

RIDDLECRAFT

In the cold, dark depths of the Misty Mountains, Bilbo Baggins finds himself trapped in a high-stakes game of riddles with the creature known as Gollum. One wrong word, and he may never see the Shire again. To survive, Bilbo must craft a riddle—not just any riddle, but one that leads unambiguously to a single, specific answer. It must be clever, clear, and precise.

Bilbo scours his memory for every riddle, proverb, and puzzle he has ever heard. Some are useful, some are misleading, and some are completely irrelevant. His goal is to choose just enough of them to craft a riddle that leads Gollum to a specific answer—but no more than necessary, lest Gollum solve it too easily.

Formally, let $S = \{\phi_1, \phi_2, \dots, \phi_n\}$ be a set of propositional logic formulas representing Bilbo's potential clues, and let C be a propositional formula representing the desired answer to the riddle. The goal is to identify the smallest subset $S' \subseteq S$ such that $S' \models C$, i.e., C is a logical consequence of only the formulas in S' .

That is, Bilbo seeks the smallest set of clues that still leads Gollum to the correct—and only—answer.

Given a set S of formulas and a conclusion C , find the smallest subset $S' \subseteq S$ such that $S' \models C$.

Prove that **RIDDLECRAFT** is NP-hard.