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Personal Life-cycle Financial Planning Decision Model

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***Abstract*—This paper derives the optimal consumption and portfolio choice pattern over the life-cycle for households facing uninsurable labor income risk, ruin risk, stochastic capital markets, and uncertain lifetime. Our model posits a dynamic utility maximized with CRRA and Epstein-Zin preferences that has access to liquid stocks, bonds, and illiquid life annuities. The empirical results of this research indicate that the annuity insurance commodity can hedge longevity risk. The investor would purchase the annuity insurance commodity, enhancing her own level of utility.**

Keywords-annuity insurance; financial planning decision model; life-cycle asset allocation

I. INTRODUCTION

Many financial economists are interested in the problem of portfolio optimization. Analytical solutions for optimal portfolio choice problems are often available in single period models. However, in a multi-period model, the portfolio optimization problem is complicated because it is inherently nonlinear. Therefore, this research proposes employing a personal life-cycle financial planning model based on financial theory in order to help the variety of investors make a comprehensive personal financial planning decision. But studying household financial problems in long-term portfolio choice models is challenging because it requires considering stochastic investment opportunity sets, illiquid assets such as labor income, housing or deferred tax accounts, and mortality risk [1].

Beginning with Merton many studies have analyzed the magnitude of hedging demands on the long term asset allocation caused by time-varying investment opportunity sets [2]. This particular string of the life-cycle literature highlights that investors should actively trade stocks, bonds, and money market over time. A second string of life-cycle articles emphasize that illiquid assets such as human capital and housing wealth play a dominant role apart from financial wealth in the total asset portfolio of the household. As human capital is a closer substitute to bonds than to stocks, young households compensate for the overinvestment in bonds by holding higher stock fractions in financial wealth. Over the life-cycle the optimal stock fraction decreases because the value of human capital declines [3].

While prior life-cycle asset allocation studies have already included mortality risk by incorporating a stochastic investment horizon, the literature has seldom considered life contingent claims to hedge the mortality risk so far. In a seminal study, Yaari finds that all assets should be annuitized because of the mortality credit if the individual is a rational investor without a bequest motive [4]. In his model, the investor is only exposed to mortality risk and all annuities are fairly priced from an actuarial standpoint. Constant life annuities are the most prominent claim contingent on the individual's survival. A constant life annuity is a financial contract between a buyer (annuitant) and a seller (insurer) that pays out a constant periodic amount for as long as the buyer is alive, in exchange for an initial premium [5].

We contribute to the prior literature by including constant life annuity markets into a realistically calibrated life-cycle asset allocation framework. This allows us to quantify the hedging demand for mortality risk and to assess the impact on the asset allocation of the financial assets stocks and bonds [3]. We use a realistic discrete-time life-cycle asset allocation and consumption model which incorporates the main three sources of risk of a household: untradeable labor income risk during work-life, risky stocks, and stochastic time of death. We assume that the utility function of the household is either of the CRRA or Epstein-Zin type and potentially includes a bequest motive. The model is used to derive the optimal dynamic consumption and asset allocation with stocks, bonds, and illiquid constant life annuities.

II. THE MODEL

A. Objective Function

Epstein and Zin define a utility function use the time separable base on CAPM and CCAPM [6]. Based on the Epstein-Zin Utility Function and with reference to Horneff, Maurer and Stamos add the Survival Probability and Bequest Motives, making it suitable for life-cycle financial planning decision model as the objective function [3]. Then objective function is described by

$$V_t = \left\{ (1 - bp_t^s) C_t^{1-1/s} + bE_t \left[p_t^s V_{t+1}^{1-g} + (1 - p_t^s) m \frac{(B_{t+1}/m)^{1-g}}{1-g} \right]^{\frac{1-1/s}{1-g}} \right\}^{\frac{1}{1-1/s}} \quad (1)$$

The model is time discrete with t (0≤t≤T + 1), where t is the adult age of the individual and can be calculated as actual age less 19. The individual lives up to T years. The individual has a subjective probability π_t^s to survive from t

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until t+1. Furthermore, the individual has Epstein-Zin utility defined over a single non-durable consumption good. Let C_t be the consumption level and B_t the bequest at time t. Where γ is the level of relative risk aversion, σ is the elasticity of intertemporal substitution, β is the discount factor and m the strength of the bequest motive.

Since π_T^s=0 reduces in T to which gives us the terminal condition for V_T.

$$V_t = \left\{ C_t^{1-1/\sigma} + \beta E_t \left[m \frac{(B_{t+1}/m)^{1-\gamma}}{1-\gamma} \right]^{\frac{1-1/\sigma}{1-\gamma}} \right\}^{\frac{1}{1-1/\sigma}} \quad (2)$$

B. Labor Income

Cocco, Gomes and Maenhout among others emphasize the importance of incorporating risky labor income into the asset allocation analysis of households [8]. Labor income risk creates the demand of the household for liquid assets to hedge adverse developments in labor income. We also include the labor income risk, and we divided people's life cycle into Accumulation period and Decumulation period. The Accumulation period is the work period from 23 to 60. The Decumulation period is the retirement period from 60 to 100. During the Accumulation period, the labor income is

$$Y_t = \exp(f(t)) I_t^p e_t^Y \quad (3)$$

$$I_t^p = I_{t-1}^p e_t^p \quad (4)$$

Y_t is the labor income at time t; K is the retirement age; f (t) is a deterministic function of age to recover the hump shape of income stream; I_t^p is a permanent income; ε_t^Y is a transitory shock of the labor income; ε_t^P is a transitory shock of the permanent income.

During the Decumulation period, the labor income is

$$Y_t = \zeta \exp(f(K)) I_K^p \ (t \geq K) \quad (5)$$

Where ζ is the constant replacement ratio.

C. Annuity Insurance

The individual can invest in an incomplete insurance market by purchasing constant real payout life annuities.

A life annuity is a financial contract between an individual and an insurer "that pays out a periodic amount for as long as the annuitant is alive, in exchange for an initial premium" [3, 5]. The actuarial premium Q_t of a life annuity with payments L starting in t + 1 is given by:

$$Q_t = La_t \quad (6)$$

Where at is the annuity factor for an individual with adult age t:

$$a_t = (1+\delta) \sum_{s=1}^{\infty} \left(\prod_{u=t}^{t+s} P_u^a \right) R_f^{-s} \quad (7)$$

Where P_u^a are the survival probabilities used by the life annuity provider, δ is the expense factor and R_f is the discount rate.

D. Capital Wealth

The individual can invest via direct investments in the two financial assets: riskless bonds and risky stocks. The real bond gross return denotes R_f, and the real risky stock return in t is R_t.

E. Mortality

To gives mortality a functional form we apply the Gompertz law for the sake of convenience and because of its widespread use in the insurance and finance literature. The functional form of the subjective force of mortality λ^s and the force of mortality for computing annuity premiums λ^a are then specified by

$$\lambda_t^i = \exp\left(\frac{t - m^i}{b^i} \right) \quad (8)$$

Parameters m^i and b^i determine the shape of the force of mortality function. The survival probabilities can now be expressed as follows:

$$p_t^i = \exp\left(-\int_0^1 \lambda_{t+s}^i ds \right) = \exp\left[-\exp\left(\frac{t - m^i}{b^i} \right) \left(\exp\left(\frac{1}{b^i} \right) - 1 \right) \right] \quad (9)$$

Additionally, we model the subjective force of mortality as linear transformation of the force of mortality derived from the average population mortality table λ_t^{pop} to analyze the impact of bad health propositions. Then we get for the subjective force of mortality and for the subjective probabilities:

$$\lambda_t^s = v\lambda_t^{pop}, p_t^s = \left(p_t^{pop} \right)^v \quad (10)$$

F. Wealth Accumulation

At each point in time the investor has to make a decision on how to spread wealth on hand Wt across bonds, stocks, annuities, and consumption. Therefore, the budget constraint is

$$W_t = B_t + S_t + Q_t + C_t \quad (11)$$

Where $M_t + S_t$ denote the value of financial wealth, M_t is the absolute wealth amount invested in bonds and S_t the amount invested in stocks. Q_t is the amount that the investor pays for an annuity and C_t is consumption. The individual's wealth on hand in t + 1 is given by

$$W_{t+1} = B_t R_f + S_t R_{t+1} + L_{t+1} + Y_{t+1} \tag{12}$$

$$L_{t+1} = L_t + Q_t / a_t \tag{13}$$

Where $M_t R_f + S_t R_{t+1}$ denotes the next period value of financial wealth, L_{t+1} is the sum of annuity payments which the investor gets from previously purchased annuities, Y_{t+1} is the labor income, L_t is the sum of all annuity payments from annuities purchased before t and Q_t / a_t is the annuity payment purchased in t.

We impose the following restrictions to borrow and to model the irreversibility of annuity purchases:

$$M_t, S_t, Q_t \geq 0 \tag{14}$$

If the individual dies, bequest H_t will be given by $H_t = M_{t-1} R_f + S_{t-1} R_t$. Thus, pension income or annuities cannot contribute to backing bequest motives.

G. The Model

The individual's problem is to make a choice each year how much to consume, how much to save in stocks and bonds, and how much to invest in life annuities. Thereby, has to be maximized under budget and short-selling restrictions. The optimal policy depends on four state variables: the permanent income I_t^p, wealth on hand W_t, annuity payouts from previously purchased annuities L_t, and age t. As an analytic solution to this problem does not exist, we use dynamic programming techniques to maximize the value function by backward induction. It follows that the model can then be rewritten as

$$V_t = \left\{ (1-\beta\pi_t^s) C_t^{1-1/\sigma} + \beta E_t \left[\pi_t^s V_{t+1}^{1-\gamma} + (1-\pi_t^s) m \frac{(B_{t+1}/m)^{1-\gamma}}{1-\gamma} \right]^{\frac{1-1/\sigma}{1-\gamma}} \right\}^{\frac{1}{1-1/\sigma}} \tag{15}$$

Subject to

$$W_t = B_t + S_t + Q_t + C_t \qquad \forall t$$

$$M_t, S_t, Q_t \geq 0 \qquad \forall t$$

$$W_{t+1} = B_t R_f + S_t R_{t+1} + L_{t+1} + \exp(f(t)) I_{t+1}^p \varepsilon_{t+1}^Y \qquad \forall t < K$$

$$W_{t+1} = M_t R_f + S_t R_{t+1} + L_{t+1} + \zeta \exp(f(K)) I_K^p \qquad \forall t \geq K$$

$$L_{t+1} = L_t + Q_t / a_t \qquad \forall t < K$$

$$L_{t+1} = (L_t + Q_t / a_t) N_{t+1}^{-1} \qquad \forall t \geq K$$

III. Positive Analysis

After introducing the personal life-cycle financial planning decision model for an annuity, we construct a multi-period wealth evolution model. An optimization problem is formulated with an objective of maximizing life time utility of consumptions and wealth. Optimal decisions are determined as a trade off between consumption and investment among an annuity, a risky and a risk-free asset. The investment targets are divided into two groups. The first group only considers risk-free and risky assets, i.e., bond and stock. The other group incorporates into the annuity insurance. The analysis results are shown in Fig. 1 to Fig. 5.

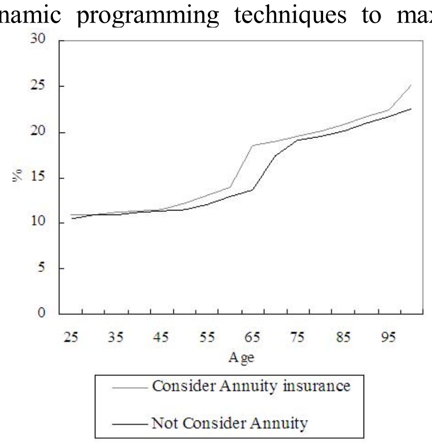

Figure 1. Consumption

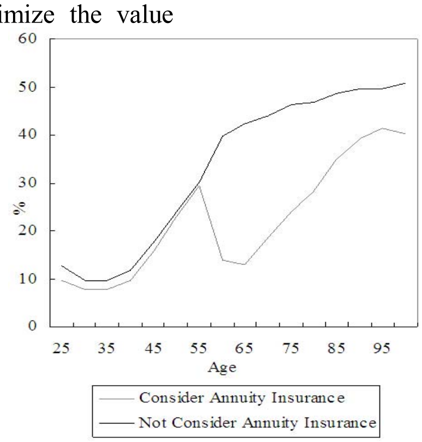

Figure 2. Bond

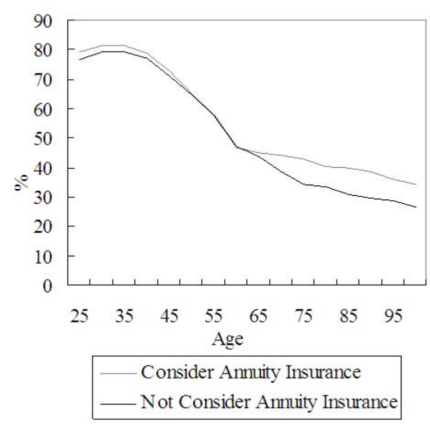

Figure 3. Stock

Fig. 1 illustrates that whether or not consider annuity insurance, both their investors at all stages of the life cycle of the optimal consumption amount of the total percentage of wealth are relatively low. However, as the investor ages sustained growth and wealth accumulation, the total amount of consumption by the percentage of the total wealth also increased. Which does not consider annuity insurance in case of optimal consumption amount of the total percentage of wealth lower than consider the case of annuity insurance consumer the amount of the percentage of the total wealth.

Fig. 2 illustrates that whether or not consider annuity insurance, both their investors at all stages of the life cycle of the optimal consumption amount of the total percentage of wealth are in the same direction of change with the investor ages grow. However, consider the case of annuity insurance optimum risk-free assets percentage of total wealth, in the 60-80 years old, because investors want to circumvent the longevity risks, so at this time of its purchase annuity insurance preferred. Therefore, investors are risk-free assets, the demand has decreased. When the investors towards the end of life, their preferred risk-free assets of the investment, risk-free assets, therefore the total percentage of wealth has become a growing phenomenon.

From Fig. 3, we can know whether or not consider annuity insurance, both their investors at all stages of the life cycle of the optimal consumption amount of the total percentage of wealth are in the reverse direction of change with the investor ages grow. During the retirement stage, because investors tend to the conservative investment strategy, so investors in the risky asset investment slowed down.

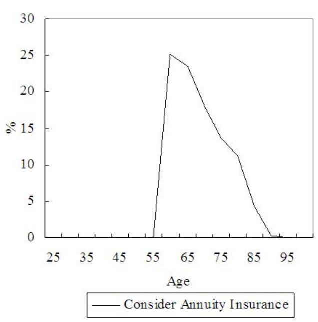

Figure 4. Annuity Insurance

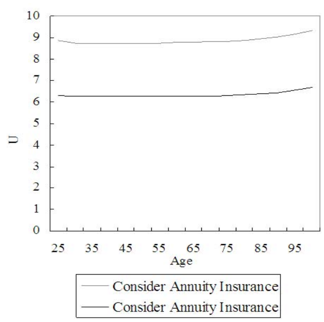

Figure 5. Utility Level

Fig. 2 illustrates that the investors in order to circumvent the elderly may face the risk of longevity, and it's around 60-year-old around in the purchase annuity insurance. But, when the investors towards the end of life, they preferred risk-free assets of the investment, will not choose to buy an annuity insurance. At this point the possible causes for the longevity risk has been eliminated and because the annuity payments can not be transferred to heirs, that is, investors can ensure that the remainder of its life by a massive transfer of wealth to their children, their assets would choose to invest in risky assets and risk-free assets.

From Fig. 5, we can know that the utility level of consider annuity insurance higher than does not consider. Therefore, this research suggests that the market average investors and annuity insurance can be incorporated into the life cycle of their own financial planning decisions, especially in wealth management retirement stage.

IV. CONCLUSION

The gradual growth of the life expectancy of people, the decrease of the fertility rate, and the low replacement ratio of the laborer retirement reveal the importance of the wealth accumulation and asset allocation during personal life-cycle. Therefore, this research proposes employing a personal life-cycle financial planning model based on financial theory in order to help the variety of investors make a comprehensive personal financial planning decision.

We introduce incomplete annuity markets into a realistically calibrated life-cycle asset allocation model and derive the optimal dynamic annuitization and asset allocation strategy during work-life and retirement. The integration of life annuities in a portfolio choice framework requires dealing with sequential real option decisions as life annuity purchases are irreversible and can happen anytime. The real option decision is based on an evaluation of the utility trade-off between the return enhancing mortality credit offered by annuities guaranteeing a constant lifelong payout and the opportunity costs related to life annuity purchases. This research firstly constructs the implied longevity yield model to investigate whether the annuity insurance can hedge the longevity risk. Next, this research employs the personal life-cycle financial decision model to dynamically conduct asset allocation. The investment targets are divided into two groups. The first group only considers risk-free and risky assets, i.e., bond and stock. The other group incorporates into the annuity insurance.

The empirical results of this research indicate that the annuity insurance commodity can hedge longevity risk. When the investment target includes annuity insurance, the investor would purchase the annuity insurance commodity, enhancing her own level of utility. Therefore, this research suggests that the investor purchase the annuity insurance in her personal life-cycle financial decision, in particular for the stage of retirement financial planning

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