KEYMASTER

You are trapped in a maze filled with keys and locked doors! Your goal is to gather keys to open corresponding gates and find the exit. The input for the problem is a graph $G = (V, E)$ where all $v \in V$ represent rooms in the maze and the edges represent corridors between rooms. Let the set of keys be denoted by $K = \{k_1, k_2, \ldots, k_n\}$ and the set of doors be denoted by $D = \{d_1, d_2, \ldots, d_n\}$. Every vertex v has a component defined by component$(v) = c$, where $c \in K \cup D \cup \{\emptyset\}$. Note that multiple nodes may have the same component, permitting duplicate keys and doors. Finally, the start and target nodes are respectively denoted by $s, t \in V$ where component$(s) =$ component$(t) = \emptyset$.

The goal is to open the doors such that a traversable path from s to t is constructed. A node u is defined as traversable if there is some path from s to u that does not pass through any locked doors, although u itself may be a locked door. Consequently, if presented with a pair of nodes x and y both traversable from s where component$(x) = k_i$ and component$(y) = d_i$, you may choose to unlock the door at node y. Should you make the choice, set both component(x) and component(y) to $\emptyset$. Thus, the key at node x is consumed and may not be used to open more than one door.

Prove that it is NP-Hard to decide whether you can open doors in a sequence such that a traversable path from s to t is constructed. The output is either TRUE or FALSE.