Risk Premia in the Bitcoin Market

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Introduction

- Cryptocurrencies are decentralized currencies that have had increasing interest as alternative investments.
- Bitcoin is one of the most prominent among the Cryptos having the highest market cap followed by Ethereum.
- Moreover, it is the only Crypto for which there is an active option market.
- There is an ongoing discussion regarding the existence or not of a risk-premium for holding these currencies.
- In this paper, we document and analyze the basic risk-premium properties of the Bitcoin market using data on options and the Bitcoin index.

Correlation

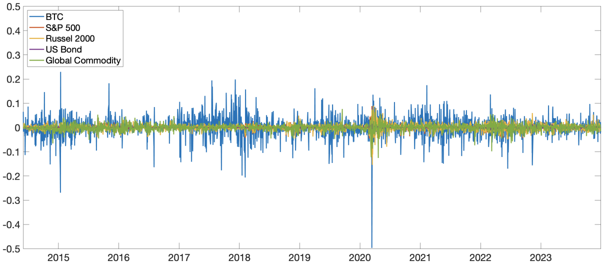


Table B21: Correlation matrix

	BTC	S&P 500	Russel 2000	US Bond	Global Commodity
BTC		-0.016	-0.013	0.003	0.064^{**}
S&P 500			0.877^{***}	-0.215^{***}	0.317^{***}
Russel 2000				-0.204^{***}	0.326^{***}
US Bond					-0.174^{***}
Global Commodity					

For equity markets, we use S&P 500 and Russel 2000 indices. For bond market, we utilize the S&P US Treasury Bond Index. For Global commodity, we use S&P GSCI index. Correlation is performed using a t-test (H_0 : no correlation). The t-statistic is calculated as $t = \frac{corr\sqrt{n-2}}{\sqrt{1-corr^2}}$, where *corr* represents the correlation coefficient and *n* is the sample size. Significance levels are denoted by $1\%(^{***})$, $5\%(^{**})$ and $10\%(^{*})$ denoting significance level.

Research Questions

1. Bitcoin has low correlation with the majority of the asset classes and, in addition, pays no dividend. Therefore, what should be the bitcoin premium? Zero, positive, time-varying?

⊡ We identify the **Implied premia** from **Bitcoin index** and **options**.

- Deribit exchange: World's biggest Bitcoin and Ethereum Options Exchange

2. Is there risk premia **variation** over time?
    - New Clustering methodology based on the time-series of risk-neutral measures.
    - State-dependency *drivers*
3. How does it compare to **premia of conventional assets**?
    - S&P 500 implied premia

Contribution

- **Risk premia** in the Bitcoin Index (BTC) market
  - Bitcoin premium (BP) and variance risk premium (BVRP)
  - Bitcoin premium $\mathrm{BP}(\cdot)$ and $\mathrm{PK}(\cdot)$ functions of returns
- **State-dependent** risk premia
  - Clustering of option-implied *risk-neutral densities* (RNDs)
  - Risk-neutral *variance* as the most important state variable
  - *Conditional* estimation of the *risk measures* during high volatility (HV) and low volatility (LV) market regimes

Main Findings

Bitcoin Premia

- Monthly Bitcoin premium $(\mathrm{BP}) \approx 66\%$ p.a.
  - Negative returns $[-60\%, -20\%]$ account for 33% of the BP
  - Positive returns $[20\%, 60\%]$ account for 48% of the BP
- Bitcoin variance risk premium $(\mathrm{BVRP}) \approx 14\%$

Market regimes

- LV: Focus on upside risk in BP
- HV: Focus on upside and downside risk in BP
- Higher BVRP during LV

Related Literature

- **Bitcoin premium decomposition**
  - Beason et al. (2022), Almeida et al. (2024)
- **State-dependent pricing kernel**
  - Rosenberg et al. (2002), Chabi-Yo et al. (2007), Branger et al. (2011), Chabi-Yo (2012), Song et al. (2016), Grith et al. (2013), Grith et al. (2017), Almeida et al. (2022), Schreindorfer et al. (2023)
- **Cryptocurrency derivatives**
  - Hou et al. (2020), Hoang et al. (2020), Chen et al. (2021), Cao et al. (2021), Foley et al. (2022), Alexander et al. (2023a), Winkel et al. (2023a), Winkel et al. (2023b), **Alexander2024-ai**, Alexander et al. (2023b)

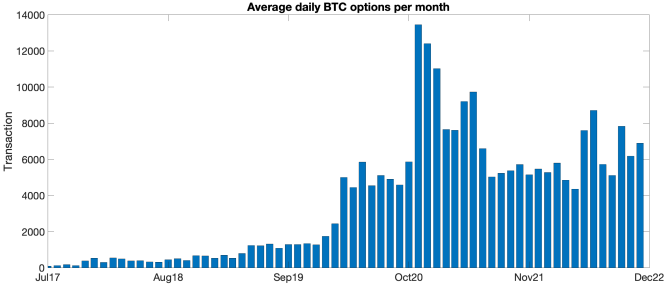
Data

- **Source**: Deribit data from Blockchain Research Center (BRC)

- *Bitcoin settlement prices* from 2014 to 2022 (3287 days)
- *Options transaction prices* from 2017 to 2022 (505 days)
  - Cash-settled European-style options with a lot size of 1 BTC
  - Divisible assets: Possible to trade fractional units of options
  - Focus one month. Contracts with time to maturity τ from 3 to 60 days and moneyness $m = K/S$ between 0.5 and 2.0; K is the strike price, S the settlement BTC price

- All instruments are denominated in U.S. Dollars

Daily transaction



*Each bar represents average transactions in one month.

	Before 2020	After 2020	Overall
Ave. Trans. Obs.	646	6356	3721

Risk Premia Methodology

- Under no-arbitrage, the **Bitcoin premium** $(\mathbf{BP})$ is

$$\mathrm{BP} := \mu_{\mathbb{P}} - \mu_{\mathbb{Q}} = \int_{-1}^{\infty} x\{p(x) - q(x)\}dx$$

  $\mu_{\mathbb{P}} = \mathbb{E}_{\mathbb{P}}(R)$ and $\mu_{\mathbb{Q}} = \mathbb{E}_{\mathbb{Q}}(R) = R^f$, with R^f risk-free rate
  $q(\cdot)$ risk-neutral density (RND) and $p(\cdot)$ physical density

  $\mathbf{BP}$ **decomposition** on return states, Beason et al. (2022)

$$\mathrm{BP}(r) = \frac{\int_{-1}^{r} x\{p(x) - q(x)\}dx}{\mathrm{BP}}$$

- **Bitcoin variance risk premium** $(\mathbf{BVRP})$

$$\mathrm{BVRP} := \sigma^2_{\mathbb{Q}} - \sigma^2_{\mathbb{P}},$$

  $\sigma^2_{\mathbb{Q}} = \mathrm{Var}_{\mathbb{Q}}(R)$ and $\sigma^2_{\mathbb{P}} = \mathrm{Var}_{\mathbb{P}}(R)$

Unconditional Estimators

$$\widehat{\mathrm{BP}} = \widehat{\mu}_{\mathbb{P}} - \widehat{\mu}_{\mathbb{Q}} \quad \text{and} \quad \widehat{\mathrm{BVRP}} = \widehat{\sigma}^2_{\mathbb{Q}} - \widehat{\sigma}^2_{\mathbb{P}}$$

$$\widehat{\mathrm{BP}}(r) = \int_{-1}^{r} x\{\widehat{p}(x) - \widehat{q}(x)\}dx/\widehat{\mathrm{BP}}$$

- **Physical moments**
  - $\widehat{\mu}_{\mathbb{P}} = \frac{365}{\tau}\int_{-1}^{\infty} x\widehat{p}(x)dx$, $r_t = S_t/S_{t-\tau} - 1$
  - $\widehat{\sigma}^2_{\mathbb{P}} = \frac{1}{T}\sum_{t=1}^{T} \mathrm{RV}_t$, $\mathrm{RV}_t = \frac{365}{\tau}\sum_{l=1}^{\tau} r^2_{d,t-l}$, $r_{d,t} = \log S_t/S_{t-1}$
- **Risk-neutral moments**
  - $\widehat{\mu}_{\mathbb{Q}} = 0$
  - $\widehat{\sigma}^2_{\mathbb{Q}} = \frac{1}{T}\sum_{t=1}^{T} \mathrm{BVIX}^2_t$, Bitcoin Volatility Index (BVIX)
- **Risk-neutral and physical densities**
  - $\widehat{p}$ smoothed histogram of returns with GEV tails
  - $\widehat{q}(r) = \frac{1}{T}\sum_{t=1}^{T} \widehat{q}_t(r)$, daily nonparametric estimates $\widehat{q}_t(r)$ (Rookley, 1997) with GEV tails (Figlewski, 2008)

Clustering

Objects: Collection of RNDs for different maturities

$\rightarrow$ RNDs $\sim$ second derivative of the (rescaled) call prices w.r.t. moneyness (Breeden et al., 1978)

$\rightarrow$ Centered-log-ratio (CLR) transformation: fcts. in Hilbert space

- **Distance metric**: L_2 distance Peng et al. (2008)

$$D(i,j) = \sqrt{\int_\tau \int_r \left[\mathrm{clr}\left\{q_i(r,\tau)\right\} - \mathrm{clr}\left\{q_j(r,\tau)\right\}\right]^2 dr d\tau}.$$

- **Clustering method**: Agglomerative hierarchical clustering algorithm (Hastie et al., 2009) with Ward linkage to the distance matrix D

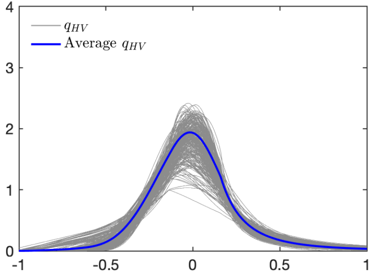
Conditional Estimators

Cluster membership $\gamma = (c_1, \ldots, c_T)'$ with $c_t \in \{c_i \mid i = \text{HV, LV}\}$

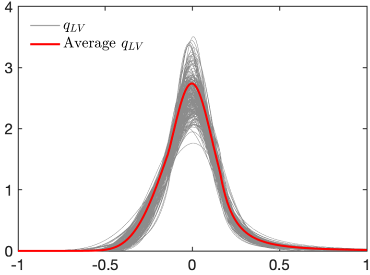
- $\widehat{q}_i(x) = \frac{1}{T_i} \sum_{\substack{t=1,\\ c_t = c_i}}^{T} \widehat{q}_t(x)$, T_i number of days in cluster i
- $\widehat{\mu}_{\mathbb{Q},i} = 0$
- $\widehat{\sigma}^2_{\mathbb{Q},i} = \frac{1}{|C_i|} \sum_{\substack{t=1,\\ c_t = c_i}}^{T} \text{BVIX}_t^2$

- $\widehat{p}_i$, smoothed histogram of $r_{t,i} = (r_t + 1)^{\widehat{\sigma}_{\mathbb{P},i}/\widehat{\sigma}_{\mathbb{P}}} - 1$, with $\widehat{\sigma}^2_{\mathbb{P},i} = \frac{1}{T_i} \sum_{\substack{t=1,\\ c_t = c_i}}^{T} \text{RV}_t$, $\widehat{\sigma}^2_{\mathbb{P}} = \frac{1}{T} \sum_{t=1}^{T} \text{RV}_t$
- $\widehat{\mu}_{\mathbb{P},i} = \frac{365}{\tau} \int_{-1}^{\infty} x \widehat{p}_i(x) dx$
- $\widehat{\sigma}^2_{\mathbb{P},i} = \frac{1}{T_i} \sum_{\substack{t=1,\\ c_t = c_i}}^{T} \text{RV}_t$

Clustering Results



(a) HV cluster



(b) LV cluster

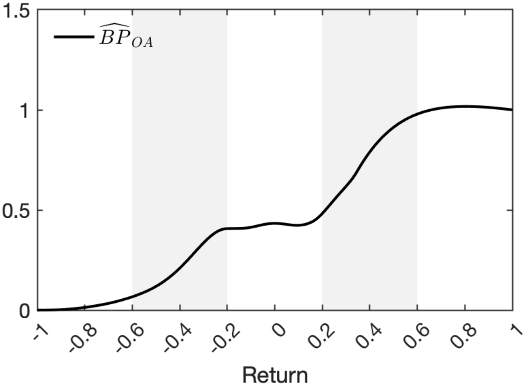
Figure: Empirical RNDs for time-to-maturity one month. Clustering relies on L_2 distance between RNDs surfaces.

Risk Premia

Panel A: Bitcoin premium						
	Based on $\widehat{q}$			Based on $R^f = 0$		
	Overall	HV	LV	Overall	HV	LV
$\widehat{\text{BP}}$	**0.66**	**0.73**	**0.55**	**0.67**	**0.69**	**0.62**
$\widehat{\mu}_{\mathbb{P}}$	0.67	0.69	0.62	0.67	0.69	0.62
$\widehat{\mu}_{\mathbb{Q}}$	0.01	-0.03	0.07	0	0	0
Panel B: Bitcoin variance risk premium						
	Based on $\widehat{q}$			Based on BVIX		
	Overall	HV	LV	Overall	HV	LV
$\widehat{\text{BVRP}}$	**0.07**	**0.04**	**0.10**	**0.14**	**0.12**	**0.17**
$\widehat{\sigma}^2_{\mathbb{Q}}$	0.63	0.80	0.43	0.71	0.88	0.50
$\widehat{\sigma}^2_{\mathbb{P}}$	0.57	0.76	0.33	0.57	0.76	0.33
Days	505	278	227	482	271	211

Estimates of the monthly unconditional and conditional $\mathrm{BP}/\mathrm{BVRP}$ (annualized). $\widehat{\mu}_{\mathbb{P}} = \frac{365}{\tau} \int_{-1}^{\infty} x\widehat{p}(x)dx$; $\widehat{\sigma}^2_{\mathbb{P}} = \frac{365}{\tau} \sum_{t=1}^{T} \text{RV}_t$. (i) $\widehat{\mu}_{\mathbb{Q}} = \frac{365}{\tau} \int_{-1}^{\infty} x\widehat{q}(x)dx$ or (ii) $\widehat{\mu}_{\mathbb{Q}} = 0$; (i) $\widehat{\sigma}^2_{\mathbb{Q}} = \frac{365}{\tau} \int_{-1}^{\infty} \left\{x - \frac{\tau}{365}\widehat{\mu}_{\mathbb{Q}}\right\}^2 \widehat{q}(x)dx$ or (ii) $\widehat{\sigma}^2_{\mathbb{Q}} = \frac{1}{T} \sum_{t=1}^{T} \text{BVIX}_t^2$

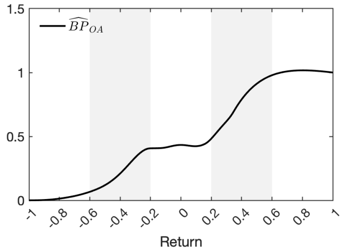
BTC Premium Decomposition



- Negative returns $[-60\%, -20\%]$ account for 33% of the BP
- Positive returns $[20\%, 60\%]$ account for 48% of the BP

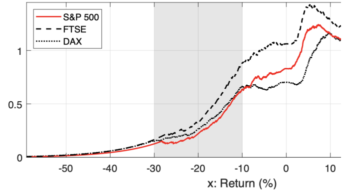
BTC vs Equity Premium Decomposition

BTC Market



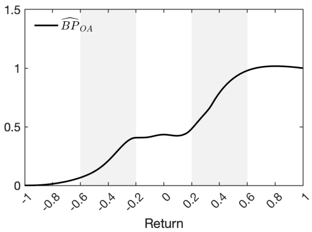
- Investors are more concerned with upside risk.
- A broader range of returns contributes positively to the BP.

SPX, FTSE, DAX Markets



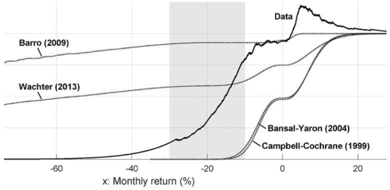
- Investors are more concerned with downside risk.
- A narrower range of returns contributes positively to the equity premium.

BTC Market



- *Habit and long-run risk*
  - Attribute a too-small fraction to medium pos. ret.
- *Rare disaster*
  - Attribute a too-large fraction to extreme and medium neg. ret.

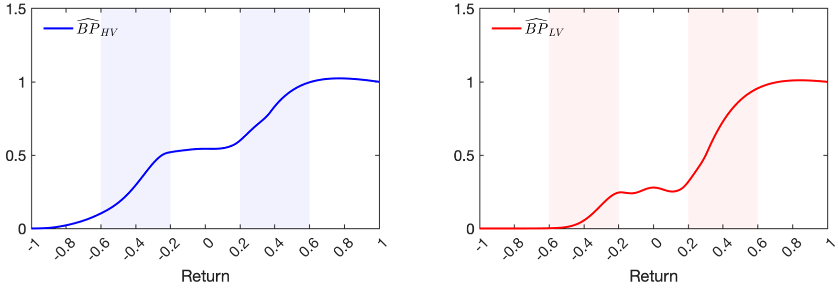
SPX Market



Model-implied S&P500 premium
Beason et al. (2022)

- *Disappointment aversion*
  - Attribute a too-small fraction to the medium pos. ret.
  - Attribute a too-large fraction to the medium neg. ret.

Market Regimes



- **HV:** Equal contribution to BP of positive and negative returns
- **LV:** Most contribution to BP attributable to positive returns

BVRP and ATM Options Returns

- Investor concerns about **volatility uncertainty in LV**

	Simple			Delta-hedged		
	Call	Put	Straddle	Call	Put	Straddle
Overall	-8.49	-37.98	-17.39	-8.82	-3.83	-17.96
Obs.	1071	599	28	1071	599	28
HV	-23.88	-22.92	-6.61	-12.79	-3.11	-4.17
Obs.	602	309	14	602	309	14
LV	11.25	-54.02	-28.16	-3.72	-4.60	-31.75
Obs.	469	290	14	469	290	14

Monthly returns of long ATM options with $|\Delta_{t,i}^{C/P}| \in [0.35, 0.65]$

Simple strategy: $r_{C,i} = \frac{(S_T - K_i)^+ - C_{t,i}}{C_{t,i}}, \; r_{P,i} = \frac{(K_i - S_T)^+ - P_{t,i}}{P_t}$

DH strategy: $r_{C,i} = \frac{C_{T,i} - \Delta_{t,i}^C S_T - C_{t,i} + \Delta_{t,i}^C S_t}{C_{t,i} - \Delta_{t,i}^C S_t}, \; r_{P,i} = \frac{P_{t,i} - \Delta_{i,}^P S_T - P_{t,i} + \Delta_{t,i}^P S_t}{P_{t,i} - \Delta_{t,i}^P S_t}$

where $\Delta_{t,i}^{C/P}$ is the option's delta calculated by BS

Options Returns

Panel A: Overall

	Call					Put					
	DITM	ITM	ATM	OTM	DOTM	DOTM	OTM	ATM	ITM	DITM	
EA	28.97	-36.15	-9.20	-40.43	Obs.	158	171	914	921	1716	1426

Panel B: HV cluster

	Call					Put					
	DITM	ITM	ATM	OTM	DOTM	DOTM	OTM	ATM	ITM	DITM	
EA	37.58	-6.72	-23.17	-82.20	-97.91	-76.48	-38.64	-23.88	-82.34	81.64	
Obs.	82	85	521	584	795	698	485	188	6	4	

Panel C: LV cluster

	Call					Put					
	DITM	ITM	ATM	OTM	DOTM	DOTM	OTM	ATM	ITM	DITM	
EA	19.69	-65.23	9.32	31.97	-72.09	-100	-87.57	-48.29	18.00	-10.66	
Obs.	76	86	393	337	921	728	313	207	7	101	

DH strategy: $r_{C,i} = \dfrac{c_{T,i} - \Delta^{C}_{t,i} S_T - c_{t,i} + \Delta^{C}_{t,i} S_t}{c_{t,i} - \Delta^{C}_{t,i} S_t}$, $r_{P,i} = \dfrac{P_{t,i} - \Delta^{P}_{i,} S_T - P_{t,i} + \Delta^{P}_{t,i} S_t}{P_{t,i} - \Delta^{P}_{t,i} S_t}$

where $\Delta^{C/P}_{t,i}$ is the option's delta calculated by BS

Conclusion

- We analyze returns on options and index to study the risk-premia structure in the Bitcoin market.
- We document that the unconditional pricing kernel implied by Bitcoin data is strongly U-shaped.
- Moreover, using a new clustering technique on risk-neutral measures we identify that variance is an important variable to condition on and study the pricing kernel under two different regimes: High Volatility (HV) and Low Volatility (LV) regimes.
- Under the HV regime, the marginal investor is more sensitive to downside risks.
- Under the LV regime, the marginal investor is concerned with upside risk and worried about hedging against variance risk.