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Dealing with the LOFAR BSR effect: A Bandpass Sawtooth Ripple due to the sub-band beamformers

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Abstract

The LOFAR station signal is processed in sub-bands of 256 kHz. The total frequency band may consist of up to 32 sub-bands, which do not have to be contiguous. Each sub-band has its own beamformer, which is only correct for its centre frequency. This causes an error pattern over the sub-band gain. Over the entire band this results in a Bandpass Sawtooth Ripple (BSR) pattern, which can have a modulation depth of a percent or more. Unfortunately, the BSR pattern strongly depends on the position in the station beam. This document describes the sawtooth, investigates its effect on various parts of LOFAR processing, and suggests possible ways of dealing with it.

1 Introduction

The signal of a LOFAR station element (dipole or tile) is digitised over a total bandwidth of 32 (or even 64) MHz. From this frequency band, a number of *sub-bands* may be selected to be processed further. These sub-bands usually have a bandwidth of 256 kHz, but other values are possible. They do not have to be contiguous, and may even overlap. [1]

Each sub-band has its own beam-former, in which the relevant signals from all station elements are added after multiplying them with a complex weight. The purpose is to 'phase up' the signals that come from a source in a given pointing direction z_0(zenith angle). Unfortunately, since a phase factor is used rather than a time delay, there will be a *phase error gradient* over the sub-band, which is zero only for its centre frequency f_0. This causes a *gain error pattern* over the sub-band, which is approximately repeated for all sub-bands. Thus, on top of other effects, the station bandpass will exhibit a gain ripple, which usually has the shape of a sawtooth. Since this ripple may have a modulation depth up to several percent, it may have serious repercussions, e.g. for the subtraction of very bright sources, and thus for the dynamic range of LOFAR observations.

The main purpose of this document is to describe and analyse this Bandpass Sawtooth Ripple (BSR) effect, and to investigate what (if anything) can be done about it. Things are illustrated by simulations, using a Glish program that was written for slightly different purposes[2]. However, the results are close enough to be relevant. Moreover, the numbers are consistent, both in general shape and in magnitude, with those calculated independently by Stefan Wijnholds and Jerome Dromer in the context of their work on the LOFAR test station (ITS). Both will be reported elsewhere.

A possible secondary use of this document is as consciousness-raising picture book. The shape and frequency-dependence of the LOFAR station beam, and especially its sidelobe pattern, will be important factors in the calibration and calibratability of LOFAR. Of course the simulations presented here are only qualitatitive, ignoring the effects of a conducting ground-plane, and mutual coupling between elements. But they do demonstrate a number of important aspects that have perhaps been under-illuminated until now.

Finally, in an appendix, the case is made for continuity in sub-band calibration, so that the entire bandpass may be characterized by the minimum number of parameters.

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[1] When sub-bands are split into channels of, say, 1 kHz, the latter lie on the same grid for all the selected sub-bands, whether they are contiguous or not. This is not relevant for the present discussion, but it mentioned nevertheless because it is an important simplifying assumption for the LOFAR calibration software.

[2] The software was developed to demonstrate the use of a one-bit phase shifter for phased arrays as used in SKA or the FAST project.

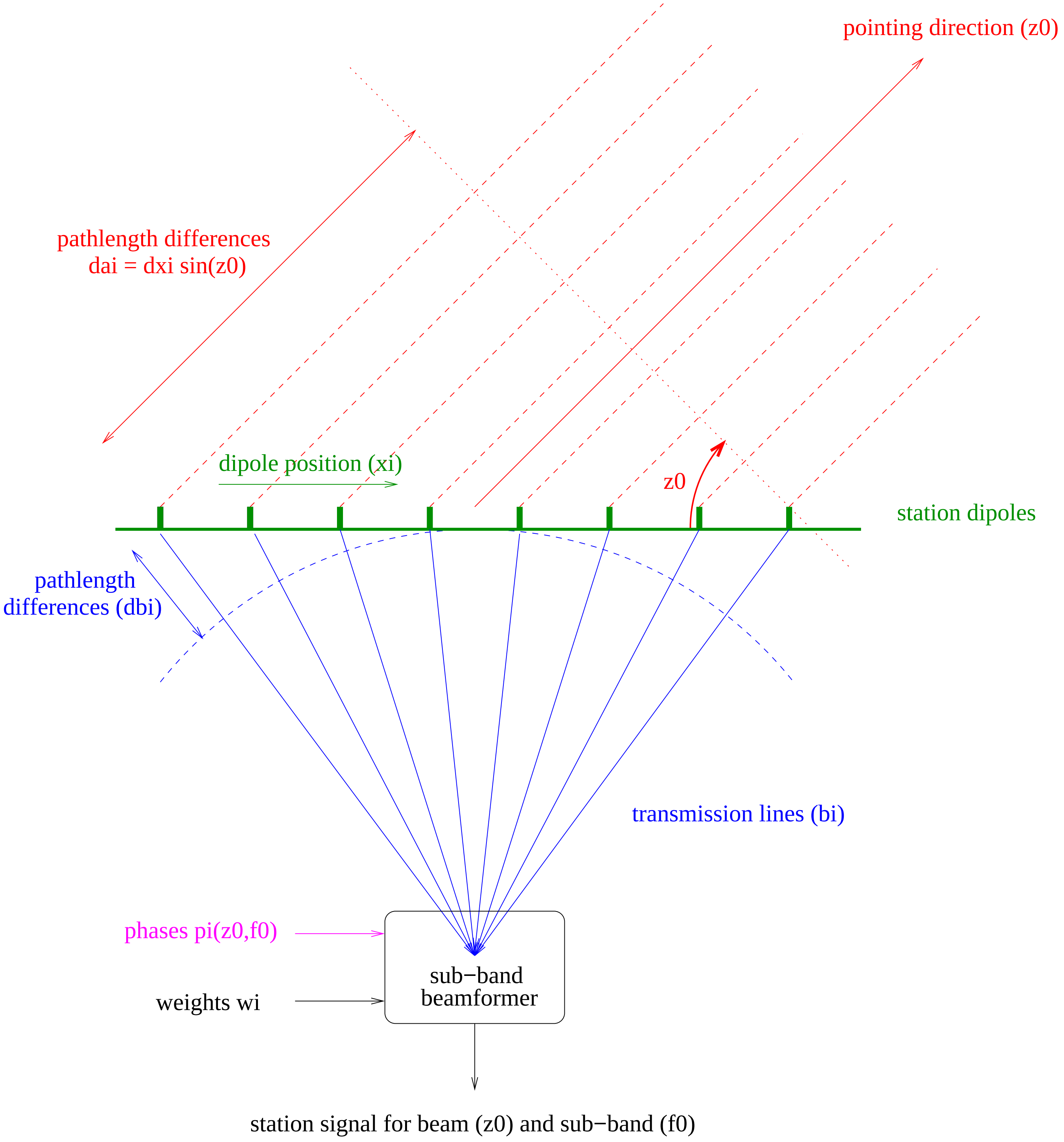

Figure 1: *Schematic diagram of a sub-band beamformer of a LOFAR station. A beam is formed in a certain pointing direction* z_0 *by compensating free-space (red) and transmission line (blue) pathlength differences by means of a phase factor, rather than a time delay. This causes an phase error gradient over the sub-band, which is zero for its centre frequency* f_0 *only.*

2 The Bandpass Sawtooth Ripple (BSR) effect

A beamformer can be seen as an adder of cosines. The sum reaches its maximum if all the cosine arguments are zero. This is the case when the input signals are 'phased up' for a particular pointing direction z_0, which may be achieved by compensating the cosine phases caused by the pathlength *differences* through the station elements to the beamformer (see fig 1). Thus, the station voltage beam response $G(z, f)$ in direction z (zenith angle), at an observing frequency f, can be written as:

$$G(z, f) = \sum_i w_i g_i \cos\left[\, 2\pi(\Delta a_i + \Delta b_i) f/c + \phi_i(f_0, z_0)\,\right] / \sum_i w_i \tag{1}$$

Note that this is a 1D cross-section of the beamshape in a plane through the zenith, calculated from only the x-coordinates x_i of the 2D dipole positions $r_i(x, y)$. The multiplicative factors $g_i(z)$ represent the element response pattern (usually something like $g_{i0}\cos(z)$), and the $w_i(r_i)$ represent an optional taper function. Neither will play a role in the present discussion. Fig 2 shows the all-sky beam of a typical LOFAR station, for various pointing directions.

The station beam is pointed in a particular direction by minimising the the cosine arguments in equ 1. This is done by compensating the sum of the free-space pathlength *difference*

$$\Delta a_i/\lambda = \Delta a_i f/c = \Delta x_i \sin(z_0) f/c \tag{2}$$

and the transmission-line pathlength *difference* $\Delta b_i/\lambda = \Delta b_i f/c$ with a beamformer phase factor

$$\phi_i(f_0, z_0) = -2\pi(\Delta x_i \sin(z_0) + \Delta b_i) f_0/c \tag{3}$$

Unfortunately, such a phase factor will nullify the cosine factors only for the centre frequency f_0 of the sub-band, and for the pointing centre z_0. Ideally, a proper time-delay should be used, which is expensive, and difficult to implement. Even the use of a frequency-dependent phase-factor $\phi(f, z_0)$ would still cause a *phase error gradient* over the sub-band for any viewing direction other than z_0. As it is, we will have to deal with the following effects:

- The phase gradient over the sub-band will NOT affect the interferometric phase *difference* between stations, provided the phase error is the same for both stations. Steps should be taken to insure that this is the case. **It may mean that station configurations should be identical....!?**
- The frequency-dependent phase errors in the contributing cosines will translate into a frequency-dependent gain error in the beamformer output. Usually this will be an approximately linear gain gradient over a sub-band, but the shape can also have 2nd order terms.

- Fortunately, the gain effect has approximately the same shape and maginitude for all sub-bands. This makes it considerably easier to do something about it.

The most general way to describe (and understand!) the gain effect is in terms of a frequency-dependent shape of the station voltage beam. This is illustrated in fig 3 and 4, where the beams for the various frequencies are plotted together. The frequency-dependent gain in a given direction z is obtained by drawing a vertical line at the relevant zenith angle, and plotting the gain for each frequency. Fig 5 shows that the general pattern is a kind of sawtooth ripple, with one tooth per sub-band. Therefore, until a better name props up, we will use the name Bandpass Sawtooth Ripple (BSR) effect.

Remark by Jerome Dromer: To first order, the gain variation ΔG over a sub-band depends only on the width of the sub-band, and not on its centre frequency.

3 From voltage beams to visibilities

In aperture synthesis, interferometers are used to sample the *visibility-function,* which is defined in the *uv-plane*. Each visibility sample $V_{ij}(f)$ is the result of a correlation between the signals from two beamformers i and j, which are usually associated with different stations. Each sample is the sum of the contributions from k sources with zenith angle z_k, each of which has its own flux $p_k(f)$ multiplied by the gain $G_i(z_k, f)$ and $G_j(z_k, f)$ (see equ 1) of two voltage beams:

$$V_{ij}(f) = \sum_k G_i G_j p_k \exp(2\pi i(z_k - z_p) b_{ij} f/c) \tag{4}$$

in which $b_{ij}$is the projected baseline between the stations as seen from the source, and $z_p$is the position of the interferometric phase centre. In the peeling technique, this will be the position of the current peeling source (see section **??** below). See also fig 7.

4 The impact of the BSR on calibration

Having described the BSR effect, we must ask ourselves whether or not it is a problem, and what, if anything, we can do about it. The short answer is that it is difficult to say, because we do not really have a good method yet to estimate the impact of this kind of effect on the final image[3]. Pending that, we can make the following statements about the impact on the various areas of LOFAR calibration:

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[3] One of the by-products of this study is exactly such a method, which can be used to estimate the impact of a wide variety of instrumental and calibration errors on the final image. This will be elaborated in another paper.

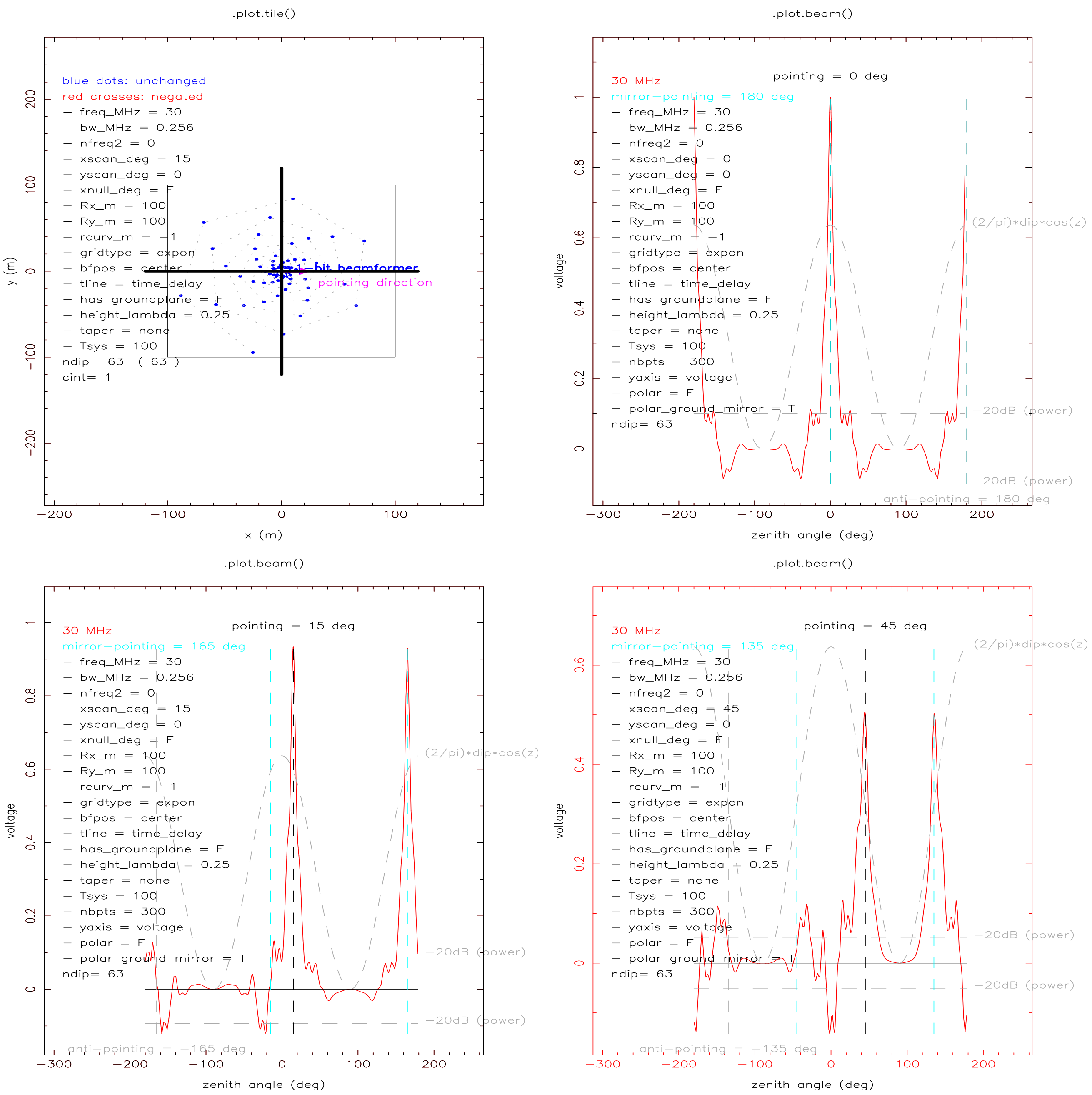

Figure 2: *The all-sky response function (voltage beam) of a singe polarisation of a typical LOFAR station, for various pointing directions. Note the relatively high side-lobes all over the sky, and the existence of a mirror-lobe looking downwards.The dipole configuration used in these simulations is slightly different from the actual LOFAR Initial Test Station (ITS), and the beamshape calculations do not include a conducting ground-plane, or mutual coupling between elements. However, the results are qualitatively correct, and entirely adequate to illustrate the BSR effect discussed in these pages.*

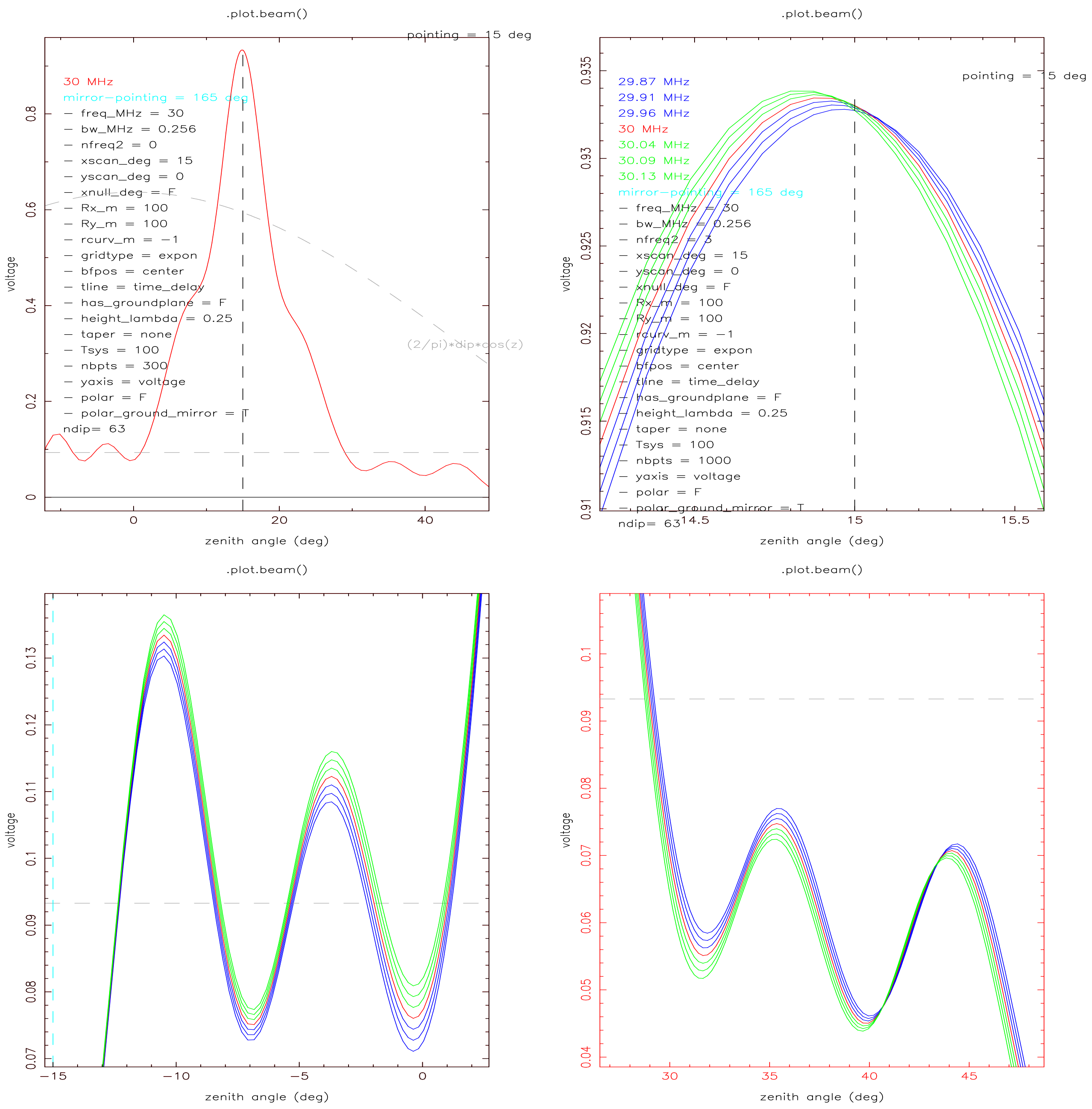

Figure 3: *An enlargement of the main lobe of the LOFAR station beam shown in fig 2, but for 7 frequencies over a 256 kHz sub-band. The centre freq is in red, the lower freqs in blue, and the higher freqs in green. The top-right panel shows that, under certain conditions, the BSR effect can be described to first order as a beam-squint, i.e. a slight pointing error as a function of frequency over the sub-band. The bottom panels show that the pattern is more complicated in the sidelobes. Therefore, it is better to discuss the BSR effect more generally in terms of station beamshape variations as a function of frequency.*

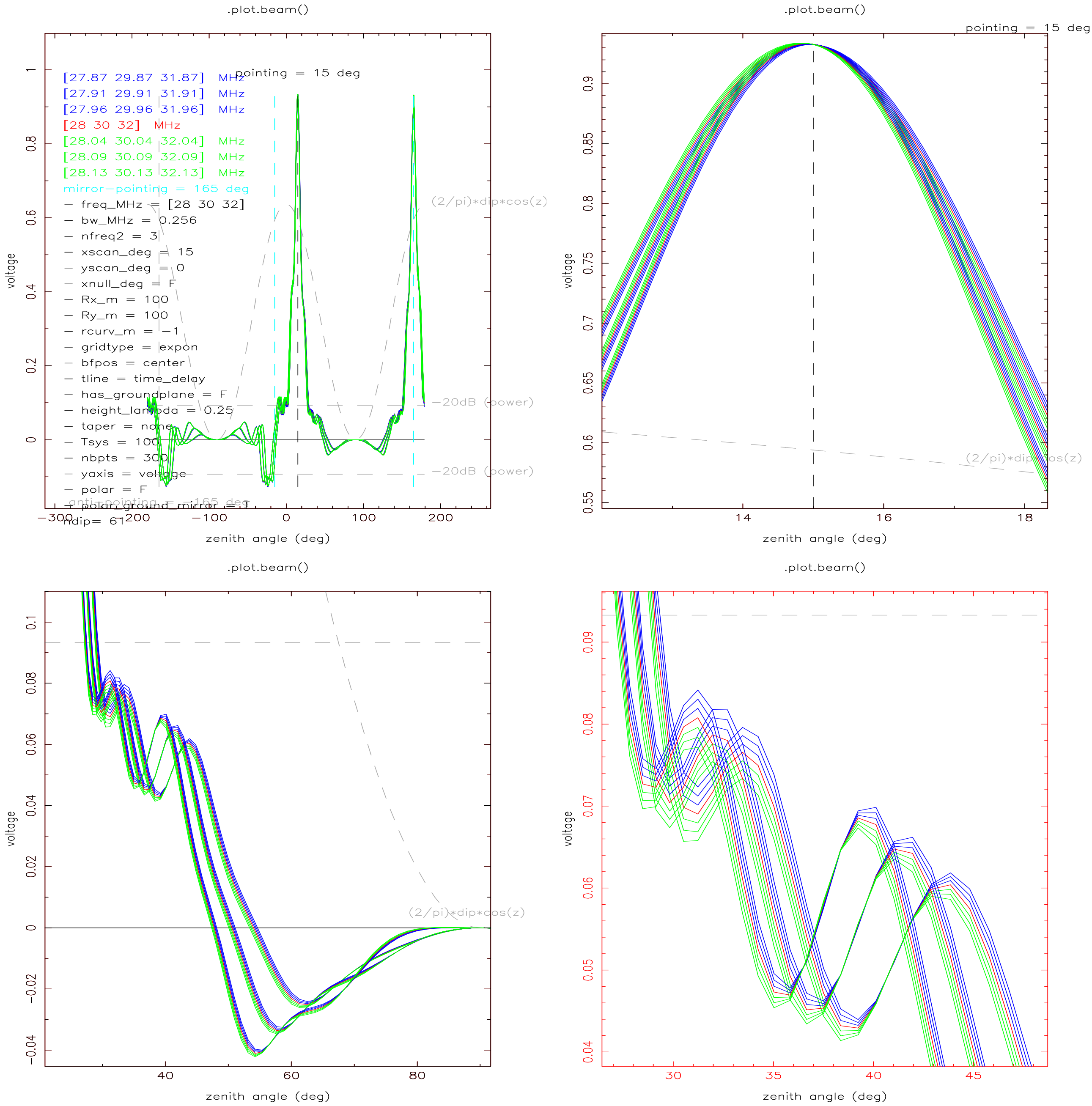

Figure 4: *As fig 3, but for 3 (non-contiguous) 256 kHz sub-bands selected from a total bandwith of about 4 MHz. The frequency-dependent voltage gain $G(z, f)$ in the direction (zenith angle $z(t)$) of a particular source can be studied by drawing a vertical line in the plot at that position. This includes the BSR effect, but also the widening of the main lobe at lower frequencies. Note that the frequency dependence gets more complicated in the sidelobes. Also note that the position of a source in the station beam, and thus its gain curve, will vary with time because of the rotation of the Earth. Again, the effects are much more pronounced in the far sidelobes, where the strongest sources will be located.*

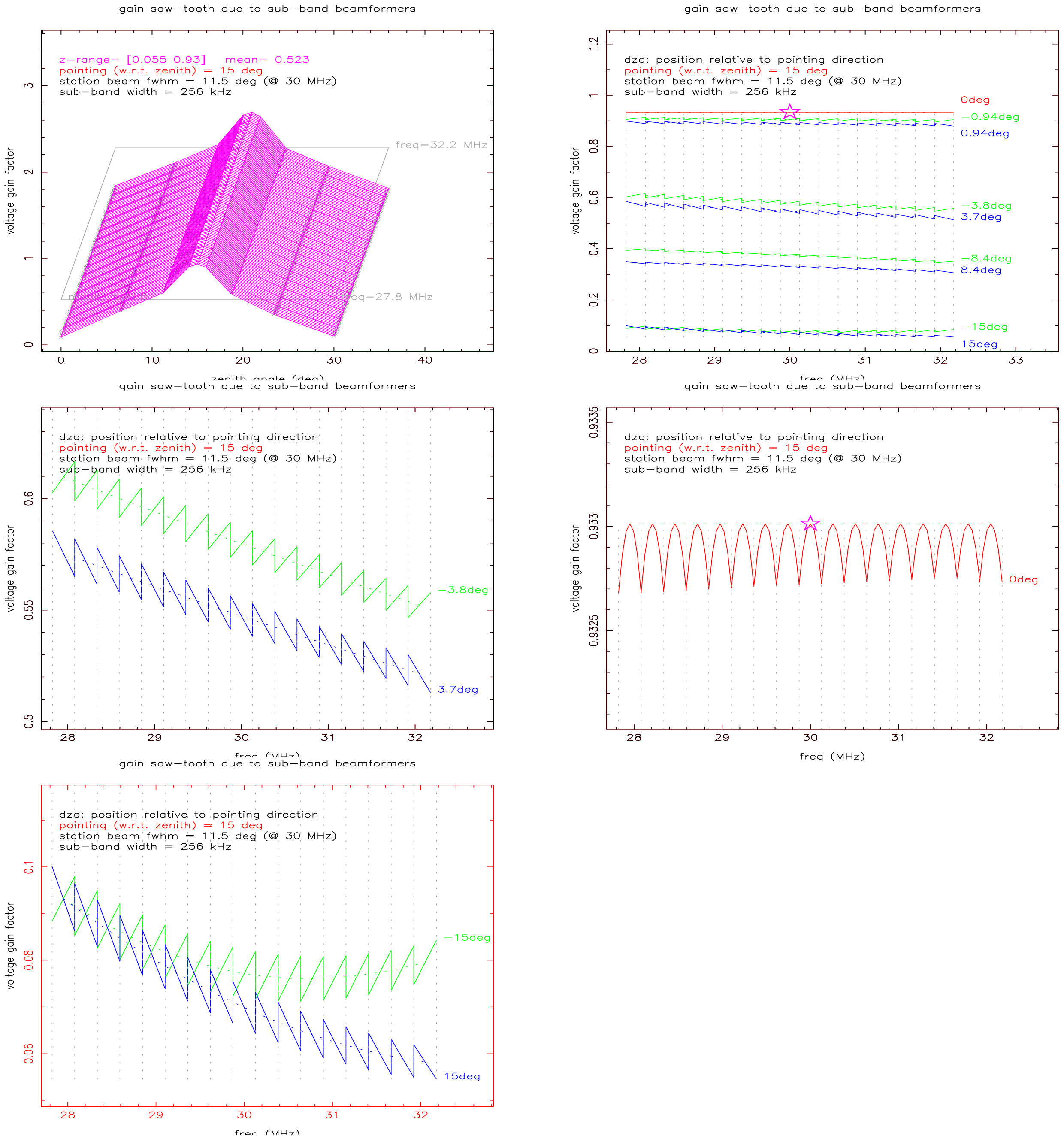

Figure 5: *The Bandpass Sawtooth Ripple (BSR) effect. The top left panel shows a 3D plot of the main lobe of the LOFAR station voltage beam as a function of frequency, for 17 contiguous sub-bands of 256 kHz each. The top right panel shows cross-sections for 9 different zenith angles in the main lobe (red is the pointing direction; the colours are consistent with those in fig 3). The remaining panels are enlargements. The generally downward slope is caused by the fact that the station beam gets narrower for higher frequencies. Calibration will be helped by the fact that the pattern is approximately the same for all sub-bands, and that the integrated error over a sub-band will average out if the pattern is triangular. The latter is generally the case, except for lobe minima or maxima like the pointing direction (red).*

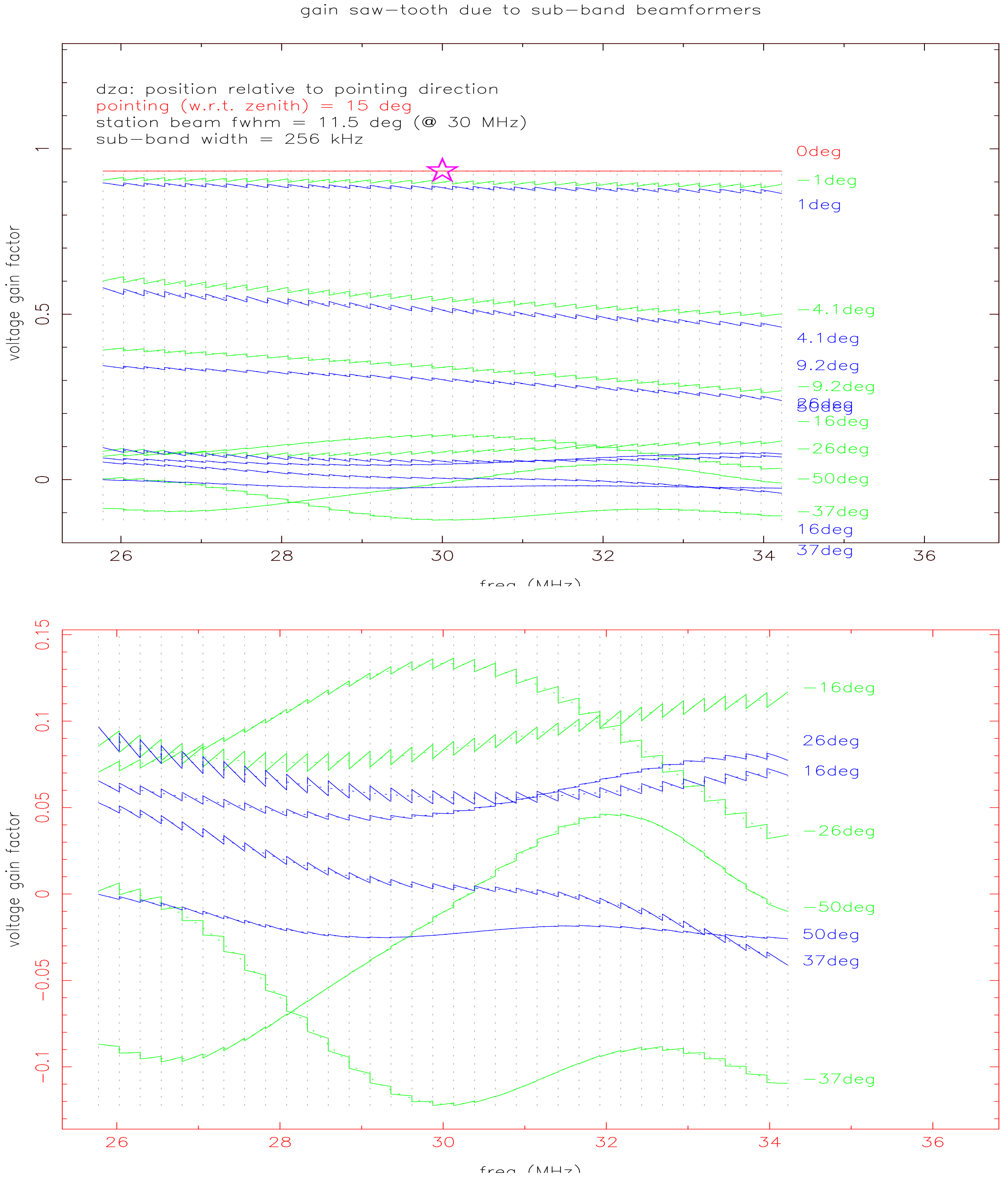

Figure 6: *Sawtooth ripples (and overall gain behaviour) for viewing directions further from the pointing centre. The bottom panel is an enlargement of the voltage gain responses in the beam sidelobes. They are important for the subtraction of (very) bright sources all over the sky. The good news is that, if we ignore the sawtooth ripple, the frequency-dependent gain over a large fractional bandwidth of* $8MHz/30MHz = 0.27$ *can be modelled with a relatively small number of parameters.*

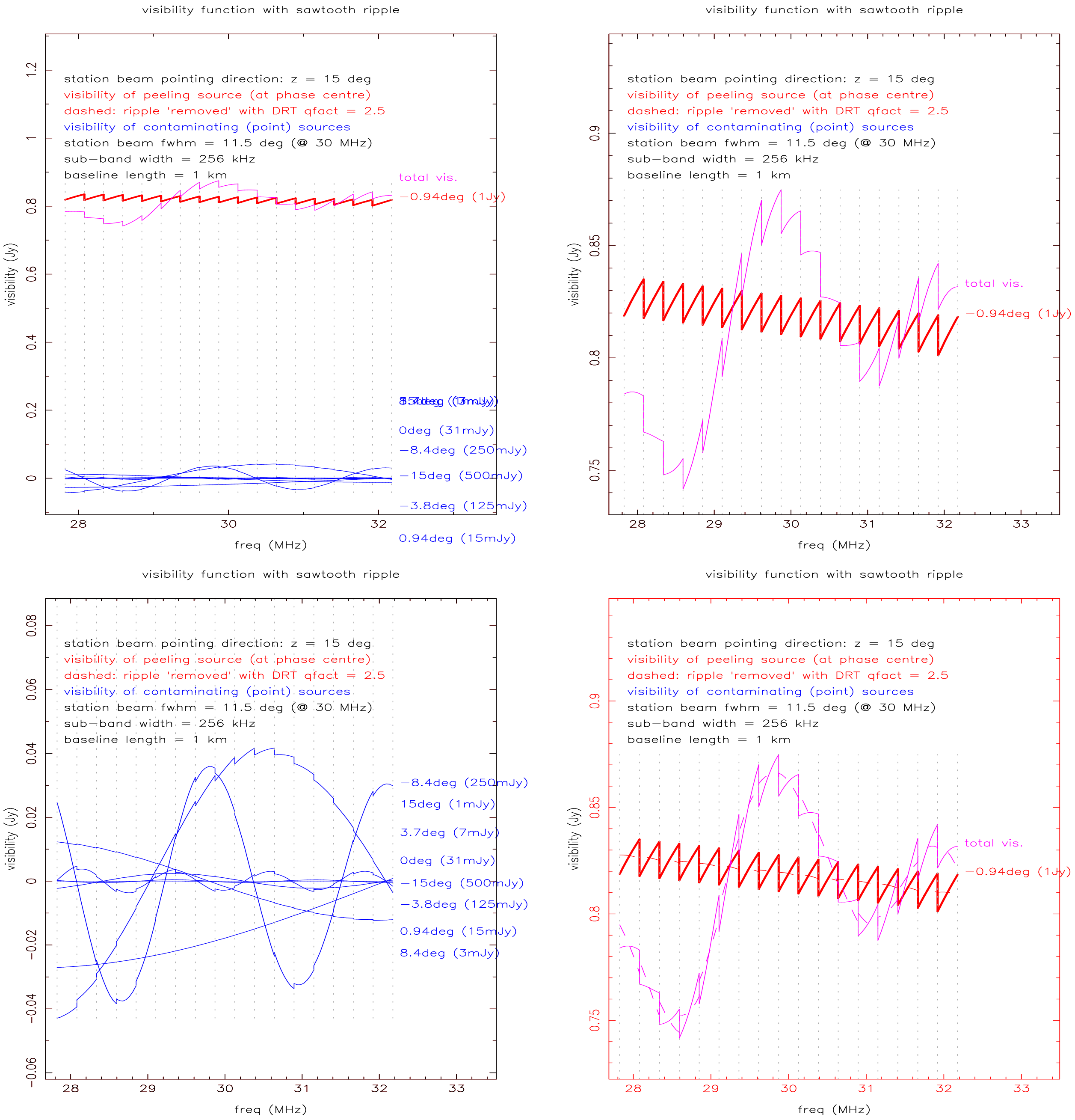

Figure 7: *The effect of sawtooth ripples on the visibility function of a 1 km baseline. See equ 4. For simplicity, it is assumed that both stations have identical beamshapes, and thus identical BSR sawtooth ripples. The visibility function (magenta) is the sum of the contributions from the peeling source (red) and a number of fainter contaminating sources (blue). The phase centre has been shifted to the position of the peeling source, which causes its visibility to be constant over the band. The bottom right panel shows the result of estimating and correcting the ripple by means of the cheap-and-dirty Direct Ripple Transform (DRT) described in section 5. Note that the BSR ripple of the contaminating sources are multiplied by a Fourier cosine, due to their distance to the phase centre. This tends to average out the ripple over the band. Thus, we only have to worry about the BSR ripple of the peeling source.*

- **Subtracting Cat I sources.** It will be assumed that, at the very least, the BSR ripple will have to be taken into account when subtracting the very brightest sources. The reason is that, even if the propagation of the BSR effect into the image may be small, it is multiplied by the very large flux of these sources. Moreover, since there are relatively few of them, their BSR effects will not cancel each other out very effectively in the image, and thus stand out as a pattern that might interfere with astrophysical interpretation. There are two possibilities to remove the BSR from the uv-data:
  - Treat the sawtooth parameter $\alpha(f,t)$ (see section 5) as a regular M.E. parameter, and use its value to include the BSR ripple in the predicted visibility values. The problem is that, for efficiency reasons, we would like to do the prediction at the low resolution (f,t) made possible by shifting the phase centre to the peeling source, while BSR prediction will require a relatively high resolution of at least two cells per sub-band. In addition, the re-sampling to full (f,t) resolution will probably be more complicated.
  - Detect the BSR ripple in the (full-resolution) residual uv-data after subtracting each peeling source, and remove it. One way of doing this is the *DRT method,* described in section 5. It is cheap-and-dirty, but works rather well for cases with $S/N > 10$. **This is the preferred method.**
- **Self-calibration,** i.e. estimating those values of M.E. parameters that will result in the best subtraction of bright sources. There are two aspects to consider:
  - The impact of the BSR ripple on the estimation of M.E. parameters. For triangular teeth, the impact will be minimal if the cell size in the frequency direction equals an integral number of sub-bands. Note that only the BSR of the peeling source will have a significant impact, because those of the contaminating sources will tend to average out by the sine-wave of the DFT (see fig 7).
  - Estimating the BSR ripple parameter $\alpha(f,t)$ as a M.E. parameter. Calibration at station-level may be able to provide an approximate start-value for all directions[4]. However, it seems unlikely that there will use such an M.E. parameter α at all, because the disadvantages in terms of increased prediction resolution and complexity.
- **Subtracting (groups of) Cat II sources.** It will be assumed (hoped) for the moment that it will not be necessary to take the BSR ripple into account when subtracting these fainter GSM sources. In any case, their instantaneous S/N is too low to estimate their BSR effectively, and we operate on the (unproven) thesis that any source that is bright enough to cause trouble, is bright enough to be tackled. Here is a number of reasons why the BSR effects do not propagate particularly well into the image, and can therefore (perhaps, hopefully) be ignored for fainter sources:
  - Since the majority of sawteeth are approximately anti-symmetric around the centre frequency of a sub-band, the BSR effect will (to first order) average out over a sub-band. However, the

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[4] Prediction of the ripple at station level will require precise knowledge of the actual values of the complex weight factors that are used for beamforming, which depends on a good calibration of the various elements. Fortunately, we do not need the absolute values, but only the relative values, which is precisely what the TV-station (calibration-beacons) scheme provides us.

  second-order effect will cause a pattern in the image[5], which is the main reason to remove it in the brightest Cat I sources.

  - Because the BSR ripple strongly depends on its position in the station beam, its magnitude and sign (!) on a particular source will vary considerably over an observation. This will not average out the BSR effect, but it will at least scramble its pattern in the map so that it becomes more noise-like.
  - Since the various station beams will have different shapes, especially far from the pointing centre, they will contribute different BSR ripples to a particular source. This will have a scrambling effect.
  - The BSR patterns of many sources are added up in the image. The total result will be noise-like, and less easily mistaken for astrophysical structure (like the EOR signature).
  - In any case, even if the unremoved BSR effect of Cat II sources does not cause any structure in the image, it will still increase the noise. Estimates of this increase are needed.

- **Imaging Cat III sources.** These are too faint, or too extended, to be identified for the GSM, and are therefore not subtracted from the uv-data. The arguments for not worrying overmuch about the BSR effect on Cat II sources apply even more strongly here. There does not seem to be any possibility for removing the BSR effect in the image: Even if it can be detected and removed at the position of a source, its effects on the rest of the rest of the image cannot be reconstructed.

Note that all these considerations assume that the sawtooth is relatively constant over the band, so that it can be adequately described by a single, or slowly varying at most, sawtooth parameter $\alpha(f,t)$.[6]

4.1 Strategies to minimize the BSR effect

Since prevention is always better than correction, LOFAR should be designed in such a way that the BSR effect is minimised. We could consider the following possibilities:

- Integrate over a sub-band. The full (1 kHz) freq resolution is only needed for the field-of view.
- Use time-delays in a sub-band beamformer. Not practical.
- Use narrower (and thus more) sub-bands. Expensive.
- Make sure that the sawtooth in the direction of each source varies sufficiently over an observation to cancel out. In practice this means that it should be at either side of the station beam pointing centre...

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[5] This pattern will be described in a separate document.

[6] Note that this requirement would rule out a one-bit phase-shifter (see section A.3).

- Since a tooth is centered on a sub-band, and we may wish to solve for a sawtooth ripple with a single parameter, it is wise to select sub-bands on a regular grid. This is automatic if the sub-bands are contiguous, and only represents a minor limitation on other selection schemes.

5 The Discrete Ripple Transform (DRT)

Here is cheap-and-dirty a method to estimate and remove the sawtooth ripple from a snippet of full-resolution residual visibility data. It can be implemented in a specialised MeqDRT node in the processing tree[7], just after the node that subtracts the contribution of a peeling source. The node will need to know about sub-band boundaries.

We define a *unit sawtooth* $S_k(f)$ as a straight line through the centre frequency f_k of sub-band k, which reaches the value $s = \pm 1$ at the sub-band edges $f - f_k = \pm\Delta f$:

$$S_k(f) = \frac{1}{\Delta f}(f - f_k) \tag{5}$$

An arbitrary visibility sample $V(f)$ is a multiplication of a smooth function $V_s(f) = v_0 + v_1 f + v_2 f^2 + \cdots$ with a sawtooth:

$$V(f) = V_s(f)(1 + \alpha R(f))$$

in which the sawtooth ripple function $R(f)$ is a succession of unit sawteeth $S_k(f)$, and α is the *ripple parameter*. For the monent, we will assume that α is a constant, i.e. that all sub-bands have identical sawteeth.

The Discrete Fourier Transform (DFT) estimates the amplitude of a given frequency component in a signal by multiplying that signal with a sine wave with the relevant period, and unit amplitude. Following this analogy, we coin the name Discrete Ripple Transform (DRT): It estimates the ripple parameter α by multiplying the residual visibility $V(f)$ with a sawtooth ripple with unit amplitude, and integrating over all sub-bands k:

$$q = \frac{1}{N}\sum_k \int_{f_k-\Delta f}^{f_k+\Delta f} V(1 + \alpha S_k) S_k df$$

In which N is the number of samples used in the multiplication. The ripple parameter α is obtained by:

$$\alpha = 2.5q \tag{6}$$

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[7] The Measurement Equation will be implemented as trees (MeqTree) of nodes, each of which implements a mathematical expression or arithmetical operation.

and the BSR ripple may be removed from the visibility by:

$$V_s = V/(1 + \alpha R)$$

NB: The factor 2.5 in equ 6 is a little empirical for the moment, but seems robust enough. Its optimum value, and the impact of the series expansion of V_s, will be investigated further in a later stage.

This technique is cheap-and-dirty, and, as illustrated in fig 7, it works quite well, even in the presence of contaminating sources. It starts breaking down when the amplitude of the peeling source is less than 10 times the noise. Again, it is hoped that the BSR only needs to be subtracted from sources that are brighter than that.

6 Conclusion

The discovery of the Bandpass Sawtooth Ripple (BSR) effect was a bit of a shock because of the possible calibration implications. It violates the often-stressed requirement that all M.E. parameters must be smooth functions of time and frequency. In this case we are saved by the fact that the awteeth have approximately the same shape for all sub-bands.

The BSR effect is quite well understood now, and we appear to have a number of viable approaches to minimize it, and to correct for it where necessary. The most important area where it may be a problem is the subtraction of very bright Cat I sources from the uv-data. Fortunately, this is precisely the area where there is sufficient S/N to use the cheap-and-dirty Discrete Ripple Transform (DRT).

More work on this subject is needed in the future. For instance:

- It has been assumed that we only have to deal with the BSR *gain* effect, since the BSR *phase* effect would cancel out in interferometric measurements. As mentioned in the introduction, this may not always be the case.
- This document limits itself to the case where all transmission lines in a LOFAR station are of equal length. This may be an unnecessarily expensive approach. If alternatives are to be considered, the impact on the BSR effect should be taken into account.

In general, we should continuously and vigilantly review the various links in the LOFAR processing chain, to check whether there are any non-smooth effects that we may have overlooked.

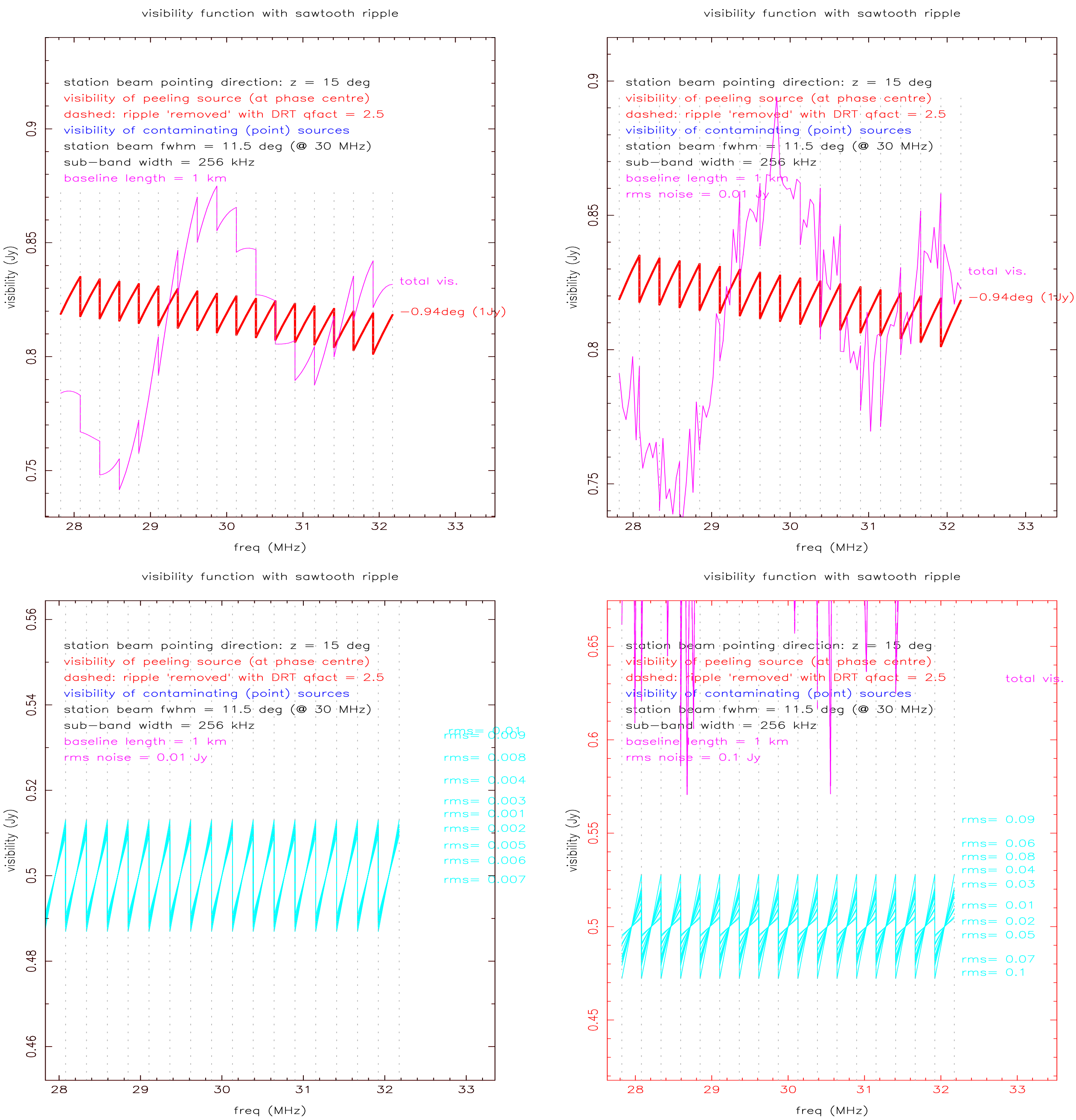

Figure 8: *Assuming that the sawtooth parameter is constant over the band, the ripple may be estimated and corrected by means of the cheap-and-dirty Direct Ripple Transform (DRT). This works quite well, even in the presence of some noise. This is demonstrated by comparing the estimated sawtooth ripples (cyan) for different noise-levels by plotting them on top of each other. The bottom-right panel shows that the DRT starts breaking down for S/N levels less than 10. Hopefully, removing the ripple may only be needed for the brightest sources, which have sufficient S/N. This is another example of the (unproved) thesis that those sources that are bright enough to cause trouble, have sufficient S/N to be tackled.*

A Appendix

A.1 Beam-squint: an unhelpful concept

If, and only if, all transmission lines have the same length b_i (i.e. if all $\Delta b_i = 0$,see fig 1), the BSR effect can be approximately described as a *'beam-squint'*, i.e. a small shift of the pointing direction as a function of frequency. The zenith angle $z_{peak}(f)$ of the peak response can be calculated from the condition:

$$f \sin(z_{peak}) - f_0 \sin(z_0) = 0 \tag{7}$$

which leads to a position shift Δz of the peak response

$$\Delta z = z_{peak} - z_0 = \arcsin(\frac{f_0}{f} \sin(z_0)) - z_0 \approx \frac{f_0 - f}{f} z_0 \tag{8}$$

The approximation is valid for small zenith angles, where we have $\sin(z) \approx z$. See also fig 3. However, rather than talking about a beam-squint, it is better to discuss this effect in terms of a frequency-dependent shape of the station voltage beam. One reason is that things are a little more complicated in the sidelobes, especially since we also have to take other effects (like the frequency-dependent beam-width, etc) into account. Moreover, the main-lobe will not only shift but also deform if the transmission lines have different lengths (which would be a lot cheaper!). A final reason is that the term *beam-squint* is confusing, because it is already used for the slightly different pointing directions of the voltage beams for the two polarizations of a station.

A.2 Unequal transmission lines

All the pictures in this document have been made for the case of equal-length transmisson lines between dipoles and beamformer. The same program is able to generate similar pictures for the case of minimum-length cables, which are only as long as the distance between a dipole and the beamformer hut. Fig 9 give a sample. It turns out that there are indeed some differences in beamshapes and sawtooth ripples, but these are relatively minor, and do not change any of the conclusions.

Therefore, there is no fundamental reason why the transmission lines between station elements and beamformer should be of equal length. If that is not a requirement, the total of 40 km of copper cable per station could be reduced to about 25 km or less. This is substantial, even if the internal cabling of a station is only 3% of the total LOFAR budget.

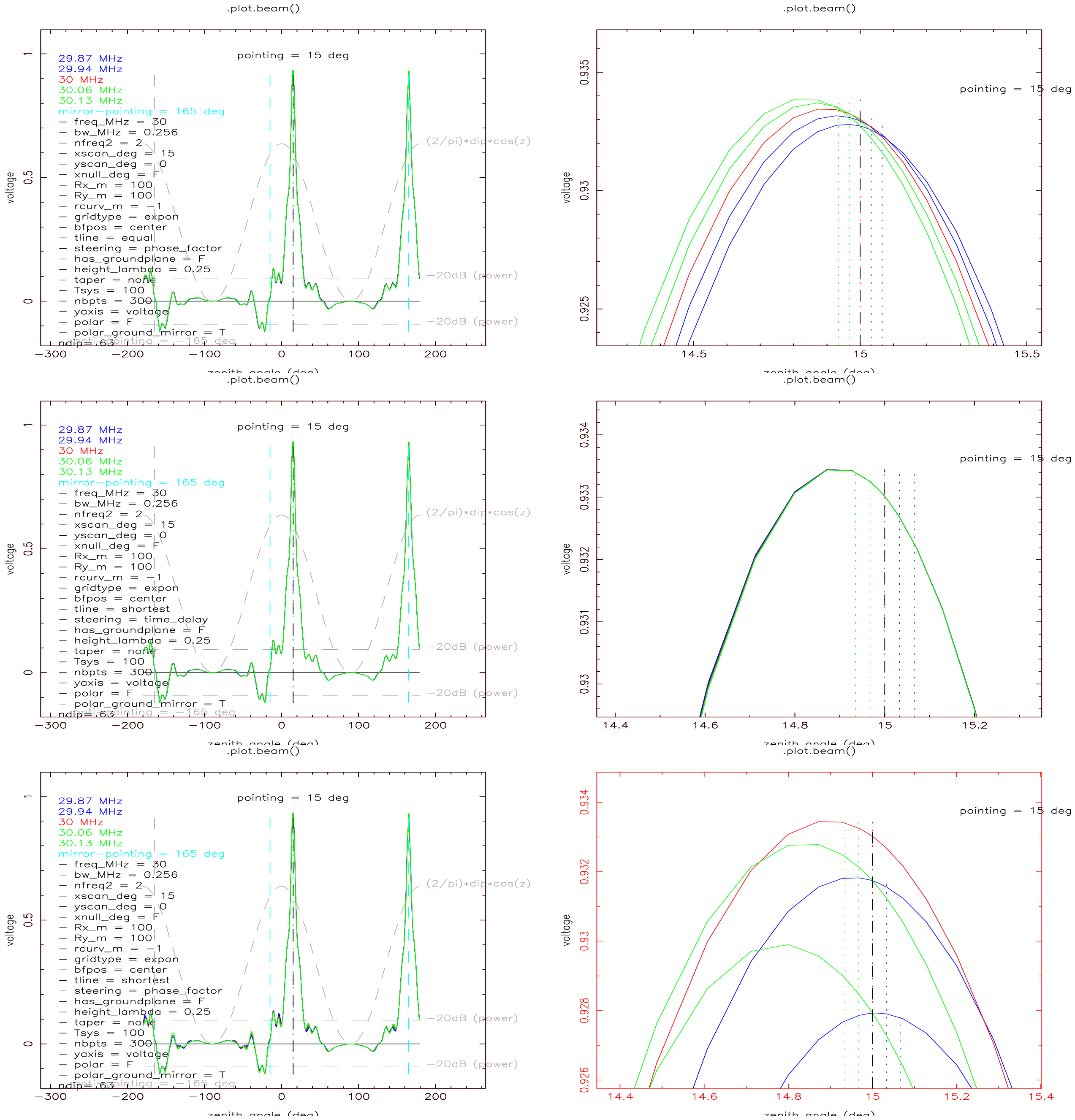

Figure 9: *The top two panels, like the other pictures in this document, have been generated with equal-length transmission lines, and beam-steering by means of a phase factor. The middle panels show the result of doing beam-steering with a proper time-delay, and the bottom ones have unequal transmission lines that are as short as possible. See section A.2 in the text.*

A.3 Use of a 1-bit phase shifter

For completeness, we mention the possibility of a one-bit phase shifter, which is by far the cheapest method to implement a beamformer. Rather than using an expensive (and quirky) vector modulator, the signal from each dipole is either added or subtracted at the input of the beamformer. In this way, the signals are not 'phased up' for a particular pointing direction, but merely prevented from being negative. For large pathlength differences, the phases of the cosines are distributed more or less randomly between $-\pi/2$ and $\pi/2$, so the resulting voltage gain can be approximated by

$$G \Rightarrow \frac{1}{\pi}\int_{-pi/2}^{pi/2} cos(\psi) d\psi = \frac{2}{\pi} = 0.63 \tag{9}$$

Another disadvantage of this approach is that the BSR sawtooth is much less regular, and can no longer be described with a single sawtooth parameter α(see section 5).

A.4 Requirement: sub-band continuity

The LOFAR calibration group has been emphasizing for a long time that it is possible to solve for many instrumental parameters by self-calibration, as long as they are smooth functions of frequency and time, i.e. functions that have only a small number of coefficients. The BSR effect decribed in this document is potentially threatening because it does not seem to meet this requirement, and it is too expensive to design the system in such a way that it does not occur. But fortunately, under the right conditions, the BSR sawtooth ripple appears to have sufficient regularity from sub-band to sub-band to be tackled.

It is important to remain vigilant in the design of the system. For instance, tracking the sky, and RFI nulling schemes imply a continuous change in the station beamformer coefficients. This is OK, as long as the changes happen very smoothly. Another area of concern is the sub-band calibration. This must be done in such a way that the overall passband over all sub-bands is a smooth function of frequency (except for the BSR sawtooth ripple, of course). Fig 10 illustrates this point.

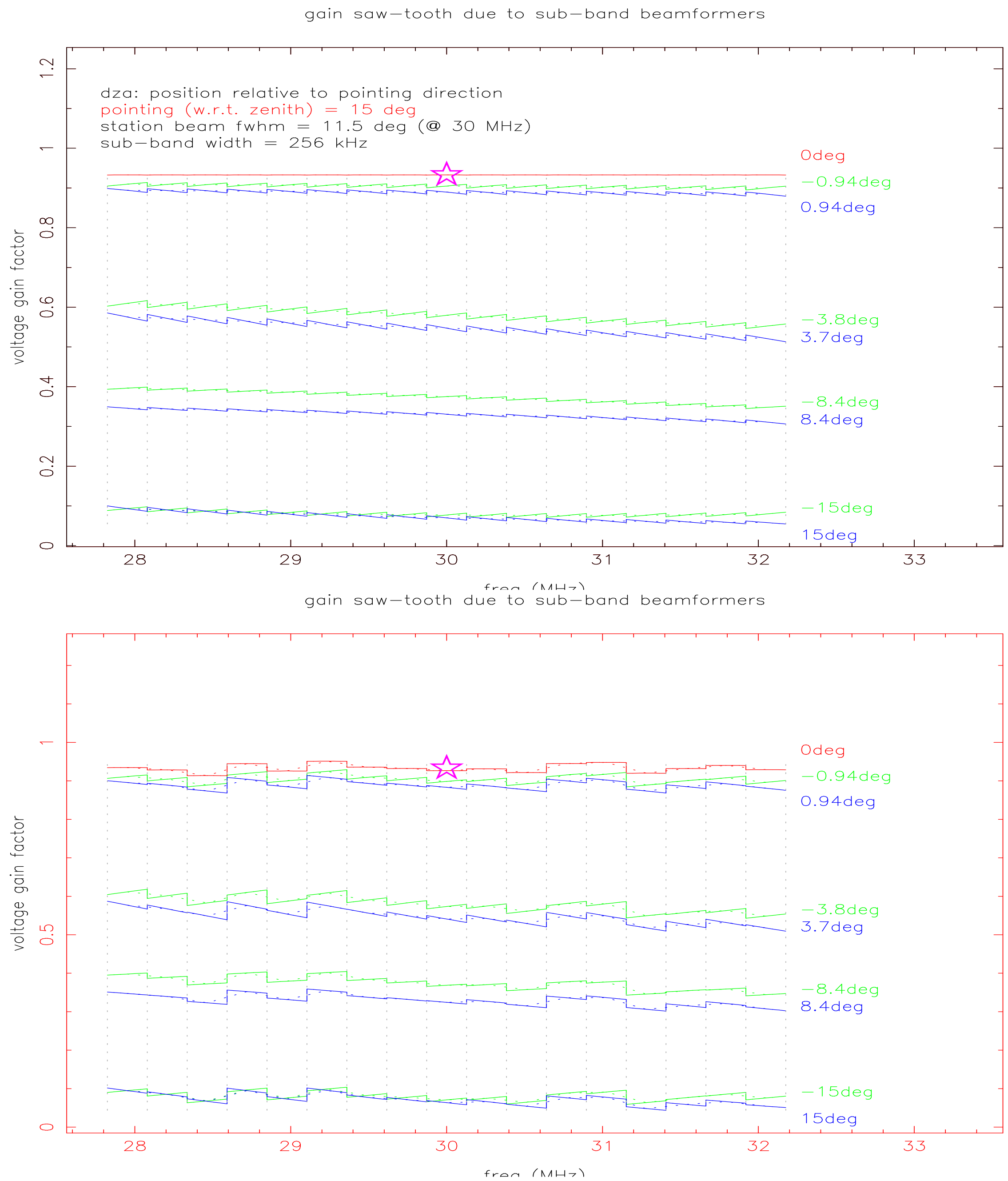

Figure 10: *One of the requirements that has been formulated for the LOFAR station is that the passband gain (except for the sawtooth ripple) is a smooth function of frequency (and time), which can be modelled with a minimum number of parameters. This point is illustrated here. It will be difficult enough to deal with the sawtooth ripple (top panel) without the extra complications of unknown gain offsets between sub-bands (bottom panel).*