Orthogonal Cutting

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Question 1

In this question, we will start with the data given in Altinatis book itself. A set of orthogonal tests are conducted to identify various parameters in orthogonal cutting of P20 mold steel. The table contains the cutting conditions (feed rate) and measured forces and chip thicknesses at each feed rate. The cutting tool had 0 rake angle. The width of cut was b = 5 mm, and the cutting speed was V = 240 m/min.

Feed rate h [mm/rev]	**Tangential force F_t [N]**	**Feed force F_f [N]**	**Measured chip thickness h_c [mm]**
0.02	350	290	0.05
0.03	480	350	0.058
0.04	590	400	0.074
0.05	690	440	0.083
0.06	790	480	0.102
0.07	890	505	0.116
0.08	980	540	0.131

Table 1: Orthogonal Cutting Test Conditions and Measurements in Plunge Turning of P20 Mold Steel

1. First, evaluate the following values, assuming an orthogonal cutting process. So, write a Matlab code to iterate through the tests and find the following for each feed rate:
   (a) Shear Angle ϕ_c
   (b) Friction angle β_a
   (c) Resultant cutting Force F
   (d) Shear stress τ_s
   (e) Normal stress on the shear plane σ_s
   (f) Total Cutting Power P_t
   (g) The cutting force coefficients K_{tc} and K_{fc}
   (h) The shear strain γ_s and strain rate $\gamma_s^{'}$ at the primary shear zone
2. Now plot the Forces vs Feed rate points and fit the following force models:
   (a) the linear model

$$F_t = K_{tc}bh + K_{te}b;$$
$$F_f = K_{fc}bh + K_{fe}b$$

   (b) the non-linear model

$$F_t = K_{tc}bh, K_t = K_T h^{-p};$$
$$F_f = K_{fc}bh, K_f = K_F h^{-p}$$

Plot all of them onto the same graph and compare. Also compare the values of the force coefficients with those found in Part 1.

3. Theoretically predict the shear angles using Merchant's Minimum Energy Principle and compare them with the ones obtained above.

Solution

Solution

For each of the tests the following sequence of calculations is performed.

- Chip thickness ratio $= r_c = \frac{h}{h_c}$
- **Shear angle** $= \phi_c = \tan^{-1} \frac{r_c \cos \alpha_r}{1 - r_c \sin \alpha_r}$
- **Friction Angle** $= \beta_a = \alpha_r + \tan^{-1} \frac{F_f}{F_t}$
- **Resultant Cutting Force** $= F = \sqrt{F_t^2 + F_f^2}$
- Shearing Force $= F_s = F \cos(\phi_c + \beta_a - \alpha_r)$
- **Shear stress** $= \tau_s = \frac{F_s}{bh/\sin(\phi_c)}$
- Normal Force on shear plane $= F_n = F \sin(\phi_c + \beta_a - \alpha_r)$
- **Normal stress on shear plane** $= \sigma_s = \frac{F_n}{bh/\sin(\phi_c)}$
- Shearing Velocity $= V_s = V \frac{\cos \alpha_r}{\cos(\phi_c - \alpha_r)}$
- Shearing Power $= P_s = F_s V_s$
- Frictional Power $= P_u = (F \sin \beta_a)(r_c V)$
- **Total Cutting Power** $= P_t = P_s + P_u$
- **Cutting Force Coefficients:** $K_{tc} = \frac{F_t}{bh}$ **and** $K_{fc} = \frac{F_f}{bh}$
- **Shear strain** $= \gamma_s = \frac{\cos \alpha_r}{\sin \phi_c \cos(\phi_c - \alpha_r)}$
- Shear plane length $= L_c = \frac{h_c}{\cos(\phi_c - \alpha_r)}$
- Assume shear zone thickness $= \Delta d = 0.15 * L_c$
- **Shear strain rate** $= \gamma_s' = \frac{V \cos \alpha_r}{\Delta d \cos(\phi_c - \alpha_r)}$

And we get the following table.

Feed rate, h [mm/rev]	Tangential force, $F_t [N]$	Feed Force, $F_f [N]$	Measured chip thickness, $h_c [mm]$	Shear angle, ϕ_c	Friction Angle, β_a	Resultant cutting force, F [N]	Shear stress $\tau_s [MPa]$	Normal Stress $\sigma_s [MPa]$	Total Cutting Power [W]	K_{tc} [N/mm^2]	K_{fc} [N/mm^2]	Shear strain γ_s	Shear strain rate γ_s' [*10^3 sec^-1]
0.02	350	290	0.05	0.380506	0.691921	454.5327	806.8966	1482.759	84000	3500	2900	2.9	32000
0.03	480	350	0.058	0.477345	0.630034	594.0539	813.3208	1627.58	115200	3200	2333.333	2.450575	27586.21
0.04	590	400	0.074	0.495552	0.595785	712.8113	781.7976	1503.674	141600	2950	2000	2.390541	21621.62
0.05	690	440	0.083	0.542189	0.567666	818.352	751.3047	1512.834	165600	2760	1760	2.26241	19277.11
0.06	790	480	0.102	0.531724	0.545985	924.3917	739.503	1376.178	189600	2633.333	1600	2.288235	15686.27
0.07	890	505	0.116	0.542951	0.516116	1023.291	739.7036	1317.063	213600	2542.857	1442.857	2.260591	13793.1
0.08	980	540	0.131	0.548241	0.503626	1118.928	723.0593	1265.99	235200	2450	1350	2.248187	12213.74

Table 2: Table containing all the values asked in Part 1.

The complete Matlab code for the solution is given at the end of solution.

Now, let us discuss the second part. Because of various varying parameters in cutting, like the shear yield stress and strain hardening of the material being machined, mechanistic models are used. The cutting forces are defined as a function of the cutting conditions and the cutting coefficients : K_{tc} and K_{fc}.

In the linear model, the cutting forces are written as follows:

$$F_t = K_{tc}bh + K_{te}b;$$
$$F_f = K_{fc}bh + K_{fe}b$$

where, the cutting coefficients (K_{tc} and K_{fc}) and edge coefficients ((K_{tc} and K_{fc})) are calibrated from experiments. Now, if the influence of the chip thickness on the friction and shear angles, and the yield shear stress are taken into account, the coefficients are expressed as a function of chip thickness and we get the following non-linear model:

$$F_t = K_{tc}bh, K_t = K_T h^{-p};$$
$$F_f = K_{fc}bh, K_f = K_F h^{-p}$$

where, the exponents (p and q) and the constants ((K_T and K_F)) are determined from experiments.

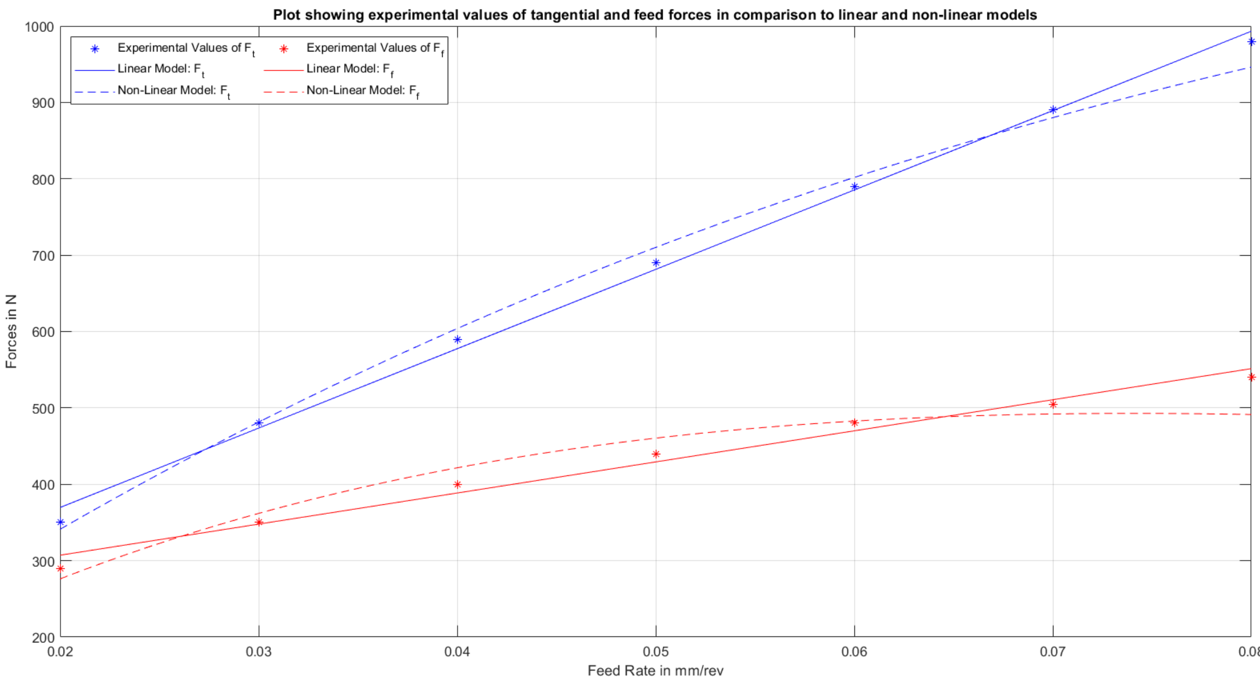

Figure 1: The plot shows the comparison between the two force models along with the experimentally found force values as asked in Part 2.

Thus we get the following plot of forces. The blue curve above is for tangential force and the red curve below is feed force.

If we want to compare the force coefficients obtained from all of them, the following graph illustrates them very nicely.

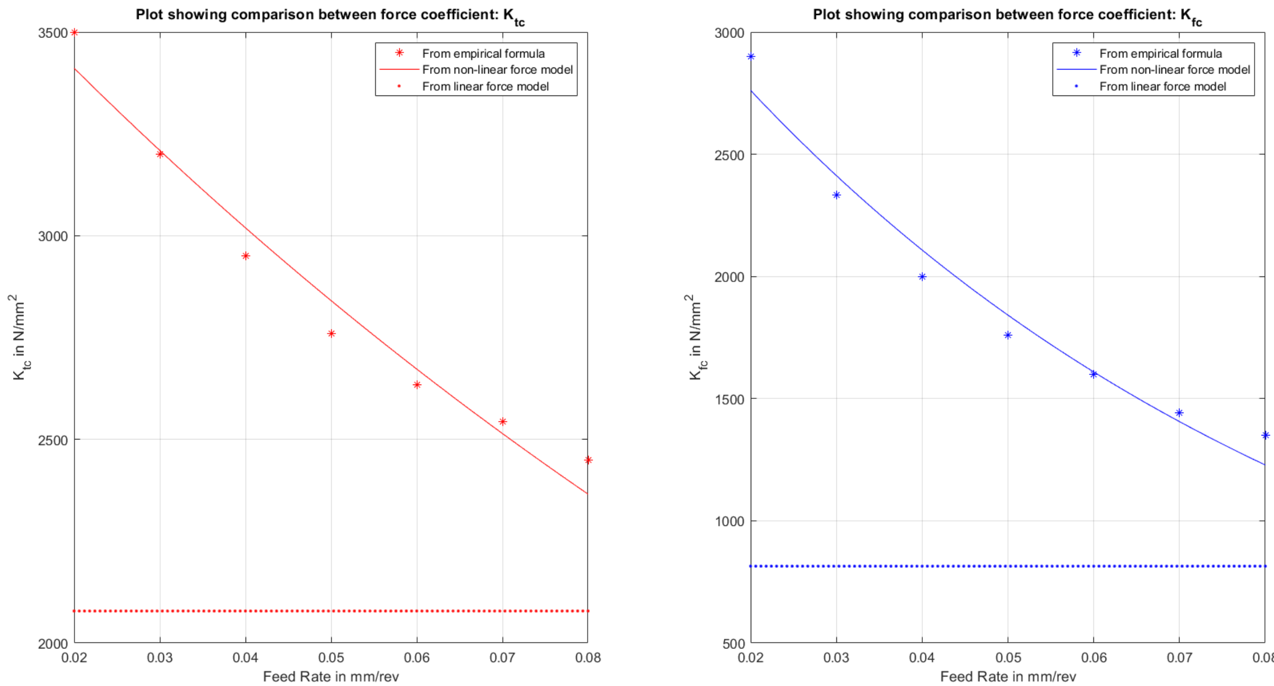

Figure 2: The plot shows the comparison between force coefficients as continuation of Part 2.

Finally, in the third part we compute the shear angle theoretically. Using Merchant's minimum energy principle:

$$\frac{dP_{tc}}{d\phi_c} = \frac{dVF_{tc}}{d\phi_c} = \frac{-V\tau_s bh\cos(\beta_a - \alpha_r)\cos(2\phi_c + \beta_a - \alpha_r)}{\sin^2\phi_c \cos^2(\phi_c + \beta_a - \alpha_r)} = 0,$$
$$\implies \cos(2\phi_c + \beta_a - \alpha_r) = 0,$$
$$\implies \phi_c = \frac{\pi}{4} - \frac{\beta_a - \alpha_r}{2}.$$

Thus, using this formula, we get the following values of ϕ_c. Previously we evaluated shear angle from orthogonal metal cutting tests and we will compare the results of this theoretical approach to that.

Feed rate, h [mm/rev]	**Shear Angle phi_c**	**Theoretical Shear Angle**
0.02	0.380506	0.439438
0.03	0.477345	0.470381
0.04	0.495552	0.487506
0.05	0.542189	0.501565
0.06	0.531724	0.512406
0.07	0.542951	0.527340
0.08	0.548241	0.533585

Table 3: Table containing the values of shear angles.

And plotting them on the same graph:

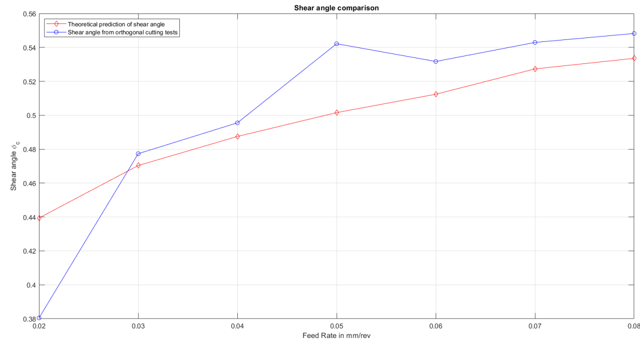

Figure 3: The plot shows the comparison between shear angles.

Finally, here is the complete Matlab solution:

```
clc;
clear;

%% Experimental Data
alpha_r = 0; % rake angle
b = 5; % width of cut in mm
V = 240/60; % cutting speed in m/sec

T = readtable('Book1.xlsx'); % table containing experimental data
h = table2array(T(:,1));
Ft = table2array(T(:,2));
Ff = table2array(T(:,3));
hc = table2array(T(:,4));
figure(1);
plot(h,Ft,'b*'); hold on;
plot(h,Ff,'r*');


%% Iterating through feed values
% Initializing the arrays
phi_c = zeros(7,1);
beta_a = zeros(7,1);
F = zeros(7,1);
tau_s = zeros(7,1);
sigma_s = zeros(7,1);
Pt = zeros(7,1);
Ktc = zeros(7,1);
Kfc = zeros(7,1);
gamma_s = zeros(7,1);
gamma_r = zeros(7,1);

for i=1:7
    rc = h(i)/hc(i); % Chip thickness ratio
    phi_c(i) = atan(rc*cos(alpha_r)/(1-rc*sin(alpha_r))); % shear angle
    beta_a(i) = alpha_r + atan(Ff(i)/Ft(i)); % Friction angle
    F(i) = sqrt(Ft(i)^2 + Ff(i)^2); % Resultant Cutting Force
    Fs = F(i)*cos(phi_c(i) + beta_a(i) - alpha_r); % Shearing Force
    tau_s(i) = Fs/(b*h(i)/sin(phi_c(i))); % Shearing stress
```

```
    Fn = F(i)*sin(phi_c(i) + beta_a(i) - alpha_r); % Normal Force on shearing Plane
    sigma_s(i) = Fn/(b*h(i)/sin(phi_c(i))); % Normal stress on shearing Plane
    Vs = V*cos(alpha_r)/cos(phi_c(i)-alpha_r); % Shearing Velocity
    Ps = Fs*Vs; % Shearing Power
    Pu = F(i)*sin(beta_a(i))*(rc*V); % Frictional Power
    Pt(i) = Ps + Pu; % Total Cutting Power
    Ktc(i) = Ft(i)/(b*h(i)); % Cutting Force Coefficients
    Kfc(i) = Ff(i)/(b*h(i));
    gamma_s(i) = cos(alpha_r)/(sin(phi_c(i))*cos(phi_c(i)-alpha_r)); % Shear strain
    Lc = hc(i)/cos(phi_c(i)-alpha_r); % shaer plane length
    del_d = 0.15*Lc; % shear zone thickness
    gamma_r(i) = V*cos(alpha_r)/(del_d*cos(phi_c(i)-alpha_r)); % shear strain rate
end

Tfin = table(h,Ft,Ff,hc,phi_c, beta_a, F, tau_s, sigma_s, Pt, Ktc, Kfc, gamma_s, gamma_r);
% Final Table as asked in Part 1

%% Curve Fitting
% Linear Model
mdFt = polyfit(h, Ft, 1); % linear regression model fit to Ft
mdFf = polyfit(h, Ff, 1); % linear regression model fit to Ff
xrange = linspace(0.02,0.08);
Ft_lin = polyval(mdFt, xrange);
Ff_lin = polyval(mdFf, xrange);
plot(xrange, Ft_lin, 'b');
plot(xrange, Ff_lin, 'r');
Ktc_lin = mdFt(1)/b;
Kte_lin = mdFt(2)/b;
Kfc_lin = mdFf(1)/b;
Kfe_lin = mdFf(2)/b;

% Non-linear Model
Ktc_fit = fit(h,Ktc,'exp1');
Kfc_fit = fit(h,Kfc,'exp1');
plot(xrange,Ktc_fit.a*exp(Ktc_fit.b*xrange)*b.*xrange,'b--')
plot(xrange,Kfc_fit.a*exp(Kfc_fit.b*xrange)*b.*xrange,'r--')

grid on;
lgd = legend('Experimental Values of F_t','Experimental Values of F_f', ...
    'Linear Model: F_t','Linear Model: F_f','Non-Linear Model: F_t','Non-Linear Model: F_f');
lgd.NumColumns = 2;
lgd.Orientation = 'horizontal';
lgd.Location = 'northwest';
xlabel('Feed Rate in mm/rev');
ylabel('Forces in N');
title('Plot showing experimental values of tangential and feed forces in comparison
        to linear and non-linear models');

% Comparing the force coefficients
figure(2)
subplot(1,2,1)
plot(h,Ktc,'r*'); hold on;
plot(xrange, Ktc_fit.a*exp(Ktc_fit.b*xrange), 'r')
plot(xrange,Ktc_lin, 'r.')
```

```
grid on;
xlabel('Feed Rate in mm/rev');
ylabel('K_{tc} in N/mm^2');
legend('From empirical formula','From non-linear force model','From linear force model')
title('Plot showing comparison between force coefficient: K_{tc}');

subplot(1,2,2)
plot(h,Kfc,'b*'); hold on;
plot(xrange, Kfc_fit.a*exp(Kfc_fit.b*xrange), 'b')
plot(xrange,Kfc_lin, 'b.')
grid on;
xlabel('Feed Rate in mm/rev');
ylabel('K_{fc} in N/mm^2');
legend('From empirical formula','From non-linear force model','From linear force model')
title('Plot showing comparison between force coefficient: K_{fc}');

%% Shear angle comparison
phi_c_merch = pi/4 + alpha_r/2 - beta_a/2;
figure(3);
plot(h,phi_c_merch,'rd-'); hold on;
plot(h,phi_c,'bo-'); hold on;
grid on;
xlabel('Feed Rate in mm/rev');
ylabel('Shear angle \phi_{c}');
legend('Theoretical prediction of shear angle','Shear angle from orthogonal cutting
        tests','Location','northwest')
title('Shear angle comparison')
```

References

1. Altintas, Y. (2012). Manufacturing Automation: Metal Cutting Mechanics, Machine Tool Vibrations, and CNC Design (2nd ed.). Cambridge: Cambridge University Press. doi:10.1017/CBO9780511843723

2. Law, M., Class Lecture, Topic: "Orthogonal Cutting." ME668A, Indian Institute of Technology, Kanpur, Jan., 2021.