A tutorial on Padé approximation, with applications to control

Second part

Claude Brezinski

We consider the **linear dynamical system**

$$\begin{aligned} x'(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

with $x(t_0) = x_0$, where $x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^p$ and $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{p\times n}, D \in \mathbb{R}^{p\times m}$.

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We assume that m and p are both smaller or equal to n.

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Modifying the input $u(t)$ according to the output vector $y(t)$ which is observed or to the *state vector*$x(t)$ is called **feedback**.

Frequency-domain approach

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The Laplace transform of our system is $s\tilde{x}(s) = A\tilde{x}(s) + B\tilde{u}(s)$ and $\tilde{y}(s) = C\tilde{x}(s) + D\tilde{u}(s)$ and, thus,

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$$\begin{aligned}
\tilde{x}(s) &= (sI - A)^{-1}B\tilde{u}(s) \\
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Model reduction consists in finding an approximation (in some sense) of this system with a **dimension** $k < n$.

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The interest of using the Laplace transform is that the dynamical behavior of a complicated system can be studied using only algebraic operations.

It holds

$$\begin{aligned} G(s) &= C(sI-A)^{-1}B = s^{-1}\sum_{i=0}^{\infty} c_i s^{-i}, \quad c_i = CA^iB \in \mathbb{R}^{p\times m} \\ &= (s^{n-1}G_0 + s^{n-1}G_1 + \cdots + G_{n-1})/\det(sI-A), \end{aligned}$$

with $G_0 = CB$ and $G_i = CB_iB$ for $i = 1, \dots, n-1$.

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The matrices B_i are given by the **Leverrier-Faddeev-Souriau formulæ**

$$\begin{aligned} B_1 &= A + a_1 I \\ B_i &= = AB_{i-1} + a_i I, \quad i = 2, \ldots, n, \end{aligned}$$

where $a_1 = -\mathrm{tr}(A)$ and $a_i = -\mathrm{tr}(AB_{i-1})$ for $i = 2, \ldots, n$.

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Obviously, approximations of G can be obtained by **Padé approximants**.

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$$H(s) = \tilde{C}(sI - \tilde{A})^{-1}\tilde{B} = -\sum_{i=0}^{\infty} h_i s^i,$$

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In fact, we look for a rational function H of the form

$$H(s) = P(s)/Q(s) = -\sum_{i=0}^{N-1} \tilde{A}_i s^i / \sum_{i=0}^{N} b_i s_i,$$

where the b_i are numbers and $\tilde{A}_i \in \mathbb{R}^{p\times m}$.

If the numbers b_i are arbitrarily chosen, we have

$$Q(s)G(s) - P(s) = \mathcal{O}(s^n),$$

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For increasing the order of approximation up to $N + M - 1$, the conditions

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if $N = Mpm$, the rational function $P(s)/Q(s)$ becomes the $[N-1/N]$ **Padé approximant** of G.

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He then went to Paris to continue his education at the *Lycée St Louis* where he spent two years preparing to sit the entrance exams to superior schools. He his mentioned as **good, diligent, very gifted. Will be received at École Polytechnique**.

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In 1886, he is received 6th at **Agrégation de Mathématiques**. There were 76 candidates, 12 were admitted. He began a career teaching in secondary schools: Limoges, Carcassonne, Montpellier.

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In 1873, the French mathematician **Charles Hermite** (1822-1901) gave a proof using the characterization of π as the smallest positive number whose half is a zero of the cosine function and it actually proves that π^2 is irrational.

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Transcendental numbers were proved to exist by the French mathematician **Joseph Liouville** (1809-1882) in 1844 by exhibiting examples of them using continued fractions.

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The transcendence of π was proved by the German mathematician **Carl Louis Ferdinand von Lindemann** (1852-1939) in 1882 by a method similar to that used nine years earlier by Hermite.

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Felix Christian Klein (1849-1925) who is best known for his work in non-euclidean geometry, for his work on the connections between geometry and group theory, and for results in function theory, and **Hermann Amandus Schwarz** (1841-1921) known for his work in complex analysis and his way of computing the solution of an elliptic boundary value problem on a domain which is the union of two overlapping subdomains. This method is still in use.

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Padé translated the Erlangen program into French.

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The thesis committee was composed by **Charles Hermite**, his thesis advisor, **Émile Picard** (1856-1941), Hermite's son- in-law, and **Paul Appell** (1855-1930), husband of the cousine of Picard... a family story!

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He proved results on their general structure and also clearly set out the connection between Padé approximants and continued fractions.

He also showed that, in a properly defined sense, the Padé approximant was the best approximant among all the rational ones

In his report on Padé's thesis, Hermite wrote

> **The detailed study of these particular cases is made with the greatest care, a real talent, and without enumerating the theorems obtained by Mr. Padé we can say that he has completely exhausted the question he wanted to treat.**
>
> **While thinking that the theory of continued fractions could be generalized under other points of view opening a more interesting and more fruitful way, and that the importance of the results to which he arrived does not correspond as much as one would wish it to his long and conscientious work, we judge that the thesis of Mr. Padé is worthy of the approval of the Faculty.**

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> **The general opinion - for my part I communicate to you that of Appell and Goursat who have spoken - is that the judgment that Mr. Hermite expressed orally (and between the lines) was too strict. It is explained by the fact that M. Hermite is much less indulgent for the work which relate to subjects which he wholly possesses than to others. For the former, he judges gladly that everything he sees is easy, which is certainly exaggerated.**

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In 1758 **Johann Heinrich Lambert** (1728-1777) found approximants which are Padé approximants, but developed no general theory.

The first who seemed to realise the full significance of the method of Padé approximants was **Joseph Louis (Giuseppe Luigi) Lagrange** (1736-1813), born in Torino, in a paper of 1776 where he related them to continued fractions.

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Padé approximants appear in the doctoral thesis of **Hermann Hankel** (1839-1873) entitled *On a special class of symmetric determinants* written in 1861.

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Other contributions were made by **Edmond Nicolas Laguerre** (1834-1886) and **Pafnouty Lvovitch Chebyshev** (1821-1894).

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Three of the five submissions received a prize, with Padé receiving the **first prize** together with half the total prize money, with smaller amounts going to the submissions judged to be worthy of second and third place.

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Cliché Pirou
Héliog L. Schutzenberger


EN CETTE MAISON EST NÉ LE 24 DÉCEMBRE 1822
CHARLES HERMITE
MEMBRE DE L'ACADÉMIE DES SCIENCES
MORT À PARIS LE 14 JANVIER 1901
ILLUSTRE MATHÉMATICIEN
DU XIX^E SIÈCLE

37

Monsieur,

L'étude de vos travaux est tellement difficile et tellement pénible que mes services présents me la rendent impossible. Votre mise en demeure de l'entreprendre cependant, sur le champ, m'oblige de vous déclarer que si vous réussissez à me la faire parvenir par ceux de vos amis qui sont membres de l'Académie, j'y répondrai en envoyant immédiatement ma démission de membre de l'Institut.

J'ai l'honneur d'être Monsieur, votre très humble et obéissant serviteur

Ch. Hermite

Paris 2 Décembre 1874

H. Padé (photo faite à Abbeville).

Beaucourt 6. 6. 1922 - A la droite de Padé, sa femme, sa fille ainée et son mari. Devant sa plus jeune fille Odette (mère de Claude Bonfils) avec Madame Vinot sur ses genoux.

ACADÉMIE D'AIX

RÉPUBLIQUE FRANÇAISE

Aix, le 8 février, 1927.

Monsieur le Grand Chancelier

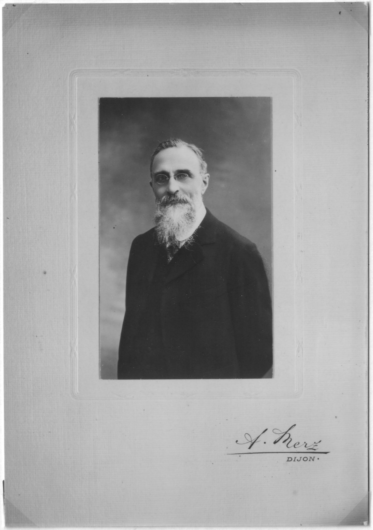
Conformément à vos instructions en date du 3 février 1927, j'ai l'honneur de vous adresser ci-joint un récépissé constatant le versement, à la Caisse du Receveur des finances de l'Arrondissement d'Aix, d'une somme de 100 frs, pour droits de Chancellerie et frais d'expédition de brevet.

Je vous serai reconnaissant de bien vouloir habiliter, pour procéder à ma réception, M. Petit-Dutaillis, Officier de la Légion d'Honneur, Directeur de l'Office des Universités, 96, Boulevard Raspail, Paris, 6e.

Veuillez agréer, je vous prie, Monsieur le Grand-Chancelier, l'hommage de mon respect.

N° 80436 H. Padé, recteur de l'Académie d'Aix, nommé officier de la Légion d'Honneur par décret du 29 janvier 1927

H. Padé vers 1920



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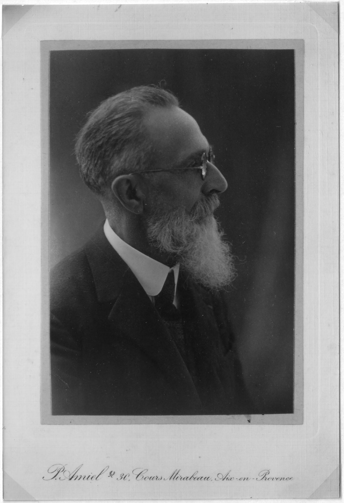

THANK YOU!