Delay Model Expansion

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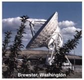
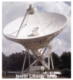
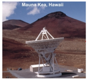






The delay model

Define $T(\vec{x}, \vec{R}, t)$ as the *absolute delay model* where

$\vec{x}$ is the receiver location (ITRF)

$\vec{R}$ is the source location α, δ (ICRF) and distance,

t is the time

Note that:

* $T \geq 0$
* T is a metric

Define $\tau(\vec{x}, \vec{R}, t) \equiv T(\vec{x}, \vec{R}, t) - T(\vec{\oplus}, \vec{R}, t)$ where $\vec{\oplus}(t)$ is the Earth center[1]. This choice of an origin sets the Doppler reference frame. For receivers on Earth, $|\tau| < 22$ ms.

[1] Most quantities discussed will be time dependent even when not made explicit.

Example: Goddard's CALC/SOLVE package

A realization of τ for VLBI that includes

* The rotation state of $\oplus$
* Atmospheric refraction
* Earth tides
* Gravitational potentials of $\odot$, $\oplus$, and the planets
* Aberration

Additional features in CalcServer

* Plate tectonics
* Near field gravitational potentials in solar system
* Wavefront curvature for near field objects
* Ocean loading

Some more definitions

* Gradient on first slot:

$$\vec{\nabla}_1 \tau(\vec{x}, \vec{R}, t) \equiv \left(\frac{\partial \tau}{\partial x_1}, \frac{\partial \tau}{\partial x_2}, \frac{\partial \tau}{\partial x_3} \right)$$

* Unit gradient vector:

$$\hat{\nabla} f = \frac{\vec{\nabla} f}{|\vec{\nabla} f|}$$

* Derivatives w.r.t. tangent plane coordinates

$$\begin{aligned} \frac{\partial}{\partial l} \tau \left(\vec{x}, \vec{R}(\alpha, \delta, D), t \right) &= \frac{1}{\cos\delta} \frac{\partial}{\partial \alpha} \tau \left(\vec{x}, \vec{R}(\alpha, \delta, D), t \right) \\ \frac{\partial}{\partial m} \tau \left(\vec{x}, \vec{R}(\alpha, \delta, D), t \right) &= \frac{\partial}{\partial \delta} \tau \left(\vec{x}, \vec{R}(\alpha, \delta, D), t \right) \end{aligned}$$

Coordinate system for baseline vectors

First, define the celestial north pole:

$$\hat{N}(t) = \hat{\nabla}_1 \tau \left(\vec{\oplus}, (0^\circ, 90^\circ, \infty), t\right)$$

Then the basis vectors for a source at $\vec{R}$ and time t are:

$$\begin{aligned} \hat{w}(t) &= -\hat{\nabla}_1 \tau(\vec{\oplus}, \vec{R}, t) \\ \hat{u}(t) &= \frac{\hat{w} \times \hat{N}}{|\hat{w} \times \hat{N}|} \\ \hat{v}(t) &= \hat{w} \times \hat{u} \end{aligned}$$

Note that the antenna coordinates are not explicitly used; all information comes via the delay model.

Calculation of a dirty image

The expression for the dirty image given measured voltages $v_p(t)$ is:

$$I^{\mathrm{D}}(l,m) = \sum_{j\neq k} \int dt\, v_j^*(t) v_k(t) e^{2\pi i\nu(\tau_k(l,m,t)-\tau_j(l,m,t))}$$

where $\tau_p(l,m,t)$ is shorthand for $\tau\left(\vec{x}_p, \vec{R}\left(\alpha(l,m), \delta(l,m), \infty\right), t\right)$ which can be expanded as

$$\tau_p(l,m,t) = \tau_p(t) + u_p l + v_p m + \mathcal{O}(l^2+m^2) + \cdots$$

where $\tau_p(t) = \tau_p(l=0, m=0, t)$. Defining the visibility as

$$V_{jk,t} = \left\langle v_j^*(t) v_k(t) e^{2\pi i\nu(\tau_k(t)-\tau_j(t))} \right\rangle$$

results in a familiar equation

$$I^{\mathrm{D}}(l,m) \approx \sum_t \sum_{j\neq k} V_{jk,t} e^{2\pi i\nu(l u_{jk,t} + m v_{jk,t})}$$

Calculation of a dirty image (continued)

The coefficients of l and m are

$$u_{jk,t} = -c\left(\frac{\partial}{\partial l}\tau_k(t) - \frac{\partial}{\partial l}\tau_j(t)\right) \quad \text{and}$$
$$v_{jk,t} = -c\left(\frac{\partial}{\partial m}\tau_k(t) - \frac{\partial}{\partial m}\tau_j(t)\right).$$

These "baseline vectors" are here self-consistently determined purely from the delay model. The third component of the baseline vector is directly related to the delay

$$w_{jk,t} = -c\left(\tau_k(t) - \tau_j(t)\right)$$

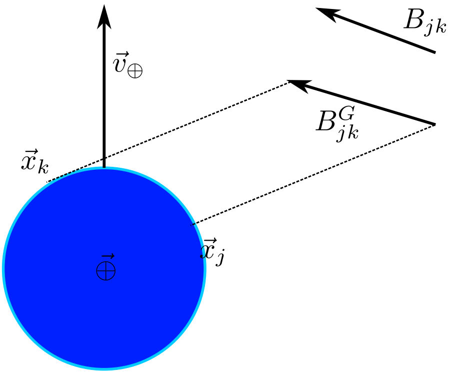
In ITRF coordinates, the baseline vector is thus

$$\vec{B} = u\hat{u} + v\hat{v} + w\hat{w}$$

This typically differs from the geometric baseline vector, $\vec{B}^G_{jk,t} = \vec{x}_k - \vec{x}_j$, by a few parts in 10^5 with aberration being the primary difference.

Example 1: aberration

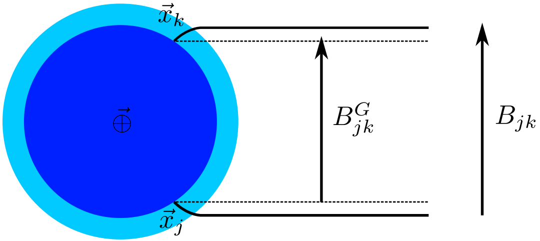
The largest difference between $\vec{B}$ and $\vec{B}^G$ when observing a distant source is annual aberration[2].



[2] Diurnal aberration is absorbed by the delay model.

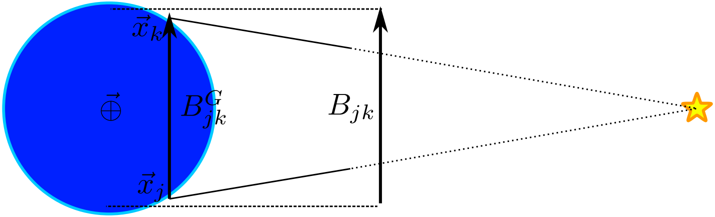
Example 2: atmospheric refraction

Refraction in the Earth's atmosphere causes the effective location of the receiver, as probed by a distant observer, to be higher in elevation.



Example 3: object in the near field

Baseline vectors for near-field objects are non-intuitive.



Effective baselines are larger than expected since the antennas are closer to the object than is $\vec{\oplus}$. The magnitude of the effect is of order $r_\oplus/D$.

Proposal: scrap the (u, v, w) vector

Instead, compute a higher order expansion of the delay model over the region of interest:

$$\begin{aligned} \tau_j(l, m, t) &= \sum_{a,b} \frac{1}{a!b!} \frac{\partial^{a+b}}{\partial l^a \partial m^b} \tau_j(t) l^a m^b \\ &= \sum_{a,b} C^j_{ab}(t) l^a m^b \end{aligned}$$

with the following correspondences:

$$\begin{aligned} C^j_{00}(t) &= -\frac{w_j}{c} = \tau_j(t) \\ C^j_{10}(t) &= -\frac{u_j}{c} \\ C^j_{01}(t) &= -\frac{v_j}{c} \end{aligned}$$

Then store C^j_{ab} for $a + b \leq N$ along with coordinates for the antenna pointing, the correlation center, and the center of the tangent plane.

Application 1: accurate model subtraction

Removal of a point source at location (l, m) of flux density S from a visibility database can be performed with precision, even for a source far from the delay center.

$$V'_{jk,t} = V_{jk,t} - S \exp\left(2\pi i\nu \sum_{a,b\neq 0,0} (C^k_{ab} - C^j_{ab}) l^a m^b\right)$$

Within an iterative cleaning process, an approximate dirty image calculation can be interleaved with this accurate model subtraction. Near field sources, if identifiable, can use their appropriate delay model against a separate far-field delay model for the remainder of the field.

Application 2: tangent plane (UV) shifting

Self-consistent shifting over large angles (without "generation loss") can be performed to high precision provided enough terms in the delay model are preserved. Two steps are:

1. Shift tangent plane from $\vec{R}_0$ to $\vec{R}_1$
   - Use model to calculate delay at various points $\vec{R}(l, m)$
   - Define new plane where $\vec{R}(l' = 0, m' = 0) = \vec{R}_1$
   - Recompute expansion coefficients, C^p_{ab}, in the new tangent plane
2. Update phases of visibilities:

$$V'_{jk,t} = V_{jk,t} \exp\left(2\pi i \nu \left(\delta\tau_k - \delta\tau_j\right)\right)$$

   where $\delta\tau_p = \tau_p(\vec{x}_p, \vec{R}_1, t) - \tau_p(\vec{x}_p, \vec{R}_0, t)$
3. Correct for non-commutation of sampling and frequency shifting

Other possible applications

* multi-look VLBI
* improved model accountability
* astrometry
* imaging (faceted, w-projection, ...)
  - low freq, wide-field imaging with long baselines most challenging
  - Faraday rotation, if modeled within τ, could predict different $\vec{B}$ values for R and L polarization!
* RFI excision

Conclusions

* A formalism for self-consistent use of the delay model was presented
* The intimate connection between this model and interferometer geometry was demonstrated
* Near-field objects naturally fit into this formalism
* The (u, v, w) baseline vectors have limitations in their current use
* A generalization of these vectors is proposed.

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