The Principles of Radio Astronomical Data Reduction

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MCCT SKADS
Summer School on Third Generation Calibration

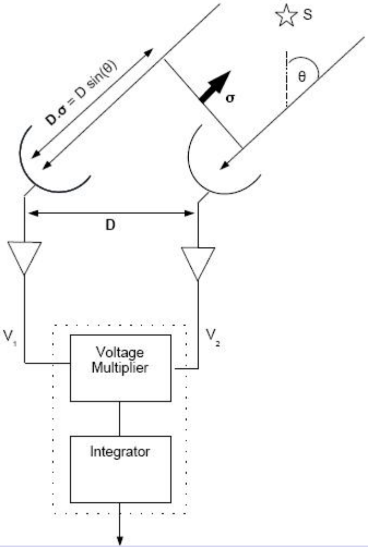
Outline

Pre-science Checklist

- Geometry
- Path Compensation (Geometrical delays AND phase center delays)
- Pointing
- Front-end noise ($T_{\rm sys}$)
- Phase switch demodulation

Correlator Units/Primary calibration

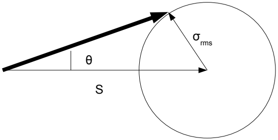
- Primary calibration ties the value of the correlator units to the flux scale.
- It accounts for moderate changes in atmospheric opacity.



- To account for more rapid variations in atmospheric opacity we can use noise injection systems to correct relative to our primary calibration value.

Secondary calibration

- In addition to the primary flux scale it is often necessary to employ a more frequent *phase* calibration.
- This secondary calibration accounts for drifts in the phase of the baselines due to things such as LO drifts.
- The phase stability of a given calibrator depends on its **flux**.



Outline

$$E_{\rm i} = [E^p\, E^q]_{\rm i}^{\rm T}$$

$$V_{\rm ij} = [V^{\rm pp}\ V^{\rm pq}\ V^{\rm qp}\ V^{\rm qq}]_{\rm ij}^{\rm T}$$

$$I = [I^{\rm pp}\ I^{\rm pq}\ I^{\rm qp}\ I^{\rm qq}]^{\rm T}$$

$$V_{\rm ij}(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(l,m){\rm e}^{-i2\pi[ul+vm+w(\sqrt{1-l^2-m^2}-1)]}\,\frac{dldm}{\sqrt{1-l^2-m^2}}$$

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$$\mathrm{K}_{\rm ij} = J_{\rm i} \otimes J_{\rm j}^\dagger$$

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Jones Formalism

$$K_{ij} = [J_i \otimes J_j^{\dagger}]^{vis,sky}$$

$$K_{ij}^{sky} = EPZ \qquad K_{ij}^{vis} = GDC$$

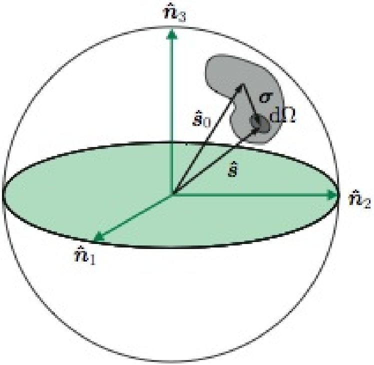
$$\mathrm{V}_{ij}^{obs} = K_{ij}^{vis} \int K_{ij}^{sky} \mathbf{I}^{sky}(\mathbf{s}) e^{-i2\pi \mathbf{D}_{\lambda} \cdot \sigma} d\Omega$$

$$\mathrm{V}^{obs} = K^{vis} S F K^{sky} \mathbf{I}^{sky}$$

Where,

$$\mathrm{V}^{obs} = \{V_m\}_{1 \leq m \leq M} \qquad \mathbf{I}^{sky} = \{I_n\}_{1 \leq n \leq N}$$

(for one correlation)



Gain

If we initially ignore the direction-dependent terms...

$$V^{obs} = K^{vis} S F I^{sky}$$

$$K^{vis}_{\rm ij} = g_{\rm i} g^*_{\rm j} = G_{\rm i} \otimes G^{\dagger}_{\rm j}$$

This can be solved using the standard self-calibration method:

$$\chi^2 = \sum_{\rm ij} w_{\rm ij} | V^{obs}_{\rm ij} - g_{\rm i} g^*_{\rm j} V^{model}_{\rm ij} |^2$$

$$V^{corr} = [K^{vis}]^{-1} V^{obs}$$

$$SFI^{sky} = V^{corr}$$

$$\underbrace{F^\dagger S^\dagger WSF}\, I^{sky} = F^\dagger S^\dagger W V^{corr}$$

- $\underbrace{\quad}$ represents the imaging properties of the telescope.
- The RHS is the *dirty map* of the image
  $\rightarrow$ if $V^{corr} = \{1\}_{1 \leq n \leq N}$ it equals the PSF.

Outline

Closure phase

When the complex gain terms differ to an extent that they may prohibit imaging entirely closure relation become important.

Phase of a baseline

$$\tilde{\phi}_{\rm ij} = \phi_{\rm ij} + \theta_{\rm i} - \theta_{\rm j} + n_{\phi,{\rm ij}}$$

Closure Phase

$$\begin{aligned}\tilde{\Phi}_{123,{\rm C}} &= \tilde{\phi}_{12} + \tilde{\phi}_{23} + \tilde{\phi}_{31} \\ &= \phi_{12} + \phi_{23} + \phi_{31} + n_{\phi,123} \\ &= \Phi_{123,{\rm C}} + n_{\phi,123}\end{aligned}$$

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Closure Phase

For N antennas $\Rightarrow$ 0.5N(N-1)-(N-1) closure phases

- We have N-1 too few closure phases to solve for all the visibility phases.
- We need an independent prediction of the phase on N-1 baselines.
- Answer: *just add it into SELF-CAL*.

Aside on Imaging

- Although it is possible to restore images using a DFT it is more usual (due to scaling constraints) to use an FFT.
- This requires the visibilities to be *re-sampled* onto a regular grid.
- Typically this is achieved using a shah function and an appropriate gridding kernel.
- The gridding kernel ensures a smooth visibility distribution and *reduces aliasing*.

The choice of gridding kernel is crucial...

$$R = \frac{\int \int_{\mathrm{A}} |\tilde{C}(l,m)|^2 dldm}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\tilde{C}(l,m)|^2 dldm} \quad (1)$$

The PSF satisfies the ODE:

$$(1-\eta^2)\frac{d^2 Y_{mn}(c,\eta)}{d^2\eta} - 2(m+1)\eta\frac{dY_{mn}(c,\eta)}{d\eta} - + \left(c^2\eta^2 + m(m+1) - \lambda_{mn}(c)\right) Y_{mn}(c,\eta) = 0,$$

...but fortunately there is a numerical approximation.

In terms of our matrix notation we can include the gridding kernel as:

$$G^{\mathrm{gc}} = F(F^{\dagger}\tilde{C})F^{\dagger}$$

In the image plane...

$$I_{wt}^{\mathrm{gc}} = F^{\dagger}\tilde{C}$$

So, infact when we make an image from our visibilities we are doing:

$$I^{dirty,psf} = w_{sum}^{-1}[I_{\mathrm{gc}}^{wt}]^{-1}F^{\dagger}G^{\mathrm{gc}}S^{\dagger}V^{corr,1}$$

(w_{sum} normalises the peak of the PSF)

Outline

DD-Effects

$$V^{obs} = SG^{\mathrm{dd}}V^{sky}$$

$$G^{\mathrm{dd}}_{\mathrm{ij}} = diag(FK^{sky})$$

- G^{dd} should have compact support.
- G^{dd} should be (approximately) unitary.
- Both are true for Primary Beam and W-projection corrections

Generalised DD effects

$$I^{dirty,psf} = [I^{wt}_{\rm dd}]^{-1}[I^{wt}_{\rm gc}]^{-1} F^{\dagger} G^{\rm gc} G^{\rm dd\dagger} S^{\dagger} W^{im} V^{corr,1}$$

$$I^{wt}_{\rm dd} = F^{\dagger} G^{\rm dd\dagger} (W^{im}) G^{\rm dd} F$$

Outline

Non-coplanar baselines

$$V_{\rm ij}(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(l,m)I(l,m){\rm e}^{-i2\pi[ul+vm+w(\sqrt{1-l^2-m^2}-1)]}\frac{dldm}{\sqrt{1-l^2-m^2}}$$

$$w\sqrt{1-l^2-m^2} \ll 1$$

$$V_{\rm ij}(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(l,m)I(l,m){\rm e}^{-i2\pi[ul+vm]}dldm$$

- However, when we have *large fields of view* or *non-coplanar antennas* we cannot make this simplification.
- We could correct this by only imaging small parts of our primary beam at once and mosaicking them...
- Or we can correct for this analytically using the **w-projection**, which is an order of magnitude faster.

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W-projection

- The effect of the **w**-term is to introduce a baseline dependent phase shift.
- In addition the *structure* of a particular object on the sky will vary as seen by individual baselines.

$$V_{\rm ij}(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(l,m)I(l,m)\mathrm{e}^{-i2\pi[ul+vm+w(\sqrt{1-l^2-m^2}-1)]}\frac{dldm}{\sqrt{1-l^2-m^2}}$$

$$V_{\rm ij}(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(l,m)I(l,m)G(l,m,w)\mathrm{e}^{-i2\pi[ul+vm]}\frac{dldm}{\sqrt{1-l^2-m^2}}$$

$$\mathrm{G(l,m,w)} = \mathrm{e}^{-i2\pi[w\sqrt{1-l^2-m^2}]}$$

$$\mathrm{G(u,v,w)} = \sin\left(\frac{\pi(u^2+v^2)}{2w}\right)/2w$$

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Time–Varying, Direction–Dependent Primary Beams

- Extremely high dynamic range imaging requires that we can no longer assume a single analytic beam model for our antennas.
- Nor can we assume that all our antennas have identical beams...
- In the case of phased arrays this was never going to be the case anyway.
- We could correct for this baseline by baseline in the image plane... but this would require two FFT steps, which makes it computationally slow.
- The PB is the autocorrelation function of the two aperture illumination functions of the consitituent antennas in our baseline - so it has finite support in Fourier space.

Outline

The ionosphere introduces an extra bit of path length, which is a function of frequency.

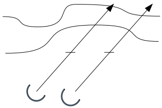
Ionospheric phase

$$\Delta x = -4\lambda^2 \, \mathrm{TEC}$$

$$\Downarrow$$

$$\phi_{\mathrm{iono}} = -25\lambda \, \mathrm{TEC}$$

One might think of the ionospheric phase as a kind of *phase primary beam*, but unlike the gain Primary Beam it does not necessarily have compact support.



The ionosphere is different along each line of sight (l,m) from each antenna (x,y). The baselines will only see the **difference** between these directions...

Differential phase

$$\Delta\phi_{\rm iono} = -25\lambda\Delta{\rm TEC} = -25\lambda({\rm TEC}_1 - {\rm TEC}_2)$$

- So each baseline gets an extra bit of phase - which moves the source position.
- If all the baselines put the source at a slightly different location then the source gets *smeared* around its true position.

Calibrating out the ionosphere

Current Methods

- Field-based calibration
- Peeling

We can calibrate out the atmosphere (to some degree) by implementing an ionospheric phase term in self-cal.

SPAM

$$\phi_{\rm i} = \phi_{\rm i}^{instr} + \phi_{\rm i,(l,m)}^{iono} - \phi_{\rm ref} - \phi_{\rm i,(l,m)}^{ambig}$$

$$\chi^2 = \sum_i \left[\left(\phi_{\rm i} - \tilde{\phi}_{\rm i}^{instr} + \tilde{\phi}_{\rm i,(l,m)}^{ambig} \right) - G_{(l,m)} \cdot (x_{\rm i} - x_{\rm ref}) \right]^2$$

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Conclusions

Calibration check-list

- Visibility Complex Gains
- Polarization Leakage
- Antenna Gains
- Ionospheric Effects

For Further Reading I

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