Types of Proof with Simple Examples

Introduction to Proofs

- - Proofs are logical arguments that establish the truth of mathematical statements.
- - Used in engineering, computer science, and logical reasoning.
- - Different methods help in verifying statements systematically.

Direct Proof

- \- Assumes a given statement is true and logically deduces the conclusion.
- \- Example: Prove that if n is even, then n^2 is even.
- \- Let $n = 2k$ (definition of even numbers).
- \- Then $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$, which is even.

Indirect Proof (Contrapositive Proof)

- - Instead of proving $P \to Q$, we prove $\neg Q \to \neg P$.
- - Example: Prove that if n^2 is odd, then n is odd.
- - Assume n is even, so $n = 2k$.
- - Then $n^2 = (2k)^2 = 4k^2$, which is even, contradicting the assumption.
- - Hence, if n^2 is odd, n must be odd.

Proof by Contradiction

- \- Assume the opposite of what you want to prove and derive a contradiction.
- \- Example: Prove that $\sqrt{2}$ is irrational.
- - Assume $\sqrt{2}$ is rational, meaning it can be written as p/q in simplest form.
- - Squaring both sides gives $2 = p^2/q^2 \rightarrow 2q^2 = p^2$, so p^2 is even.
- - Thus, p is even (since squares of odd numbers are odd).
- - Let $p = 2k$, substituting gives $q^2 = 2k^2$, so q^2 is even, making q even.
- - This contradicts the assumption that p/q is in simplest form.
- - Hence, $\sqrt{2}$ is irrational.

Proof by Mathematical Induction

- - Used for proving statements about natural numbers.
- - Steps:
- 1. Base Case: Verify the statement for the smallest value.
- 2. Inductive Hypothesis: Assume it holds for n = k.
- 3. Inductive Step: Prove it holds for n = k+1.
- - Example: Prove that 1 + 2 + ... + n = n(n+1)/2.

Paradoxes in Logic

- - Paradoxes are statements that contradict themselves or challenge intuition.
- - **Liar Paradox**: "This statement is false." (If true, then false; if false, then true.)
- - **Russell's Paradox**: The set of all sets that do not contain themselves—does it contain itself?
- - **Zeno's Paradoxes**: Infinite divisions preventing motion (e.g., Achilles and the Tortoise).
- - Paradoxes highlight limitations and complexities in logic.

Thank you