Linear Discriminent Analysis

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Abstract

The main aim of the project is to build library function for LDA which can be implemented on hardware. An input of 32x32 binary image matrix is considered here, accordingly two classes are created to separate pixel values of 0 and 1. Here the concept of within-class distance and between-class distance is taken into consideration. The final output here are all the eigenvectors of the matrix $S_w^{-1}S_B$. These eigenvectors can be further used for applications for image compression, pattern recognition etc.

Keywords:

1. Linear Discriments
2. Eigen Values
3. Eigen Vectors
4. Dimension Reduction
5. Classified

1 Introduction:

Definition Of LDA:
The goal of the LDA technique is to project the original data matrix onto lower dimensional space. In order to do so, three steps have to be performed.

1. To calculate the separability between different classes, which is called between-class variance.
2. To calculate the distance between mean and samples of each class, which is called within-class variance.
3. To construct lower dimensional space which maximizes the between-class variance and minimizes within-class variance.

2 Literature Review:

LDA can be performed by two approaches. The two approaches are:

1. Class-Dependent Transformation
2. Class-Independent Transformation

2.1 Class-Dependent Transformation

This type of approach involves maximizing the ratio of between class variance to within class variance. The main objective is to maximize this ratio so that adequate class separability is obtained. The class-specific type approach involves using two optimizing criteria for transforming the data sets independently.

2.2 Class-Independent Transformation

This approach involves maximizing the ratio of overall variance to within class variance. This approach uses only one optimizing criterion to transform the data setsand hence all data points irrespective of their class identity are transformed using this transform. this type of LDA, each class is considered as a separate class against all other classes.

The mathematical operations needed for LDA:
Input : Dataset with C number of classes.
1.Solving for within-class variance matrix:

$$S_W = \frac{1}{N-1}\sum_{i=1}^{C} S_i$$

$$\text{where } S_i = \sum_{x \in \omega_i} (x-\mu_i)(x-\mu_i)^T$$

$$\text{and } \mu_i = \frac{1}{N_i}\sum_{x \in \omega_i} x$$

2.Solving for between-class variance matrix:

$$S_B = \sum_{i=1}^{C} N_i(\mu_i - \mu)(\mu_i - \mu)^T$$

where $\mu = \frac{1}{N} \sum_{forallx} x = \frac{1}{N} \sum_{forallx} N_i \mu_i$

3:Generating EigenValues and EigenVectors

$$S_w^{-1} S_B w_i = \lambda_i w_i$$

where λ_i = Eigen Values of corresponding class and w_i = contains eigen vectors according to eigen values

3 Hardware Implementation

3.1 Inbuilt HDL CODER

3.1.1 Problems Faced

Few problems were faced by us while working with inbuilt HDL coder.

1. Matrics can't be used as input as well as output
2. The verilog code generated was over 2000 lines
3. The for loops generated by HDL coder were not synthesizable by Xilinx
4. The code failed to work on inputs and other calculations involving floating points.

3.2 Manual Approach

While computing the LDA for a given matrix, in-between we have to deal with floating numbers like adding/subtraction/multiplication/division to floating number. So we need to represent those floating num-bers in Binary IEEE Standards. Floating-point solves a number of representation problems. Fixed-point has a fixed window of representation, which limits it from representing both very large and very small numbers. Also, fixed-point is prone to a loss of precision when two large numbers are divided. Floating-point, on the other hand, employs a sort of sliding window of precision appropriate to the scale of the number. This allows it to represent numbers from 1,000,000,000,000 to 0.0000000000000001 with ease, and while maximizing precision (the number of digits) at both ends of the scale.

IEEE floating point numbers have 3 basic compo- nent in it: (1)Mantissa (2)Exponent and (3)sign The following table shows the layout for single (32-bit) and double (64-bit) precision floating-point values. The number of bits for each field are shown, followed by the bit ranges in square brackets. 00 = least-significant bit. Here Sign bit represent the sign of Floating number, Exponent Bits represent the integer part of the Floating number, Fraction Bits represent the fraction part of the floating number.
1)In order to compute Addition, Subtraction, and Multiplication of floating Num-

ber in verilog, we have used FPU(floating point unit) module of verilog which takes a single clock cycle to compute this operations.
2)Apart from this operations, for Square-root and Division operation we have used in-built IP-Core(Intellectual property-core : it is an is a reusable unit of logic, cell, or integrated circuit layout design that is the intellectual property of one party.) libraries which takes 40-50 clock cycles to compute this operations.

4 Comparision

Eigen Values In-Built	Eigen Values Project
1	6
2	7

Eigen Vectors In-Built	Eigen Vectors Project
$w = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$	$w = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$

5 Conclusion:

After the first simulation we observe that error between in-built code and our code is neglible or zero. This is because we just have two classes(0 and1) and thus we get only two eigenvectors corresponding to these classes. Another observation is that out of these vectors one with the maximum eigenvalue separates the data into two classes more precisely thus fulfilling the motive of LDA technique.

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