a summary of the primary differences between exchange-traded and OTC derivatives.

Exchange-traded derivatives are:

- Traded at a centralized location, an exchange.
- Traded by exchange members (market makers).
- Based on standardized contracts and have lower trading costs.
- Subject to the trading rules of the exchange (i.e., are more regulated).
- Backed by the central clearinghouse to minimize counterparty credit risk. They also require deposits by both parties at initiation, and additional deposits when a position decreases in value.
- More liquid.
- More transparent, as all transactions are known to the exchange and to regulators.

OTC derivatives (not subject to the central clearing mandate) are:

- Custom instruments.
- Less liquid and have higher transaction costs.
- Less transparent.
- Subject to counterparty risk.
- More difficult to clear and settle.
- Subject to higher trading costs.
- Not subject to requirements for the deposit of collateral.

Derivatives in dealer markets that are subject to the central clearing mandate have reduced counterparty risk, are subject to more disclosure of trades, and are easier to clear and settle, but are still customizable and are contracts with dealers or financial intermediaries.



MODULE QUIZ 66.1

1. Which of the following statements *most accurately* describes a derivative security? A derivative:
 - A. always increases risk.
 - B. has no expiration date.
 - C. has a payoff based on an asset value or interest rate.
2. Which of the following statements about exchange-traded derivatives is *least accurate*? Exchange-traded derivatives:
 - A. are liquid.
 - B. are standardized contracts.
 - C. carry significant default risk.

LOS 66.a

A derivative is a security that derives its value from value of another security or variable at a specific future date. The security or variable that determines the value of a derivative security is referred to as the underlying.

Basic features of a derivative include the underlying, the price specified in the contract, the contract size, and the settlement date. The price is typically set so the contract has zero value at initiation to both parties. Contracts may be deliverable or cash-settled.

LOS 66.b

Exchange-traded derivatives are standardized and backed by a central clearinghouse that takes the opposite position to each side of a trade, guaranteeing the payments promised under the contract.

Over-the-counter (OTC) derivatives can be customized to fit the needs of the counterparties. OTC markets are largely unregulated and less transparent than exchange markets. Some OTC markets are subject to a central clearing mandate that reduces counterparty credit risk.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 66.1

1. **C** A derivative's value is derived from another asset or an interest rate. (LOS 66.a)
2. **C** Exchange-traded derivatives have relatively low default risk (counterparty credit risk) because the clearinghouse stands between the counterparties involved, in most contracts. (LOS 66.b)

READING 67

FORWARD COMMITMENT AND CONTINGENT CLAIM FEATURES AND INSTRUMENTS

MODULE 67.1: FORWARDS AND FUTURES



Video covering this content is available online.

LOS 67.a: Define forward contracts, futures contracts, swaps, options (calls and puts), and credit derivatives and compare their basic characteristics.

Forward Contracts

In a **forward contract** between two parties, one party (the buyer) commits to buy and the other party (the seller) commits to sell a physical or financial asset at a specific price on a specific date (the settlement date) in the future.

The buyer has long exposure to the underlying asset in that he will make a profit on the forward if the price of the underlying at the settlement date exceeds the forward price, and have a loss if the price of the underlying at the settlement date is less than the forward price. The results are opposite for the seller of the forward, who has short exposure to the underlying asset.

Futures Contracts

A **futures contract** is quite similar to a forward contract but is standardized and exchange-traded. The primary ways in which forwards and futures differ are that futures trade in a liquid secondary market, are subject to greater regulation, and trade in markets with more disclosure (transparency). Futures are backed by a central clearinghouse and require daily cash settlement of gains and losses, so that counterparty credit risk is minimized.

On a futures exchange, **margin** is cash or other acceptable collaterals that both the buyer and seller must deposit. Unlike margin in bond or stock accounts, there is no loan involved and, consequently, there are no interest charges. This collateral provides protection for the clearinghouse. At the end of each trading day, the margin balance in a futures account is adjusted for any gains and losses in the value of the futures position based on the new settlement price, a process called the **mark-to-market** or marking-

to-market. The settlement price is calculated as the average price of trades over a period at the end of the trading session.

Initial margin is the amount of cash or collateral that must be deposited in a futures account before a trade may be made. Initial margin per contract is relatively low and is approximately one day's maximum expected price fluctuation on the total value of the assets covered by the contract.

Maintenance margin is the minimum amount of margin that must be maintained in a futures account. If the margin balance in the account falls below the maintenance margin through daily mark to market from changes in the futures price, the account holder must deposit additional funds to bring the margin balance back up to the *initial* margin amount, or the exchange will close out the futures position. This is different from a margin call in an equity account, which requires investors only to bring the margin back up to the maintenance margin amount. Futures margin requirements are set by the exchange.

To illustrate the daily mark-to-market for futures, consider a contract for 100 ounces of gold that settles on May 15. The initial margin amount is \$5,000 and the maintenance margin is \$4,700.

On Day 0

- A buyer and seller make a trade at the end of the day at a price of \$1,950 per ounce and both parties deposit the initial margin of \$5,000 into their accounts.

On Day 1, the settlement price falls to \$1,947.50. The seller has gains and the buyer has losses.

- The exchange will credit the seller's account for $(1,950 - 1,947.50) \times 100 = \250 , increasing the margin balance to \$5,250.
- The exchange will deduct $(1,950 - 1,947.50) \times 100 = \250 from the buyer's account, decreasing the margin balance to \$4,750. Because \$4,750 is more than the maintenance (minimum) margin amount of \$4,700, no additional deposit is required.

On Day 2, the settlement price falls to \$1,945. Again, the seller has gains and the buyer has losses.

- The exchange will credit the seller's account for $(1,947.50 - 1,945) \times 100 = \250 , increasing their margin balance to \$5,500.
- The exchange will deduct $(1,947.50 - 1,945) \times 100 = \250 from the buyer's account, decreasing the margin balance to \$4,500. Because \$4,500 is less than the maintenance (minimum) margin amount of \$4,700, the buyer must deposit $5,000 - 4,500 = \$500$ into their margin account to return it to the initial margin amount of \$5,000.
- At the end of Day 2, both parties have futures positions at the new settlement price of \$1,945 per ounce.

Many futures contracts have **price limits**, which are exchange-imposed limits on how much each day's settlement price can change from the previous day's settlement price. Exchange members are prohibited from executing trades at prices outside these limits. If the equilibrium price at which traders would willingly trade is above the upper limit

or below the lower limit, trades cannot take place. Some exchanges have **circuit breakers**; in this case, when a futures price reaches a limit price, trading is suspended for a short period.



MODULE QUIZ 67.1

1. Which type of contract always requires daily marking to market of gains and losses?
 - A. Futures contracts only.
 - B. Forward contracts only.
 - C. Both futures and forward contracts.
2. Compared to a futures contract, an otherwise identical forward contract *most likely* has greater:
 - A. liquidity.
 - B. transparency.
 - C. counterparty risk.

MODULE 67.2: SWAPS AND OPTIONS



Video covering
this content is
available online.

Swaps

Swaps are agreements to exchange a series of payments on multiple settlement dates over a specified time period (e.g., quarterly payments for two years). At each settlement date, the two payments are netted so that only one net payment is made. The party with the greater liability at each settlement date pays the net difference to the other party.

Swaps trade in a dealer market and the parties are exposed to counterparty credit risk, unless the market has a central counterparty structure to reduce counterparty risk. In this case, margin deposits and mark-to-market payments may also be required to further reduce counterparty risk.

We can illustrate the basics of a swap with a simple fixed-for-floating interest rate swap for two years with quarterly interest payments based on a **notional principal** amount of \$10 million. In such a swap, one party makes quarterly payments at a fixed rate of interest (the **swap rate**) and the other makes quarterly payments based on a floating **market reference rate**.

The swap rate is set so that the swap has zero value to each party at its inception. As expectations of future values of the market reference rate change over time, the value of the swap can become positive for one party and negative for the other party.

Consider an interest rate swap with a notional principal amount of \$10 million, a fixed rate of 2%, and a floating rate of the 90-day secured overnight financing rate (SOFR). At each settlement date, the fixed-rate payment will be $\$10 \text{ million} \times 0.02/4 = \$50,000$. The floating-rate payment at the end of the first quarter will be based on 90-day SOFR at the initiation of the swap, so that both payments are known at the inception of the swap.

If, at the end of the first quarter, 90-day SOFR is 1.6%, the floating-rate payment at the second quarterly settlement date will be $\$10 \text{ million} \times 0.016 / 4 = \$40,000$. The fixed-rate payment is again \$50,000, so at the end of the second quarter the fixed-rate payer will pay the net amount of \$10,000 to the other party.

A company with 2-year floating-rate quarterly-pay note outstanding could enter such a swap as the fixed-rate payer, converting its floating-rate liability into a fixed-rate liability. It now makes fixed interest rate payments and can use the floating-rate payments from the counterparty to make the payments on its floating-rate debt. By entering into the swap, the company can hedge the interest rate risk (uncertainty about future quarterly rates) of their existing floating-rate liability.

As we will see in our reading on swap valuation, a swap can be constructed from a series of forward contracts in which the underlying is a floating rate and the forward price is a fixed rate. Each forward settles on one of the settlement dates of the swap. At each settlement date, the difference between the fixed and the floating rates would result in a net payment, just as with a swap. Often, interest rate forwards settle at the beginning of the quarter rather than the end; the cash flows are the present value equivalents of the end-of-quarter swap payments.

Credit Swaps

One type of swap that is structured a bit differently is a **credit default swap (CDS)**. With a CDS, the protection buyer makes fixed payments on the settlement dates and the protection seller pays only if the underlying (a reference security) has a **credit event**. This could be a bond default, a corporate bankruptcy, or an involuntary restructuring.

When a credit event occurs, the protection seller must pay an amount that offsets the loss in value of the reference security. The fixed payments represent the yield premium on the reference bonds that compensates bondholders for the expected loss from default, the probability of default times the expected loss in the event of default (or other credit event). The protection buyer is essentially paying the yield premium on the reference security for insurance against default.

The holder of a risky bond can hedge its default risk by entering a CDS as the protection buyer. The protection seller receives the default risk premium (credit spread) and takes on the risk of default, resulting in risk exposure similar to that of holding the reference bond.

Options

The two types of options of interest to us here are **put options** and **call options** on an underlying asset. We introduce them using option contracts for 100 shares of a stock as the underlying asset.

A put option gives the buyer the right (but not the obligation) to sell 100 shares at a specified price (the **exercise price**, also referred to as the **strike price**) for specified period of time, the **time to expiration**. The put seller (also called the *writer* of the option) takes on the obligation to purchase the 100 shares at the price specified in the option, if the put buyer exercises the option.

Note the “one-way” nature of options. If the exercise price of the puts is \$25 at the expiration of the option, and the shares are trading at or above \$25, the put holder will not exercise the option. There is no reason to exercise the put and sell shares at \$25 when they can be sold for more than \$25 in the market. This is the outcome for any

stock price greater than or equal to \$25. Regardless of whether the stock price at option expiration is \$25 or \$1,000, the put buyer lets the option expire, and the put seller keeps the proceeds from the sale.

If the stock price is below \$25, the put buyer will exercise the option and the put seller must purchase 100 shares for \$25 from the put buyer. On net, the put buyer essentially receives the difference between the stock price at expiration and \$25 (times 100 shares).

A call option gives the buyer the right (but not the obligation) to buy 100 shares at a specified price (the exercise price) for a specified period of time. The call seller (writer) takes on the obligation to sell the 100 shares at the exercise price, if the call buyer exercises the option.

LOS 67.b: Determine the value at expiration and profit from a long or a short position in a call or put option.

Unlike forwards, futures, and swaps, options are sold at a price (they do not have zero value at initiation). The price of an option is also referred to as the **option premium**.

At expiration the payoff (value) of a call option to the owner is $\text{Max}(0, S - X)$, where S is the price of the underlying at expiration and X is the exercise price of the call option. The $\text{Max}()$ function tells us that if $S < X$ at expiration, the option value is zero, that is, it expires worthless and will not be exercised.

At expiration the payoff (value) of a put option to the owner is $\text{Max}(0, X - S)$, where S is the price of the underlying at expiration and X is the exercise price of the put option. A put has a zero value at expiration unless $X - S$ is positive.

For the buyer of a put or call option, the profit at expiration is simply the difference between the value (payoff) of the option at expiration and the premium the investor paid for the option.

Because the seller (writer) of an option receives the option premium, the profit to the option seller at expiration is the amount of the premium received minus the option payoff at expiration. The writer loses the payoff at expiration and will have a loss on the option if the payoff is greater than the premium received.

Note the risk exposures of call and put buyers and writers. The buyer of a put or call has no further obligation, so the maximum loss to the buyer is simply the amount they paid for the option. The writer of a call option has exposure to an unlimited loss because the maximum price of the underlying, S , is (theoretically) unlimited, so that the payoff $S - X$ is unlimited. The payoff on a put option is $X - S$, so if the lower limit on S is zero, the maximum payoff on a put option is the exercise price, X .

Call Option Profits and Losses

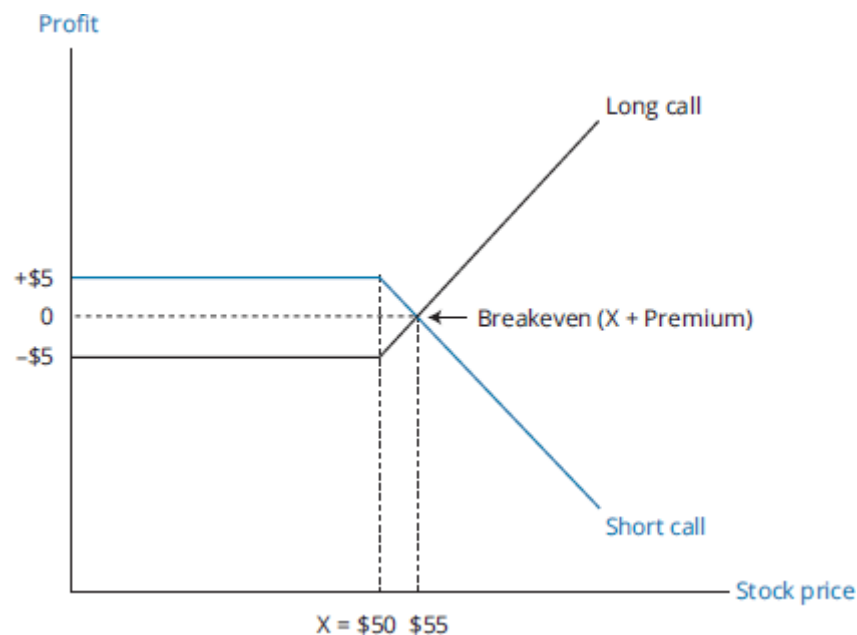
Consider a call option with a premium of \$5 and an exercise price of \$50. This means the buyer pays \$5 to the writer. At expiration, if the price of the stock is less than or equal to the \$50 exercise price, the option has zero value, the buyer of the option is out

\$5, and the writer of the option is ahead \$5. When the stock's price exceeds \$50, the option starts to gain (breakeven will come at \$55, when the value of the stock equals the exercise price plus the option premium). Conversely, as the price of the stock moves upward, the seller of the option starts to lose (negative figures will start at \$55, when the value of the stock equals the exercise price plus the option premium).

An illustration of the profit or loss at expiration for the buyer (long) and writer (short) of this call option, as a function of the stock price, is presented in Figure 67.1. This profit/loss diagram indicates the following:

- The maximum loss for the buyer of a call is the \$5 premium (at any $S \leq \$50$).
- The breakeven point for the buyer and seller is the exercise price plus the premium (at $S = \$55$).
- The profit potential to the buyer of the option is unlimited, and, conversely, the potential loss to the writer of the call option is unlimited.
- The call holder will exercise the option whenever the stock's price exceeds the exercise price at the expiration date.
- The greatest profit the writer can make is the \$5 premium (at any $S \leq \$50$).
- The sum of the profits between the buyer and seller of the call option is always zero; thus, trading options is a *zero-sum game*. One party's profits equal the other party's losses.

Figure 67.1: Profit/Loss Diagram for a Call Option



Put Option Profits and Losses

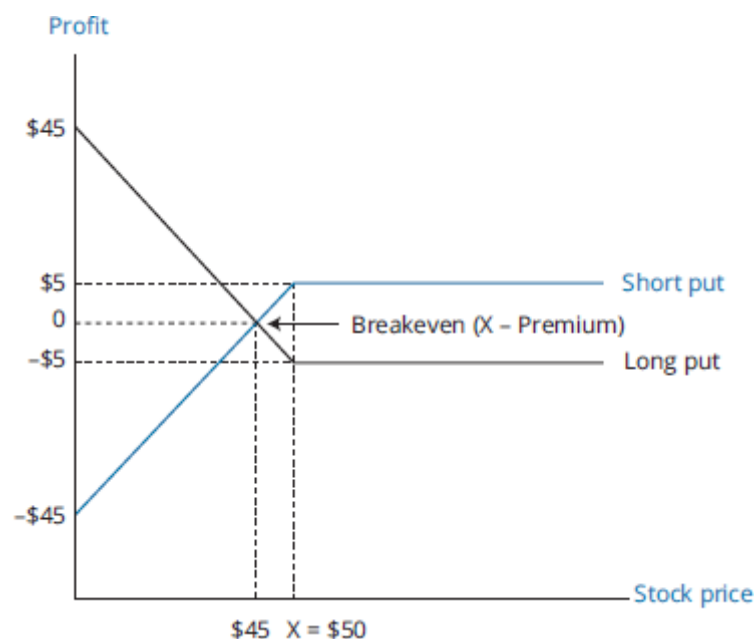
To examine the profits and losses associated with trading put options, consider a put option with a \$5 premium and a \$50 exercise price. The buyer pays \$5 to the writer. When the price of the stock at expiration is greater than or equal to the \$50 exercise price, the put has zero value. The buyer of the option has a loss of \$5 and the writer of the option has a gain of \$5. When the stock price is less than \$50, the put option has a

positive payoff. Breakeven will come at \$45, when the value of the stock equals the exercise price less the option premium. At a stock price below \$45, the put seller will have a loss.

Figure 67.2 shows the profit/loss diagram for the buyer (long) and seller (short) of the put option that we have been discussing. This profit/loss diagram illustrates that:

- The maximum loss for the buyer of the put is the \$5 premium (at any $S \geq \$50$).
- The maximum gain to the buyer of the put is limited to the exercise price less the premium ($\$50 - \$5 = \$45$). The potential loss to the writer of the put is the same amount.
- The breakeven price for the put buyer (seller) is the exercise price minus the option premium ($\$50 - \$5 = \$45$).
- The maximum profit for the writer is the \$5 premium ($S \geq \50).
- The profit (loss) of the put buyer will always equal the loss (profit) of the put writer.

Figure 67.2: Profit/Loss Diagram for a Put Option



EXAMPLE: Option profit calculations

Suppose that both a call option and a put option have been written on a stock with an exercise price of \$40. The current stock price is \$42, and the call and put premiums are \$3 and \$0.75, respectively.

Calculate the profit to the long and short positions for both the put and the call with an expiration day stock price of \$35 and with a price at expiration of \$43.

Answer:

Profit will be computed as ending option value – initial option cost.

Stock at \$35:

- Long call: $\$0 - \$3 = -\$3$. The option has no value, so the buyer loses the premium paid.
- Short call: $\$3 - \$0 = \$3$. Because the option has no value, the call writer's gain equals the premium received.
- Long put: $\$5 - \$0.75 = \$4.25$. The buyer paid \$0.75 for an option that is now worth \$5.
- Short put: $\$0.75 - \$5 = -\$4.25$. The seller received \$0.75 for writing the option, but the option will be exercised so the seller will lose \$5 at expiration.

Stock at \$43:

- Long call: $-\$3 + \$3 = \$0$. The buyer paid \$3 for the option, and it is now in the money by \$3. Hence, the net profit is zero.
- Short call: $\$3 - \$3 = \$0$. The seller received \$3 for writing the option and now faces a $-\$3$ valuation because the buyer will exercise the option, for a net profit of zero.
- Long put: $-\$0.75 - \$0 = -\$0.75$. The buyer paid \$0.75 for the put option and the option now has no value.
- Short put: $\$0.75 - \$0 = \$0.75$. The seller received \$0.75 for writing the option and it has zero value at expiration.

A buyer of puts or a seller of calls has short exposure to the underlying (will profit when the price of the underlying asset decreases). A buyer of calls or a seller of puts has long exposure to the underlying (will profit when the price of the underlying asset increases).

LOS 67.c: Contrast forward commitments with contingent claims.

A **forward commitment** is a legally binding promise to perform some action in the future. Forward commitments include forward contracts, futures contracts, and most swaps.

A **contingent claim** is a claim (to a payoff) that depends on a particular event. Options are contingent claims; the event is the price of the underlying being above or below the exercise price. Credit default swaps are also considered contingent claims because the payment by the protection seller depends on a credit event occurring.



MODULE QUIZ 67.2

1. Interest rate swaps are:
 - A. highly regulated.
 - B. equivalent to a series of forward contracts.
 - C. contracts to exchange one asset for another.
2. A call option is:
 - A. the right to sell at a specific price.
 - B. the right to buy at a specific price.

- C. an obligation to buy at a certain price.
3. At expiration, the exercise value of a put option is:
- A. positive if the underlying asset price is less than the exercise price.
 - B. zero only if the underlying asset price is equal to the exercise price.
 - C. negative if the underlying asset price is greater than the exercise price.
4. At expiration, the exercise value of a call option is the:
- A. underlying asset price minus the exercise price.
 - B. greater of zero or the exercise price minus the underlying asset price.
 - C. greater of zero or the underlying asset price minus the exercise price.
5. An investor writes a put option with an exercise price of \$40 when the stock price is \$42. The option premium is \$1. At expiration the stock price is \$37. The investor will realize a:
- A. loss of \$2.
 - B. loss of \$3.
 - C. profit of \$1.
6. Which of the following derivatives is a forward commitment?
- A. Stock option.
 - B. Interest rate swap.
 - C. Credit default swap.

KEY CONCEPTS

LOS 67.a

Forward contracts obligate one party to buy, and another to sell, a specific asset at a specific price at a specific time in the future.

Futures contracts are much like forward contracts, but are exchange-traded, liquid, and require daily settlement of any gains or losses.

A call option gives the holder the right, but not the obligation, to buy an asset at a specific price at some time in the future.

A put option gives the holder the right, but not the obligation, to sell an asset at a specific price at some time in the future.

In an interest rate swap, one party pays a fixed rate and the other party pays a floating rate, on a given amount of notional principal. Swaps are equivalent to a series of forward contracts based on a floating rate of interest.

A credit default swap is a contract in which the protection seller provides a payment if a specified credit event occurs.

LOS 67.b

Call option value at expiration is $\text{Max}(0, \text{underlying price} - \text{exercise price})$ and profit or loss is $\text{Max}(0, \text{underlying price} - \text{exercise price}) - \text{option cost}$ (premium paid).

Put value at expiration is $\text{Max}(0, \text{exercise price} - \text{underlying price})$ and profit or loss is $\text{Max}(0, \text{exercise price} - \text{underlying price}) - \text{option cost}$.

A call buyer (call seller) benefits from an increase (decrease) in the value of the underlying asset.

A put buyer (put seller) benefits from a decrease (increase) in the value of the underlying asset.

LOS 67.c

A forward commitment is an obligation to buy or sell an asset or make a payment in the future. Forward contracts, futures contracts, and most swaps are forward commitments.

A contingent claim is a derivative that has a future payoff only if some future event takes place (e.g., asset price is greater than a specified price). Options and credit derivatives are contingent claims.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 67.1

1. **A** Futures contracts are marked to market daily. Forward contracts typically are not, but could be if there is central clearing party. (LOS 67.a)
2. **C** Forward contracts involve counterparty risk; futures contracts trade through a clearinghouse. Because futures contracts trade on organized exchanges, they have greater liquidity and transparency than forward contracts. (LOS 67.a)

Module Quiz 67.2

1. **B** A swap is an agreement to buy or sell an underlying asset periodically over the life of the swap contract. It is equivalent to a series of forward contracts. (LOS 67.a)
2. **B** A call gives the owner the right to call an asset away (buy it) from the seller. (LOS 67.a)
3. **A** The exercise value of a put option is positive at expiration if the underlying asset price is less than the exercise price. Its exercise value is zero if the underlying asset price is greater than or equal to the exercise price. The exercise value of an option cannot be negative because the holder can allow it to expire unexercised. (LOS 67.b)
4. **C** If the underlying asset price is greater than the exercise price of a call option, the value of the option is equal to the difference. If the underlying asset price is less than the exercise price, a call option expires with a value of zero. (LOS 67.b)
5. **A** Because the stock price at expiration is less than the exercise price, the buyer of the put option will exercise it against the writer. The writer will have to pay \$40 for the stock and can only sell it for \$37 in the market. However, the put writer collected the \$1 premium for writing the option, which reduces the net loss to \$2. (LOS 67.b)

6. **B** This type of custom contract is a forward commitment. (LOS 67.c)

READING 68

DERIVATIVE BENEFITS, RISKS, AND ISSUER AND INVESTOR USES

MODULE 68.1: USES, BENEFITS, AND RISKS OF DERIVATIVES



Video covering
this content is
available online.

LOS 68.a: Describe benefits and risks of derivative instruments.

Advantages of Derivatives

Derivative instruments offer several potential advantages over cash market transactions, including the following:

Ability to Change Risk Allocation, Transfer Risk, and Manage Risk

We have discussed these benefits in our introduction to derivative contracts. Some examples of ways that risk exposures can be altered using derivatives, without any cash market securities transactions, are:

- A portfolio manager can increase or decrease exposure to the risk and return of a market index.
- A manufacturer can hedge the exchange rate risk of anticipated receipts or payments.
- The issuer of a floating-rate note can change that exposure to a fixed-rate obligation.

Derivative instruments can be used to create risk exposures that are not available in cash markets. Consider the following examples of changing an existing risk profile:

- The owner of common stock can buy puts that act as a floor on the sale price of their shares, reducing the downside risk of the stock by paying the cost of the puts.
- An investor can acquire the upside potential of an asset without taking on its downside risk by buying call options.

Information Discovery

Derivatives prices and trading provide information that cash market transactions do not.

- Options prices depend on many things we can observe (interest rates, price of the underlying, time to expiration, and exercise price) and one we cannot, the expected

future price volatility of the underlying. We can use values of the observable variables, together with current market prices of derivatives, to estimate the future price volatility of the underlying that market participants expect.

- Futures and forwards can be used to estimate expected prices of their underlying assets.
- Interest rate futures across maturities can be used to infer expected future interest rates and even the number of central bank interest rate changes over a future period.

Operational Advantages

Compared to cash markets, derivatives markets have several operational advantages. Operational advantages of derivatives include greater ease of short selling, lower transaction costs, greater potential leverage, and greater liquidity.

- *Ease of short sales.* Taking a short position in an asset by selling a forward or a futures contract may be easy to do. Difficulty in borrowing an asset and restrictions on short sales may make short positions in underlying assets problematic or more expensive.
- *Lower transaction costs.* Transaction costs can be significantly lower with commodities derivatives, where transportation, storage, and insurance add costs to transactions in physical commodities. Entering a fixed-for-floating swap to change a floating-rate exposure to fixed rate is clearly less costly than retiring a floating-rate note and issuing a fixed-rate note.
- *Greater leverage.* The cash required to take a position in derivatives is typically much less than for an equivalent exposure in the cash markets.
- *Greater liquidity.* The low cash requirement for derivatives transactions makes very large transactions easier to handle.

Improved Market Efficiency

Low transaction costs, greater liquidity and leverage, and ease of short sales all make it less costly to exploit securities mispricing through derivatives transactions and improve the efficiency of market prices.

Risks of Derivatives

Implicit Leverage

The implicit leverage in derivatives contracts gives them much more risk than their cash market equivalents. Just as we have shown regarding the leverage of an equity investment on margin, a lower cash requirement to enter a trade increases leverage. Futures margins, according to the CME Group, are typically in the 3% to 12% range, indicating leverage of 8:1 to 33:1. With required cash margin of 4%, a 1% decrease in the futures price decreases the cash margin by 25%.

A lack of transparency in derivatives contracts and securities that combine derivative and cash market exposures (structured securities) may lead to situations in which the purchasers do not well understand the risks of derivatives or securities with embedded derivatives.

Basis Risk

Basis risk arises when the underlying of a derivative differs from a position being hedged with the derivative. For a manager with a portfolio of 50 large-cap U.S. stocks, selling a forward with the S&P 500 Index as the underlying (in an amount equal to the portfolio value) would hedge portfolio risk, but would not eliminate it because of the possibility that returns on the portfolio and returns on the index may differ over the life of the forward. Basis risk also arises in a situation where an investor's horizon and the settlement date of the hedging derivative differ, such as hedging the value of a corn harvest that will occur on September 15 by selling corn futures that settle on October 1. Again the hedge may be effective but will not be perfect, and the corn producer is said to have basis risk.

Liquidity Risk

Derivative instruments have a special type of liquidity risk when the cash flows from a derivatives hedge do not match the cash flows of the investor positions. As an example, consider a farmer who sells wheat futures to hedge the value of her wheat harvest. If the future price of wheat increases, losses on the short position essentially offset the extra income from the higher price that will come at harvest (as intended with a hedge), but these losses may also cause the farmer to get margin calls during the life of the contract. If the farmer does not have the cash (liquidity) to meet the margin calls, the position will be closed out and the value of the hedge will be lost.

Counterparty Credit Risk

We have discussed counterparty credit risk previously. Here we note additionally that different derivatives and positions have important differences in the existence or amount of counterparty risk. The seller of an option faces no counterparty credit risk; once the seller receives the option premium there is no circumstance in which the seller will be owed more at settlement. On the other hand, the buyer of an option will be owed money at settlement if the option is in the money; thus, the buyer faces counterparty credit risk. In contrast, both the buyer and seller of a forward on an underlying asset may face counterparty credit risk.

In futures markets the deposit of initial margin, the daily mark-to-market, and the guarantee of the central clearinghouse all reduce counterparty risk. With forwards there may be no guarantees, or the terms of the forward contract may specify margin deposits, a periodic mark-to-market, and a central clearing party to mitigate credit risk.

Systemic Risk

Widespread impact on financial markets and institutions may arise from excessive speculation using derivative instruments. Market regulators attempt to reduce systemic risk through regulation, for example the central clearing requirement for swap markets to reduce counterparty credit risk.

Derivatives Use by Issuers

Corporate users of derivative instruments are considered issuers of derivatives. A non-financial corporation may have risks associated with changes in asset and liability values as well as earnings volatility from changes in various underlying securities or interest rates. Some examples are:

- A corporation may have income in a foreign currency and hedge the exchange rate risk with forwards to smooth earnings reported in their domestic currency.
- A corporation may use fair value reporting for its fixed-rate debt, and that value changes as interest rates change. By entering an interest rate swap as the floating-rate payer, the corporation has essentially converted the fixed-rate liability to a floating-rate liability that has much lower duration so that its balance sheet value is less sensitive to changes in interest rates.
- A corporation with a commodity-like product may carry its inventory at fair market value, leading to fluctuations in the value reported on the balance sheet over time as the market price of their product changes. By selling forward contracts on an underlying that matches well with their product, the firm will have gains or losses on the forwards that offset decreases or increases in reported inventory value. With the market value of the forward position also reported on the balance sheet, total assets will have less variation from changes in the market price of their product.

Accounting rules may permit **hedge accounting**. Hedge accounting allows firms to recognize the gains and losses of qualifying derivative hedges at the same time they recognize the corresponding changes in the values of assets or liabilities being hedged. Issuer hedges against the effects of a changing price or value of a derivative's underlying are classified by their purpose.

- A hedge of the domestic currency value of future receipts in a foreign currency using forwards is termed a **cash flow hedge**. A swap that converts a floating-rate liability to a fixed-rate liability is also considered a cash flow hedge (cash flows for interest payments are more certain).
- A **fair value hedge** is one that reduces (offsets) changes in the values of the firm's assets or liabilities. Our examples of a firm that uses derivatives to hedge against changes in the balance sheet value of its inventory, and a firm that uses an interest rate swap to decrease the volatility of debt values on its balance sheet, are considered fair value hedges.
- A **net investment hedge** is one that reduces the volatility of the value of the equity of a company's foreign subsidiary reported on its balance sheet. Foreign currency forwards or futures can be used to hedge changes in the reported value of the subsidiary's equity due to changes in exchange rates.

Derivatives Use by Investors

As we have seen, investors can hedge, modify, or increase their exposure to the risk of an underlying asset or interest rate with derivatives positions, either forward commitments or contingent claims. Some examples are:

- An investor can buy silver forwards to gain exposure to the price of silver, with no or low funds initially required.
- An investor can increase the duration of their bond portfolio by entering an interest rate swap as the floating-rate payer/fixed-rate receiver, which is similar to issuing floating-rate debt and buying a fixed-rate bond with the proceeds.
- An equity portfolio manager can modify their market risk exposure temporarily at low cost, increasing it by buying equity index futures or decreasing it by selling equity index futures. Alternatively, the portfolio manager could decrease downside risk and preserve upside potential by buying puts on an equity index.



MODULE QUIZ 68.1

1. Which of the following *most accurately* describes a risk of derivative instruments?
 - A. Derivatives make it easier for market participants to take short positions.
 - B. The underlying of a derivative might not fully match a position being hedged.
 - C. Volatility in underlying asset prices is implied by the prices of options on those assets.
2. Uses of derivatives by investors *most likely* include:
 - A. hedging against price risk for inventory held.
 - B. modifying the risk exposure of a securities portfolio.
 - C. stabilizing the balance sheet value of a foreign subsidiary.

KEY CONCEPTS

LOS 68.a

Advantages of derivatives include the ability to change or transfer risk; information discovery about the expected prices or volatility of underlying assets or interest rates; operational advantages such as ease of short sales, low transaction costs, and greater leverage and liquidity; and improved market efficiency.

Risks of derivatives include implicit leverage, basis risk from inexact hedges, liquidity risk from required cash flows, counterparty credit risk, and systemic risk for financial markets.

LOS 68.b

Derivatives uses by issuers include managing risks associated with changes in asset and liability values as well as earnings volatility from changes in various underlying securities or interest rates.

Derivatives uses by investors include hedging, modifying, or increasing their exposure to the risk of an underlying asset or interest rate.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 68.1

1. **B** Basis risk arises when the underlying of a derivative differs from a position being hedged. Ease of taking short positions with derivatives compared to their underlying assets, and the information about implied volatility that is revealed by option prices, are two of the advantages of derivative instruments. (LOS 68.a)
2. **B** Modifying the risk exposure of a securities portfolio is an example of derivatives use by investors. Hedging against price risk for inventory and stabilizing the balance sheet value of a foreign subsidiary are examples of derivatives use by issuers. (LOS 68.b)

READING 69

ARBITRAGE, REPLICATION, AND THE COST OF CARRY IN PRICING DERIVATIVES

MODULE 69.1: ARBITRAGE, REPLICATION, AND CARRYING COSTS



Video covering this content is available online.

LOS 69.a: Explain how the concepts of arbitrage and replication are used in pricing derivatives.

In contrast to valuing risky assets as the (risk-adjusted) present value of expected future cash flows, the valuation of derivative securities is based on a **no-arbitrage condition**. *Arbitrage* refers to a transaction in which an investor purchases one asset or portfolio of assets at one price and simultaneously sells an asset or portfolio of assets that has the same future payoffs, regardless of future events, at a higher price, realizing a risk-free gain on the transaction.

While arbitrage opportunities may be rare, the reasoning is that when they do exist, they will be exploited rapidly. Therefore, we can use a no-arbitrage condition to determine the current value of a derivative, based on the known value of a portfolio of assets that has the same future payoffs as the derivative, regardless of future events. Because there are transaction costs of exploiting an arbitrage opportunity, small differences in price may persist when the arbitrage gain is less than the transaction costs of exploiting it.

We can illustrate no-arbitrage pricing with a 1-year forward contract, with a forward price of $F_0(1)$, on an Acme share that pays no dividends and is trading at a current price, S_0 , of \$30.

Consider two strategies to own an Acme share at $t = 1$:

- *Portfolio 1*: Buy a pure discount bond with a yield of 5% that pays $F_0(1)$ at $t = 1$. The current cost of the bond is $F_0(1)/1.05$. Additionally, enter a forward contract on one Acme share at $F_0(1)$ as the buyer. The forward has a zero cost, so the cost of Portfolio 1 is $F_0(1)/1.05$.

At $t = 1$ the bond pays $F_0(1)$, which will buy an Acme share at the forward price, so that the payoff on Portfolio 1 is the value of one share at $t = 1$, S_1 .

- *Portfolio 2:* Buy a share of Acme at $S_0 = 30$ and hold it for one year. Cost at $t = 0$ is \$30.

At $t = 1$ the value of the Acme share is S_1 , and this is the payoff for Portfolio 2.

The no-arbitrage condition (law of one price) requires that two portfolios with the same payoff in the future for any future value of Acme have the same cost today. Because our two portfolios have a payoff of S_1 , they must have the same cost at $t = 0$ to prevent arbitrage. That is, $F_0(1)/1.05 = \$30$, so we can solve for the no-arbitrage forward price as $F_0(1) = 30(1.05) = 31.50$.

To better understand the no-arbitrage condition, we will consider two situations in which the forward price is not at its no-arbitrage value: $F_0(1) > 31.50$ and $F_0(1) < 31.50$.

- If the forward contract price is 32 ($F_0(1) > 31.50$), the profitable arbitrage is to sell the forward (because the forward price is “too high”) and buy a share of stock. At $t = 1$, deliver the share under the forward contract and receive 32, for a return of $32/30 - 1 = 6.67\%$, which is higher than the risk-free rate.

We can also view this transaction as borrowing 30 at the risk-free rate (5%) to buy the Acme share at $t = 0$, and at $t = 1$ paying 31.50 to settle the loan. The share delivered under the forward has a contract price of 32, so the arbitrageur has an arbitrage profit of $32 - 31.50 = 0.50$ with no risk and no initial cost.

- If the forward contract price is 31 ($F_0(1) < 31.50$), the profitable arbitrage is to buy the forward and sell short an Acme share at $t = 0$. The proceeds of the short sale, 30, can be invested at the risk-free rate to produce $30(1.05) = 31.50$ at $t = 1$. The forward contract requires the purchase of a share of Acme for 31, which the investor can return to close out the short position. The profit to an arbitrageur is $31.50 - 31 = 0.50$. With no cash investment at $t = 0$, the investor receives an arbitrage profit of 0.50 at $t = 1$.

When the forward price is “too high,” the arbitrage is to sell the forward and buy the underlying asset. When the forward price is “too low,” the arbitrage is to buy the forward and sell (short) the underlying asset. In either case, the actions of arbitrageurs will move the forward price toward its no-arbitrage level until arbitrage profits are no longer possible.

Replication refers to creating a portfolio with cash market transactions that has the same payoffs as a derivative for all possible future values of the underlying.

Our arbitrage example for Acme forwards will serve to illustrate replication.

A long forward on an Acme share can be replicated by borrowing 30 at 5% to purchase an Acme share, and repaying the loan on the settlement date of the forward. At settlement ($t = 1$), the payoff on the replication is $S_1 - 30(1.05) = S_1 - 31.50$ (value of one share minus the repayment of the loan), the same as the payoff on a long forward at 31.50, for any value of Acme shares at settlement.

A short forward on an Acme share can be replicated by shorting an Acme share and investing the proceeds of 30 at 5%. At settlement the investor receives 31.50 from the investment of short sale proceeds, and must buy a share of Acme for S_1 . The payoff on the replicating portfolio is $31.50 - S_1$, the same as the payoff on a short forward at 31.50, for any value of Acme shares at settlement.

These replications allow us to calculate the no-arbitrage forward price of an asset, just as we did in our example using Acme shares. Because our replicating portfolio for a long forward has the same payoff as a long forward at time = T , the payoff at settlement on a portfolio that is long the replicating portfolio and short the forward must be zero to prevent arbitrage. For this strategy, when the forward is priced at its no-arbitrage value the payoff at time = T is:

$$S_T - S_0(1 + Rf)^T - [S_T - F_0(T)] = 0$$

so that $F_0(T) - S_0(1 + Rf)^T = 0$ and $F_0(T) = S_0(1 + Rf)^T$.

For a portfolio that is short the replicating portfolio and long the forward, the payoff at time T is:

$$S_0(1 + Rf)^T - S_T + [S_T - F_0(T)] = 0$$

so that $S_0(1 + Rf)^T - F_0(T) = 0$ and $F_0(T) = S_0(1 + Rf)^T$.

The forward price that will prevent arbitrage is $S_0(1 + Rf)^T$, just as we found in our example of a forward contract on an Acme share.

LOS 69.b: Explain the difference between the spot and expected future price of an underlying and the cost of carry associated with holding the underlying asset.

When we derived the no-arbitrage forward price for an asset as $F_0(T) = S_0(1 + Rf)^T$, we assumed there were no benefits of holding the asset and no costs of holding the asset, other than the opportunity cost of the funds to purchase the asset (the risk-free rate of interest).

Any additional costs or benefits of holding the underlying asset must be accounted for in calculating the no-arbitrage forward price. There may be additional costs of owning an asset, especially with commodities, such as storage and insurance costs. For financial assets, these costs are very low and not significant.

There may also be monetary benefits to holding an asset, such as dividend payments for equities and interest payments for debt instruments. Holding commodities may have non-monetary benefits, referred to as **convenience yield**. If an asset is difficult to sell short in the market, owning it may convey benefits in circumstances where selling the asset is advantageous. For example, a shortage of the asset may drive prices up temporarily, making sale of the asset in the short term profitable.

We denote the present value of any costs of holding the asset from time 0 to settlement at time T (e.g., storage, insurance, spoilage) as $PV_0(\text{cost})$, and the present value of any cash flows from the asset or convenience yield over the holding period as $PV_0(\text{benefit})$.

Consider first a case where there are storage costs of holding the asset, but no benefits. For an asset with no costs or benefits of holding the asset, we established the no-arbitrage forward price as $S_0(1 + R_f)^T$, the cost of buying and holding the underlying asset until time T . When there are storage costs to hold the asset until time T , an arbitrageur must both buy the asset and pay the present value of storage costs at $t = 0$. This increases the no-arbitrage price of a 1-year forward to $[S_0 + PV_0(\text{cost})](1 + R_f)^T$. Here we see that *costs of holding an asset increase its no-arbitrage forward price*.

Next consider a case where holding the asset has benefits, but no costs. Returning to our example of a 1-year forward on a share of Acme stock trading at 30, now consider the costs of buying and holding an Acme share that pays a dividend of \$1 during the life of the forward contract. In this case, an arbitrageur can now borrow the present value of the dividend (discounted at R_f), and repay that loan when the dividend is received. The cost to buy and hold Acme stock with an annual dividend of \$1 is $[30 - PV_0(1)](1.05) = 30(1.05) - 1$. This illustrates that *benefits of holding an asset decrease its no-arbitrage forward price*.

The no-arbitrage price of a forward on an asset that has both costs and benefits of holding the asset is simply $[S_0 + PV_0(\text{costs}) - PV_0(\text{benefit})](1 + R_f)^T$.

We can also describe these relationships when costs and benefits are expressed as continuously compounded rates of return. Recall from Quantitative Methods that given a stated annual rate of r with continuous compounding, the effective annual return is $e^r - 1$, and the relationships between present and future values of S for a 1-year period are $FV = Se^r$ and $PV = Se^{-r}$. For a period of T years, $FV = Se^{rT}$ and $PV = Se^{-rT}$. With continuous compounding the following relationships hold:

- With no costs or benefits of holding the underlying asset, the no-arbitrage price of a forward that settles at time T is S_0e^{rT} , where r is the stated annual risk-free rate with continuous compounding.
- With storage costs at a continuously compounded annual rate of c , the no-arbitrage forward price until time T is $S_0e^{(r+c)T}$.
- With benefits, such as a dividend yield, expressed at a continuously compounded annual rate of b , the no-arbitrage forward price is until time T is $S_0e^{(r+c-b)T}$.

EXAMPLE: No-arbitrage price with continuous compounding

Consider a stock index trading at 1,550 with a dividend yield of 1.3% (continuously compounded rate) when the risk-free rate is 3% (continuously compounded rate). Calculate the no-arbitrage 6-month forward price of the stock index.

Answer:

The no-arbitrage price of a long 6-month forward is $1,550 \times e^{(0.03 - 0.013)(0.5)} = 1,563.23$.

Forward Contracts on Currencies

Recall from Economics that we defined the no-arbitrage price of a forward on a currency as the forward price that satisfies the equality:

$$\text{forward exchange rate (p/b)} = \frac{1 + \text{interest rate}_{\text{price currency}}}{1 + \text{interest rate}_{\text{base currency}}} \times \text{spot exchange rate}$$

We can use this no-arbitrage forward rate to examine how an arbitrage profit can be made when the exchange rate in a forward contract is greater or less than the no-arbitrage forward exchange rate. The forward exchange rate depends on the spot exchange rate and the *difference* between the interest rates on the base and price currencies.

Consider a situation at $t = 0$ where the risk-free rate in euros is 3%, the risk-free rate in U.S. dollars is 2%, and the current USD/EUR exchange rate is 1.10. We will examine the arbitrage transactions that establish this relationship by looking at the trades for an investor based in the United States that seeks to profit from the higher interest rate on euros. The investor borrows 100 USD for one year at 2%, exchanges the USD for euros, invests the euros for one year at 3%, and then exchanges the resulting euros for USD. At the end of one year the arbitrageur will have $100/1.10 \times 1.03 = 93.64$ euros and owe $100(1.02) = 102$ USD.

As these transactions have no net cost, there should be no gain from this transaction relative to simply investing the USD for one year at 2%. If this is the case, the 93.64 euros should equal 102 USD. This is the case if the exchange rate at the end of the year is $102/93.64$, which equals a USD/EUR exchange rate of 1.0893. This is the no-arbitrage forward rate. From the formula we saw in Economics we can arrive at the same solution by $1.10 \times (1.02/1.03) = 1.0893$.

If the arbitrageur has a forward contract to buy USD with a price of $1/1.0893 = 0.9180$ euros, he can exchange the 93.64 euros for $93.64/0.9180 = 102$ USD, which is the amount owed on the original loan of 100 USD. The depreciation of the euro in the forward price just offsets the higher euro interest, and the arbitrage transaction returns zero. With a forward exchange rate greater than 1.0893, the arbitrage would have a profit, and with a forward exchange rate less than 1.0893, an arbitrageur could profit from the opposite transactions.

If we convert the effective annual rates to equivalent stated annual rates with continuous compounding, we get $R_{\text{USD}} = \ln 1.02 = 1.98\%$ and $R_{\text{EUR}} = \ln 1.03 = 2.96\%$. In this case we can say: forward exchange rate = $1.10 \times e^{(0.0198 - 0.0296)} = 1.0893$.



MODULE QUIZ 69.1

- Derivatives pricing models use the risk-free rate to discount future cash flows because these models:
 - are based on portfolios with certain payoffs.
 - assume that derivatives investors are risk-neutral.
 - assume that risk can be eliminated by diversification.
- Arbitrage prevents:
 - market efficiency.

- B. earning returns higher than the risk-free rate of return.
 - C. two assets with identical payoffs from selling at different prices.
3. The underlying asset of a derivative is *most likely* to have a convenience yield when the asset:
- A. is difficult to sell short.
 - B. pays interest or dividends.
 - C. must be stored and insured.
4. An investor can replicate a long forward on a stock that pays no dividends by:
- A. selling the underlying short and investing the proceeds at the risk-free rate.
 - B. buying the underlying in the spot market and holding it.
 - C. borrowing at the risk-free rate to buy the underlying.
5. The forward price of a commodity will *most likely* be equal to the current spot price if the:
- A. convenience yield equals the storage costs as a percentage.
 - B. convenience yield is equal to the risk-free rate plus storage costs as a percentage.
 - C. risk-free rate equals the storage costs as a percentage minus the convenience yield.

KEY CONCEPTS

LOS 69.a

Valuation of derivative securities is based on a no-arbitrage condition. When the forward price is too high, the arbitrage is to sell the forward and buy the underlying asset. When the forward price is too low, the arbitrage is to buy the forward and sell short the underlying asset. Arbitrage will move the forward price toward its no-arbitrage level.

Replication refers to creating a portfolio with cash market transactions that has the same payoffs as a derivative for all possible future values of the underlying. Replication allows us to calculate the no-arbitrage forward price of an asset.

LOS 69.b

Assuming no costs or benefits of holding the underlying asset, the forward price that will prevent arbitrage is the spot price compounded at the risk-free rate over the time until expiration.

The cost of carry is the benefits of holding the asset minus the costs of holding the asset.

Greater costs of holding an asset increase its no-arbitrage forward price.

Greater benefits of holding an asset decrease its no-arbitrage forward price.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 69.1

1. **A** Derivatives pricing models use the risk-free rate to discount future cash flows because they are based on arbitrage relationships that are theoretically riskless. (LOS 69.a)
2. **C** Arbitrage forces two assets with the same expected future value to sell for the same current price. (LOS 69.a)
3. **A** Convenience yield refers to nonmonetary benefits from holding an asset. One example of convenience yield is the advantage of owning an asset that is difficult to sell short when it is perceived to be overvalued. Interest and dividends are monetary benefits. Storage and insurance are carrying costs. (LOS 69.b)
4. **C** Borrowing S_0 at R_f to buy the underlying asset at S_0 has a zero cost and pays the spot price of the underlying asset minus the loan repayment of at time $= T$ of $S_0(1 + R_f)^T$, which is the same payoff as a long forward at $F_0 = S_0(1 + R_f)^T$, the no-arbitrage forward price. (LOS 69.a)
5. **B** When the opportunity cost of funds (R_f) and storage costs just offset the benefits of holding the commodity, the no-arbitrage forward price is equal to the current spot price of the underlying commodity. (LOS 69.b)

READING 70

PRICING AND VALUATION OF FORWARD CONTRACTS AND FOR AN UNDERLYING WITH VARYING MATURITIES

MODULE 70.1: FORWARD CONTRACT VALUATION



Video covering this content is available online.

LOS 70.a: Explain how the value and price of a forward contract are determined at initiation, during the life of the contract, and at expiration.

Consider a forward contract that is initially priced at its no-arbitrage value of $F_0(T) = S_0(1 + R_f)^T$. At initiation, the value of such a forward is: $V_0(T) = S_0 - F_0(T)(1 + R_f)^{-T} = 0$.

At any time during its life, the value of the forward contract to the buyer will be $V_t(T) = S_t - F_0(T)(1 + R_f)^{-(T-t)}$. This is simply the current spot price of the asset minus the present value of the forward contract price.

This value can be realized by selling the asset short at S_t and investing $F_0(T)(1 + R_f)^{-(T-t)}$ in a pure discount bond at R_f . These transactions end any exposure to the forward; at settlement, the proceeds of the bond will cover the cost of the asset at the forward price, and the asset can be delivered to cover the short position.

At expiration, time T , the value of a forward to the buyer is $= S_T - F_0(T)(1 + R_f)^{-(T-T)} = S_T - F_0(T)$. The long buys an asset valued at S_T for the forward contract price of $F_0(T)$, gaining if $S_T > F_0(T)$, losing if $S_T < F_0(T)$. If the forward buyer has a gain, the forward seller has an equal loss, and vice versa.

In the more general case, when there are costs and benefits of holding the underlying asset, the value of a forward to the buyer at time $t < T$ is:

$$V_t(T) = [S_t + PV_t(\text{costs}) - PV_t(\text{benefit})] - F_0(T)(1 + R_f)^{-(T-t)}$$

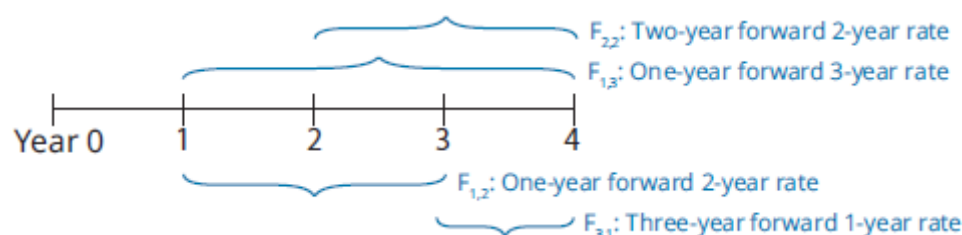
LOS 70.b: Explain how forward rates are determined for interest rate forward

contracts and describe the uses of these forward rates.

Forward rates are yields for future periods. The rate of interest on a 1-year loan to be made two years from today is a forward rate.

The notation for forward rates must identify both the length of the loan period and how far in the future the money will be loaned (or borrowed). 1y1y or $F_{1,1}$ is the rate for a 1-year loan one year from now; 2y1y or $F_{2,1}$ is the rate for a 1-year loan to be made two years from now; the 2-year forward rate three years from today is 3y2y or $F_{3,2}$; and so on.

Figure 70.1: Forward Rates



For money market rates the notation is similar, with 3m6m denoting a 6-month rate three months in the future.

Recall that spot rates are zero-coupon rates. We will denote the YTM (with annual compounding) on a zero-coupon bond maturing in n years as Z_n .

An **implied forward rate** is the forward rate for which the following two strategies have the same yield over the total period:

- Investing from $t = 0$ to the forward date, and rolling over the proceeds for the period of the forward.
- Investing from $t = 0$ until the end of the forward period.

As an example, lending for two years at Z_2 would produce the same ending value as lending for one year at Z_1 and, at $t = 1$, lending the proceeds of that loan for one year at $F_{1,1}$. That is, $(1 + Z_2)^2 = (1 + Z_1)(1 + F_{1,1})$. When this condition holds, $F_{1,1}$ is the implied (no-arbitrage) forward rate.

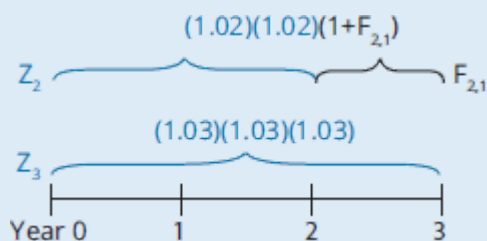
EXAMPLE: Implied forward rate

Consider two zero-coupon bonds, one that matures in two years and one that matures in three years, when $Z_2 = 2\%$ and $Z_3 = 3\%$. Calculate the implied 1-year forward rate two years from now, $F_{2,1}$.

Answer:

As illustrated in Figure 70.2, lending for three years at Z_3 should be equivalent to lending for two years at Z_2 and then for the third year at $F_{2,1}$.

Figure 70.2: Implied Forward Rate



Lending \$100 for two years at Z_2 (2%) results in a payment of $\$100(1.02)^2 = \104.04 at $t = 2$, while lending \$100 for three years at Z_3 (3%) results in a payment of $\$100(1.03)^3 = \109.27 . The forward interest rate $F_{2,1}$ must be $109.27/104.04 - 1 = 5.03\%$, the implied forward rate from $t = 2$ to $t = 3$.

An example of an interest rate derivative is a **forward rate agreement (FRA)**, in which the fixed-rate payer (long) will pay the forward rate on a notional amount of principal at a future date, and the floating-rate payer will pay a future reference rate times that same amount of principal. In practice, only the net amount is exchanged.

Consider a 3-month forward on a 6-month MRR (F_{3m6m}) with a notional principal of \$1 million. At settlement in three months, the buyer receives (or pays) the present value of $(\text{realized 6-month MRR} - 1\%)/2 \times \1 million .

We divide by 2 because MRRs are typically annualized rates. We take the present value of the difference in interest because the settlement payment is at the beginning of the 6-month period, whereas the interest savings would be at the end of the period.

Assume that the current 3-month MRR is 1.0% and 9-month MRR is 1.2%. Adjusting for periodicity, the no-arbitrage condition for the value of F_{3m6m} is:

$$1 + 0.012\left(\frac{9}{12}\right) = \left[1 + 0.01\left(\frac{3}{12}\right)\right]\left[1 + F_{3m,6m}\left(\frac{6}{12}\right)\right]$$

The implied forward rate, F_{3m6m} , as an annualized rate, is:

$$F_{3m,6m} = \left[\frac{1 + 0.012\left(\frac{9}{12}\right)}{1 + 0.01\left(\frac{3}{12}\right)} - 1 \right] \times \frac{12}{6} = 0.013$$

Now let's examine the payoff to the fixed-rate payer in an F_{3m6m} FRA with a notional principal of \$1 million when the 6-month MRR three months from now is 1.5%.

Because the realized 6-month MRR is greater than the forward rate, the fixed-rate payer (floating-rate receiver) will have a gain.

The payment to the fixed-rate payer is the present value (discounted at 6-month MRR) of the interest differential between two 6-month loans, one at 1.3% and one at 1.5% (both annualized rates). The fixed-rate payer in the FRA receives:

$$\$1 \text{ million} \times \left(\frac{0.015 - 0.013}{2} \right) \left(\frac{1}{1 + (0.015/2)} \right) = \$992.56$$

FRAs are used primarily by financial institutions to manage the volatility of their interest-sensitive assets and liabilities. FRAs are also the building blocks of interest rate swaps over multiple periods. An FRA is equivalent to a single-period swap. Multiple-period swaps are used primarily by investors and issuers to manage interest rate risk.



MODULE QUIZ 70.1

1. Two parties agree to a forward contract to exchange 100 shares of a stock one year from now for \$72 per share. Immediately after they initiate the contract, the price of the underlying stock increases to \$74 per share. This share price increase represents a gain for:
 - A. the buyer.
 - B. the seller.
 - C. neither the buyer nor the seller.
2. The forward rate 2y3y represents the interest rate on a loan period that begins:
 - A. 2 years from now and ends 3 years from now.
 - B. 2 years from now and ends 5 years from now.
 - C. 3 years from now and ends 5 years from now.
3. Given zero-coupon bond yields for 1, 2, and 3 years, an analyst can *least likely* derive an implied:
 - A. 1-year forward 1-year rate.
 - B. 2-year forward 1-year rate.
 - C. 2-year forward 2-year rate.

KEY CONCEPTS

LOS 70.a

The value of a forward contract at initiation is zero.

During its life, the value of a forward contract to the buyer is the spot price of the asset minus the present value of the forward contract price, and the value to the seller is the present value of the forward contract price minus the spot price of the asset.

At expiration, the value of a forward contract to the buyer is the spot price of the asset minus the forward contract price, and the value to the seller is the forward contract price minus the spot price of the asset.

LOS 70.b

An implied forward rate is the forward rate for which the following two strategies have the same yield over the total period:

- Investing from $t = 0$ to the forward date, and rolling over the proceeds for the period of the forward.
- Investing from $t = 0$ until the end of the forward period.

In a forward rate agreement (FRA), the fixed-rate payer (long) will pay the forward rate on a notional amount of principal at a future date, and the floating-rate payer will pay a future reference rate times that same amount of principal. FRAs are used primarily by

financial institutions to manage the volatility of their interest-sensitive assets and liabilities.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 70.1

1. **A** If the value of the underlying is greater than the forward price, this increases the value of the forward contract, which represents a gain for the buyer and a loss for the seller. (LOS 70.a)
2. **B** 2y3y is the 2-year forward 3-year rate, covering a period that begins two years from now and extends for three years after that. (LOS 70.b)
3. **C** The forward rate $F_{2,2}$ extends four years into the future and cannot be derived using zero-coupon yields that only extend three years. From zero-coupon bond yields for 1, 2, and 3 years, we can derive implied forward rates $F_{1,1}$, $F_{1,2}$, and $F_{2,1}$. (LOS 70.b)

READING 71

PRICING AND VALUATION OF FUTURES CONTRACTS

MODULE 71.1: FUTURES VALUATION

LOS 71.a: Compare the value and price of forward and futures contracts.



Video covering this content is available online.

While the *price* of a forward contract is constant over its life when no mark-to-market gains or losses are paid, its *value* will fluctuate with changes in the value of the underlying. The payment at settlement of the forward reflects the difference between the (unchanged) forward price and the spot price of the underlying.

The price and value of a futures contract *both* change when daily mark-to-market gains and losses are settled. Consider a futures contract on 100 ounces of gold at \$1,870 purchased on Day 0. The following illustrates the changes in contract price and value with daily mark-to-market payments.

Day 0	Price = settlement price of 1,870	MTM value = 0
Day 1	Settlement price = 1,875 \$500 addition to margin account New futures price = 1,875	MTM value = \$500 MTM value = 0
Day 2	Settlement price = 1,855 \$2,000 deduction from margin account New futures price = 1,855	MTM value = -\$2,000 MTM value = 0

The change in the futures price to the settlement price each day returns its value to zero. Prices of forward contracts for which mark-to-market gains and losses are settled daily will also be adjusted to the settlement price.

Interest rate futures contracts are available on many market reference rates. We may view these as exchange-traded equivalents to forward rate agreements. One key difference is that interest rate futures are quoted on a price basis. For a market reference rate from time A to time B , an interest rate futures price is stated as follows:

$$\text{futures price} = 100 - (100 \times \text{MRR}_{A, B-A})$$

For example, if the futures price for a 6-month rate six months from now is 97, then $MRR_{6m, 6m} = 3\%$.

Like other futures contracts, interest rate futures are subject to daily mark-to-market. The **basis point value (BPV)** of an interest rate futures contract is defined as:

$$BPV = \text{notional principal} \times \text{period} \times 0.01\%$$

If the contract in our example is based on notional principal of €1,000,000, its BPV is $€1,000,000 \times (0.0001 / 2) = €50$. This means a one basis point change in the MRR will change the futures contract value by €50.

LOS 71.b: Explain why forward and futures prices differ.

For pricing, the most important distinction between futures and forwards is that with futures, mark-to market gains and losses are paid each day. Gains above initial margin can be withdrawn from a futures account and losses that reduce margin deposits below their maintenance level require payments into the account. Forwards most often have no mark-to-market cash flows, with gains or losses settled at contract expiration. Forwards typically do not require or provide funds in response to fluctuations in value during their lives.

If interest rates are constant or uncorrelated with futures prices over time, the prices of futures and forwards are the same. A positive correlation between interest rates and the futures price means that (for a long position) daily settlement provides funds (excess margin) when rates are high and they can earn more interest, and requires funds (margin deposits) when rates are low and opportunity cost of deposited funds is less. Because of this, futures are theoretically more attractive than forwards when interest rates and futures prices are positively correlated, and less attractive than forwards when interest rates and futures prices are negatively correlated.

Because of the short maturity of most forwards and the availability of funds at near risk-free rates, differences between equivalent forwards and futures are not observed in practice. Additionally, derivative dealers in some markets with central clearing are required to post margin and may require derivative investors to post mark-to-market margin payments as well.

A separate issue arises for interest rate forwards and futures settlement payments. Recall that the payoff on an interest rate forward is the present value (at the beginning of the forward period) of any interest savings (at the end of the forward period) from the difference between the realized MRR and the forward MRR. Because the realized MRR is the discount rate for calculating the payment for a given amount of future interest savings, the payment for an increase in the MRR will be less than the payment for an equal decrease in the MRR, as the following example will illustrate.

Consider a \$1 million interest rate future on a 6-month MRR priced at 97.50 (an MRR of 2.5%) that settles six months from now. Each basis point change in the (annualized) MRR will change the value of the contract by $0.0001 \times 6/12 \times \$1 \text{ million} = \$50$. If the MRR at settlement is either 2.51% or 2.49%, the payoff on the future at the end of one year is either \$50 higher or \$50 lower than when the MRR at settlement is 2.5%.

Compare this result with the payoffs for an otherwise equivalent forward, F_{6m6m} , priced at 2.5%.

If the MRR at settlement is 2.51%, the long receives $50/(1 + 0.0251/2) = \$49.3803$.

If the MRR at settlement is 2.49%, the long must pay $50/(1 + 0.0249/2) = \$49.3852$.

The value of the forwards exhibit convexity. An increase in rates decreases the forward's value by less than a decrease in the interest rate increases the forward's value, just as we saw with bonds. Also just as with bonds, the convexity effect for the value of forwards increases for longer periods. The convexity of forwards is termed **convexity bias** and forwards and futures prices can be significantly different for longer-term interest rates.



MODULE QUIZ 71.1

1. For a forward contract on an asset that has no costs or benefits from holding it to have zero value at initiation, the arbitrage-free forward price must equal the:
 - A. expected future spot price.
 - B. future value of the current spot price.
 - C. present value of the expected future spot price.
2. For a futures contract to be more attractive than an otherwise equivalent forward contract, interest rates must be:
 - A. uncorrelated with futures prices.
 - B. positively correlated with futures prices.
 - C. negatively correlated with futures prices.

KEY CONCEPTS

LOS 71.a

For a forward contract on which no mark-to-market gains or losses are paid, the forward price is constant over its life, but the contract's value will fluctuate with changes in the value of the underlying.

For a futures contract, the price and value both change when daily mark-to-market gains and losses are settled. The change in the futures price to the settlement price each day returns its value to zero.

Unlike forward rate agreements, interest rate futures are quoted on a price basis:

$$\text{futures price} = 100 - (100 \times \text{MRR}_{A, B-A})$$

LOS 71.b

Because gains and losses on futures contracts are settled daily, prices of forwards and futures that have the same terms may be different if interest rates are correlated with futures prices. Futures are more valuable than forwards when interest rates and futures prices are positively correlated and less valuable when they are negatively correlated. If interest rates are constant or uncorrelated with futures prices, the prices of futures and forwards are the same.

Convexity bias can result in price differences between interest rate futures contracts and otherwise equivalent forward rate agreements.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 71.1

1. **B** For an asset with no holding costs or benefits, the forward price must equal the future value of the current spot price, compounded at the risk-free rate over the term of the forward contract, for the contract to have a value of zero at initiation. Otherwise an arbitrage opportunity would exist. (LOS 71.a)
2. **B** If interest rates are positively correlated with futures prices, interest earned on cash from daily settlement gains on futures contracts will be greater than the opportunity cost of interest on daily settlement losses, and a futures contract is more attractive than an otherwise equivalent forward contract that does not feature daily settlement. (LOS 71.b)

READING 72

PRICING AND VALUATION OF INTEREST RATES AND OTHER SWAPS

MODULE 72.1: SWAP VALUATION



Video covering this content is available online.

LOS 72.a: Describe how swap contracts are similar to but different from a series of forward contracts.

In a simple interest-rate swap, one party pays a floating rate and the other pays a fixed rate on a notional principal amount. Consider a 1-year swap with quarterly payments, one party paying a fixed rate and the other a floating rate equal to a 90-day market reference rate (MRR). At each payment date the difference between the swap fixed rate and the MRR is paid to the party that owes the least, that is, a net payment is made from one party to the other.

We can separate these payments into a known payment and three unknown payments that are equivalent to the payments on three forward rate agreements (FRAs). Let MRR_n represent the floating rate payment (based on the 90-day MRR) owed at the end of quarter n and F be the fixed payment owed at the end of each quarter. We can represent the swap payment to be received by the fixed-rate payer at the end of period n as $MRR_n - F$. We can replicate each of these payments to (or from) the fixed-rate payer in the swap with a forward contract, specifically a long position in an FRA with a contract rate equal to the swap fixed rate and a settlement value based on the 90-day MRR.

We illustrate this separation below for a 1-year fixed-for-floating swap with a fixed rate of F and floating-rate payments for period n of MRR_n . Note that if the fixed rate and MRR are quoted as annual rates, the payments will be $(MRR_n - F)$ times one-fourth of the notional principal.

First payment (90 days from now) = $MRR_1 - F$ which is known at time zero because the payment 90 days from now is based on the 90-day MRR at time zero and the swap fixed rate, F , both of which are known at the initiation of the swap.

Second payment (180 days from now) is equivalent to a long position in an FRA with contract rate F that settles in 180 days and pays $MRR_2 - F$.

Third payment (270 days from now) is equivalent to a long position in an FRA with contract rate F that settles in 270 days and pays $MRR_3 - F$.

Fourth payment (360 days from now) is equivalent to a long position in an FRA with contract rate F that settles in 360 days and pays $MRR_4 - F$.

Note that a forward on a 90-day MRR that settles 90 days from now, based on the 90-day MRR at that time, actually pays the present value of the difference between the fixed rate F and the 90-day MRR 90 days from now (times the notional principal amount). Thus, the forwards in our example actually pay on days 90, 180, and 270. However, the amounts paid are equivalent to the differences between the fixed-rate payment and floating-rate payment that are due when interest is actually due on days 180, 270, and 360, which are the amounts we used in the example.

Therefore, we can describe an interest-rate swap as equivalent to a series of forward contracts, specifically FRAs, each with a forward contract rate equal to the swap fixed rate. However, there is one important difference. Because the forward contract rates are all equal in the FRAs that are equivalent to the swap, these would not be zero-value forward contracts at the initiation of the swap. Recall that forward contracts are based on a contract rate for which the value of the forward contract at initiation is zero. There is no reason to suspect that the swap fixed rate results in a zero value forward contract for each of the future dates. Instead, a swap is most likely to consist of some forwards with positive values and some forwards with negative values. The sum of their values will equal zero at initiation.

Finding the swap fixed rate that gives the swap a zero value at initiation, which is also known as the **par swap rate**, is not difficult if we follow our principle of no-arbitrage pricing. The fixed rate payer in a swap can replicate that derivative position by borrowing at a fixed rate and lending the proceeds at a variable (floating) rate. For the swap in our example, borrowing at the fixed rate F and lending the proceeds at the 90-day MRR will produce the same cash flows as the swap. At each date, the payment due on the fixed-rate loan is F_n and the interest received on lending at the floating rate is MRR_n .

LOS 72.b: Contrast the value and price of swaps.

As with FRAs, the *price* of a swap is the fixed rate of interest specified in the swap contract (the par swap rate) and the *value* depends on how expected future floating rates change over time. At initiation, a swap has zero value because the present value of the fixed-rate payments equals the present value of the expected floating-rate payments.

We can solve for the no-arbitrage fixed rate, termed the **par swap rate**, from the following equality:

$$\frac{MRR_1}{1+S_1} + \frac{MRR_2}{(1+S_2)^2} + \frac{MRR_3}{(1+S_3)^3} + \frac{MRR_4}{(1+S_4)^4} = \frac{F}{1+S_1} + \frac{F}{(1+S_2)^2} + \frac{F}{(1+S_3)^3} + \frac{F}{(1+S_4)^4}$$

where S_1 through S_4 are the current effective spot rates for 90, 180, 270, and 360 days, MRR_1 through MRR_4 are the forward 90-day rates implied by the spot rates, and F is the fixed rate payment.

Given the current spot rates (S_1 to S_4), we can calculate the implied (expected) forward rates (MRRs), and then solve for F , the fixed rate that will give the swap a value of zero.

An increase in expected future 90-day rates will produce an increase the value of the fixed-rate payer position in a swap, while a decrease in expected rates will decrease the value of that position. At any point in time, the value of the fixed-rate payer side of a swap will be the present value of the expected future floating-rate payments, minus the present value of the future fixed-rate payments. This calculation is based on the spot rates and implied future 90-day rates at that point in time and can be used for any required mark-to-market payments.



MODULE QUIZ 72.1

1. Which of the following is *most* similar to the floating-rate receiver position in a fixed-for-floating interest-rate swap?
 - A. Buying a fixed-rate bond and a floating-rate note.
 - B. Buying a floating-rate note and issuing a fixed-rate bond.
 - C. Issuing a floating-rate note and buying a fixed-rate bond.
2. The price of a fixed-for-floating interest-rate swap:
 - A. is specified in the swap contract.
 - B. is paid at initiation by the floating-rate receiver.
 - C. may increase or decrease during the life of the swap contract.

KEY CONCEPTS

LOS 72.a

In a simple interest-rate swap, one party pays a floating rate and the other pays a fixed rate on a notional principal amount. The first payment is known at initiation and the rest of the payments are unknown. The unknown payments are equivalent to the payments on FRAs. The par swap rate is the fixed rate at which the sum of the present values of these FRAs equals zero.

LOS 72.b

The price of a swap is the fixed rate of interest specified in the swap contract. The value depends on how expected future floating rates change over time. An increase in expected future short-term future rates will increase the value of the fixed-rate payer position in a swap, and a decrease in expected future rates will decrease the value of the fixed-rate payer position.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 72.1

1. **B** The floating-rate receiver (fixed-rate payer) in a fixed-for-floating interest-rate swap has a position similar to issuing a fixed-coupon bond and buying a floating-rate note. (LOS 72.a)
2. **A** The price of a fixed-for-floating interest-rate swap is defined as the fixed rate specified in the swap contract. Typically a swap will be priced such that it has a value of zero at initiation and neither party pays the other to enter the swap. (LOS 72.b)

READING 73

PRICING AND VALUATION OF OPTIONS

MODULE 73.1: OPTION VALUATION



Video covering this content is available online.

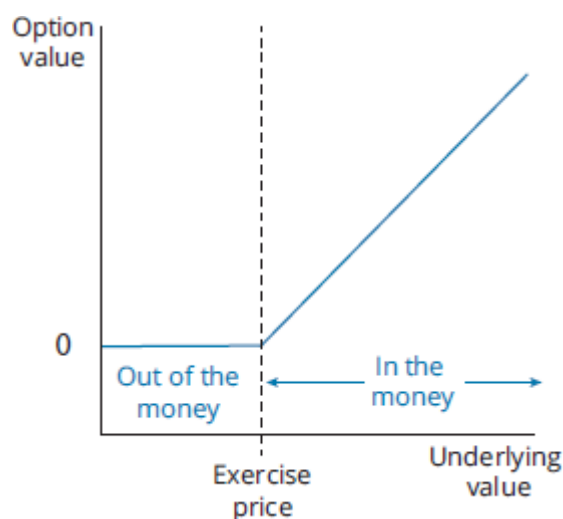
LOS 73.a: Explain the exercise value, moneyness, and time value of an option.

Moneyness refers to whether an option is *in the money* or *out of the money*. If immediate exercise of the option would generate a positive payoff, it is in the money. If immediate exercise would result in a loss (negative payoff), it is out of the money. When the current asset price equals the exercise price, exercise will generate neither a gain nor loss, and the option is *at the money*.

The following describes the conditions for a **call option** to be in, out of, or at the money. S is the price of the underlying asset and X is the exercise price of the option.

- *In-the-money call options.* If $S - X > 0$, a call option is in the money. $S - X$ is the amount of the payoff a call holder would receive from immediate exercise, buying a share for X and selling it in the market for a greater price S .
- *Out-of-the-money call options.* If $S - X < 0$, a call option is out of the money.
- *At-the-money call options.* If $S = X$, a call option is said to be at the money.

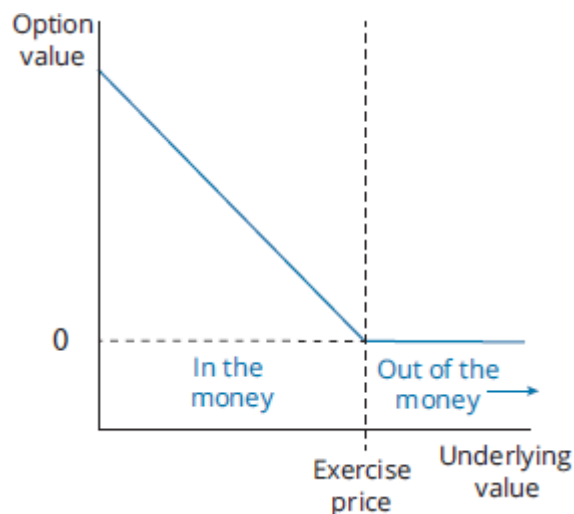
Figure 73.1: Call Option Moneyness



The following describes the conditions for a **put option** to be in, out of, or at the money.

- *In-the-money put options.* If $X - S > 0$, a put option is in the money. $X - S$ is the amount of the payoff from immediate exercise, buying a share for S and exercising the put to receive X for the share.
- *Out-of-the-money put options.* When the stock's price is greater than the exercise price, a put option is said to be out of the money. If $X - S < 0$, a put option is out of the money.
- *At-the-money put options.* If $S = X$, a put option is said to be at the money.

Figure 73.2: Put Option Moneyness



EXAMPLE: Moneyness

Consider a July 40 call and a July 40 put, both on a stock that is currently selling for \$37/share. Calculate how much these options are in or out of the money.



PROFESSOR'S NOTE

A July 40 call is a call option with an exercise price of \$40 and an expiration date in July.

Answer:

The call is \$3 out of the money because $S - X = -\$3.00$. The put is \$3 in the money because $X - S = \$3.00$.

We define the **exercise value** (or **intrinsic value**) of an option as the maximum of zero and the amount that the option is in the money. That is, the exercise value is the amount an option is in the money, if it is in the money, or zero if the option is at or out of the money. The exercise value is the value of the option if exercised immediately.

Prior to expiration, an option has **time value** in addition to its exercise value. The time value of an option is the amount by which the **option premium** (price) exceeds the exercise value and is sometimes called the *speculative value* of the option. This relationship can be written as:

option premium = exercise value + time value

At any point during the life of an option, its value will typically be greater than its exercise value. This is because there is some probability that the underlying asset price will change in an amount that gives the option a positive payoff at expiration greater than the current exercise value. Recall that an option's exercise value (to a buyer) is the amount of the payoff at expiration and has a lower bound of zero.

When an option reaches expiration, there is no time remaining and the time value is zero. This means the value at expiration is either zero, if the option is at or out of the money, or its exercise value, if it is in the money.

LOS 73.b: Contrast the use of arbitrage and replication concepts in pricing forward commitments and contingent claims.

To model forward commitments, we used no-arbitrage pricing based on an initial value of zero to both parties. With options, however, the initial values of options are positive; the buyer pays a premium (the option price) to the writer (seller). Another difference is that where forward commitments have essentially unlimited gains or losses for both parties (except to the extent that prices are constrained by zero), options are one-sided: Potential losses for the buyer, and potential gains for the writer, are limited to the premium paid. For these reasons, the no-arbitrage approach we use for pricing contingent claims is different from the model we use for forward commitments.

The following is some terminology that we will use to determine the minimum and maximum values for European options:

S_t = price of the underlying stock at time t

X = exercise price of the option

$T-t$ = time to expiration

c_t = price of a European call at any time t prior to expiration at time $= T$

p_t = price of a European put at any time t prior to expiration at time $= T$

R_f = risk-free rate

Upper Bound for Call Options

The maximum value of a European call option at any time t is the time- t share price of the underlying stock. This makes sense because no one would pay more for the right to buy an asset than the asset's market value. It would be cheaper simply to buy the underlying asset. At time $t = 0$, the upper boundary condition for European call options is $c_0 \leq S_0$, and at any time t during a European call option's life, the upper boundary condition is $c_t \leq S_t$.

Upper Bound for Put Options

Logically the value of a put option cannot be more than its exercise price. This would be its exercise value if the underlying stock price goes to zero. However, because European puts cannot be exercised prior to expiration, their maximum value is the

present value of the exercise price discounted at the risk-free rate. Even if the stock price goes to zero and is expected to stay at zero, the put buyer will not receive the intrinsic value, X , until the expiration date.

At time $t = 0$, the upper boundary condition can be expressed for European put options as:

$$p_0 \leq X (1 + R_f)^{-T}$$

At any time t during a European put option's life, the upper boundary condition is:

$$p_t \leq X (1 + R_f)^{-(T-t)}$$

Lower Bounds for Options

Theoretically, no option will sell for less than its intrinsic value and no option can take on a negative value. For European options, however, the lower bound is not so obvious because these options are not exercisable immediately.

To determine the lower bounds for European options, we can examine the value of a portfolio in which the option is combined with a long or short position in the stock and a pure discount bond.

For a *European call option*, construct the following portfolio:

- A long at-the-money European call option with exercise price X , expiring at time T .
- A long discount bond priced to yield the risk-free rate that pays X at option expiration.
- A short position in one share of the underlying stock priced at $S_0 = X$.

The current value of this portfolio is $c_0 - S_0 + X(1 + R_f)^{-T}$.

At expiration time T , this portfolio will pay $c_T - S_T + X$. That is, we will collect $c_T = \max[0, S_T - X]$ on the call option, pay S_T to cover our short stock position, and collect X from the maturing bond.

- If $S_T \geq X$, the call is in-the-money, and the portfolio will have a zero payoff because the call pays $S_T - X$, the bond pays $+X$, and we pay $-S_T$ to cover our short position. That is, the time $t = T$ payoff is: $S_T - X + X - S_T = 0$.
- If $S_T < X$, the call is out-of-the-money, and the portfolio has a positive payoff equal to $X - S_T$ because the call value, c_T , is zero, we collect X on the bond, and pay $-S_T$ to cover the short position. So, the time $t = T$ payoff is: $0 + X - S_T = X - S_T$.

No matter whether the option expires in-the-money, at-the-money, or out-of-the-money, the portfolio value will be equal to or greater than zero. We will never have to make a payment.

To prevent arbitrage, any portfolio that has no possibility of a negative payoff cannot have a negative value. Thus, we can state the value of the portfolio *at time* $t = 0$ as:

$$c_0 - S_0 + X(1 + Rf)^{-T} \geq 0$$

which allows us to conclude that:

$$c_0 \geq S_0 - X(1 + Rf)^{-T}$$

Combining this result with the earlier minimum on the call value of zero, we can write:

$$c_0 \geq \text{Max}[0, S_0 - X(1 + Rf)^{-T}]$$

Note that $X(1 + Rf)^{-T}$ is the present value of a pure discount bond with a face value of X .

For a *European put option* we can derive the minimum value by forming the following portfolio at time $t = 0$:

- A long at-the-money European put option with exercise price X , expiring at T .
- A short position on a risk-free bond priced at $X(1 + Rf)^{-T}$, equivalent to borrowing $X(1 + Rf)^{-T}$.
- A long position in a share of the underlying stock priced at S_0 .

At expiration time T , this portfolio will pay $p_T + S_T - X$. That is, we will collect $p_T = \text{Max}[0, X - S_T]$ on the put option, receive S_T from the stock, and pay X on the bond (loan).

- If $S_T > X$, the payoff will equal: $p_T + S_T - X = S_T - X$.
- If $S_T \leq X$, the payoff will be zero.

Again, a no-arbitrage argument can be made that the portfolio value must be zero or greater, because there are no negative payoffs to the portfolio.

At time $t = 0$, this condition can be written as:

$$p_0 + S_0 - X(1 + Rf)^{-T} \geq 0$$

and rearranged to state the minimum value for a European put option at time $t = 0$ as:

$$p_0 \geq X(1 + Rf)^{-T} - S_0$$

We have now established the minimum bound on the price of a European put option as:

$$p_0 \geq \text{Max}[0, X(1 + Rf)^{-T} - S_0]$$

Figure 73.3 summarizes what we now know regarding the boundary prices for European options at any time t prior to expiration at time $t = T$.

Figure 73.3: Lower and Upper Bounds for Options

Option	Minimum Value	Maximum Value
European call	$c_t \geq \text{Max}[0, S_t - X(1 + Rf)^{-(T-t)}]$	S_t
European put	$p_t \geq \text{Max}[0, X(1 + Rf)^{-(T-t)} - S_t]$	$X(1 + Rf)^{-(T-t)}$



PROFESSOR'S NOTE

For the exam, know the price limits in Figure 73.3. You will not be asked to derive them, but you may be expected to use them.

LOS 73.c: Identify the factors that determine the value of an option and describe how each factor affects the value of an option.

There are six factors that determine option prices.

1. **Price of the underlying asset.** For call options, the higher the price of the underlying, the greater its exercise value and the higher the value of the option. Conversely, the lower the price of the underlying, the less its exercise value and the lower the value of the call option. In general, call option values increase when the value of the underlying asset increases.

For put options this relationship is reversed. An increase in the price of the underlying reduces the value of a put option.

2. **The exercise price.** A higher exercise price decreases the values of call options and a lower exercise price increases the values of call options.

A higher exercise price increases the values of put options and a lower exercise price decreases the values of put options.

3. **The risk-free rate of interest.** An increase in the risk-free rate will increase call option values, and a decrease in the risk-free rate will decrease call option values.

An increase in the risk-free rate will decrease put option values, and a decrease in the risk-free rate will increase put option values.



PROFESSOR'S NOTE

One way to remember the effects of changes in the risk-free rate is to think about present values of the payments for calls and puts. These statements are strictly true only for in-the-money options, but it's a way to remember the relationships. The holder of a call option will pay in the future to exercise a call option and the present value of that payment is lower when the risk-free rate is higher, so a higher risk-free rate increases a call option's value. The holder of a put option will receive a payment in the future when the put is exercised and an increase in the risk-free rate decreases the present value of this payment, so a higher risk-free rate decreases a put option's value.

4. **Volatility of the underlying.** Volatility is what makes options valuable. If there were no volatility in the price of the underlying asset (its price remained constant), options would always be equal to their exercise values and time or speculative value would be zero. An increase in the volatility of the price of the underlying asset increases the values of both put and call options and a decrease in volatility of the price of the underlying decreases both put values and call values.
5. **Time to expiration.** Because volatility is expressed per unit of time, longer time to expiration effectively increases expected volatility and increases the value of a call

option. Less time to expiration decreases the time value of a call option so that at expiration its value is simply its exercise value.

For most put options, longer time to expiration will increase option values for the same reasons. For some European put options, however, extending the time to expiration can decrease the value of the put. In general, the deeper a put option is in the money, the higher the risk-free rate, and the longer the current time to expiration, the more likely that extending the option's time to expiration will decrease its value.

To understand this possibility consider a put option at \$20 on a stock with a value that has decreased to \$1. The exercise value of the put is \$19 so the upside is very limited, the downside (if the price of the underlying subsequently increases) is significant, and because no payment will be received until the expiration date, the current option value reflects the present value of any expected payment. Extending the time to expiration would decrease that present value. While overall we expect a longer time to expiration to increase the value of a European put option, in the case of a deep in-the-money put, a longer time to expiration could decrease its value.

6. **Costs and benefits of holding the asset.** If there are benefits of holding the underlying asset (dividend or interest payments on securities or a convenience yield on commodities), call values are decreased and put values are increased. The reason for this is most easily understood by considering cash benefits. When a stock pays a dividend, or a bond pays interest, this reduces the value of the asset. Decreases in the value of the underlying asset decrease call values and increase put values.

Positive storage costs make it more costly to hold an asset. We can think of this as making a call option more valuable because call holders can have long exposure to the asset without paying the costs of actually owning the asset. Puts, on the other hand, are less valuable when storage costs are higher.



MODULE QUIZ 73.1

1. The price of an out-of-the-money option is:
 - A. less than its time value.
 - B. equal to its time value.
 - C. greater than its time value.
2. The lower bound for the value of a European put option is:
 - A. $\text{Max}(0, S - X)$.
 - B. $\text{Max}[0, X(1 + R_f)^{-(T-t)} - S]$.
 - C. $\text{Max}[0, S - X(1 + R_f)^{-(T-t)}]$.
3. A decrease in the risk-free rate of interest will:
 - A. increase put and call option prices.
 - B. decrease put option prices and increase call option prices.
 - C. increase put option prices and decrease call option prices.

KEY CONCEPTS

LOS 73.a

If immediate exercise of an option would generate a positive payoff, the option is in the money. If immediate exercise would result in a negative payoff, the option is out of the money. An option's exercise value is the greater of zero or the amount it is in the money. Time value is the amount by which an option's price is greater than its exercise value. Time value is zero at expiration.

LOS 73.b

The approach for pricing contingent claims is different from the model for forward commitments because contingent claims have one-sided payoffs and values at initiation that are not equal to zero. A replication model for European options is based on the value of a portfolio in which the option is combined with a pure discount bond and a long or short position in the underlying.

LOS 73.c

Factors that determine the value of an option:

Increase in:	Effect on Call Option Values	Effect on Put Option Values
Price of underlying asset	Increase	Decrease
Exercise price	Decrease	Increase
Risk-free rate	Increase	Decrease
Volatility of underlying asset	Increase	Increase
Time to expiration	Increase	Increase, except some European puts
Costs of holding underlying asset	Increase	Decrease
Benefits of holding underlying asset	Decrease	Increase

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 73.1

1. **B** Because an out-of-the-money option has an exercise value of zero, its price is its time value. (LOS 73.a)
2. **B** The lower bound for a European put ranges from zero to the present value of the exercise price less the current stock price, where the exercise price is discounted at the risk-free rate. (LOS 73.b)
3. **C** A decrease in the risk-free rate will decrease call option values and increase put option values. (LOS 73.c)

READING 74

OPTION REPLICATION USING PUT–CALL PARITY

MODULE 74.1: PUT-CALL PARITY

LOS 74.a: Explain put–call parity for European options.



Video covering
this content is
available online.

Our derivation of **put-call parity** for European options is based on the payoffs of two portfolio combinations: a fiduciary call and a protective put.

A *fiduciary call* is a combination of a call with exercise price X and a pure-discount, riskless bond that pays X at maturity (option expiration). The payoff for a fiduciary call at expiration is X when the call is out of the money, and $X + (S - X) = S$ when the call is in the money.

A *protective put* is a share of stock together with a put option on the stock. The expiration date payoff for a protective put is $(X - S) + S = X$ when the put is in the money, and S when the put is out of the money.



PROFESSOR'S NOTE

When working with put-call parity, it is important to note that the exercise prices on the put and the call and the face value of the riskless bond are all equal to X .

If at expiration S is greater than or equal to X :

- The protective put pays S on the stock while the put expires worthless, so the payoff is S .
- The fiduciary call pays X on the bond portion while the call pays $(S - X)$, so the payoff is $X + (S - X) = S$.

If at expiration X is greater than S :

- The protective put pays S on the stock while the put pays $(X - S)$, so the payoff is $S + (X - S) = X$.
- The fiduciary call pays X on the bond portion while the call expires worthless, so the payoff is X .

In either case, the payoff on a protective put is the same as the payoff on a fiduciary call. Our no-arbitrage condition holds that portfolios with identical payoffs regardless

of future conditions must sell for the same price to prevent arbitrage. We can express the put-call parity relationship as:

$$c + X(1 + R_f)^{-T} = S + p$$

Equivalencies for each of the individual securities in the put-call parity relationship can be expressed as:

$$S = c - p + X(1 + R_f)^{-T}$$

$$p = c - S + X(1 + R_f)^{-T}$$

$$c = S + p - X(1 + R_f)^{-T}$$

$$X(1 + R_f)^{-T} = S + p - c$$

Note that the options must be European style and the puts and calls must have the same exercise price and time to expiration for these relations to hold.

The single securities on the left-hand side of the equations all have exactly the same payoffs as the portfolios on the right-hand side. The portfolios on the right-hand side are the **synthetic** equivalents of the securities on the left. For example, to synthetically produce the payoff for a long position in a share of stock, use the following relationship:

$$S = c - p + X(1 + R_f)^{-T}$$

This means that the payoff on a long stock can be synthetically created with a long call, a short put, and a long position in a risk-free discount bond.

The other securities in the put-call parity relationship can be constructed in a similar manner.



PROFESSOR'S NOTE

After expressing the put-call parity relationship in terms of the security you want to synthetically create, the sign on the individual securities will indicate whether you need a long position (+ sign) or a short position (- sign) in the respective securities.

EXAMPLE: Call option valuation using put-call parity

Suppose that the current stock price is \$52 and the risk-free rate is 5%. You have found a quote for a 3-month put option with an exercise price of \$50. The put price is \$1.50, but due to light trading in the call options, there was not a listed quote for the 3-month, \$50 call. Estimate the price of the 3-month call option.

Answer:

Rearranging put-call parity, we find that the call price is:

$$\text{call} = \text{put} + \text{stock} - \text{present value (X)}$$

$$\text{call} = \$1.50 + \$52 - \frac{\$50}{1.05^{0.25}} = \$4.11$$

This means that if a 3-month, \$50 call is available, it should be priced at (within transaction costs of) \$4.11 per share.

LOS 74.b: Explain put-call *forward* parity for European options.

Put-call-forward parity is derived with a forward contract rather than the underlying asset itself. Consider a forward contract on an asset at time T with a contract price of $F_0(T)$. At contract initiation the forward contract has zero value. At time T , when the forward contract settles, the long must purchase the asset for $F_0(T)$. The purchase (at time = 0) of a pure discount bond that will pay $F_0(T)$ at maturity (time = T) will cost $F_0(T)(1 + R_f)^{-T}$.

By purchasing such a pure discount bond and simultaneously taking a long position in the forward contract, an investor has created a synthetic asset. At time = T the proceeds of the bond are just sufficient to purchase the asset as required by the long forward position. Because there is no cost to enter into the forward contract, the total cost of the synthetic asset is the present value of the forward price, $F_0(T)(1 + R_f)^{-T}$.

The put-call-forward parity relationship is derived by substituting the synthetic asset for the underlying asset in the put-call parity relationship. Substituting $F_0(T)(1 + R_f)^{-T}$ for the asset price S_0 in $S + p = c + X(1 + R_f)^{-T}$ gives us:

$$F_0(T)(1 + R_f)^{-T} + p_0 = c_0 + X(1 + R_f)^{-T}$$

which is put-call-forward parity at time 0, the initiation of the forward contract, based on the principle of no arbitrage. By rearranging the terms, put-call-forward parity can also be expressed as:

$$p_0 - c_0 = [X - F_0(T)](1 + R_f)^{-T}$$

Application of Options Theory to Corporate Finance

We can view the claims of a firm's equity holders and debt holders as a call option and a put option, respectively. Consider a firm that has a value of V_t at time = t and has issued debt in the form of a zero-coupon bond that will pay D at time = T . At time = T , if $V_T > D$ the equity holders receive $V_T - D$ and if $V_T < D$, the firm is insolvent and equity holders receive nothing. The payoff to the equity holders at time = T can be written as $\text{Max}(0, V_T - D)$ which is equivalent to a call option with the firm value as the underlying and an exercise price of D .

At time = T , if $V_T > D$ the debt holders receive D and if $V_T < D$, the firm is insolvent and debt holders receive V_T . The payoff to the debt holders at time = T can be written as $\text{Min}(V_T, D)$. This is equivalent to a portfolio that is long a risk-free bond that pays D at $t = T$, and short (has sold) a put option on the value of the firm, V_T , with an exercise price of D . If $V_T > D$ the portfolio pays D and the put expires worthless, and if $V_T < D$ the portfolio pays $D - (D - V_T) = V_T$ and the debtholders effectively pay $D - V_T$ on the short put position.



MODULE QUIZ 74.1

1. The put-call parity relationship for European options must hold because a protective put will have the same payoff as a(n):
 - A. covered call.
 - B. fiduciary call.
 - C. uncovered call.
2. The put-call-forward parity relationship *least likely* includes:
 - A. a risk-free bond.
 - B. call and put options.
 - C. the underlying asset.

KEY CONCEPTS

LOS 74.a

A fiduciary call (a call option and a risk-free zero-coupon bond that pays the strike price X at expiration) and a protective put (a share of stock and a put at X) have the same payoffs at expiration, so arbitrage will force these positions to have equal prices: $c + X(1 + R_f)^{-T} = S + p$. This establishes put-call parity for European options.

Based on the put-call parity relation, a synthetic security (stock, bond, call, or put) can be created by combining long and short positions in the other three securities.

$$c = S + p - X(1 + R_f)^{-T}$$

$$p = c - S + X(1 + R_f)^{-T}$$

$$S = c - p + X(1 + R_f)^{-T}$$

$$X(1 + R_f)^{-T} = S + p - c$$

LOS 74.b

Because we can replicate the payoff on an asset by lending the present value of the forward price at the risk-free rate and taking a long position in a forward, we can write put-call-forward parity as:

$$c_0 + X(1 + R_f)^{-T} = F_0(T)(1 + R_f)^{-T} + p_0$$

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 74.1

1. **B** Given call and put options on the same underlying asset with the same exercise price and expiration date, a protective put (underlying asset plus a put option) will have the same payoff as a fiduciary call (call option plus a risk-free bond that will pay the exercise price on the expiration date) regardless of the underlying asset price on the expiration date. (LOS 74.a)
2. **C** The put-call-forward parity relationship is $F_0(T)(1 + RFR)^{-T} + p_0 = c_0 + X(1 + R_f)^{-T}$, where $X(1 + R_f)^{-T}$ is a risk-free bond that pays the exercise price on the

expiration date, and $F_0(T)$ is the forward price of the underlying asset. (LOS 74.b)

READING 75

VALUING A DERIVATIVE USING A ONE-PERIOD BINOMIAL MODEL

MODULE 75.1: BINOMIAL MODEL FOR OPTION VALUES

LOS 75.a: Explain how to value a derivative using a one-period binomial model.



Video covering this content is available online.

Recall from Quantitative Methods that a **binomial model** is based on the idea that, over the next period, some value will change to one of two possible values (binomial). To construct a one-period binomial model for pricing an option, we need:

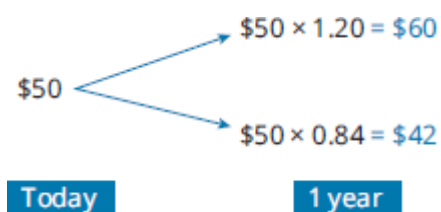
- A value for the underlying at the beginning of the period.
- An exercise price for the option. The exercise price can be different from the value of the underlying. We assume the option expires one period from now.
- Returns that will result from an up-move and a down-move in the value of the underlying over one period.
- The risk-free rate over the period.

For now we do not need to consider the probabilities of an up-move or a down-move. Later in this reading we will examine one-period binomial models with risk-neutral probabilities.

One-Period Binomial Model for a Call Option

As an example, we can model a call option with an exercise price of \$55 on a stock that is currently valued (S_0) at \$50. Let us assume that in one period the stock's value will either increase (S_u) to \$60 or decrease (S_d) to \$42. We state the return from an up-move (R^u) as $\$60 / \$50 = 1.20$, and the return from a down-move (R^d) as $\$42 / \$50 = 0.84$.

Figure 75.1: One-Period Binomial Tree



The call option will be in the money after an up-move or out of the money after a down-move. Its value at expiration after an up-move, c_1^u , is $\text{Max}(0, \$60 - \$55) = \$5$. Its value after a down-move, c_1^d , is $\text{Max}(0, \$42 - \$55) = 0$.

Now we can use no-arbitrage pricing to determine the initial value of the call option (c_0). We do this by creating a portfolio of the option and the underlying stock, such that the portfolio will have the same value following either an up-move (V_1^u) or a down-move (V_1^d) in the stock. To value a call option, we create a portfolio that is *short* a call (i.e., the call writer) and *long* a number of shares of the stock that we will denote as h . We must solve for the h that results in $V_1^u = V_1^d$.

- The initial value of our portfolio, V_0 , is $hS_0 - c_0$ (remember we are short the call option).
- The portfolio value after an up-move, V_1^u , is $hS_1^u - c_1^u$.
- The portfolio value after a down-move, 9.

In our example, $V_1^u = h(\$60) - \5 , and $V_1^d = h(\$42) - 0$. Setting $V_1^u = V_1^d$ solving for h , we get:

$$\begin{aligned} h(\$60) - \$5 &= h(\$42) \\ h(\$60) - h(\$42) &= \$5 \\ h &= \$5 / (\$60 - \$42) = 0.278 \end{aligned}$$

This result, the number of shares of the underlying we would buy for each call option we would write, is known as the **hedge ratio** for this option.

With $V_1^u = V_1^d$, the value of the portfolio after one period is known with certainty. This means we can say that either V_1^u or V_1^d must equal V_0 compounded at the risk-free rate for one period. In this example, $V_1^d = 0.278(\$42) = \11.68 , or $V_1^u = 0.278(\$60) - \$5 = \$11.68$. Let us assume the risk-free rate over one period is 3%. Then $V_0 = \$11.68 / 1.03 = \11.34 .

Now we can solve for the value of the call option, c_0 . Recall that $V_0 = hS_0 - c_0$, so $c_0 = hS_0 - V_0$. Here, $c_0 = 0.278(\$50) - \$11.34 = \$2.56$.



PROFESSOR'S NOTE

When we “create a portfolio” to construct this model, we do not assume any cash has changed hands to buy the underlying or sell the call. We simply assume a portfolio exists that is made up of these positions.

One-Period Binomial Model for a Put Option

We can model a put option using the same technique. The key difference is that the model portfolio consists of *long* positions in both a put and the underlying stock.

As an example, using the same assumptions that the stock is currently valued at \$50, and in one period its value will either increase to \$60 (for a return of 1.20) or decrease to \$42 (for a return of 0.84), we can value a put option with an exercise price of \$48.

The put option will be in the money after a down-move or out of the money after an up-move. Its value at expiration after a down-move, p_1^d , is $\text{Max}(0, \$48 - \$60) = 0$.

Again we want to find the hedge ratio h , the number of shares at which our portfolio has the same value after an up-move or a down-move, $V_1^u = V_1^d$.

- The initial value of our portfolio, V_0 , is $hS_0 + p_0$ (remember we are long the put).
- The portfolio value after an up-move, V_1^u , is $hS_1^u + p_1^u$.
- The portfolio value after a down-move, V_1^d , is $hS_1^d + p_1^d$.

Setting $V_1^u = V_1^d$, we get:

$$h(\$60) = h(\$42) + \$6$$

$$h(\$60) - h(\$42) = \$6$$

$$h = \$6 / (\$60 - \$42) = 0.333$$

As before, we can determine the value of the portfolio one period from now using either V_1^u or V_1^d because they will be equal:

$$V_1^u = 0.333(\$60) = \$20, \text{ or } V_1^d = 0.333(\$42) + \$6 = \$20$$

Discounting this value to $t = 0$ at the risk-free rate of 3% gives us:

$$V_0 = \$20 / 1.03 = \$19.42$$

Finally we can solve for p_0 , the value of the put option:

$$V_0 = hS_0 + p_0, \text{ so } p_0 = V_0 - hS_0$$

$$p_0 = \$19.42 - 0.333(\$50) = \$2.75$$

LOS 75.b: Describe the concept of risk neutrality in derivatives pricing.

Another approach to constructing a one-period binomial model involves risk-neutral probabilities of an up-move or a down-move. Consider a share of stock currently priced at \$30. The size of the possible price changes, and the probabilities of these changes occurring, are as follows:

$$R^u = \text{up-move factor} = 1.15$$

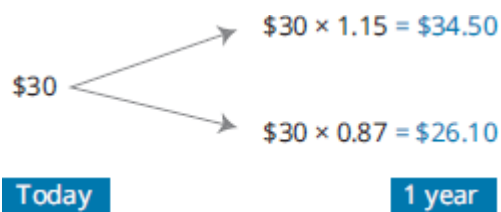
$$R^d = \text{down-move factor} = \frac{1}{R^u} = \frac{1}{1.15} = 0.87$$

$$\pi_U = \text{risk-neutral probability of an up-move} = 0.715$$

$$\pi_D = \text{risk-neutral probability of a down-move} = 1 - \pi_U = 1 - 0.715 = 0.285$$

Note that the down-move factor is the reciprocal of the up-move factor, and the probability of an up-move is one minus the probability of a down-move. The one-period binomial tree for the stock is shown in Figure 75.2. The beginning stock value of \$30 is to the left, and to the right are the two possible end-of-period stock values, $30 \times 1.15 = \$34.50$ and $30 \times 0.87 = \$26.10$.

Figure 75.2: One-Period Binomial Tree



The risk-neutral probabilities of an up-move and a down-move are calculated from the sizes of the moves and the risk-free rate:

$$\pi_U = \text{risk-neutral probability of an up-move} = \frac{1 + R_f - R^d}{R^u - R^d}$$

$$\pi_D = \text{risk-neutral probability of a down-move} = 1 - \pi_U$$

where:

R_f = risk-free rate

R^u = size of an up-move

R^d = size of a down-move



PROFESSOR'S NOTE

These two probabilities are not the actual probabilities of the up- and down-moves. They are risk-neutral pseudo probabilities. The calculation of risk-neutral probabilities does not appear to be required for the Level I exam.

We can calculate the value of an option on the stock by:

- Calculating the payoffs of the option at expiration for the up-move and down-move prices.
- Calculating the expected payoff of the option in one year as the (risk-neutral) probability-weighted average of the up-move and down-move payoffs.
- Calculating the PV of the expected payoff by discounting at the risk-free rate.

EXAMPLE: Calculating call option value with risk-neutral probabilities

Use the binomial tree in Figure 75.2 to calculate the value today of a 1-year call option on a stock with an exercise price of \$30. Assume the risk-free rate is 7%, the current value of the stock is \$30, and the up-move factor is 1.15.

Answer:

First, we need to calculate the down-move factor and risk-neutral the probabilities of the up- and down-moves:

$$R^d = \text{size of down-move} = \frac{1}{R^u} = \frac{1}{1.15} = 0.87$$

$$\pi_U = \text{risk-neutral probability of an up-move} = \frac{1 + 0.07 - 0.87}{1.15 - 0.87} = 0.715$$

$$\pi_D = \text{risk-neutral probability of a down-move} = 1 - 0.715 = 0.285$$

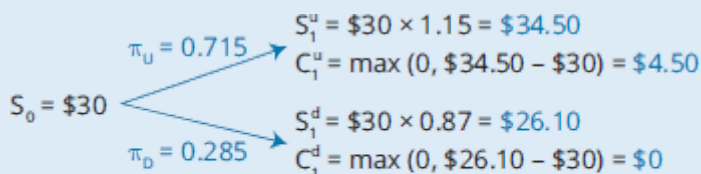
Next, determine the payoffs on the option in each state. If the stock moves up to \$34.50, a call option with an exercise price of \$30 will pay \$4.50. If the stock moves

down to \$26.10, the call option will expire worthless. The option payoffs are illustrated in the following figure.

Let the stock values for the up-move and down-move be S_1^u and S_1^d and for the call values, c_1^u and c_1^d .

One-Period Call Option With $X = \$30$

The expected value of the option in one period is:



Today

1 year

$$E(\text{call option value in 1 year}) = (\$4.50 \times 0.715) + (\$0 \times 0.285) = \$3.22$$

The value of the option today, discounted at the risk-free rate of 7%, is:

$$C_0 = \frac{\$3.22}{1.07} = \$3.01$$

We can use the same basic framework to value a one-period put option. The only difference is that the payoff to the put option will be different from the call payoffs.

EXAMPLE: Valuing a one-period put option on a stock

Use the information in the previous example to calculate the value of a put option on the stock with an exercise price of \$30.

Answer:

If the stock moves up to \$34.50, a put option with an exercise price of \$30 will expire worthless. If the stock moves down to \$26.10, the put option will be worth \$3.90.

The risk-neutral probabilities are 0.715 and 0.285 for an up- and down-move, respectively. The expected value of the put option in one period is:

$$E(\text{put option value in 1 year}) = (\$0 \times 0.715) + (\$3.90 \times 0.285) = \$1.11$$

The value of the option today, discounted at the risk-free rate of 7%, is:

$$P_0 = \frac{\$1.11}{1.07} = \$1.04$$

In practice, we would construct a binomial model with many short periods and have many possible outcomes at expiration. However, the one-period model is sufficient to illustrate the concept and method.

Note that the actual probabilities of an up-move and a down-move do not enter directly into our calculation of option value. The size of the up-move and down-move, along with the risk-free rate, determines the risk-neutral probabilities we use to calculate the expected payoff at option expiration. Remember, the risk-neutral probabilities come

from constructing a hedge that creates a certain payoff. Because their calculation is based on an arbitrage relationship, we can discount the expected payoff based on risk-neutral probabilities, using the risk-free rate.



MODULE QUIZ 75.1

1. To construct a one-period binomial model for valuing an option, are probabilities of an up-move or a down-move in the underlying price required?
 - A. No.
 - B. Yes, but they can be calculated from the returns on an up-move and a down-move.
 - C. Yes, the model requires estimates for the actual probabilities of an up-move and a down-move.
2. In a one-period binomial model based on risk neutrality, the value of an option is *best* described as the present value of:
 - A. a probability-weighted average of two possible outcomes.
 - B. a probability-weighted average of a chosen number of possible outcomes.
 - C. one of two possible outcomes based on a chosen size of increase or decrease.
3. A one-period binomial model for option pricing uses risk-neutral probabilities because:
 - A. the model is based on a no-arbitrage relationship.
 - B. they are unbiased estimators of the actual probabilities.
 - C. the buyer can let an out-of-the-money option expire unexercised.

KEY CONCEPTS

LOS 75.a

A one-period binomial model for pricing an option requires the underlying asset's value at the beginning of the period, an exercise price for the option, the asset prices that will result from an up-move and a down-move, and the risk-free rate.

A portfolio of the underlying asset hedged with a position in an option can be created such that the portfolio has the same value for both an up-move and a down-move. Because the portfolio's value at the end of the period is certain, that value must be the portfolio's initial value compounded at the risk-free rate. The number of units of the underlying required to construct such portfolios is the hedge ratio.

LOS 75.b

To determine the value of an option using the concept of risk neutrality, we calculate its payoffs for both an up-move and a down-move, calculate the expected payoff as a weighted average using the risk-neutral probabilities of an up-move and a down-move, and discount this expected payoff for one period at the risk-free rate.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 75.1

1. **A** A one-period binomial model can be constructed based on replication and no-arbitrage pricing, without regard to the probabilities of an up-move or a down-move. (LOS 75.a)
2. **A** In a one-period binomial model based on risk-neutral probabilities, the value of an option is the present value of a probability-weighted average of two possible option payoffs at the end of a single period, during which the price of the underlying asset is assumed to move either up or down to specific values. (LOS 75.b)
3. **A** Because a one-period binomial model is based on a no-arbitrage relationship, we can discount the expected payoff at the risk-free rate. (LOS 75.b)

TOPIC QUIZ: DERIVATIVES

You have now finished the Derivatives topic section. Please log into your Schweser online dashboard and take the Topic Quiz on this section. The Topic Quiz provides immediate feedback on how effective your study has been for this material. Questions are more exam-like than typical Module Quiz or QBank questions; a score of less than 70% indicates that your study likely needs improvement. These tests are best taken timed; allow 1.5 minutes per question.

APPENDIX

Rates, Returns, and Yields

A **holding period return (HPR)**, or holding period yield (HPY), can be for a period of any length and is simply the percentage increase in value over the period, which is calculated as:

$$\text{HPR} = \text{ending value} / \text{beginning value} - 1$$

1. If an investor puts \$2,000 into an account and 565 days later it has grown in value to \$2,700, the 565-day HPY is $2,700 / 2,000 - 1 = 35\%$.
2. If an investor buys a share of stock for \$20/share, receives a \$0.40 dividend, and sells the shares after nine months, the nine-month HPY is $(22 + 0.40) / 20 - 1 = 12\%$.

An HPR for a given period is also the **effective yield** for that period.

An **effective annual yield** is the HPR for a one-year investment or the HPY for a different period converted to its annual equivalent yield.

3. If the six-month HPR is 2%, the effective annual yield is $1.02^2 - 1 = 4.040\%$.
4. If the 125-day HPR is 1.5%, the effective annual yield is $1.015^{365/125} - 1 = 4.443\%$.
5. If the two-year HPR (two-year effective rate) is 9%, the effective annual yield is $1.09^{1/2} - 1 = 4.4031\%$.

Compounding Frequency

Sometimes the “rate” on an investment is expressed as a **simple annual rate** (or *stated rate*)—the annual rate with no compounding of returns. The number of compounding periods per year is called the **periodicity** of the rate. For a periodicity of one, the stated rate and the effective annual rate are the same. When the periodicity is greater than one (more than one compounding period per year), the effective annual rate is the effective rate for the sub-periods, compounded for the number of sub-periods.

6. A bank CD has a stated annual rate of 6% with annual compounding (periodicity of 1); the effective annual rate is 6% and a \$1,000 investment will return $\$1,000(1.06) = \$1,060$ at the end of one year.
7. A bank CD has a stated annual rate of 6% with semiannual compounding (periodicity of 2); the effective annual rate is $(1 + 0.06 / 2)^2 - 1 = 6.09\%$ and a \$1,000 investment will return $\$1,000(1.0609) = \$1,060.90$ at the end of one year.
8. A bank CD has a stated annual rate of 6% with quarterly compounding (periodicity of 4); the effective annual rate is $(1 + 0.06 / 4)^4 - 1 = 6.136\%$ and a \$1,000 investment will return $\$1,000(1.06136) = \$1,061.36$ at the end of one year.

Note that increasing compounding frequency increases the effective annual yield for any given stated rate. In the limit, as compounding periods get shorter (more frequent),

compounding is *continuous*. A stated rate of r %, with continuous compounding, results in an effective annual return of $e^r - 1$.

9. A bank CD has a stated annual rate of 6%, continuously compounded; its effective annual yield is $e^{0.06} - 1 = 6.184\%$ and a \$1,000 investment will return $\$1,000(1.06184) = \$1,061.84$ at the end of one year.

Bond Quotations and Terminology

The **coupon rate** on a bond is the total cash coupon payments made over one year as a percentage of face value.

10. A bond with a face value of \$1,000 that pays a coupon of \$50 once each year (an annual-pay bond) has a coupon rate of $50 / 1,000 = 5\%$ and we say it has a periodicity of 1.
11. A bond with a face value of \$1,000 that pays a coupon of \$25 twice each year (a semiannual-pay bond) has a coupon rate of $(25 + 25) / 1,000 = 5\%$ and we say it has a periodicity of 2.
12. A bond with a face value of \$1,000 that pays a coupon of \$12.50 (1.25%) four times each year (a quarterly-pay bond) has a coupon rate of $(12.50 + 12.50 + 12.50 + 12.50) / 1,000 = 5\%$ and we say it has a periodicity of 4.

The **current yield** on a bond is the coupon rate divided by the bond price as a percentage of face value or, alternatively, the sum of the coupon payments for one year divided by the bond price.

13. A bond with a stated coupon rate of 5% that is selling at 98.54% of face value has a current yield of $5 / 98.54 = 5.074\%$.
14. A bond that is trading at \$1,058 and makes annual coupon payments that sum to \$50 has a current yield of $50 / 1,058 = 4.726\%$.

The **yield to maturity** (YTM) of a bond, on an *annual basis*, is the effective annual yield and is used for bonds that pay an annual coupon. For bonds that pay coupons semiannually, we often quote the YTM on a *semiannual basis*, that is, two times the effective semiannual yield. To compare the yields of two bonds, we must calculate their YTM on the same basis.

15. A bond with a YTM of 5% on a semiannual basis has a YTM on an annual basis (effective annual yield) of $(1 + 0.05 / 2)^2 - 1 = 5.0625\%$.
16. A bond with a YTM of 5% on an annual basis has a YTM on a semiannual basis of $(1.05^{1/2} - 1) \times 2 = 4.939\%$.

Internal Rate of Return (IRR)

The internal rate of return is the discount rate that makes the PV of a series of cash flows equal to zero. This calculation must be done with a financial calculator. We can use the IRR for calculating the return on a capital project, the YTM on a bond, and the money weighted rate of return for a portfolio.

17. For the YTM of an annual-pay bond (YTM on an annual basis) on a coupon date with N years remaining until maturity, we calculate the annual IRR that satisfies:

$$-\text{bond price} + \frac{\text{coupon 1}}{1 + \text{IRR}} + \frac{\text{coupon 2}}{(1 + \text{IRR})^2} + \dots + \frac{\text{coupon } N + \text{face value}}{(1 + \text{IRR})^N} = 0$$

18. For the YTM of a semiannual-pay bond on a coupon date with N years remaining until maturity, we calculate the IRR that satisfies:

$$-\text{bond price} + \frac{\text{coupon 1}}{1 + \frac{\text{IRR}}{2}} + \frac{\text{coupon 2}}{\left(1 + \frac{\text{IRR}}{2}\right)^2} + \dots + \frac{\text{coupon } 2N + \text{face value}}{\left(1 + \frac{\text{IRR}}{2}\right)^{2N}} = 0$$

After solving for $\text{IRR} / 2$, which is the IRR for semiannual periods, we must multiply it by 2 to get the bond's YTM on a semiannual basis.

Money Market Securities

For some money market securities, such as U.S. T-bills, price quotations are given on a bond discount (or simply discount) basis. The discount yield is the percentage discount from face value of a T-bill, annualized based on a 360-day year, and is therefore not an effective yield but simply an annualized discount from face value.

19. A T-bill that will pay \$1,000 at maturity in 180 days is selling for \$984, a discount of $1 - 984 / 1,000 = 1.6\%$. The annualized discount is $1.6\% \times 360 / 180 = 3.2\%$.

20. A 120-day T-bill is quoted at a discount yield of 2.83%, its price is $[1 - (0.0283 \times 120 / 360)] \times 1,000 = \990.57 . Its 120-day *holding period return* is $1,000 / 990.57 - 1 = 0.952\%$. Its *effective annual yield* is $(1,000 / 990.57)^{365/120} - 1 = 2.924\%$.

21. HPY on a 30-day loan at a quoted market reference rate of 1.8% is $0.018 \times 30 / 360 = 0.15\%$ so the interest on a \$10,000 loan is $10,000 \times 0.0015 = \$15$.

A related yield is the **money market yield (MMY)**, which is HPY annualized based on a 360-day year.

22. A 120-day discount security with a maturity value of \$1,000 that is priced at \$995 has a money market yield of $(1,000 / 995 - 1) \times 360 / 120 = 1.5075\%$.

Spot and Forward Rates

Forward rates are rates for a loan to be made in a future period. They are quoted based on the period of the loan. For loans of one year, we write 1y1y for a 1-year loan to be made one year from today and 2y1y for a 1-year loan to be made two years from today.

Spot rates are discount rates for single payments to be made in the future (such as for zero-coupon bonds).

23. Given a 3-year spot rate expressed as a compound annual rate (S_3) of 2%, a 3-year bond that makes a single payment of \$1,000 in three years has a current value of $1,000 / (1 + 0.02)^3 = \$942.32$.

An N -year spot rate is the geometric mean of the individual annual forward rates:

$$S_N = [(1 + S_1)(1 + 1y1y)(1 + 2y1y)\dots(1 + Ny1y)]^{1/N} - 1$$

and the annualized forward rate for $M - N$ periods, N periods from now is:

$$N y(M - N) y = \left[\frac{(1 + S_M)^M}{(1 + S_N)^N} \right]^{\frac{1}{M - N}} - 1$$

24. Given $S_5 = 2.4\%$ and $S_7 = 2.6\%$, $5y2y = [(1.026)^7 / (1.024)^5]^{1/2} - 1 = 3.1017\%$, which is approximately equal to $(7 \times 2.6\% - 5 \times 2.4\%) / 2 = 3.1\%$.

Par yields reflect the coupon rate that a hypothetical bond at each maturity would need to have to be priced at par, given a specific spot curve.

25. With spot rates of 1%, 2%, and 3%, a 3-year annual par bond will have a payment that will satisfy the following:

$$\frac{PMT}{1.01} + \frac{PMT}{(1.02)^2} + \frac{PMT + 100}{(1.03)^3} = 100$$

The payment is 2.96 and the par bond coupon rate is 2.96%.

FORMULAS

duration gap = Macaulay duration – investment horizon

Modified duration (annual-pay bond):

$$\text{ModDur} = \text{MacDur} / (1 + \text{YTM})$$

Modified duration (semiannual-pay bond):

$$\text{ModDur}_{\text{SEMI}} = \text{MacDur}_{\text{SEMI}} / (1 + \text{YTM} / 2)$$

$$\text{approximate modified duration} = \frac{V_- - V_+}{2V_0 \Delta \text{YTM}}$$

$$\text{approximate convexity} = \frac{V_- + V_+ - 2V_0}{(\Delta \text{YTM})^2 V_0}$$

where:

V_- = price of the bond if YTM is *decreased* by ΔYTM

V_+ = price of the bond if the YTM is *increased* by ΔYTM

V_0 = current price of the bond

$$\text{portfolio duration} = W_1 D_1 + W_2 D_2 + \dots + W_N D_N$$

where:

W_i = full price of bond i divided by the total value of the portfolio

D_i = duration of bond i

N = number of bonds in the portfolio

$$\text{effective duration} = \frac{V_- - V_+}{2V_0 \Delta \text{Curve}}$$

$$\text{effective convexity} = \frac{V_- + V_+ - 2V_0}{(\Delta \text{Curve})^2 V_0}$$

expected loss = probability of default \times loss given default

$$\text{debt service coverage ratio} = \frac{\text{net operating income}}{\text{debt service}}$$

$$\text{loan-to-value ratio} = \frac{\text{current mortgage amount}}{\text{current appraised value}}$$

no-arbitrage forward price: $F_0(T) = S_0 (1 + R_f)^T$

payoff to long forward at expiration = $S_T - F_0(T)$

value of forward at time t : $V_t(T) = [S_t + PV_t(\text{costs}) - PV_t(\text{benefit})] - F_0(T) (1 + R_f)^{-(T-t)}$

exercise value of a call = $\text{Max}[0, S - X]$

exercise value of a put = $\text{Max}[0, X - S]$

option value = exercise value + time value

put-call parity: $c + X(1 + R_f)^{-T} = S + p$

put-call-forward parity: $F_0(T)(1 + R_f)^{-T} + p_0 = c_0 + X(1 + R_f)^{-T}$

INDEX

SYMBOLS

30/360 convention, 52

A

actual/actual convention, 52

affirmative covenant, 4

agencies, 142

agency bonds, 43

amortization period, 168

amortizing loan, 9

analytical durations, 125

annualized holding period rate of return, 92

asset-backed commercial paper (ABCP), 33

auctions, 43

B

backup lines of credit, 32

balloon payment, 10, 180

basis point, 11

basis point value (BPV), 232

bearer bond, 15

benchmark spread, 66

best-efforts offering, 25

bilateral repo, 37

binomial model, 259

bond, 1

bond equivalent yield, 75

bond indenture, 4

bridge financing, 32

bullet structure, 9

business model, 147

business risk, 147

C

- callable bond, 13
- call option, 243
- call options, 200
- call period, 13
- call protection, 180
- call risk, 13
- capacity, 129
- capital, 129
- capital-indexed bond, 12
- capital market security, 2
- carrying value, 93
- cash flow hedge, 213
- cash market transaction, 189
- cash-settled contract, 188
- central bank funds market, 33
- central bank funds rate, 33
- central clearinghouse (CCH), 191
- central clearing mandate, 192
- central counterparty (CCP), 192
- certificate of deposit (CD), 33
- character, 129
- circuit breakers, 199
- CMBS-level call protection, 180
- collateral, 4, 129
- collateralized debt obligation (CDO), 169
- collateralized mortgage obligations (CMOs), 177
- collateral manager, 169
- commercial mortgage-backed securities (CMBSs), 179
- commercial paper (CP), 32
- conditional pass-through covered bond, 166
- conditions, 129
- constant-yield price trajectory, 54
- contingency provision, 12
- contingency provisions, 2
- contingent claim, 204
- contingent convertible bond, 14
- contraction risk, 173
- contract size, 187
- convenience yield, 219
- conversion price, 14

- conversion ratio, 14
- conversion value, 14
- convexity, 110
- convexity bias, 233
- corporate credit ratings (CCRs), 152
- corporate family ratings (CFRs), 152
- corporate governance, 148
- country, 130
- coupon, 1, 9
- coupon rate and frequency, 2
- covenant, 4
- covenants, 129
- covered bond, 165
- credit default swap (CDS), 200
- credit event, 200
- credit facilities, 22
- credit-linked note, 11
- credit migration risk, 132
- credit rating, 132, 152
- credit risk, 129
- credit spread, 11
- credit spread risk, 134
- credit tranching, 166
- cross-default provision, 5
- currency, 130
- current yield, 63
- cut-off yield, 43

D

- day count methods, 52
- debt management policy, 42
- debt service coverage ratio (DSCR), 179
- debt-to-income ratio (DTI), 175
- debut issuer, 25
- default risk, 130
- deferred coupon bond, 12
- deliverable contract, 188
- derivative, 187
- developed market (DM), 41
- domestic bond, 15

domestic debt, 42
duration gap, 98

E

EBITDA, 149
economic flexibility factors, 142
economic growth and stability, 142
economies and financial markets, 159
effective annual yield, 269
effective convexity (EffCon), 122
effective duration, 121
effective duration (EffDur), 122
effective yield, 269
embedded option, 2, 12
emerging market (EM), 41
empirical durations, 125
Eurobond, 15
excess spread, 166
exchange-traded derivatives, 191
exercise price, 200
exercise value, 245
expected loss (EL), 130
extension risk, 173
external debt, 42
external stability, 142
external status factors, 142

F

face value, 1
factoring, 32
fair value hedge, 213
fallen angels, 22
fiduciary call, 253
fiscal flexibility factors, 141
fiscal strength, 142
fixed-income security, 1
flat price, 52
floater, 11

- floating-rate note (FRN), 2, 11
- floating-rate note yields, 73
- floating-rate tranche, 178
- foreign bond, 15
- forward commitment, 204
- forward contract, 187, 197
- forward price, 187
- forward rate, 81
- forward rate agreement (FRA), 227
- forward rates, 85
- forward yield curve, 85
- free cash flow (FCF), 149
- full price, 52
- fully amortizing, 9
- funds from operations (FFO), 149
- futures contract, 197

G

- general collateral repo, 35
- general obligation bonds, 143
- general obligation bonds (GO bonds), 43
- general obligation (GO) bond, 143
- global bond, 15
- government sector banks/financing institutions, 142
- green bond, 11
- G-spread, 66

H

- haircut, 35
- hard-bullet covered bond, 166
- hard commodities, 190
- hedge accounting, 213
- hedged, 189
- hedge ratio, 260
- holding period return, 269
- horizon yield, 92

I

- implied forward rate, 226
- income yield, 63
- incurrence test, 5
- index-linked bond, 11
- industry competition, 147
- inflation-linked bond, 11
- initial margin (futures), 198
- initial margin (repo), 34
- institutions and policy factors, 141
- interbank funds, 33
- interest-indexed bond, 12
- interest-only (IO) security, 178
- interpolated spreads (I-spreads), 66
- intrinsic value, 245
- investment grade, 133
- investment horizon, 92
- investors, 158
- issuer, 1
- issuers, 158

J

- junior debt, 2

K

- key rate duration, 124

L

- leveraged loan, 11
- lien, 4
- lines of credit, 31
- linker, 11
- loan, 1
- loan-level call protection, 180
- loan-to-value ratio (LTV), 175, 180

local and regional government authorities, 43
lockout or revolving period, 168
loss given default (LGD), 130
loss severity, 130

M

Macauley duration, 97
maintenance margin, 198
margin, 197
market reference rate, 199
market reference rate (MRR), 2, 11
mark-to-market, 198
master repurchase agreement, 35
matrix pricing, 55
maturity, 2
modified duration (ModDur), 101
monetary effectiveness factors, 142
money convexity (MoneyCon), 112
money duration, 103
money duration (MoneyDur), 112
money market security, 2
money market yield, 271
moneyness, 243
mortgage pass-through security, 176
multiple-price auction, 44

N

negative covenant, 5
negative pledge clause, 5
net cash flow from operating activities (CFO), 149
net investment hedge, 213
no-arbitrage, 217
no-arbitrage condition, 217
no-arbitrage price, 80
non-investment grade, 133
nonrecourse loanrecourse loan, 174
non-sovereign government debt, 43, 142
notching, 152

notional principal, 199

novation, 191

O

on-the-run, 44

option-adjusted price, 65

option-adjusted spread (OAS), 68

option-adjusted yield, 65

option-free bond, 12

option premium, 201, 245

original issue discount (OID) bond, 16

overcollateralization, 166

P

parallel shift, 114

par bond yield curve, 85

pari passu, 5, 152

pari passu provision, 5

par swap rate, 238

partial duration, 124

partially amortizing bond, 10

partially hedged, 189

par value, 1, 2

pass-through rates, 176

payment-in-kind (PIK) bond, 11

periodicity, 62

perpetual bond, 2

planned amortization class (PAC) tranches and support tranches, 179

pledge, 4

prepayment penalty, 174

prepayment risk, 173

price limits, 198

price risk, 95

price value of a basis point (PVBP), 104

primary dealers, 44

primary market, 25

principal, 2

principal amount, 2

principal-only (PO) security, 178
principal value, 1
priority of claims, 151
private placement, 25
probability of default (POD), 130
public offering, 25
pure discount bond, 2
puttable bond, 13
put-call-forward parity, 255
put-call parity, 253
put option, 244
put options, 200

Q

quasi-government bonds, 43
quoted margin (QM), 73

R

rating agencies, 152
recovery rate, 131
regional governments, 143
registered bonds, 15
reinvestment risk, 95
replication, 218
repo rate, 34
repurchase date, 34
repurchase (repo) agreement, 33
required margin, 73
reserve currency, 41
residential mortgage loan, 174
residual tranche, 178
retained cash flow (RCF), 149
revenue bond, 143
revenue bonds, 43, 143
reverse repo, 36
running yield, 63

S

- secondary market, 25
- secured (asset-backed) loans, 32
- secured bond, 4
- secured debt, 151
- securitization, 157
- securitized mortgage, 176
- semiannual bond basis, 62
- senior debt, 2
- seniority, 2
- seniority ranking, 151
- sequential pay CMO, 177
- settlement date, 187
- shelf registration, 25
- simple annual rate, 269
- simple yield, 64
- single-price auction, 43
- sinking fund provision, 10
- soft-bullet covered bond, 166
- soft commodities, 190
- sovereign debt, 141
- special purpose entity (SPE), 159
- special purpose vehicle (SPV), 159
- speculating, 189
- spot curve, 80
- spot price, 188
- spot rates, 67, 79
- spot rate yield curve, 85
- stated (coupon) rate, 270
- step-up coupon bond, 11
- street convention, 63
- strike price, 200
- strip curve, 85
- structural subordination, 152
- subordinated debentures, 151, 154
- subordinated debt, 2
- subordination, 166
- support tranche, 179
- supranational bonds, 43
- supranational issuers, 142
- swap execution facility (SEF), 192

swap rate, 199
Swaps, 199
syndicate, 22
synthetic, 254

T

tenor, 2
term to maturity, 2
three sources of returns, 91
time to expiration, 200
time tranching, 174
time value, 245
tranches, 166
Treasury Inflation-Protected Securities (TIPS), 12
tri-party repos, 37
true yield, 63

U

underlying, 187
underlying asset, 187
underwritten offering, 25
unsecured bond, 4
unsecured debt, 151

V

value a one-period put option, 264
variation margin, 35

W

warrant, 14
waterfall, 167
waterfall structure, 10
weighted average coupon (WAC), 176
weighted average maturity (WAM), 176

Y

- yield, 2
- yield curve, 3
- yield curve for coupon bonds, 85
- yield spread, 66
- yield to call, 64
- yield to maturity (YTM), 49
- yield-to-maturity (YTM), 270
- yield to worst, 64

Z

- zero-coupon bond, 2
- zero-coupon bonds, 12
- zero-coupon rates, 79
- zero curve, 85
- zero-volatility spread (Z-spread), 67
- Z-tranche, 178