line search

$$\gamma' = \underset{\gamma \in [0,1]}{\operatorname{argmin}} \; \underbrace{KL(q^t + \gamma(s - q^t) \| p)}_{\tilde{D}_{KL}(\gamma)} \qquad = \nabla_\gamma f(\tilde{q}_t)$$

grad descent,

$$\gamma \leftarrow \gamma - \eta \nabla_\gamma \tilde{D}_{KL}(\gamma)$$

func. grad,

$$\nabla_{q} f(q) = \log \frac{q}{p}$$

$$-g_q = \langle \nabla f(q), s \rangle - \langle \nabla f(q), q \rangle$$

$$\nabla_\gamma KL(q^t + \gamma(s - q^t) \| p)$$

$$\int \underbrace{(q^t + \gamma(s - q^t))}_{\tilde{q}_t} \log \left( \frac{q^t + \gamma(s - q^t)}{p} \right) dq$$

$$= \nabla_\gamma \int \tilde{q}_t \log \frac{\tilde{q}_t}{p}$$

$$= \int \left( \underbrace{\nabla_\gamma \tilde{q}_t}_{s - q^t} \right) \log \frac{\tilde{q}_t}{p} + \cancel{\tilde{q}_t} \nabla_\gamma \log \frac{\tilde{q}_t}{p}$$

$$\nabla_\gamma \log \frac{\tilde{q}_t}{p} \to \frac{1}{\cancel{\tilde{q}_t}} \nabla_\gamma \hat{q}_t \to (s - q^t)$$

$$= \int (s - q^t) \log \frac{\tilde{q}_t}{p} + (s - q^t)$$

$$= \int (s - q^t) \left( 1 + \log \frac{\tilde{q}_t}{p} \right)$$

$$= \int s \log \frac{\tilde{q}_t}{p} - \int q^t \log \frac{\tilde{q}_t}{p}$$

$$= \left\langle s, \log \frac{\tilde{q}_t}{p} \right\rangle - \left\langle q^t, \log \frac{\tilde{q}_t}{p} \right\rangle$$

$$= \mathbb{E}_s [\log \tilde{q}_t - \log p] - \mathbb{E}_{q^t} [\log \tilde{q}_t - \log p]$$

$$= \mathbb{E}_s [\nabla_{\tilde{q}_t} f(\tilde{q}_t)] - \mathbb{E}_{q^t} [\nabla_{\tilde{q}_t} f(\tilde{q}_t)]$$