Concurrency: Foundations and Algorithms

Mutual Exclusion

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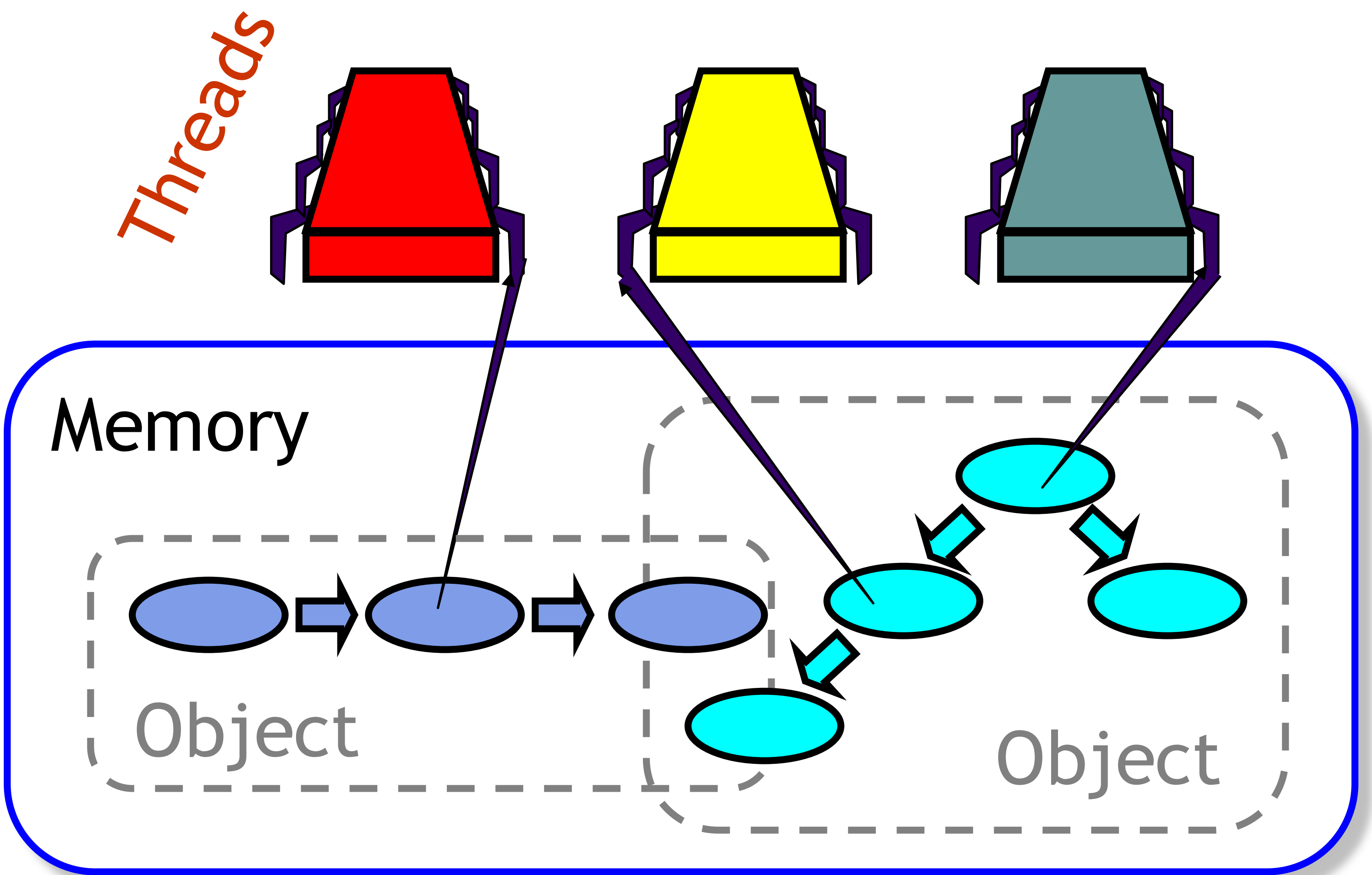
*Based on slides by **Maurice Herlihy** and **Nir Shavit***

Review

- Model of computation
- Asynchrony
- Mutual exclusion

Concurrent Computation



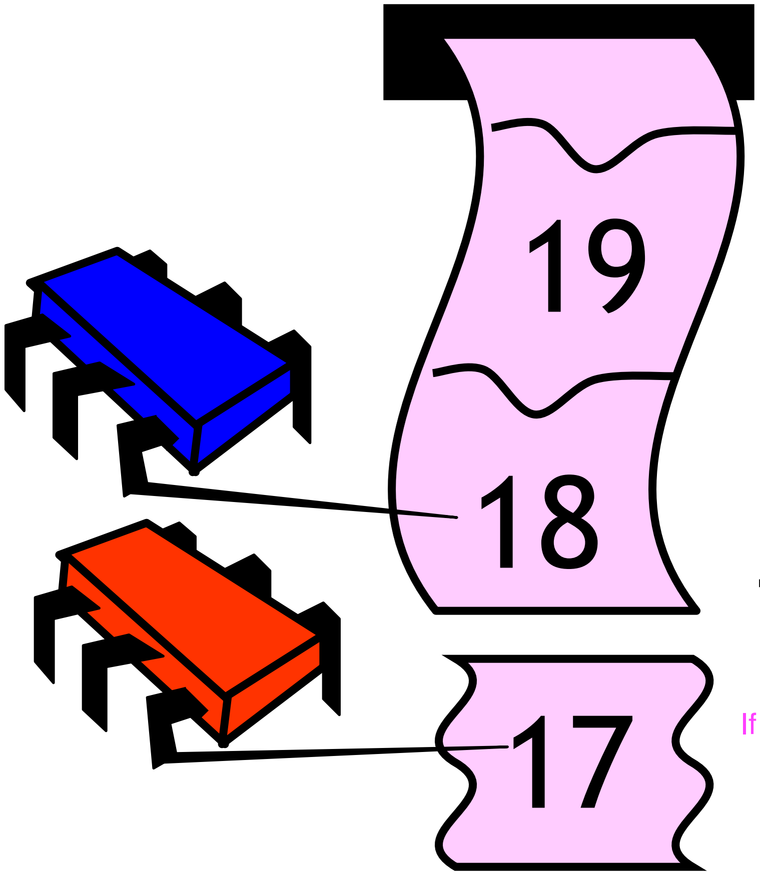
Asynchrony

- Sudden unpredictable delays
  - Cache misses (*short*)
  - Page faults (*long*)
  - Scheduling quantum used up (*really long*)

Parallel Primality Testing

- Challenge
  - Print primes from 1 to 10^{10}
- Given
  - Ten-processor multiprocessor
  - One thread per processor
- Goal
  - Get ten-fold speedup (or close)

Shared Counter



Each thread takes a number

If it's a prime, print it, otherwise get another number

Procedure for Thread *i*

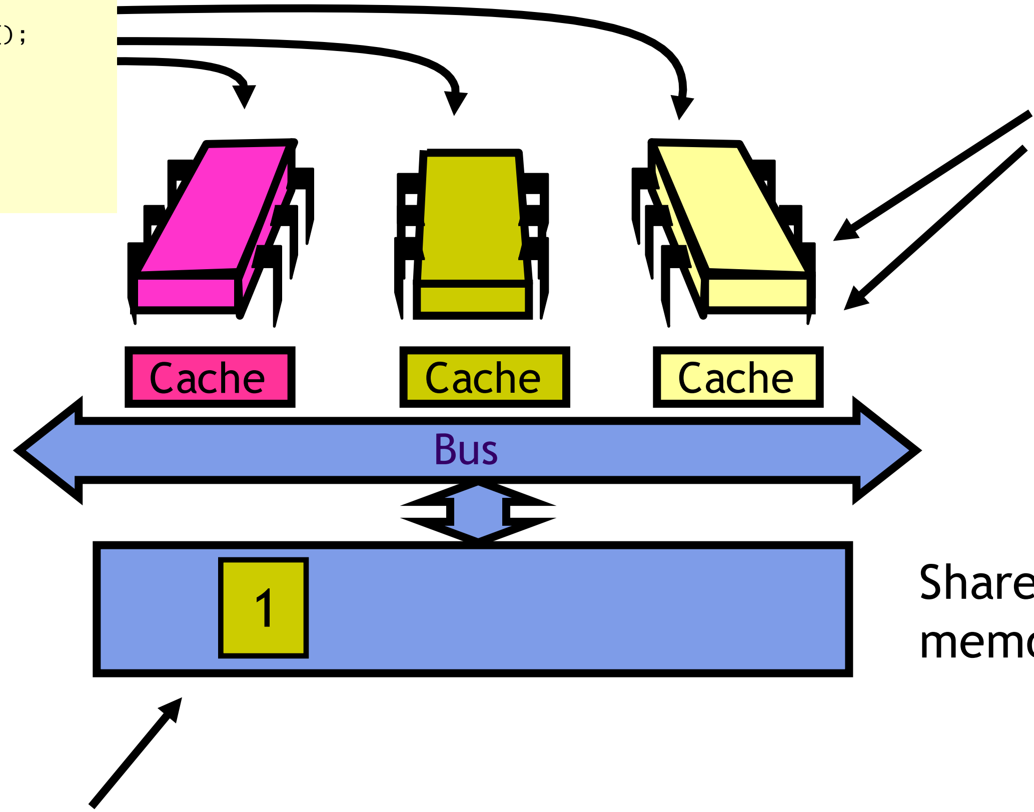
```
Counter counter = new Counter(1);

void primePrint {
  while (true) {
    long j = counter.getAndIncrement();
    if (j > 10^10)
      break;
    if (isPrime(j))
      print(j);
  }
}
```

Where Things Reside

```
Counter counter = new Counter(1);
void primePrint {
  while (true) {
    long j = counter.getAndIncrement();
    if (j > 10^10)
      break;
    if (isPrime(j))
      print(j);
  }
}
```

Code



Challenge

```
public class Counter {
  private long value;

  public long getAndIncrement() {
    long temp = value;
    value = temp + 1;
    return temp;
  }
}
```

Make these steps *atomic* (indivisible)

Mutual Exclusion

- Today we will try to formalize our understanding of mutual exclusion

- We will also use the opportunity to show you how to argue about and prove various properties in an asynchronous concurrent setting

Mutual Exclusion

- Formal problem definitions
- Solutions for 2 threads
- Solutions for *n* threads
- Fair solutions
- Inherent costs

Warning

- You will never use these protocols
  - Get over it
- You are advised to understand them
  - The same issues show up everywhere
  - Except hidden and more complex

Why is Concurrent Programming so Hard?

- Try preparing a seven-course banquet
  - By yourself
  - With one friend
  - With twenty-seven friends…
- Before we can talk about programs
  - Need a language
  - Describing time and concurrency

What is Time?

- *"Absolute, true and mathematical time, of itself and from its own nature, flows equably without relation to anything external."* **(I. Newton, 1689)**

- *"Time is, like, Nature's way of making sure that everything doesn't happen all at once."* **(Anonymous, circa 1968)**

Time

Events

- An **event** a_0 of thread A is
  - Instantaneous
  - No simultaneous events (break ties)

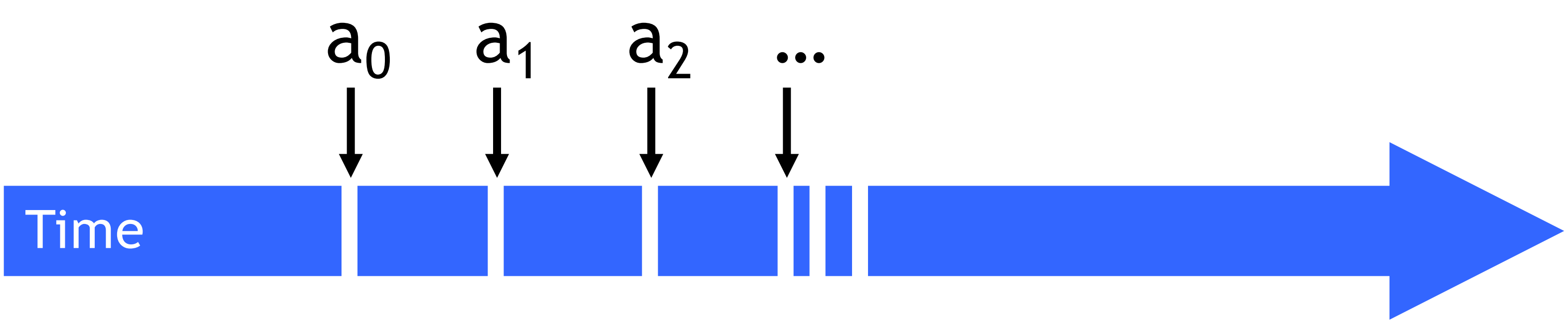
Threads

- A **thread** A is (formally) a sequence a_0, a_1, … of events

processor instructions. read/write/calculate....

  - "Trace" model
  - Notation: $a_0 \rightarrow a_1$ indicates order

precedence

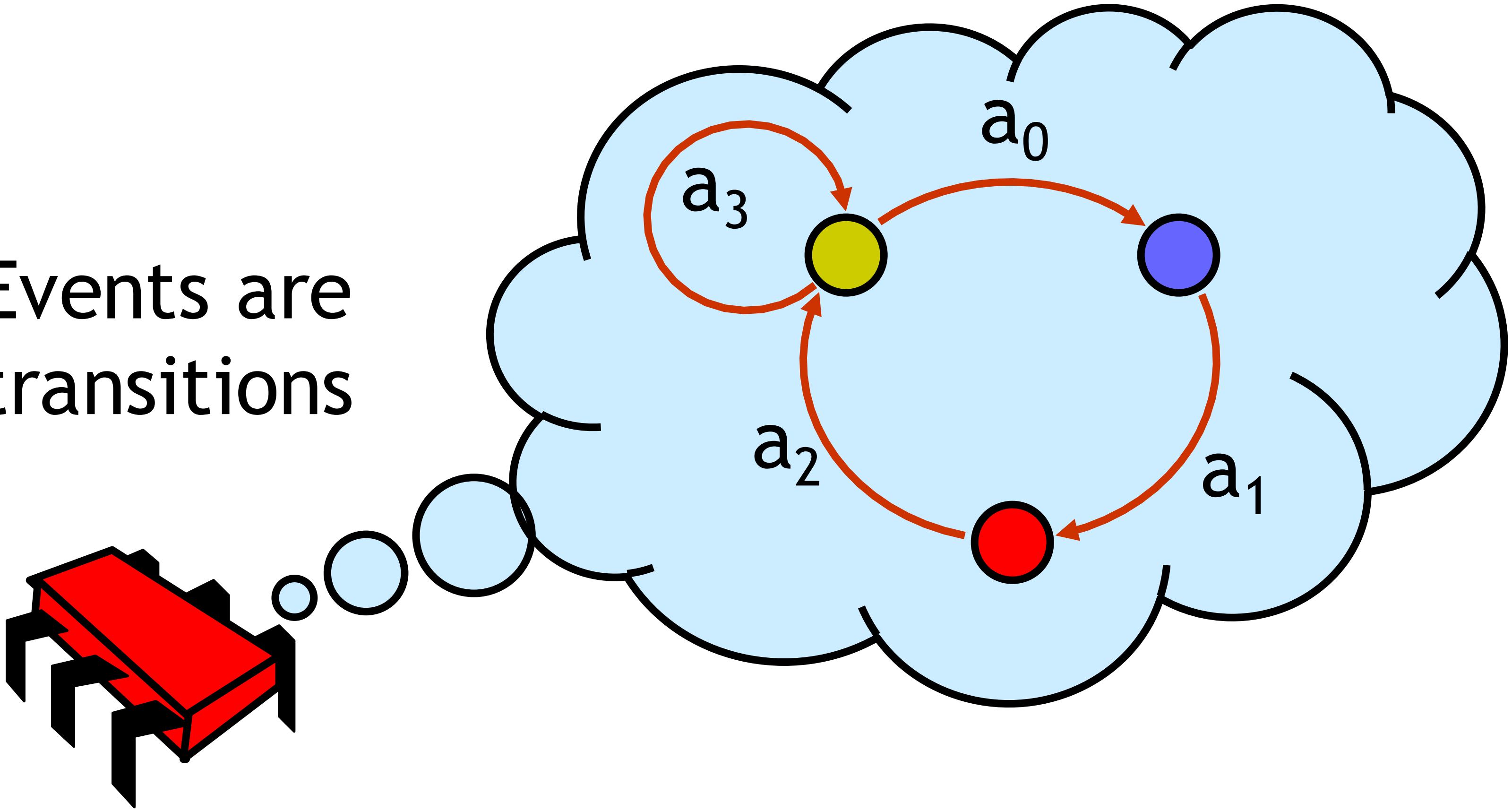


Example Thread Events

- Assign to shared variable
- Assign to local variable
- Invoke method
- Return from method
- Lots of other things…

Threads are State Machines

Events are transitions



States

Data associated with the thread

- Thread state
  - Program counter
  - Local variables

Composed of many threads

- System state
  - Object fields (shared variables)
  - Union of thread states

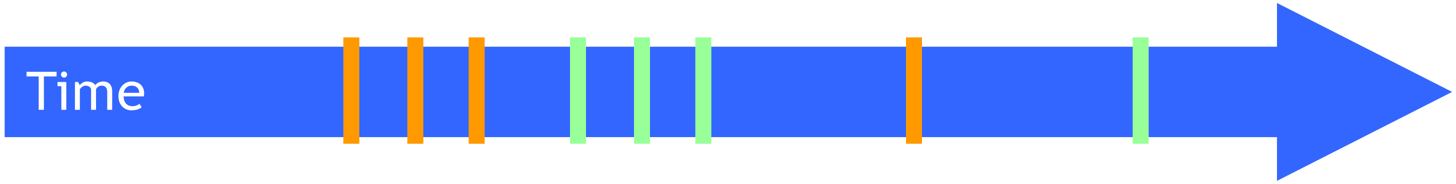
Concurrency

- Thread A

- Thread B

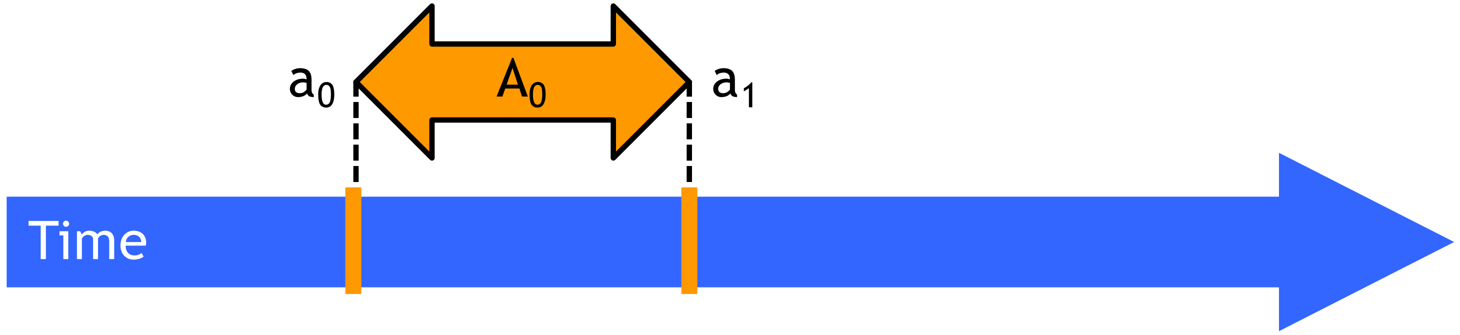
Interleavings

- Events of two or more threads
  - Interleaved
  - Not necessarily independent (why?)



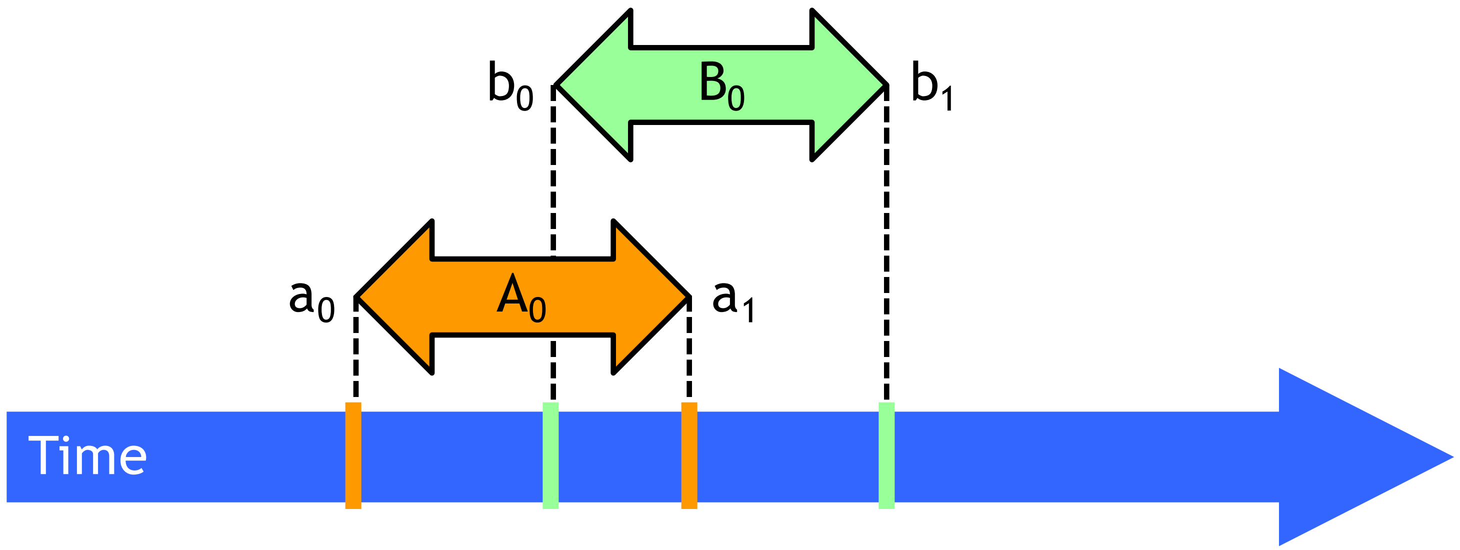
Intervals

- An **interval** $A_0=(a_0,a_1)$ is
  - Time between events a_0 and a_1



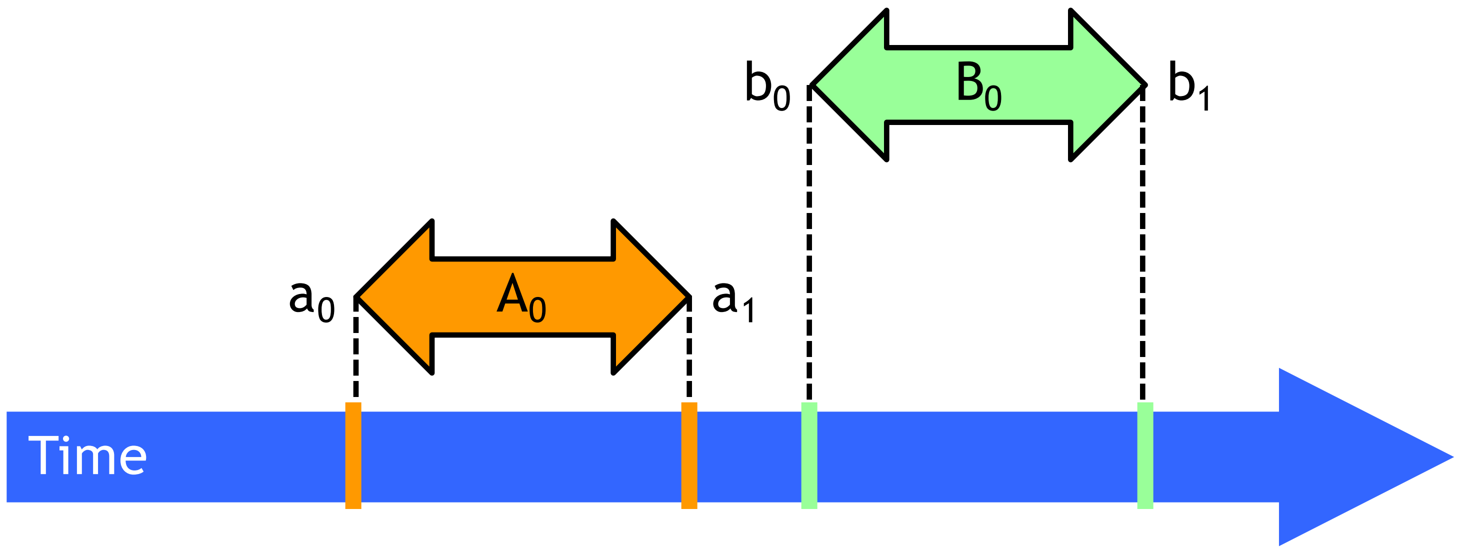
Intervals may Overlap

Nonempty intersection



Intervals may be Disjoint

- Precedence
  - Interval A_0 **precedes** interval B_0

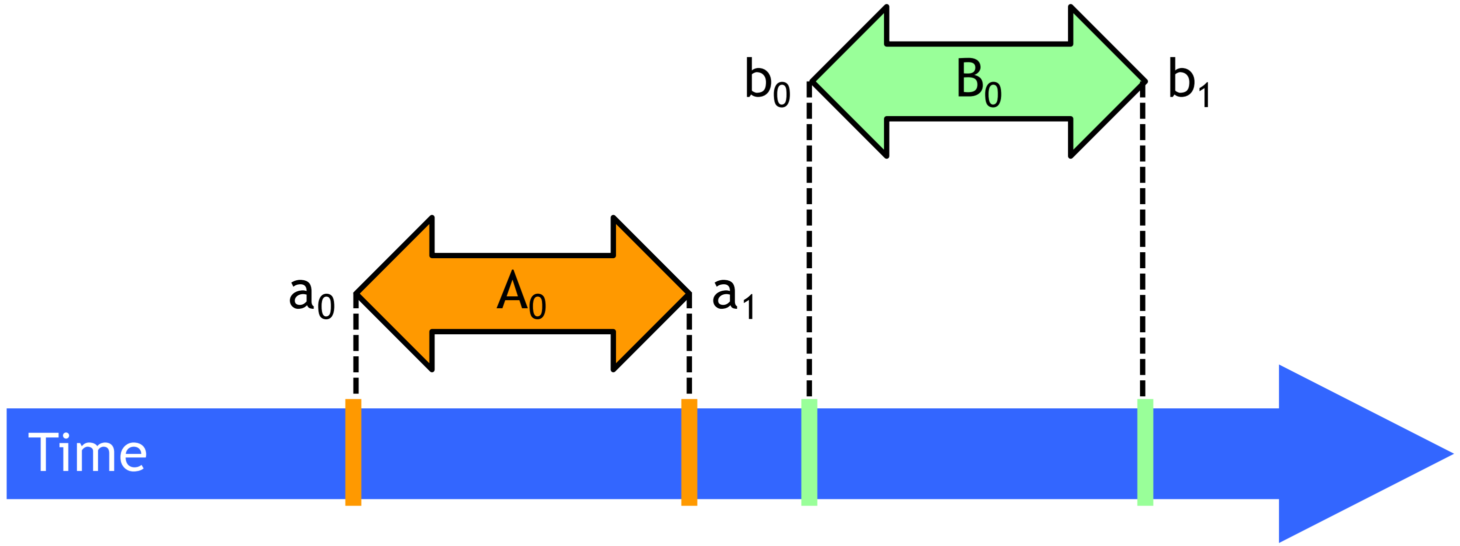


Precedence

- Notation: $A_0 \rightarrow B_0$

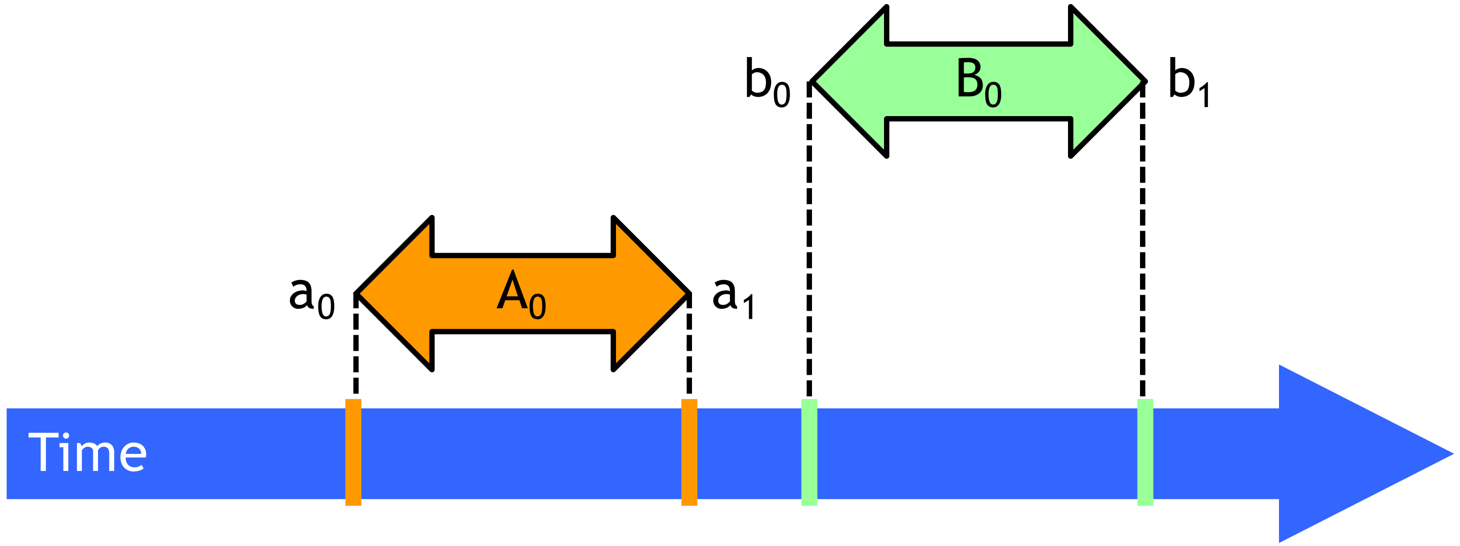
A0 happens before B0 when the last event
of A happens before the first event of B

- Formally:
  - End event of A_0 before start event of B_0
  - Also called “happens before” or “precedes”



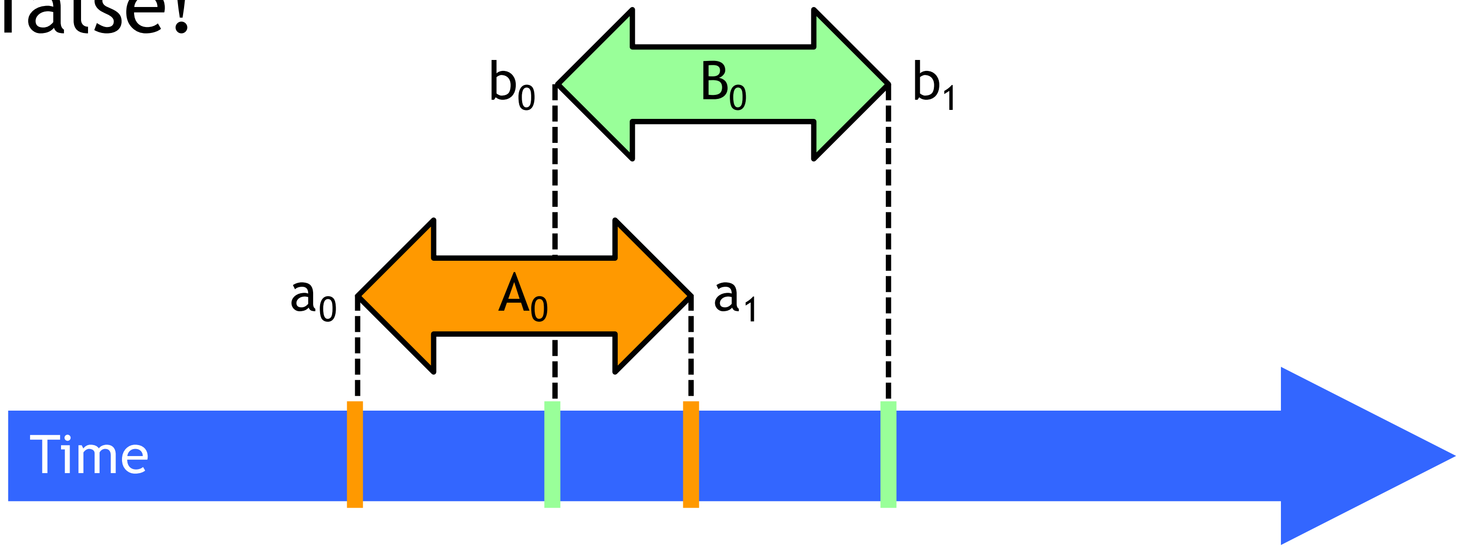
Precedence Ordering

- Remark: $A_0 \rightarrow B_0$ is just like saying
  - 1066 AD ➔ 1492 AD
  - Middle Ages ➔ Renaissance
- Oh wait, what about this week vs. this month?



Precedence Ordering

- Never true that A ➔ A
- If A ➔ B then not true that B ➔ A
- If A ➔ B and B ➔ C then A ➔ C
- Funny thing: A ➔ B and B ➔ A might both be false!



Partial Orders
(you may know this already)

- **Irreflexive**
  - Never true that A ➔ A
- **Antisymmetric**
  - If A ➔ B then not true that B ➔ A
- **Transitive**
  - If A ➔ B and B ➔ C then A ➔ C

Total Orders
(you may know this already)

- Also
  - Irreflexive
  - Antisymmetric
  - Transitive
- Except that for every distinct A, B
  - Either A ➔ B or B ➔ A

Repeated Events

```
while (mumble) {
   a0; a1;
}
```

a_0^k — *k*^th^ occurrence of event a_0

A_0^k — *k*^th^ occurrence of interval $A_0=(a_0,a_1)$

Implementing a Counter

```
public class Counter {
  private long value;

  public long getAndIncrement() {
    long temp = value;
    value = temp + 1;
    return temp;
  }
}
```

Make these steps *atomic* (indivisible)

Locks (Mutual Exclusion)

```
public interface Lock {

  public void lock();

  public void unlock();

}
```

Acquire lock

Release lock

Class that implements this Interface will
ensure mutual exclusion

Using Locks

```
public class Counter {
  private long value;
  private Lock lock;
  public long getAndIncrement() {
    int temp;
    lock.lock();
    try {
      temp = value;
      value = value + 1;
    } finally {
      lock.unlock();
    }
    return temp;
  }
}
```

Acquire lock

Critical section

Release lock (no matter what)

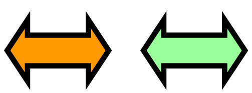
Using Locks

```
private Lock lock;
…
while{mumble}{
   <non critical section>
   lock.lock();
   <critical section>
   lock.unlock();
}
…
```

- Only well formed executions!
- What are the required properties of locks?

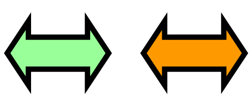
Mutual Exclusion

- Let CS_i^k be thread i's kth critical section execution
- And CS_j^m thread j's mth critical section execution
- Then either



$CS_i^k \rightarrow CS_j^m$

or



$CS_j^m \rightarrow CS_i^k$

Deadlock-Free

- If some thread calls **lock()**
  - And never returns
  - Then other threads must complete **lock()** and **unlock()** calls infinitely often
- System as a whole makes progress
  - Even if individuals starve

Lockout-Free

- If some thread calls lock()
  - It will eventually return
- Individual threads make progress

Two-Thread vs. *n*-Thread

- Two-thread solutions first
  - Illustrate most basic ideas
  - Fits on one slide
- Then *n*-thread solutions

Read/write shared variables (registers)

Two-Thread Conventions

```
class … implements Lock {
  …
  // Thread-local index, 0 or 1
  public void lock() {
    int i = ThreadID.get();
    int j = 1 - i;
  …
  }
}
```

Henceforth: `i` is current thread, `j` is other thread

LockOne

For 2 threads

volatile: will disable reordering/optimizations from compiler to guarantee order. use it when dealing with shared data.

```
class LockOne implements Lock {
  private volatile boolean[]
    flag = new boolean[2];
  public void lock() {
    flag[i] = true;
    while (flag[j]) {}
  }
  public void unlock() {
    flag[i] = false;
  }
}
```

Set my flag

Wait for other flag to go false

No longer interested

LockOne Satisfies Mutual Exclusion

- Assume CS_A^j overlaps CS_B^k
- Consider each thread's (j^{th} and k^{th}) last read and write in the **lock()** method before entering
- Derive a contradiction

From the Code

$\text{write}_A(\text{flag[A]=true}) \rightarrow \text{read}_A(\text{flag[B]==false}) \rightarrow \text{CS}_A$

$\text{write}_B(\text{flag[B]=true}) \rightarrow \text{read}_B(\text{flag[A]==false}) \rightarrow \text{CS}_B$

```
class LockOne implements Lock {
private volatile boolean[] flag = new boolean[2];
public void lock() {
  flag[i] = true;
  while (flag[j]) {}
}
```

From the Assumption

$\text{read}_A(\text{flag[B]==false}) \rightarrow \text{write}_B(\text{flag[B]=true})$

$\text{read}_B(\text{flag[A]==false}) \rightarrow \text{write}_A(\text{flag[A]=true})$

Combining

Assumptions

$read_A(flag[B]==false) \rightarrow write_B(flag[B]=true)$

$read_B(flag[A]==false) \rightarrow write_A(flag[A]=true)$

From the code

$write_A(flag[A]=true) \rightarrow read_A(flag[B]==false)$

$write_B(flag[B]=true) \rightarrow read_B(flag[A]==false)$

Cycle!

Deadlock Freedom

- LockOne fails deadlock-freedom
  - Concurrent execution can deadlock

Time

`flag[i] = true;`	`flag[j] = true;`
`while (flag[j]) {}`	`while (flag[i]){}`

  - Sequential executions OK

LockTwo

```
public class LockTwo implements Lock {
  private volatile int victim;
  public void lock() {
    victim = i;
    while (victim == i) {}
  }

  public void unlock() {}
}
```

Let other go first

Wait for permission

Nothing to do

LockTwo Claims

- Satisfies mutual exclusion
  - If thread `i` in CS
  - Then `victim == j`
  - Cannot be both `i` and `j`
- Not deadlock free
  - Sequential execution deadlocks
  - Concurrent execution does not

```
{
  victim = i;
  while (victim == i) {}
}
```

Peterson's Algorithm

Combines flag and victim

```
public void lock() {
  flag[i] = true;
  victim = i;
  while (flag[j] && victim == i) {}
}
public void unlock() {
  flag[i] = false;
}
```

Announce I'm interested

Defer to other

Wait while other interested & I'm the victim

No longer interested

Mutual Exclusion

```
public void lock() {
  flag[i] = true;
  victim = i;
  while (flag[j] && victim == i) {}
}
```

- If thread 0 in critical section
  - `flag[0] == true`
  - `victim == 1`
- If thread 1 in critical section
  - `flag[1] == true`
  - `victim == 0`

Cannot both be true

Deadlock Free

```
public void lock() {
  …
  while (flag[j] && victim == i) {}
}
```

- Thread blocked
  - Only at **`while`** loop
  - Only if it is the victim
- One or the other must not be the victim

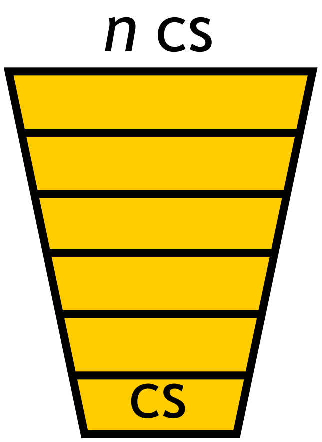
Lockout Free

```
public void lock() {
  flag[i] = true;
  victim = i;
  while (flag[j] && victim == i) {}
}
public void unlock() {
  flag[i] = false;
}
```

- Thread `i` blocked only if `j` repeatedly re-enters with `flag[j] == true` and `victim == i`
- When `j` re-enters, it sets `victim` to `j`, so `i` gets in

The Filter Algorithm for *n* Threads

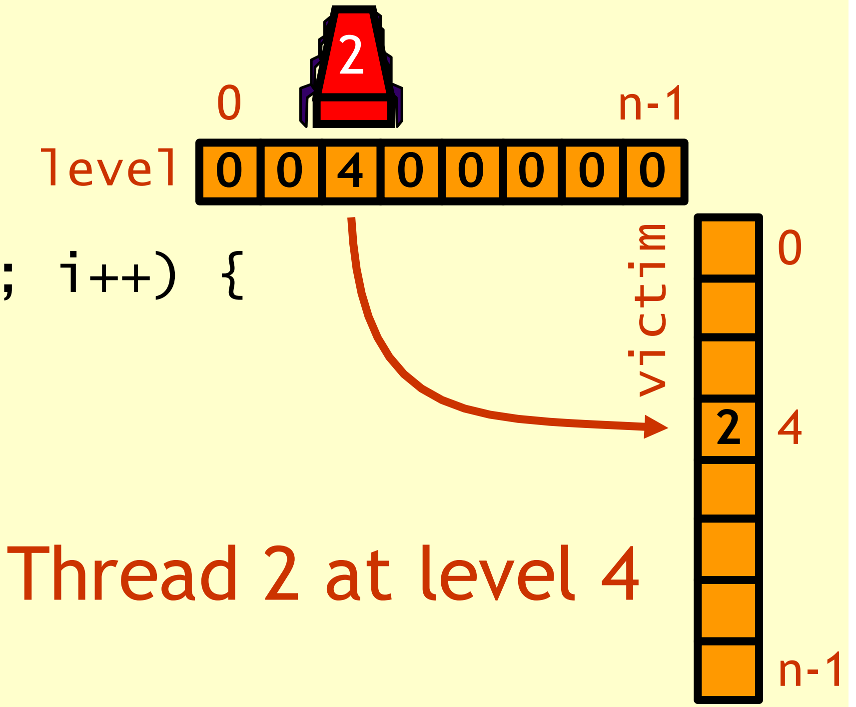
- There are *n-1* “waiting rooms” called levels
- At each level
  - At least one enters level
  - At least one blocked if many try
- Only one thread makes it through



Filter

```
class Filter implements Lock {
  volatile int[] level;   // level[i] for thread i
  volatile int[] victim;  // victim[L] for level L

  public Filter(int n) {
      level = new int[n];
      victim = new int[n];
      for (int i = 0; i < n; i++) {
        level[i] = 0;
      }
  }
  …
}
```



Filter

```
class Filter implements Lock {
  …
  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i] = L;
      victim[L] = i;
      while ((∃k != i | level[k] >= L) &&
        victim[L] == i) {}
    }
  }
  public void unlock() { level[i] = 0; }
}
```

One level at a time

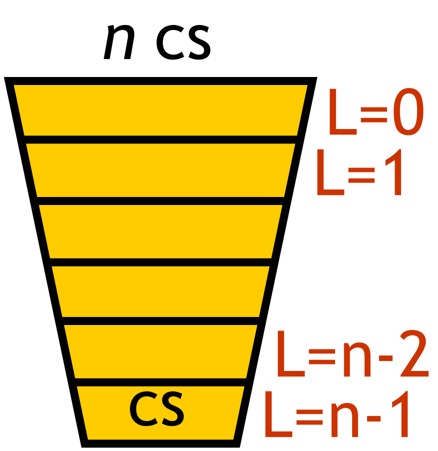
Announce intention to enter level L

Give priority to anyone but me

Wait as long as someone else is at same or higher level and I'm designated victim (enter level L when loop complete)

Claim

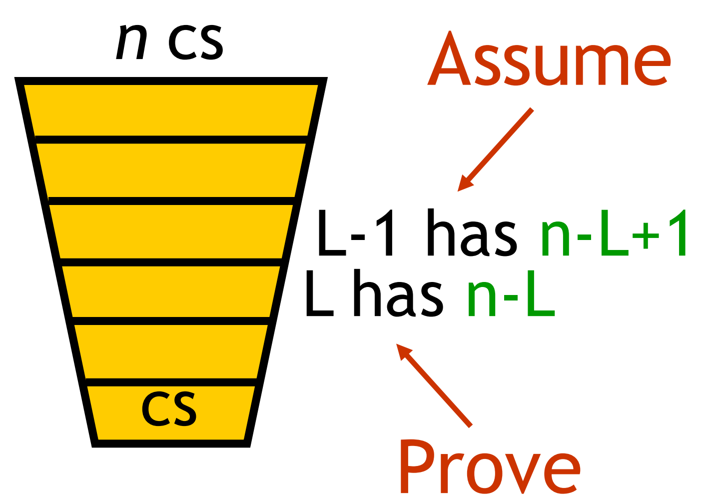
- Start at level L=0
- At most n-L threads enter level L
- Mutual exclusion at level L=n-1



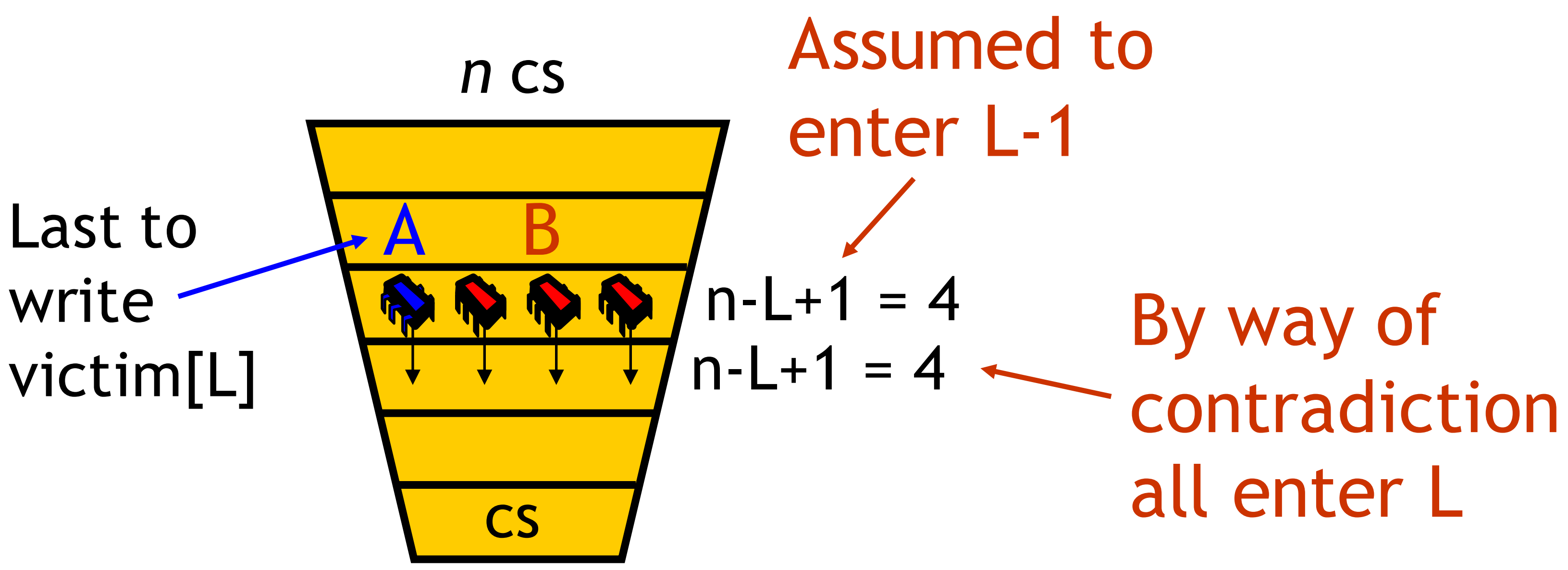
Induction Hypothesis

- No more than n-L+1 threads at level L-1
- Induction step: by contradiction

- Assume all at level L-1 enter level L
- A last to write victim[L]
- B is any other thread at level L



Proof Structure



- Show that A must have seen B at level L and since victim[L] == A could not have entered

Observations

1. $\text{write}_B(\text{level[B]=L}) \rightarrow \text{write}_B(\text{victim[L]=B})$ (code)
2. $\text{write}_A(\text{victim[L]=A}) \rightarrow \text{read}_A(\text{level[B]})$ (code)
3. $\text{write}_B(\text{victim[L]=B}) \rightarrow \text{write}_A(\text{victim[L]=A})$

   (by assumption: A is the last thread to write victim[L])

```
public void lock() {
  for (int L = 1; L < n; L++) {
    level[i] = L;
    victim[L] = i;
    while ((∃k != i | level[k] >= L) && victim[L] == i) {}
  }
}
```

Combining Observations

1. $\text{write}_B(\text{level}[B]=L) \rightarrow \text{write}_B(\text{victim}[L]=B)$ (code)
3. $\text{write}_B(\text{victim}[L]=B) \rightarrow \text{write}_A(\text{victim}[L]=A)$ (ass.)
2. $\text{write}_A(\text{victim}[L]=A) \rightarrow \boxed{\text{read}_A(\text{level}[B])}$ (code)

   Thus, A read level[B] ≥ L, A was last to write victim[L], so it could not have entered level L!

```
public void lock() {
  for (int L = 1; L < n; L++) {
    level[i] = L;
    victim[L] = i;
    while ((∃k != i | level[k] >= L) && victim[L] == i) {}
  }
}
```

No Lockout

- Filter Lock satisfies “no lockout” properties
  - Just like Peterson algorithm at any level
  - So no one starves (no lockout)
- But what about fairness?
  - Threads can be overtaken by others

Bounded Waiting

- Want stronger fairness guarantees
- Thread not “overtaken” too much
- Need to adjust definitions…

- Divide `lock()` method into 2 parts
  - Doorway interval (**D_A**), always finishes in finite number of steps
  - Waiting interval (**W_A**), may take unbounded number of steps

r-Bounded Waiting

typically you want r to be 1

- For threads A and B
  - If $D_A^k \rightarrow D_B^j$
    - A's kth doorway precedes B's jth doorway
  - Then $CS_A^k \rightarrow CS_B^{j+r}$
    - A's kth critical section precedes B's (j+r)th critical section
    - B cannot overtake A by more than r times

- First-come-first-served if **r = 0**

Fairness Again

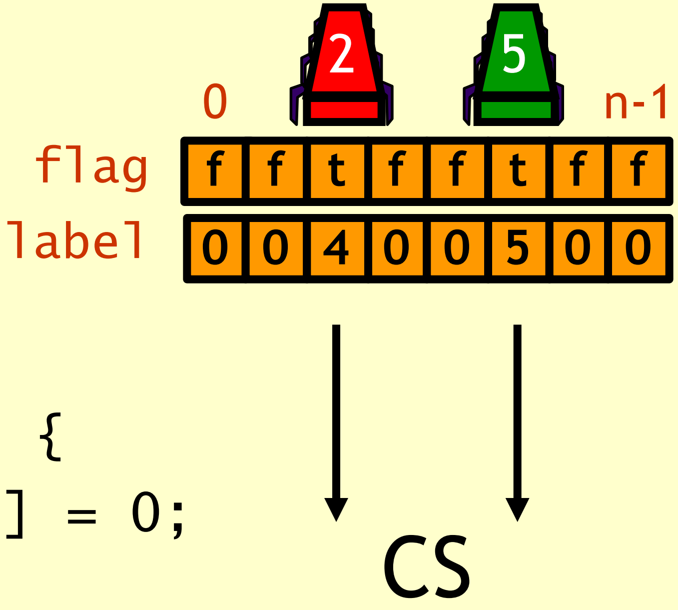
- Filter Lock satisfies some properties
  - No one starves (no lockout)
  - But very weak fairness
    - Not *r*-bounded for any *r*!
  - That's pretty lame…

Bakery Algorithm

- Provides First-Come-First-Served
- How?
  - Take a “number”
  - Wait until lower numbers have been served
- Lexicographic order
  - (a,i) > (b,j)
    - If a > b, or a == b and i > j

Bakery Algorithm

```
class Bakery implements Lock {
  volatile boolean[] flag;
  volatile int[] label;
  public Bakery (int n) {
    flag  = new boolean[n];
    label = new int[n];
    for (int i = 0; i < n; i++) {
       flag[i] = false; label[i] = 0;
    }
  }
 …
```



Bakery Algorithm

Doorway lines are completed in a finite number of steps

Doorway

```
class Bakery implements Lock {
  …
  public void lock() {
    flag[i] = true;
    label[i] = max(label[0],…,label[n-1])+1;
    while ((∃k | flag[k])
      && (label[i],i) > (label[k],k)) {}
  }
  …
}
```

I'm interested

Take increasing label (read labels in some arbitrary order)

Someone is interested…

…with lower label in lexicographic order

Bakery Algorithm

```
class Bakery implements Lock {
  …
  public void unlock() {
    flag[i] = false;
  }
}
```

No longer interested

No Deadlock

- There is always one thread with earliest label
- Ties are impossible (why?)

because we have a total order

First-Come-First-Served

- If $D_A \rightarrow D_B$ then A's label is earlier
  - $\text{write}_A(\text{label}[A]) \rightarrow \text{read}_B(\text{label}[A]) \rightarrow \text{write}_B(\text{label}[B]) \rightarrow \text{read}_B(\text{flag}[A])$
- So B is locked out while flag[A] == true

```
class Bakery implements Lock {
  public void lock() {
    flag[i] = true;
    label[i] = max(label[0],…,label[n-1])+1;
    while ((∃k | flag[k]) && (label[i],i) > (label[k],k)) {}
  }
}
```

Mutual Exclusion

- Suppose A and B in CS together
- Suppose A has earlier label
- When B entered, it must have seen
  flag[A] == false or label[A] > label[B]

```
class Bakery implements Lock {
  public void lock() {
    flag[i] = true;
    label[i] = max(label[0],…,label[n-1])+1;
    while ((∃k | flag[k]) && (label[i],i) > (label[k],k)) {}
  }
}
```

Mutual Exclusion

- Labels are strictly increasing so B must have seen flag[A] == false
  - $\text{labeling}_B \rightarrow \text{read}_B(\text{flag[A]}) \rightarrow \text{write}_A(\text{flag[A]}) \rightarrow \text{labeling}_A$
- Which contradicts the assumption that A has an earlier label

Deep Philosophical Question

- The Bakery Algorithm is
  - Succinct
  - Elegant
  - Fair
- Q: So why isn't it practical?
- A: Well, you have to read *n* distinct variables

you have to go through all arrays. cost will increase linear to the number of threads

Shared Memory

- Shared read/write memory locations called registers (historical reasons)
- Come in different flavors
  - Multi-Reader-Single-Writer (`flag[]`)
  - Multi-Reader-Multi-Writer (`victim[]`)
  - Not interesting: SRMW and SRSW

Theorem

- At least *n* MRSW (multi-reader/single-writer, like `flag[]`) registers are needed to solve deadlock-free mutual exclusion

Summary of Lecture

- In the 1960's many incorrect solutions to lockout-free mutual exclusion using RW-registers were published…
- Today we know how to solve FIFO *n* thread mutual exclusion using *2n* RW-registers