# LOFAR Imager: taking Direction Dependent Effects into account using A-Projection

Cyril Tasse, Ger van Diepen, Joris van Zwieten, Bas van der Tol

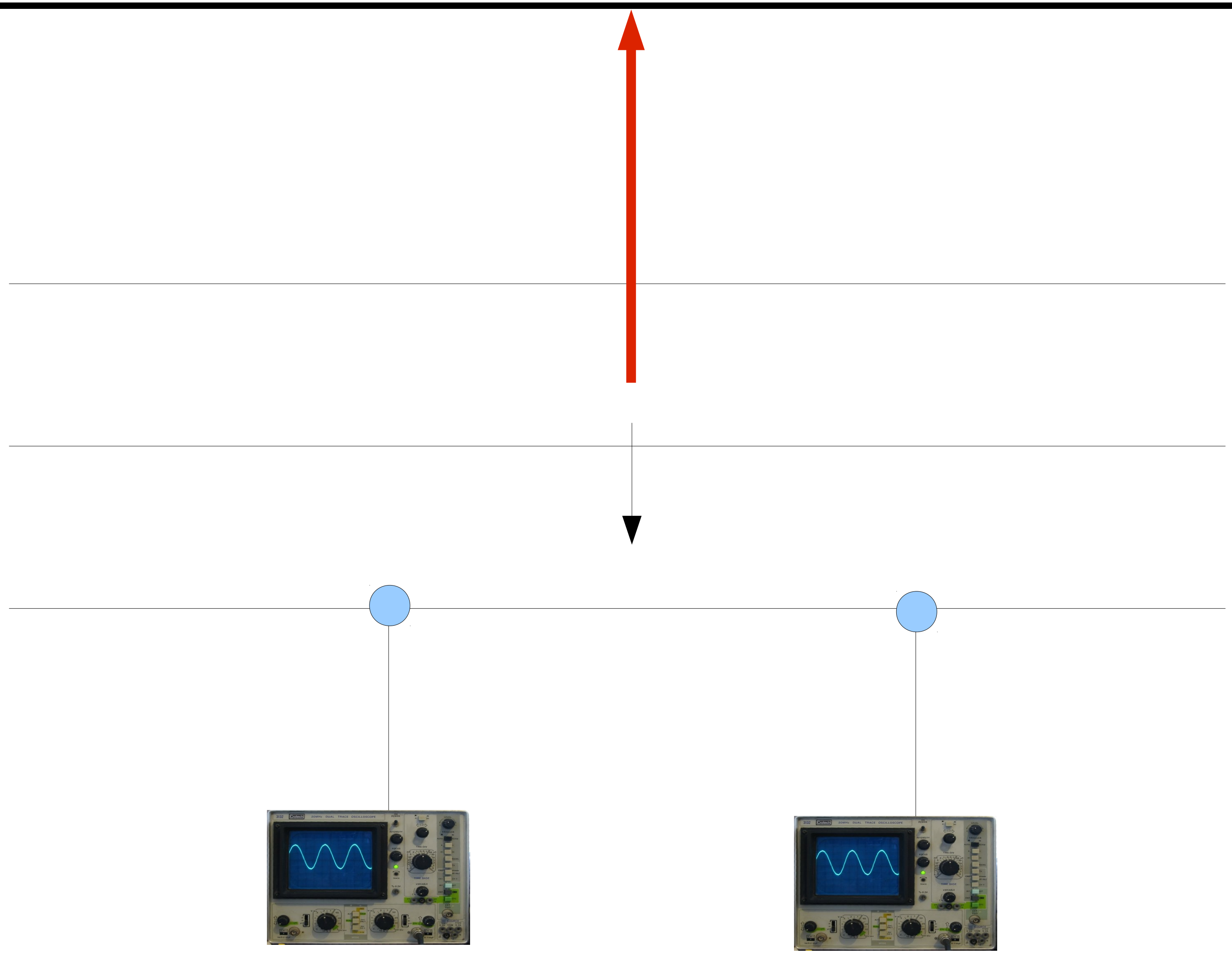
Sanjay Bhatnagar, Urvashi Rau, Kumar Golap

# Outline

- Imaging for the dummies
- UV-Brick
- A-Projection

# Principle

FOR DUMMIES



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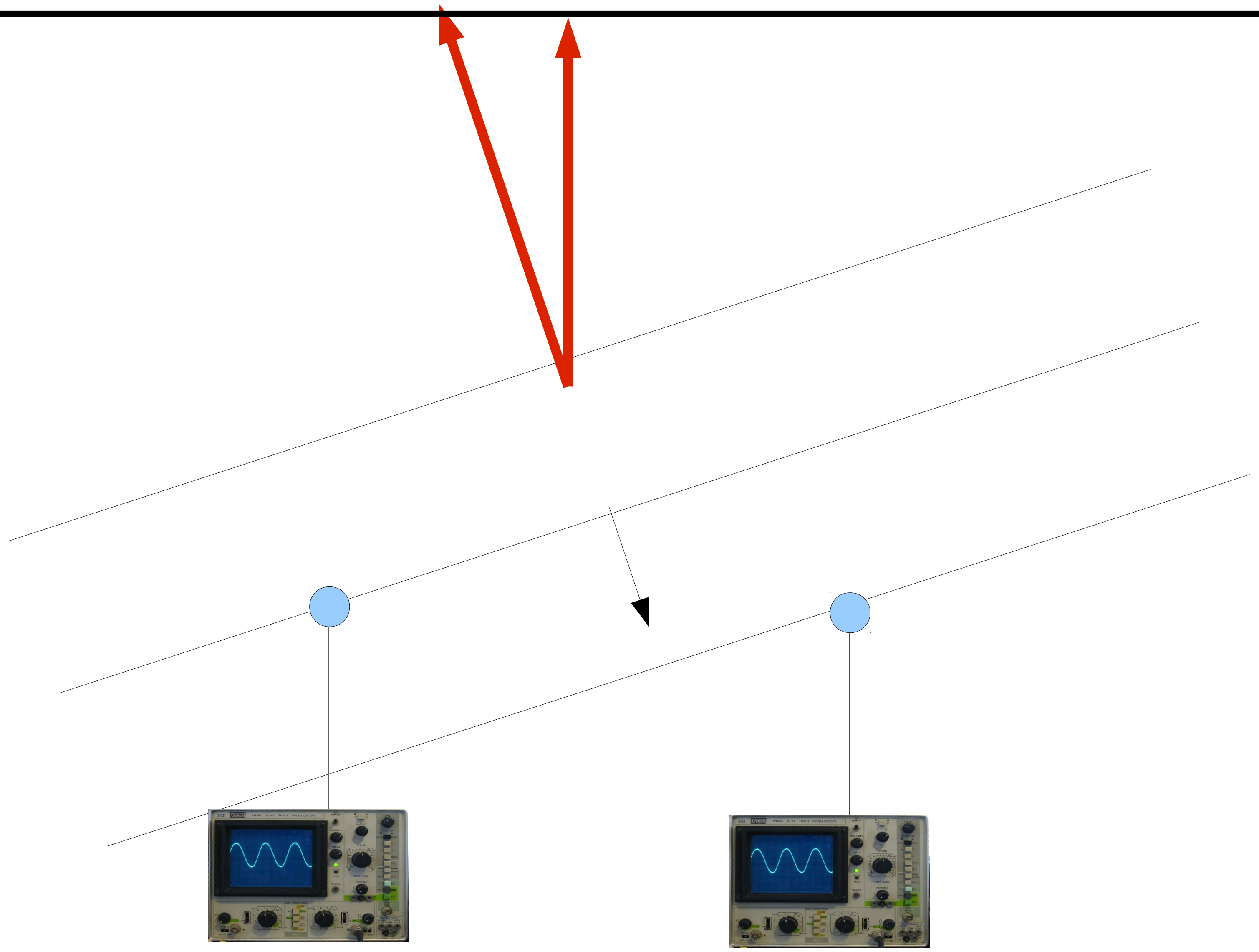

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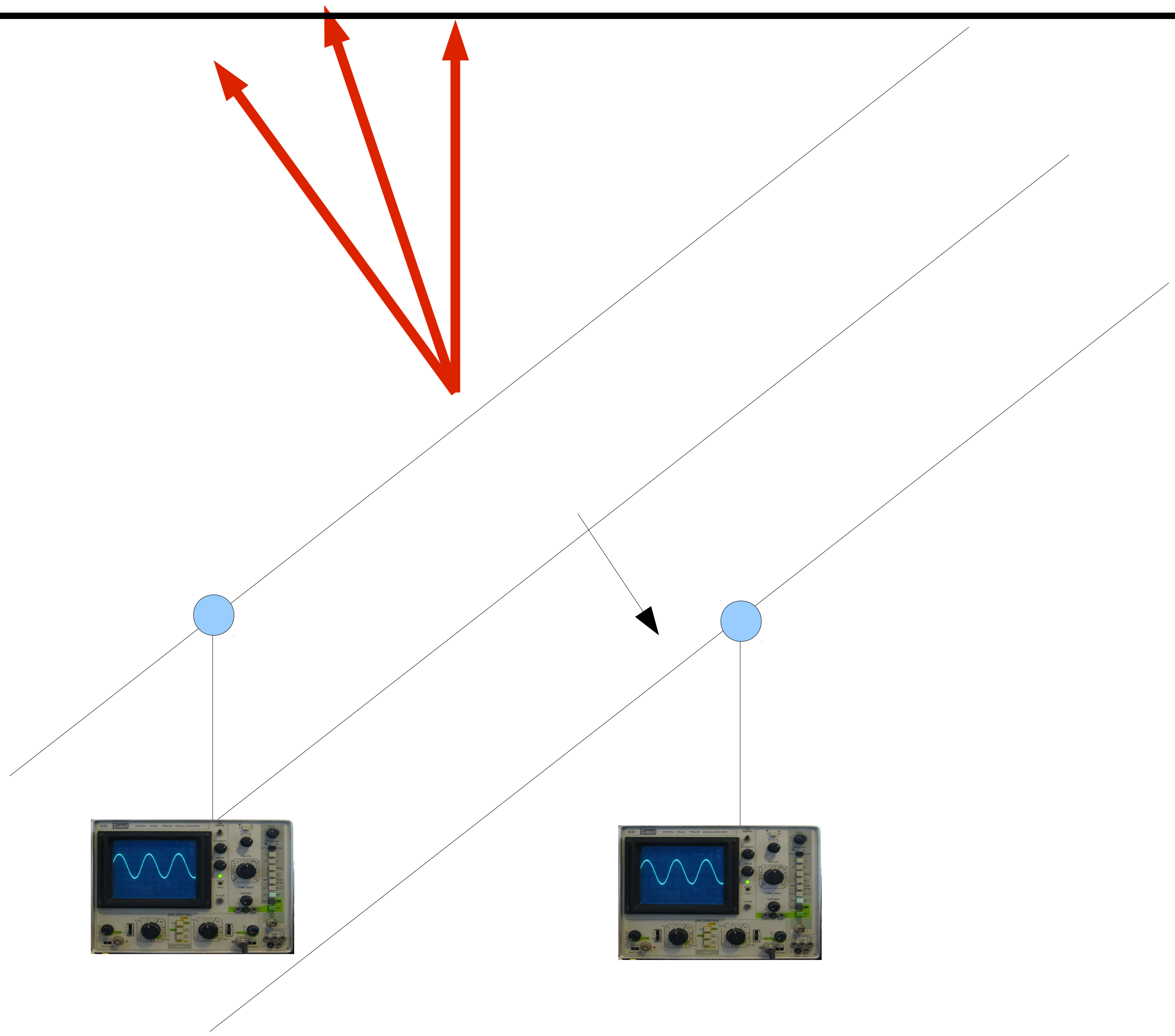

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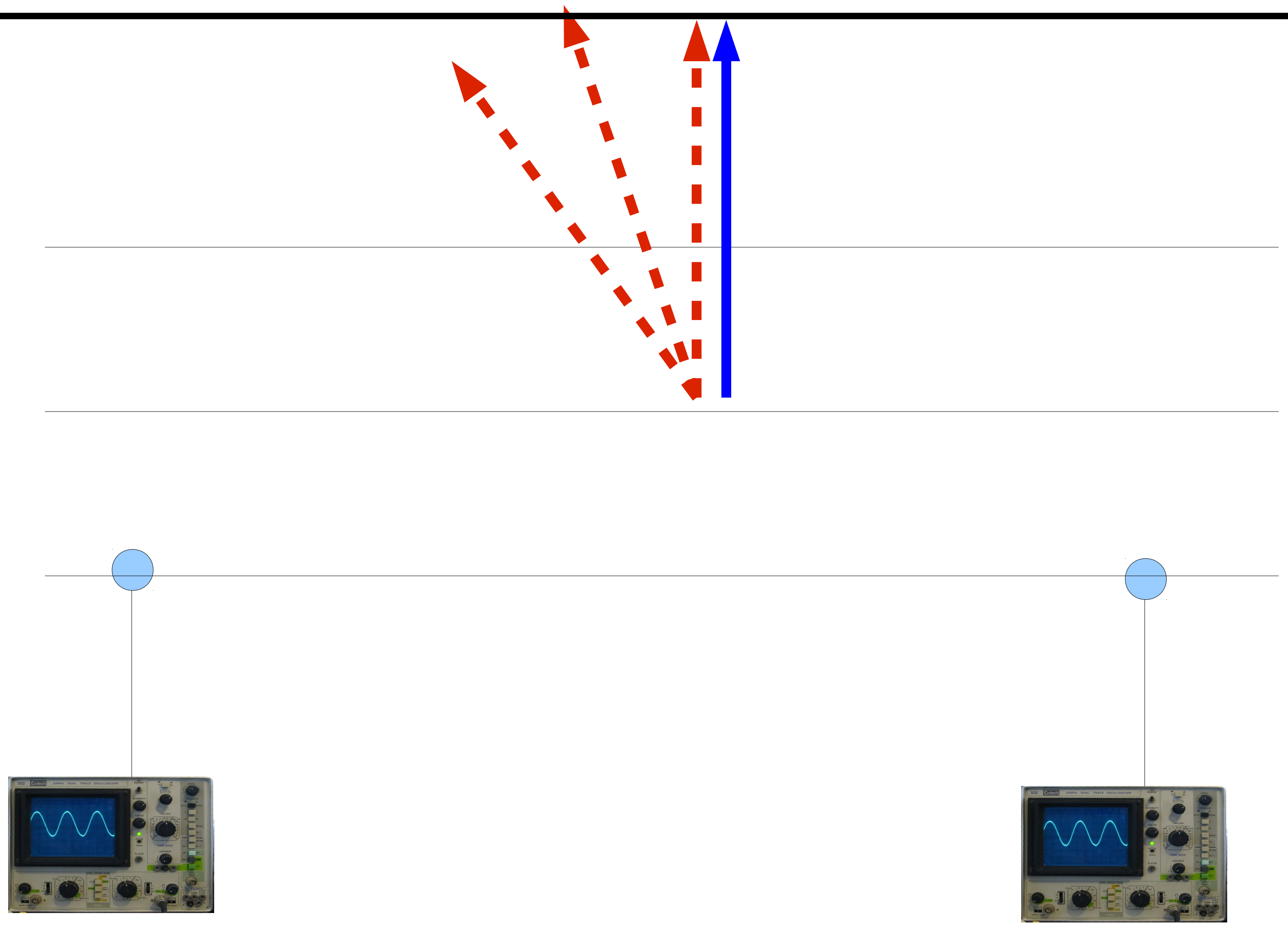

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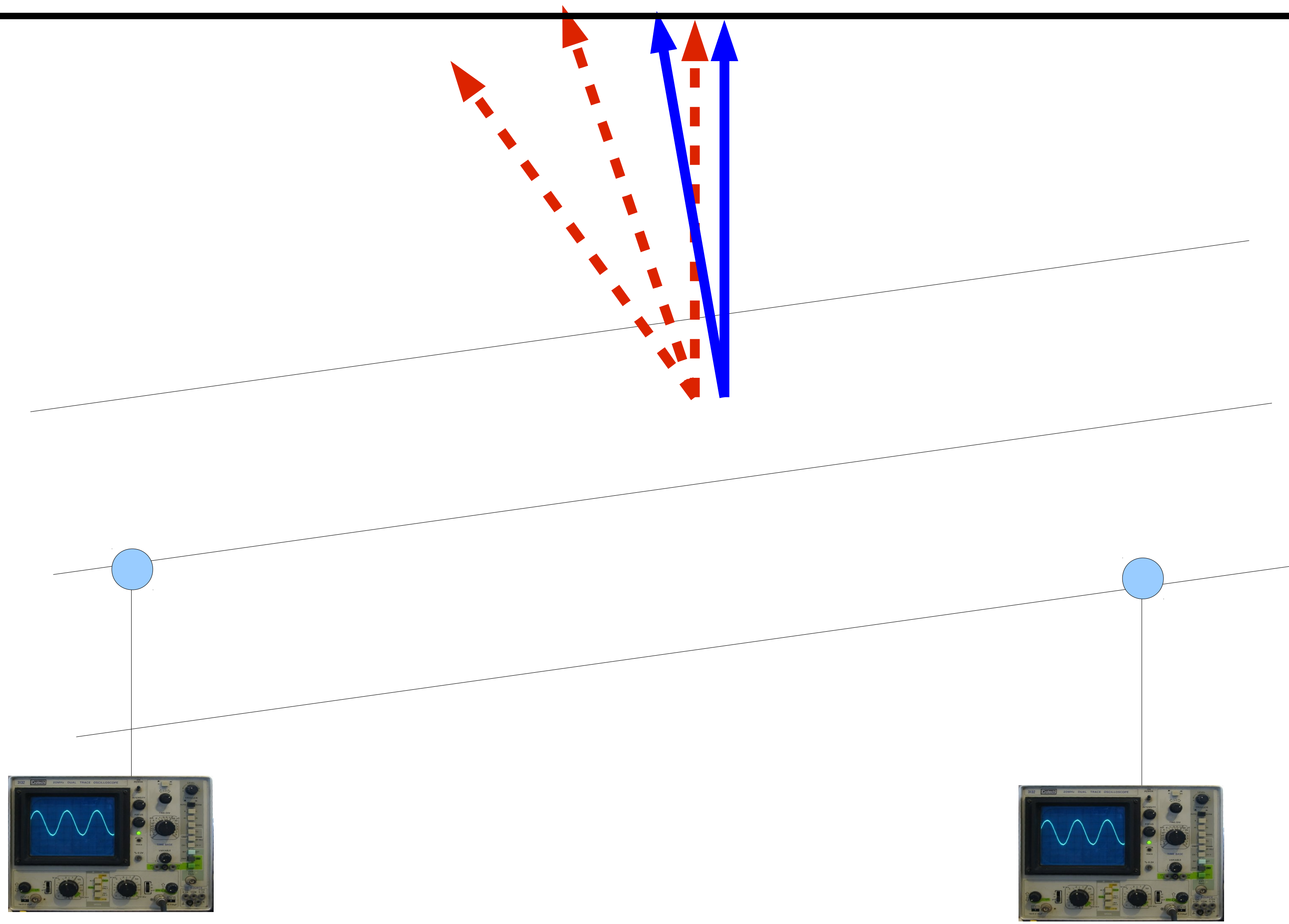

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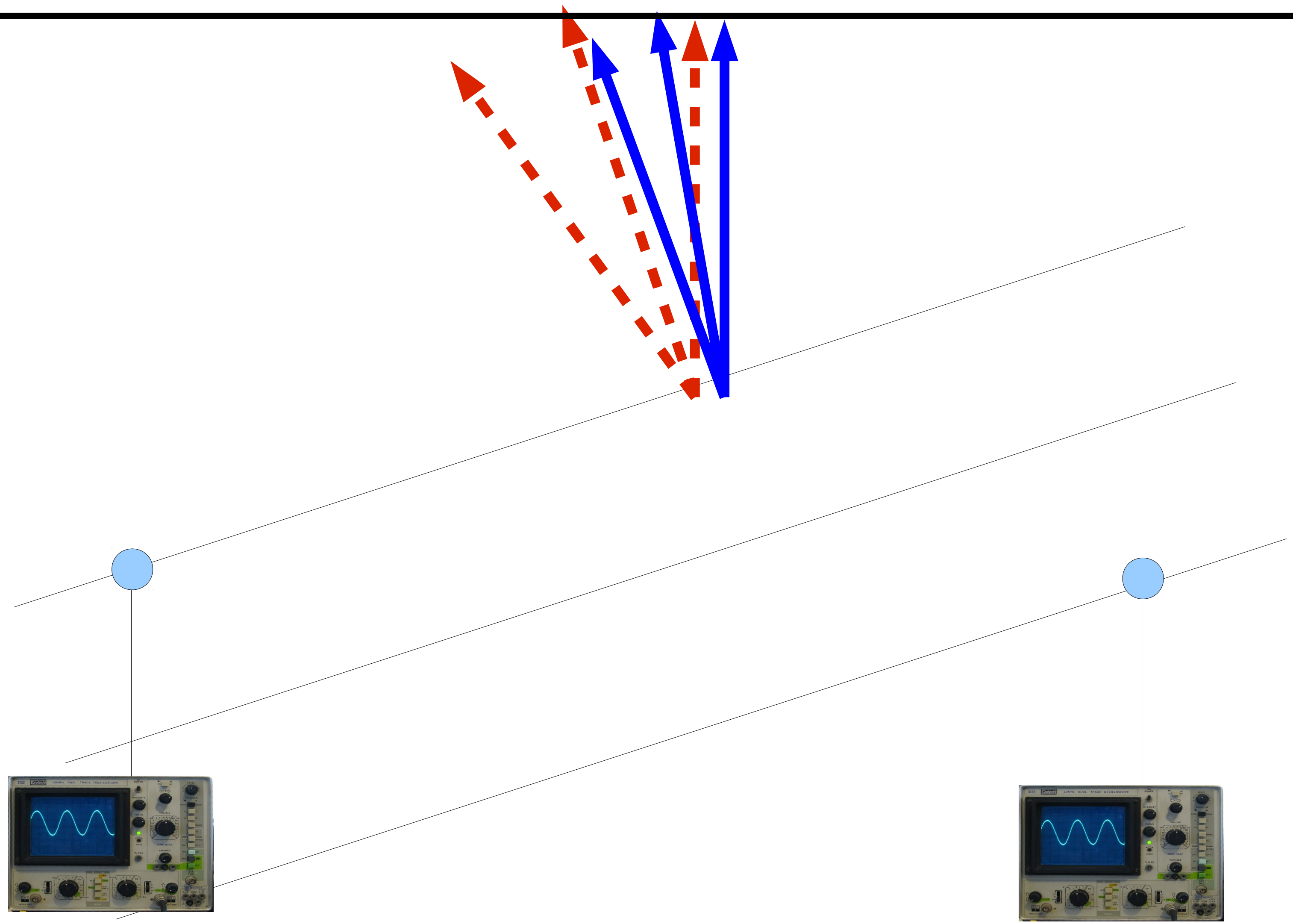

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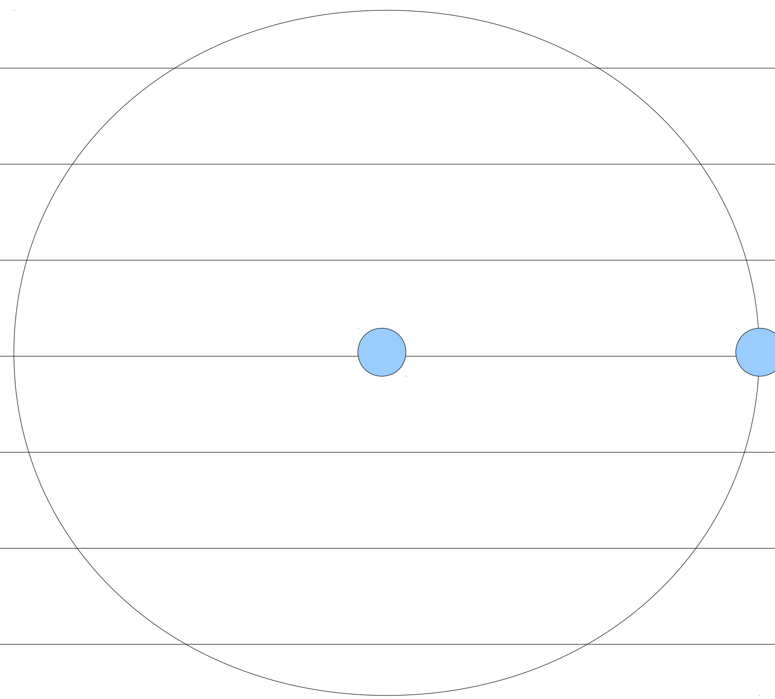

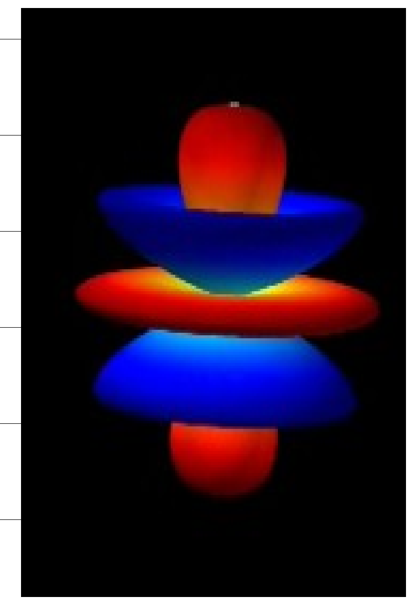

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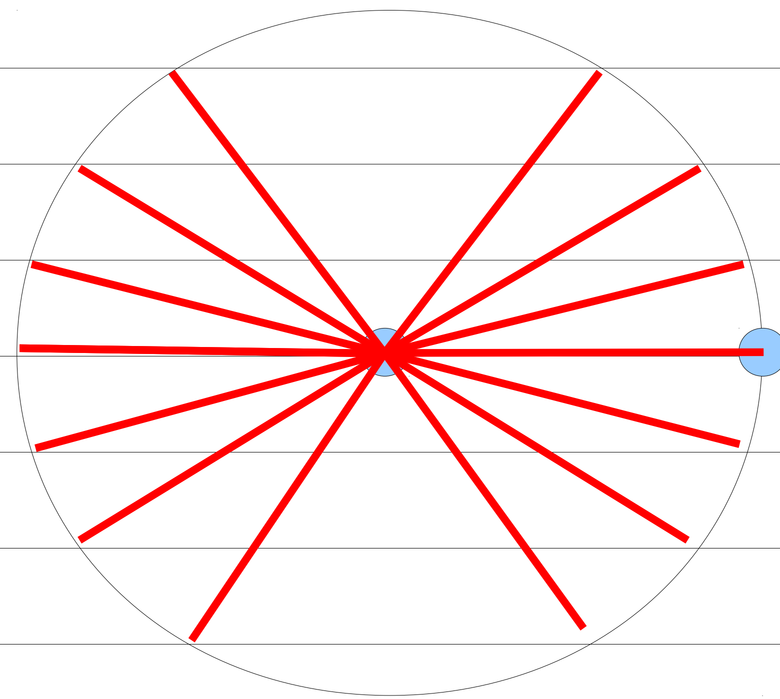

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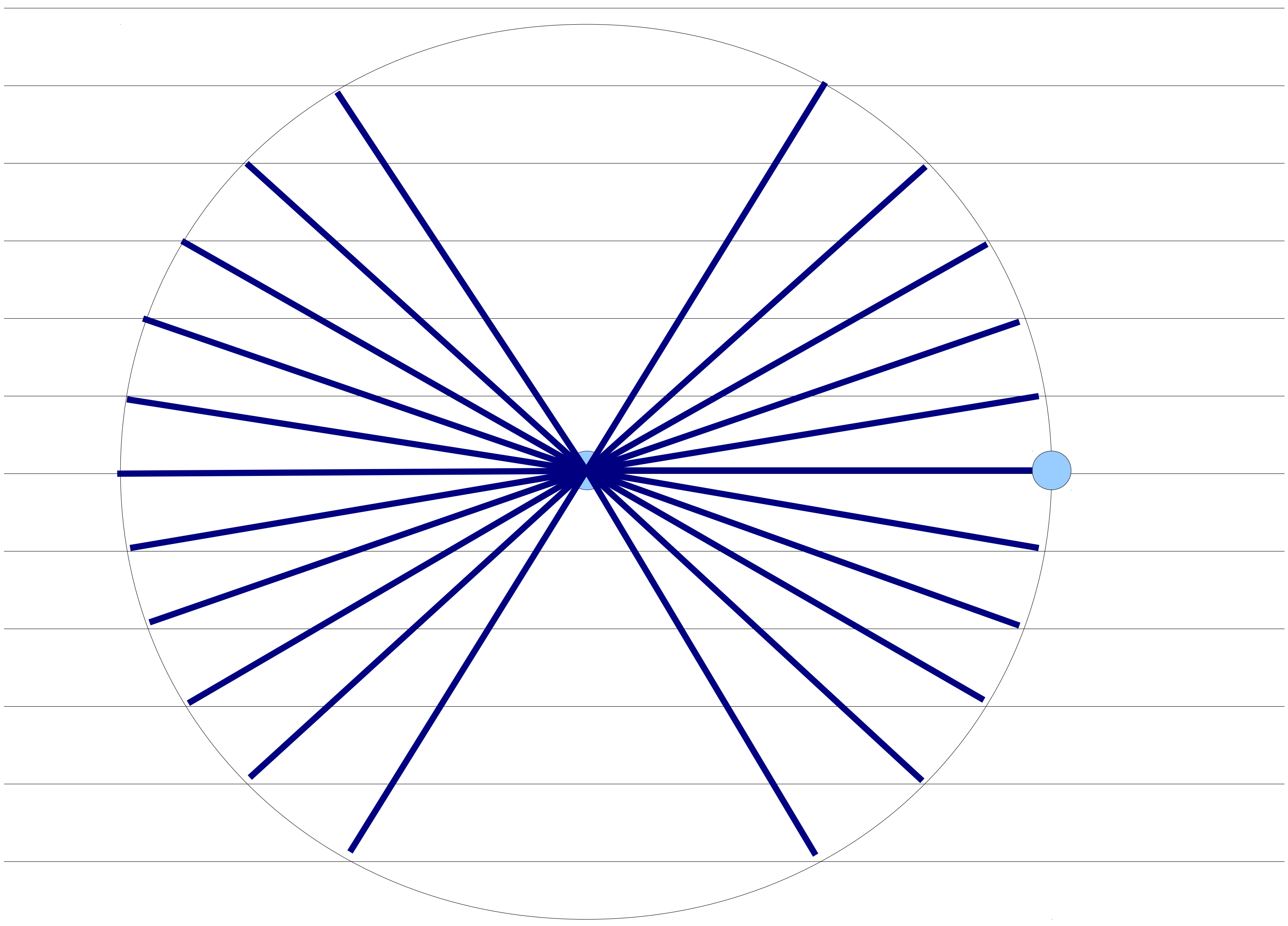
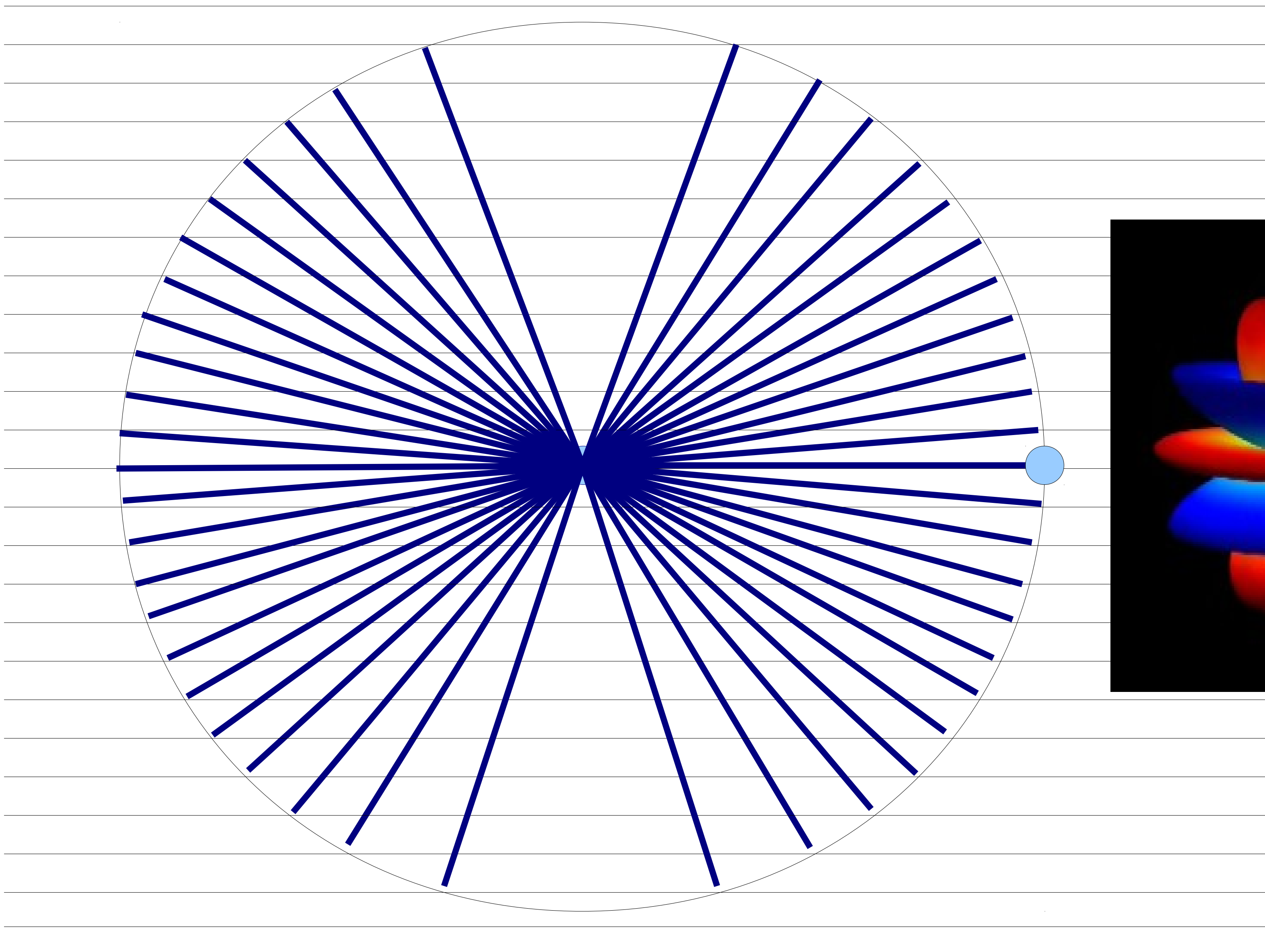
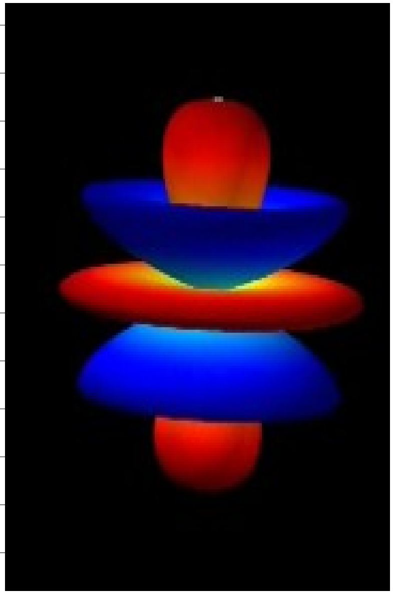


# Principle

**Resolution = Wavelength / Distance**





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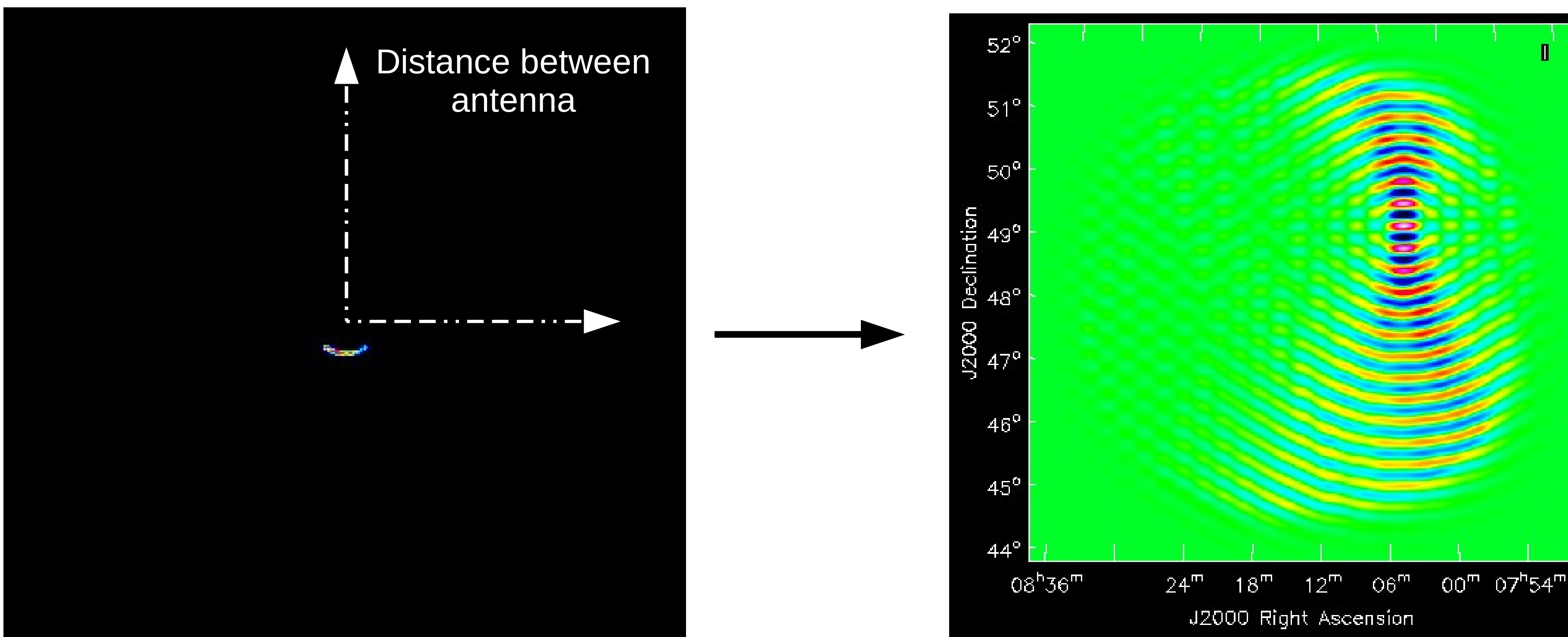
**- Each baseline “draws” a fringe on the sky**

**- The superposition of the information of many baseline “draws” the image.**



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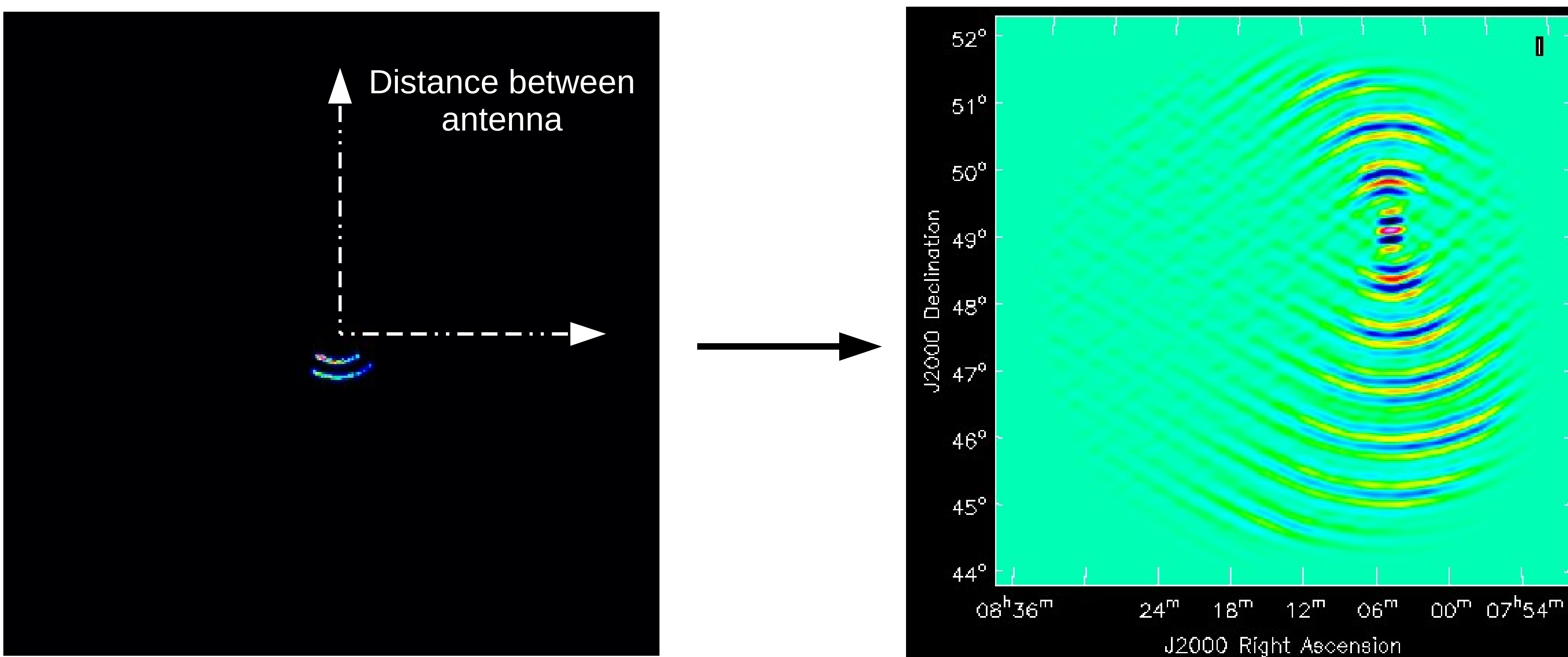

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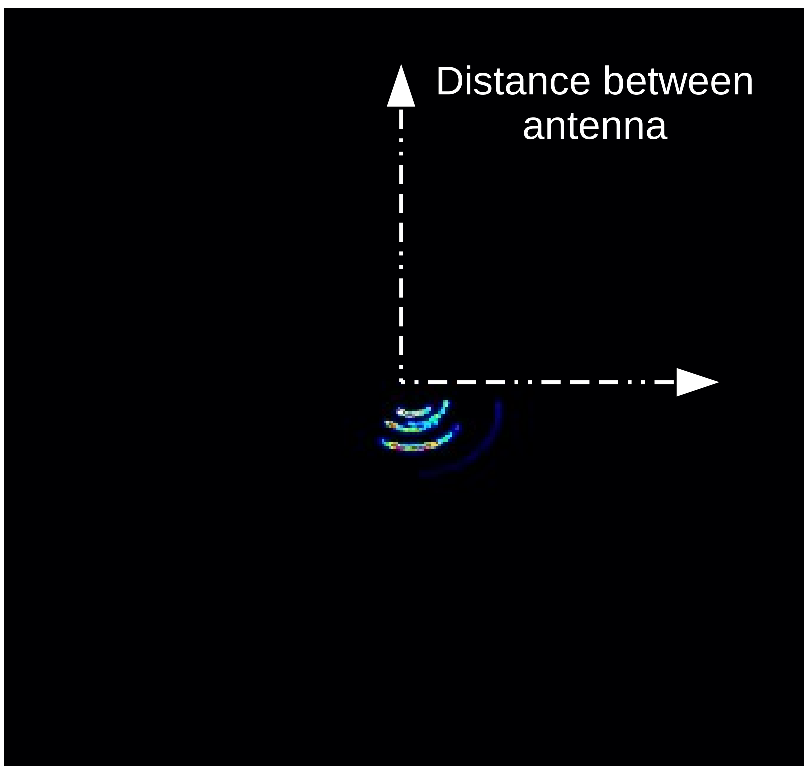


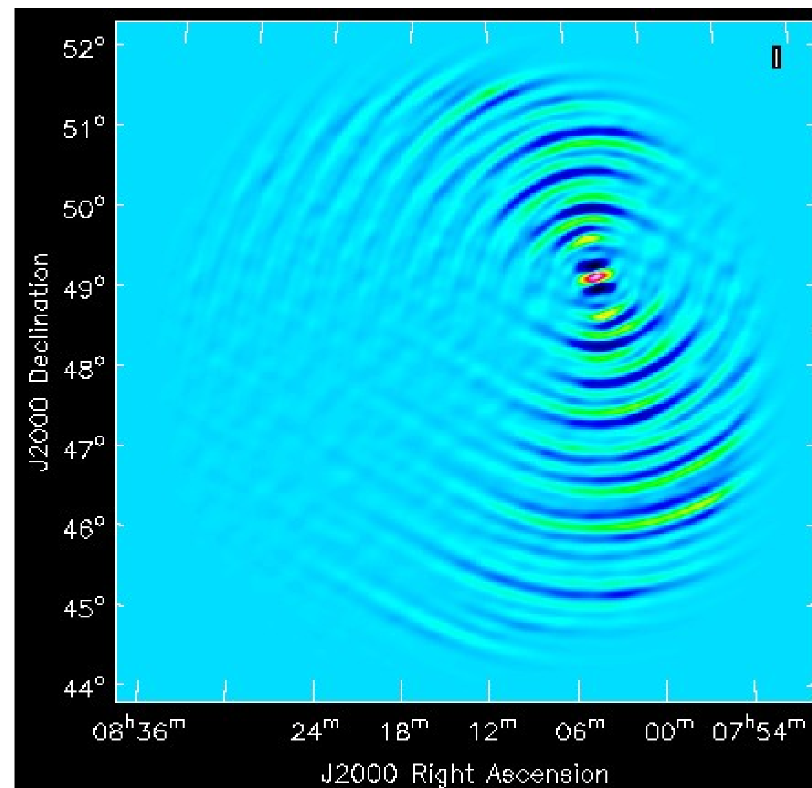

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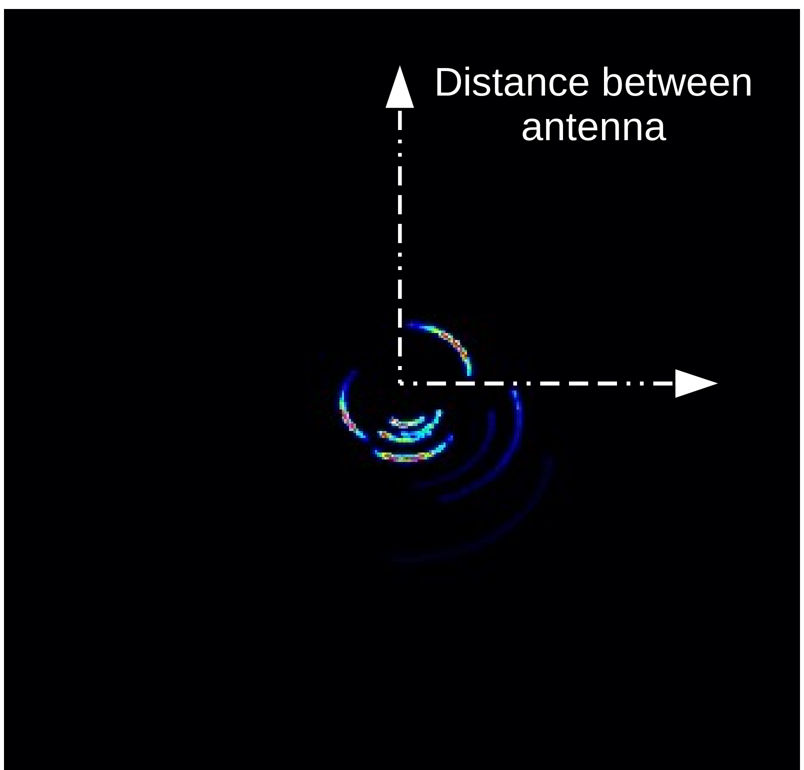
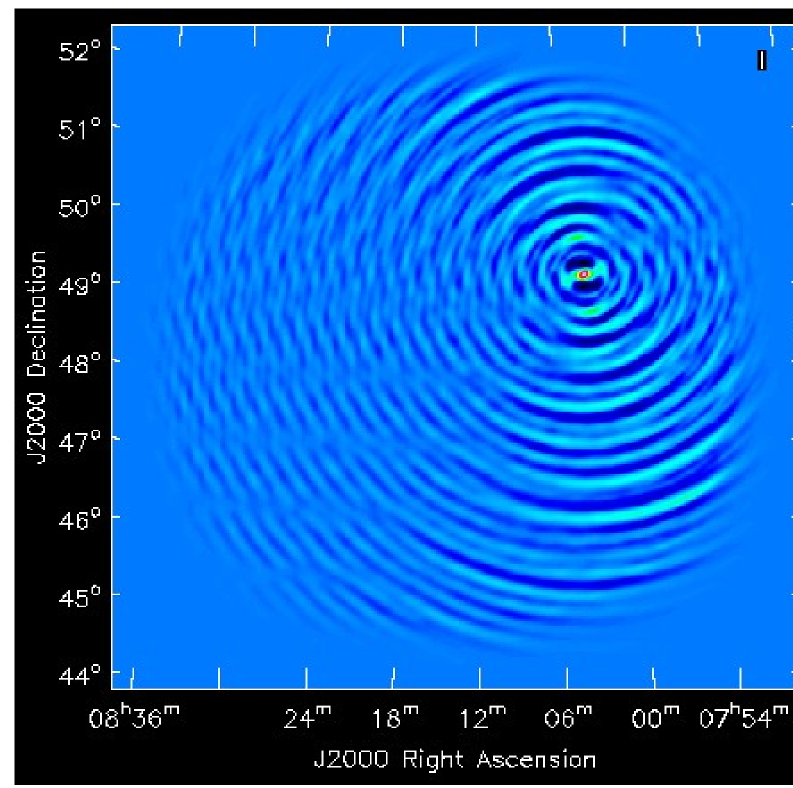
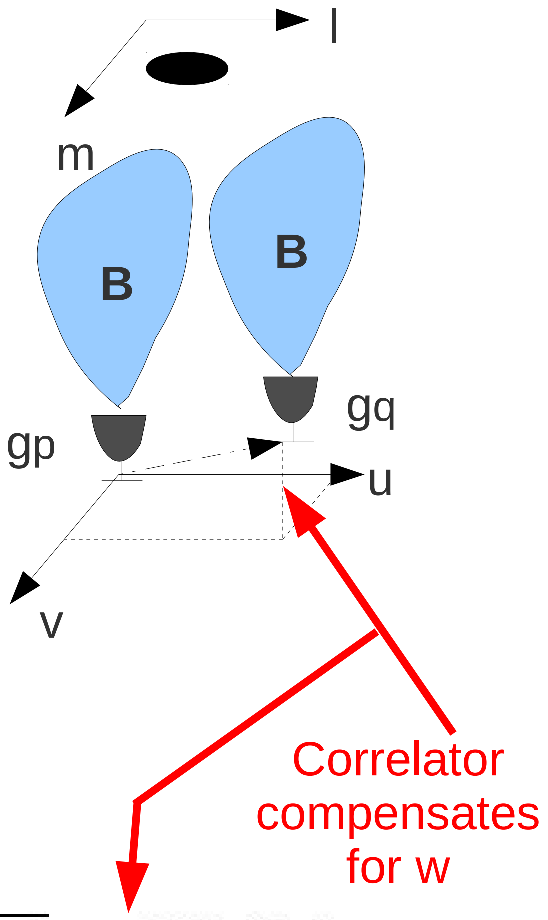


# Traditional Calibration and imaging (scalar)




$$V_{pq} = (g_p . g_q^*) \int_S \mathrm{B(l,m).I(l,m)}$$
$$. \exp\left(-2\pi i(u_{pq} l + v_{pq} m + w_{pq}.(\sqrt{1-l^2-m^2}-1))\right) dl.dm$$

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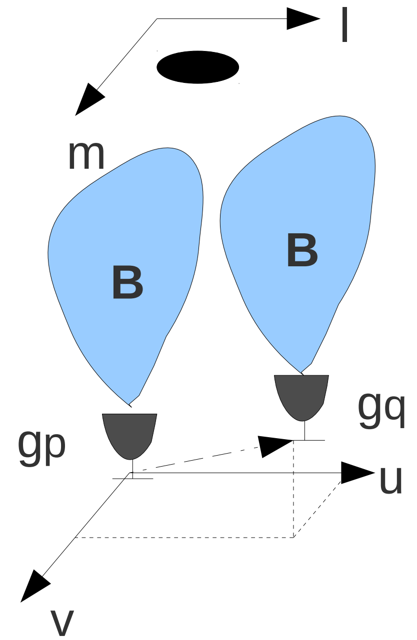


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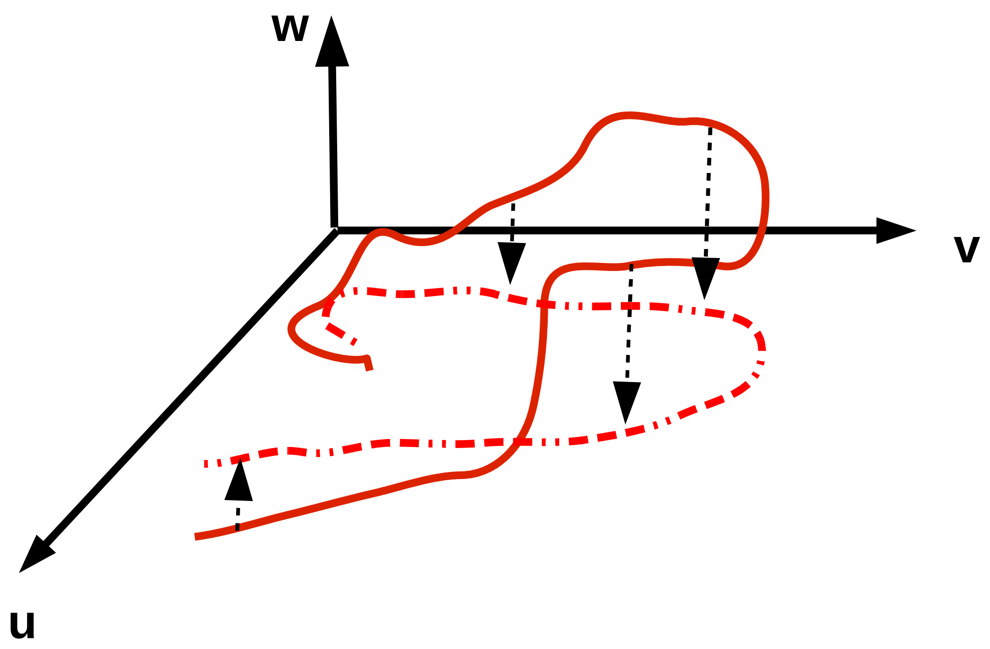
**Small field of view**

# Gridding in practice?



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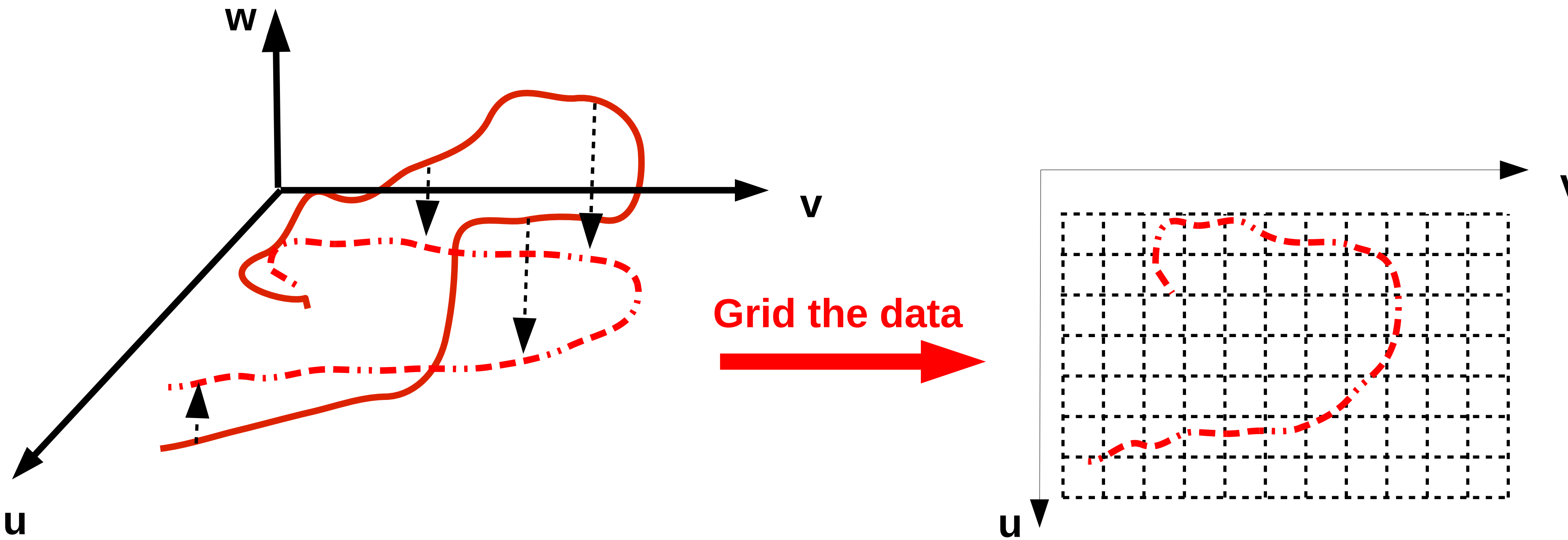

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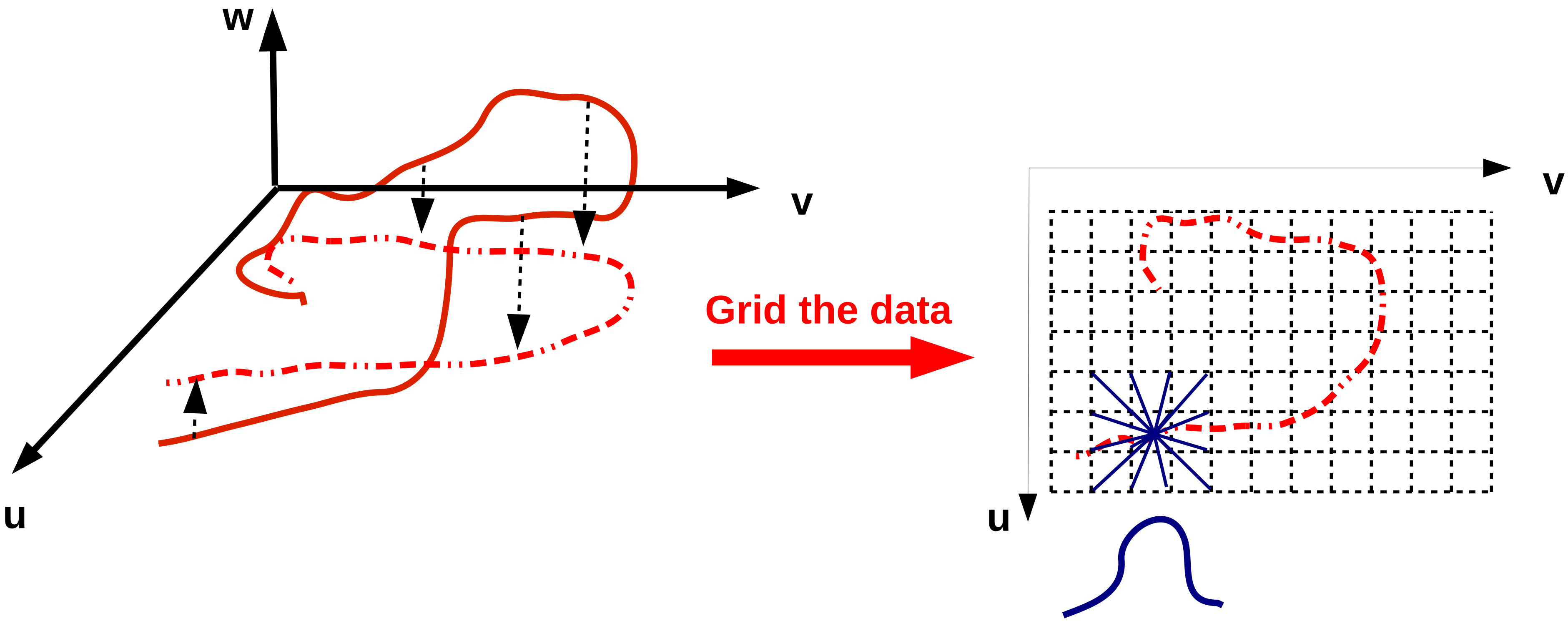

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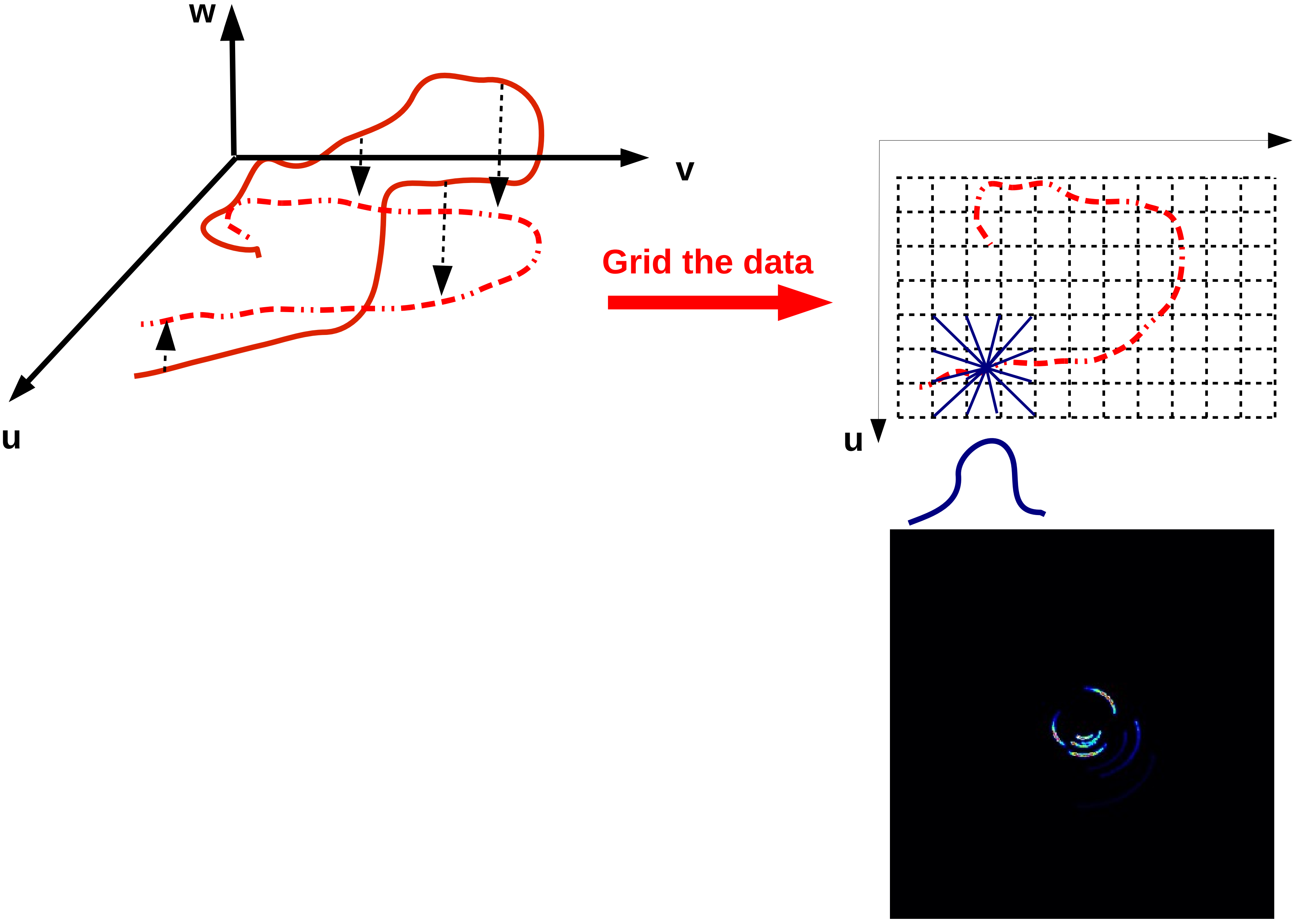

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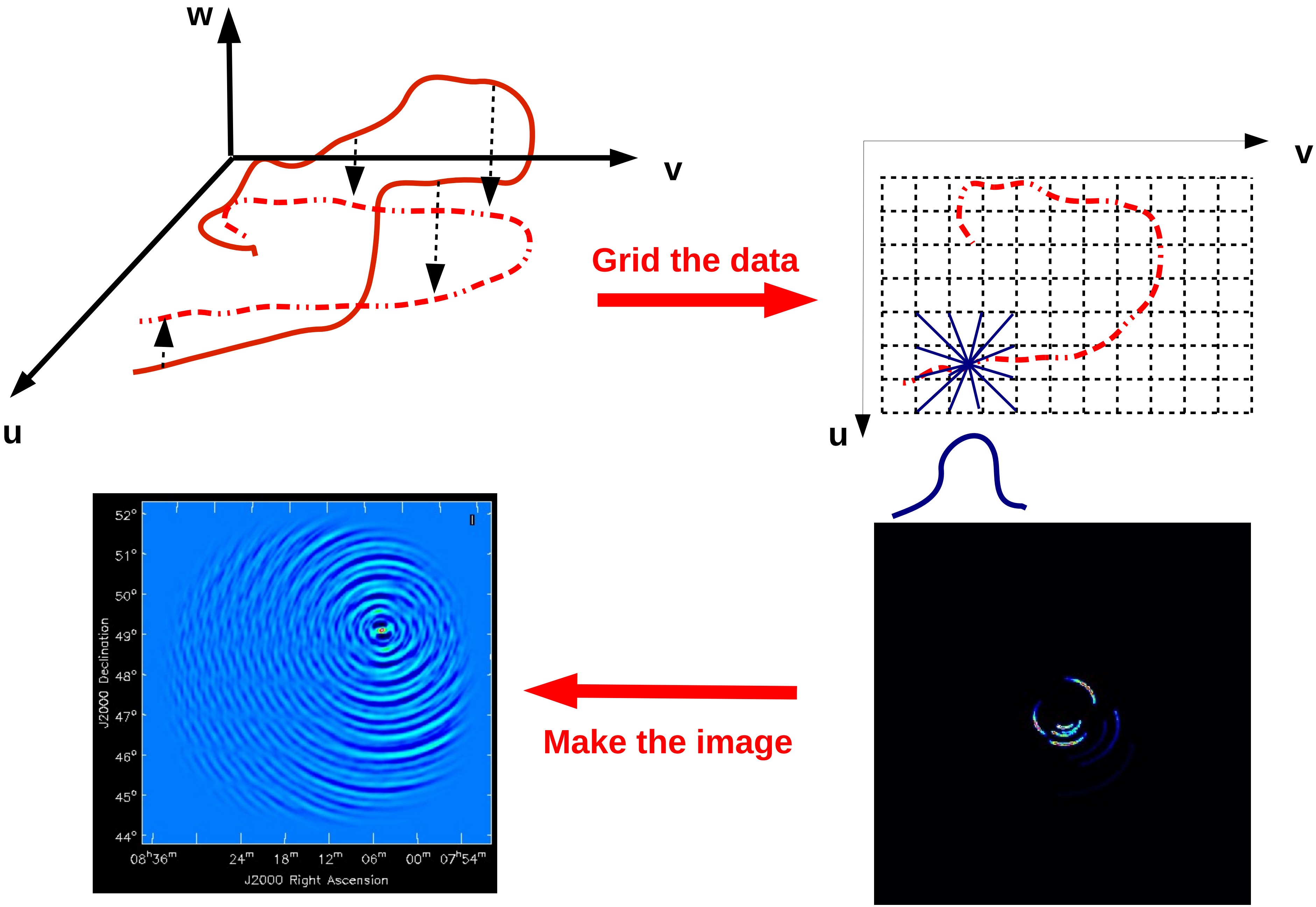
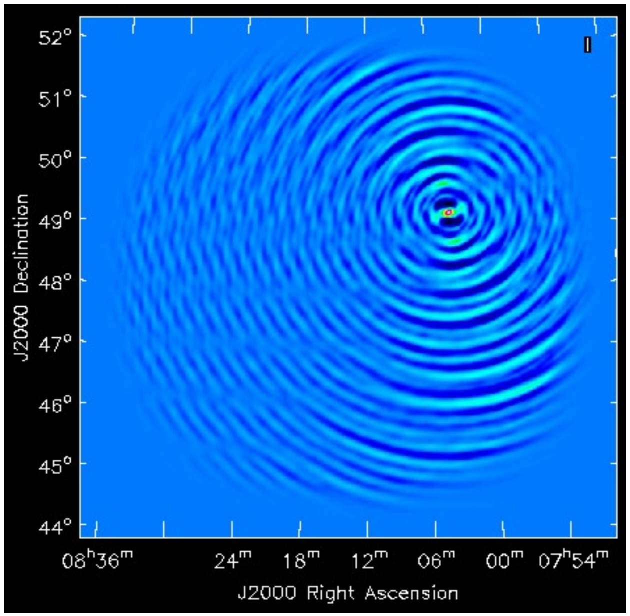
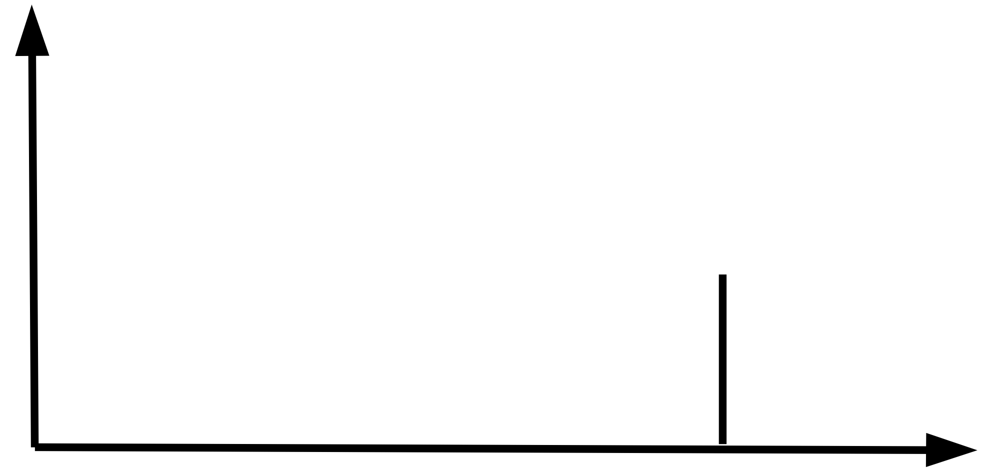


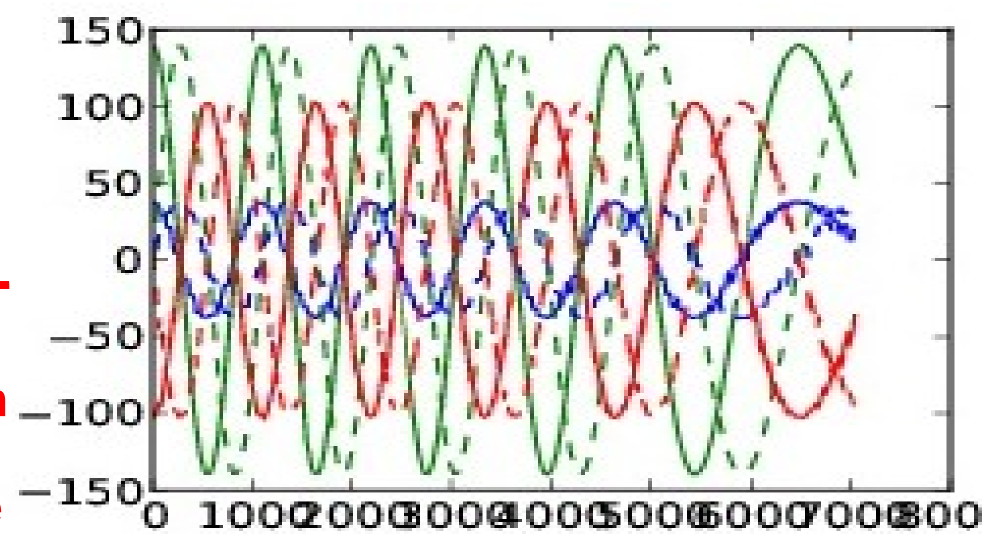
# Deconvolution?



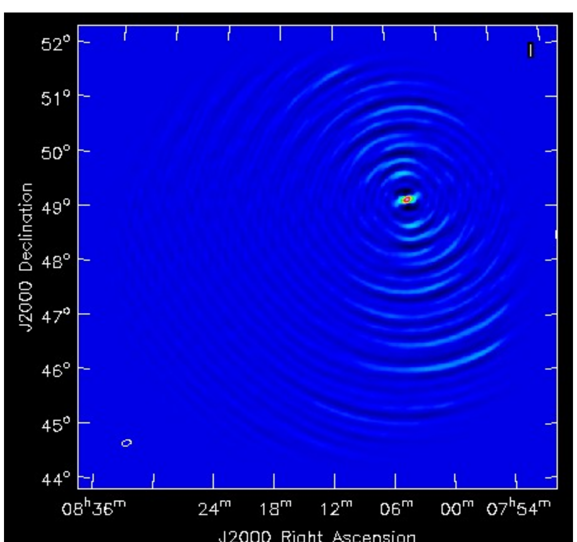

Minor Cycle



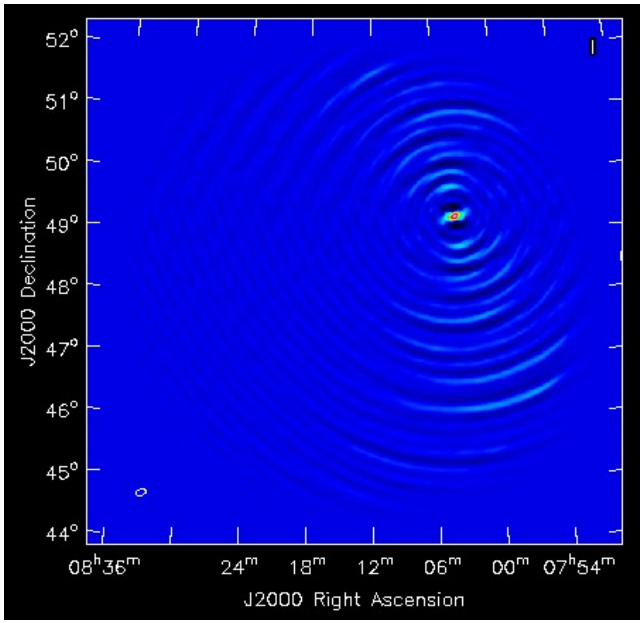
Fourier Transform




Fourier Transform the difference



FOR DUMMIES

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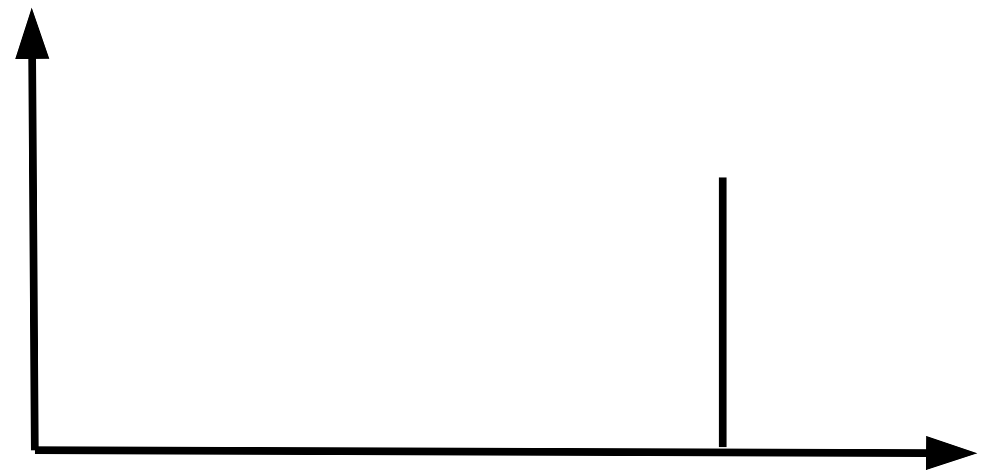
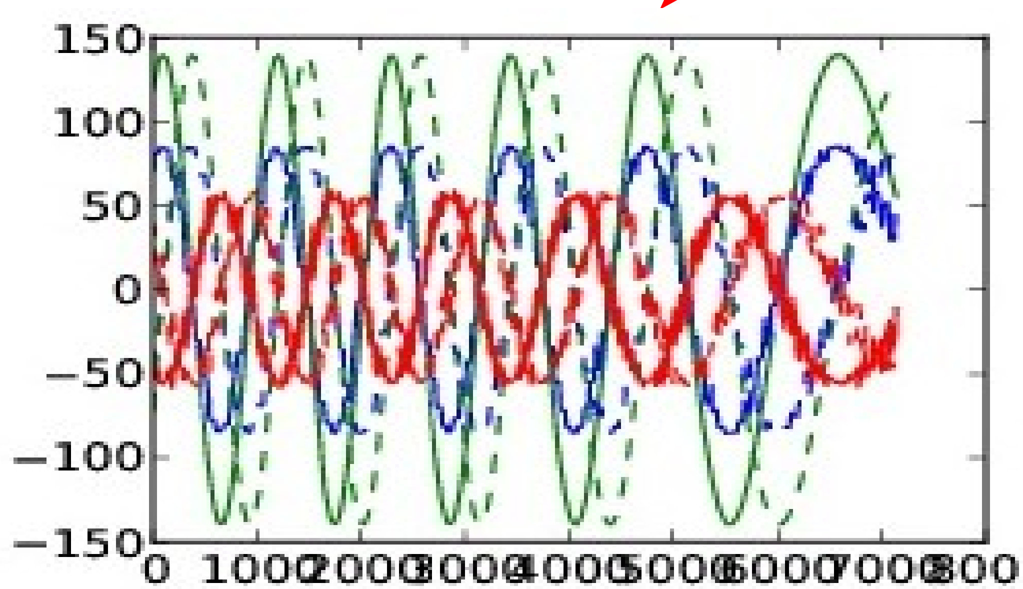


Fourier Transform



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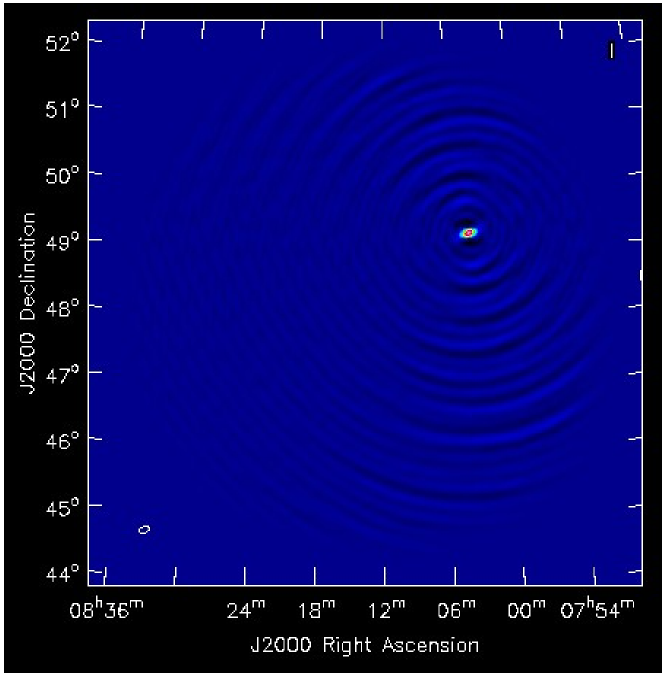
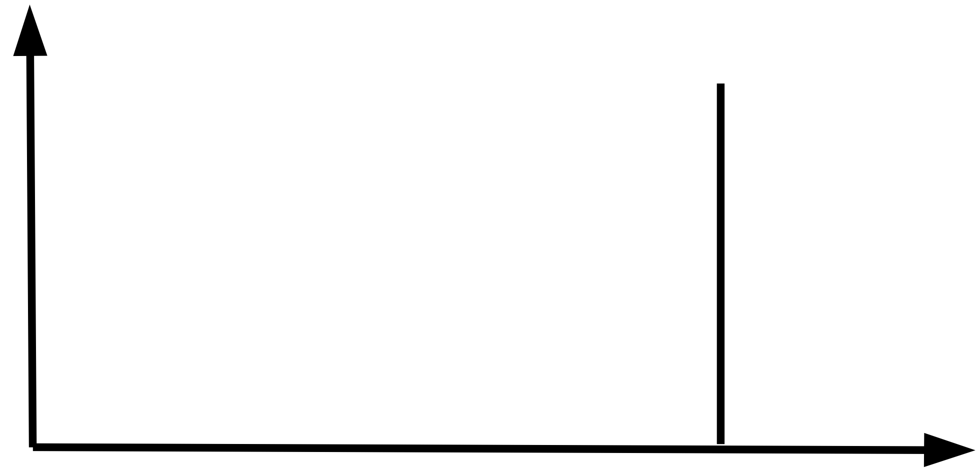



**Minor Cycle**



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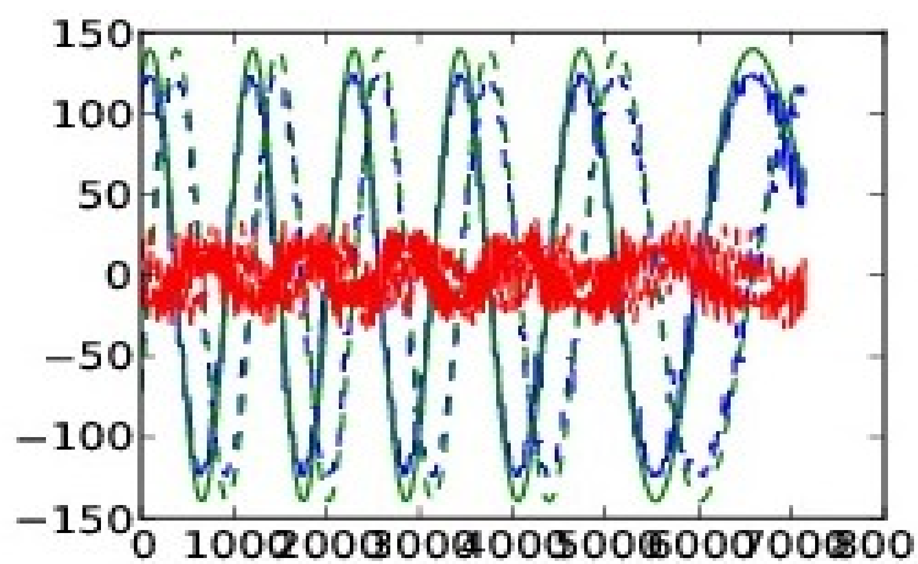

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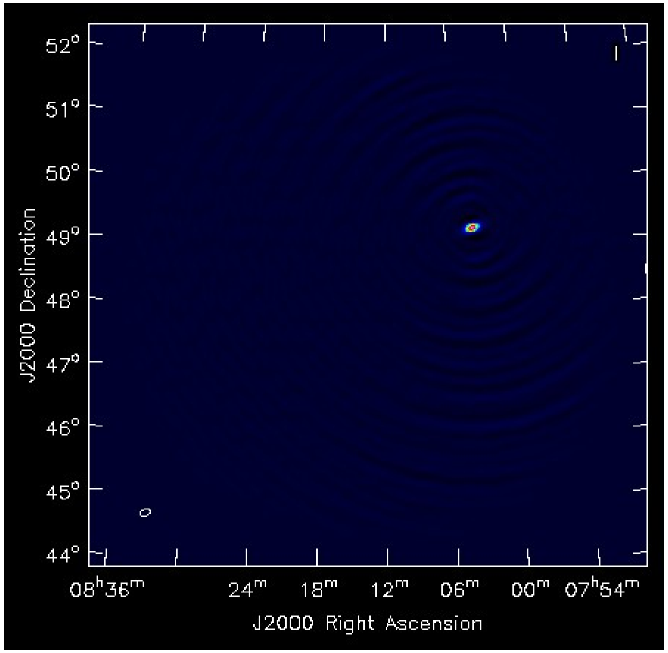


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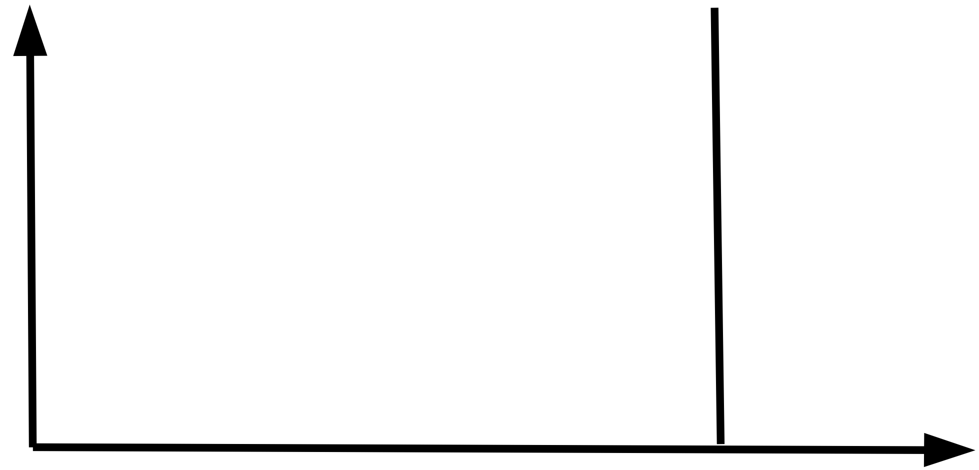

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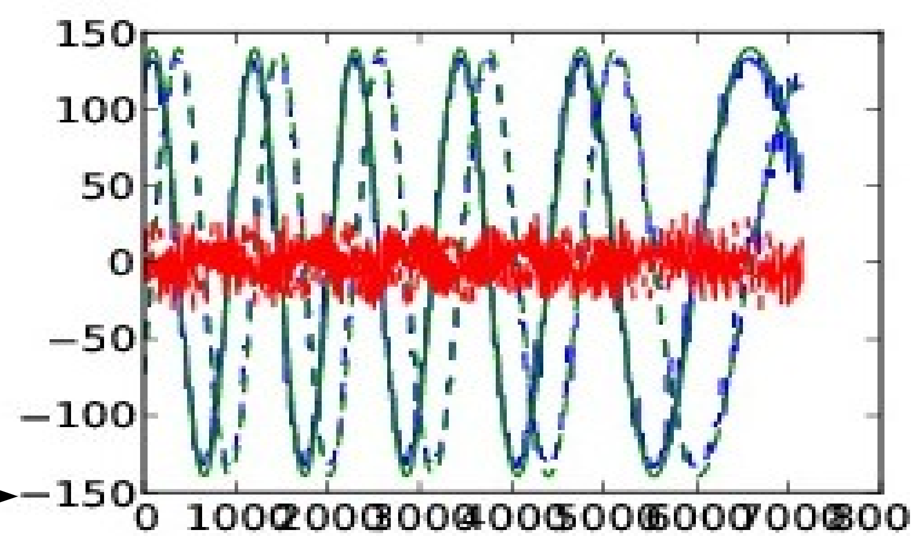
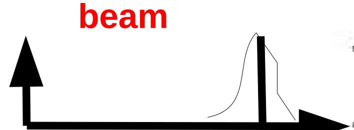
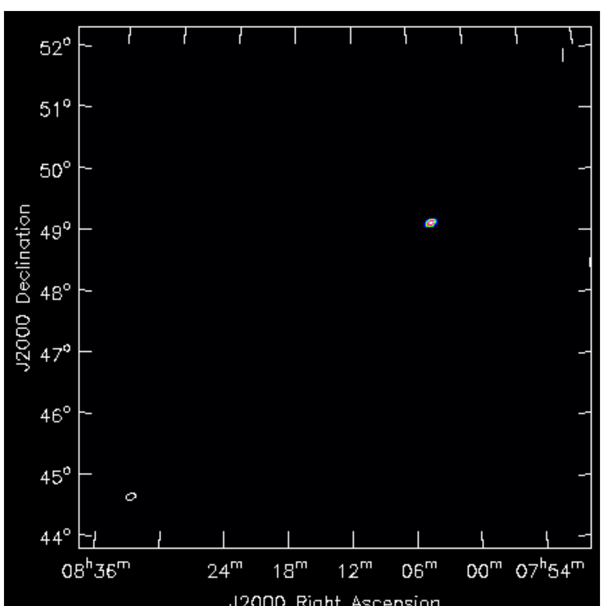



**Fourier Transform the difference and convolve with restoring beam**

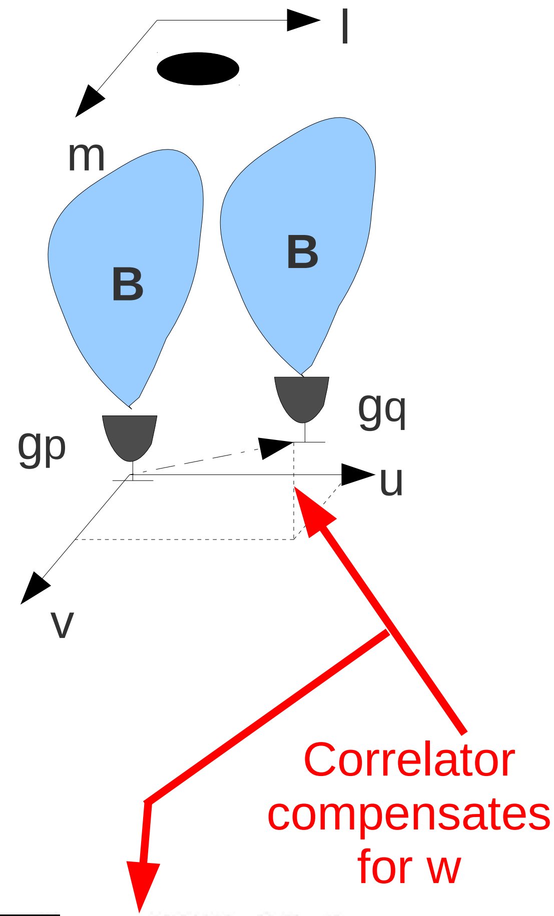




# Next Talk

**Presentation of UV-Brick by Iniyan**

# Traditional Calibration and imaging (scalar)

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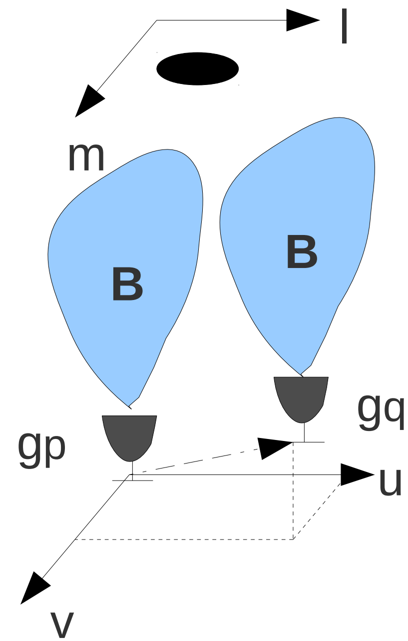


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**Small field of view**

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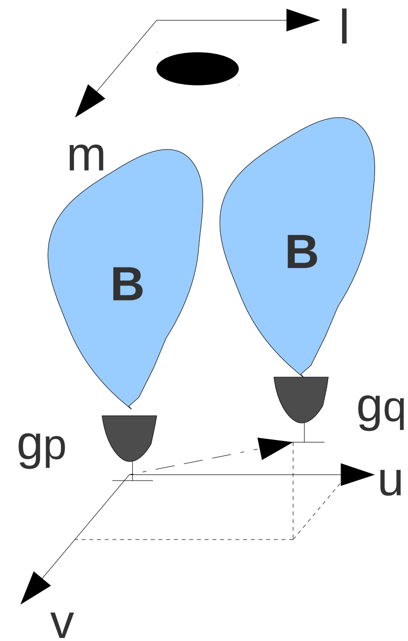


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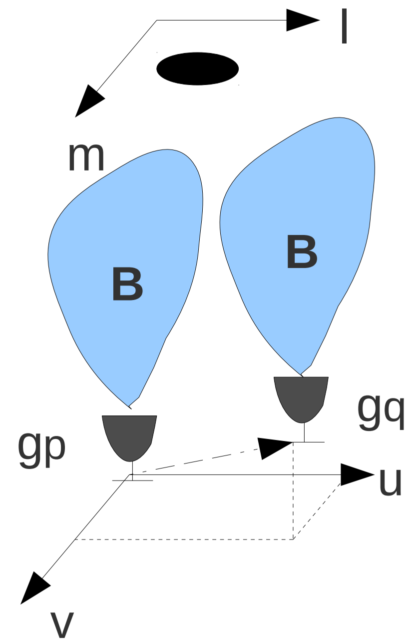


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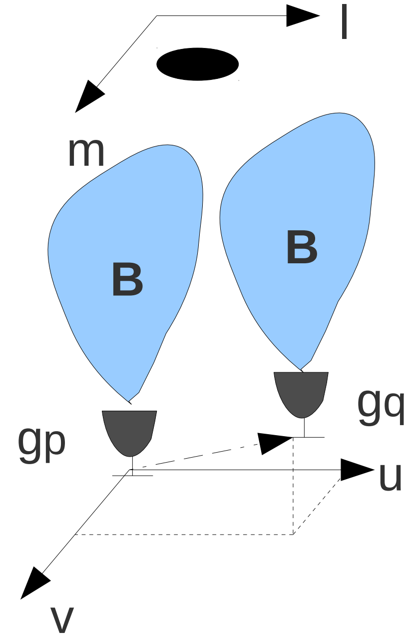
**Small field of view**

- Imaging

$$\mathrm{I(l,m)} = \frac{1}{\mathrm{B(l,m)}} \mathrm{FT}\left(\frac{V(u,v)}{[g.g^*](u,v)}\right)$$

# Traditional Calibration and imaging (scalar)

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**Small field of view**

## - Imaging

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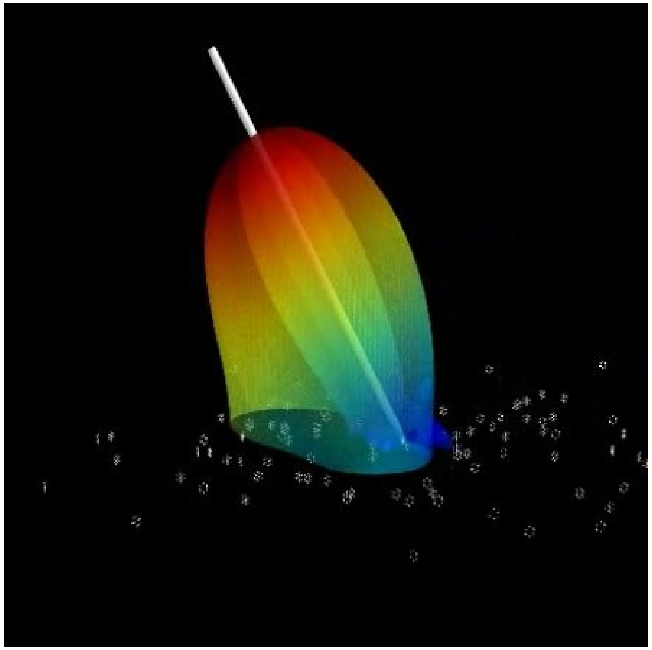
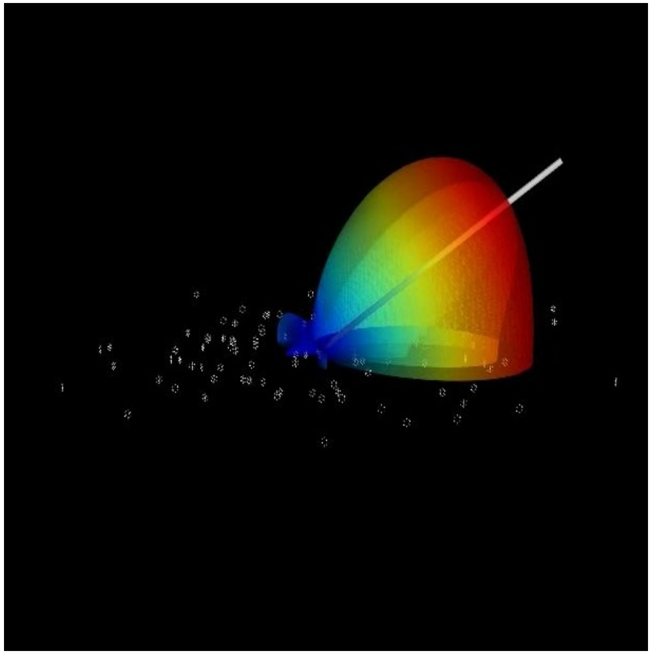
Beam correction in the image plane

# … When Direction Dependent Effects (DDE) become a problem : Beam



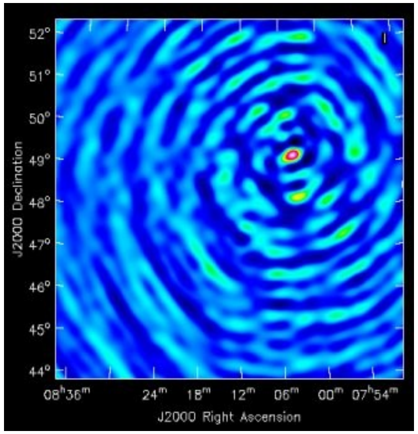


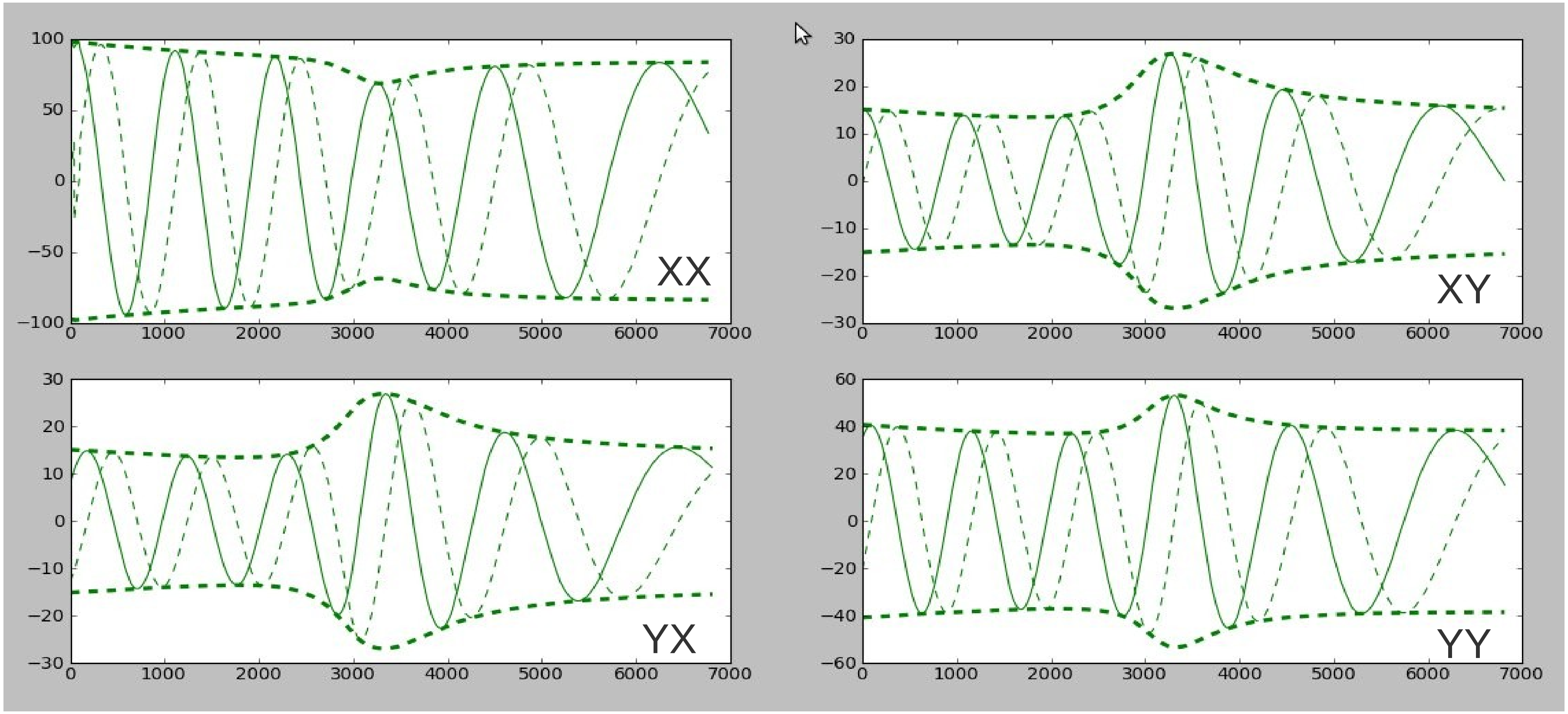
**LOFAR stations are phased arrays**

- Beam is variable in frequency and time
- Beam can be station-dependent

# … When Direction Dependent Effects (DDE) become a problem : Beam

**One off-axis source IQUV=(100, 40, 20 10)**



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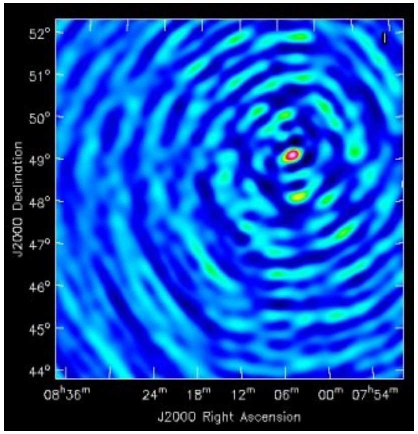
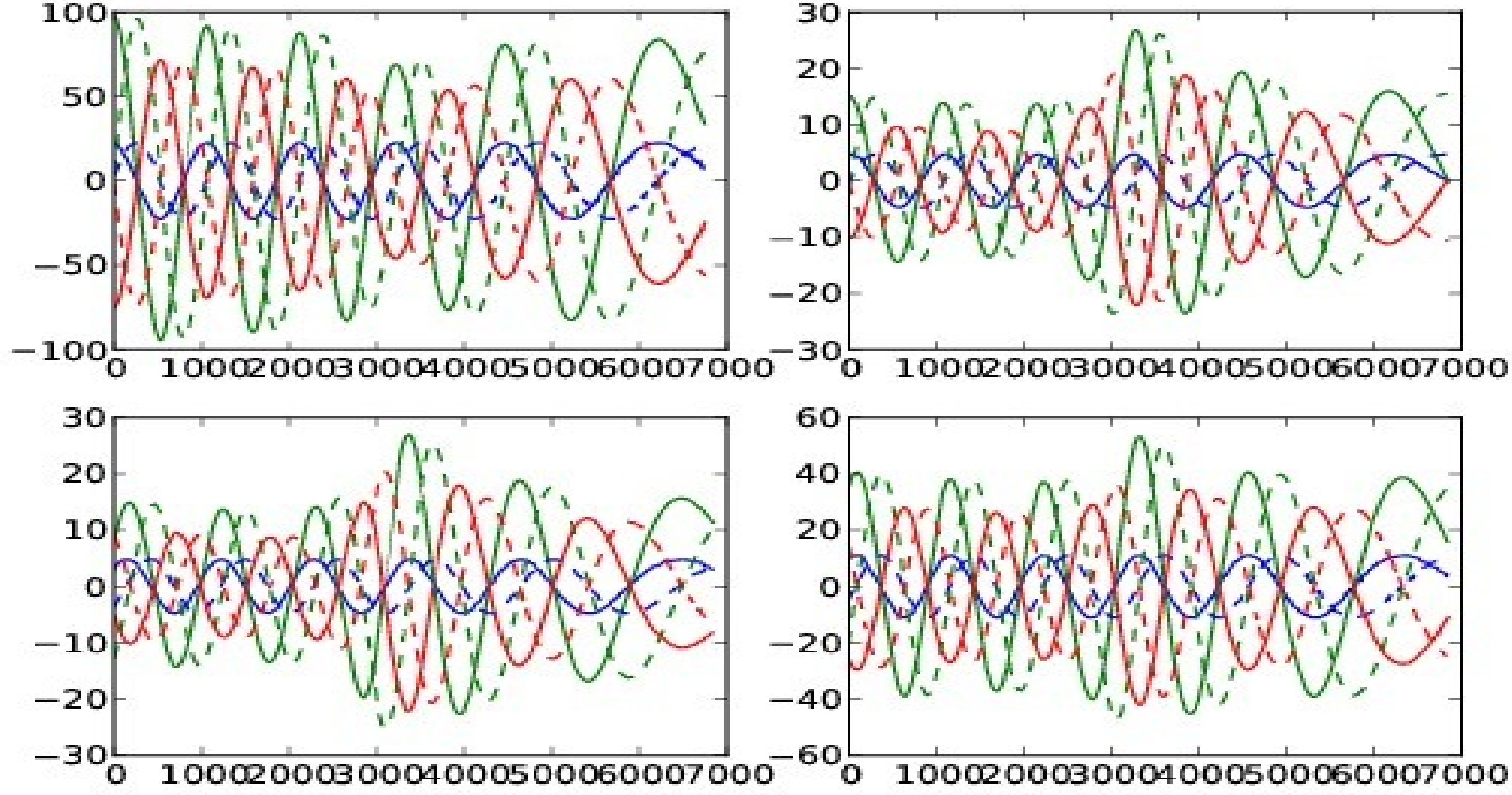



**One off-axis source IQUV=(100, 40, 20 10)**

**“Traditional” imager removes visibility with constant amplitude**



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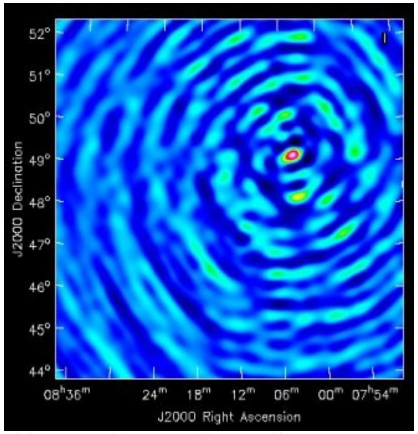


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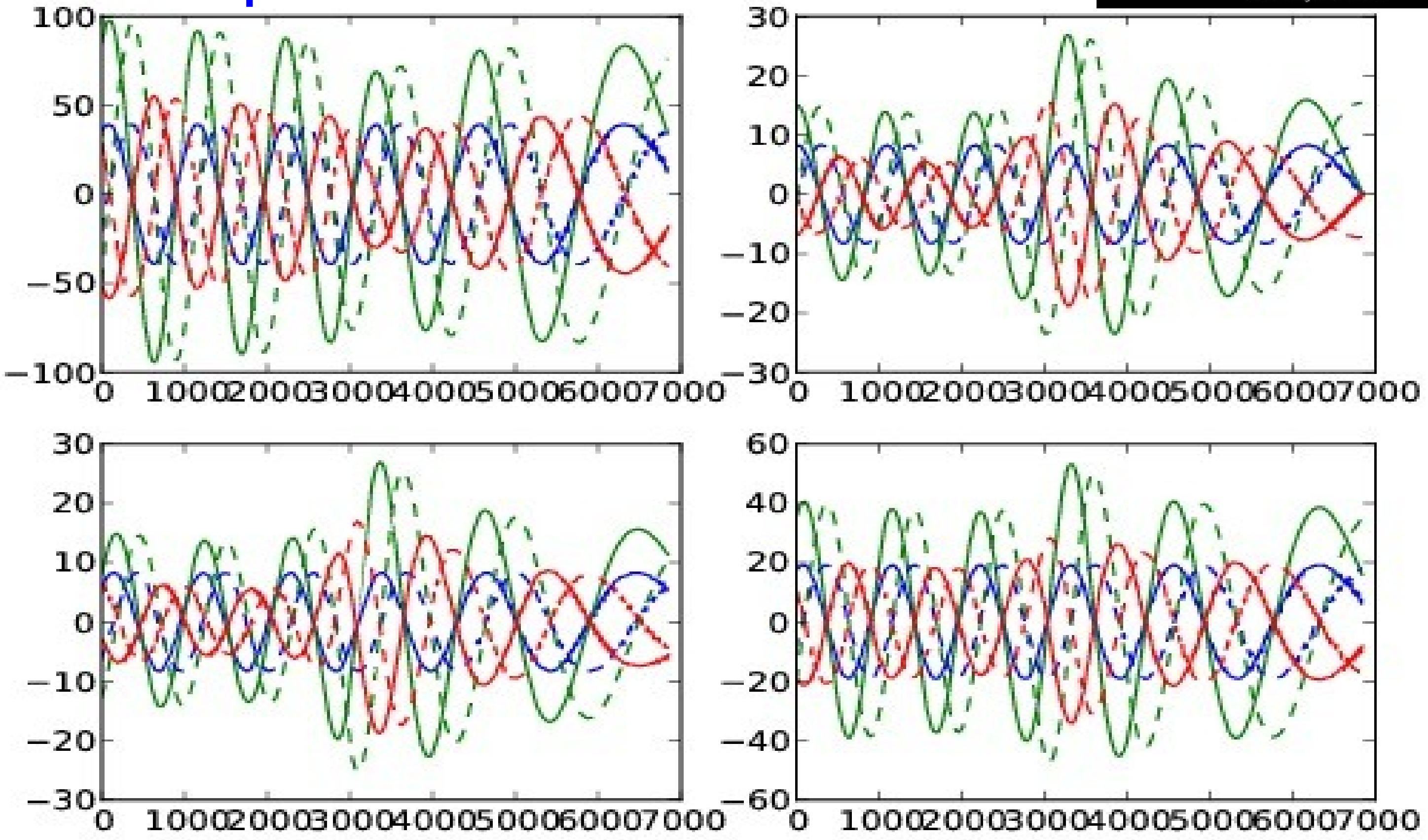

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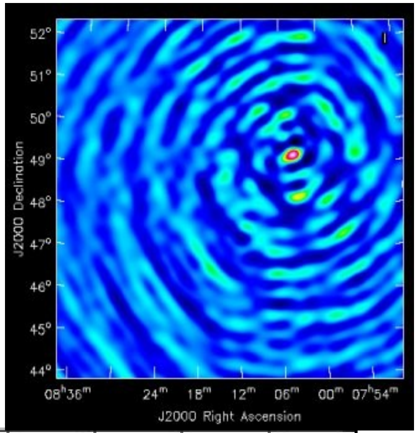


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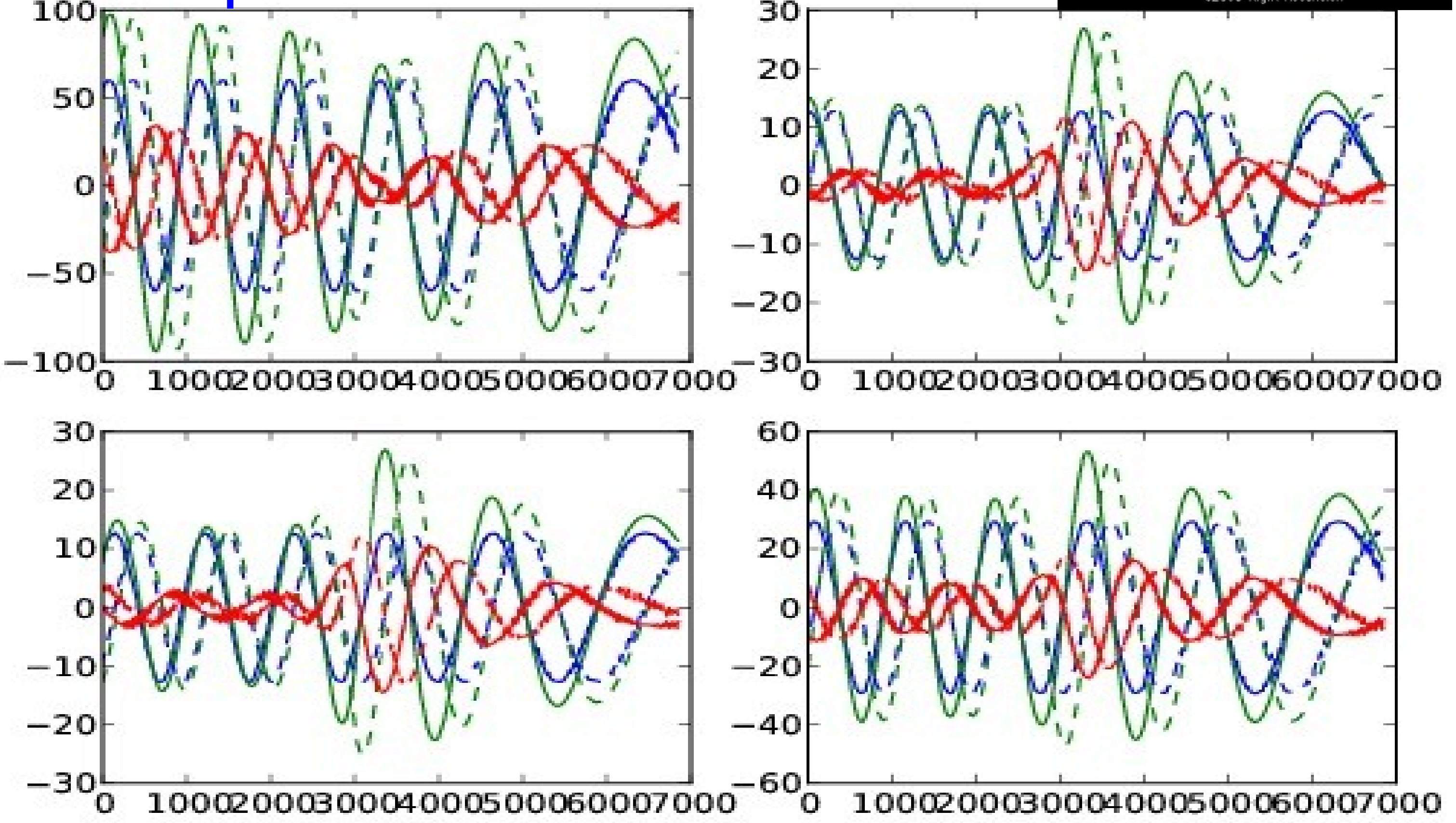

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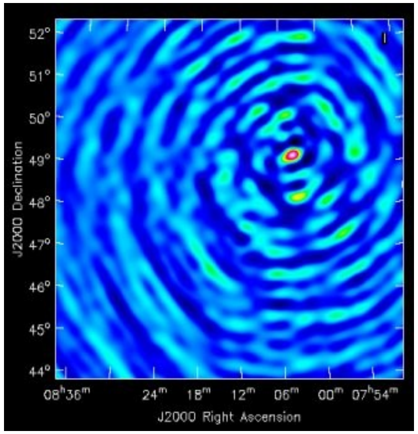


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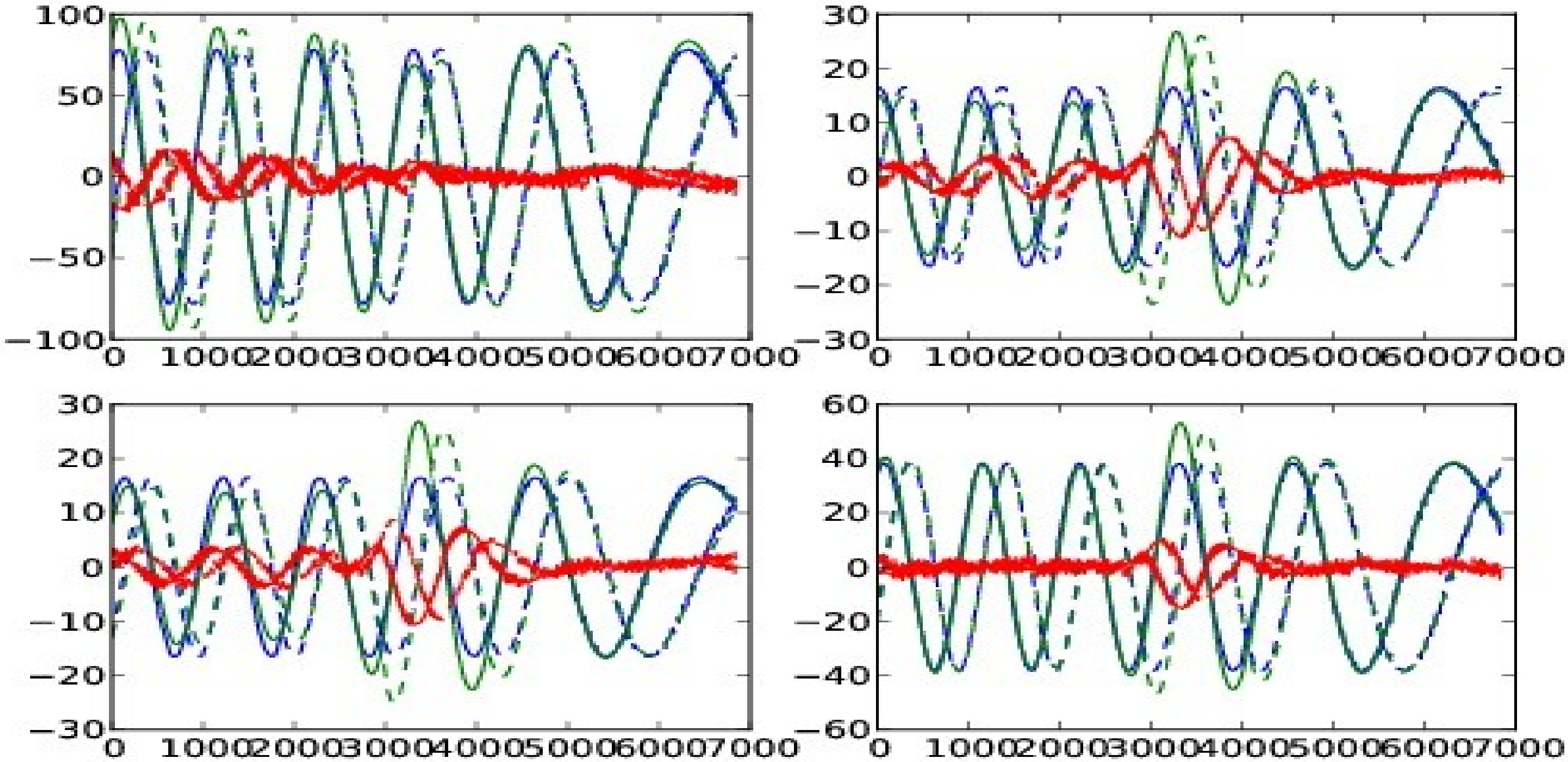
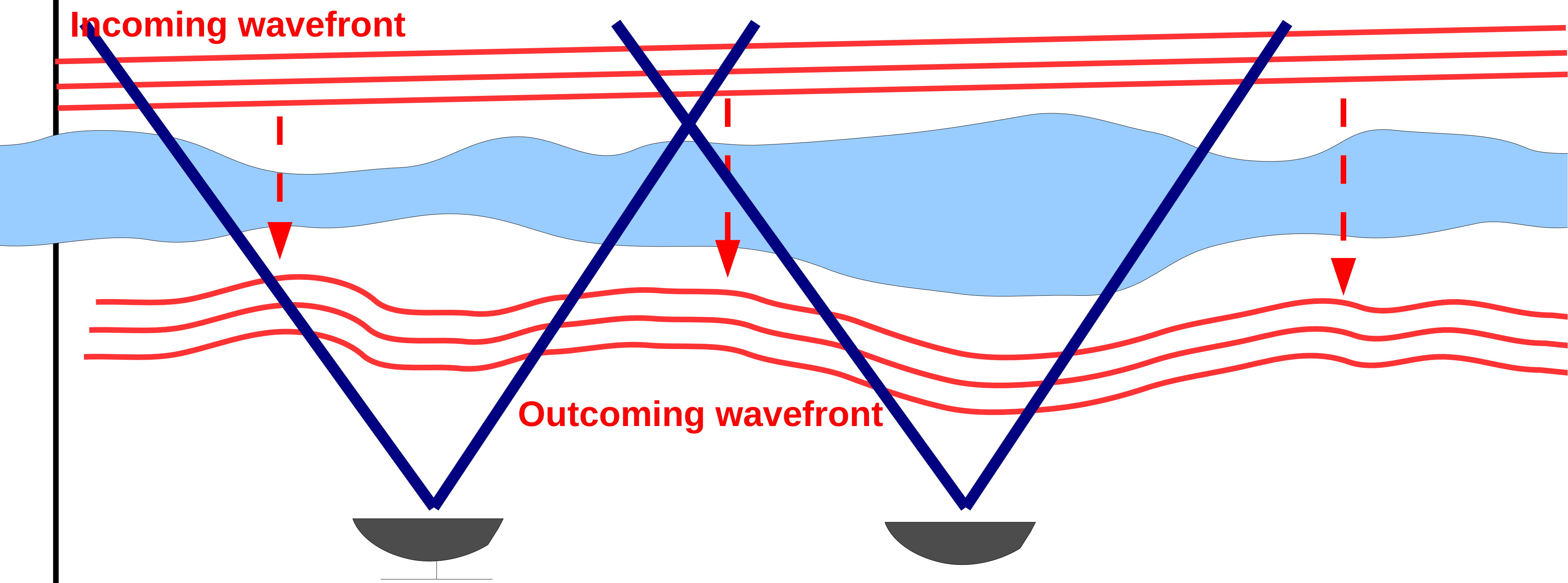


# … When Direction Dependent Effects (DDE) become a problem : Ionosphere




**Big field of view : station, direction, time and frequency dependent**

**Other direction dependent effects :**

- Projection of the dipoles on the sky
- Faraday rotation

**+ Effect on the polarisation**

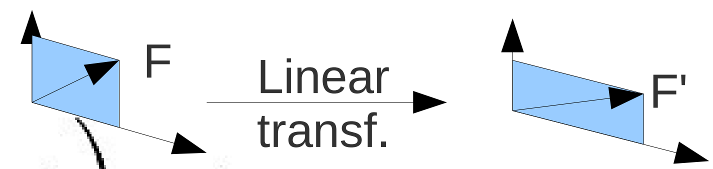
# The Measurement Equation

Hamaker 1996

Direction independent

Direction dependent

Source coherency




$$V_{pq} = G_p \left( \sum_{i=1}^{N} \mathrm{B}_{pi} \mathrm{K}_{pi} \mathrm{I}_{pi} F_i \; . \; F_i^{+} \mathrm{I}_{qi}^{+} \mathrm{K}_{qi}^{+} \mathrm{B}_{qi}^{+} \right) G_q^{+}$$

**[Voltage antenna p] x [ Voltage antenna q]***

Beam

Geometrical delay +Correlator

Ionosphere

Electric field

$$\mathrm{K}_p \mathrm{K}_q^{+} = \exp(-2i\pi\phi_{pq}) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\phi_{pq} = u_{pq} l + v_{pq} m + w_{pq} (\sqrt{1 - l^2 - m^2} - 1)$$

# The “*Vec*” Operator

If $\boldsymbol{A} = [\boldsymbol{a}_1 \boldsymbol{a}_2 \cdots \boldsymbol{a}_n]$

Columns of a Matrix

And $\operatorname{vec}(\boldsymbol{A}) = \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \vdots \\ \boldsymbol{a}_n \end{bmatrix}$

then $\operatorname{vec}(\boldsymbol{A}\boldsymbol{X}\boldsymbol{B}) = (\boldsymbol{B}^T \otimes \boldsymbol{A}) \operatorname{vec}(\boldsymbol{X})$

# The "*Vec*" Operator

If $\boldsymbol{A} = [\boldsymbol{a}_1 \boldsymbol{a}_2 \cdots \boldsymbol{a}_n]$

Columns of a Matrix

And $\text{vec}(\boldsymbol{A}) = \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \vdots \\ \boldsymbol{a}_n \end{bmatrix}$

then $\text{vec}(\boldsymbol{AXB}) = (\boldsymbol{B}^T \otimes \boldsymbol{A}) \, \text{vec}(\boldsymbol{X})$

**Beam (4*4)**

$$\text{Vec}(V_{pq}) = (G_q^* \otimes G_p) \int_S (E_{q,\vec{s}}^* \otimes E_{p,\vec{s}}).\text{Vec}(F_{\vec{s}}.F_{\vec{s}}^+).\exp\left(i\ \vec{b_{pq}}.\vec{s}\right)\ d\vec{s}$$

**3D FT**

# A-Projection

Bhatnagar 08

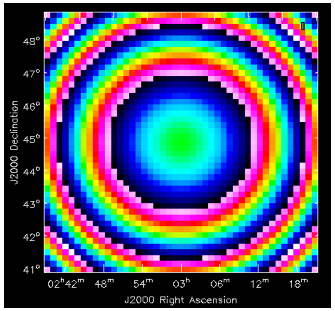
**Convolution function (4*4)**

**Beam (4*4)** **W term (scalar)**

$$\mathrm{Vec}(V_{pq}) = (G_q^* \otimes G_p)\mathrm{FT}\left[\left(E^*_{q,\vec{s}} \otimes E_{p,\vec{s}}.\exp\left(-2\pi i w_{pq}.\left(\sqrt{1-l^2-m^2}-1\right)\right)\right)\right]$$

$$\star \int_S \mathrm{Vec}(X_{\vec{s}}).\exp\left(-2\pi i(u_{pq}l + v_{pq}m)\right)dl.dm$$

**Convolution** **2D FFT**

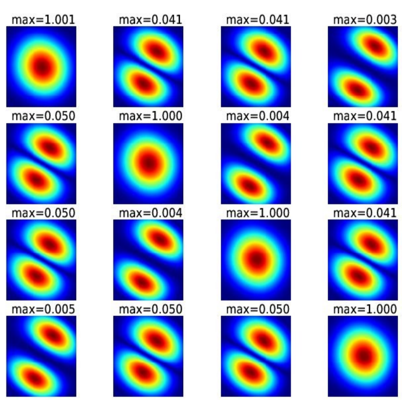



**This is an EXACT map from sky plane to the Visibilities in the UVW space!**

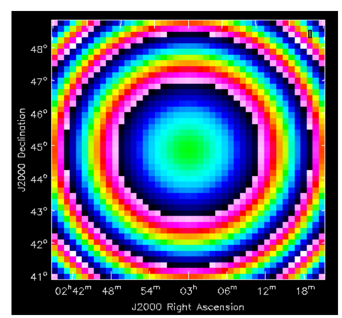
# A-Projection

Bhatnagar 08

FT(




X



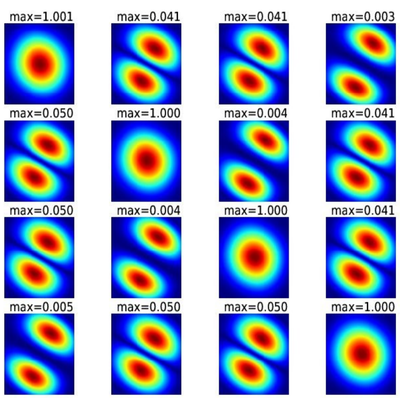

)

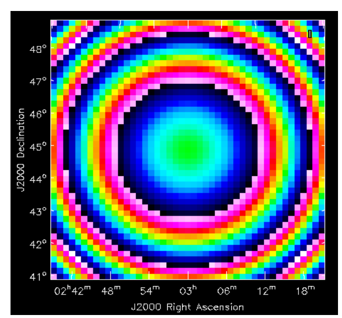
$$\ast \begin{bmatrix} \text{VisXX} \\ \text{VisXY} \\ \text{VisYX} \\ \text{VisYY} \end{bmatrix}$$

# A-Projection

Bhatnagar 08

$$\begin{bmatrix} VisXX \\ VisXY \\ VisYX \\ VisYY \end{bmatrix}$$

FT(


X


)

$*$

$$\begin{bmatrix} GridXX \\ GridXY \\ GridYX \\ GridYY \end{bmatrix}$$

# A-Projection

Bhatnagar 08

$$FT\left(\begin{bmatrix} GridXX \\ GridXY \\ GridYX \\ GridYY \end{bmatrix}\right) \rightarrow \begin{bmatrix} ImXX \\ ImXY \\ ImYX \\ ImYY \end{bmatrix}$$

$$\downarrow$$

I, Q, U, V

# A-Projection

Bhatnagar 08

**The inverse map is approximative! (based on pseudo-inverse)**

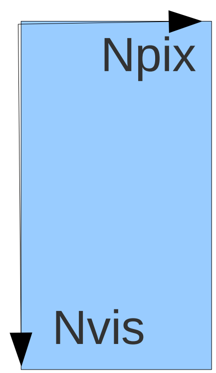
$$\mathrm{Vec}(V_{pq}) = (G_q^* \otimes G_p)\mathrm{FT}\left[\left(E_{q,\vec{s}}^* \otimes E_{p,\vec{s}}.\exp\left(-2\pi i w_{pq}.\left(\sqrt{1-l^2-m^2}-1\right)\right)\right)\right]$$

$$\star \int_S \mathrm{Vec}(X_{\vec{s}}).\exp\left(-2\pi i (u_{pq} l + v_{pq} m)\right) dl.dm$$

**This equation is linear in Sky**

$$V^M = \mathbf{A} I^M$$

**A=S.AW.F =**

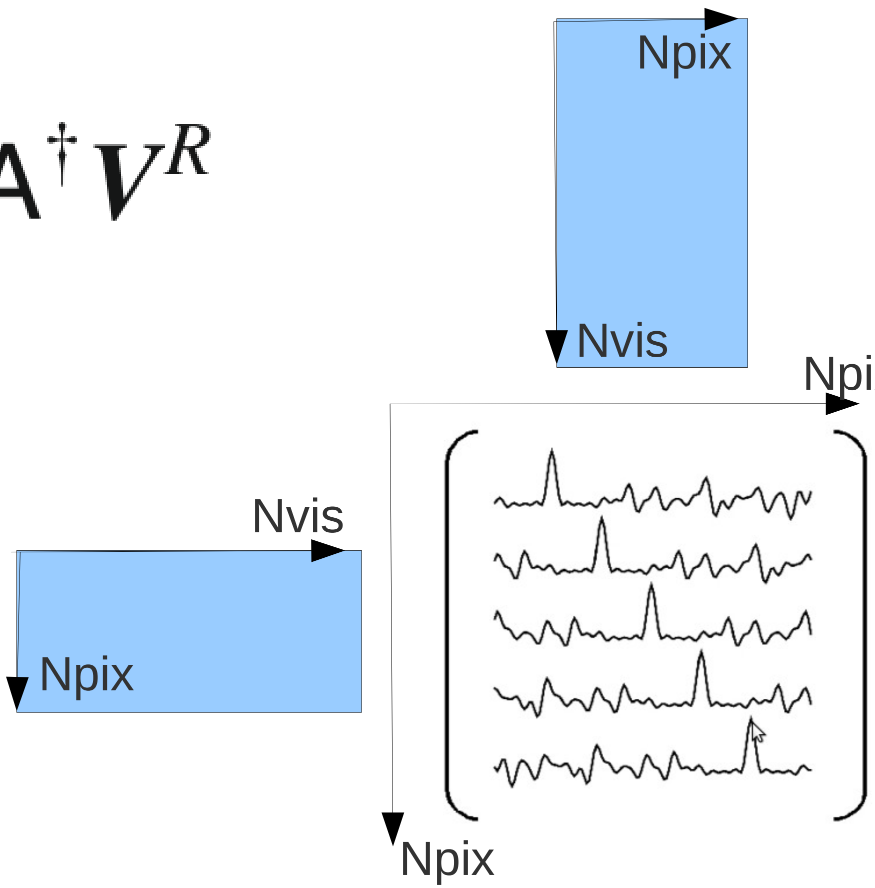


# A-Projection

Bhatnagar 08

**The inverse map is approximative! (based on pseudo-inverse)**

$$I^R = \left[A^\dagger A\right]^{-1} A^\dagger V^R$$




$= A^H A$

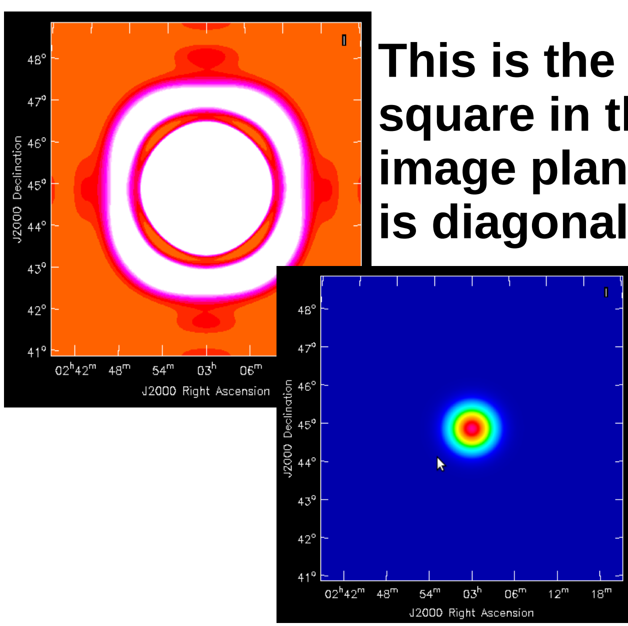
See Urvashi Rau PhD thesis

# A-Projection

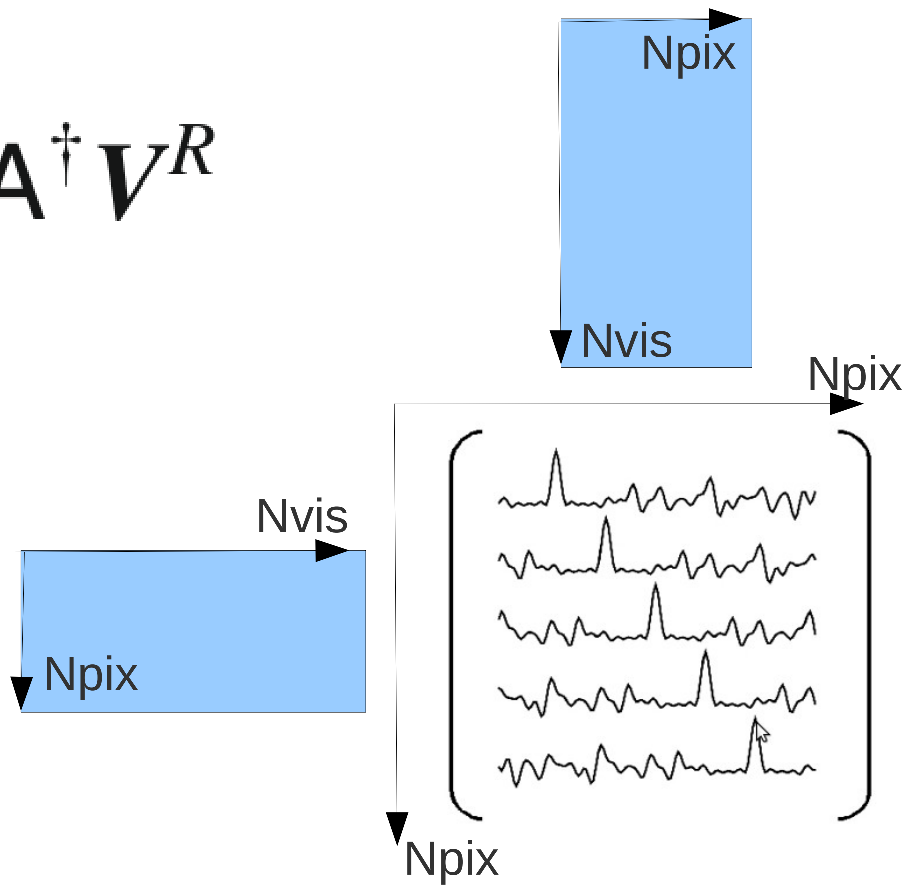
Bhatnagar 08

**The inverse map is approximative! (based on pseudo-inverse)**

$$I^R = \left[A^\dagger A\right]^{-1} A^\dagger V^R$$



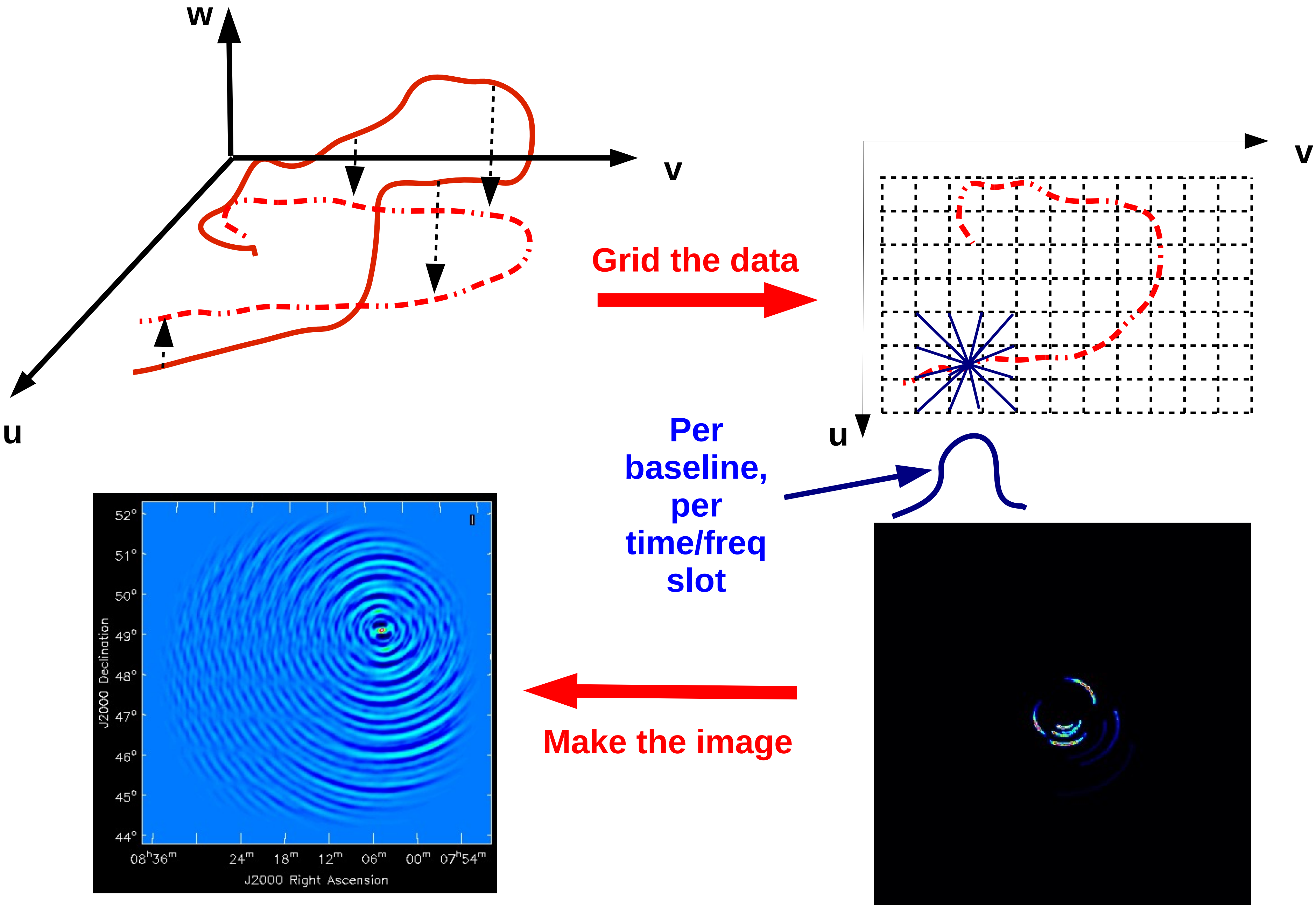

**This is the beam square in the image plane if $A^H A$ is diagonal**



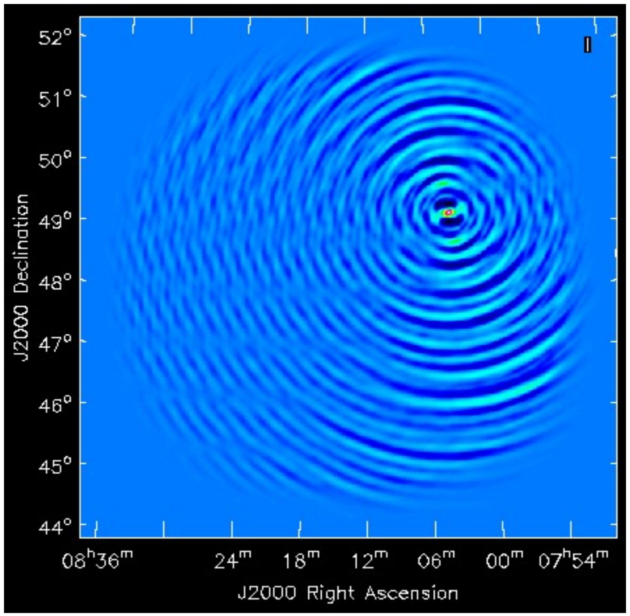

**$= A^H A$**

See Urvashi Rau PhD thesis

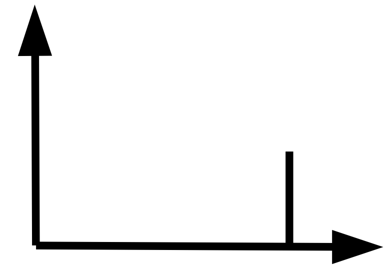
# Gridding in practice?

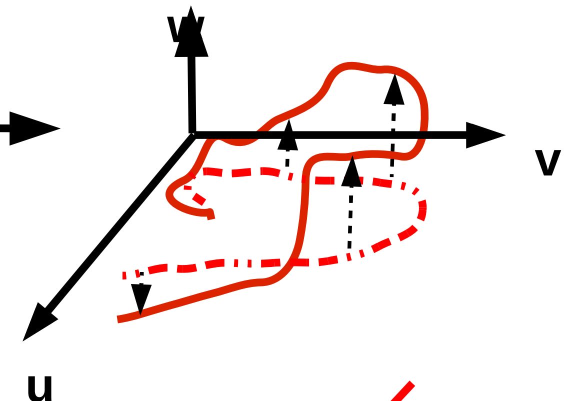


# Deconvolution?

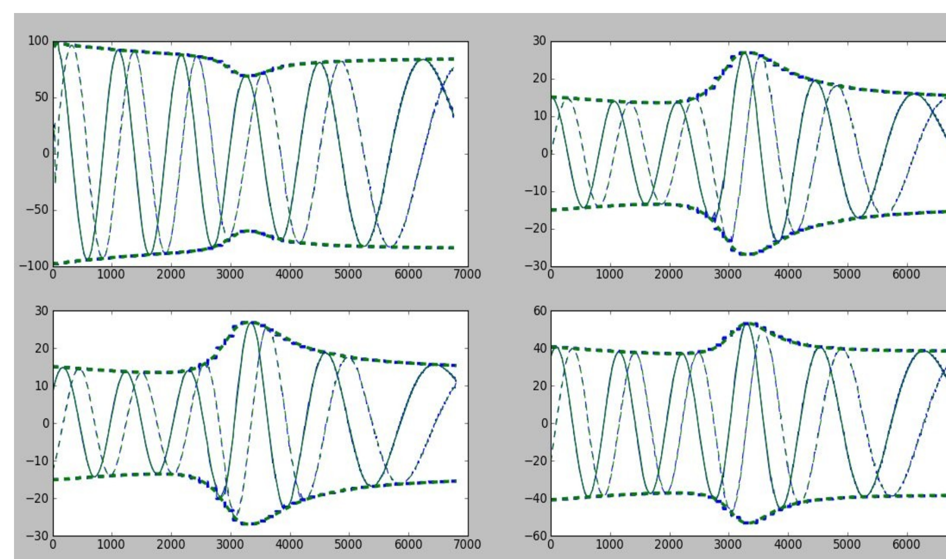



Minor Cycle

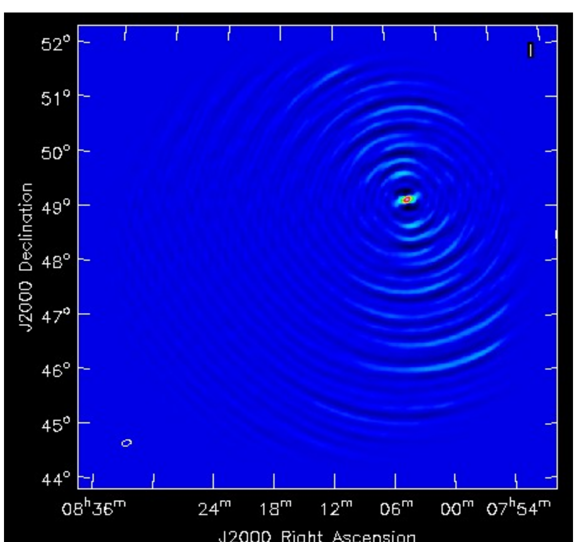





Fourier Transform

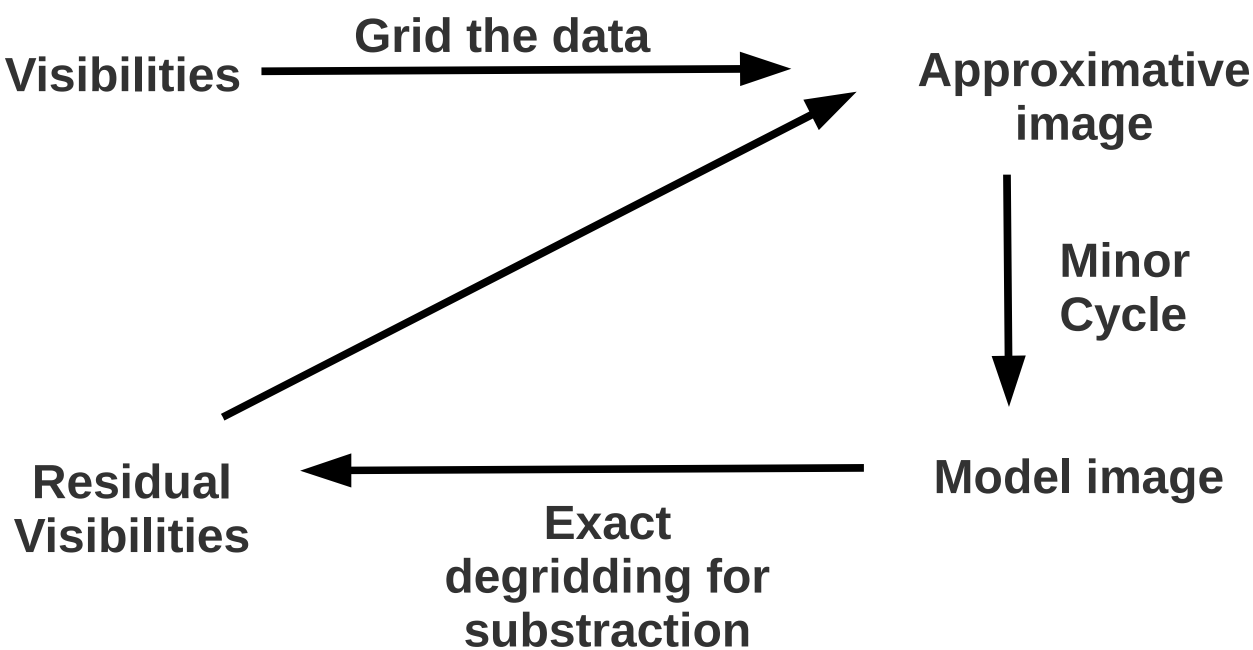


Fourier Transform the difference



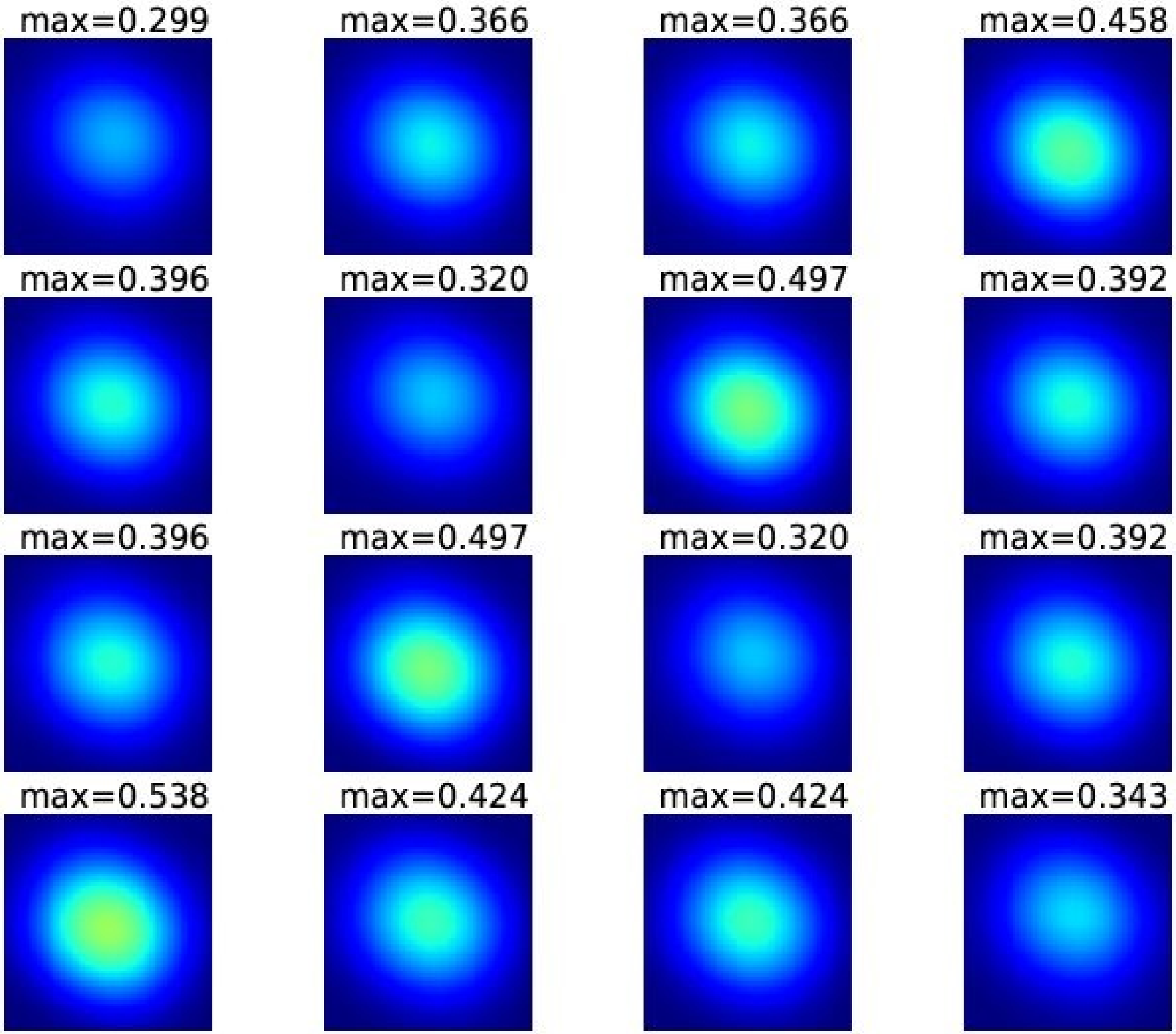
# JAWS: the practice

- Plug in the casa architecture
- Full Polarization
- Convolution function is mapped by i,j,t, nu
- Ionosphere easy to plug in
- Will run in parallel




After a number of iteration, the flux in the clean component converges to the true values (to be studied)

# LOFAR Beam: The Mueller Matrix varying over the image plane

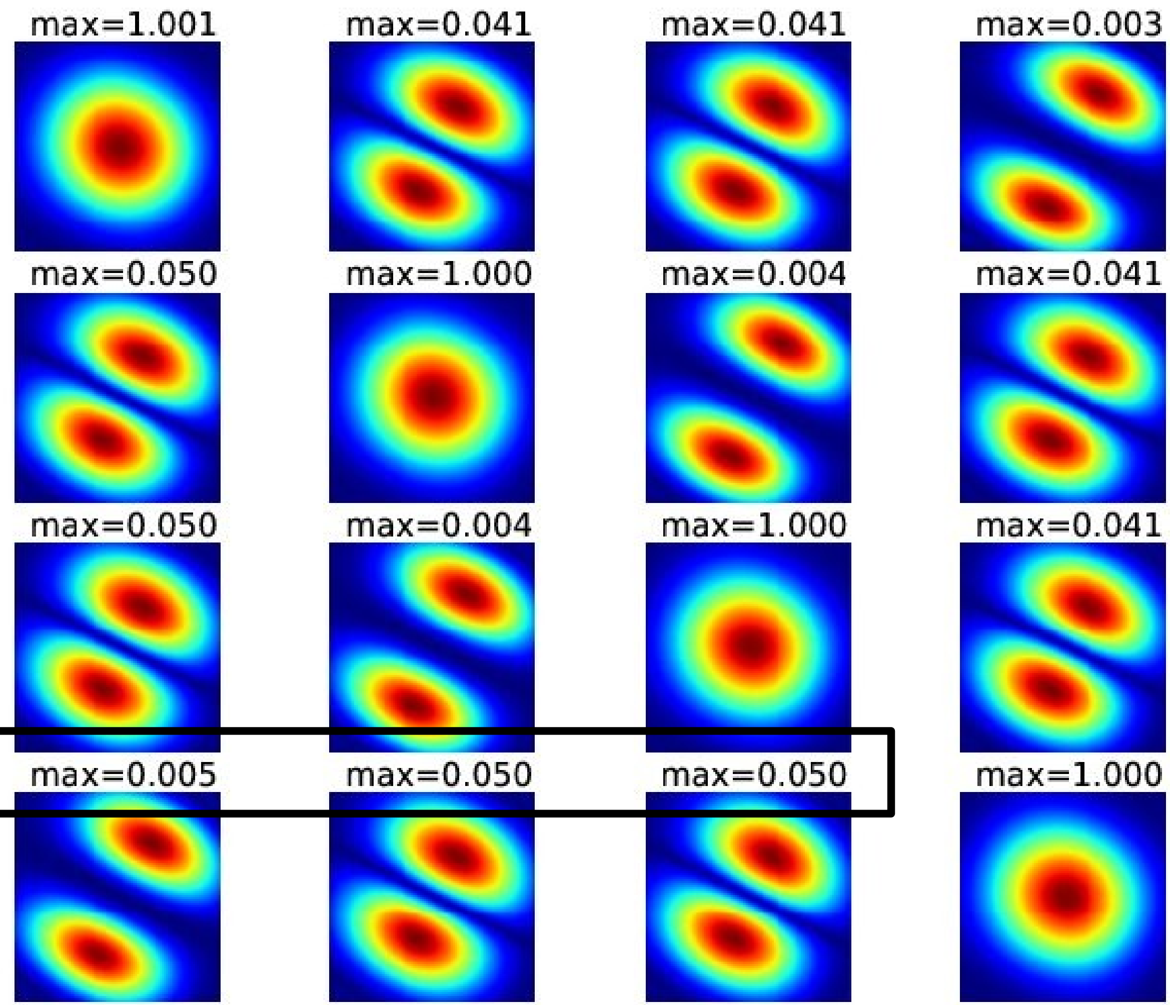



One pair of antennae, one time and frequency value

# LOFAR Beam: The Mueller Matrix varying over the image plane

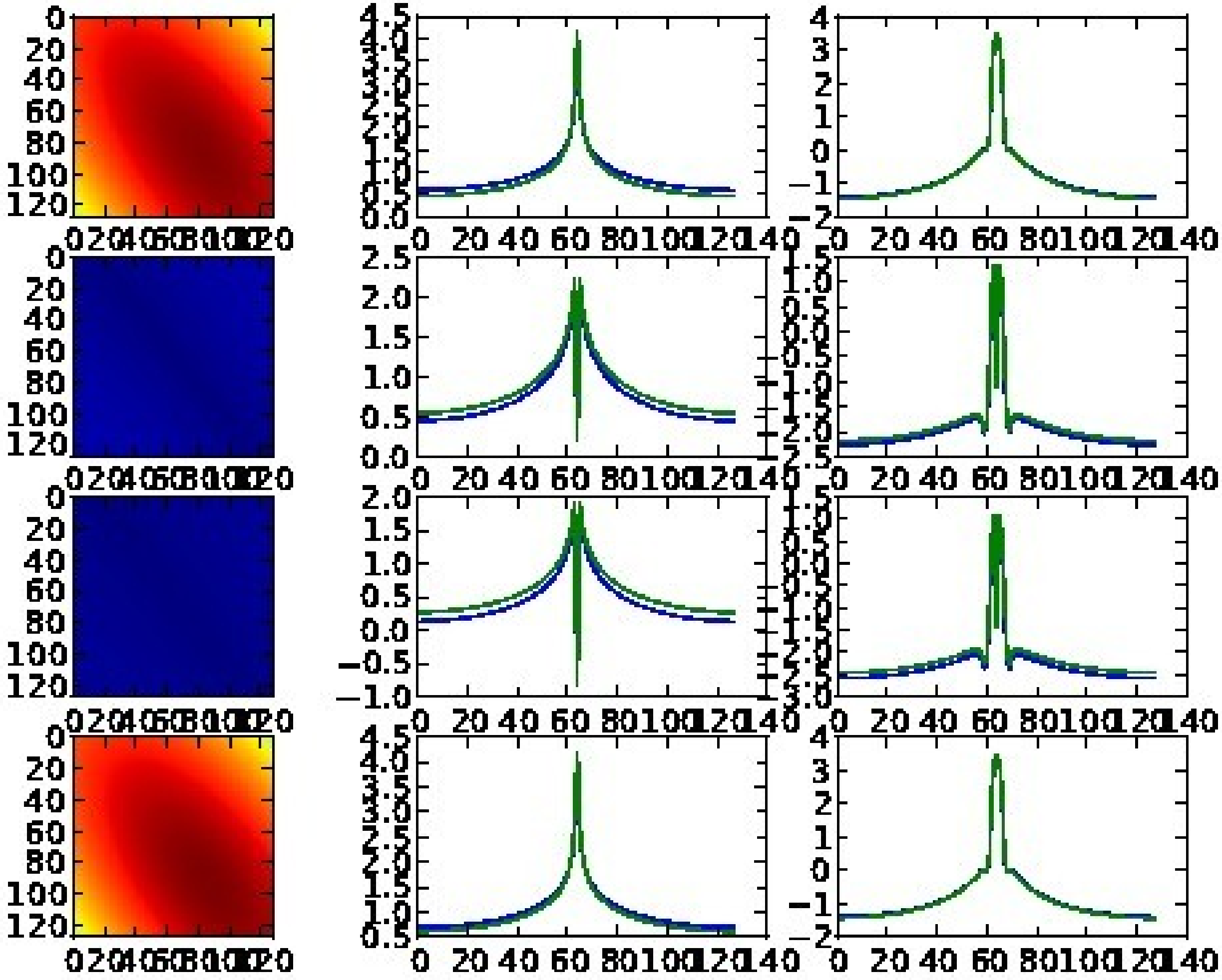
**Beam bormalized by Beam Jones matrix at the center of the field (we correct the visibilities accordingly before the imaging)**

**Off axis small but perhaps not negligible for the degridding ?**




**!!! Color bar is adapted to the image here otherwise you don't see anything!!!**

# … When Direction Dependent Effects (DDE) become a problem : Beam



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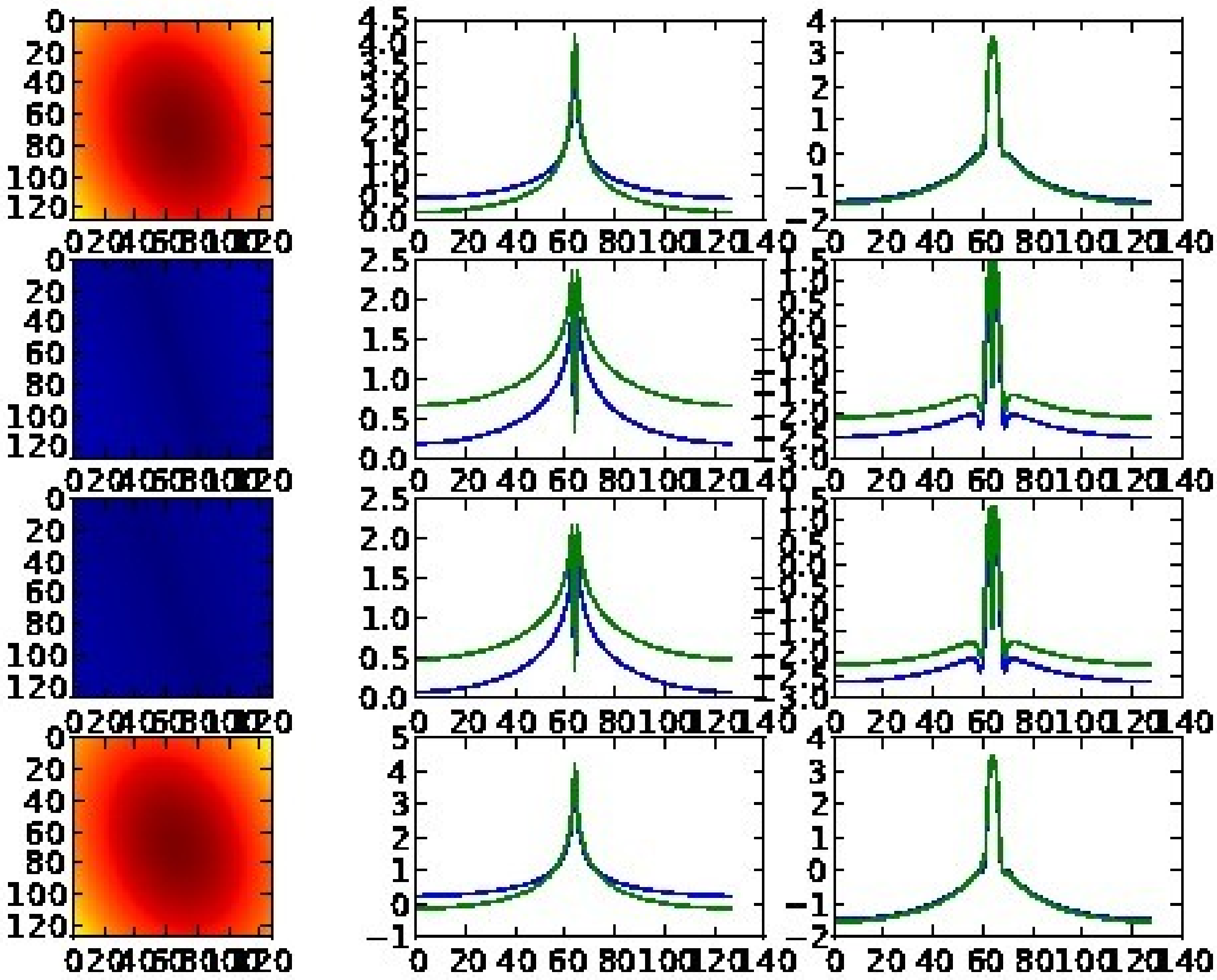

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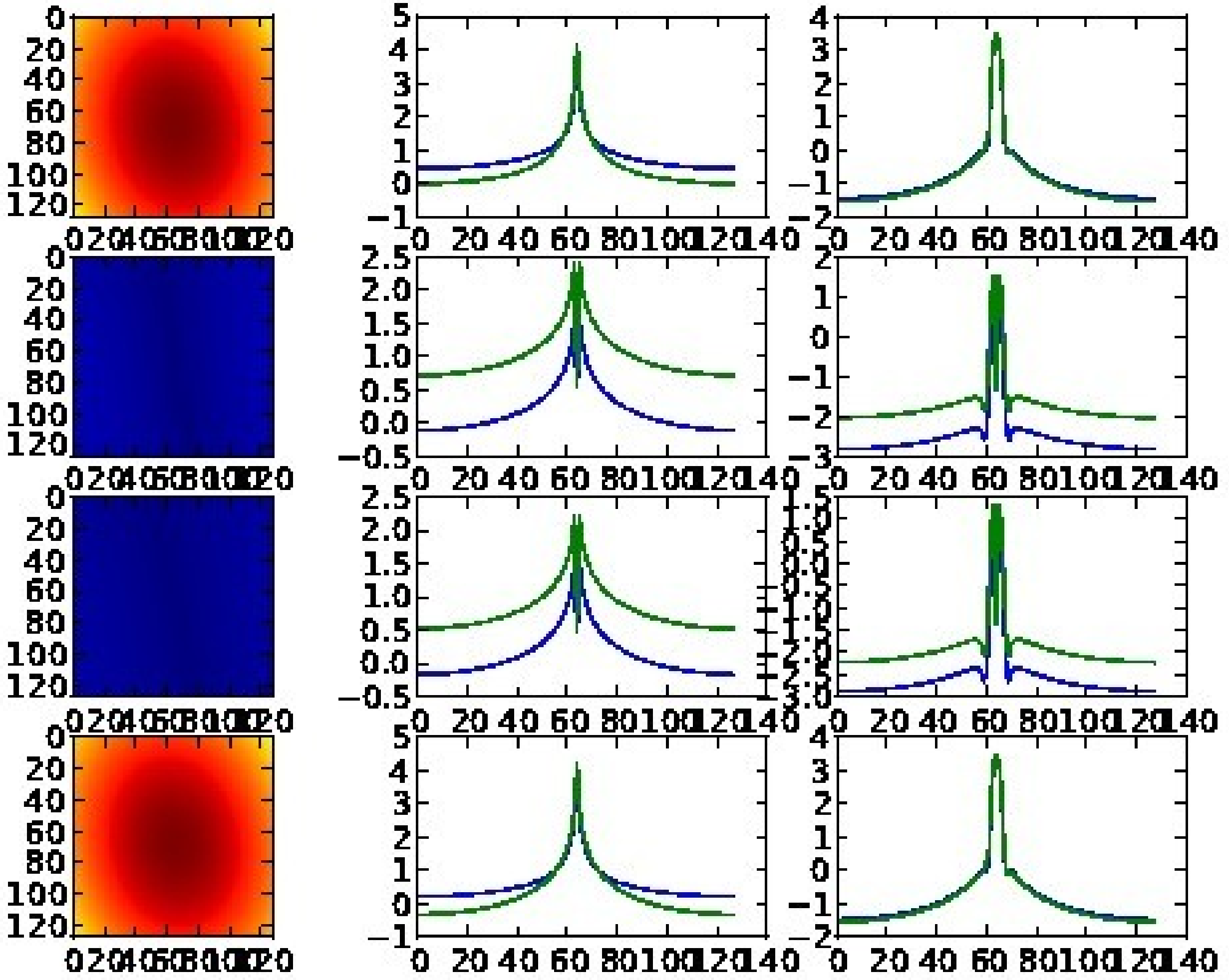

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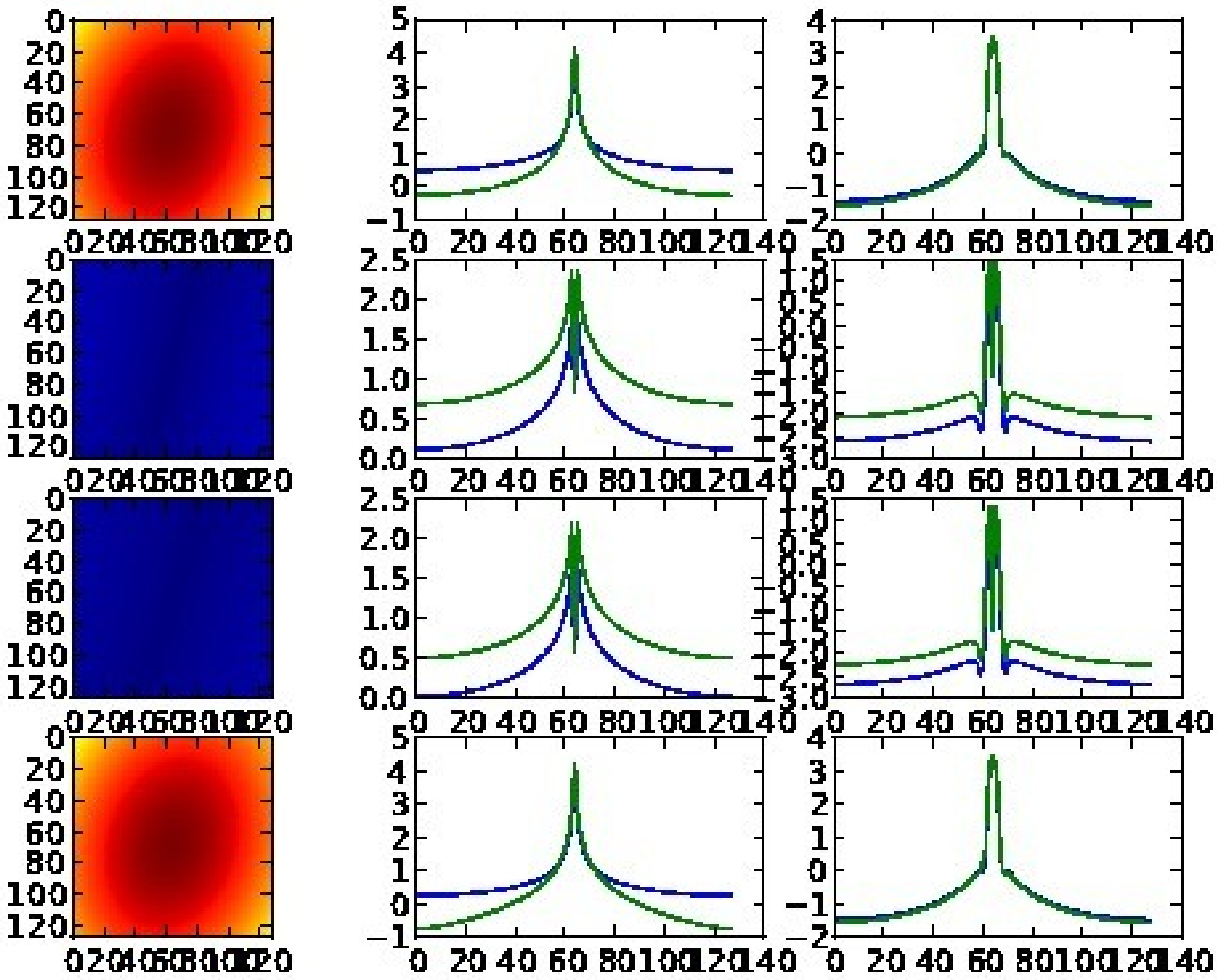
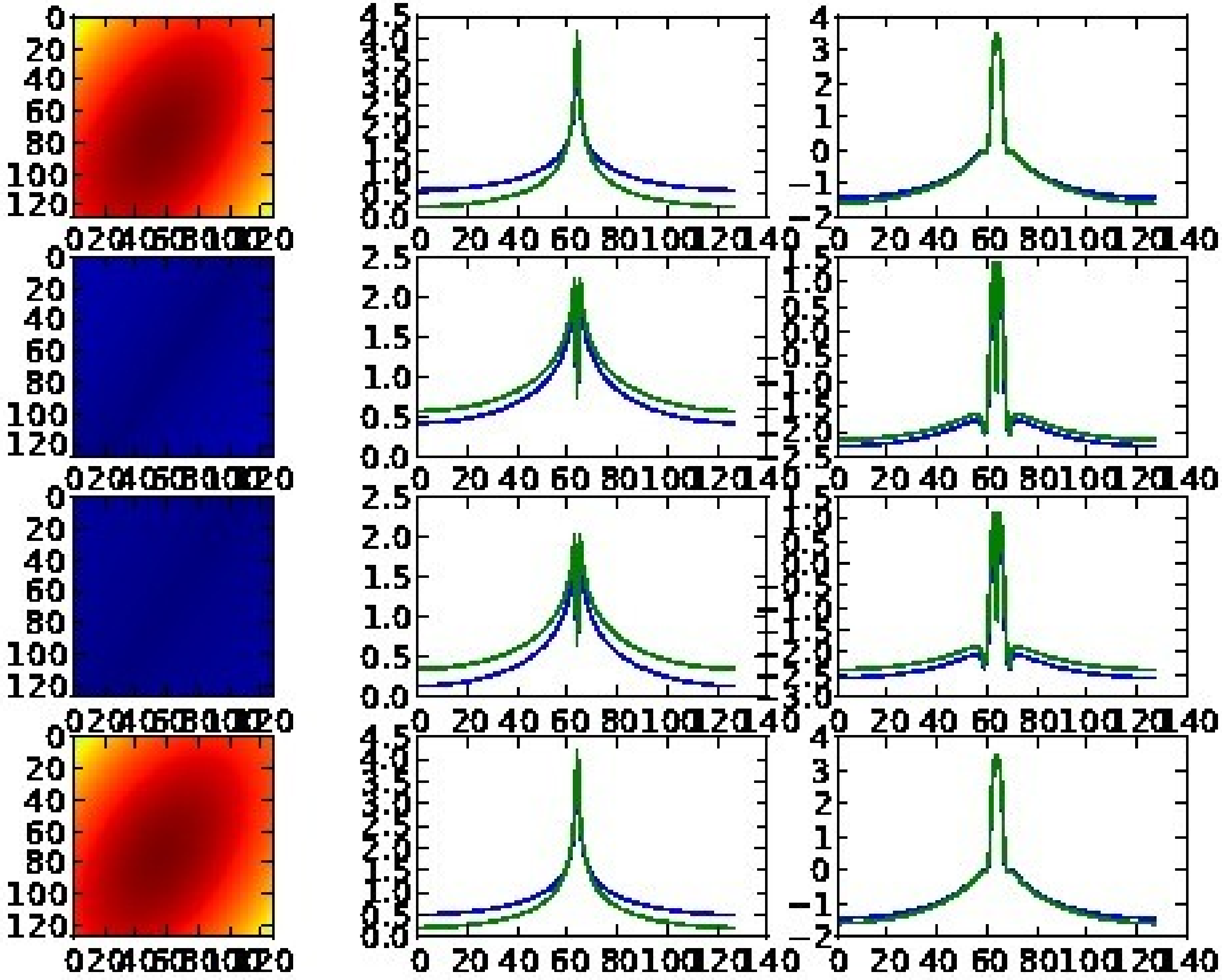


# … When Direction Dependent Effects (DDE) become a problem : Beam



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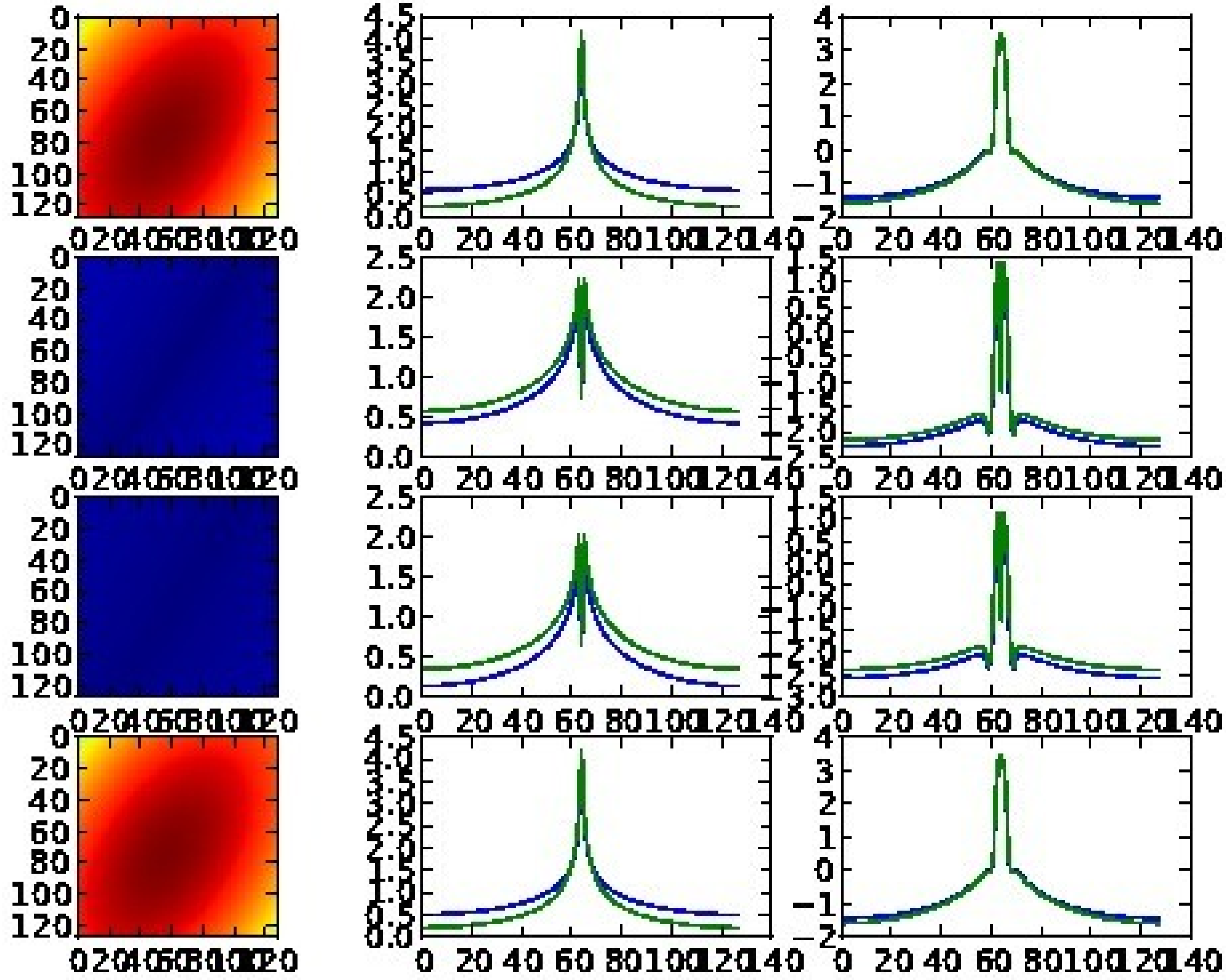
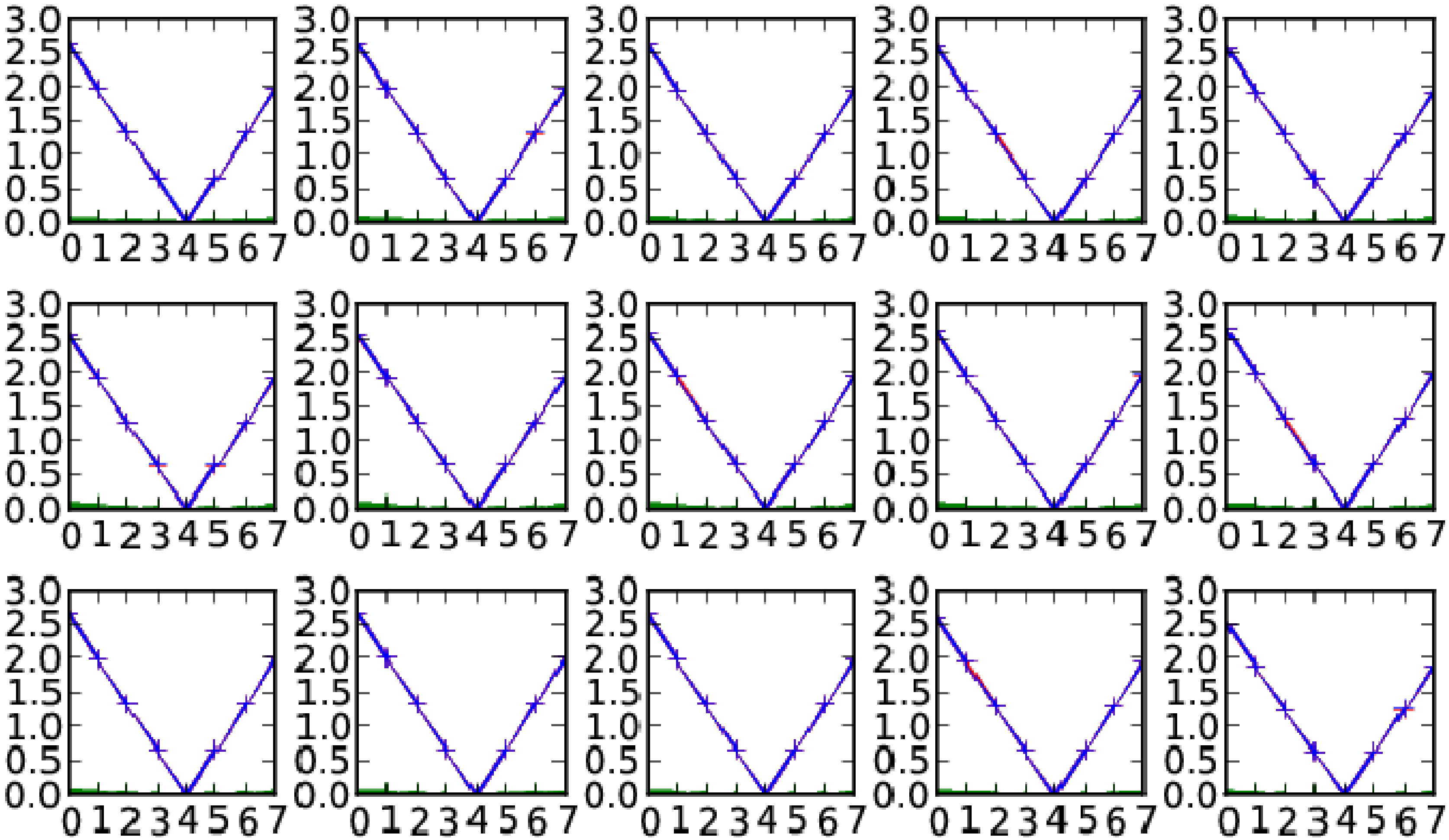
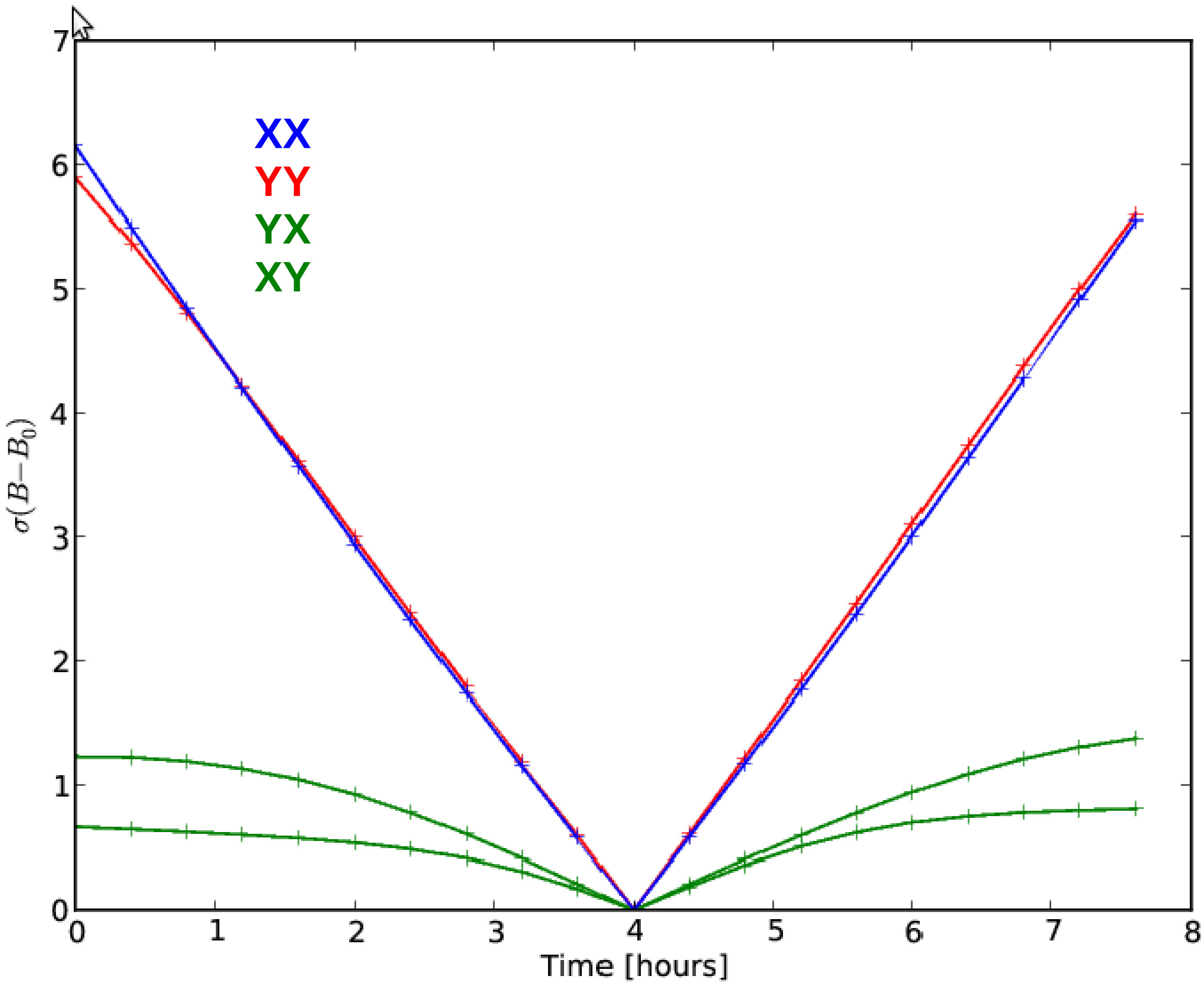


# … When Direction Dependent Effects (DDE) become a problem : Beam




Beam variability across a subband during a 6 hours observation (ordinate in per thousand)

# … When Direction Dependent Effects (DDE) become a problem : Beam



# JAWS: the practice

**How many convolution function?**
**- One convolution every 10 minutes**
**- 8 hour oberving run**
**- 45 antenna: 990 baselines**
**- 16 Mueller elements**
**- 1 complex number pert pixel**
**- Average size 30*30 pixel**

**= 1216 Tbytes**

# JAWS: the practice

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**- One convolution every 10 minutes**
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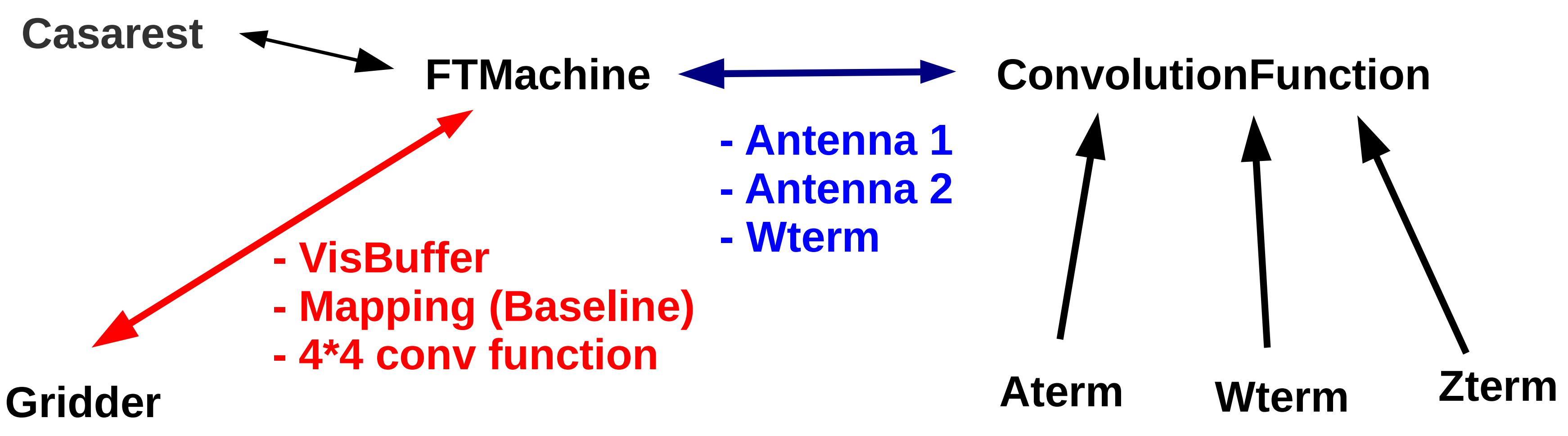
**- Average size 30*30 pixel**

**= 1216 Tbytes**

**→ We compute the convolution functions on the fly**

**- We compute and store the Aterm and Wterm at the minimum resolution**



# JAWS: the practice

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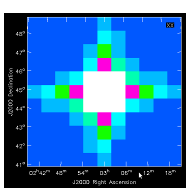
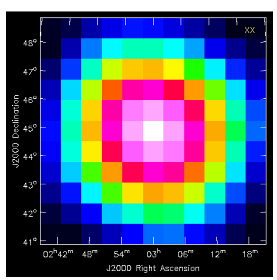
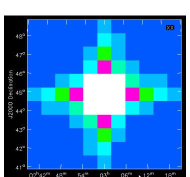
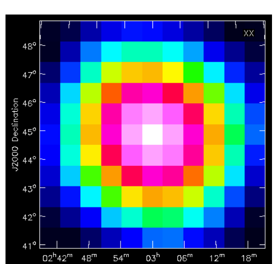
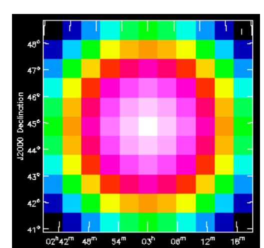



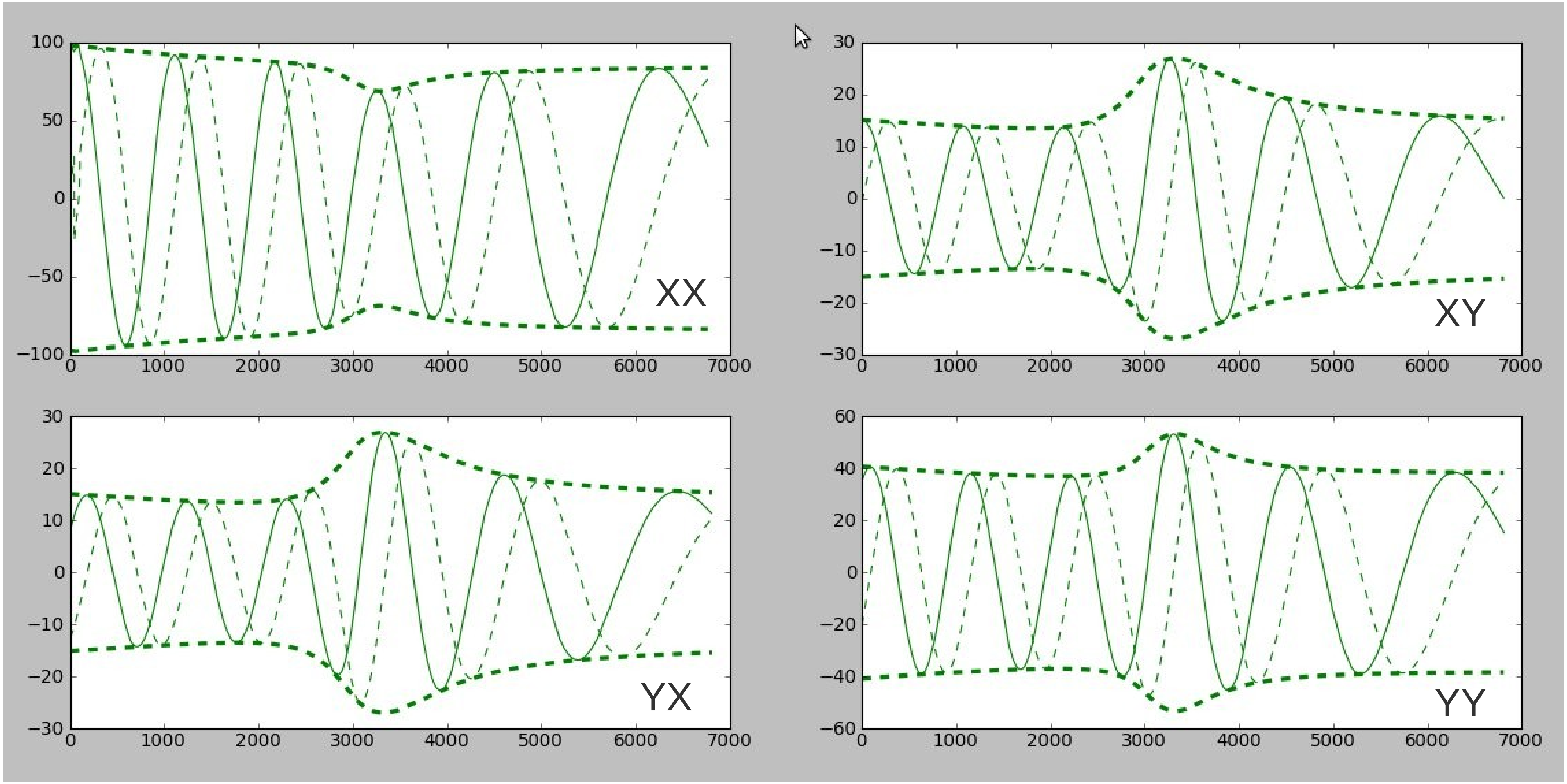





# Mathematical framework-works

## One off-axis source IQUV=(100, 40, 20 10)

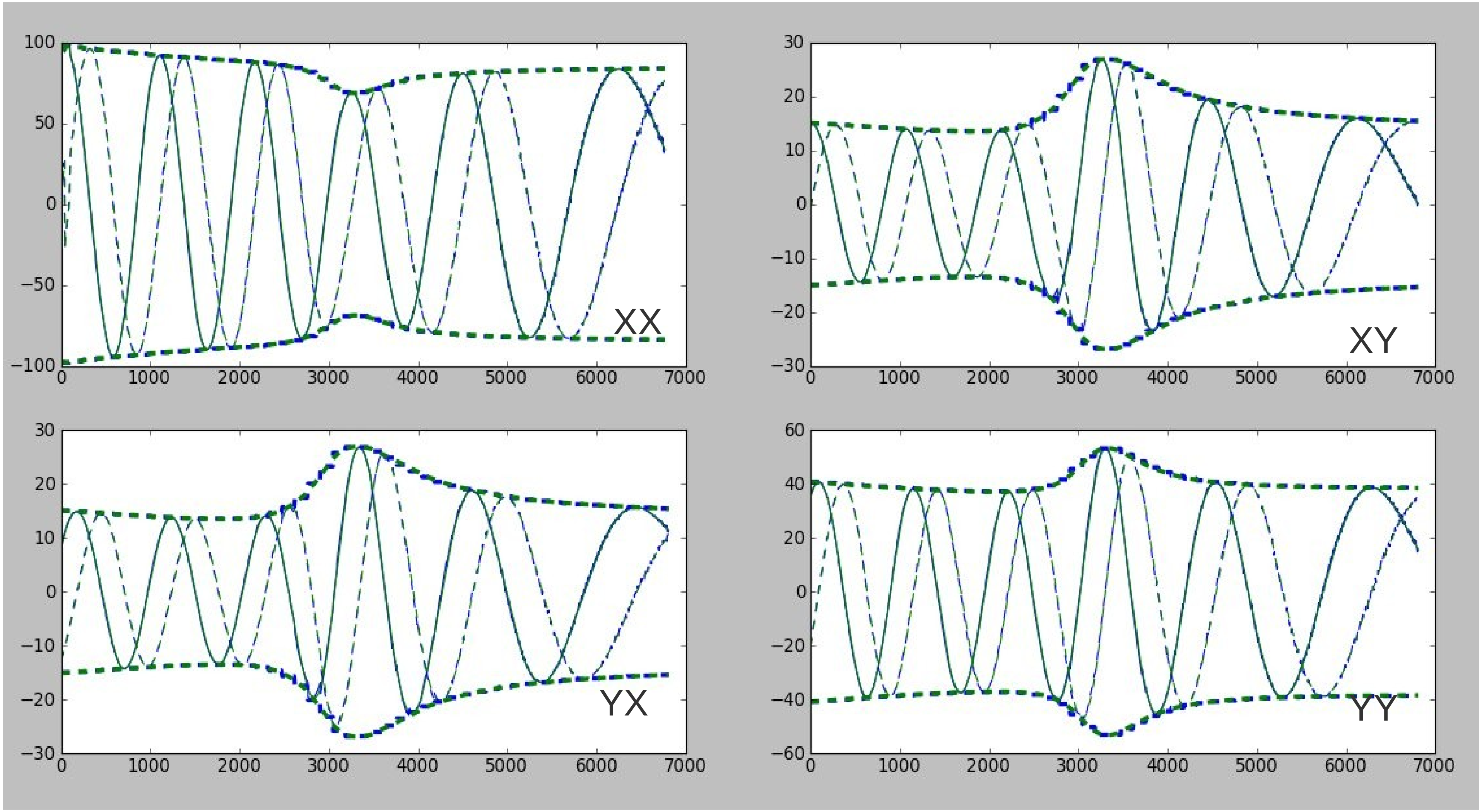
BBS predict (DFT)



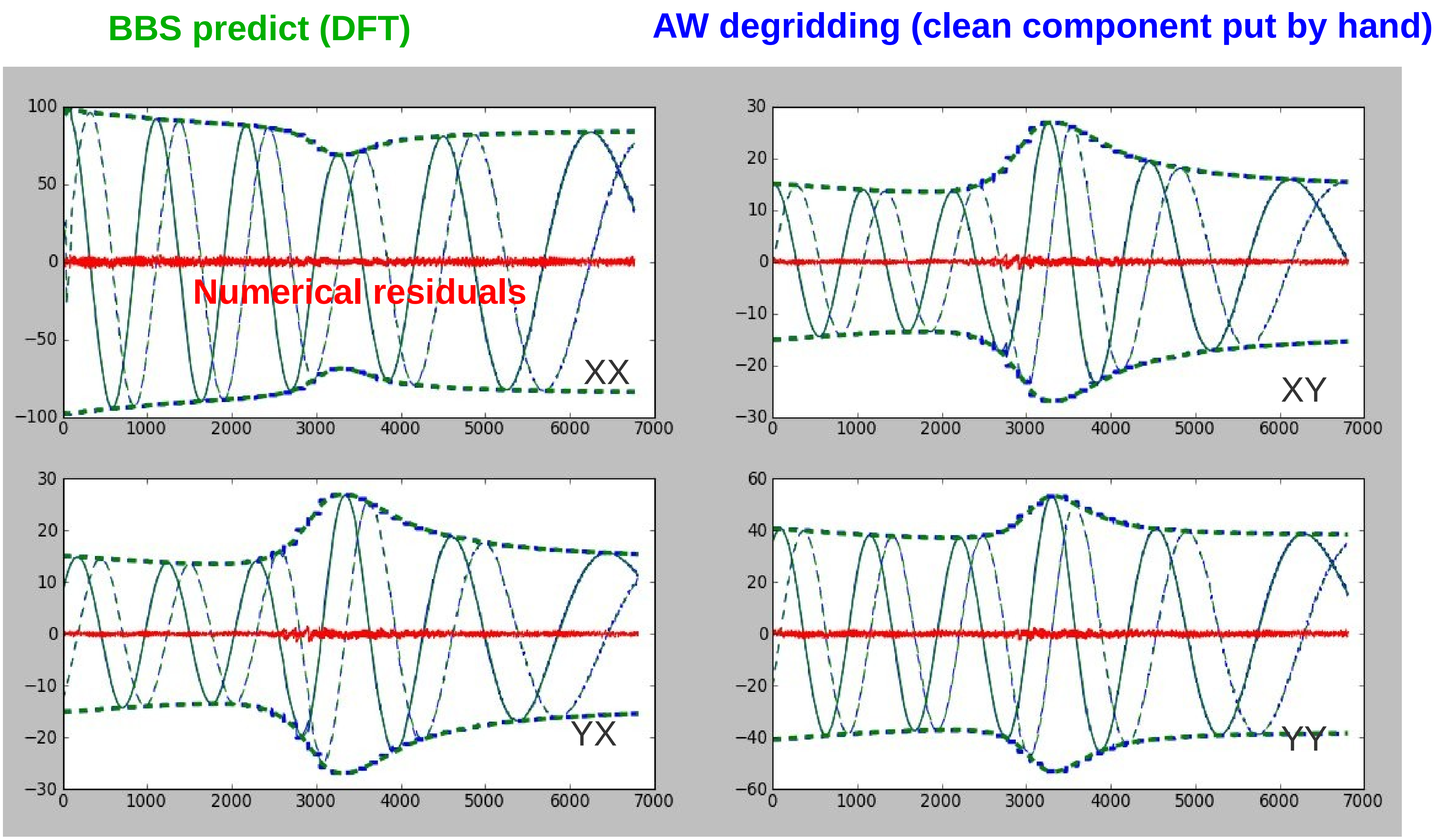
# Mathematical framework-works

BBS predict (DFT)

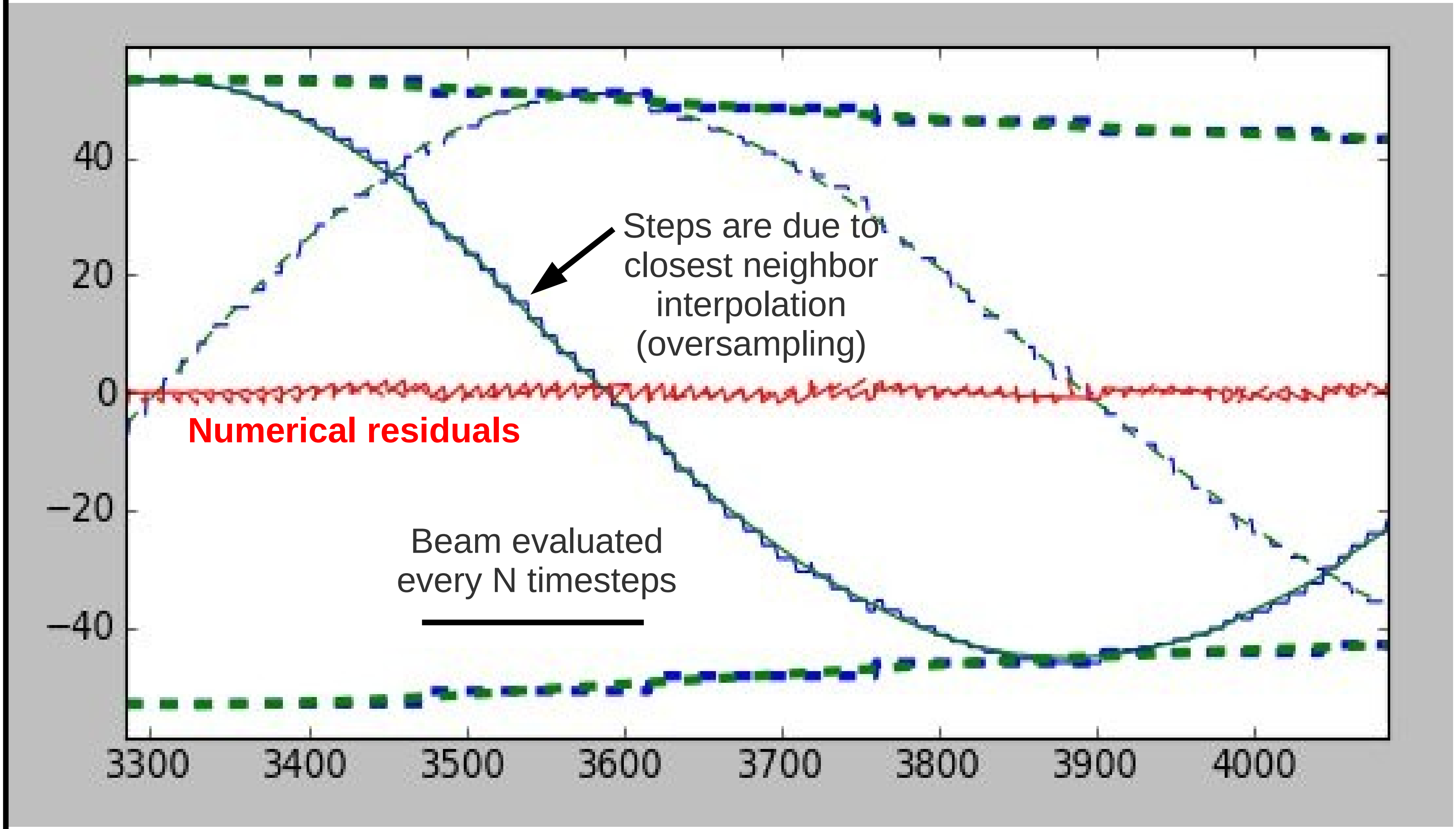
AW degridding (clean component put by hand)



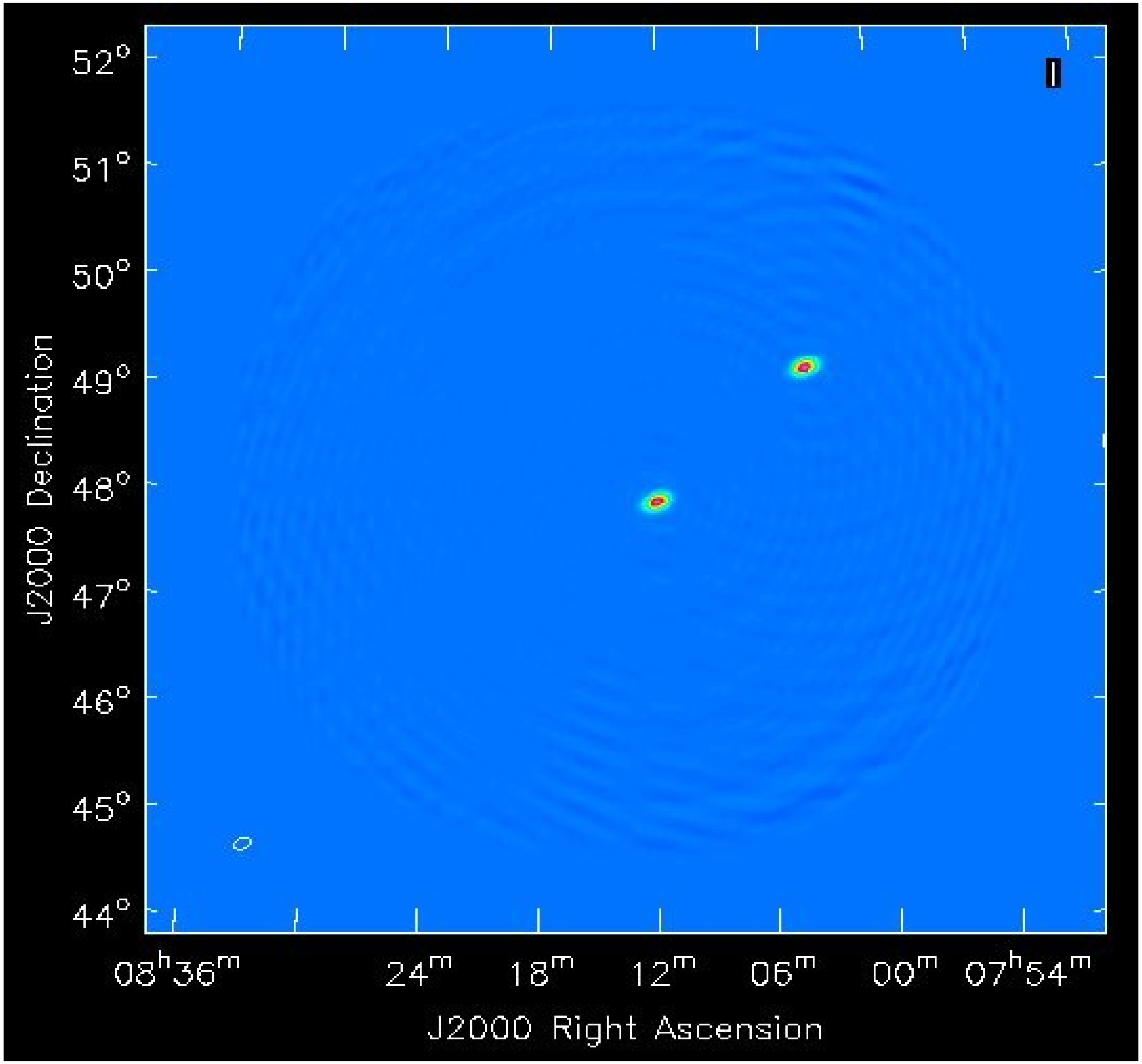
# Mathematical framework-works



# Mathematical framework-works

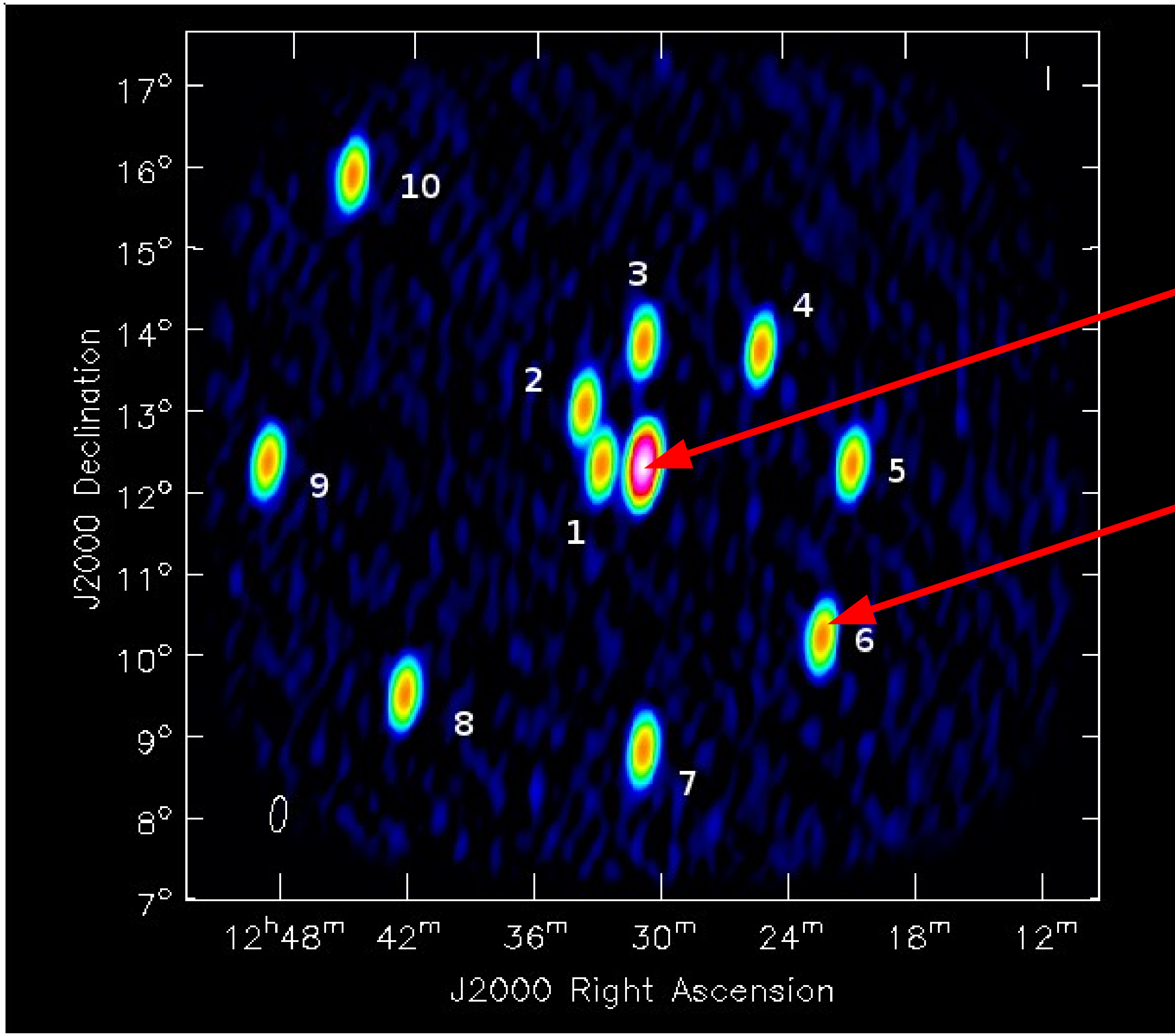


# Mathematical framework-works




**Recovered IQUV=(100, 40, 20 10) fluxes to better than 1%**

# Mathematical framework-works




10 Jy Source

1Jy source

Recovered flux better than 1%

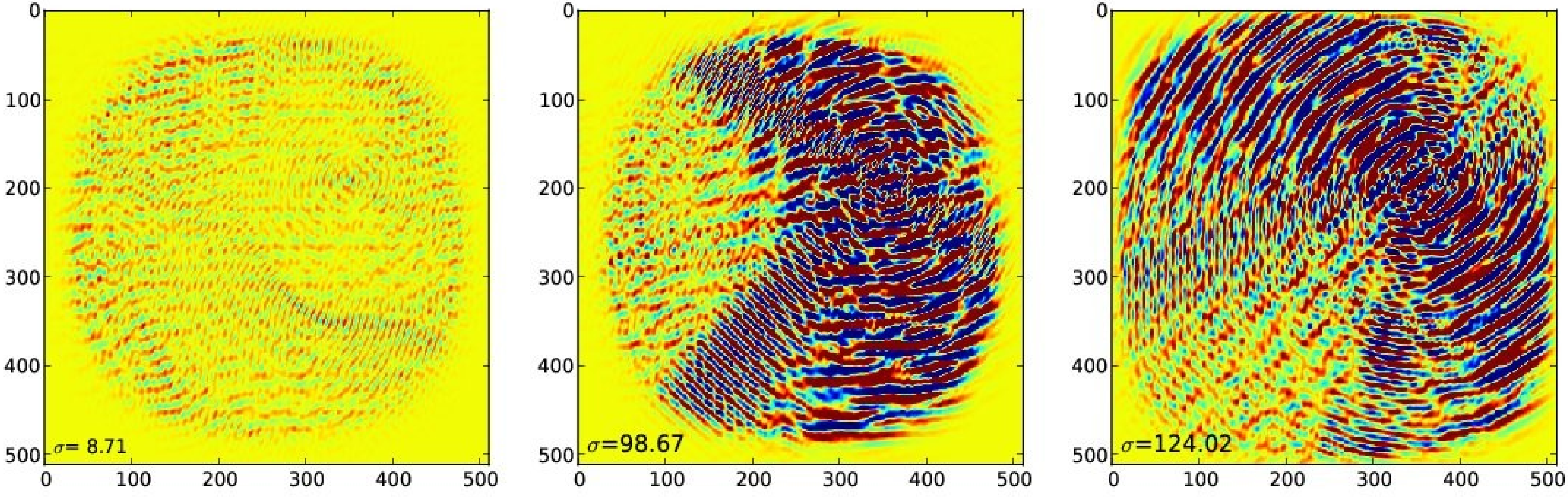
Francesco Da Gasperin

# Mathematical framework-works

**Same simulated dataset with one off-axis source and the beam (IQUV=100,40,20,10)**

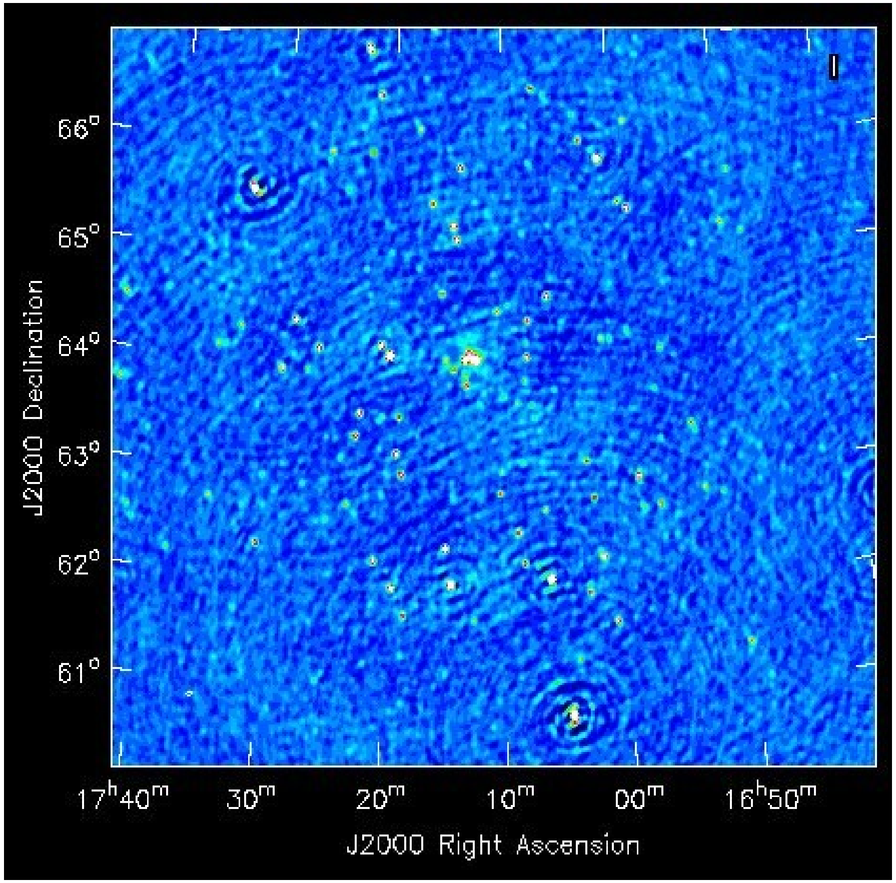
---

**Residual images, Stokes I**

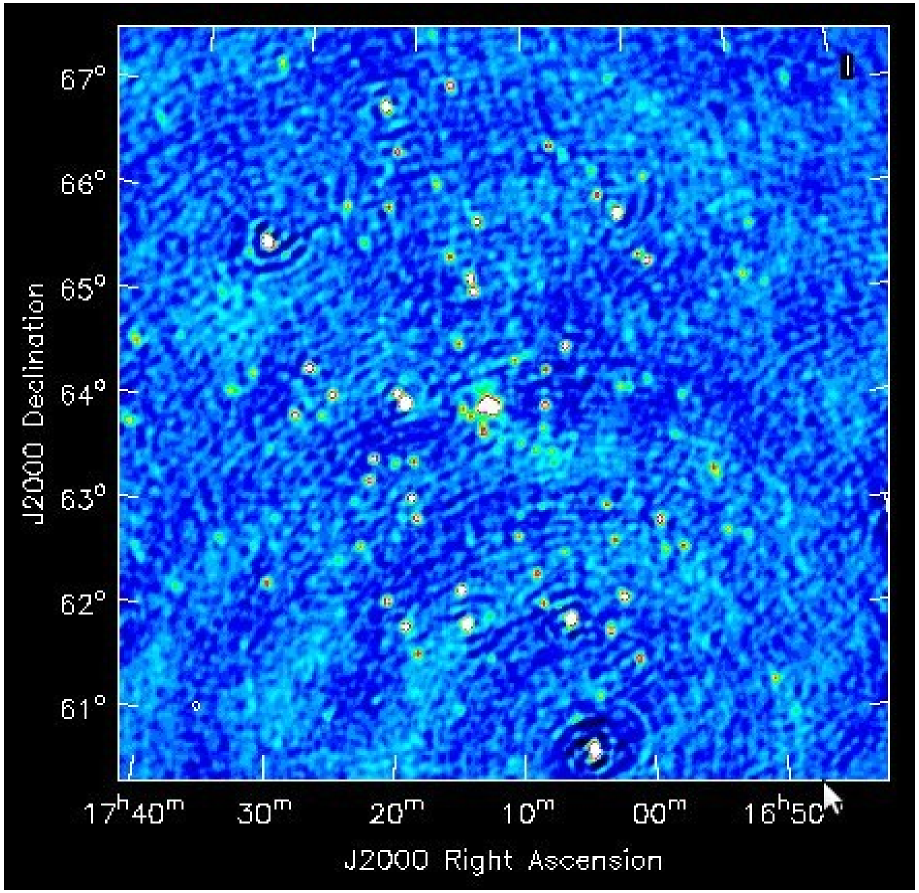



**AW projection** | **AW, only diag terms** | **W projection only**

# On real data (A2255)




**Casa**




**JAWS**

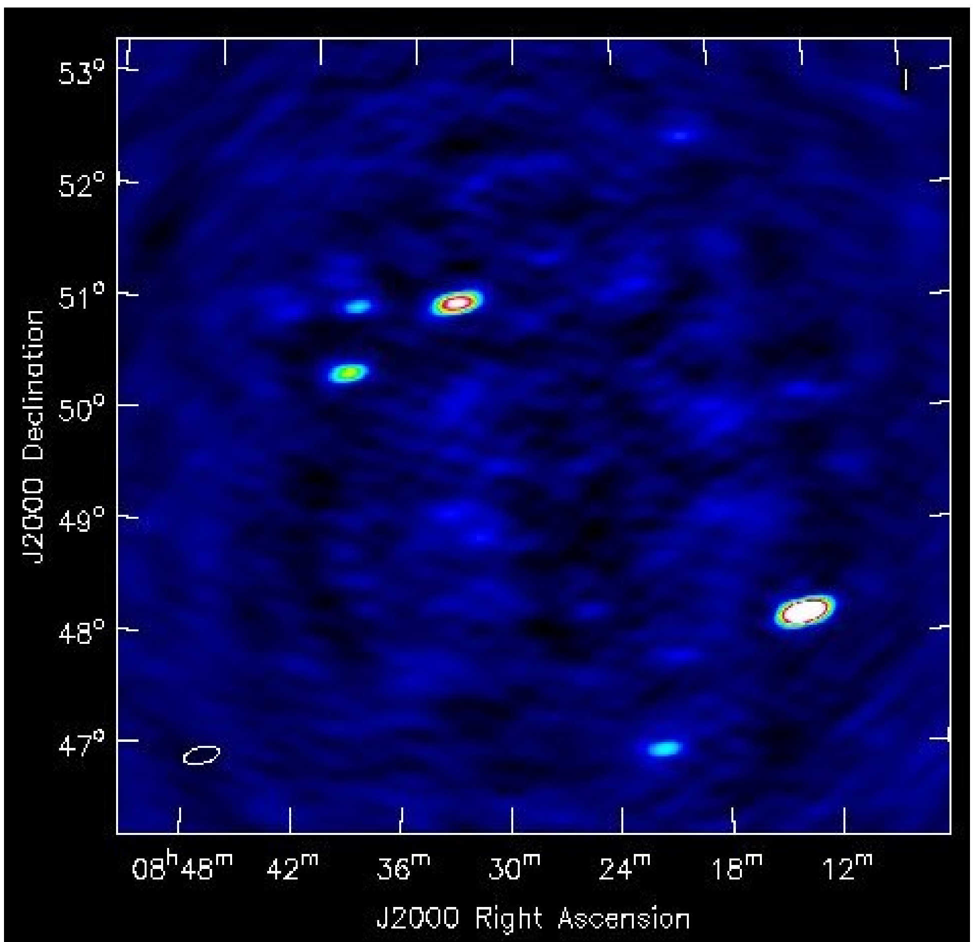
Roberto Pizzo

# On real data (3C196)

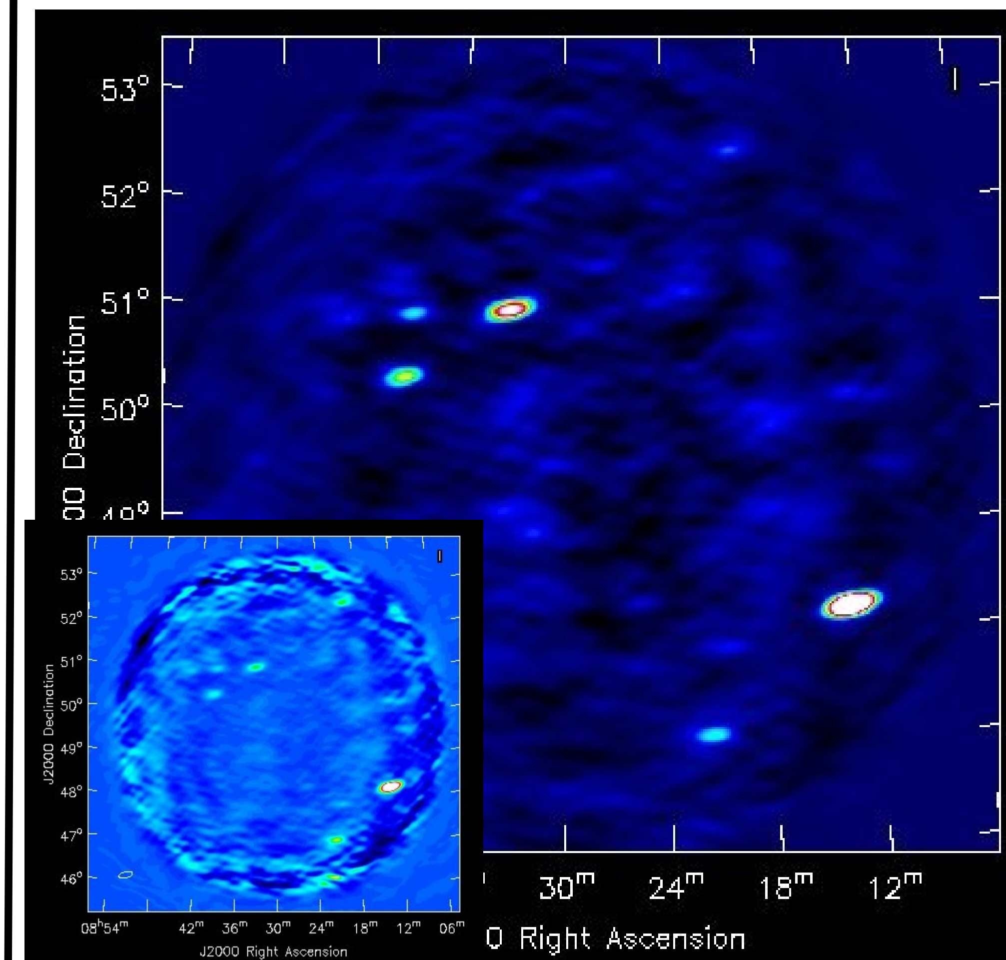
## 3C196 off axis ~150MHz

- Calibrated using 3C196+2 sources sources
- AW visibility estimates for those. Little difference?

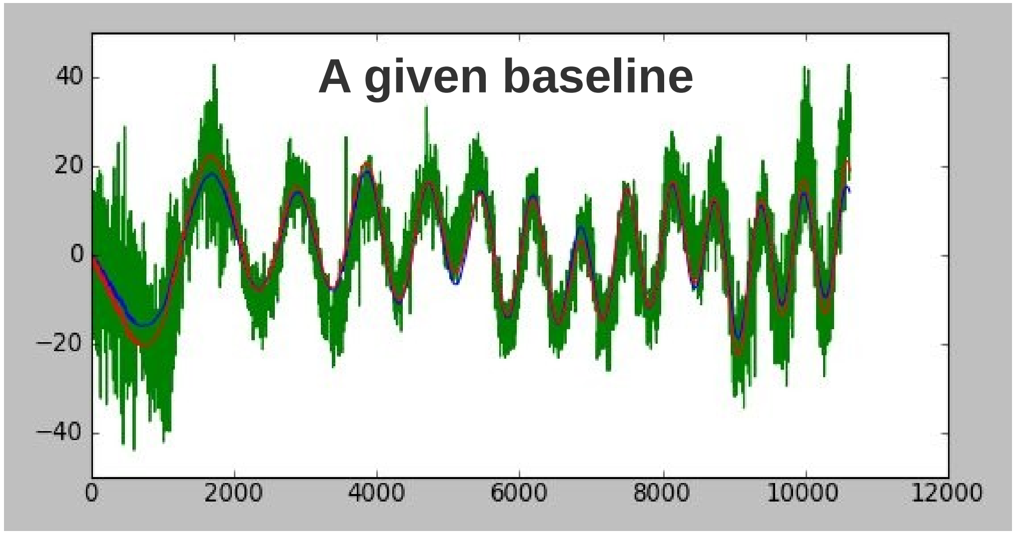
**NOT Taking the beam into account**



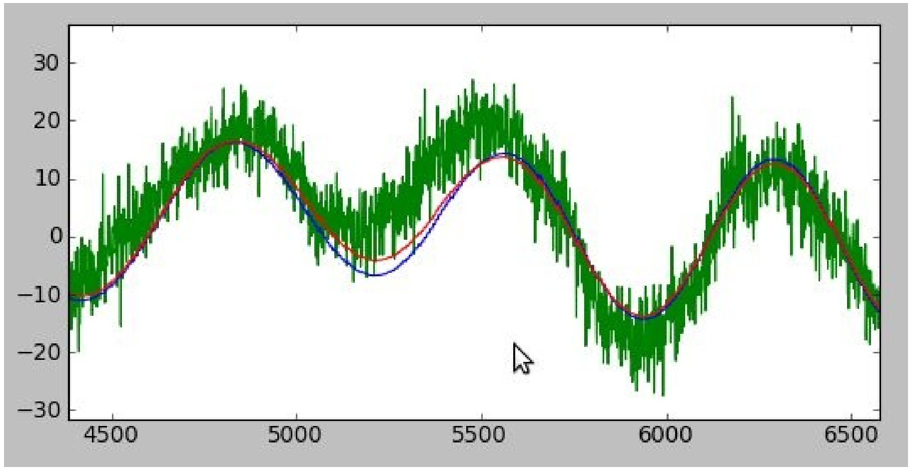

**Taking the beam into account**

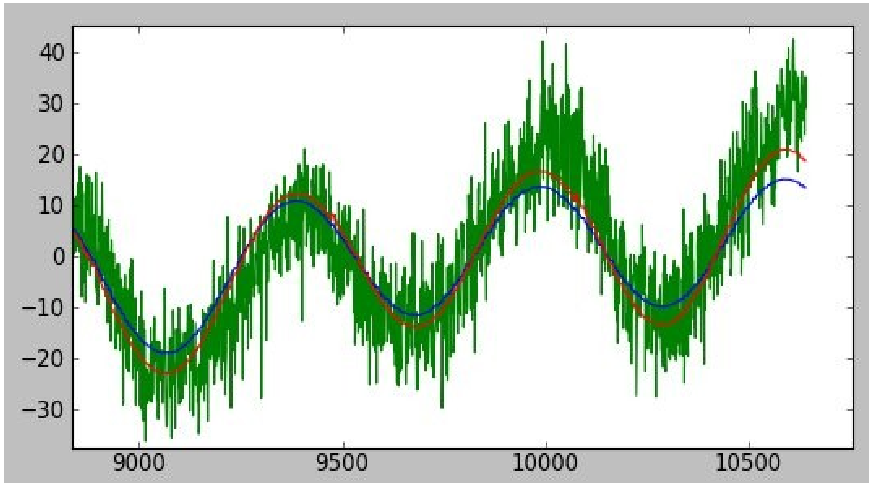


# On real data (3C196)



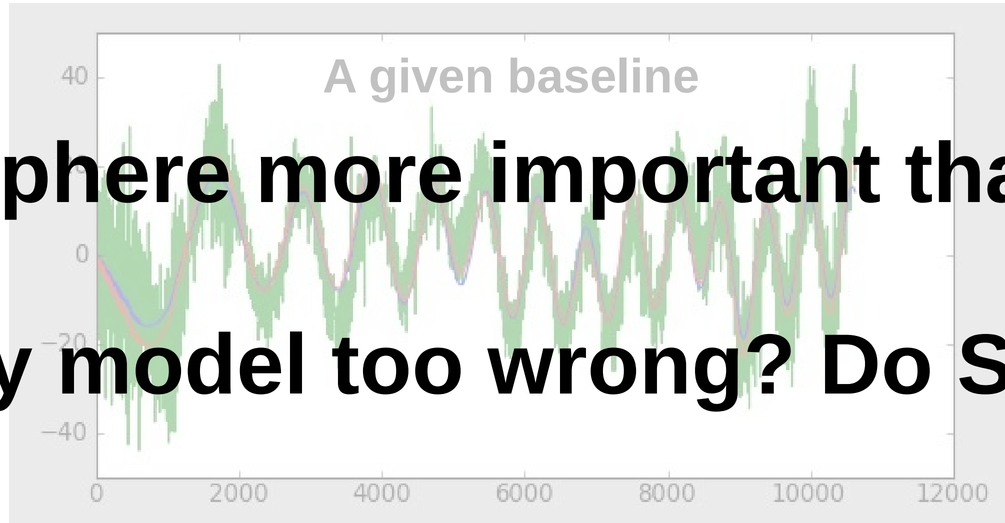

**Beam taken into account**




**No Beam taken into account**

# On real data (3C196)



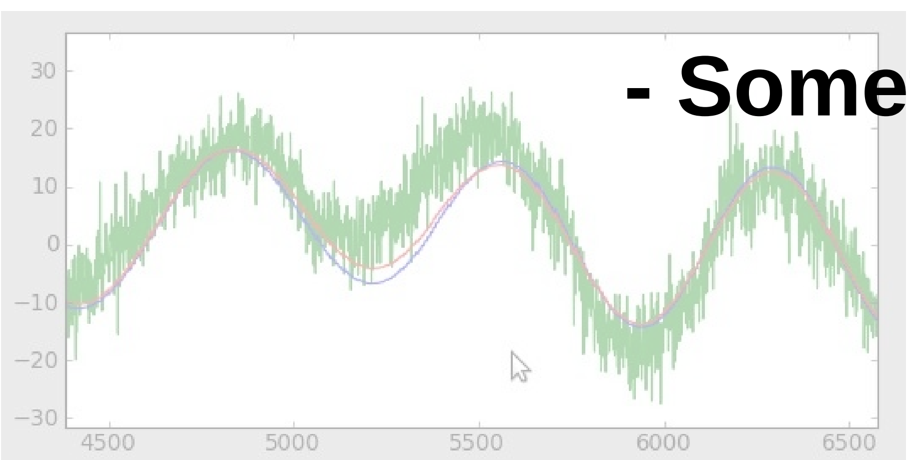

- Ionosphere more important than beam ?

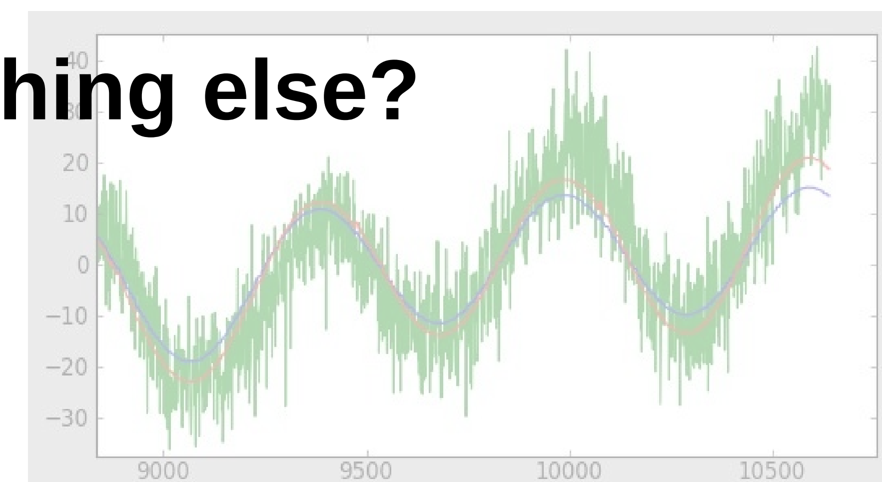
- Sky model too wrong? Do SelfCal?

- Beam model too wrong?

- Something else?

Beam taken into account

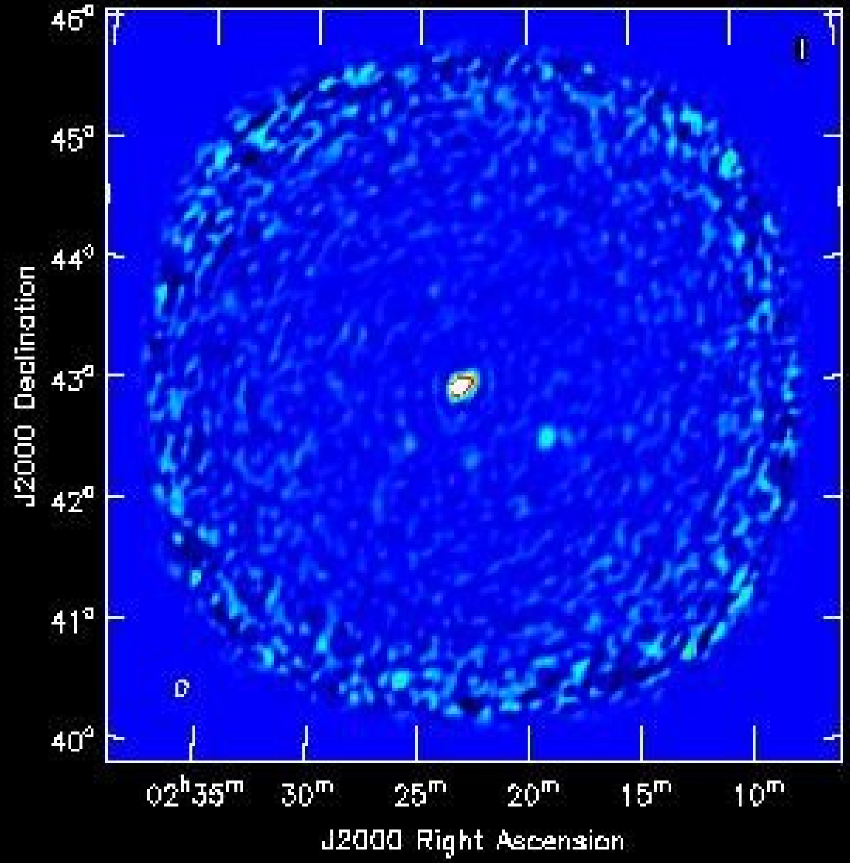




No Beam taken into account

# JAWS: 3C66

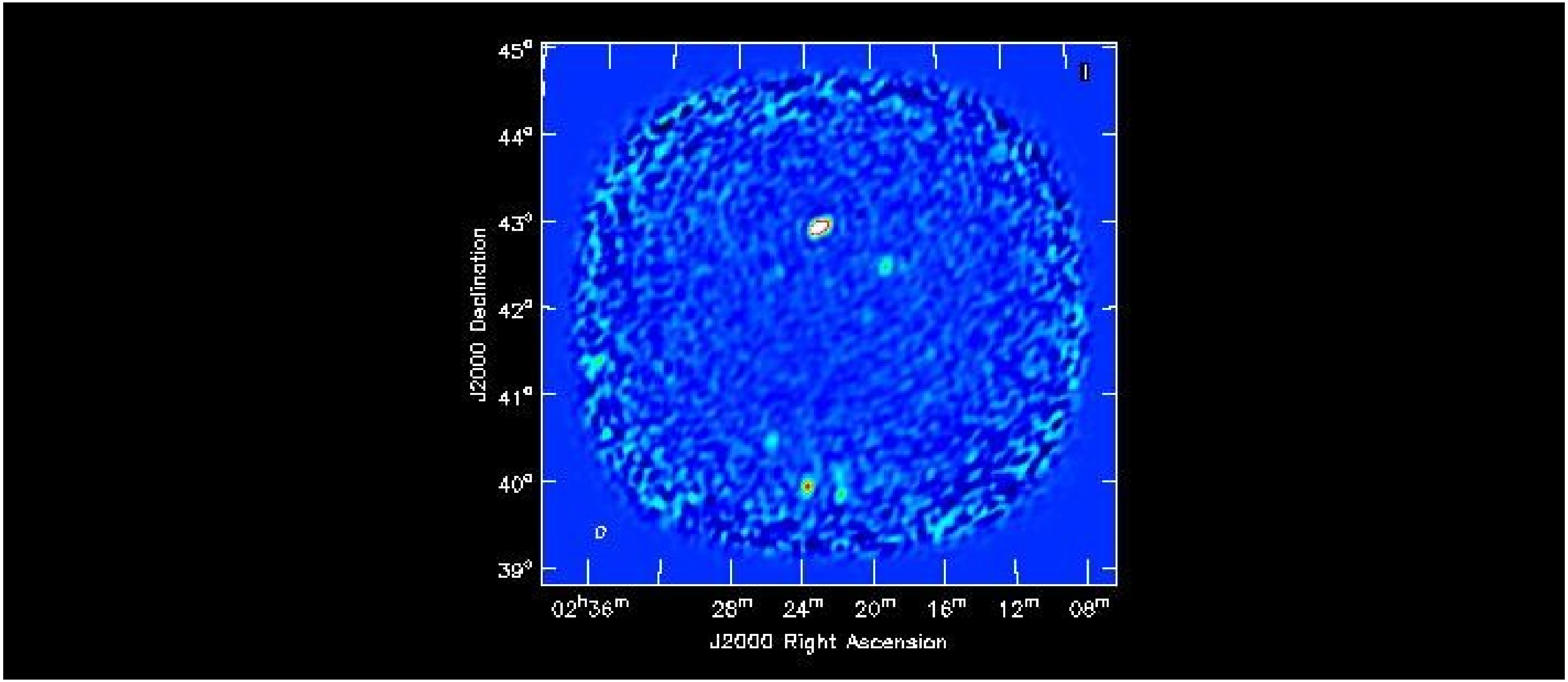
**Flux = 63 Jy**




Aleksandar Sulevski

# JAWS: 3C66

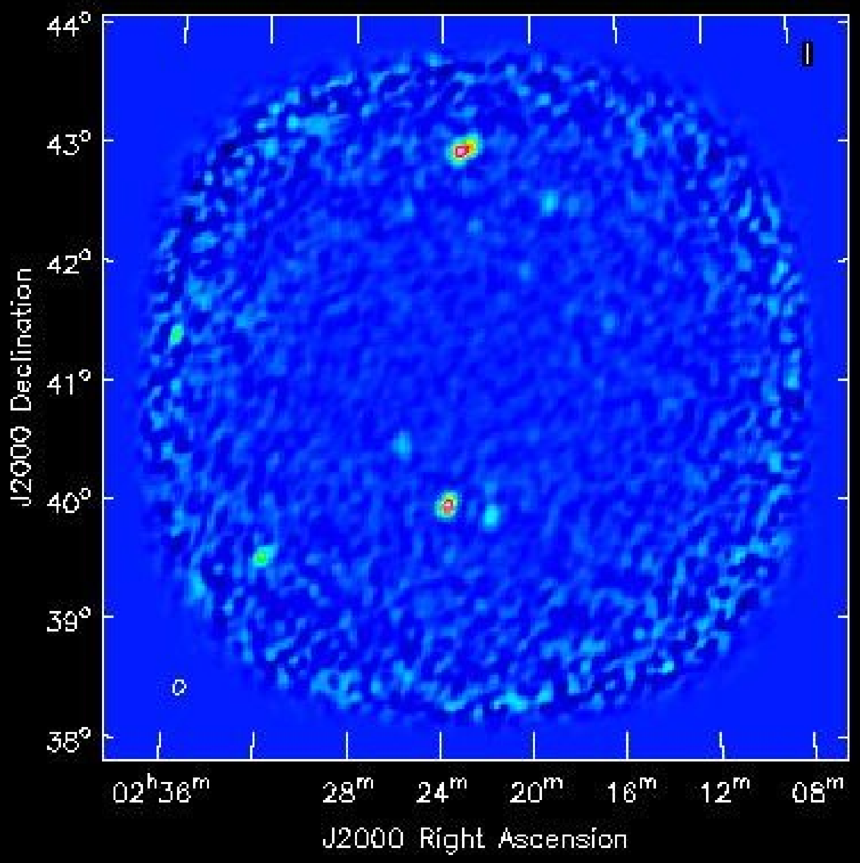
**Flux = 65 Jy**




Aleksandar Sulevski

# JAWS: 3C66

**Flux = 51 Jy**




Aleksandar Sulevski

# Conclusion and Next steps

**<u>Conclusion:</u>**
- **Full Polarisation Framework based on Measurement Equation is working**
- **Very flexible**
- **Effect will be seen at higher dynamical range?**

---

**<u>Next steps:</u>**
- **Optimise code**
- **Study convergence major cycle & SelfCal**
- **Ionosphere phase screen model**
- **Full Multi-Frequency cleaning**
- **Faraday Rotation?**

**… Start doing serious survey science**