Ionospheric Tomography for Low-Frequency Wide-Field Arrays

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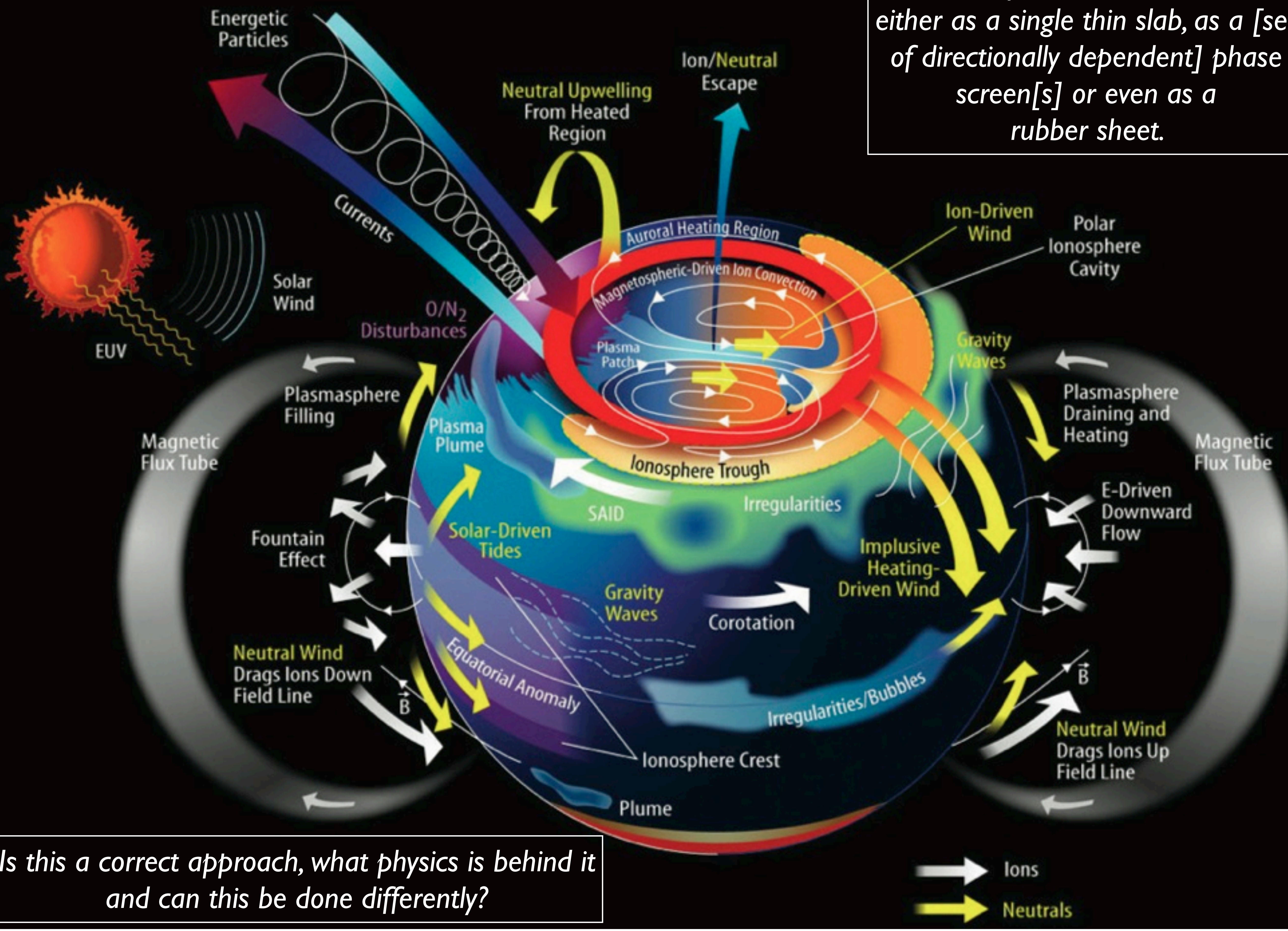
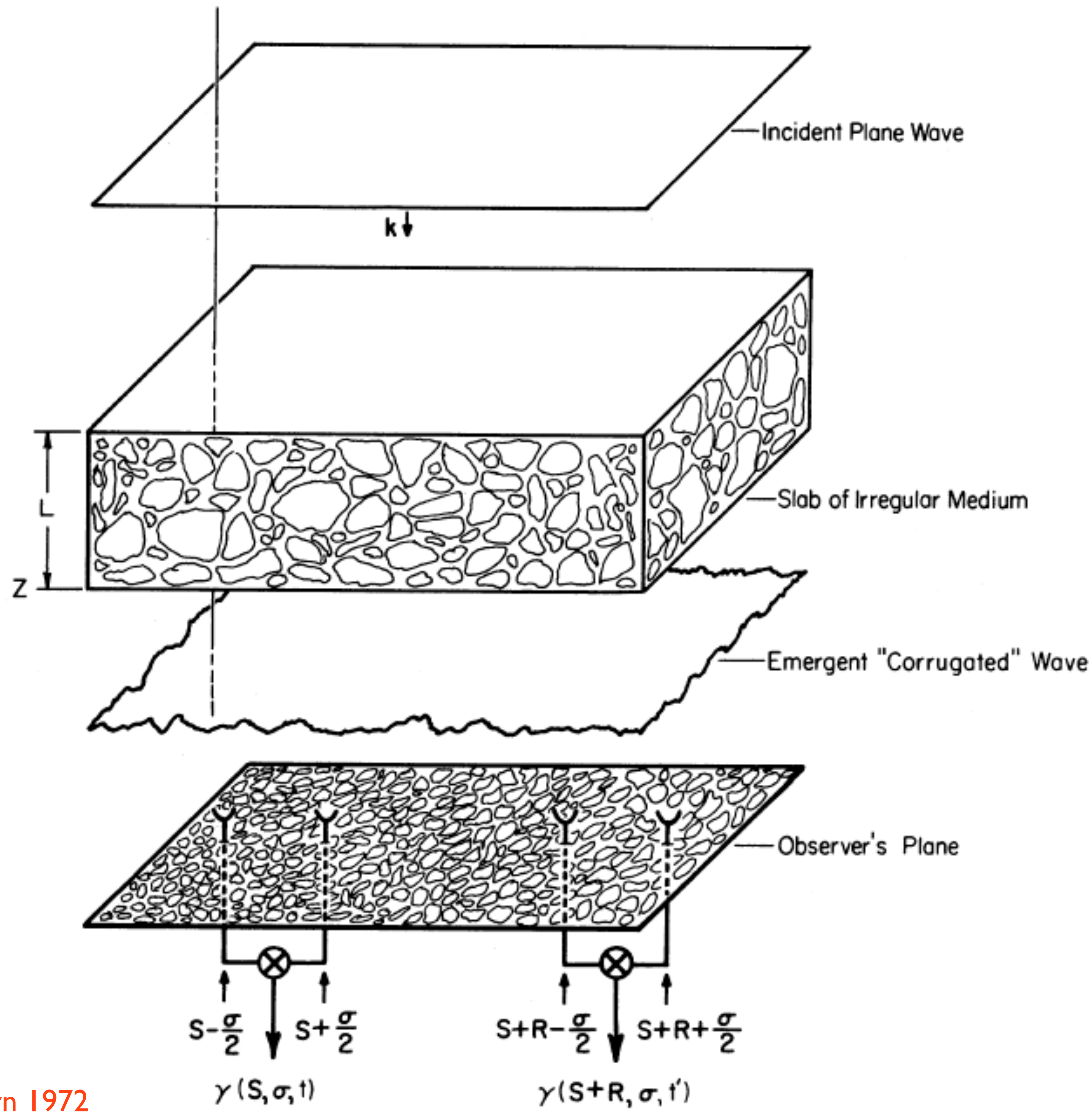
The ionosphere is often modeled either as a single thin slab, as a [set of directionally dependent] phase screen[s] or even as a rubber sheet.



Is this a correct approach, what physics is behind it and can this be done differently?

Ionospheric Phase Corrugations

Wavefront corrugation by the ionosphere



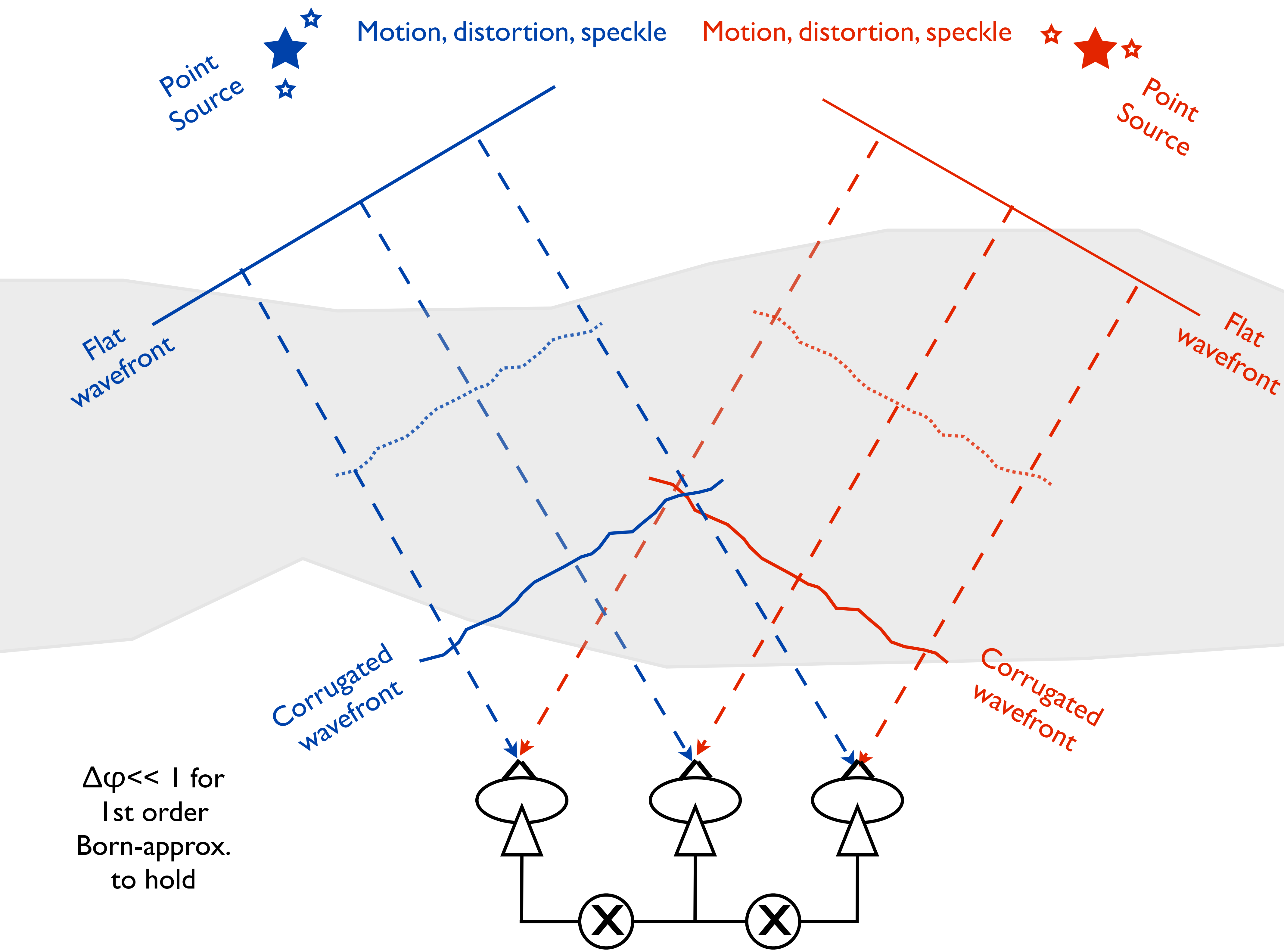
Point Source EM Wave

Ionosphere

Interferometer measurement
of corrugated wavefront

Cronyn 1972

Direction-Dependent Image Distortion



Δφ<< 1 for 1st order Born-approx. to hold

Fourier relation between flat wavefront and point source

↓

Wavefront is modulated by ionosphere

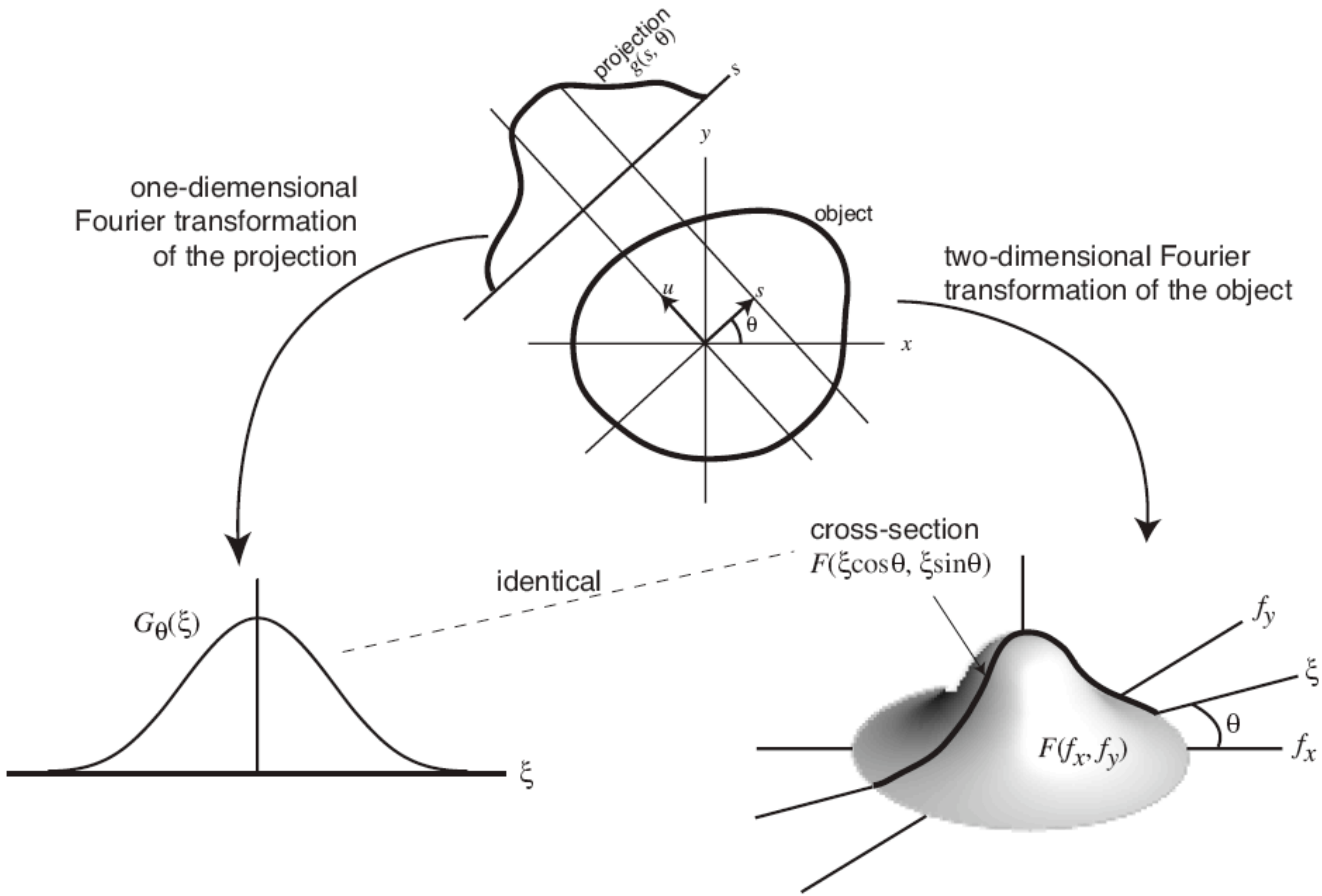
↓

Fourier relation between corrugated wavefront and a shifted, distorted point source possibly with a speckle halo

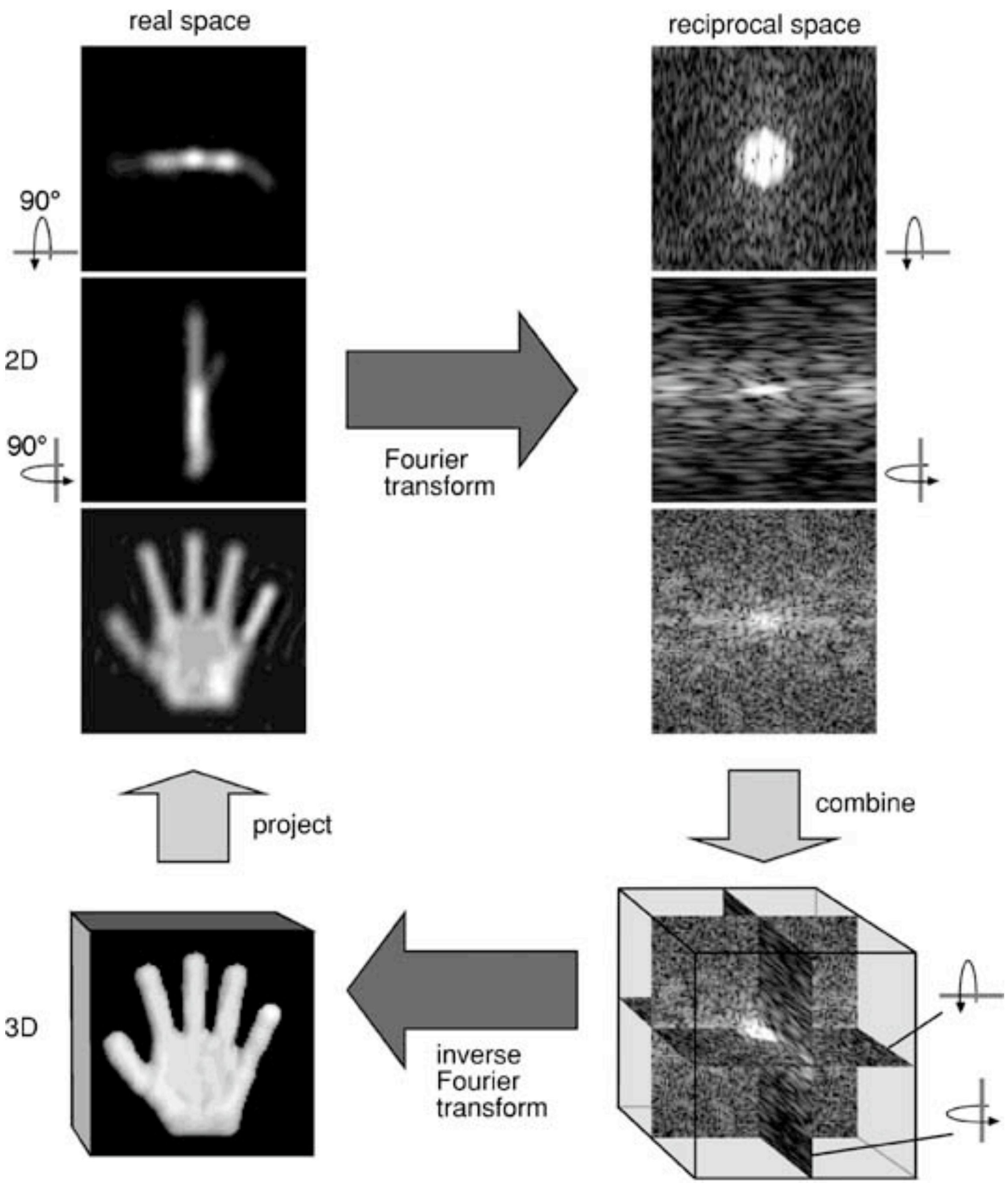
$$\delta I^{(s)}(s_u, s_v) = \langle \tilde{E}^{(s)*}(s_u, s_v)\tilde{E}^{(s)}(s_u, s_v)\rangle_{t}$$

Tomography

How to model this self-consistently: Fourier Slice Theorem



Radon projection



Tomography 101

1) Take 2D images of many projections of e.g. a hand (Radon projection)

2) FT them

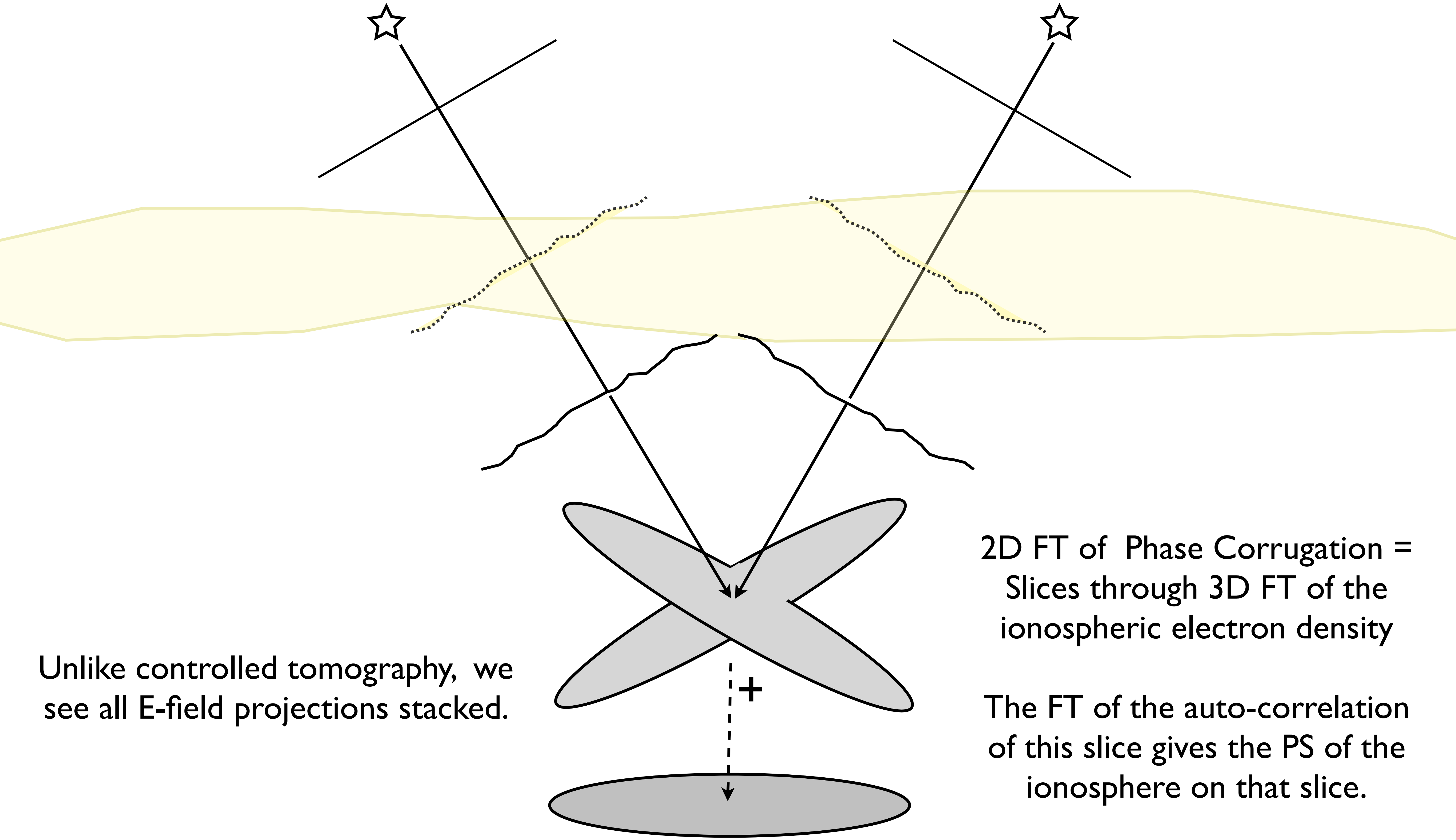
3) Combine 2D FTs into a single 3D FT. (Take projection angles into account)

4) 3D FT this to a 3D image of hand.

That is all there is to tomography

Phase distortion = $\delta\psi \approx k \int \delta n(\mathbf{r})\, ds$ = Radon Projection

Tomography can be used!



Unlike controlled tomography, we see all E-field projections stacked.

2D FT of Phase Corrugation = Slices through 3D FT of the ionospheric electron density

The FT of the auto-correlation of this slice gives the PS of the ionosphere on that slice.

Power-spectrum Tomography

Scattered Intensity — 2D Sky Intensity — 3D power-spectrum

$$\delta I^{(s)}(s_u, s_v) = \frac{1}{s_w^2} \iint I^{(i)}(s_{0,u}, s_{0,v}) \, |\tilde{\Phi}(\mathbf{s} - \mathbf{s}_0)|^2 ds_{0,u} ds_{0,v}$$

Δφ<< 1 for 1st order Born-approx. to hold

Directional Cosines

$$s_w^2 = 1 - s_u^2 + s_v^2$$

Geometric Term due to Curve Sky and Planar Array

Rescaled Intensity

$$\delta J^{(s)}(s_u, s_v) \equiv s_w^2 \cdot \delta I^{(s)}(s_u, s_v)$$

Koopmans 2010

Power-spectrum Tomography

A spherical surface slice through an 3D intensity cylinder

$$J^{(i)}_{3D}(\mathbf{s}_0) \equiv I^{(i)}_{3D}(\mathbf{s}_0) \cdot \delta(s_{0,w} - \sqrt{1 - s^2_{0,u} - s_{0,v}})$$

Rescaled scattering as a 3D convolution

$$\delta J^{(s)}(s_u, s_v) = \iiint J^{(i)}_{3D}(\mathbf{s}_0)\, |\tilde{\Phi}(\mathbf{s} - \mathbf{s}_0)|^2 d\mathbf{s}_0$$

Koopmans 2010

Power-spectrum Tomography

Using the Hankel Transform

$$\mathcal{H}(u,v;w) = 2\pi \int_0^1 e^{-2\pi i w \sqrt{1-s^2}} J_0(s \cdot u_{2\mathrm{D}})\; s\, ds$$

Fourier Transform of the phase-shifted w-term

one can show that the power-spectrum can be extracted from a model intensity and the scattered intensity

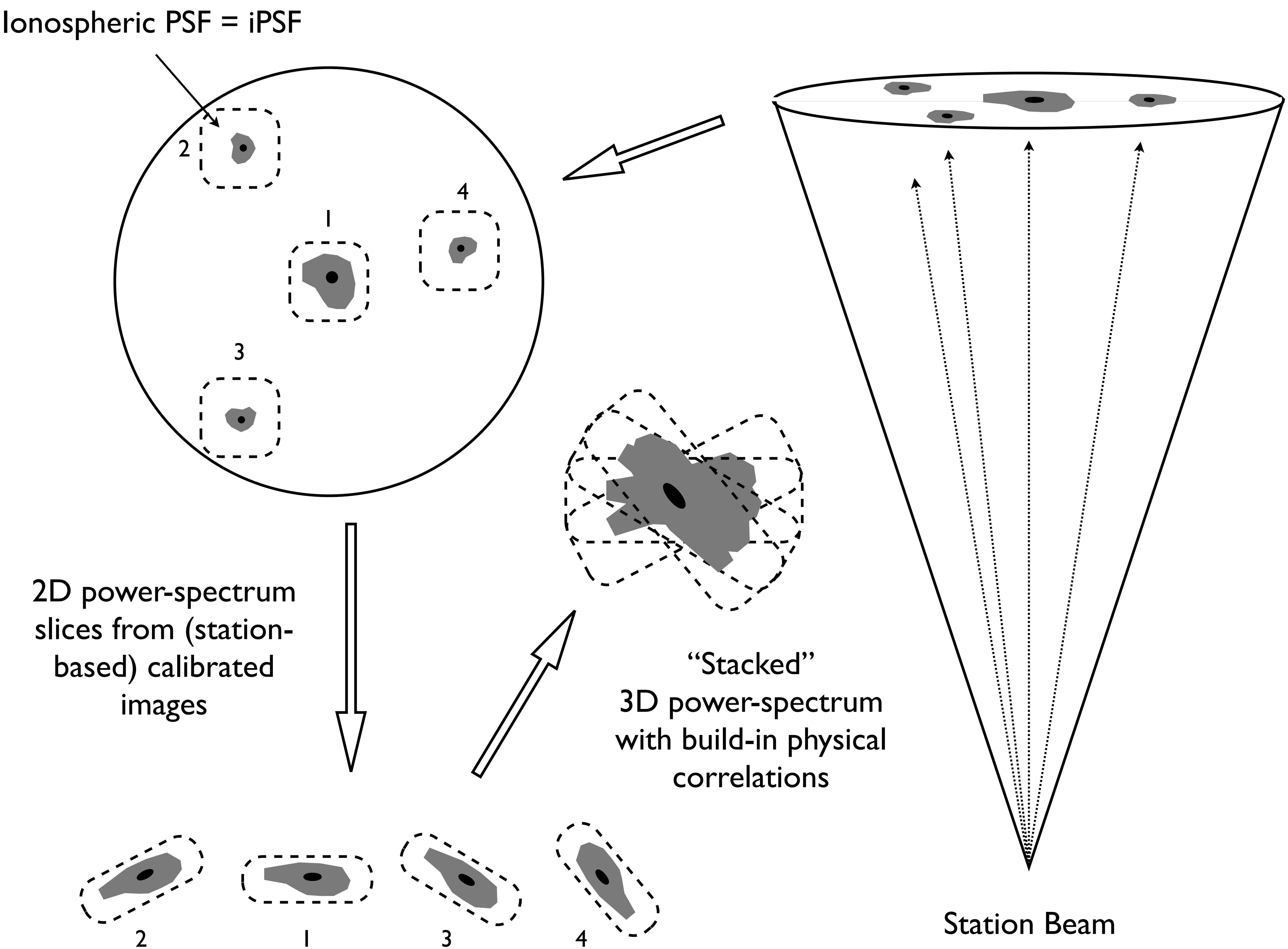
This is holography

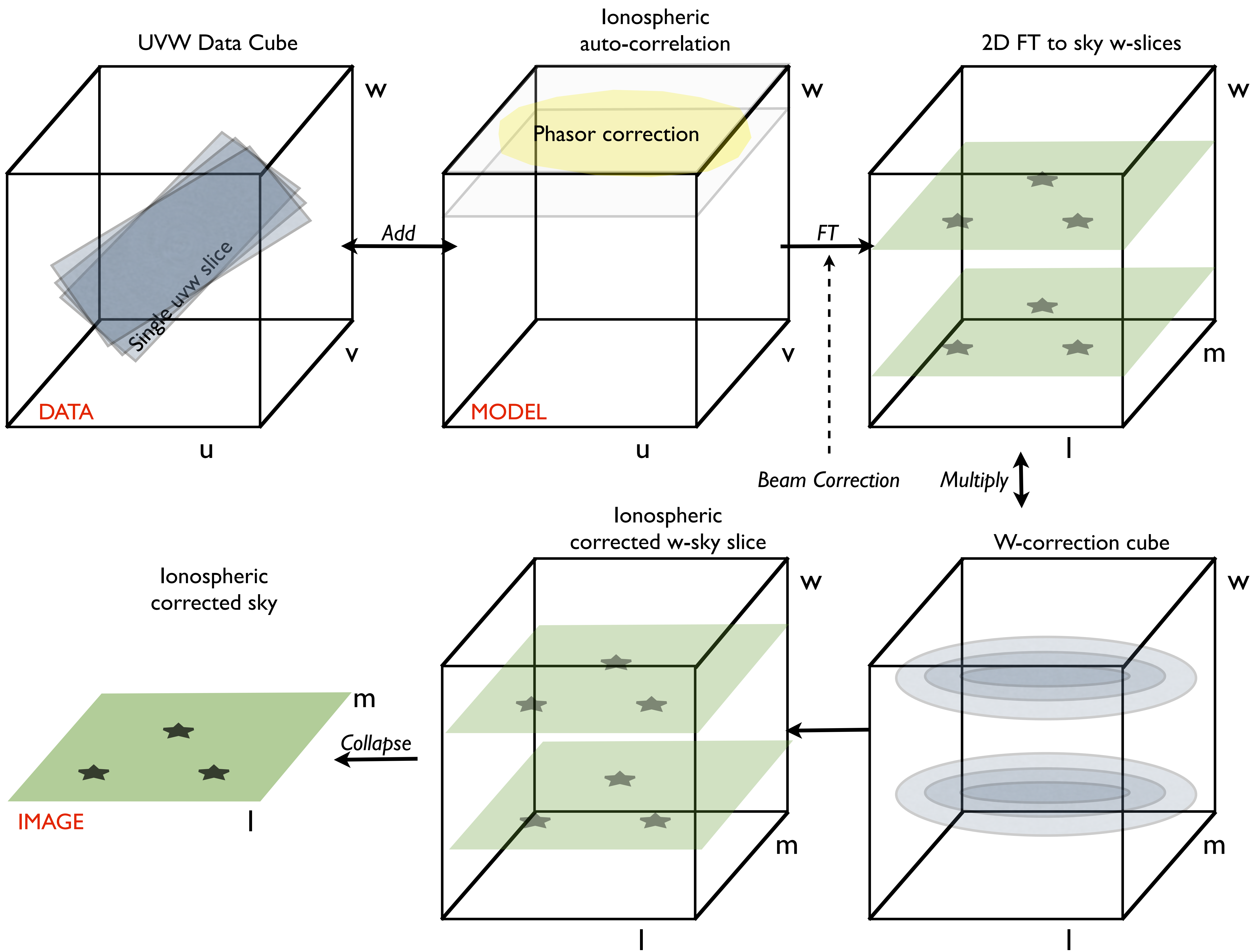
$$|\tilde{\Phi}(\mathbf{s})|^2 = \mathcal{F}^{-1}\left[\frac{\delta \tilde{J}^{(s)}(u,v)}{\tilde{I}^{(i)}(u,v) \otimes \mathcal{H}(u,v;w)}\right]$$

FT of sky residuals divided by FT of the sky model multiplied with a phase-shifted w-term

Note that the 3D power-spectrum is build up from interference of different w-slices

Koopmans 2010


Ionospheric PSF = iPSF
2
4
1
3
2D power-spectrum slices from (station-based) calibrated images
2
1
3
4
"Stacked" 3D power-spectrum with build-in physical correlations
Station Beam


UVW Data Cube
w
v
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Single uvw slice
DATA
Add
Ionospheric
auto-correlation
Phasor correction
w
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MODEL
FT
Beam Correction
2D FT to sky w-slices
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m
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Multiply
W-correction cube
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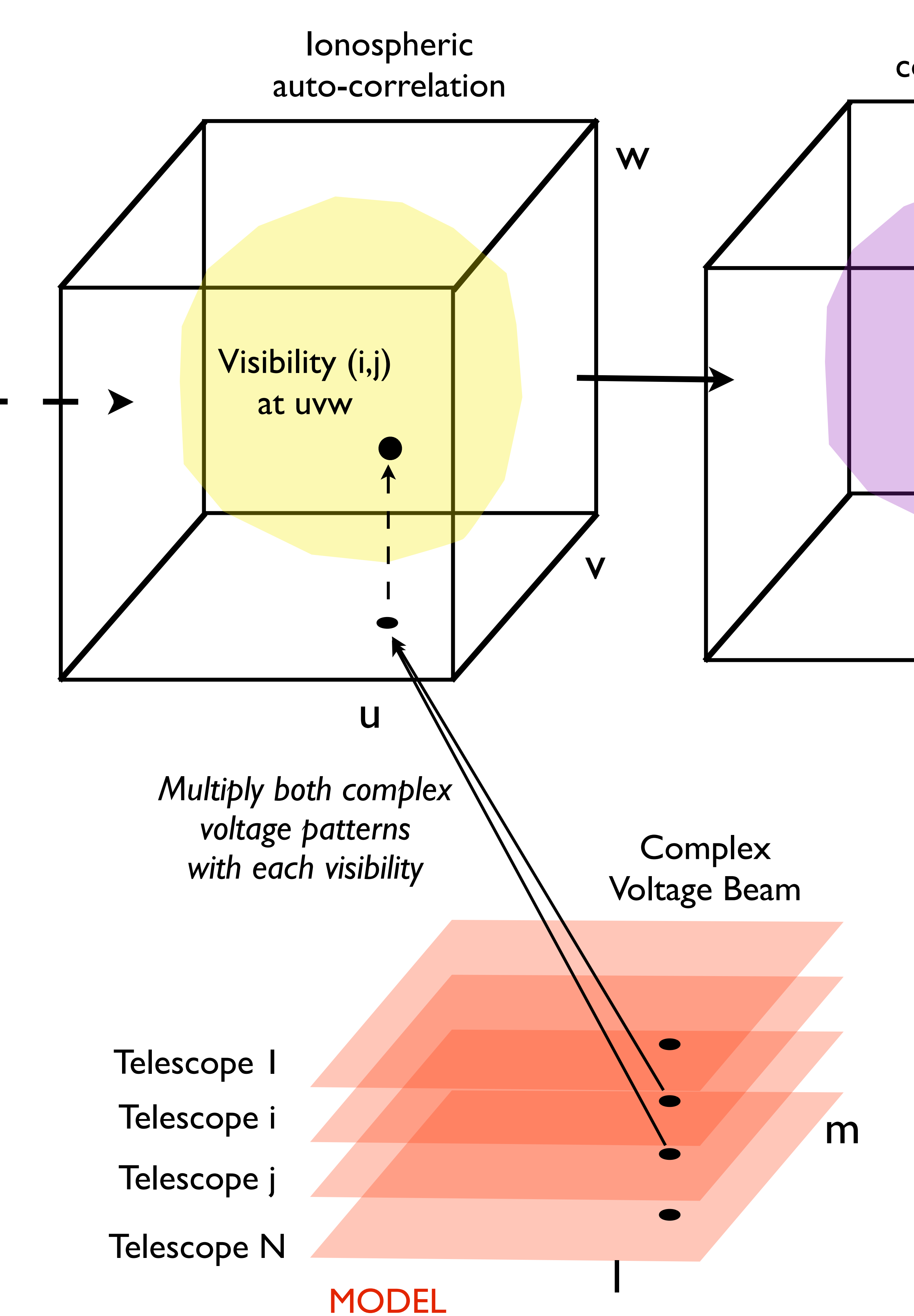
Ionospheric
corrected w-sky slice
w
m
l
Collapse
Ionospheric
corrected sky
m
l
IMAGE


Ionospheric
auto-correlation
w
Visibility (i,j)
at uvw
v
u
Beam
corrected uvw data
w
v
u
FT
2D FT to sky w-slices
w
m
l
Multiply both complex
voltage patterns
with each visibility
Complex
Voltage Beam
Telescope 1
Telescope i
Telescope j
Telescope N
m
l
MODEL

Build a Model the Ionospheric Power-Spectrum through Tomography: From Model to Data

(1) Build a 3D wedge (on an adaptive grid if needed) of the ionospheric PS as function of (l,m,n, [t]) or auto-correlation of the ionospheric refractive index. This model can be regularized in Fourier space and time. This model describe the iPSF for all directions in the beam (1st order Born approx.), as function of the PS grid-values.

(2) For each L/GSM model component direction (assuming some compact support), take the slice through the 3D PS model perpendicular to that direction (or it's FT for AC).

(3) Fourier Transform that slice in 2D and apply the phaso-correction to the L/GSM model component in the sampled UV space. This can be done for each frequency as well by simple scalings of the corruptions.

(4) Repeat over (3) and sum all iPSF-corrupted L/GSM model components in uv-space to build a data-model. One can also do this for "patches" of the sky so only a number of slices and their FTs are needed. The data will dictate the size of these patches.

(5) Compare this data-model to that observed to provide a penalty-function value.

(6) Minimize the penalty-function by varying the grid-values in (1) and repeating (2)-(5) till convergence. By regularizing in (l,m,n,[t]) one can make the model as smooth as required by physics (e.g. observed spatial and time correlations).

Summary & Conclusions

- The ionosphere causes spatial, time & freq. dependent phase fluctuations of the measured visibilities.

- For low-frequency large FoV interferometers (e.g. LOFAR, PAPER, MWA) this causes a spatially varying ionospheric scattering PSF

- To correct for this (in principle) a 3D model of the ionosphere is required.

- Since phase-fluctuations are Radon projections of the ionospheric refractive index (or electron density), tomographic (and holographic) methods can be used to model the ionosphere above the interferometric array.

- The FT of the phase corrugation of a point source with a plane EM wave is (neglecting curvature) a tilted slice through the power-spectrum of the ionosphere (Fourier slice theorem).

- By observing many compact sources in the FoV, one can attempt to build up a 3D PS model of the ionosphere slice-by-slice. This model is the equivalent of the self-cal loop for directionally independent errors.