Revised Schedule

 $9:00 \sim 10:30$ session 1 $10:30 \sim 11:00$ coffee $11:00 \sim 12:30$ session 2 $12:30 \sim 13:30$ lunch $13:30 \sim 15:00$ session 3 $15:00 \sim 15:30$ coffee $15:30 \sim 17:30$ session 4 $17:30 \sim 9:30$ beer & homework

ME1: ME Of A Point Source

ntro1: MegTree Basics

The Measurement Equation: Putting the "Meq" into MeqTrees!

- The Measurement Equation tells you what you can expect to observe with an interferometer, given a sky and the properties of your instrument.
- Absolutely crucial for simulating and calibrating the next generation of radio telescopes; everything literally revolves around it.
- **Therefore:** no-one gets any beer tonight until we achieve full harmony and understanding!

ME1: ME Of A Point Source

ME1: Measurement Equation Of a (Polarized) Point Source

Objectives:

- Aligning our terminology!
- Mapping your existing intuition about interferometry onto ME concepts
- Implementing some MEs using MeqTrees.

ME1: ME Of A Point Source

Survey Results...

- 1. Radio interferometry...
 - [1] heard of it
 - [11] basic knowledge
 - [6] do it all the time
 - [3] I was doing it when Oleg was in diapers

Therefore...

...you know practically everything about the Measurement Equation!

On The Other Hand...

3. The Measurement Equation...

- [1] never heard of it
- [2] heard of it
- [10] know what it looks like, never actually used it
- [6] know it pretty well
- [2] I use Jones matrices to do my taxes

Conclusion:

The ME is no longer one of these unknown knowns -- the things we don't know we know -- that Donald Rumsfeld didn't know he knew.

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A Wafer-Thin Slice of Physics: Jones Matrices

- * J is called a Jones matrix.
- * **J** is obviously cumulative:

$$\vec{v} = J_n(J_{n-1}(\dots J_1\vec{e})) = (\prod_{i=n}^{1} J_i)\vec{e} = J\vec{e}$$

where $J_1...J_n$ describes the full signal path. * Do remember that matrices, in general, do not commute.

NB: What if something is non-linear?.. ...we can also write down an equation: $\vec{v} = \mathcal{J}(\vec{e})$...but this is to be avoided if at all possible.

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A Wafer-Thin Slice of Physics: EM Field Propagation

Pick an *xyz* frame with *z* along the direction of propagation. The EM field can be described by the complex vector $\vec{e} = \begin{pmatrix} e_x \\ e_y \end{pmatrix}$ The fundamental assumption is **LINEARITY**: 1. Propagation through a medium is linear \Rightarrow can be fully described by a 2x2 complex matrix: $\vec{e} = J\vec{e}$ i.e. $\begin{pmatrix} e_x \\ e_y \end{pmatrix} = \begin{pmatrix} \Box & \Box \\ \Box & \Box \end{pmatrix} \begin{pmatrix} e_x \\ e_y \end{pmatrix}$ 2. Receptor voltages $\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$ are also linear w.r.t. \vec{e} $\Rightarrow \vec{v} = J\vec{e}$

A Wafer-Thin Slice of Physics: Correlations & Visibilities

An interferometer measures *correlations* btw voltages \vec{v}_p, \vec{v}_q : $v_{xx} = \langle v_{px} v_{qx}^*, v_{xy} = \langle v_{px} v_{qy}^*, v_{yx} = \langle v_{py} v_{qx}^*, v_{yy} = \langle v_{py} v_{qy}^* \rangle$ It is convenient to represent these as a matrix product: $V_{pq} = \langle \vec{v}_p \vec{v}_q^\dagger = \langle \begin{pmatrix} v_{px} \\ v_{py} \end{pmatrix} (v_{qx}^* v_{qy}^*) = \begin{pmatrix} v_{xx} & v_{xy} \\ v_{yx} & v_{yy} \end{pmatrix}$ ($\langle : \text{time/freq averaging}; t: \text{conjugate-and-transpose}$) V_{pq} is also called the *visibility matrix*. Now let's assume that all radiation arrives from a single point, and designate the "source" E.M. vector by \vec{e} .

A Wafer-Thin Slice of Physics: The M.E. Emerges

Antennas p,q then measure: $\vec{v}_p = J_p \vec{e}$, $\vec{v}_q = J_q \vec{e}$ where J_p, J_q are Jones matrices describing the signal paths from the source to the antennas.

Then
$$\mathbf{V}_{pq} = \langle (\mathbf{J}_{p} \vec{e}) (\mathbf{J}_{q} \vec{e})^{\dagger} = \langle \mathbf{J}_{p} (\vec{e} \vec{e}^{\dagger}) \mathbf{J}_{q}^{\dagger} = \mathbf{J}_{p} \langle \vec{e} \vec{e}^{\dagger} \ \mathbf{J}_{q}^{\dagger}$$

making use of $(\mathbf{A}\mathbf{B})^{\dagger} = \mathbf{B}^{\dagger} \mathbf{A}^{\dagger}$, and assuming \mathbf{J}_{p} is constant over $\langle \ \rangle$
The inner quantity is known as the *source coherency*:

$$\mathbf{B} = \langle \vec{e} \, \vec{e}^{\dagger} = \frac{1}{2} \begin{pmatrix} I + Q & U \pm iV \\ U \mp iV & I - Q \end{pmatrix} \iff (I, Q, U, V)$$

which we can also call the *source brightness*. Thus:

$$\boldsymbol{V}_{pq} = \boldsymbol{J}_{p} \boldsymbol{B} \boldsymbol{J}_{q}^{T}$$







Why is this great?

- A complete and mathematically elegant framework for describing all kinds of signal propagation effects.
- ...including those at the antenna, e.g.:
 - beam & receiver gain
 - dipole rotation

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- receptor cross-leakage
- Effortlessly incorporates polarization:
 - think in terms of a **B** matrix and never worry about polarization again.
- Applies with equal ease to heterogeneous arrays, by using different Jones chains.

Why is this even greater?

• Most effects have a very simple Jones representation:

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Observing a point source with a perfect instrument

Even w/o instrumental effects, we still have geometry, so:

 $V_{pq} = K_p B K_q^{\dagger}$ $K_p \text{ is the phase shift term, a scalar Jones matrix:}$ $K_p = \begin{pmatrix} e^{-i\phi_p} & 0\\ 0 & e^{-i\phi_p} \end{pmatrix} \equiv e^{-i\phi_p}$ Antenna phase ϕ_p accounts for the pathlength difference: $\phi_p = 2\pi (u_p l + v_p m + w_p (n-1))$

where u_p, v_p, w_p are antenna coordinates (in wavelengths), and l, m, n are the direction cosines for the source.

 $(n=\sqrt{1-l^2-m^2}, \text{ for "small" fields } n \rightarrow 1.)$

The (familiar?) Scalar Case

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'Classic' (scalar) visibility of a source: $v_{pq} = Ie^{-i\phi_{pq}}$

where ϕ_{na} is the interferometer phase difference:

 $\phi_{pq} = 2\pi (u_{pq}l + v_{pq}m + w_{pq}(n-1))$

Baseline coordinates $\vec{u}_{pq} = (u_{pq}, v_{pq}, w_{pq})$ have a very simple relationship to antenna coordinates.





K: Breaking The Coherency Barrier

- The interferometer phase term is traditionally considered separately; however, it fits the Jones formalism like a glove.
- **K** combines two physical effects:
 - pathlength difference
 - time delays (i.e. fringe stopping)
- Scalar matrices commute with everything, so we're allowed to "merge" these two effects and shift the resulting *K*'s around.
- K is scalar only for co-located receivers:
 moral: keep your dipoles together!
- Forget about the Fourier Transform for now...

Building a Tree: Matrix Multiplication

22

$$\boldsymbol{V}_{pq} = \boldsymbol{K}_{p} \boldsymbol{B} \boldsymbol{K}_{q}^{\dagger}$$

See ME1/demo1-predict-ps.py

ME1: ME Of A Point Source

- The Meq.MatrixMultiply node implements matrix multiplication.
- We repeat this for all interferometers (all *p-q* pairs), in a **for** loop.





Building a Tree: Where to get meta-data?

Where do uvw's come from?

- *uvw*'s can be computed by a **Meq.UVW** node.
- This requires antenna positions, time, and the phase centre RA/Dec:
 - time: comes from the MS grid (via the request)
 - phase centre RA/Dec: from MS sub-tables
 - antenna positions: from MS sub-tables
- Don't want to hard-code this stuff, else our script will be tied to a particular MS.
- Need a way to get observational parameters from the MS and put them into the tree.

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Building a Tree: Attaching an init-script

26

- An *init-script* is a Python script that is executed *on the kernel* side by the VisDataMux node. The script name is specified as part of the I/O request.
- It can provide a *MS header handler* (among other things).
- The MS header contains all observational parameters.
- The header handler can put the required values into "placeholder" nodes (finding them by name).
- The same script can be used for all trees, as long as our placeholder nodes follow the same naming convention.
- i.e. "ra", "dec" for phase center
- "x0", "y0", "z0" for array center
- "x:p", "y:p", "z:p" for position of antenna #p.
- Placeholders are created as constants, e.g. "ns.ra<<0".





We'll use the AIPS++ imager...

2. AIPS++...

[5] heard of it[2] tried to run it once[9] succeeded in running it once[5] have used it in anger[0] invented it

- We have enough expertise in this room...
- ...no-one to blame this time though.



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TDL Jobs: Doing other useful stuff

- _test_forest() is a "TDL job".
- More jobs can be added by defining functions called _tdl_job_foo(), all these will be automatically placed into the "Exec" menu.
- Jobs can contain arbitrary Python code...
 ...including calling the shell...
 - ...e.g. to run Glish and call the AIPS++ imager.
- See ME1/demo2-predict-ps-image.py



Time Variability On the Cheap

22

Something like: $g = e^{iA\sin(Bt+C)}$

- Remember that we get a "time grid" with each request.
- The Meq.Time node returns f(t) = t, combine it with a Meq.Sin node to compute A*sin(Bt+C)
- Generate random *A*,*B*,*C*'s per dipole (using Python's **random** module)
- Meq.Polar builds x*exp(iy)
- Run the tree (load up the bookmarks).
- Make a map.

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 note that we can now select a column to image, the new simulation is in DATA, the old one is in MODEL_DATA.

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Some Performance Considerations

$\boldsymbol{V}_{pq} = \boldsymbol{G}_{p} \boldsymbol{K}_{p} \boldsymbol{B} \boldsymbol{K}_{q}^{\dagger} \boldsymbol{G}_{q}^{\dagger}$

- This does N_{time} xN_{freq} x4 individual matrix multiplications.
- **B** is constant, **G**'s are variable in time, and **K**'s are variable in time-freq.
- If we reorder the terms as follows (*K* commutes):

$\boldsymbol{V}_{pq} = \boldsymbol{K}_{p} (\boldsymbol{G}_{p} \boldsymbol{B} \boldsymbol{G}_{q}^{\dagger}) \boldsymbol{K}_{q}^{\dagger}$

- we end up with $N_{time} x^2 + N_{time} x N_{free} x^2$ ops.
- Is this a good idea?

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To Quote Ancient Wisdom...

"Premature optimization is the root of all evil." -- Donald Knuth

"More computing sins are committed in the name of efficiency (without necessarily achieving it) than for any other single reason – including blind stupidity."

-- W.A. Wulf

- Don't worry about optimizing for calculations until you establish that performance is a problem...
- ...and where the problem lies.
- Jones terms don't always commute, so think carefully before moving them around.
- But if you can reshuffle them, significant CPU savings may in fact result.

ME1: ME Of A Point Source Exercise 1: Instrumental Polarization

- Use ME1/demo3-predict-ps-gain.py as a starting point.
- Make the source unpolarized, with *I*=1Jy.
- Make the gain amplitudes frequency-dependent (and reset the gain phases to 0):

$$\boldsymbol{G}_{p} = \begin{pmatrix} 1 + a_{p}(v - v_{0}) & 0 \\ 0 & 1 - a_{p}(v - v_{0}) \end{pmatrix}$$

- pick random a 's in the range [1e-10,1e-9]
- For v_0 , create placeholder (ns.freq0<<0)
- Produce a per-channel map, look at Q fluxes.

Exercise 2: Alt-Az Mounts

- A "perfect" instrument has an equatorial mount, i.e. stationary sky.
- With an alt-az mount, the sky rotates relative to each antenna, so we must add a rotation term to our M.E.

$$\boldsymbol{V}_{pq} = \boldsymbol{G}_{p} \boldsymbol{P}_{p} \boldsymbol{K}_{p} \boldsymbol{B} \boldsymbol{K}_{q}^{\dagger} \boldsymbol{P}_{q}^{\dagger} \boldsymbol{G}_{q}^{\dagger}$$

$$\mathbf{P}_{p} = \begin{pmatrix} \cos \gamma_{p} & -\sin \gamma_{p} \\ \sin \gamma_{p} & \cos \gamma_{p} \end{pmatrix} \equiv \operatorname{Rot} \gamma_{p}$$



MEL: ME Of A Point Source **Exercise 2: Alt-Az Mounts** • Use ME1/demo3-predict-ps-gain.py as a starting point. • Use *I*=1 Jy, *Q*=.2 Jy • Insert the gain terms from Exercise 1. • Add a **P** term to model sky rotation. • Produce MFS and per-channel maps of *IQUV* flux. • Hint: **Meq.ParAngle** computes the P.A. as a function of time. It expects a **radec** child (phase center), and an **xyz** child (station position). $V_{pq} = G_p P_p K_p B K_q^{T} P_q^{T} G_q^{T}$ $P_p = \begin{pmatrix} \cos \gamma_p & -\sin \gamma_p \\ \sin \gamma_p & \cos \gamma_p \end{pmatrix}$





- Simulate a transient source, make an MFS and a per-channel map.
 For bonus points, make the source narrow-band.



41