# Assignment 1 for ME627

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Assigned: January 23, 2021
Due: before midnight on January 28, 2021

**Instructions:** This assignment will be graded on 100 marks on the Mookit platform, but those marks will be scaled so that the assignment carries 10 percent of the overall course marks. You are allowed to discuss principles with me or your classmates, but not details. In general, you can talk to people on the phone but not exchange numerical results or files. If you have finished working on the assignment before the deadline, please submit (upload) a single pdf file. Submission of extra pdf files, or of files that are not pdf, will attract a penalty of 10 marks per file upto a maximum possible penalty of 500 marks; and only one pdf file will be evaluated. Your aim is to submit a file that indicates clearly that you understand the problem and have obtained results; how you obtained them; what the results are; and what numerical code you have used, if any. You can include scanned handwritten text, Maple/Mathematica calculations, Matlab code and output including plots, sketches, and any other material that you consider relevant. But: <u>one pdf file</u>. In that pdf file, please place your solutions sequentially and start each problem on a fresh page (or be penalized). Please understand the principle: if you increase my workload in useless non-academic ways, you will pay. If you follow instructions, grading will be liberal.

1. Consider the system
$$\ddot{x} + x + x^3 = 0, \quad x(0) = A,\ \dot{x}(0) = 0.$$
Find an expression for the time period in the form of an integral, and find an approximate value for the integral assuming $A$ is small compared to unity. [20 marks]

2. Now consider the system
$$\ddot{x} + x^3 = 0, \quad x(0) = A,\ \dot{x}(0) = 0.$$
What can you say about the time period? [20 marks]

3. Consider the $5 \times 5$ matrix $A$ defined in terms of its elements as
$$A_{ij} = \sin(i)\cos(j).$$
Use Matlab to compute its eigenvalues and eigenvectors. [20 marks]

4. Consider the two-dimensional system given by
$$\dot{x}_1 = -\sin(x_1)(1 + x_2),$$
$$\dot{x}_2 = x_1 + x_2.$$
What can you say about its fixed points? Choose three such fixed points (if you can find that many), linearize the system about those fixed points, and compute the eigenvalues and eigenvectors for the corresponsing linearized flows. Take some bounded region of the phase plane that includes at least three fixed points (if you can find that many), and sketch the phase portrait in that region. [40 marks]