Prediction of Shear Angle in Oblique Cutting with Maximum Shear Stress and Minimum Energy Principles

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A new shear angle prediction theory is proposed for oblique cutting operations. Oblique cutting mechanics are described by two components of shear angle, two angles defining direction of resultant cutting force, and chip flow angle. The five unknown parameters describe the geometry of chip deformation, velocities and forces in oblique cutting. When combined with the material dependent shear stress and average chip—rake face friction coefficient, cutting forces in three Cartesian directions can be predicted. In this paper, the mechanics of oblique cutting are described by five expressions. Three of the expressions are derived from the kinematics of oblique cutting, and the remaining two are derived either by applying Maximum Shear Stress *or* Minimum Energy Principle *on the process. Unlike the previous solutions, the proposed methods do not require any intuitive or empirical assumptions, but use only the material properties, tool geometry and the physical laws of deformation. The oblique cutting parameters and forces predicted by the proposed models agree well with the empirical and experimental results reported in the classical cutting literature. The proposed models are experimentally verified in predicting forces in helical end milling which has oblique cutting mechanics.*

1 Introduction

Classical analysis of cutting has been studied for more than half a century (Komanduri, 1993). Early works focused on the mechanics of orthogonal cutting where the velocity is perpendicular to the cutting edge. The most common models assume that the chip is separated from the blank along a thin plane, which is oriented with a shear angle measured from the direction of cutting velocity. It is assumed that the shear stress in the shear plane is constant and known, and the work hardening effects are neglected. The chip slides on the rake face of the cutting tool with a known average friction coefficient between the tool and workpiece materials. When coupled with the rake angle of the tool and cutting conditions, the shear angle, friction coefficient and shear stress describe the mechanics of orthogonal cutting, i.e. cutting force, torque, power and energy produced during the material removal process. Here, the fundamental unknown is the prediction of shear angle as a function of rake angle of the tool and the friction coefficient. Probably, the first attempt in finding the relationship was proposed by Krystof (1939), who applied Maximum Shear Stress Criterion to the orthogonal cutting. Later, Merchant (1945) applied widely accepted Minimum Energy Principle on cutting power. They found that the shear angle increases with the increase in the rake angle and decrease in the friction coefficient. Lee and Shaffer (1951) applied fundamental laws of plasticity on cutting by using Slip-Line Fields, and found an identical relationship obtained by Krystof. Although these models neglected the work hardening, thick shear zone, and sticking and sliding friction zones on the rake face, they led to the comprehensive understanding of practical machining operations such as turning, drilling, milling and others. However, these practical operations have an oblique cutting geometry, where the cutting edge is inclined from the direction of cutting velocity. Hence, unlike orthogonal cutting, the oblique operations have three dimensional cutting mechanics.

Merchant (1944) modeled the relationship between shear angle and chip flow direction by considering the velocities of chip, shear and workpiece in oblique cutting. Later, Stabler (1951) proposed an empirical chip flow rule, which assumes that the chip flow and inclination angles are approximately equal. Based on the chip flow rule, Stabler (1951) derived a shear angle equation in oblique cutting. Later, Armarego and Brown (1969) proposed a new shear angle relationship based on the modified chip flow rule (Russell and Brown, 1966). Other researchers followed a similar route, by focusing on the prediction of chip flow angle using empirical or intuitive rules (Jawahir, 1993). The chip flow angle in oblique cutting is predicted by combining the empirical/intuitive assumptions and geometric conditions (Luk, 1972; Lin and Oxley, 1972; Usui et al., 1978; Lin et al., 1982; Armarego, 1985; Budak, 1996). Chisholm and Rapier (1951) predicted the chip flow angle using the geometry of oblique cutting, and mixture of maximum shear stress and minimum energy principles. However, such a mixture is against the fundamental laws of physics, and therefore led to inaccurate chip flow angle predictions as discussed in detail in section 3. The recent research attempts are either experimental identification of shear, chip flow and friction angles (Shamoto and Moriwaki, 1994; Budak et al., 1996), or numerical modeling of the complex machining process with Finite Element techniques (Strenkowski and Carroll, 1985; Ueda and Manabe, 1993; Moriwaki et al., 1993; Obikawa and Usui, 1996). In short, a sound theoretical prediction of oblique cutting process without resorting to any empirical rules or intuitive assumptions has not been achieved since Krystof (1939) and Merchant (1945) derived the shear angle and the geometric relations in orthogonal cutting.

The present paper presents a new shear angle prediction model for oblique cutting by assuming a straight thin shear zone,

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where the work hardening is neglected. The main objective of the proposed model is to understand the mechanics of oblique cutting and to realize quick and theoretically sound cutting force predictions without relying on intuitive assumptions, empirical chip flow rules and large mechanistic cutting data bases. As in the case of orthogonal cutting theories (Krystof, 1939; Merchant, 1945; and Lee and Shaffer, 1951), the analysis is based on the identification of shear angle in oblique cutting using the laws of physics, as opposed to relying on empirical chip flow rules. The oblique cutting mechanics are described by the five angles, which show the directions of shear, resultant force and chip flow. First, the three fundamental kinematics relations of oblique cutting are derived based on the geometry, as reported by previous researchers. The required additional two expressions are derived by applying Maximum Shear Stress or Minimum Energy Principle separately. The five unknown oblique cutting parameters are evaluated by numerical solution of the five equations which are based on the kinematics of oblique cutting, and the laws of deformation. When the oblique geometry is neglected, the expressions lead to either Lee and Shaffer's (1951) or Merchant's (1945) shear angle law in orthogonal cutting when Maximum Shear Stress or Minimum Energy Principle is used, respectively. The simulated shear angle, chip flow angle and resultant force direction are in good agreement with both experimental and empirical results published previously. The analytical model helps to explain the physical mechanism behind the empirical rules, as well as assisting in the analytical modeling of precision machining operations (Liu and Dornfeld, 1996). Furthermore, the present theory is applied to prediction of milling forces, and its practical validity is verified by the milling experiments.

Henceforth, the paper is presented as follows. The geometry, forces and velocity fields in oblique cutting are presented in section 2. The proposed shear angle prediction laws and their relation to previous research are given in section 3, followed by experimental and simulation based verifications in section 4. The application of the present oblique cutting models to prediction of milling forces is presented in section 5. The paper is concluded with the summary of contributions in classical oblique cutting analysis.

2 Oblique Cutting Geometry

While the velocity is perpendicular to the cutting edge in orthogonal cutting, it has an *oblique* or inclination angle i in the oblique cutting operations, see Fig. 1. In Fig. 1, x axis is perpendicular to the cutting edge but lies on the cut surface; y is aligned with the cutting edge; and z is perpendicular to the xy plane. In orthogonal cutting, angle $i = 0$, and all force, shear, friction and chip flow vectors lie on the xz plane. However, the directions of shear, friction, chip flow and resultant cutting force have components in all Cartesian coordinates (x, y, z) in oblique cutting.

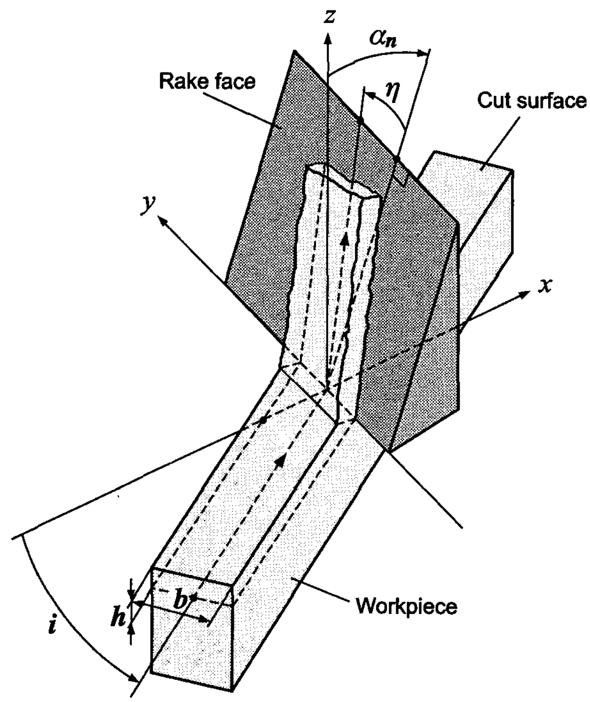


Fig. 1 Oblique cutting process

The important planes in oblique cutting are the shear plane, rake face, cut surface xy, and normal plane xz or P_n. The normal plane is useful to define the directions of velocity and force vectors in oblique cutting, hence all velocity and force vectors are projected on the normal plane. In Fig. 2, the angle between the shear and xy planes is called *normal shear angle* ϕ_n. The shear velocity lies on the shear plane, but makes an oblique

Nomenclature

i = inclination angle
α_n = normal rake angle
α_e = effective rake angle
β = friction angle
β_n = normal friction angle
ϕ_n = normal shear angle
ϕ_i = oblique shear angle
ϕ_e = effective shear angle
θ_n = normal angle of resultant cutting force direction
θ_i = oblique angle of resultant cutting force direction
η = chip flow angle
R = resultant cutting force
f = friction force
N = normal force to rake face
F_s = shear force
F_c = cutting force component in cutting direction
F_t = cutting force component in thrust (z) direction
F_r = cutting force component in normal direction to F_c and F_t
U = cutting power
F'_c, F'_t, F'_r, U' = non-dimensional F_c, F_t, F_r and U
K_c, K_t, K_r = cutting force coefficients in cutting, thrust and normal directions
F_x, F_y, F_z = milling forces in feed, normal and axial directions
τ = shear yield stress of workpiece material
A_s = shear area
b = width of cut
h = depth of cut
$\mathbf{V}_w$ = cutting velocity
$\mathbf{V}_c$ = chip flow velocity
$\mathbf{V}_s$ = shear velocity
r_l = chip length ratio
ν = interpolation ratio for maximum shear stress model
ζ = step in steepest descent method for minimum energy model
P_n = normal plane to cutting edge
P_v = velocity plane which includes $\mathbf{V}_w$, $\mathbf{V}_c$ and $\mathbf{V}_s$

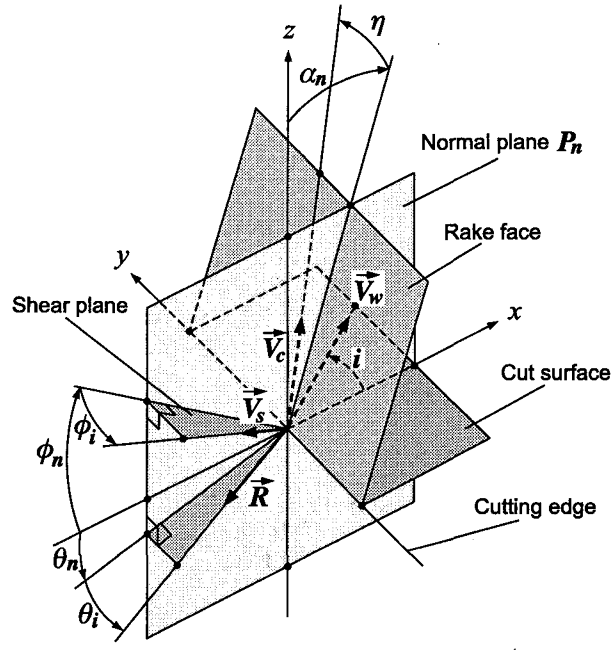


Fig. 2 Oblique cutting geometry

shear angle ϕ_i with the vector normal to the cutting edge on the normal plane. The sheared chip moves over the rake face plane with a *chip flow angle* of η measured from a vector on the rake face but normal to the cutting edge. Note that this normal vector also lies on the normal plane P_n. The friction force between the chip and the rake face is collinear with the direction of chip flow (Stabler, 1951). The angle between the z axis and normal vector on the rake face is defined as *normal rake angle* α_n. The friction force on the rake face ($\mathbf{f}$) and normal force to the rake ($\mathbf{N}$) form the resultant cutting force $\mathbf{R}$ with a friction angle of β, see Fig. 3. The resultant force vector ($\mathbf{R}$) has an acute projection angle of θ_i with the normal plane P_n, which in turn has an in plane angle of $\theta_n + \alpha_n$ with the normal force N. Here, θ_n is the angle between x axis and projection of $\mathbf{R}$ on P_n. The following geometric relations can be derived from Fig. 3:

$$f = R \sin \beta = R \frac{\sin \theta_i}{\sin \eta} \rightarrow \sin \theta_i = \sin \beta \sin \eta \qquad (1)$$

$$f = N \tan \beta = N \frac{\tan (\theta_n + \alpha_n)}{\cos \eta} \rightarrow$$

$$\tan (\theta_n + \alpha_n) = \tan \beta \cos \eta. \qquad (2)$$

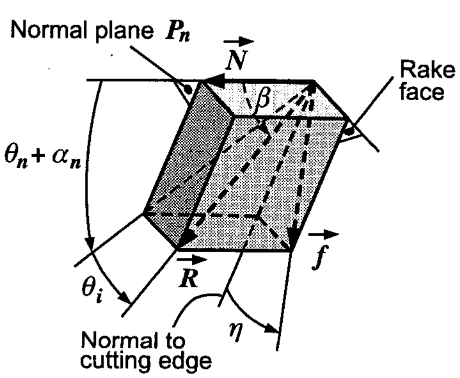


Fig. 3 Resultant force and its friction and normal components on the rake face

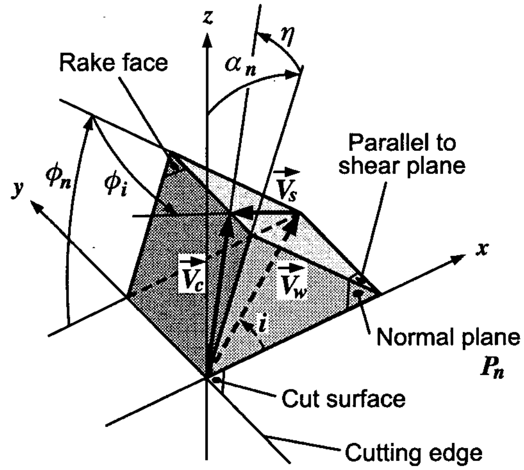


Fig. 4 The geometry of cutting (V_w), shear (V_s) and chip velocity vectors (V_c)

The velocities of chip ($\mathbf{V}_c$), shear ($\mathbf{V}_s$), and cutting ($\mathbf{V}_w$) form the velocity plane P_v as shown in Fig. 4. Each velocity vector can be defined by its Cartesian components:

$$\mathbf{V}_w = (V_w \cos i, \quad V_w \sin i, \quad 0)$$

$$\mathbf{V}_c = (V_c \cos \eta \sin \alpha_n, \quad V_c \sin \eta, \quad V_c \cos \eta \cos \alpha_n)$$

$$\mathbf{V}_s = (-V_s \cos \phi_i \cos \phi_n, \quad -V_s \sin \phi_i, \quad V_s \cos \phi_i \sin \phi_n)$$

By eliminating V_w, V_c and V_s from the velocity relation:

$$\mathbf{V}_s = \mathbf{V}_c - \mathbf{V}_w$$

the following geometric relation between the shear and the chip flow directions can be obtained (Merchant, 1944).

$$\tan \eta = \frac{\tan i \cos (\phi_n - \alpha_n) - \cos \alpha_n \tan \phi_i}{\sin \phi_n} \qquad (3)$$

One of the objectives of oblique cutting analysis is to predict the cutting forces for a given shear yield stress of the material and friction coefficient between the tool's rake face and chip material. Here, the shear yield stress in the shear plane, and friction coefficient on the rake face are assumed to be constant. The relationship among the forces, shear stress and friction coefficient is solved by modeling the cutting mechanics with oblique cutting tool geometry. There are 5 unknown oblique cutting parameters, which define the directions of resultant force (θ_n, θ_i), shear velocity (ϕ_n, ϕ_i) and chip flow (η). In addition to three equations (1), (2) and (3) obtained from the oblique cutting geometry, additional two expressions are required to solve 5 unknown angles. The new expressions must be based on the physics of shear deformation.

3 Prediction of Shear Angle in Oblique Cutting

As reviewed in the introduction, there have been numerous proposed solutions based on the empirical chip flow direction (Stabler, 1951; Russell and Brown, 1966) and other empirical assumptions (Luk, 1972; Lin and Oxley, 1972; Budak et al., 1996). A novel approach is proposed here, which parallels to the maximum shear stress (Krystof, 1939; Lee and Shaffer, 1951) and minimum energy (Merchant, 1945) principles used in two dimensional orthogonal cutting mechanics. The focus here is the prediction of shear direction based on the laws of mechanics, but not the combined empirical predictions and geometric relations.

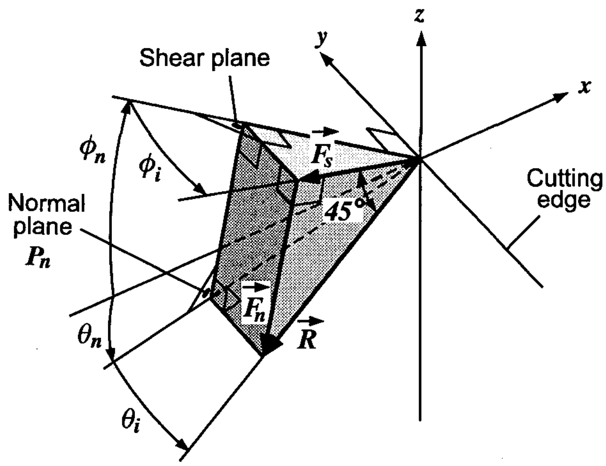


Fig. 5 Projection of resultant force (R) on the shear plane (F_s) in accordance with the Maximum Shear Stress Criterion

Maximum Shear Stress Principle. Krystof (1939) applied maximum shear stress criterion to predict the direction of shear angle in orthogonal cutting, i.e. $\phi_n = \pi/4 - \beta + \alpha_n$. Later, Lee and Shaffer (1951) achieved the same orthogonal shear angle relationship using the slip-line field solution. Both assume that the shear occurs in the direction of maximum shear stress, i.e. the angle between the shear velocity and resultant force directions is 45 deg, see Fig. 5. The same principle can be applied to the oblique cutting, i.e. *the resultant force (**R**) makes a 45 deg acute angle with the direction of shear.*

$$F_s = R(\cos\theta_i \cos(\theta_n + \phi_n)\cos\phi_i + \sin\theta_i \sin\phi_i) = R\cos(45\text{ deg})$$

Furthermore, the same principle dictates that the projection of **R** to the shear plane coincides with the shear direction, i.e. *the component of resultant force in the direction normal to the shear on the shear plane must be zero;* otherwise the shear stress in the shear direction is not the maximum on the shear plane.

$$R(\cos\theta_i \cos(\theta_n + \phi_n)\sin\phi_i - \sin\theta_i \cos\phi_i) = 0$$

The two expressions provide the necessary relationships between the shear and resultant force directions:

$$\sin\phi_i = \sqrt{2}\sin\theta_i \tag{4}$$

$$\cos(\phi_n + \theta_n) = \frac{\tan\theta_i}{\tan\phi_i} \tag{5}$$

By solving the five equations (1) to (5), the five unknown angles (ϕ_n, ϕ_i, θ_n, θ_i, η), which describe the mechanics of oblique cutting, can be obtained. However, direct analytical solution of the equations are rather difficult, hence it is solved by employing an iterative numerical method. The numerical solution is obtained according to the block diagram shown in Fig. 6. Note that the angles of friction (β), rake (α_n) and inclination (i) are known from the geometry and material tests, and considered as inputs to the system. The iterative solution is started by assuming an initial value for the chip flow angle, i.e. $\eta = i$ as proposed by Stabler (1951). The direction (θ_n, θ_i) of resultant force vector **R** is obtained from Eqs. (1) and (2). Similarly, the shear direction angles (ϕ_n, ϕ_i) can be evaluated from Eqs. (4) and (5), followed by evaluating new chip flow angle η_e from the velocity Eq. (3). The true chip flow angle is searched iteratively using the following interpolation algorithm:

$$\eta(k) = \nu\eta(k-1) + (1-\nu)\eta_e$$

where k is the iteration counter, and the interpolation ratio ν is selected within the range of $0 < \nu < 1$. ν is dynamically updated for the rapid convergence, i.e. decreased if $\eta(k)$ oscillates, and increased if its value moves in the same direction. The iteration is continued until the chip flow angle converges within 10^{-12} percent. When the three dimensional oblique cutting model introduced above is applied to two dimensional orthogonal cutting, it yields the same shear angle expression proposed by Krystof (1939), and Lee and Shaffer (1951) who used maximum shear stress principle on orthogonal cutting (i.e. $i = \theta_i = \phi_i = 0 \rightarrow \phi_n = \pi/4 - \beta + \alpha_n$). Note that with identity (4), Eq. (5) reduces to $\cos(\phi_n + \theta_n) = 1/\sqrt{2}$ in orthogonal cutting.

Minimum Energy Principle. Merchant (1945) proposed shear angle prediction theory by applying minimum energy principle to orthogonal cutting. The same principle is extended to oblique cutting here. From the geometry, the shear force is expressed as a projection of **R** in the direction of shear (Fig. 2):

$$F_s = R[\cos(\theta_n + \phi_n)\cos\theta_i \cos\phi_i + \sin\theta_i \sin\phi_i]$$

or as a product of shear stress and shear plane area (Fig. 1):

$$F_s = \tau A_s = \tau\left(\frac{b}{\cos i}\right)\left(\frac{h}{\sin\phi_n}\right)$$

where A_s, b and h are the shear area, the width of cut and the depth of cut, respectively. By equating the two shear force expressions, the resultant force is derived.

$$R = \frac{\tau bh}{[\cos(\theta_n + \phi_n)\cos\theta_i \cos\phi_i + \sin\theta_i \sin\phi_i]\cos i \sin\phi_n} \tag{6}$$

The cutting power (U) in oblique cutting can be expressed as a function of R, see Fig. 2,

$$U = F_c V_w = R(\cos\theta_i \cos\theta_n \cos i + \sin\theta_i \sin i)V_w$$

and the non-dimensional power (U') is obtained by substituting Eq. (6) for R,

$$U' = \frac{U}{V_w \tau bh} = \frac{\cos\theta_n + \tan\theta_i \tan i}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n} \tag{7}$$

where the term $V_w \tau bh$ is constant. The minimum energy principle requires that the cutting power must be minimum for a unique shear angle solution. Since the direction of shear is characterized by the shear angles ϕ_n and ϕ_i:

$$\left.\begin{aligned} \partial U'/\partial\phi_n &= 0 \\ \partial U'/\partial\phi_i &= 0 \end{aligned}\right\} \tag{8}$$

which provides two more equations in addition to the three geometric relations given by Eqs. (1), (2) and (3). Hence, the five unknown angles (ϕ_n, ϕ_i, θ_n, θ_i, η), which describe the mechanics of oblique cutting, can be obtained. However, analytical solution of five equations is rather difficult, therefore the solution is obtained using a numerical iteration technique. The algorithm starts with an initial value of chip flow angle $\eta = i$ (Stabler, 1951), followed by the evaluation of remaining angles from Eqs. (1), (2), (4) and (5). After calculating the initial values of θ_n, θ_i, ϕ_n and ϕ_i, the cutting power (U') is obtained from Eq. (7). By changing the shear angles slightly, i.e. $\phi_n + \Delta\phi_n$ and $\phi_i + \Delta\phi_i$, the steepest descent direction ($\Delta U'/\Delta\phi_n$, $\Delta U'/\Delta\phi_i$) is evaluated. The shear angles are changed by a step ζ in the steepest descent direction so that the cutting energy approaches to the minimum value.

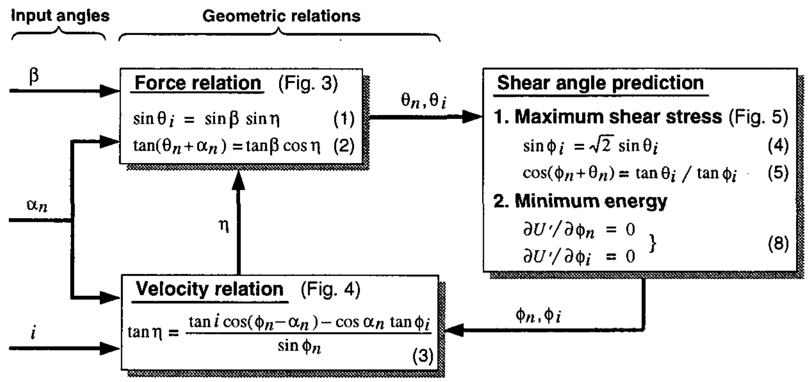


Fig. 6 The block diagram of solving oblique cutting parameters

$$\begin{Bmatrix} \phi_n(k) \\ \phi_i(k) \end{Bmatrix} = \begin{Bmatrix} \phi_n(k-1) \\ \phi_i(k-1) \end{Bmatrix} - \zeta \begin{Bmatrix} \Delta U'/\Delta\phi_n \\ \Delta U'/\Delta\phi_i \end{Bmatrix}$$

The numerical iteration is continued in accordance with Fig. 6 until the non-dimensional cutting power (U') converges to a minimum value. When the three dimensional oblique cutting model introduced above is applied to two dimensional orthogonal cutting, it yields the same shear angle expression proposed by Merchant (1945) (i.e. $i = \theta_i = \phi_i = 0 \rightarrow \phi_n = \pi/4 - \beta/2 + \alpha_n/2$).

Critical Review of Oblique Cutting Literature. To the best of authors' knowledge, the proposed solutions of oblique cutting mechanics, i.e. prediction of shear, force and chip flow directions using the maximum shear stress and minimum energy principles, are novel contributions to the classical metal cutting literature. There has been only one notable previous attempt to develop a theoretical prediction of oblique cutting process without resorting to empirical and intuitive assumptions. Chisholm and Rapier (1951) used the "geometric relation" found by Stabler (1951),

$$\tan(\phi_n + \beta_n) = \frac{\cos\alpha_n \tan i}{\tan\eta - \sin\alpha_n \tan i} \tag{9}$$

where $\tan\beta_n = \tan\beta \cos\eta$. This equation can also be derived from the geometric relations (1)–(3) and one of the maximum shear stress criteria (5) presented here. Chisholm and Rapier numerically evaluated the chip flow angle (η) by minimizing the cutting energy. However, their chip flow angle prediction results were not in agreement with the Stabler's chip flow rule, $\eta = i$. The discrepancy seems to be due to mixture of parameters obtained from the maximum shear stress and minimum energy principles in evaluating the oblique cutting parameters. In order to determine the shear direction in the three dimensional space, one pair of two relations, i.e. Eqs. (4) and (5) or Eqs. (8), are required, and each pair must be used together. Although Chisholm and Rapier (1951) proposed a noted shear angle solution for oblique cutting, they separated each pair of equations and used one from the maximum shear stress equations and another from the minimum energy equations. As a result, two fundamental physical principles were mixed in solving the shear angles. In this paper, the shear angle is solved either using purely maximum shear stress, or minimum energy principle but not both.

The remaining literature in oblique cutting relies on empirical and intuitive assumptions. Stabler (1951) proposed a shear angle solution by combining geometric relation (9) and his empirical chip flow rule ($\eta = i$).

$$\phi_n = \frac{\pi}{4} - \beta_n + \frac{\alpha_n}{2}$$

Later, Armarego and Brown (1969) used the same relation (9) and modified chip flow rule of Russell and Brown (1966),

$$\tan\eta = \tan i \cos\alpha_n \tag{10}$$

and obtained the following shear angle relation:

$$\tan(\phi_n + \beta_n) = \frac{1}{1 - \tan\alpha_n}$$

The difference between these empirical shear angle solutions in oblique cutting and Merchant's or Lee-Shaffer's solution in orthogonal cutting has been discussed as a "knotty problem" (Stabler, 1951; Armarego, 1994). However this is because the previous solutions relied heavily on empirical rules, and not based on physics of shear deformation.

Luk (1972) and Lin and Oxley (1972) predicted the chip flow angle η by utilizing shear angle identified in orthogonal cutting. Both assumed that the normal shear angle ϕ_n is empirically equal to the shear angle in orthogonal cutting by citing Armarego and Brown (1964), and then derived the chip flow angle by using Stabler's equation (9). Usui et al. (1978) also employed orthogonal cutting data of the shear angle, and assumed that the relation among the effective shear (ϕ_e), rake (α_e) and friction angles (β) in the velocity plane are equal to the relation observed in orthogonal cutting, which is based on the concept of effective rake angle proposed by Stabler (1951). However, the velocity plane does not include resultant cutting force, hence their method is not accurate as supported by the experimental results of Brown and Armarego (1964). They predicted the chip flow angle which minimizes the cutting energy. Although the minimum energy method provides two sufficient equations (8), they derived only one equation and thus employed the intuitive assumption to utilize the orthogonal cutting data. Lin et al. (1982) argued that the above semi-theoretical models were over sophisticated and proposed a more empirical and simple model. They employed Stabler's chip flow rule (1951) and an empirical rule found by Brown and Armarego (1964) that cutting force components in the cutting and thrust directions are nearly independent of inclination angle i. From cutting experiments, Armarego (1982) observed that the chip length ratio (r_l) is independent of the inclination angle (i) when $i < 40$ deg. He assumed that the chip length ratio obtained in orthogonal cutting tests are equal in the velocity plane of the oblique cutting. Later, Budak et al. (1996) solved the chip flow angle numerically by utilizing this rule, Stabler's equation (9) and shear angle measured in orthogonal cutting.

In summary, since Stabler (1951) proposed the chip flow rule, it seems that the previous research has put the main emphasis on the prediction of chip flow by assuming empirical and intuitive rules. On the other hand, two new methods, which are based on the laws of physics, are introduced here for evaluating the shear direction.

4 Results of Oblique Cutting Simulations

Oblique cutting process is simulated by the proposed two models, and compared with the experimental data published by

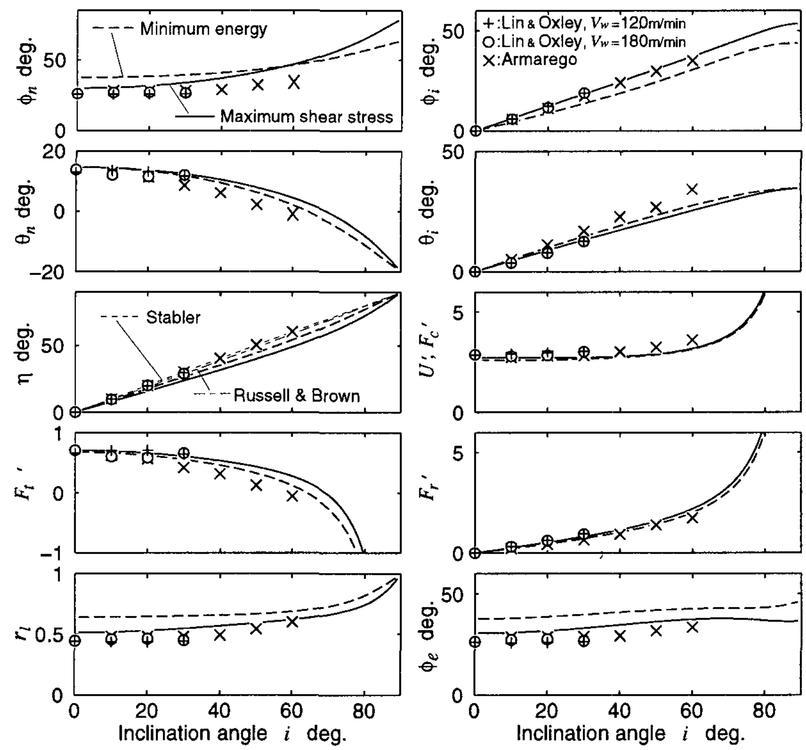


Fig. 7 Effects of inclination angle i on oblique cutting parameters. The proposed model predictions are compared against empirical rules (Stabler, 1951; Russell and Brown, 1966) and experimental data published by Lin and Oxley (1972) and Armarego (1982). Simulation conditions: α_n = 20 deg, β = 34.6 deg. Lin and Oxley's data (1972): S1214 steel, α_n = 20 deg, β = 32.5–35.5 deg (Ave. 34.1 deg), h = 0.5 mm, b = 5 mm, V_w = 120, 180 m/min. Armarego's data (1982): 60655-T6 aluminum alloy, α_n = 20 deg, β = 33.5–40 deg (Ave. 35.1 deg), h = 0.06–0.32 mm, b = 6.25 mm, V_w = 0.25 m/min, edge force components are removed by extrapolating the cutting force data measured at various chip loads.

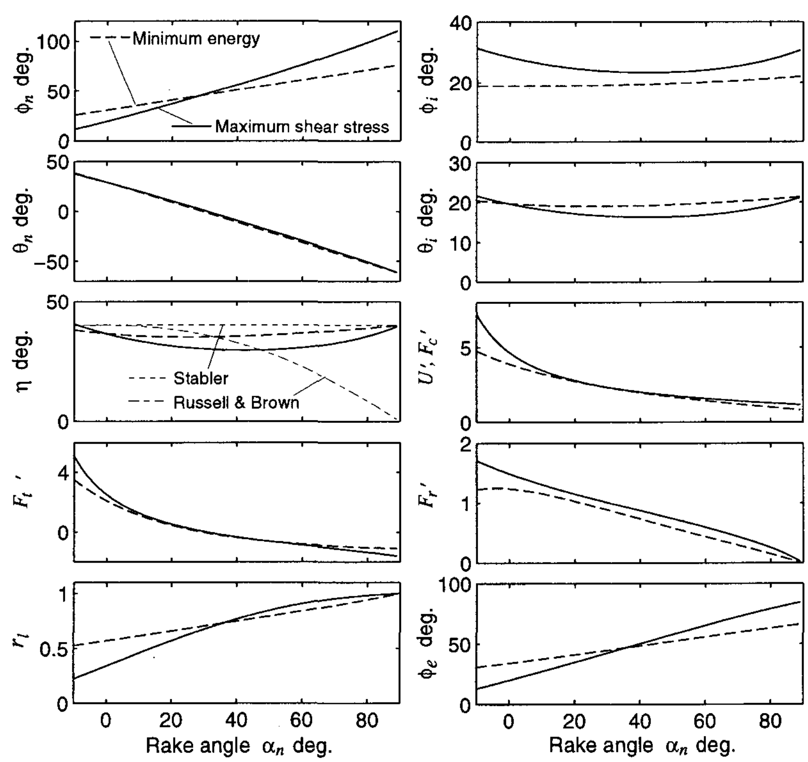


Fig. 8 Effect of normal rake angle α_n on oblique cutting parameters. The proposed model predictions are compared against the empirical rules reported by Stabler (1951), Russell and Brown (1966). Simulation conditions: i = 40 deg, β = 34.6 deg.

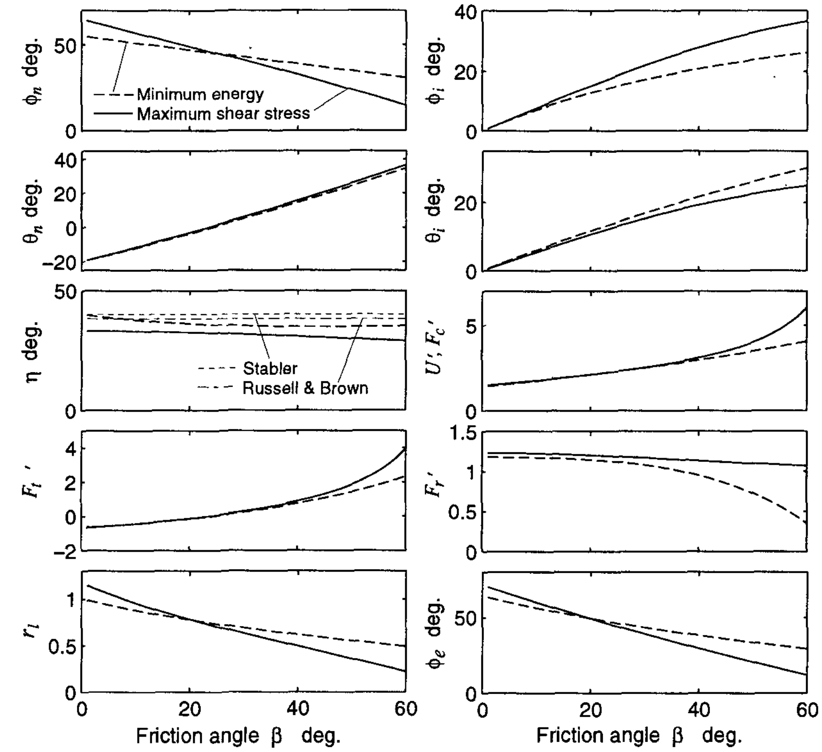


Fig. 9 Effect of friction angle β on oblique cutting parameters. The proposed model predictions are compared against the empirical rules reported by Stabler (1951), Russell and Brown (1966). Simulation conditions: i = 40 deg, α = 20 deg.

Lin and Oxley (1972) and Armarego (1982), and the empirical chip flow rules of Stabler ($\eta = i$, 1951) and Russell and Brown [Eq. (10), 1966].

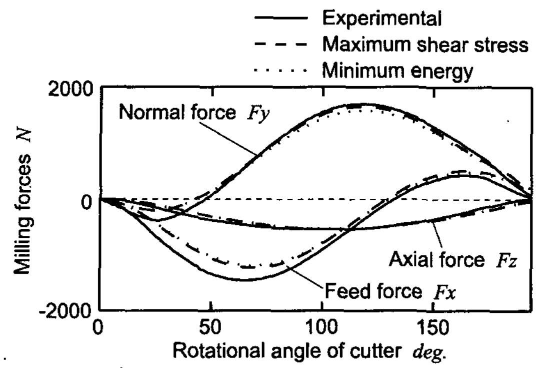
Figure 7: The oblique cutting parameters and forces (see Appendix) are estimated for a wide range of inclination angles. The normal rake (α_n) and friction angle (β) are set to 20 and 34.6 degrees, respectively. Lin and Oxley (1972) provided the experimental data for chip flow (η), normal shear (ϕ_n) and normal friction angles (β_n), as well as the three cutting force components (F_c, F_t, F_r). The remaining Lin and Oxley's data shown in the figure (ϕ_i, ϕ_e, θ_n, θ_i, U', F_c', F_t', F_r', β, r_l) are evaluated from the geometric relations in oblique cutting. ϕ_n, ϕ_e, θ_n, θ_i, U', F_c', F_t' and F_r' in Armarego's experimental data are also derived from the geometric relations. The nondimensional power U' is equal to the nondimensional power force F_c' as mentioned in Appendix. The proposed maximum shear stress and minimum energy principles are used to predict oblique shear angles (ϕ_n, ϕ_i), chip flow angle (η) and direction of resultant force (θ_n, θ_i) observed during cutting experiments at different inclination angles. The predicted values of angles and forces are all in good agreement with the published experimental data.

The empirical chip flow rules of Stabler ($\eta = i$), and Russell and Brown (10) agree better with the experimental results than the theoretical predictions in this case. However, the experimental results of chip flow angle often vary by over 10 degrees from both the empirical chip flow rules, see Brown and Armarego (1964) and Shaw et al. (1952). Considering this deviation of experimental data, the predicted chip flow angles are in good agreement with both the experimental data and the empirical chip flow rules. Although the prediction errors are smaller than the deviation, the under-prediction of the chip flow angle can be explained as follows. Kobayashi and Thomsen (1959) conducted a wide range of experiments which indicated that the shear angle tends to be over-predicted especially when the minimum energy principle is applied (Merchant, 1945). A similar trend is expected in oblique cutting. The over-predicted normal shear angle is considered to lead to under-prediction of the chip flow angle, see Fig. 4 and Eq. (3).

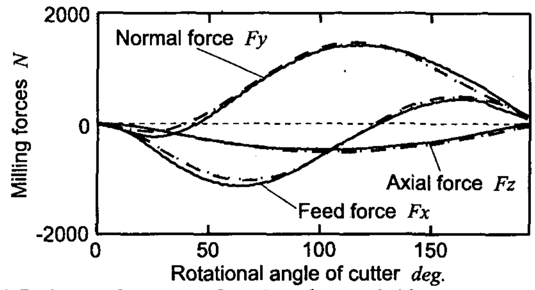
The overall results also agree with the empirical trends reported by the previous researchers. Brown and Armarego (1964), and Armarego (1982) reported that the power force (F_c), the normal shear angle (ϕ_n) and the chip length ratio (r_l) are independent of inclination angle (i). These observations are approximately true especially at low inclination angle, judging from the simulated and experimental results. On the other hand, Usui's assumption on the effective rake and shear angles does not agree with either the simulated or experimental results, because the effective shear angle ϕ_e stays almost constant although the effective rake angle α_e is increased considerably with the inclination angle.

The proposed theory is also helpful to understand the basic mechanics of oblique cutting directly, such as the influence of inclination, normal rake and friction angles on the unknown angles, which represent the oblique cutting process. For example, the influence of inclination angle can be visualized by using the maximum shear stress principle as follows. Imagine that the inclination angle (i) is increased suddenly after all angles are stabilized, the velocity relation is changed first according to the block diagram shown in Fig. 6, i.e. the chip flow angle (η) is increased, see Fig. 4. Then, the increase in η leads to increase in θ_i while decrease in θ_n (Fig. 3), and furthermore these cause increase in ϕ_i and ϕ_n (Fig. 5). All influences of input angles on the process and relationship among the angles can be seen in Figs. 3–5 based on the block diagram shown in Fig. 6 without any calculation.

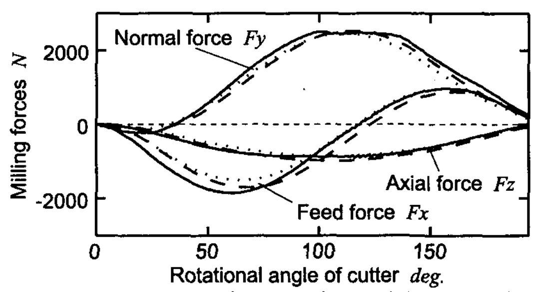
Figure 8: The effect of normal rake angle (α_n) on oblique cutting parameters is simulated, and compared against empirical model predictions here. The results obtained by the maximum shear stress and the minimum energy models are different as the normal rake angle is negative or excessively large. Same discrepancy exists between Krystof's (1939) or Lee-Shaffer's



a) Rake angle: αn=0 *deg.*, Feed rate: 0.1016 *mm/tooth.*



b) Rake angle: αn=5 *deg.*, Feed rate: 0.1016 *mm/tooth.*



c) Rake angle: αn=12 *deg.*, Feed rate: 0.2032 *mm/tooth.*

Fig. 10 Experimental and simulated milling forces. Single flute slotting, work: Ti6Al4V, axial depth: 7.62 mm, speed: 30 m/min, helix: 30 deg, diameter: 19.05 mm. Simulation conditions: τ = 613 MPa, β = 19.1 + 0.29 α_n (deg) (Budak et al., 1966)

(1951) theory which is based on the maximum shear stress criterion, and Merchant's (1945) minimum energy solution in orthogonal cutting. It is expected from the orthogonal cutting literature (Komanduri, 1993; Kobayashi and Thomsen, 1959) that the minimum energy model will give better results when the rake angle is larger than the friction angle, and the maximum shear stress model will be better in many cases when the rake angle is smaller.

Shaw et al. (1952) reported that the chip flow angle η decreases as rake angle α_n is increased. Brown and Armarego (1964) and Russell and Brown (1966) confirmed this tendency by their experiments, and proposed the empirical chip flow rule given in Eq. (10). The proposed oblique cutting theories agree well with the experiments reported in the literature. The chip flow rule of Russell and Brown deviates significantly from the Stabler's and the proposed solutions at rake angles greater than 30 degrees, mainly because their empirical rule was calibrated from experimental data conducted at a rake angle range of 0 to 30 degrees. Based on physics, the chip flow angle should approach to the inclination angle at extremely large normal rake angle, because $\mathbf{V}_c \rightarrow \mathbf{V}_w$ and $\mathbf{V}_s \rightarrow 0$ when $\alpha_n \rightarrow 90$ deg, see Fig. 4.

Fig. 9: The effect of average friction angle (β) on oblique cutting parameters is simulated, and compared favorably against empirical model predictions here. Furthermore, the predicted tendency of chip flow angle η agrees with the experimental results reported by Shaw et al. (1952), i.e. "η increases as the friction characteristics of the metal cut improve," although the effect is not significant as it has been neglected in most of the previous studies (Stabler, 1951; Russell and Brown, 1966).

5 Application to Milling Force Prediction

The oblique cutting parameters are predicted by the proposed model, and then the cutting force coefficients are obtained from the equations shown in Appendix. The milling forces in the feed (F_x), normal (F_y) and axial (F_z) directions are simulated by using these coefficients. The milling force expressions can be found in Altintas and Lee (1996). The simulation results are compared with the experimental data in Fig. 10. The shear yield stress and the friction angle are set to τ = 613 MPa and $\beta = 19.1 + 0.29\alpha_n$ (deg.) in the simulations, which are identified in the orthogonal cutting tests for the same work and tool materials (Budak et al., 1996). The milling forces simulated by both the maximum shear stress and minimum energy models are all in good agreement with the experimental results, regardless of the various rake angles of the milling cutter. Note that the present simulation of milling forces requires only the milling conditions and the material properties, i.e. the shear yield stress and the friction angle, unlike any conventional mechanistic models which require large amount of milling tests to calibrate cutting constants for each cutter design.

The proposed models allow the design and analysis of complex milling tools for force, chatter and surface finish prediction without relying on experimentally calibrated cutting coefficients (Altintas and Lee, 1996).

6 Conclusion

Two new solutions are contributed to the classical analysis of oblique cutting mechanics. The mechanics of oblique cutting are defined by five expressions. Three of the expressions are obtained from the geometry of oblique cutting, and the remaining two are derived by applying either *Maximum Shear Stress or Minimum Energy Principle.* For a given tool geometry, average friction angle between the cutting tool and work material and average shear yield stress of the work material, the proposed theories can predict the shear angles (ϕ_n, ϕ_i), chip flow direction (η) and the direction of resultant force (θ_n, θ_i) in oblique machining operations. The oblique cutting parameters predicted by the two proposed solutions are in good agreement with the empirical and experimental data published in the literature. Furthermore, the proposed model is applied to prediction of milling forces and successfully verified against the experimental data. Unlike the previous research reported in the literature, the proposed solution is purely analytical, based on the laws of physics, and free of any intuitive assumptions, empirical chip flow rules and large orthogonal or mechanistic cutting data bases.

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APPENDIX

Cutting Force Components

The cutting force components are derived as projections of resultant cutting force R by using the resultant force direction (θ_n, θ_i), and then they are expressed as functions of shear yield stress τ by substituting Eq. (6) for R.

The non-dimensional force F_c' in the cutting direction is defined as the force divided by shear yield stress and cut area (τbh), or as cutting force coefficient (K_c) divided by shear yield stress. Thus, it is the same as the non-dimensional cutting power U' (see Eq. (7)),

$$F_c' = \frac{F_c}{\tau bh} = \frac{K_c}{\tau} = U'$$

and the other non-dimensional forces and cutting coefficients in the thrust (z) direction (F_t', K_t) and the normal direction to the other two (F_r', K_r) are given by the following equations.

$$F_t' = \frac{F_t}{\tau bh} = \frac{K_t}{\tau} = \frac{R \cos\theta_i \sin\theta_n}{\tau bh}$$

$$= \frac{\sin\theta_n}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\cos i \sin\phi_n}$$

$$F_r' = \frac{F_r}{\tau bh} = \frac{K_r}{\tau} = \frac{R(\cos\theta_i \cos\theta_n \sin i - \sin\theta_i \cos i)}{\tau bh}$$

$$= \frac{\cos\theta_n \tan i - \tan\theta_i}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$