Main beam representation in non-regular arrays

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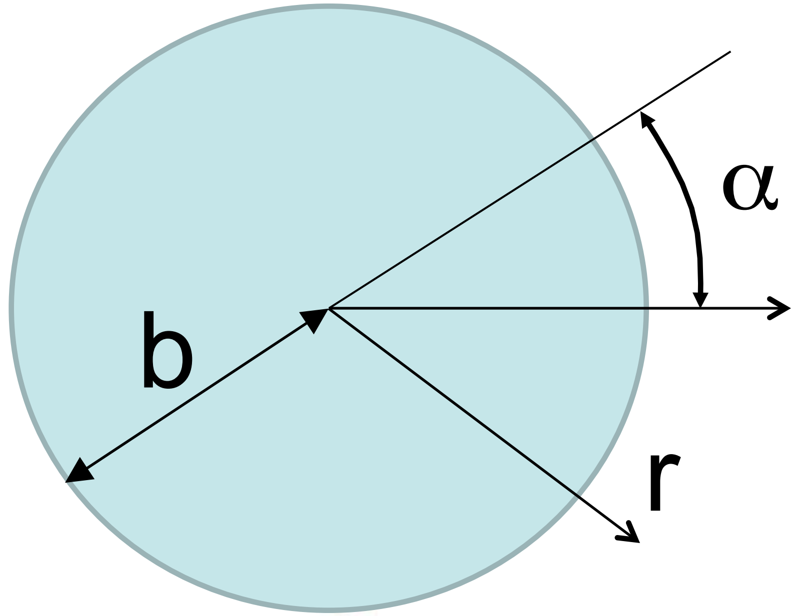
2 parts

Preliminary: Patterns of apertures – a review

Next: analysis of aperture arrays with mutual coupling

$$\vec{f} = \int_S \vec{I}(\vec{r})\, e^{j\,k\,\hat{u}\,.\,\vec{r}}\, dS$$



$$I_x(r,\alpha) \simeq \sum_{n=-N}^{N} a_{nx}(r)\; e^{j\,n\,\alpha}$$

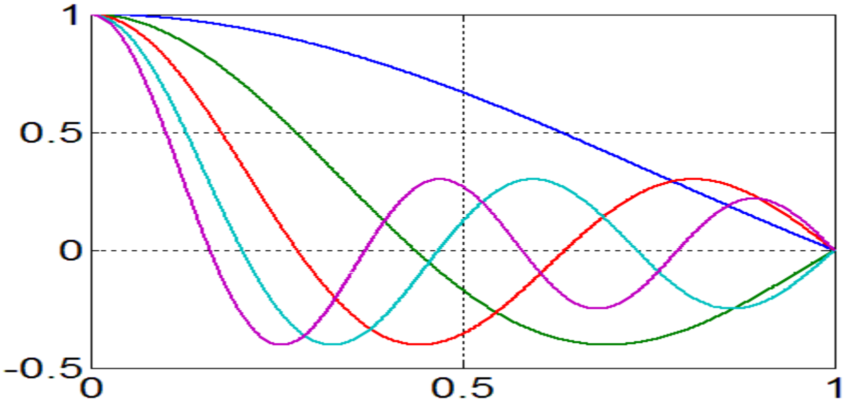
$$a_{nx}(r) = \frac{1}{2\,\pi} \int_0^{2\pi} I_x(r,\alpha)\, e^{-j\,n\,\alpha}\, d\alpha$$

Algarve meeting, 2011

Fourier-Bessel series

$$a_{nx}(r) \simeq \sum_{m=1}^{M} c_{mnx}\, J_n(\lambda_n^m\, r/b)$$

$$c_{mnx} = \frac{A_{mnx}}{\pi\, b^2\, J_{n+1}^2(\lambda_n^m)}$$



$$\begin{aligned} A_{mnx} &= 2\pi \int_0^b J_n(\lambda_n^m\, r/b)\, a_{nx}(r)\, r\, dr \\ &= \int_S I_x(r,\alpha)\, J_n(\lambda_n^m\, r/b)\, e^{-j\, n\, \alpha}\, dS \end{aligned}$$

Algarve meeting, 2011

Zernike series

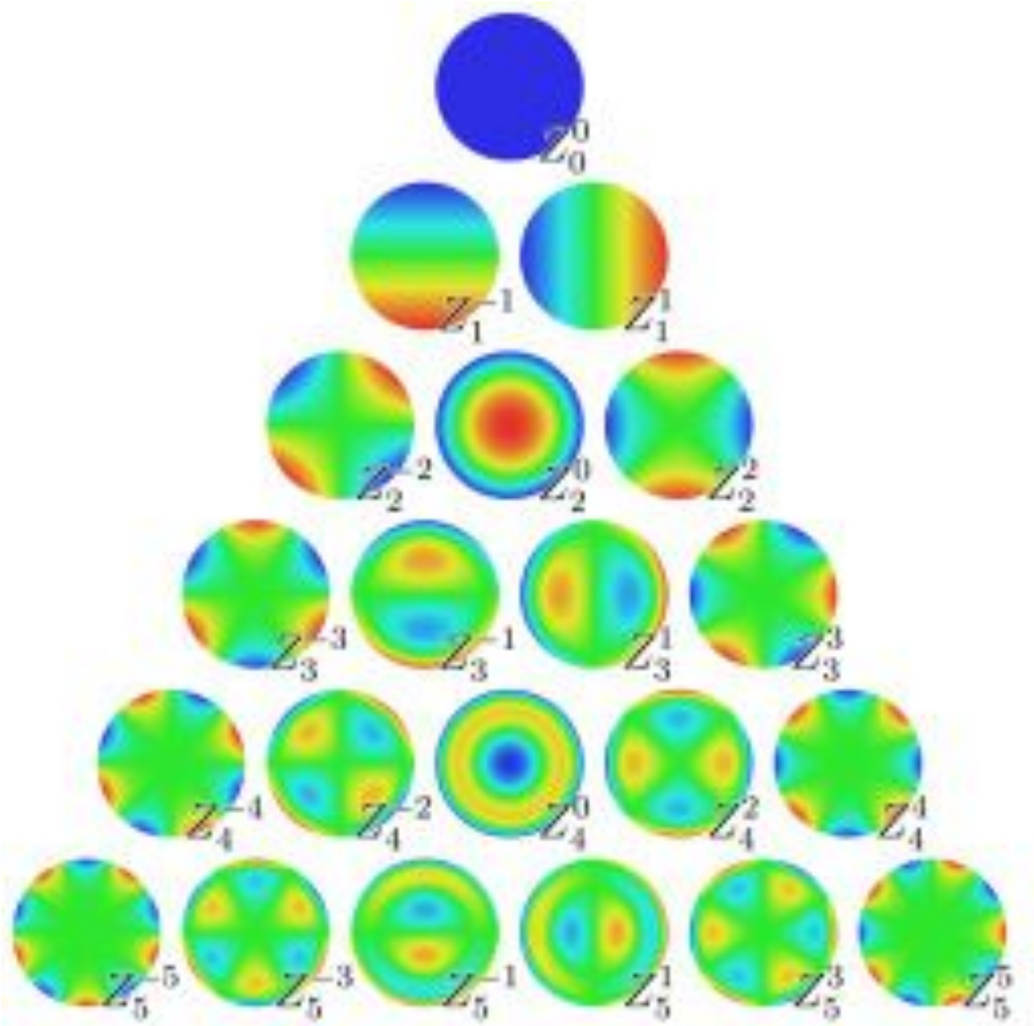
$$a_{nx}(r) \simeq \sum_{m=0}^{M} z_{mnx}\, F_m^{|n|}(r/b)$$

$$z_{mnx} = \frac{(|n| + 2m + 1)}{\pi\, b^2}\, B_{mnx}$$

$$\begin{aligned} B_{mnx} &= 2\pi \int_0^b F_m^{|n|}(r/b)\, a_{nx}(r)\, r\, dr \\ &= \int_S I_x(r,\alpha)\, F_m^{|n|}(r/b)\, e^{-j\, n\, \alpha}\, dS \end{aligned}$$

NB: the Zernike function is a special case of the Jacobi Function

Zernike functions



Picture from Wikipedia

Radiation pattern

$$f_x(\theta,\phi) = \int_S I_x(r,\alpha)\, e^{j\,k\,(u_{x'}\,x' + u_{y'}\,y')}\, dS$$

$$= \sum_{n=-N}^{N} \underbrace{\int_0^b a_{nx}(r)\, 2\pi\, j^n\, e^{j\,n\,\phi}\, J_n(k\,r\,\sin\theta)\, r dr}_{\text{Hankel transform}}$$

F.T. of Bessel

$$f_x(\theta,\phi) \simeq \sum_{n=-N}^{N} \sum_{m=1}^{M} 2j^n \frac{\lambda_n^m}{J_{n+1}(\lambda_n^m)} A_{mnx}\, \mathbf{e}^{\mathbf{j\,n}\,\phi}$$

$$\boxed{\frac{\mathbf{J_n}(\mathbf{k\,b}\ \sin\theta)}{(\lambda_\mathbf{n}^\mathbf{m})^\mathbf{2} - (\mathbf{k\,b}\ \sin\theta)^\mathbf{2}}}$$

FT of Zernike

$$f_x(\theta,\phi) \simeq 2 \sum_{n=-N}^{N} \sum_{m=0}^{M} (|n| + 2m + 1)\, B_{mnx}\, j^n\, \mathbf{e}^{\mathbf{j\,n}\,\phi}$$

$$(-1)^s \boxed{\frac{\mathbf{J_{|n|+2m+1}}(\mathbf{k\,b}\,\sin\theta)}{\mathbf{k\,b}\,\sin\theta}}$$

Sparse polynomial

$$f_x \simeq \sum_{n=-N}^{N} \sum_{m=0}^{M} C_{mnx} \boxed{\left(\frac{\mathbf{b}}{\lambda} \sin\theta\right)^{\mathbf{2m+|n|}}} \; \mathbf{e}^{\mathbf{j\,n}\,\phi}$$

$$C_{mnx} = D_{mn} \int_S I_x \left(\frac{2\pi\, r}{b}\right)^{2m+|n|} e^{-j\,n\,\alpha}\, dS$$

$$D_{mn} = (-1)^{m+s} \frac{(1/2)^{2m+|n|}}{m!\,(m+|n|)!}\, j^n$$

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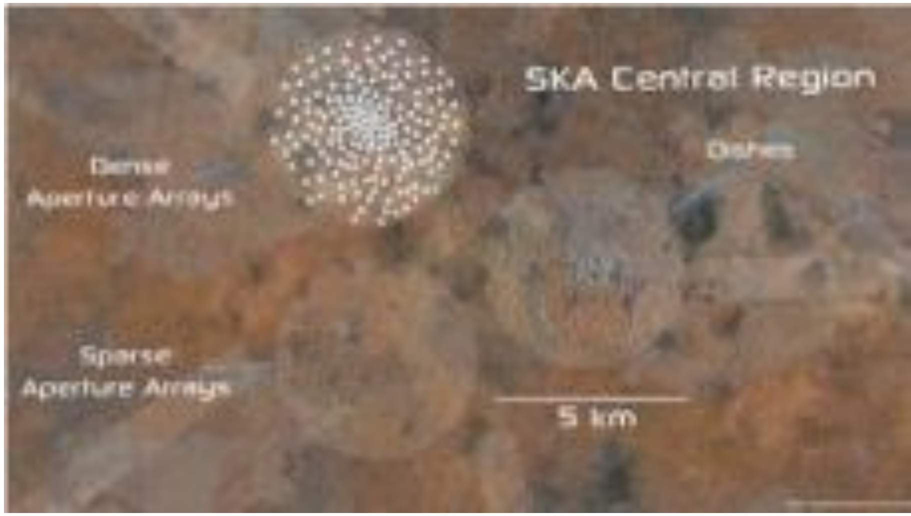
Context: SKA AA-lo

Type of element
Bowtie
Spiral
Log-periodic

Non-regular: max effective area with min nb. elts w/o grating lobes.

Parameter	Specification
Low frequency	70 MHz
High frequency	450 MHz
Nyquist sampling frequency	100 MHz
Number of stations	50 => 250
Antennas per station	10.000



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Problem statement

Goal: pattern representation for all modes of operation at station level.

Too many antennas vs. number of calibration sources

→ Calibrate the main beam and first few sidelobes
Suppress far unwanted sources using interferometric methods (open).

→ Compact representations of patterns, inspired from radiation from apertures, including effects of mutual coupling

Specific to SKA AAlo

- Fairly circular stations (hexagonal would be OK)
- Relatively dense
- Weak amplitude tapering – some space tapering
- Irregular => all EEP's very different
- Even positions are not 100 % reliable (within a few cm)
- Correlation matrix not available
- Nb. of beam coefficients << nb. Antennas
- Restrict to main beam and first few sidelobes

Even assuming identical EEP's and find 1 amplitude coefficient per antenna is way too many coefficients

Outline

1. Limits of traditional coupling correction
2. Array factorization
3. Array factors: series representations
4. Reduction through projection
5. Scanning

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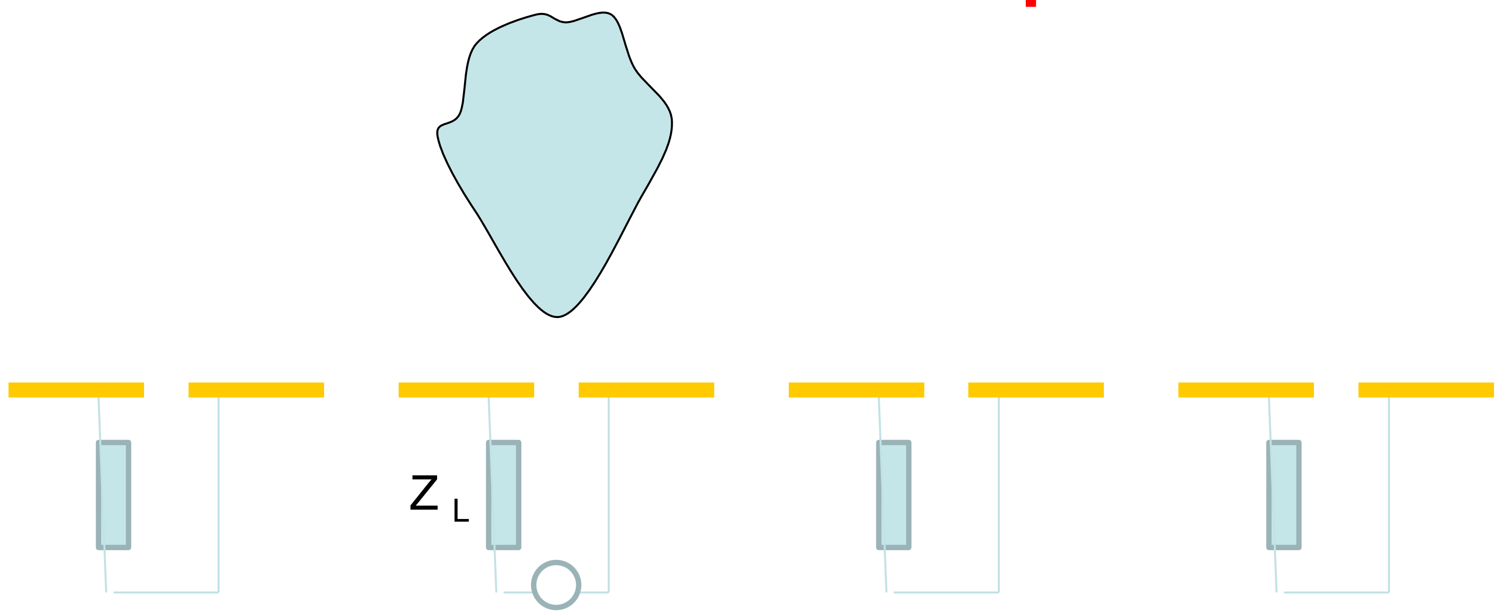
Embedded element pattern



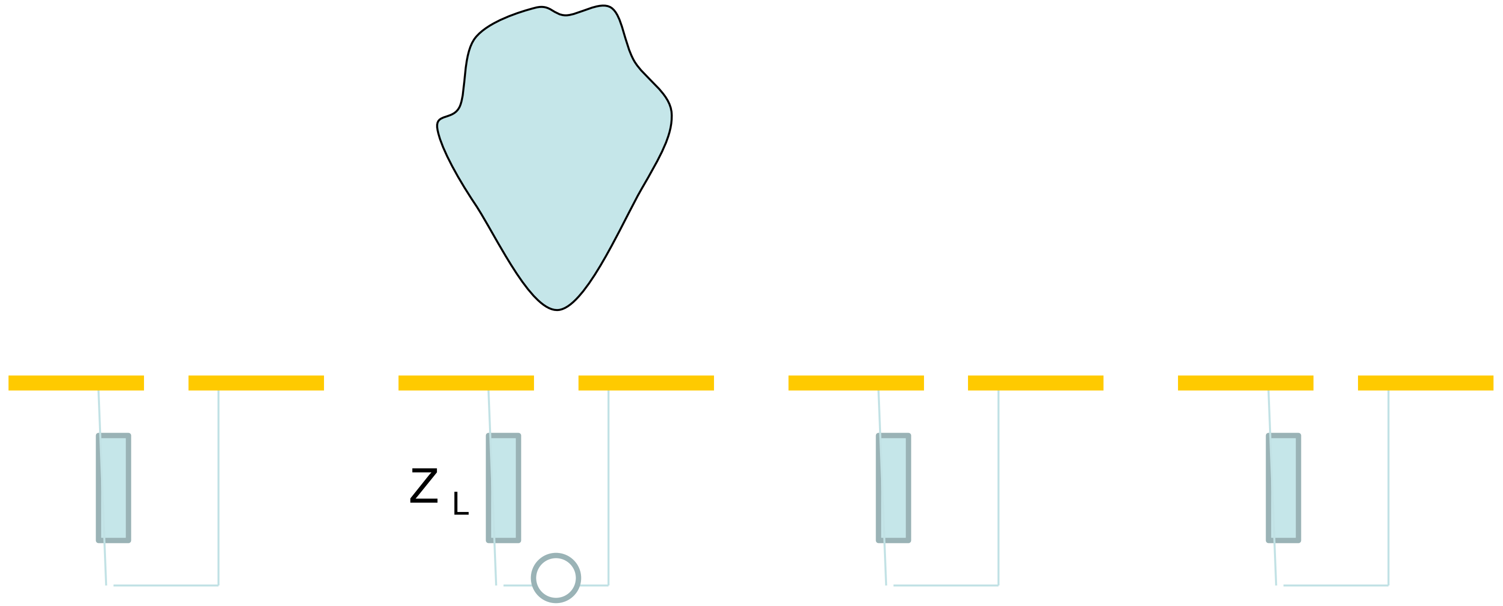
$$(\mathbf{Z}_d + \mathbf{Z}_L)\,\mathbf{g}^{nc} \simeq (\mathbf{Z} + \mathbf{Z}_L)\,\mathbf{g}^{e}$$

Impedance
isolated elt

Array impedance
matrix

To get voltages in uncoupled case:
multiply voltage vector to the left by matrix

Embedded element pattern



After correction, we are back to original problem, with (zoomable, shiftable) array factor

Gupta, I., and A. Ksienski (1983), Effect of mutual coupling on the performance of adaptive arrays, IEEE Trans. Antennas Propag., 31(5), 785–791.

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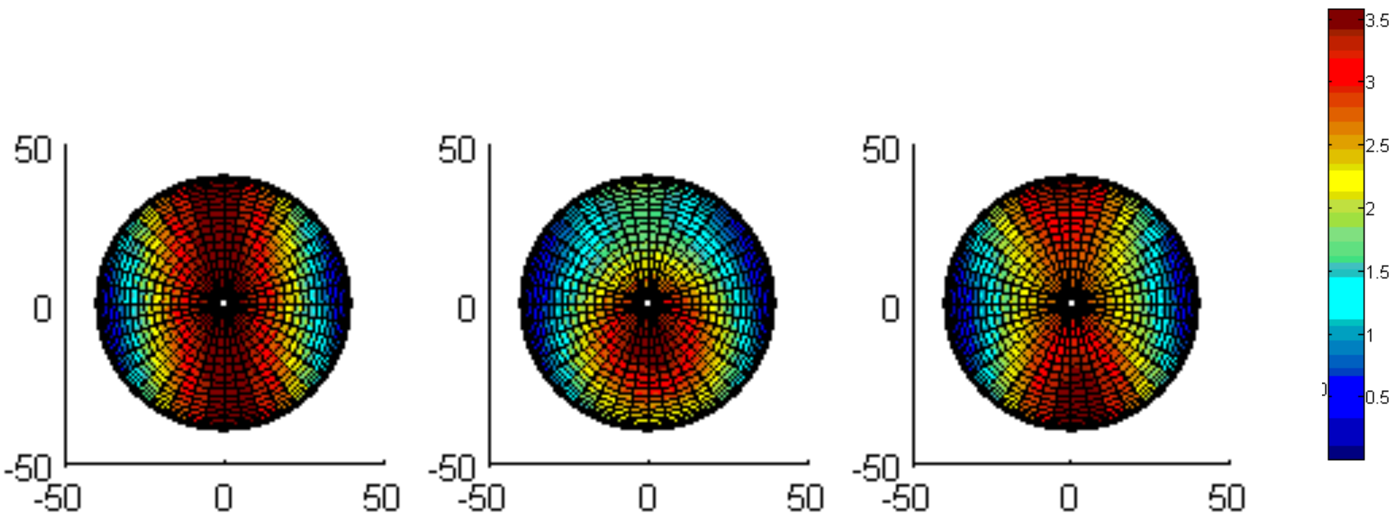
Mutual coupling correction

Half-wave dipole

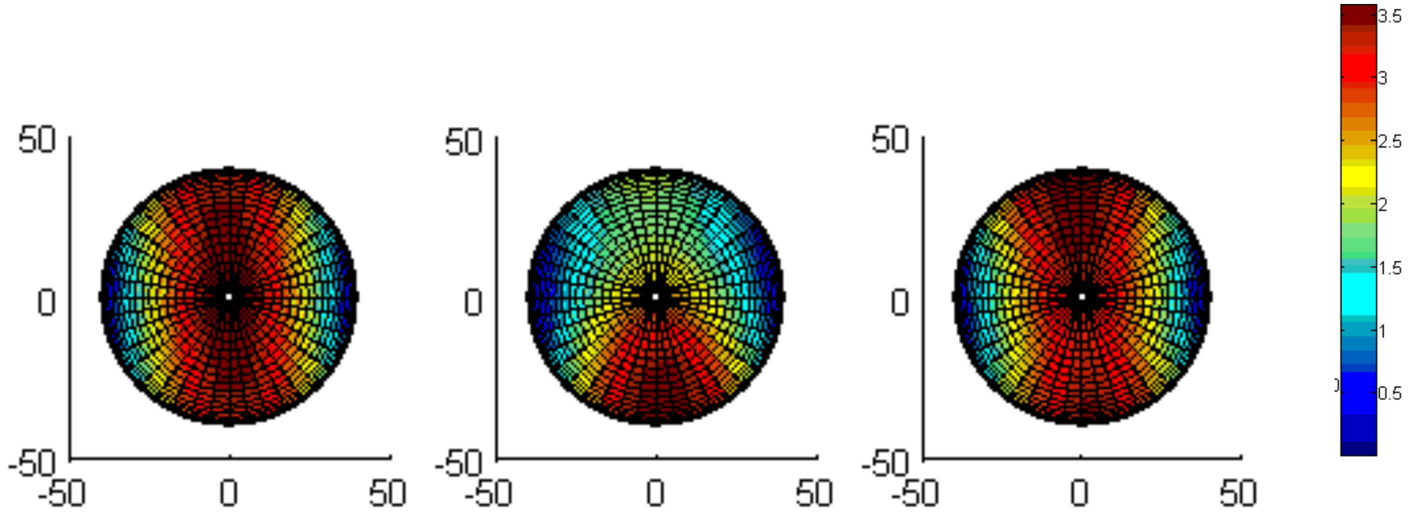


Isolated **Embedded** **Corrected**

Mutual coupling correction

Bowtie antenna

l=1.2 m, λ=3.5 m

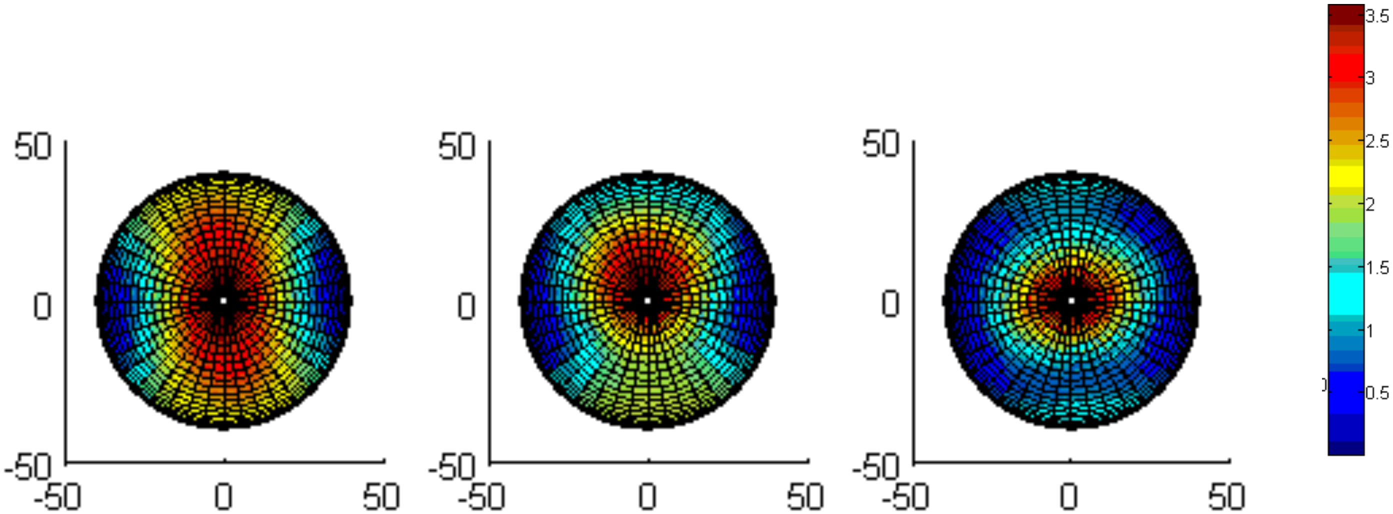


Isolated **Embedded** **Corrected**

Mutual coupling correction

Bowtie antenna

l=1.2 m, λ=1.5 m



Isolated **Embedded** **Corrected**

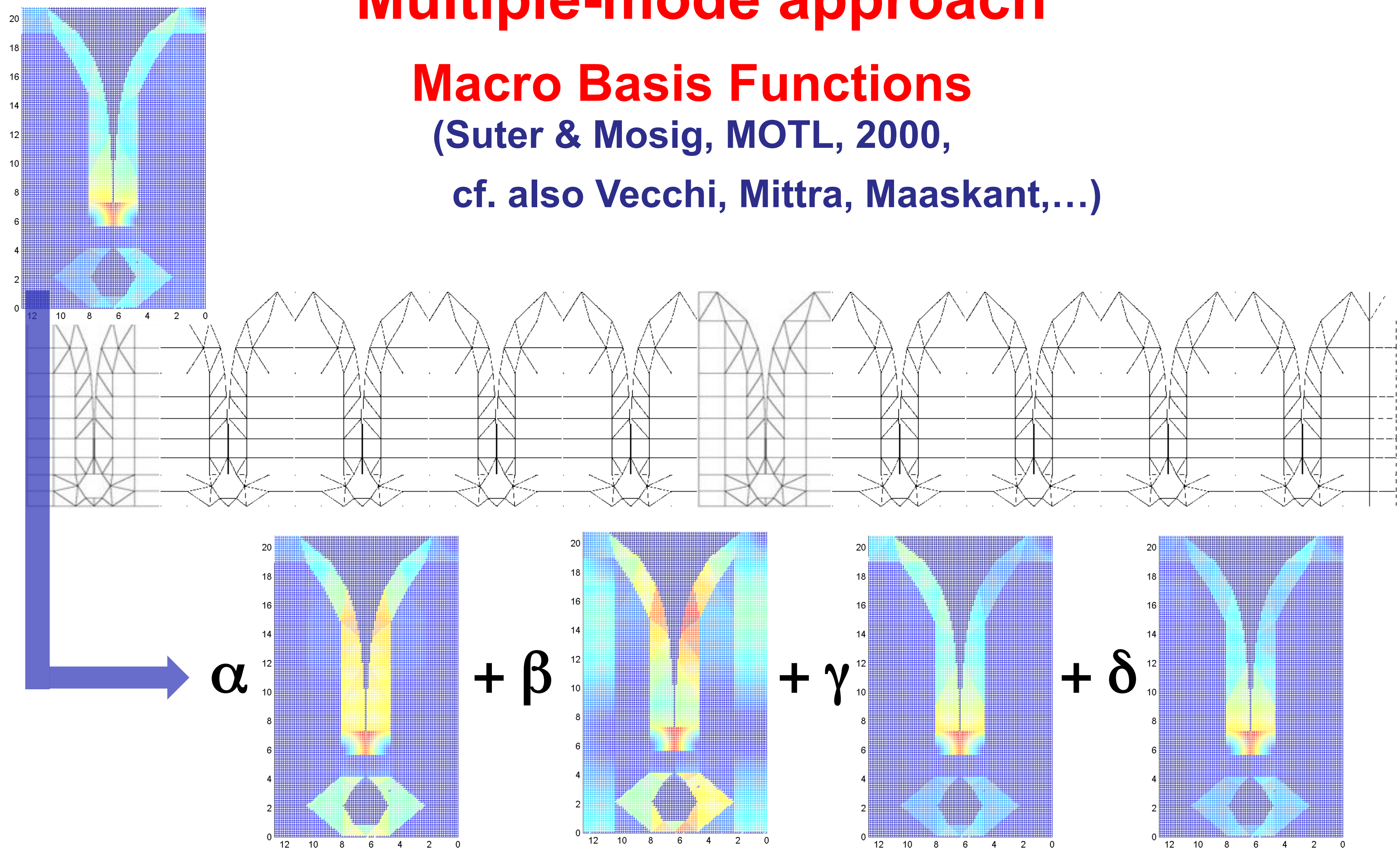
SINGLE MODE assumption not valid for l ≲ λ/2

Multiple-mode approach

Macro Basis Functions

(Suter & Mosig, MOTL, 2000,

cf. also Vecchi, Mittra, Maaskant,…)



MBF 1 MBF 2 MBF 3 MBF 4

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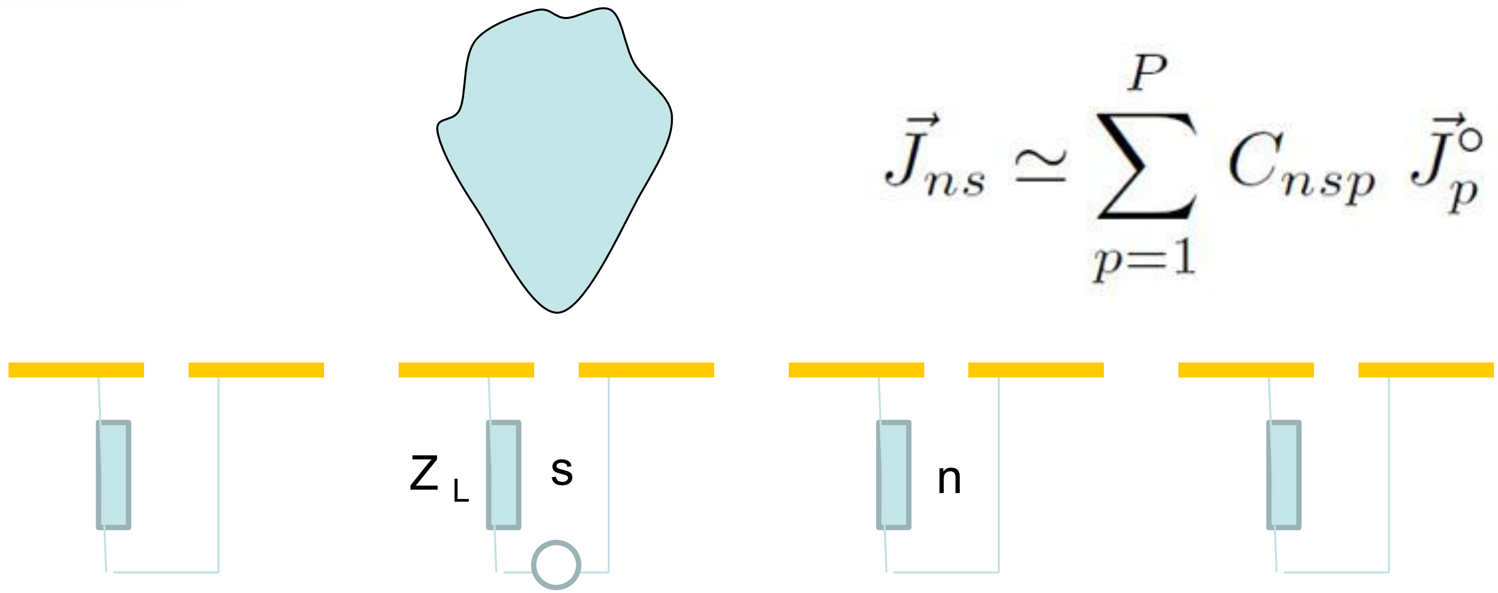
Array factorisation



$$\vec{J}_{ns} \simeq \sum_{p=1}^{P} C_{nsp}\, \vec{J}_p^{\circ}$$

	Antenna index		
MBF 1	C_{111}	C_{211}	C_{311}
MBF 2	C_{122}	C_{222}	C_{322}
⋮			

Coefficients for **IDENTICAL** current distribution

$$\vec{F}_s \simeq \sum_{p=1}^{P} \sum_{s=1}^{N} A_s \sum_{n=1}^{N} C_{nsp}\, e^{j\,k\,\hat{u}.\vec{r}_n}\, \vec{F}_p^{\circ}$$

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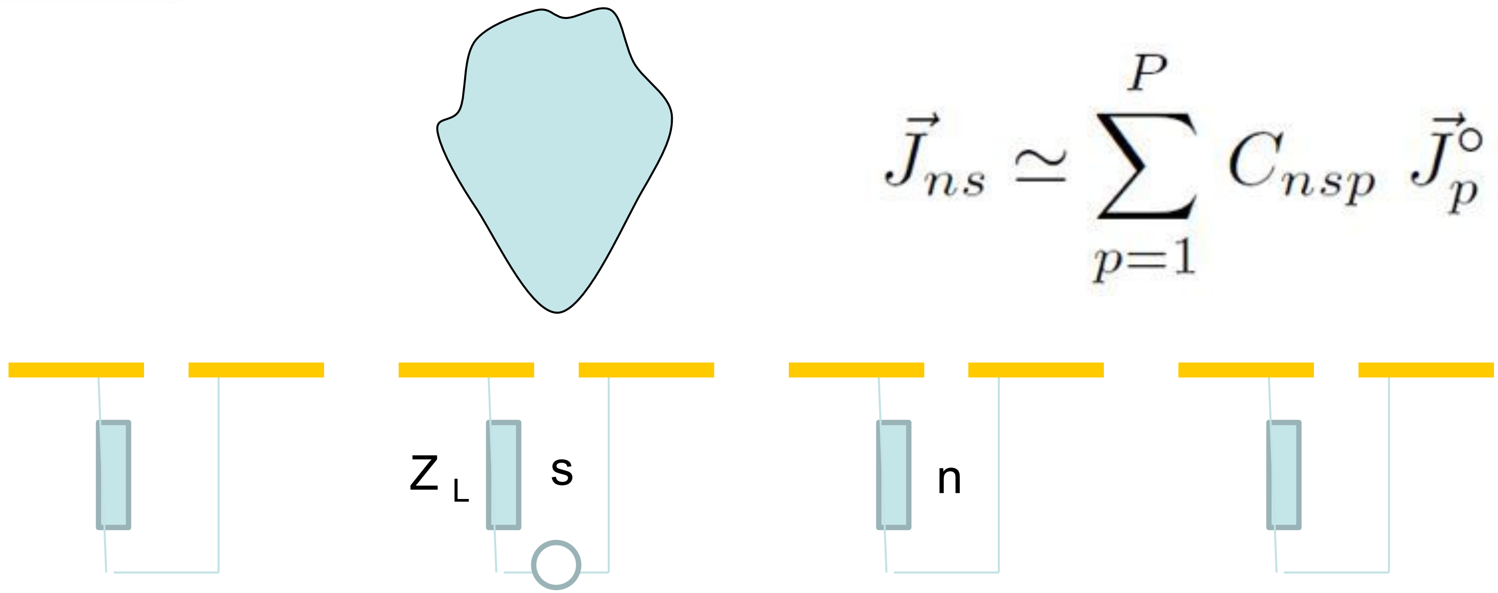
Array factorisation



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MBF 1	C_{111}	C_{211}	C_{311}
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⋮			

Coefficients for **IDENTICAL** current distribution

$$\vec{F}_s \simeq \sum_{p=1}^{P} \sum_{n=1}^{N} \left(\sum_{s=1}^{N} A_s\, C_{nsp} \right) e^{j\,k\,\hat{u}.\vec{r}_n}\; \vec{F}_p^{\circ}$$

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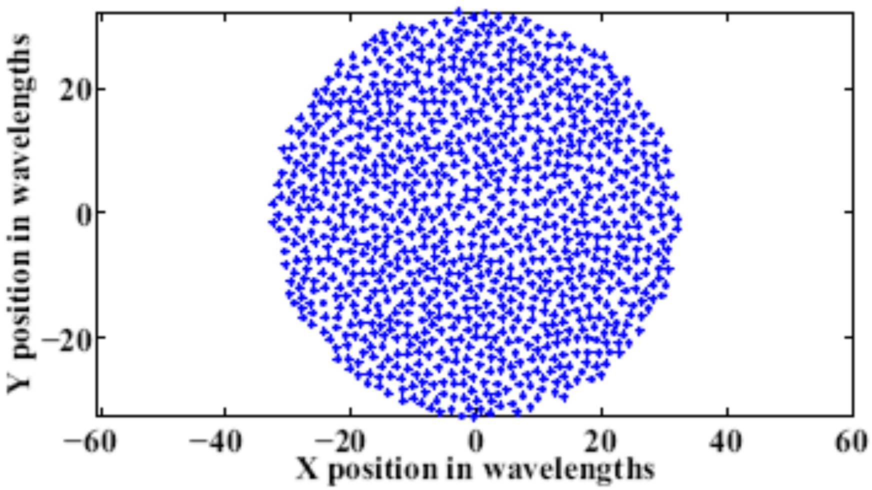
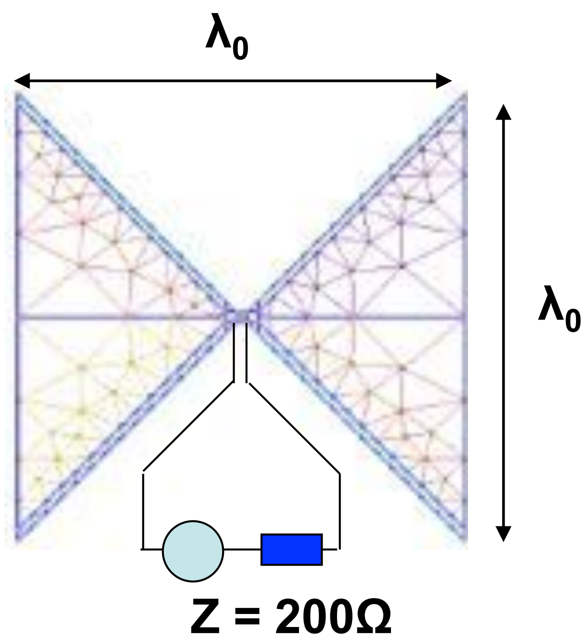
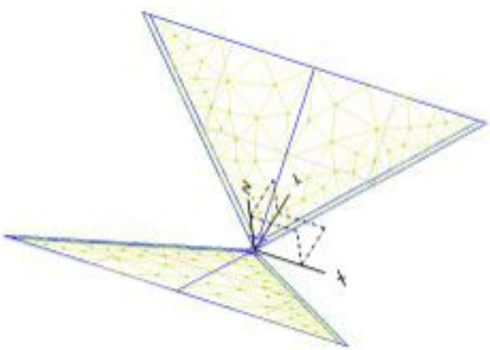
Example array



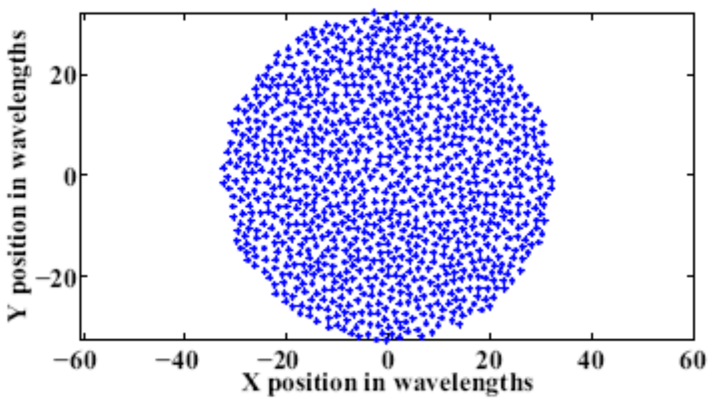
- Array radius = $30\lambda_0$.
- Number of elements = 1000.



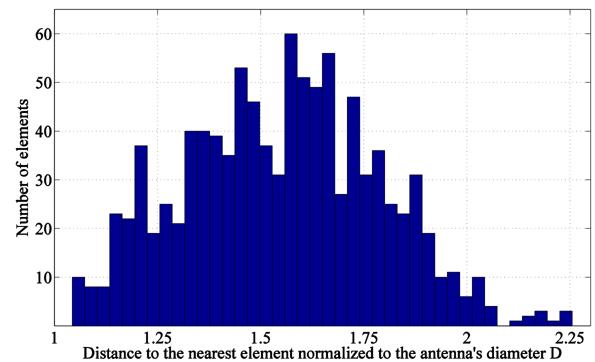
- Distance to ground plane = $\lambda_0/4$.
- No dielectric.

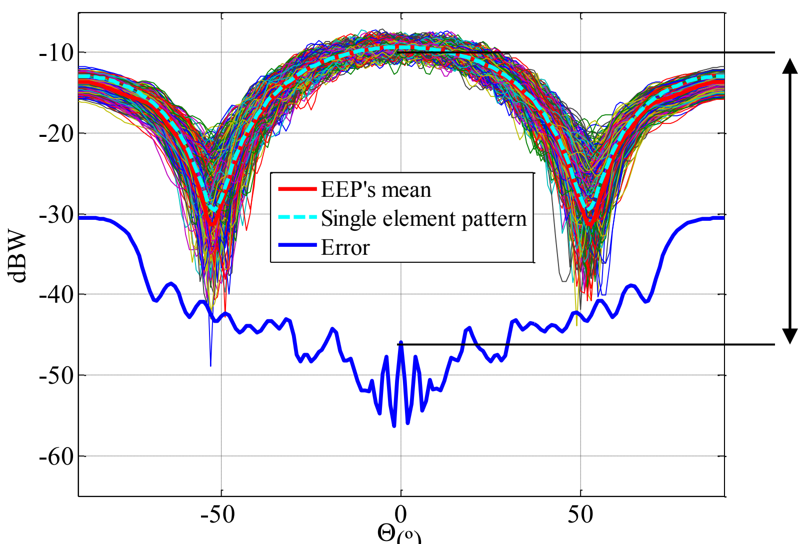


Random arrangement



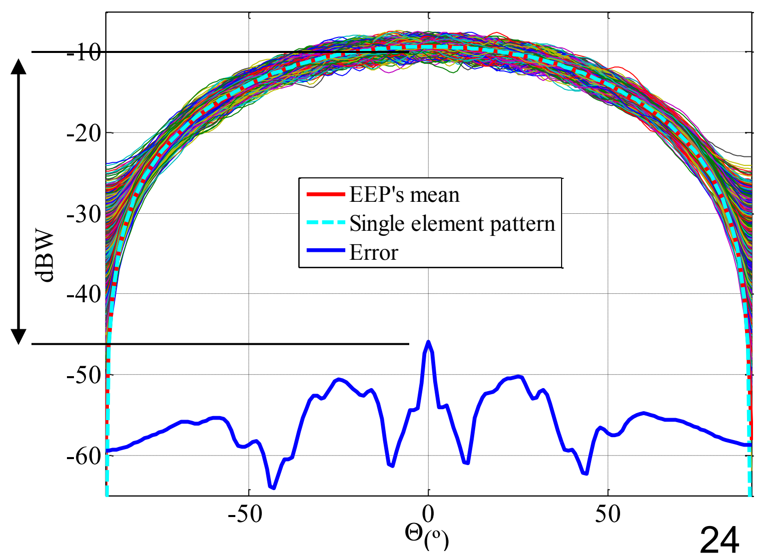
Random configuration





E-plane

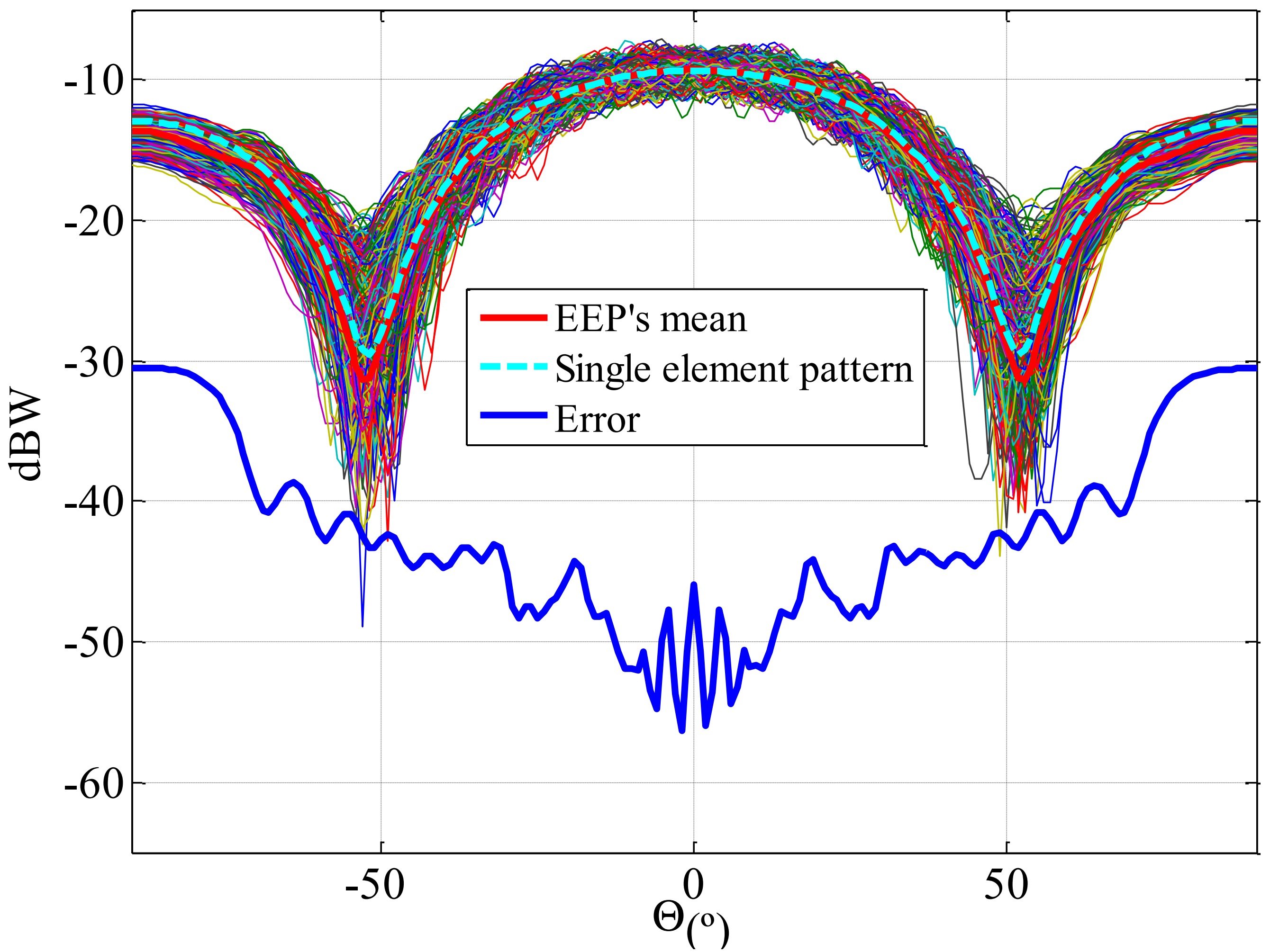
~ 35 dB



H-plane

$$e = 10\log_{10}\left(\left|\vec{E}_{mean}(\theta,\phi) - \vec{E}_{single}(\theta,\phi)\right|^2\right)$$

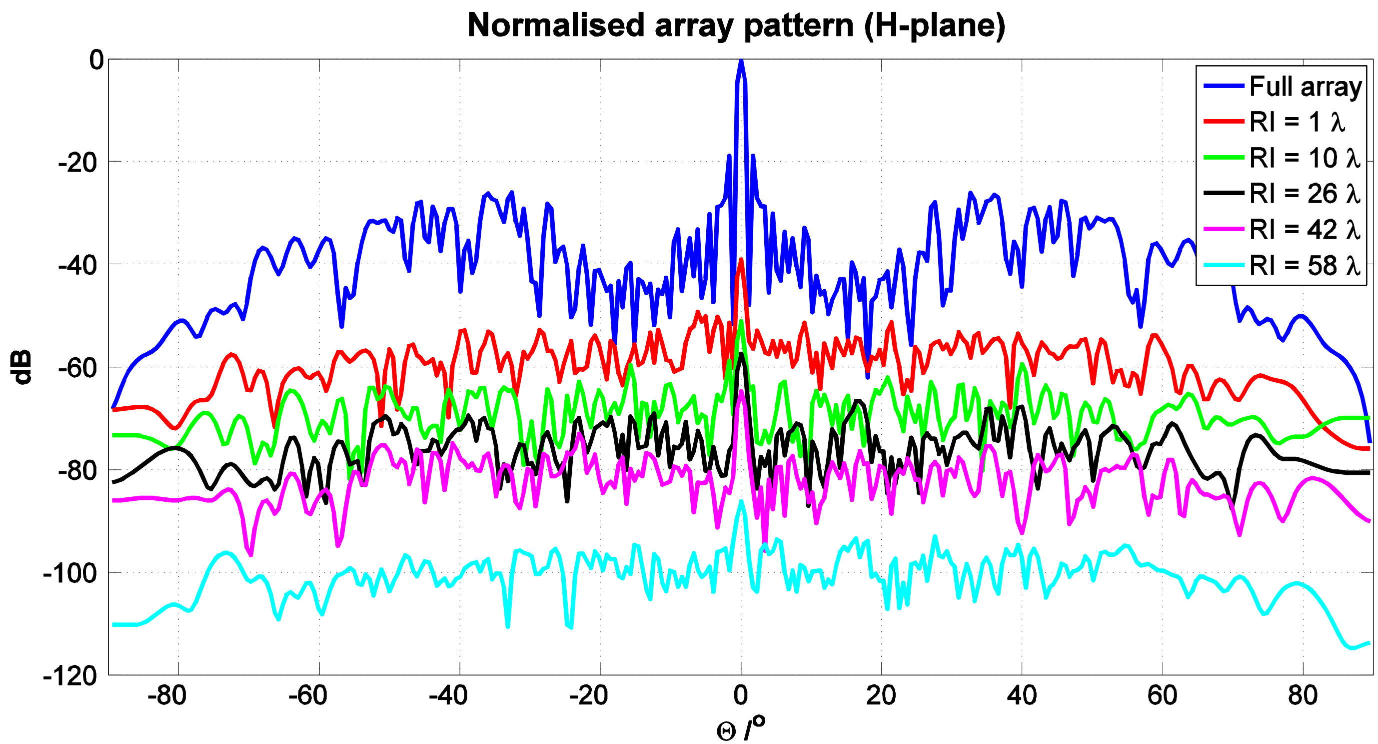
Quasi-random arrangement



Radius of Influence

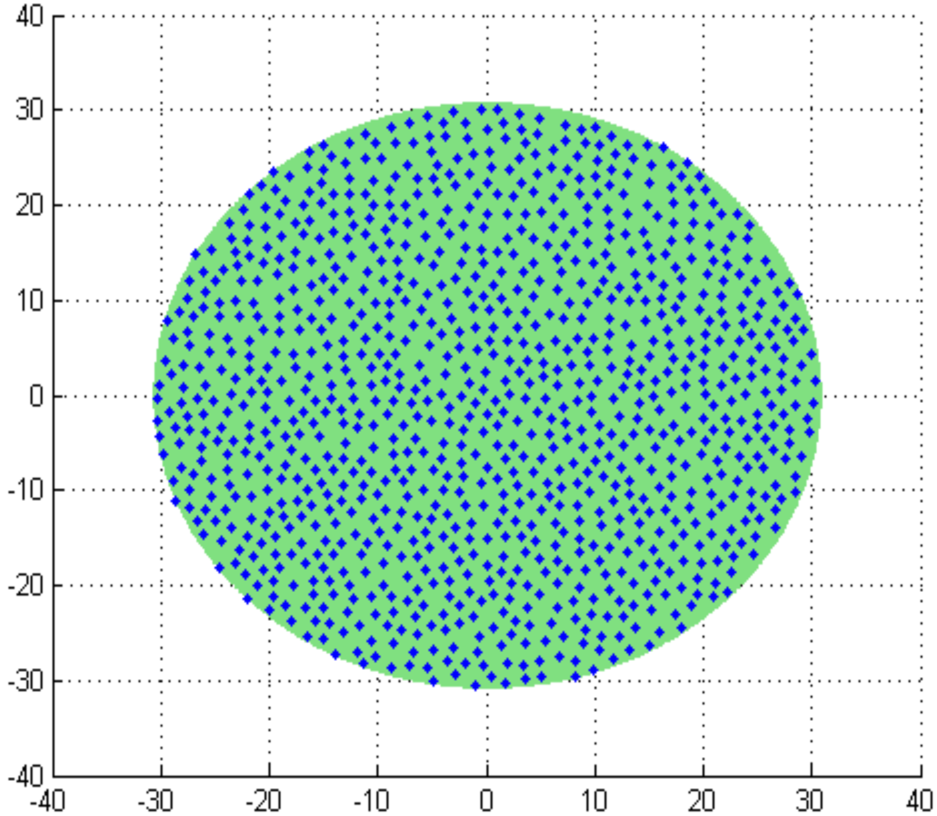
$$e_i(\theta,\phi) = 10\log_{10}\left(\frac{\left|\vec{E^h}_{full}(\theta,\phi) - \vec{E^h}_i(\theta,\phi)\right|^2 + \left|\vec{E^v}_{full}(\theta,\phi) - \vec{E^v}_i(\theta,\phi)\right|^2}{\max\left(\left|\vec{E^h}_{full}(\theta,\phi)\right|^2 + \left|\vec{E^v}_{full}(\theta,\phi)\right|^2\right)}\right)$$

1000 elements

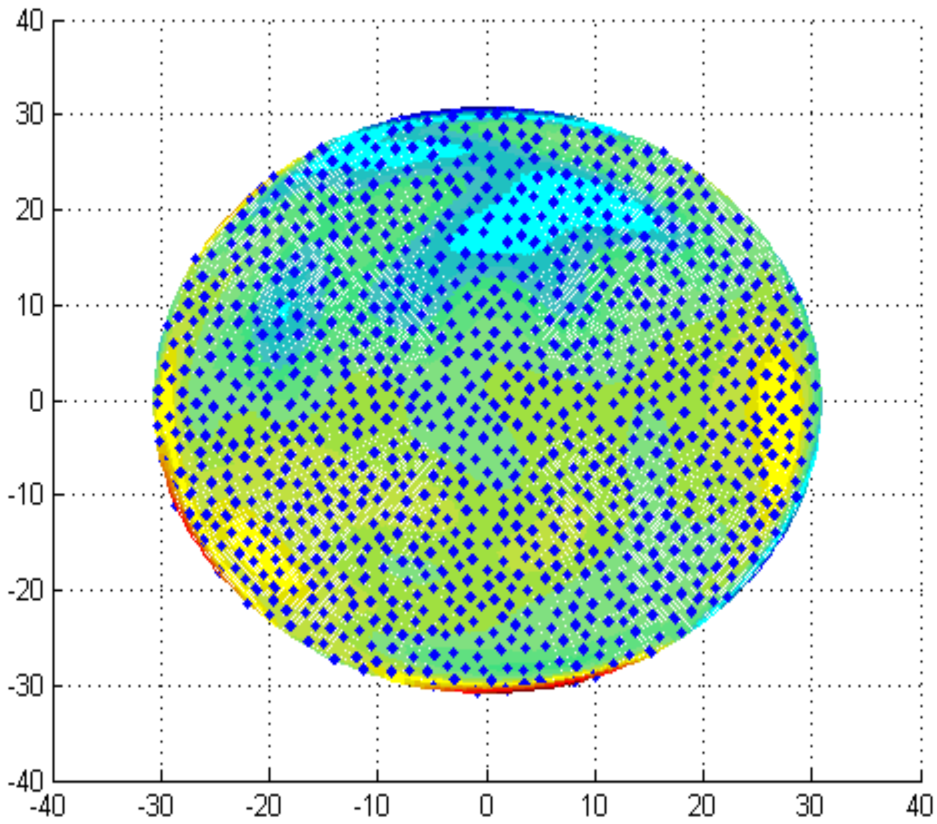


H-plane

Aperture sampling (1)

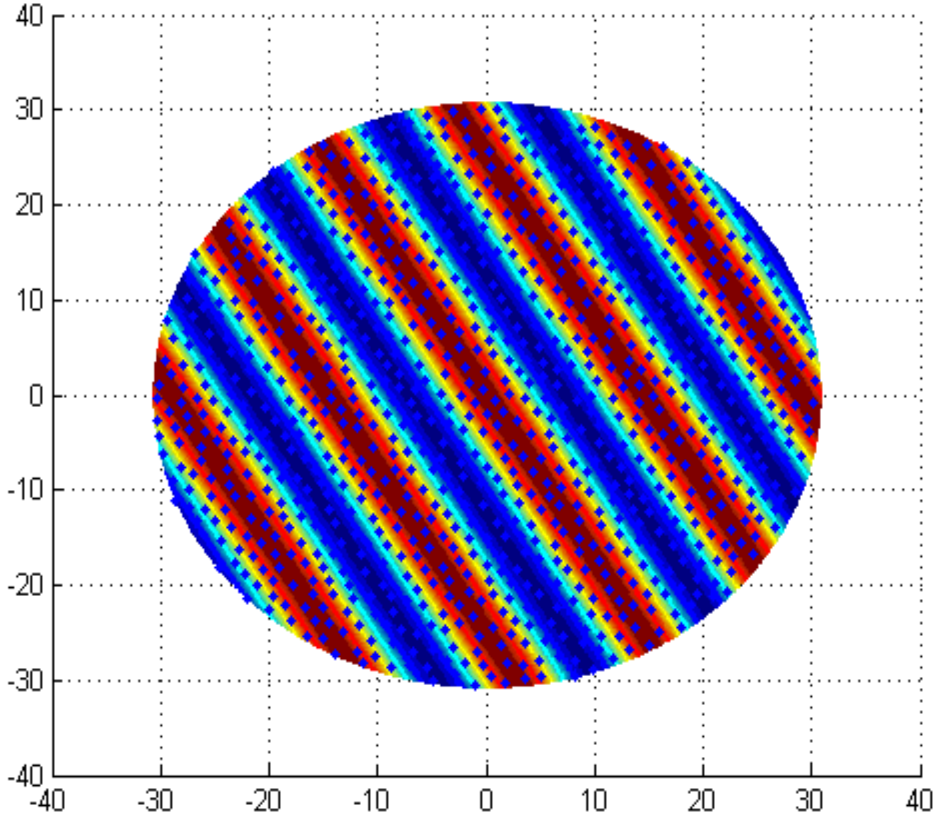


Aperture sampling (2)

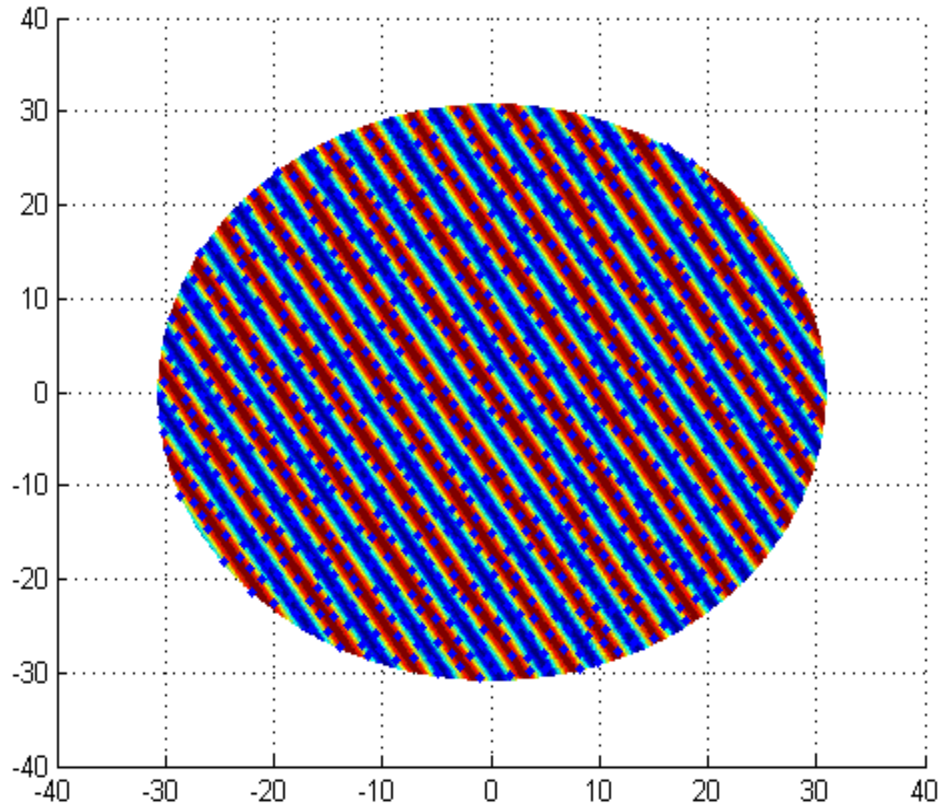


Define a local density
(several definitions possible)

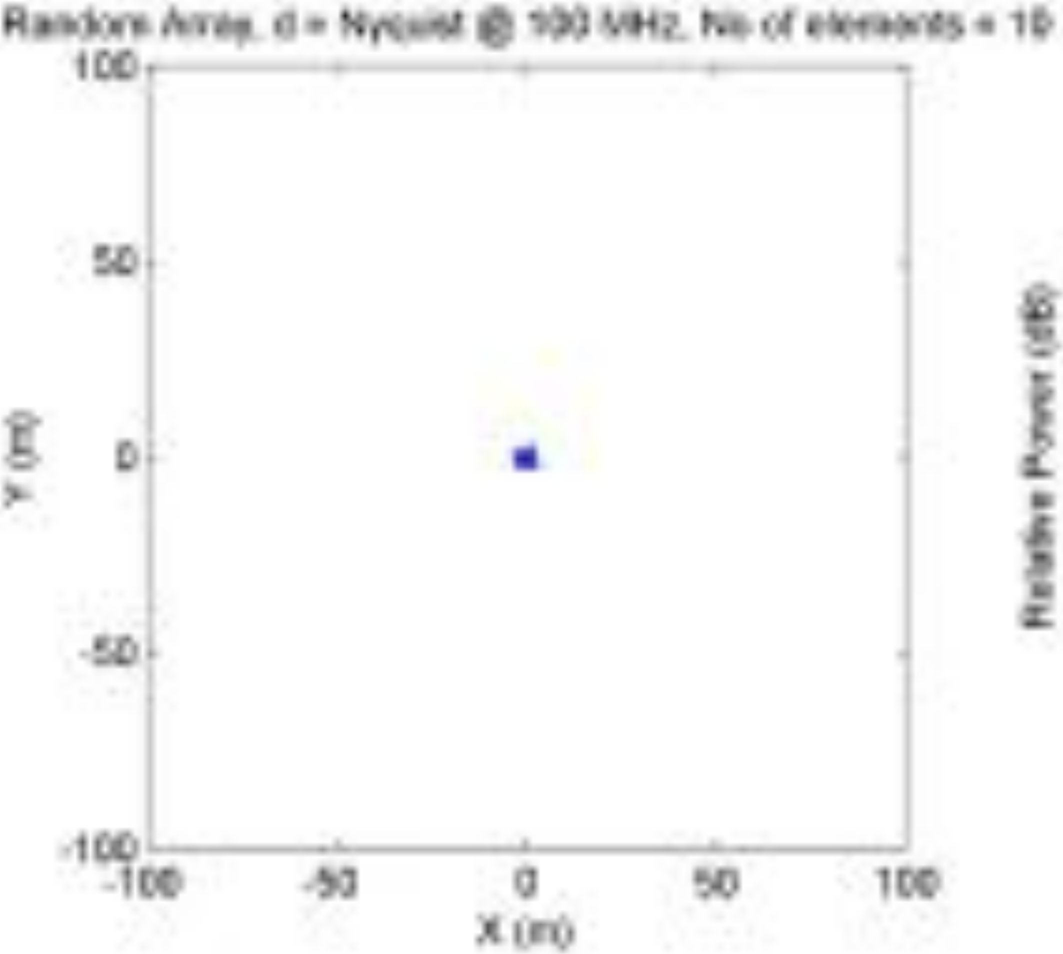
Aperture scanning (1)

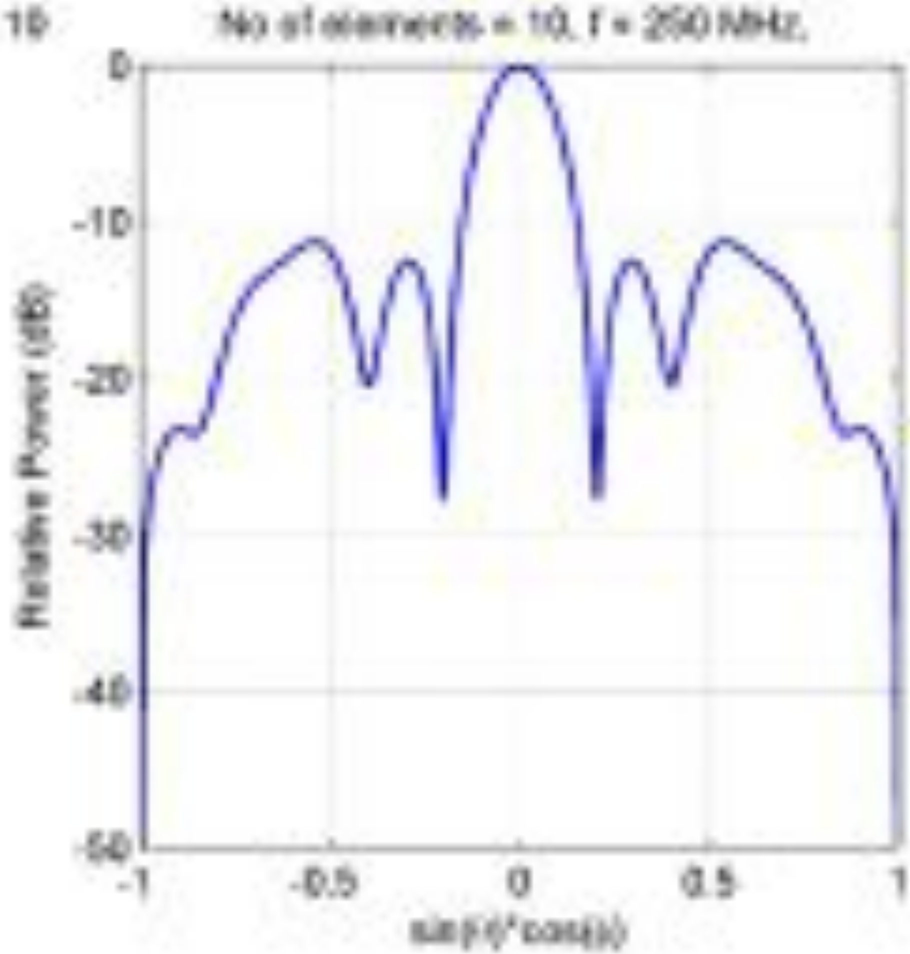


Aperture scanning (2)

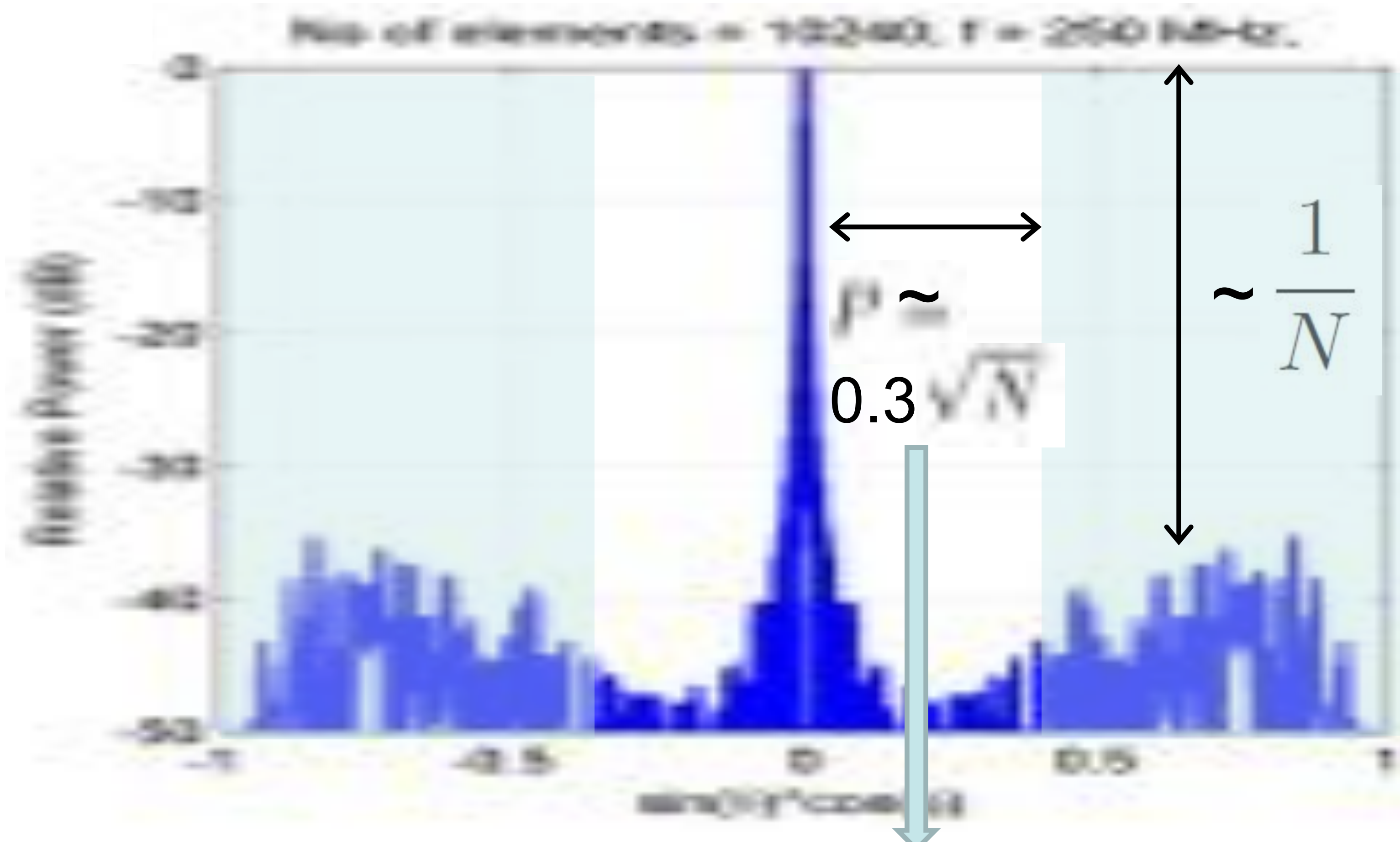


Patterns versus size of array



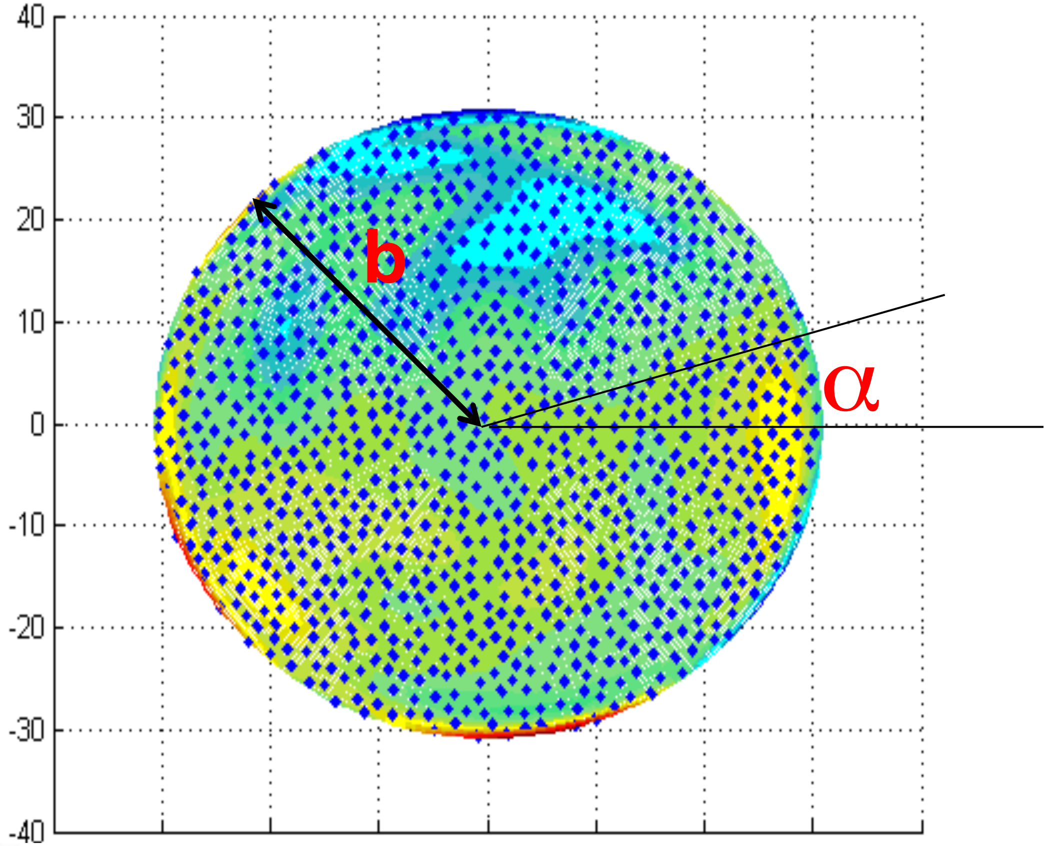


Coherent & incoherent regimes



Number of sidelobes in "coherent" regime

Aperture field representation



$$f(r,\alpha) \simeq \sum_{n=-N}^{N} a_n(r)\, e^{j\,n\,\alpha} \qquad a_n(r) = \frac{1}{2\pi}\int_0^{2\pi} f(r,\alpha)\, e^{-j\,n\,\alpha}\, d\alpha$$

Pattern representation

$$F(\theta,\phi) = \int_S f(r,\alpha)\, e^{j\,k\,(u_x\,x+u_y\,y)}\, dS$$
$$= \sum_{n=-N}^{N} 2\pi\, j^n\, e^{j\,n\,\phi} \int_0^b a_n(r)\, J_n(k\,r\,\sin\theta)\, r\,dr$$

Angle from broadside

Polynomial decomposition

$$F \simeq \sum_{n=-N}^{N} \sum_{p=0}^{P} C_{n,p} \left(\frac{b}{\lambda} \sin\theta \right)^{p} e^{j\,n\,\phi}$$

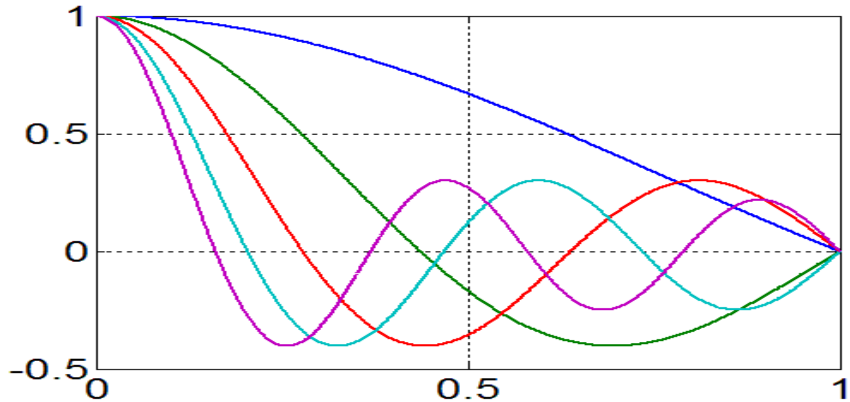
$$C_{n,p} = D_{n,p} \sum_{i=1}^{M} A_i \; (2\pi\, r_i/b)^p \; e^{-j\,n\,\alpha_i}$$

$$D_{n,p} = (-1)^s \frac{(-1/2)^p}{m!\,(m+|n|)!}\, j^n$$

Fourier-Bessel decomposition

$$F(\theta,\phi) = \sum_{n=-N}^{N} \sum_{m=1}^{M} 2j^{n} \frac{\lambda_n^m}{J_{n+1}(\lambda_n^m)} A_{mn}\, e^{j\,n\,\phi} \frac{J_n(k\,b\,\sin\theta)}{(\lambda_n^m)^2 - (k\,b\,\sin\theta)^2}$$

$$A_{mn} = \sum A_i\; J_n(\lambda_n^m\, r_i/b)\; e^{-j\,n\,\alpha_i}$$



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Fourier-Bessel decomposition

$$F(\theta,\phi) = \sum_{n=-N}^{N} \sum_{m=1}^{M} 2j^n \frac{\lambda_n^m}{J_{n+1}(\lambda_n^m)} A_{mn}\, e^{j\,n\,\phi} \frac{J_n(k\,b\,\sin\theta)}{(\lambda_n^m)^2 - (k\,b\,\sin\theta)^2}$$

$$A_{mn} = \sum_i A_i\; J_n(\lambda_n^m\, r_i/b)\; e^{-j\,n\,\alpha_i}$$

A. Aghasi, H. Amindavar, E.L. Miller and J. Rashed-Mohassel, "Flat-top footprint pattern synthesis through the design of arbitrarily planar-shaped apertures," IEEE Trans. Antennas Propagat., Vol. 58, no.8, pp. 2539-2551, Aug. 2010.

Zernike-Bessel decomposition

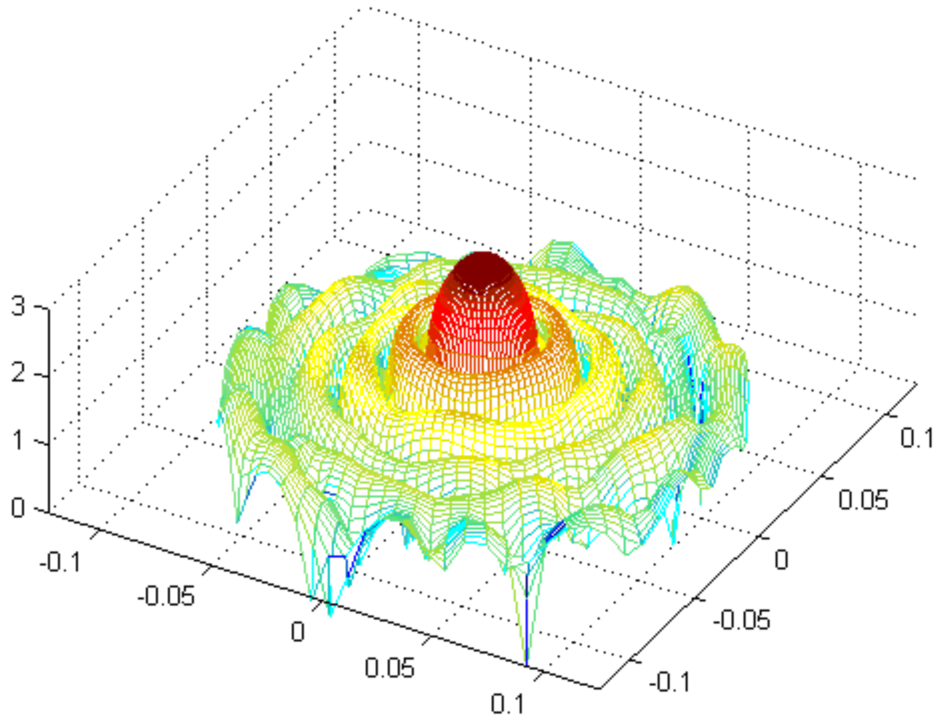
$$F(\theta,\phi) = 2 \sum_{n=-N}^{N} \sum_{m=0}^{M} (|n|+2m+1)\, B_{mn}\, j^{n}\, e^{j\,n\,\phi} (-1)^{s} \frac{J_{|n|+2m+1}(k\,b\,\sin\theta)}{k\,b\,\sin\theta}$$

$$B_{mn} = \sum_{i} A_i\, F_m^{|n|}(r/b)\, e^{-j\,n\,\alpha_i}$$

Y. Rahmat-Samii and V. Galindo-Israel, “Shaped reflector antenna analysis using the Jacobi-Bessel series,” IEEE Trans. Antennas Propagat., Vol. 28, no.4, pp. 425-435, Jul. 1980.

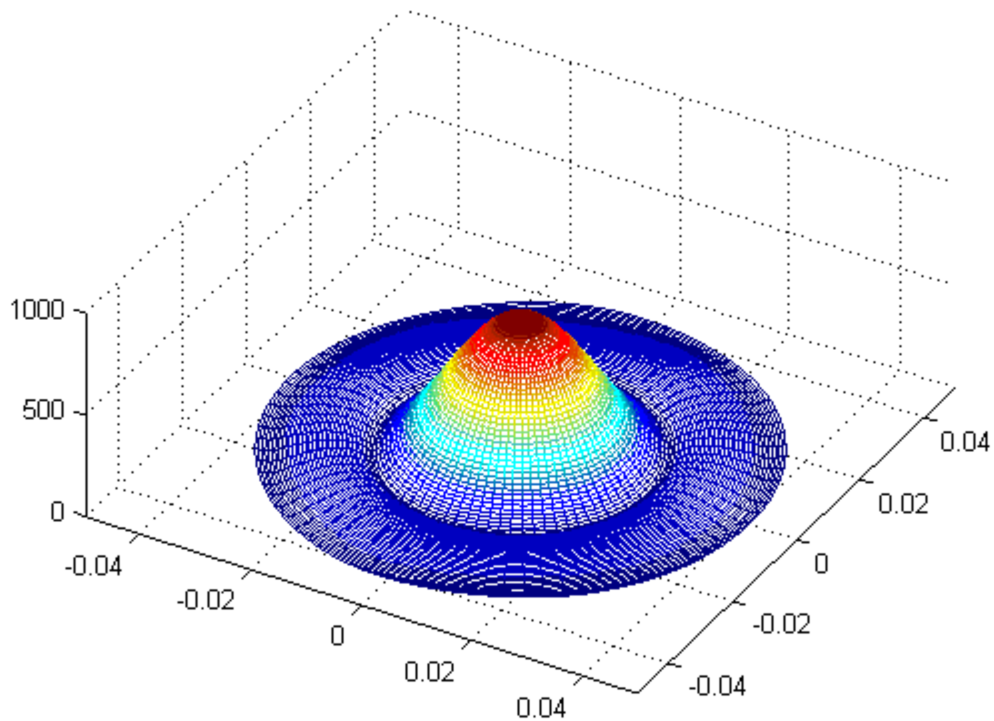
Polynomial	Fourier-Bessel	Zernike-Bessel
Fast functions	Good 1st order	Good 1st order
weaker at low orders	weak direct convergence	fast direct convergence

Array factor



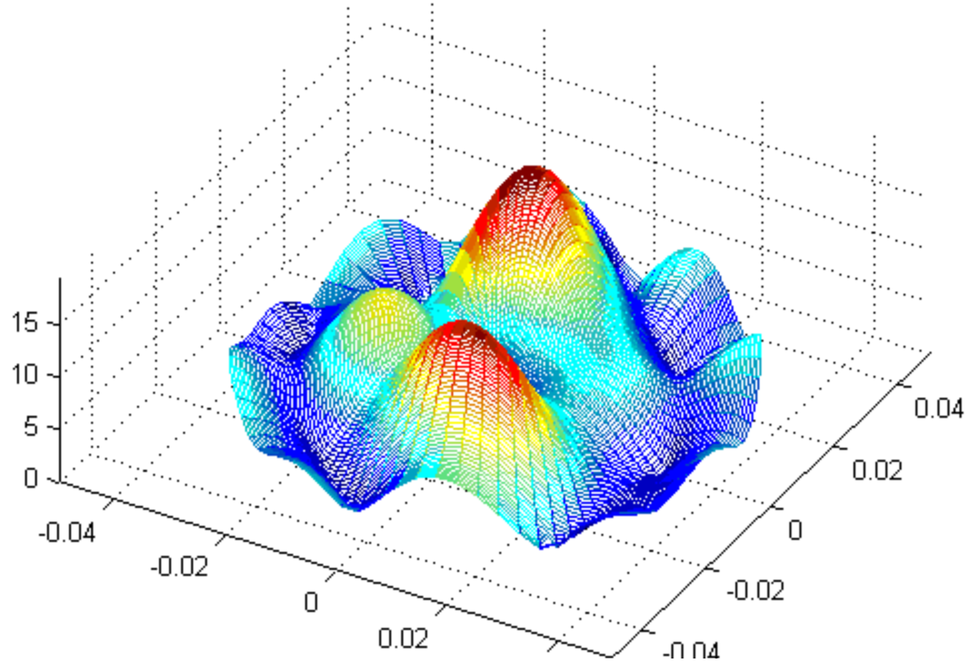
with apodization

Array factor



with apodization

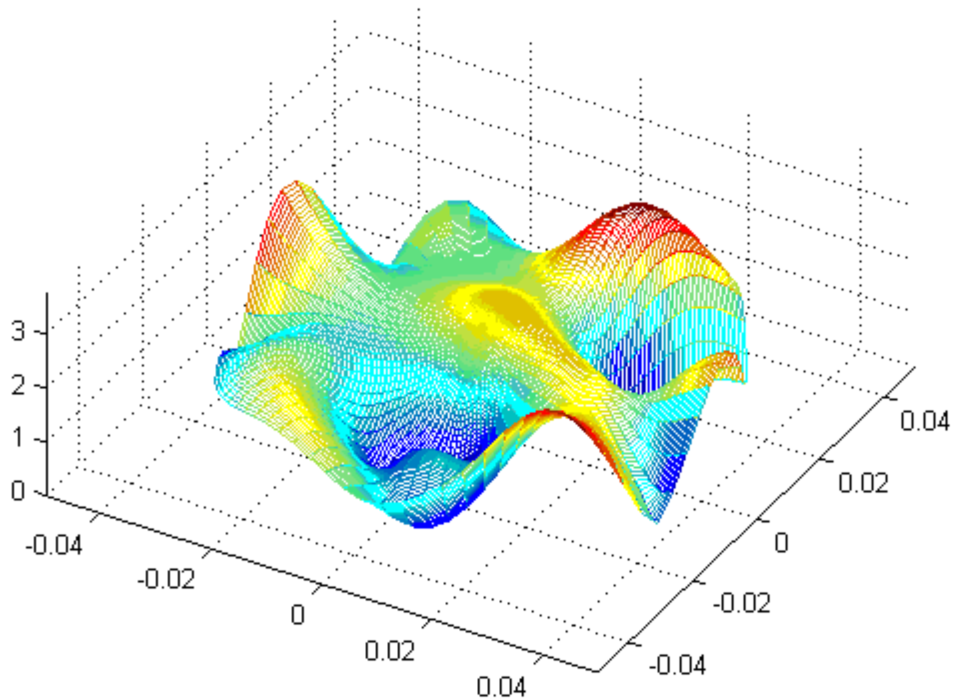
Apodization function *w(r)* extracted



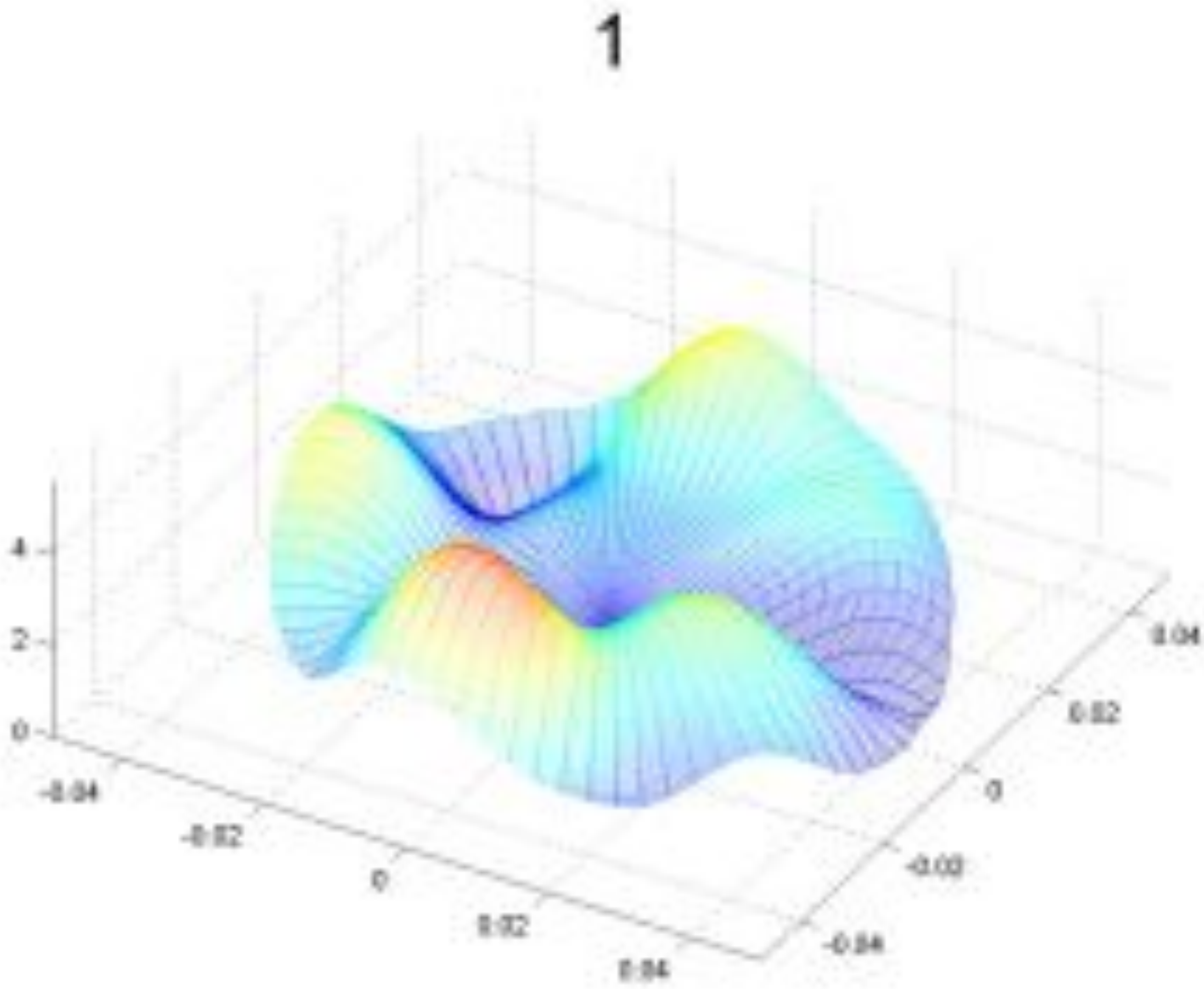
$$F_0(\theta, \alpha) = 2\pi\, q \int_0^b w(r)\, J_0(k\, r\, \sin\theta)\, r\, dr$$

$$q = N/(\pi\, b^2)$$

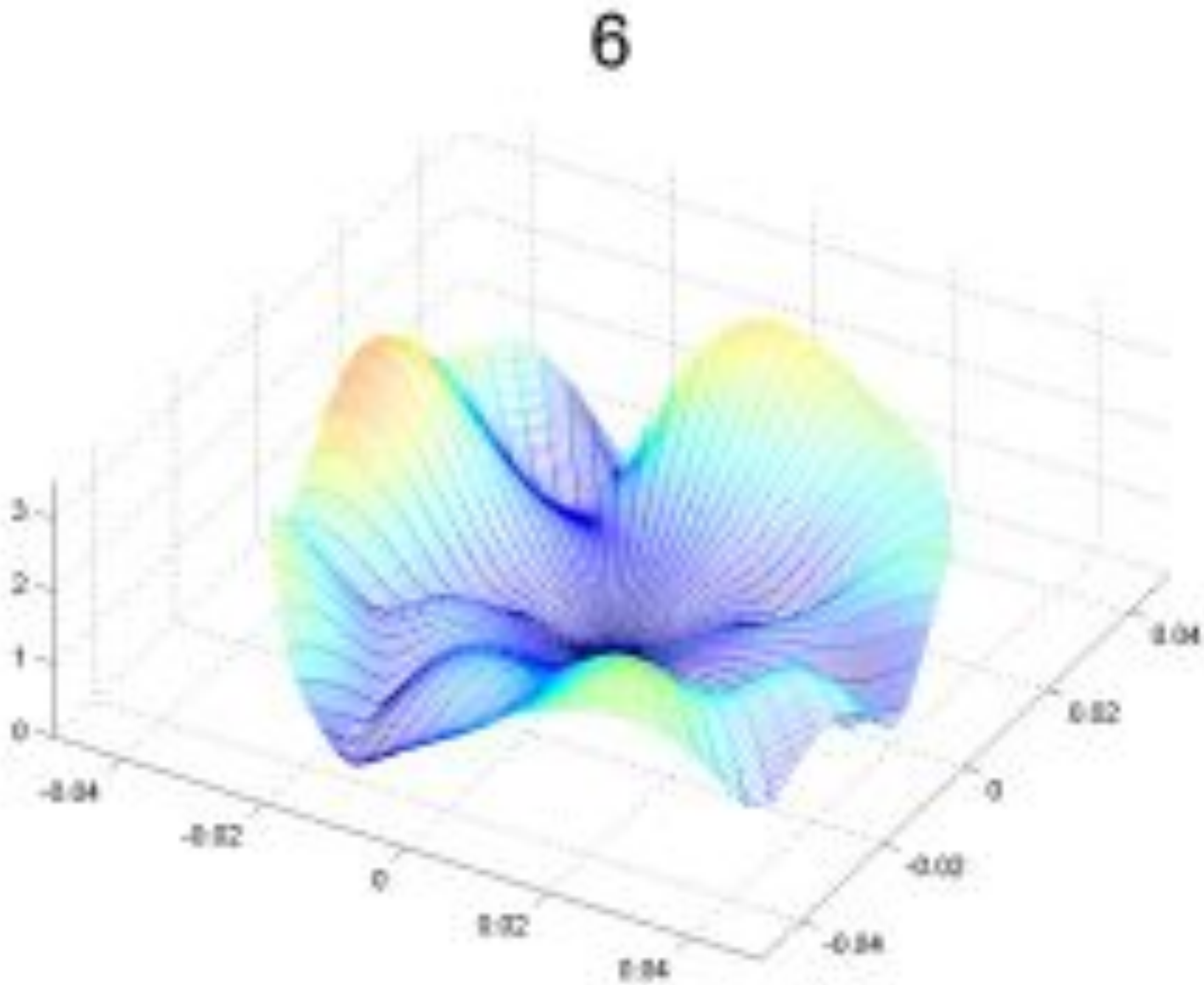
Approximate array factor extracted



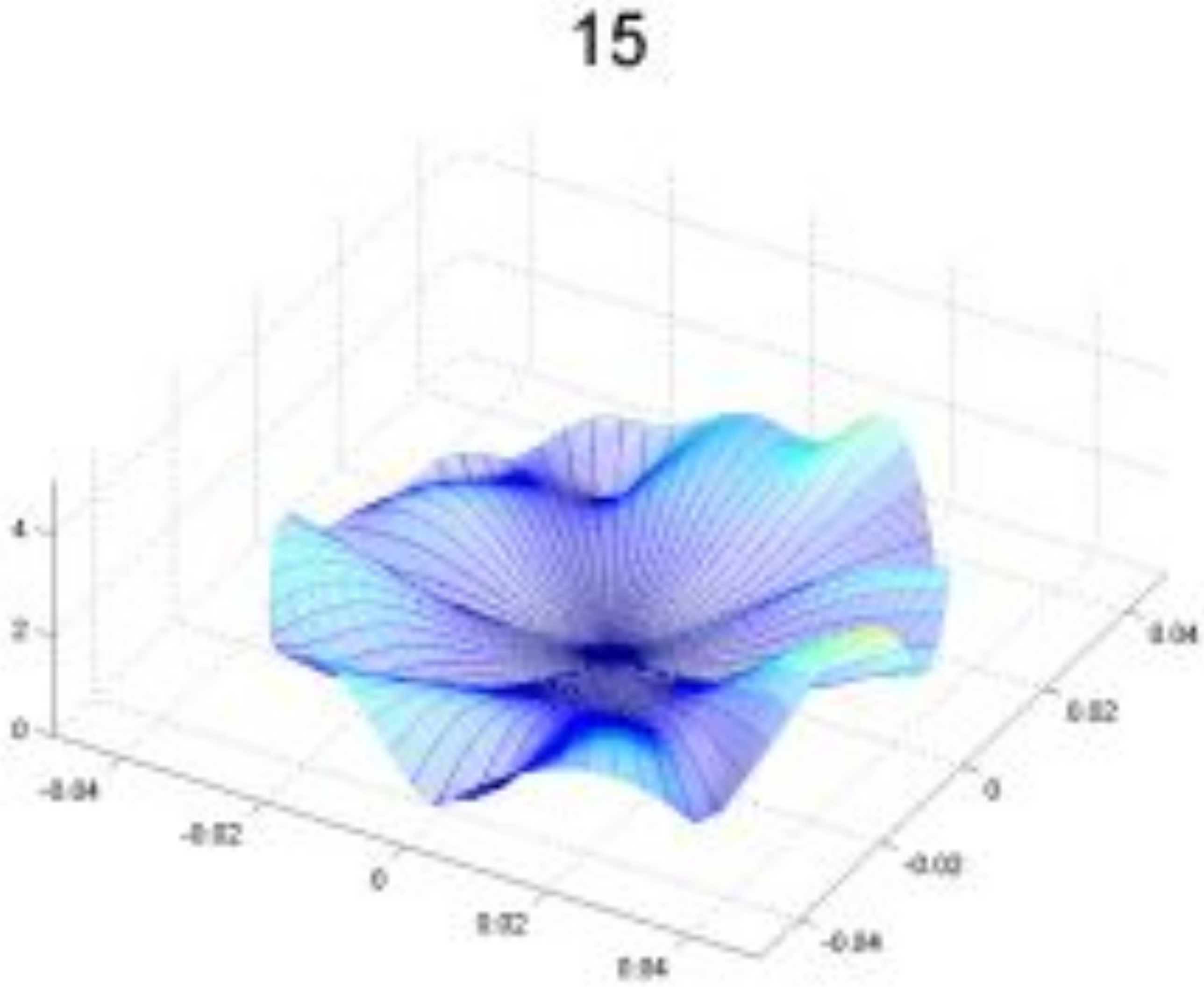
20 % error on amplitudes
λ/4 error on positions (at 300 MHz)



Residual error with Z-B approach



Residual error with Z-B approach



Residual error with Z-B approach

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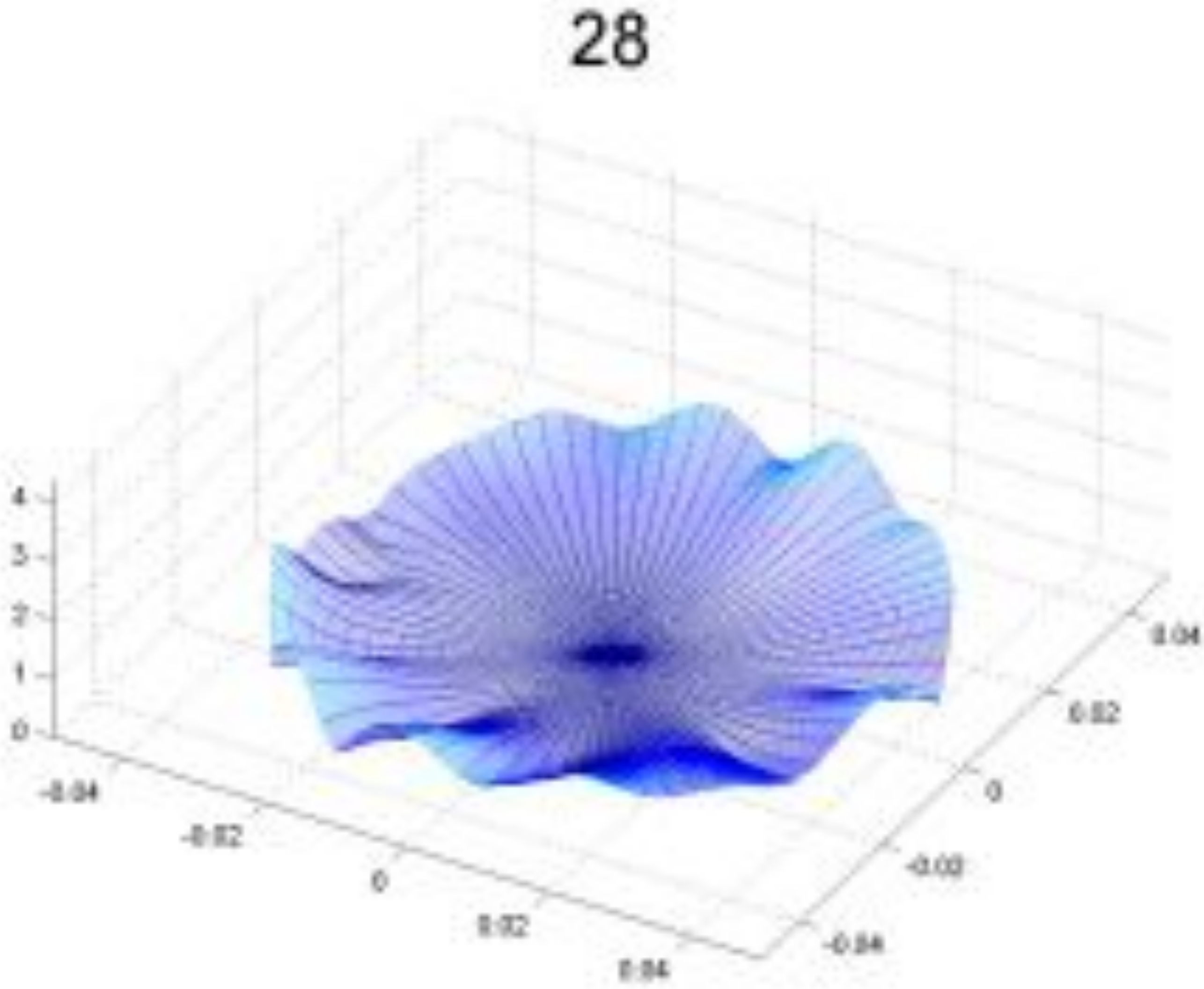
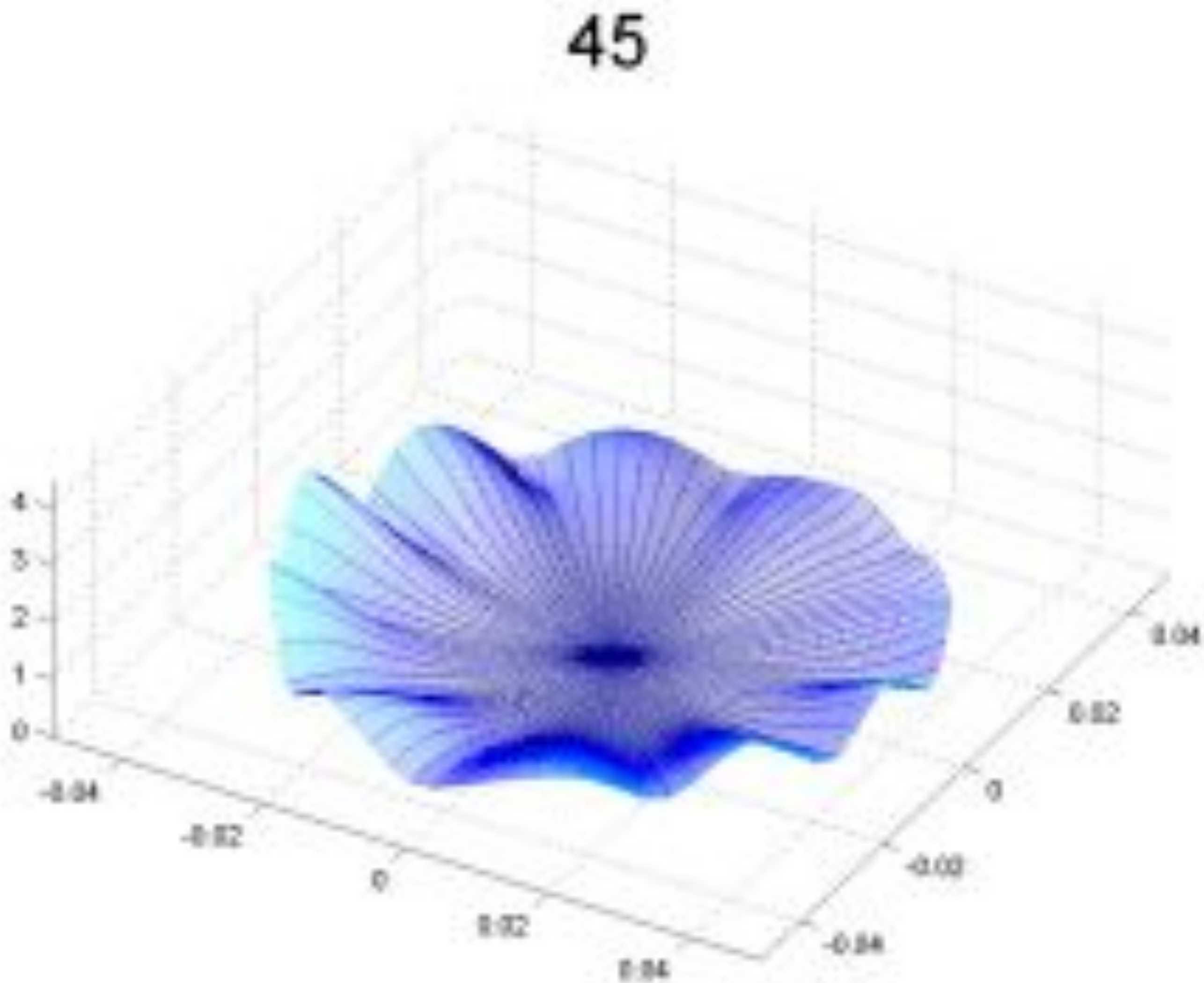
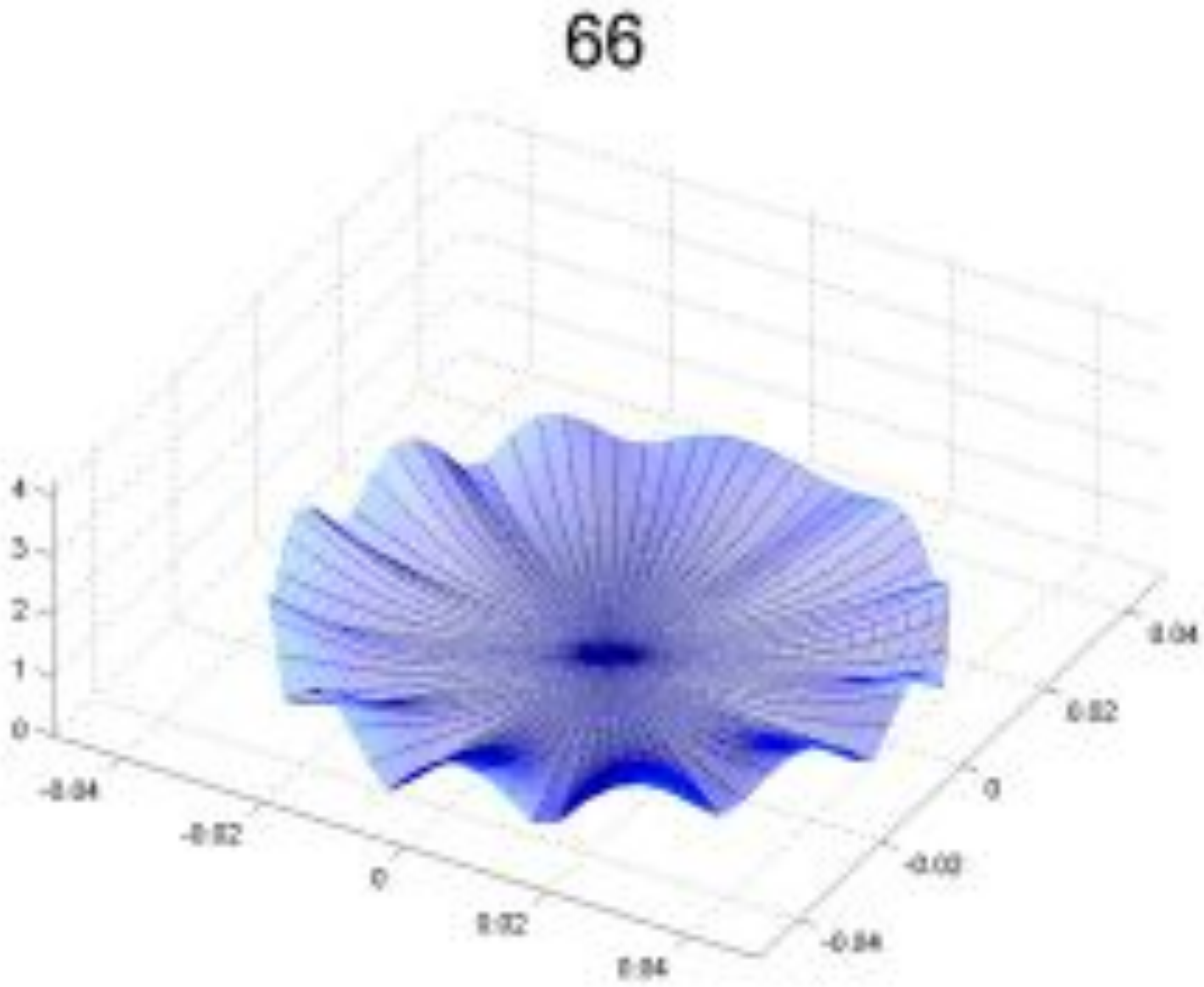


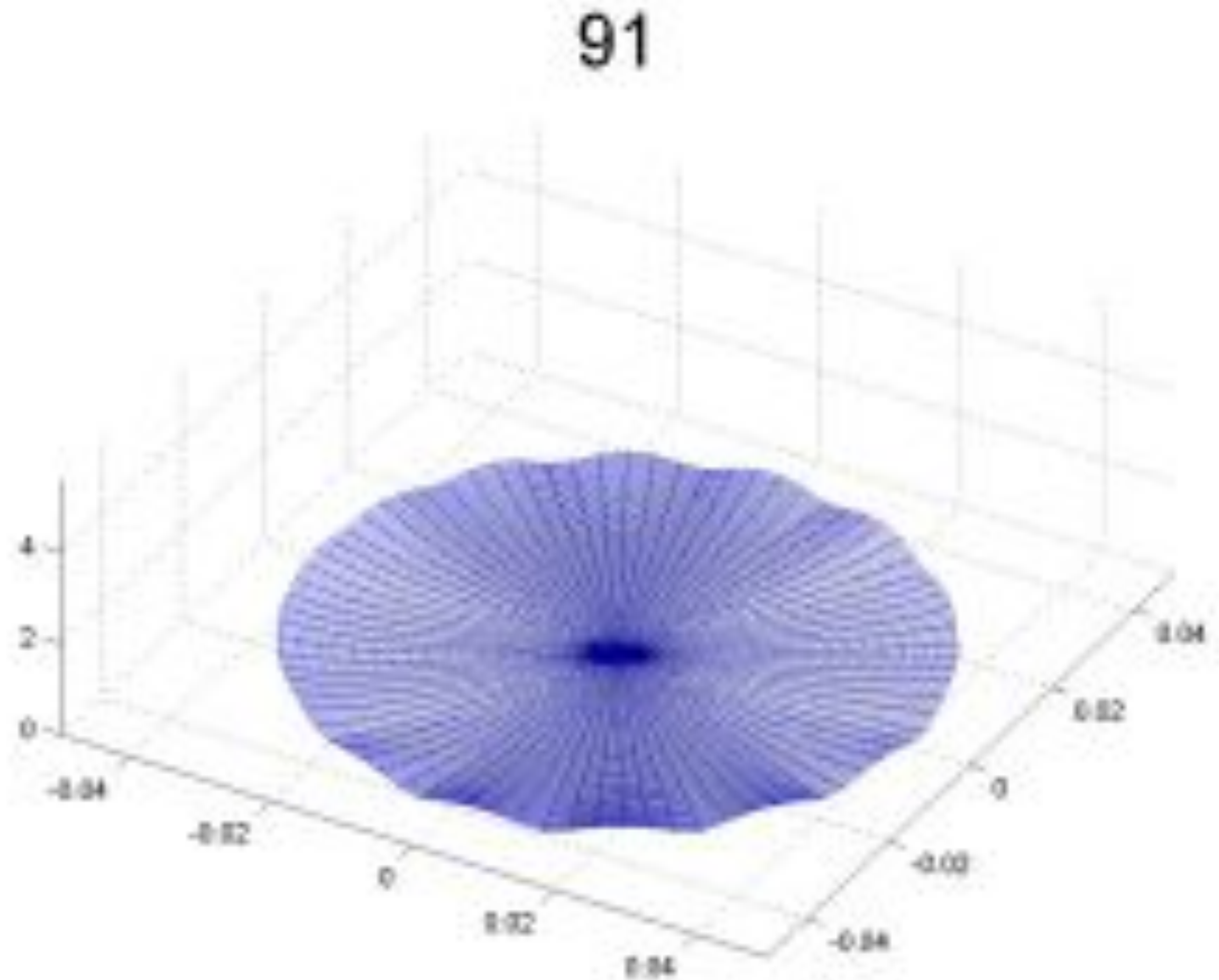
Residual error with Z-B approach



Residual error with Z-B approach

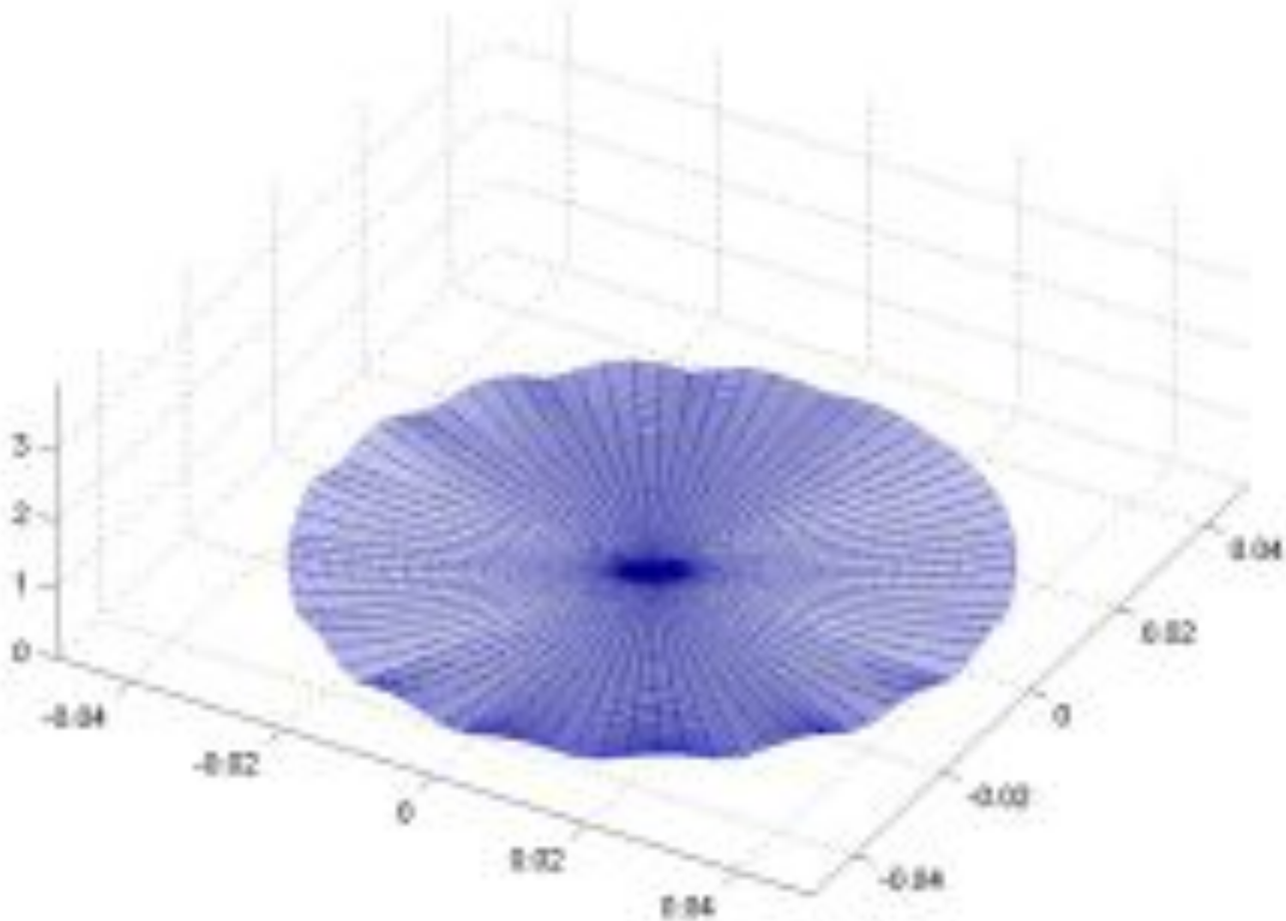


Residual error with Z-B approach

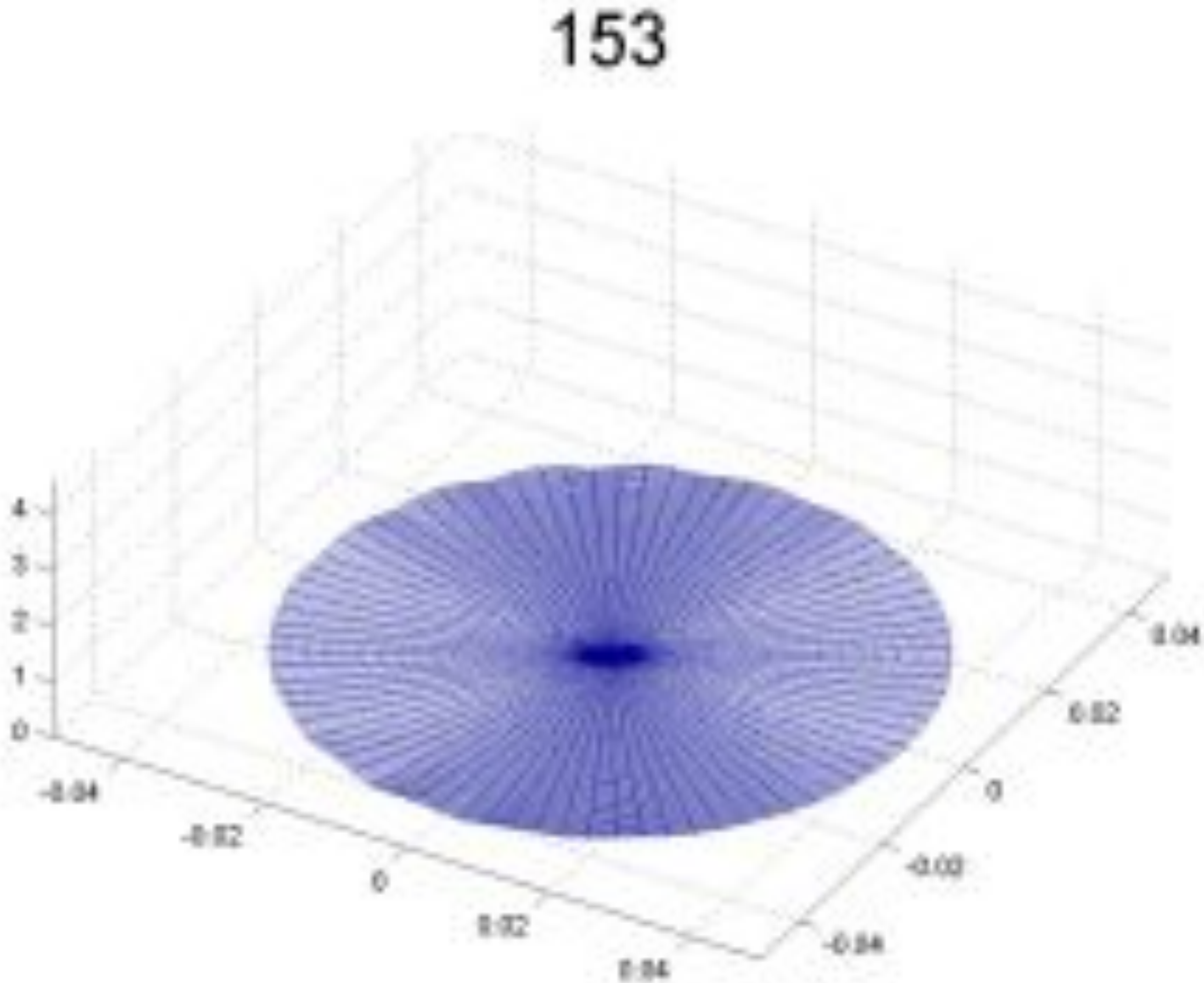


Residual error with Z-B approach

120

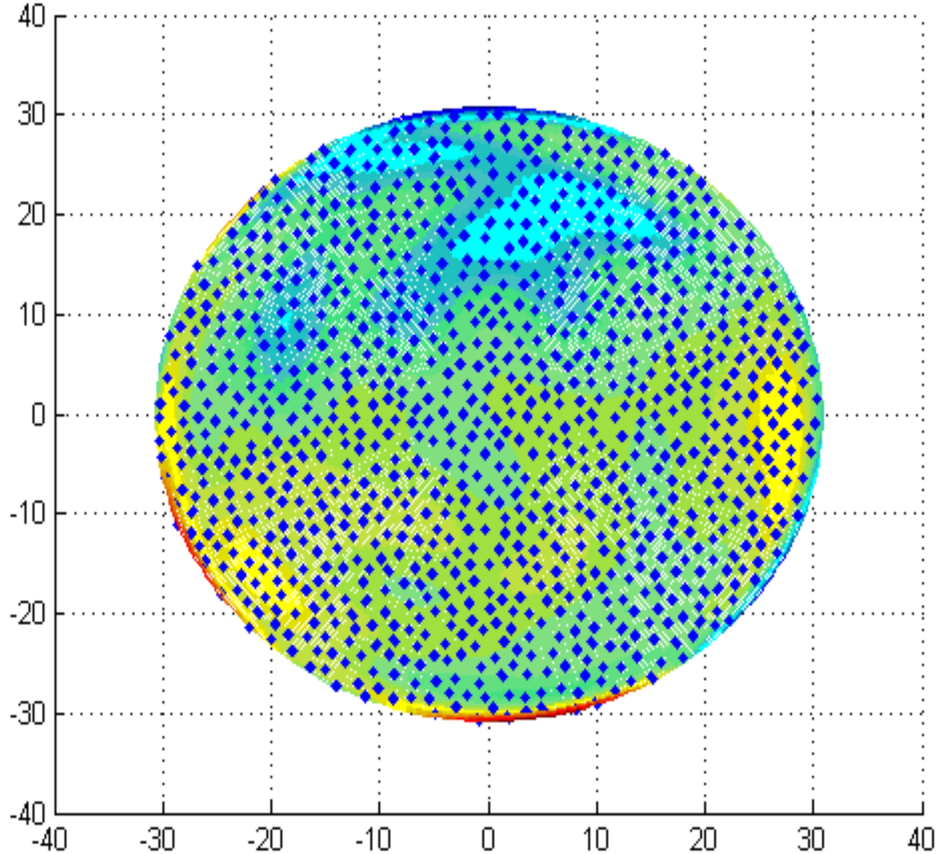


Residual error with Z-B approach



Residual error with Z-B approach

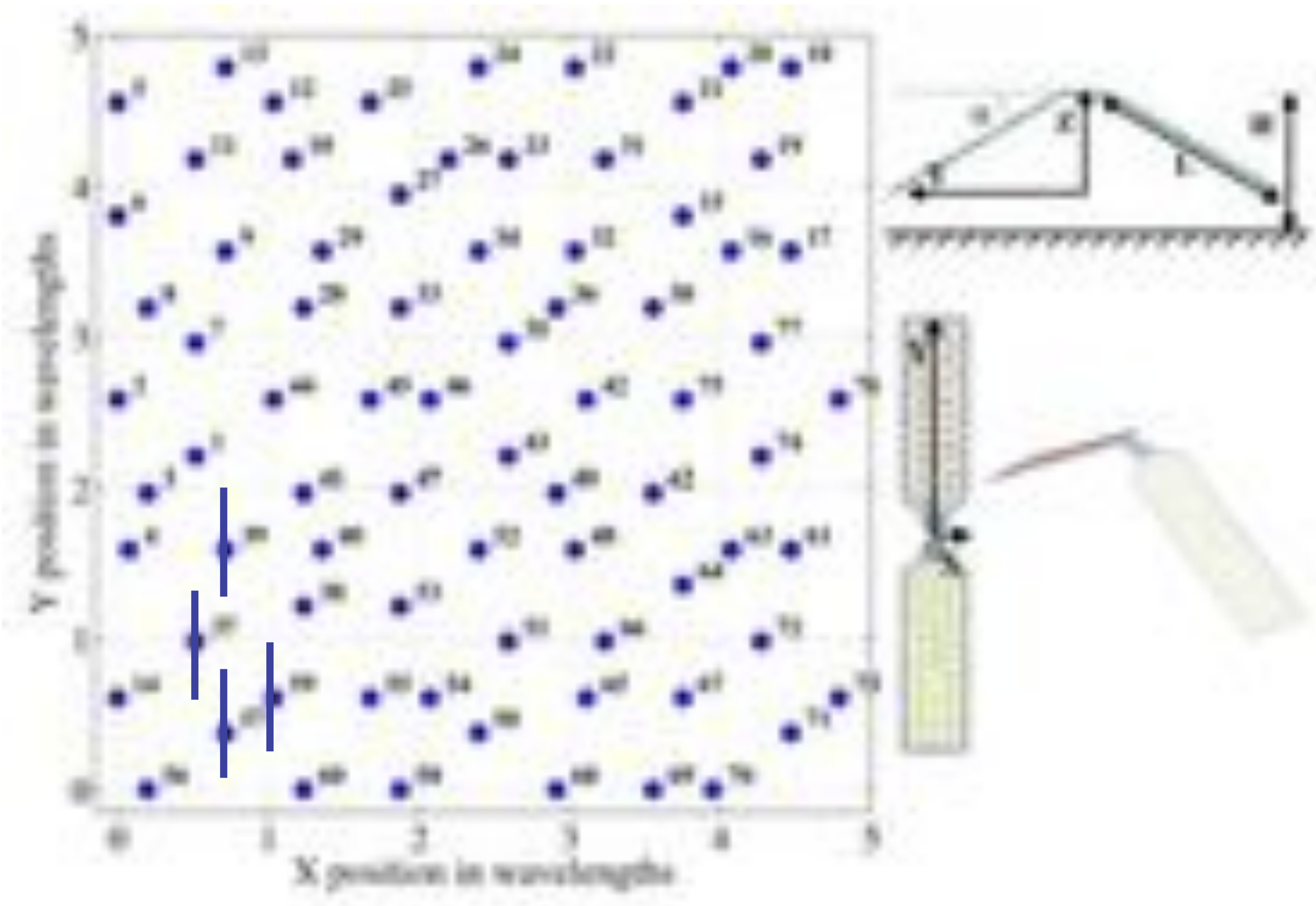
Density function



The number of terms tells the "resolution" with which density is observed

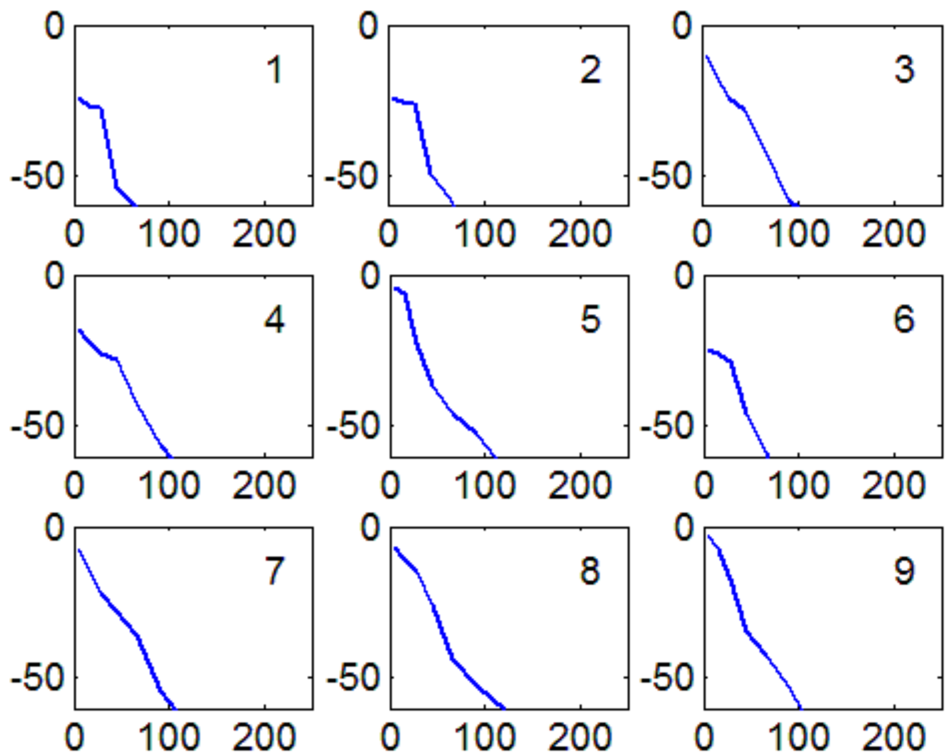
CALIM 2011

Array of wideband dipoles



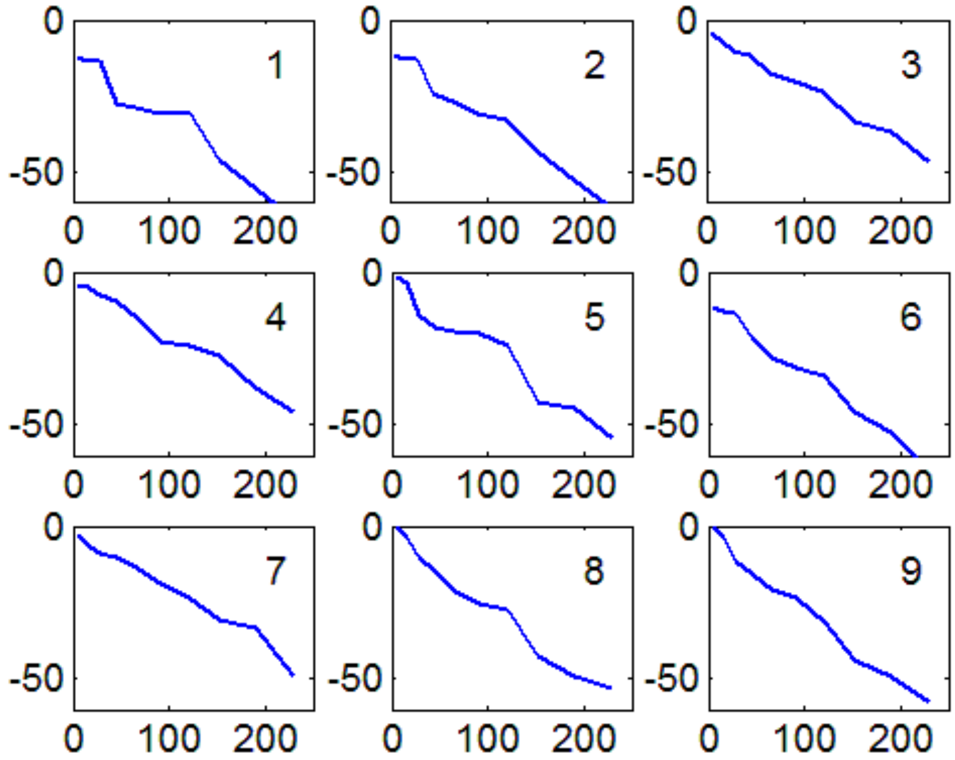
Algarve meeting, 2011

AF convergence



Over just main beam

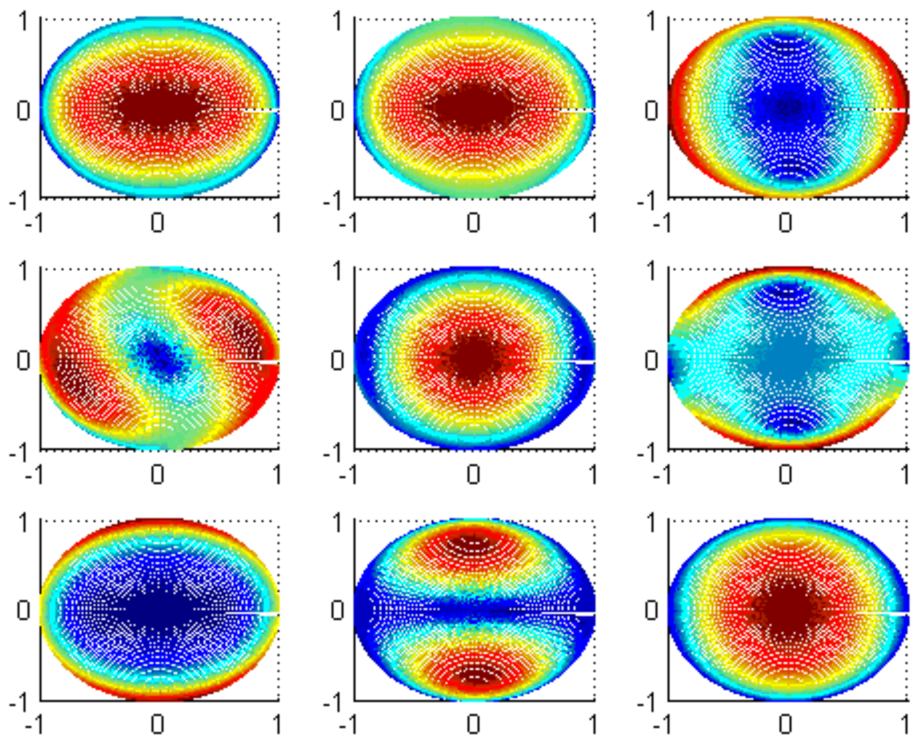
AF convergence



Main beam + 1st sidelobe

Algarve meeting, 2011

AF convergence



Project on 1, 2, 3 MBF patterns at most

Pattern projections

$$\vec{F}_p^\circ = \alpha\ \vec{F}_0^\circ + \vec{F}_{p,res.}^\circ$$

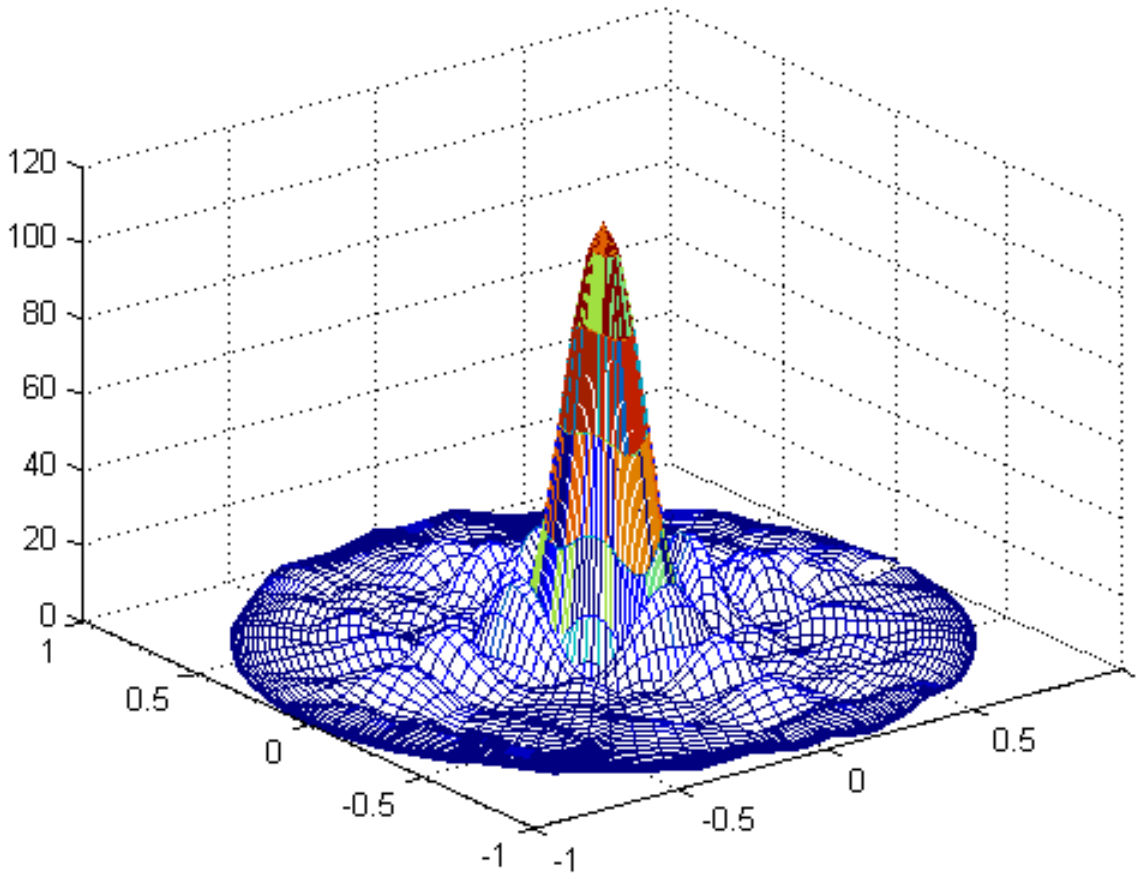
$$\alpha = \frac{\left\langle \vec{F}_p^\circ, \vec{F}_0^\circ \right\rangle}{\left\langle \vec{F}_0^\circ, \vec{F}_0^\circ \right\rangle}$$

$\longrightarrow$ Power radiated by MBF pattern 0

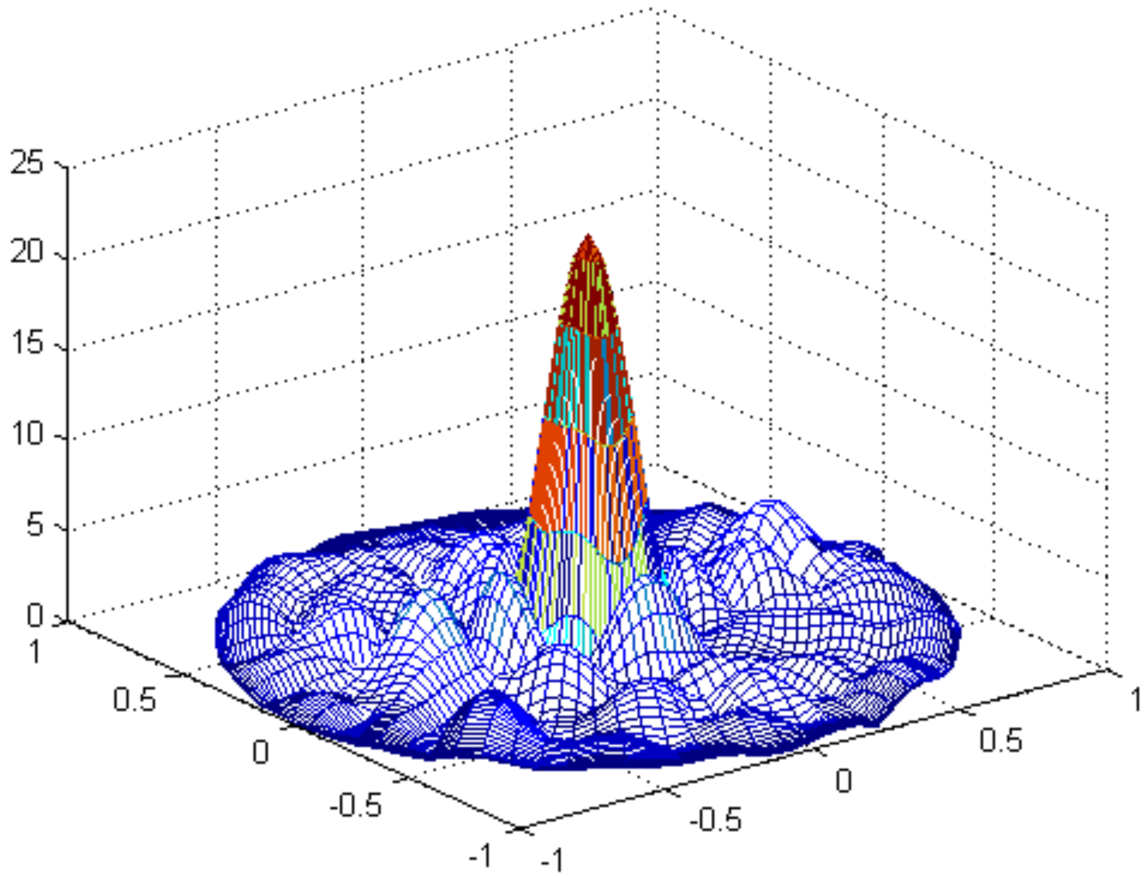
$$C_0\,\vec{F}_0^\circ + C_p\,\vec{F}_p^\circ \longrightarrow \simeq\ (C_0 + \alpha\,C_p)\,\vec{F}_0^\circ$$

NB: can be projected on more than 1 (orthogonal) MBFs

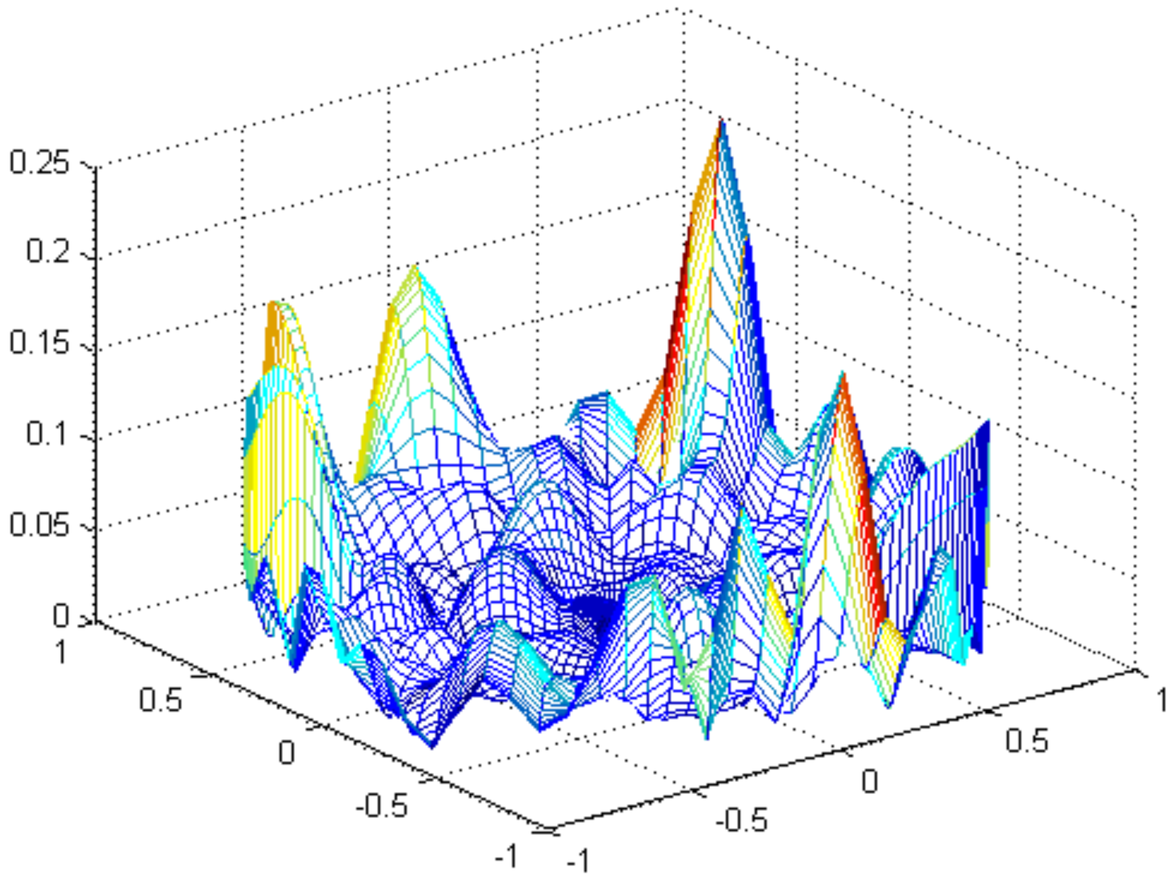
Full pattern



Error w/o MC

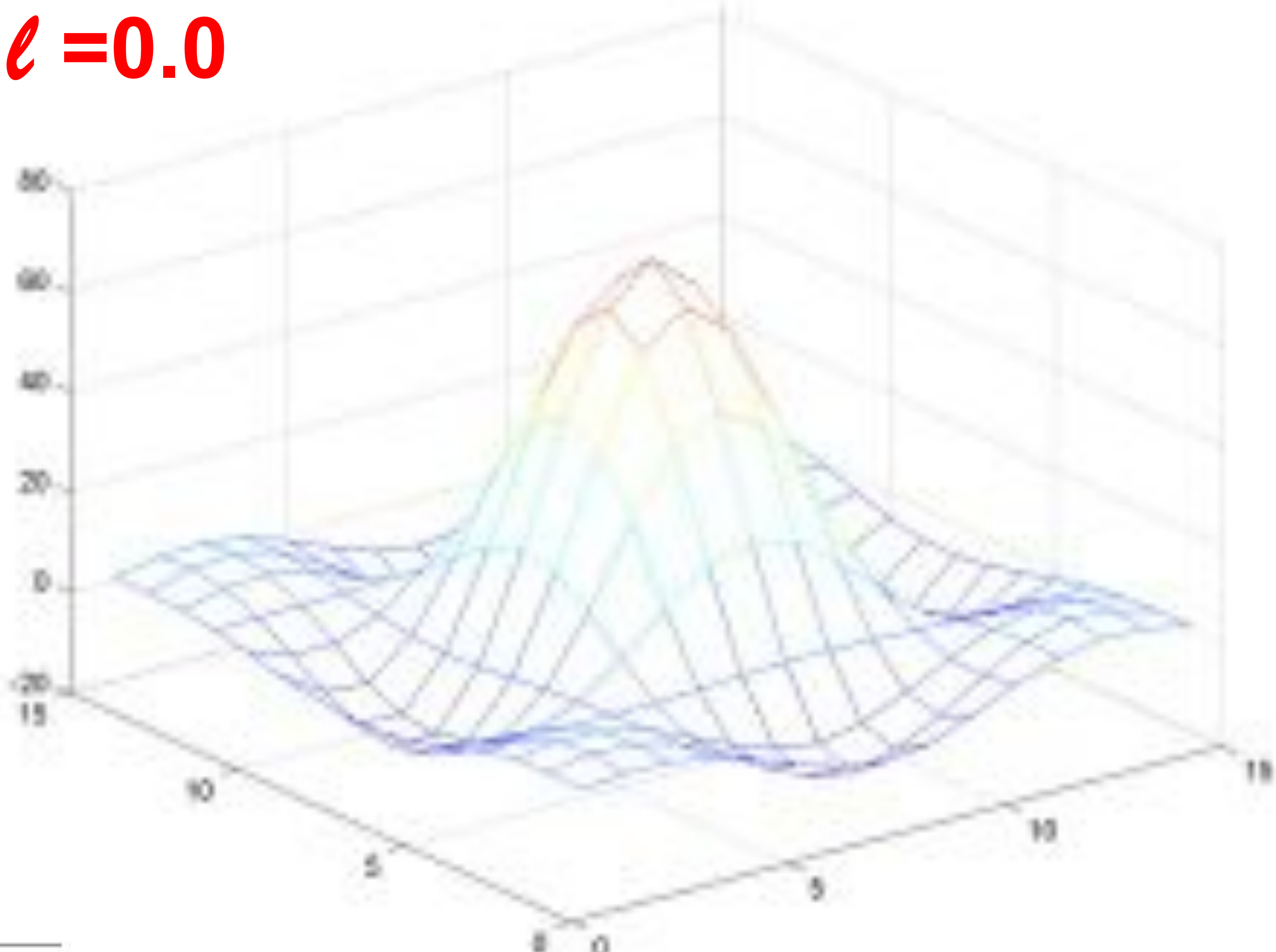


Error after pattern projection



Only 1 pattern used here (pattern of “primary”)

ℓ =0.0



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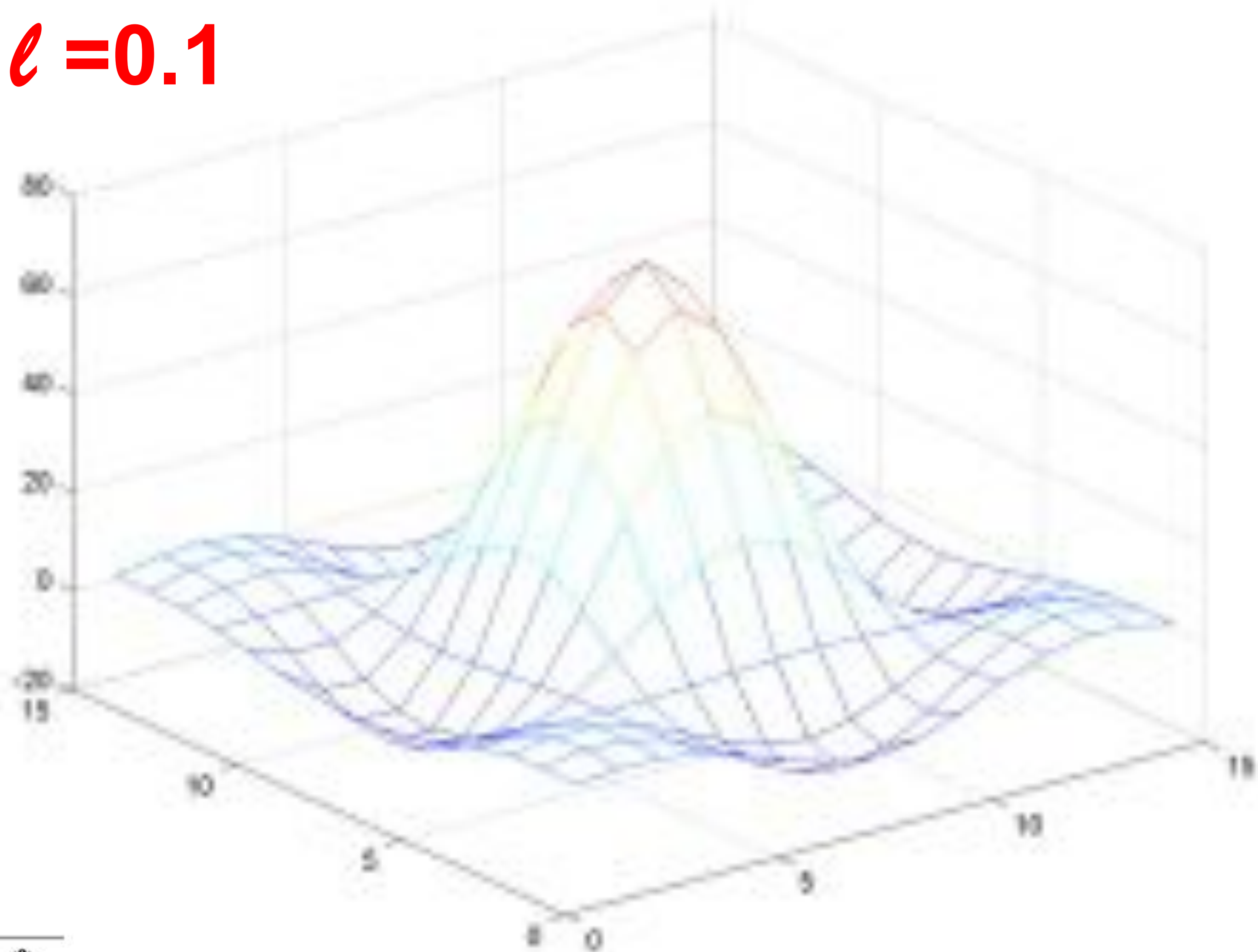
ℓ =0.1



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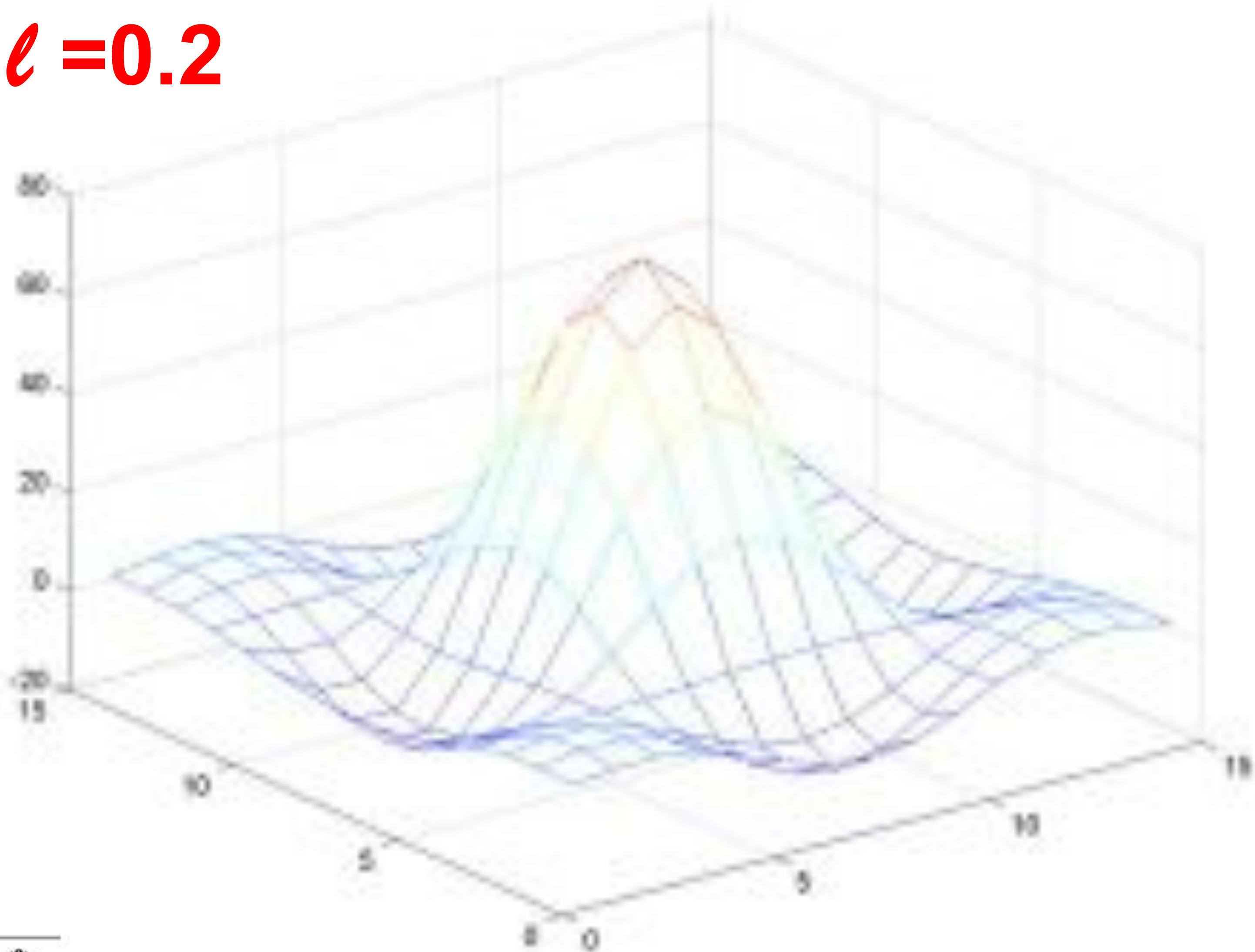
ℓ =0.2



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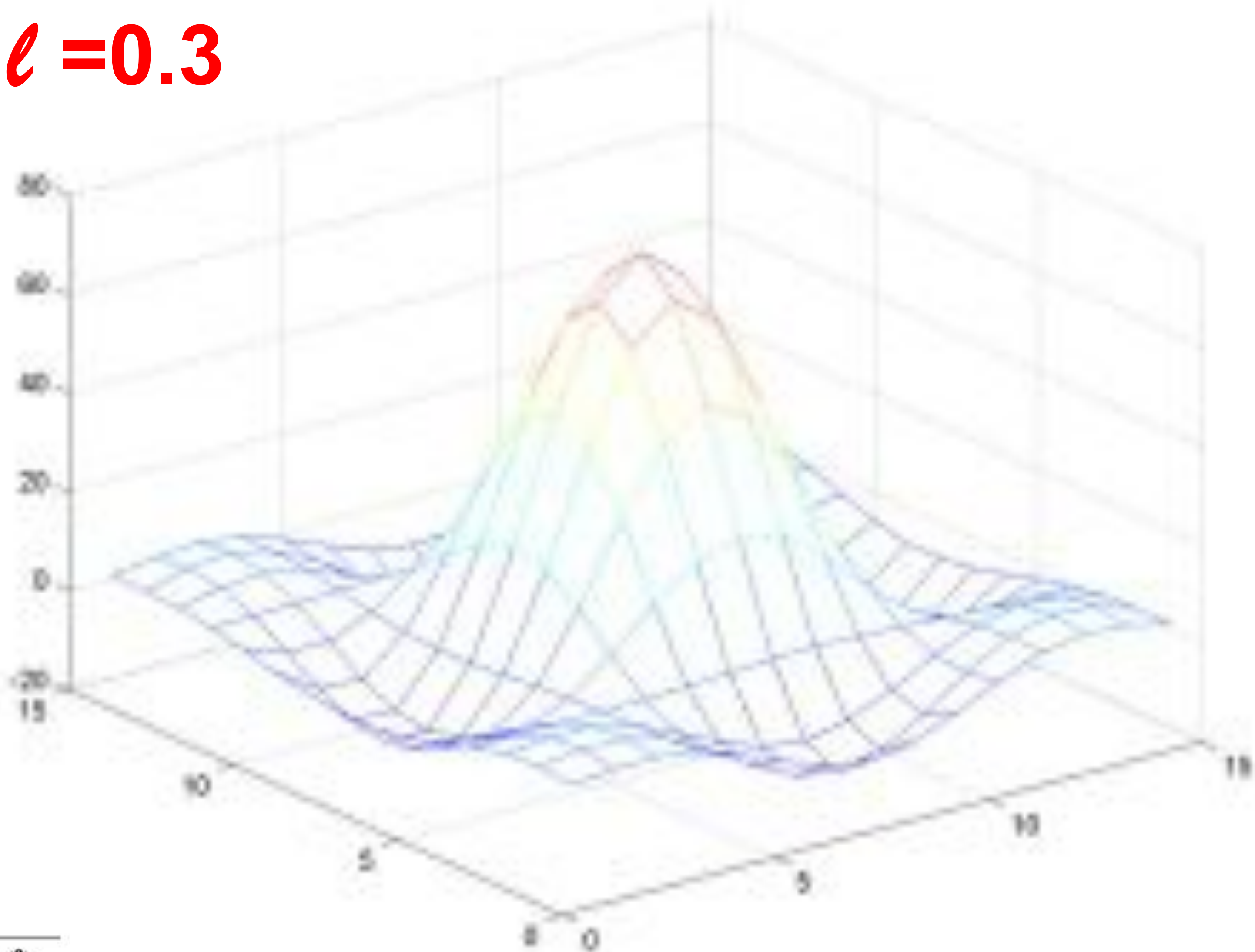
ℓ =0.3



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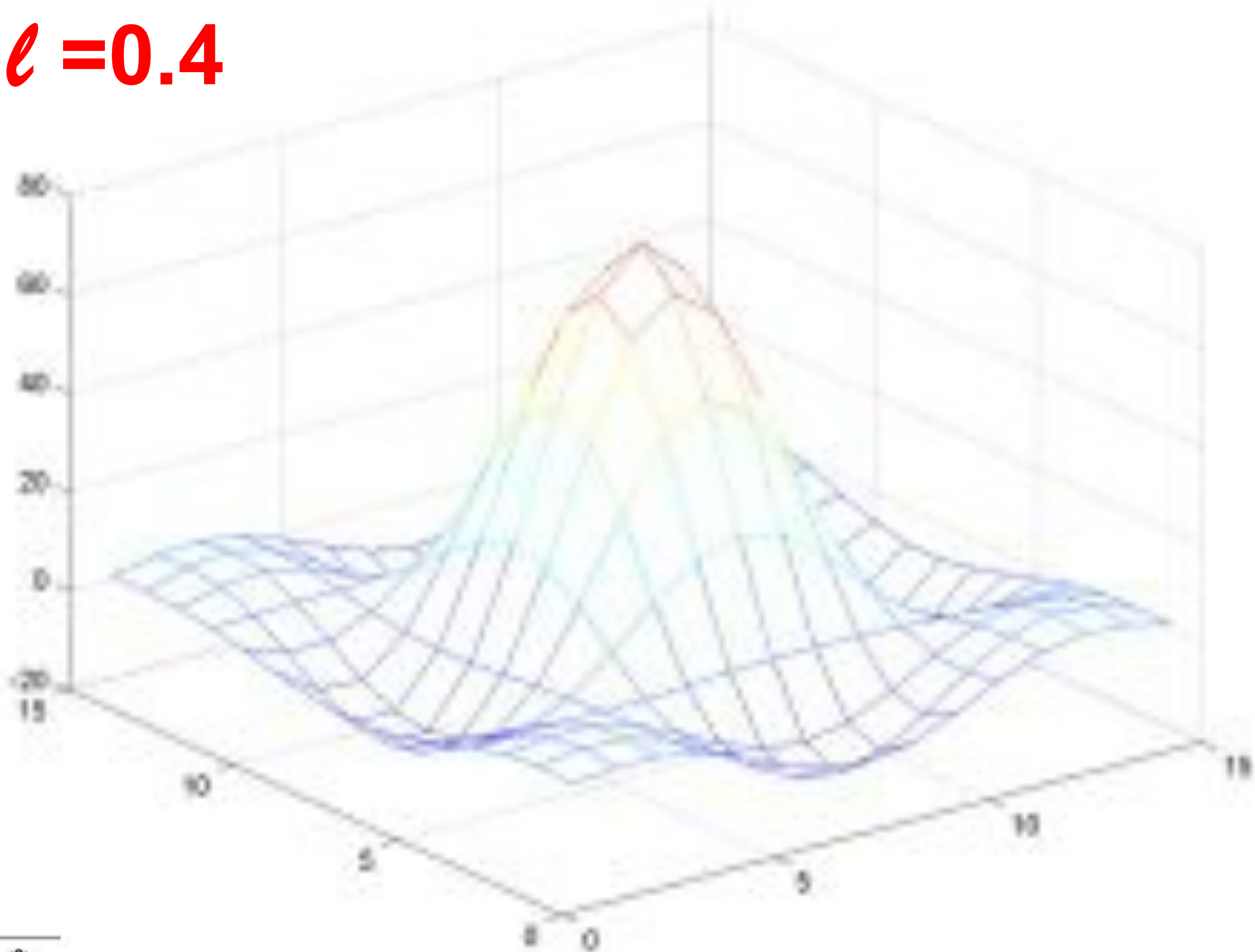
ℓ =0.4



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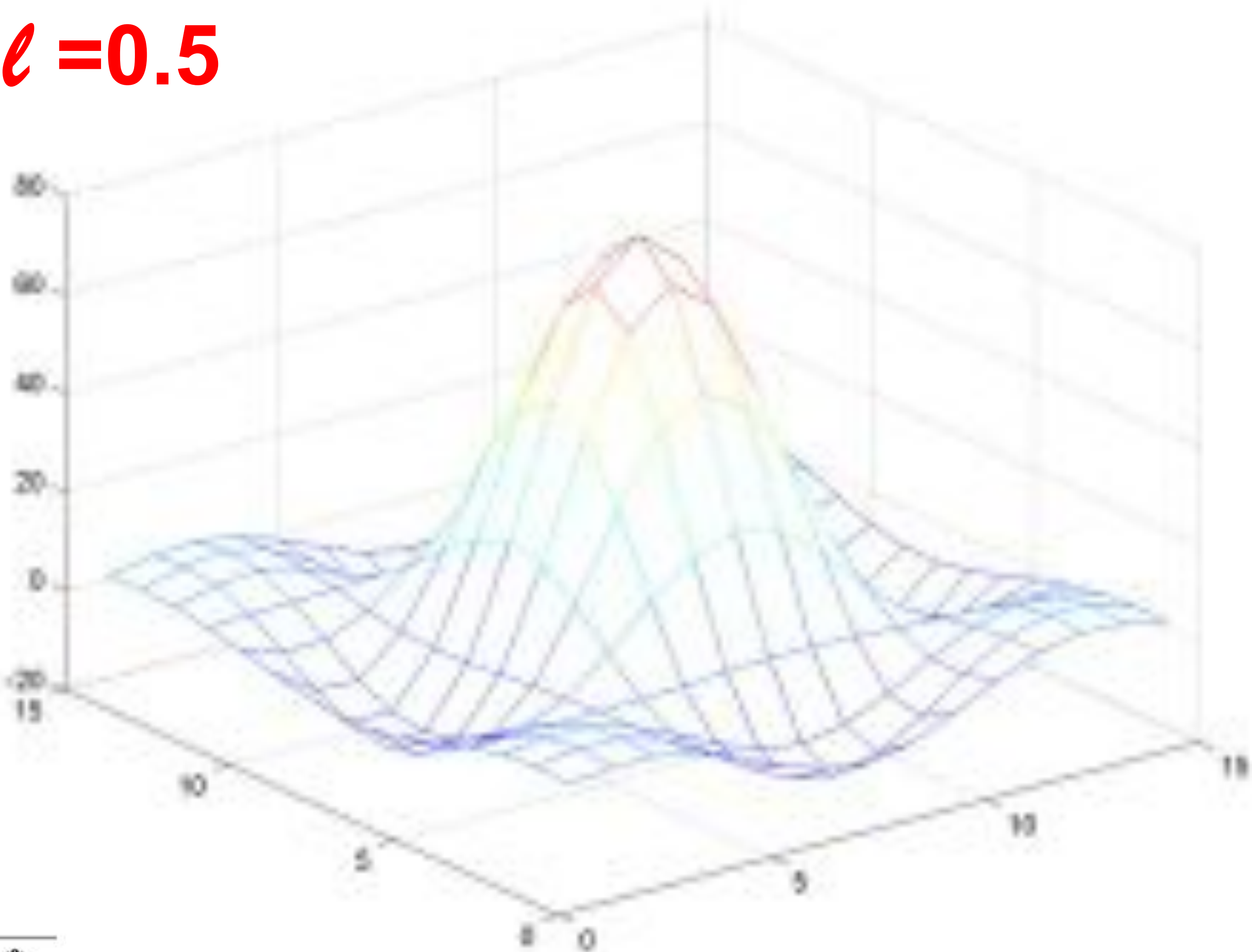
ℓ =0.5



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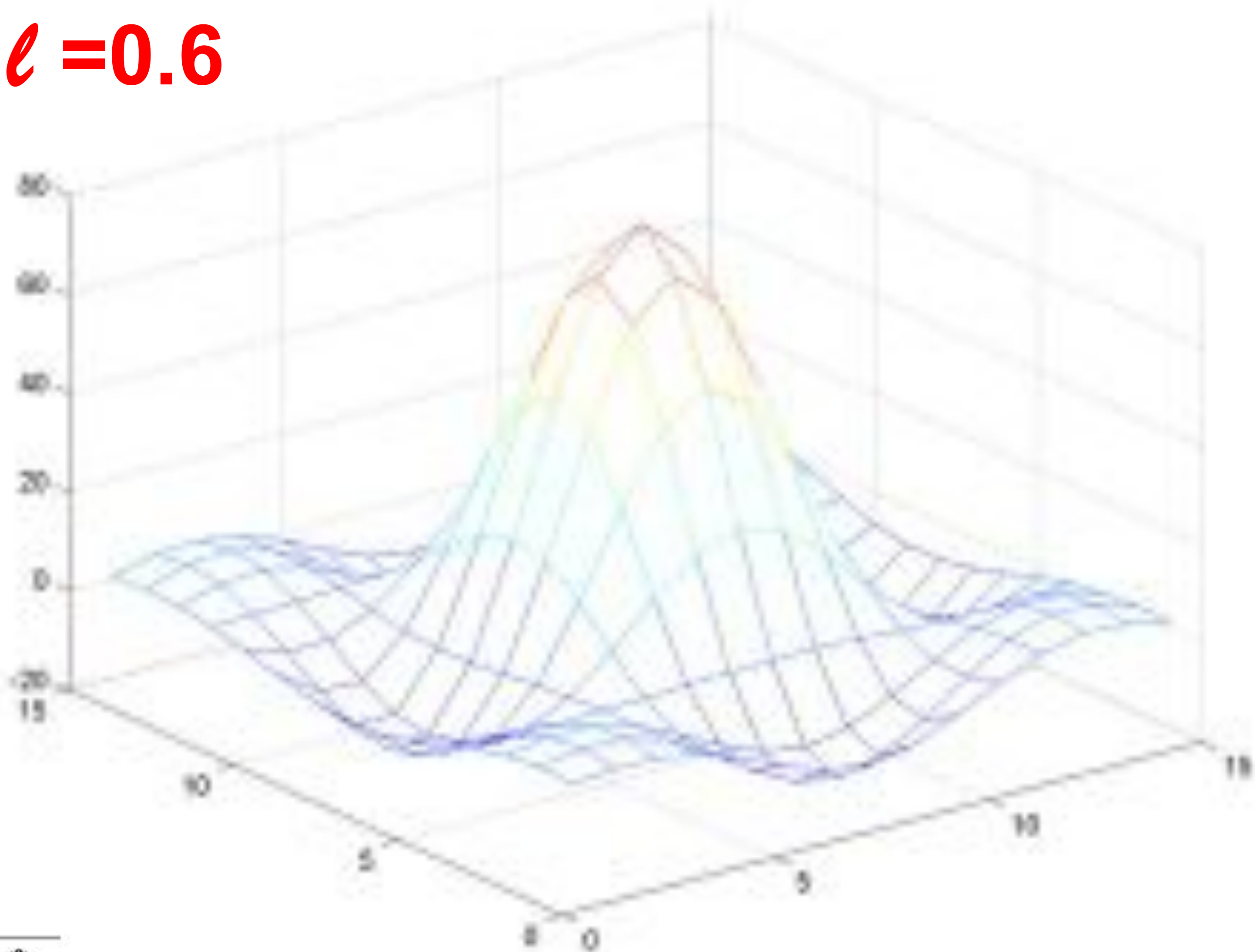
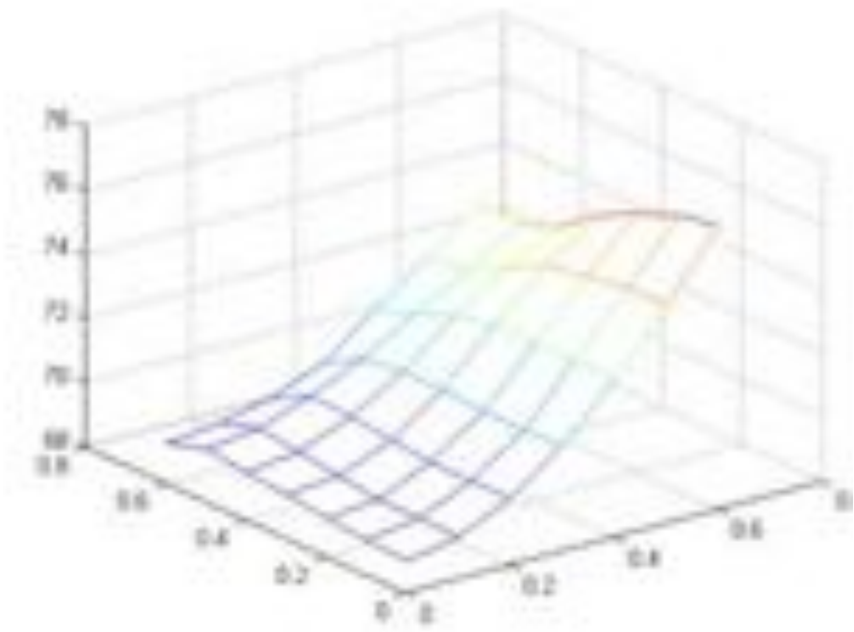
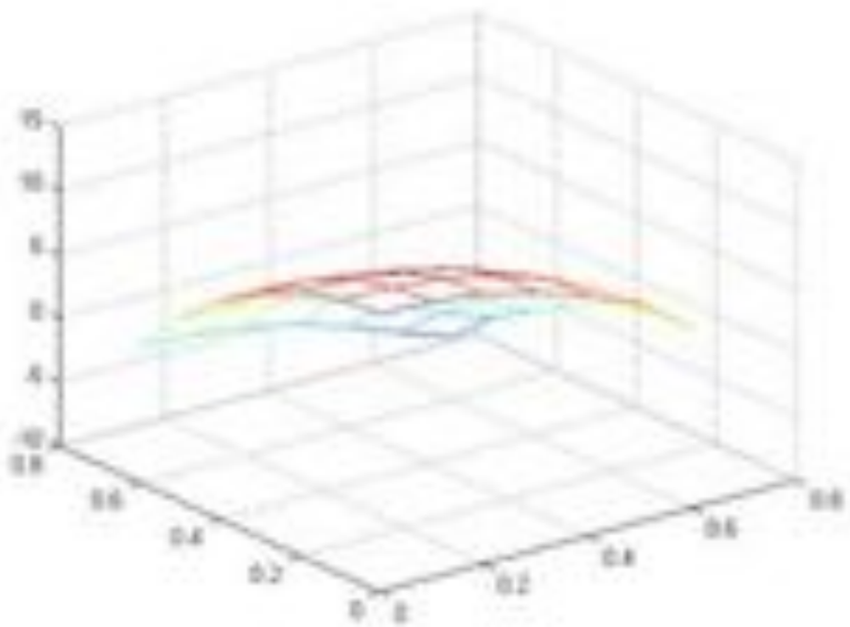
$\ell = 0.6$



Variation of maximum



Real



Imaginary

Conclusion

- Representation based on array factorization
- Projection of patterns of MBF on 1 or 2 of them
- Representation of array factors with functions use for apertures (done here with 1 array factor)
- Array factor slowly varying when shifted upon scanning

(to be confirmed with more elements and other elt types)