

Q1. Explain the following with suitable terms

i. Tautology

⇒ A tautology is a logical statement that is always true regardless of the truth values of its components. It is valid purely by its structure. Example:

" $p \vee \neg p$ " (p or not p) is always true since one part must be true irrespective of p 's value.

ii. Contradiction

⇒ A contradiction is a logical statement that is always false regardless of the truth values of its components. Example:

" $p \wedge \neg p$ " is always false since both parts cannot be simultaneously true or false.

iii. Contingency:

⇒ A contingency is a logical statement whose truth value depends on the specific truth values of its components. It is neither a tautology nor a contradiction.

Example: " $p \rightarrow q$ " (if p , then q) can be true or false depending on p & q

Q2. Write a note on applications of well ordering principle.

\Rightarrow The Well Ordering Principle states that every non-empty set of positive integers has a least element. This fundamental concept has several key applications:

1. Mathematical Induction: Induction relies on the Well Ordering Principle to ensure that if a counterexample exists, it has a minimal element, leading to a contradiction.
2. Algorithm Termination: Algorithms that operate on decreasing positive integers are proven to terminate using the principle since the sequence must reach a minimum.
3. Division Algorithm: The principle is used to prove the Division Algorithm by identifying a minimal remainder in the set of possible values.
4. Number Theory Proofs: It is applied to prove the existence of greatest common divisors, prime factorizations, and other number theory concepts.

5. Constructive Proofs: The principle exists in constructive proofs by defining objects based on minimality conditions.