ENTMOOT

The shadow of Isengard looms, and Saruman's engines cut deep into the ancient woods of Fangorn. Trees fall, rivers run black, and yet the Ents slumber in thought. Merry and Pippin, who have witnessed the destruction firsthand, know that time is short. They must convince the Ents to go to war.

But the Ents are slow to rouse and deeply rooted in their ways. An Ent will only awaken if all of its neighboring Ents have already resolved to march. Fortunately, Merry and Pippin are granted the chance to personally inspire and rouse up to k Ents of their choosing. Once roused, an Ent will awaken and, in turn, may rouse others. Can consensus among the Ents be forged in time to save the ancient woods?

Formally, let the forest be represented as an undirected graph $G = (V, E)$, where each node $v \in V$ corresponds to an Ent, and edges indicate neighboring Ents. Initially, all Ents are *undecided* (not yet roused). At time $t = 0$, Merry and Pippin may personally rouse up to k Ents of their choosing. Then, at each subsequent time step $t > 0$, any undecided Ent becomes roused if *all* of its neighboring Ents have already been roused.

Given a graph G and an integer k, does there exist a set $S \subseteq V$ with $|S| \leq k$ such that rousing the Ents in S eventually results in **all** Ents becoming roused to fight?

Prove that **ENTMOOT** is NP-Complete.

1