LOFAR Imager: taking Direction Dependent Effects into account using A-Projection

Cyril Tasse, Ger van Diepen, Joris van Zwieten, Bas van der Tol

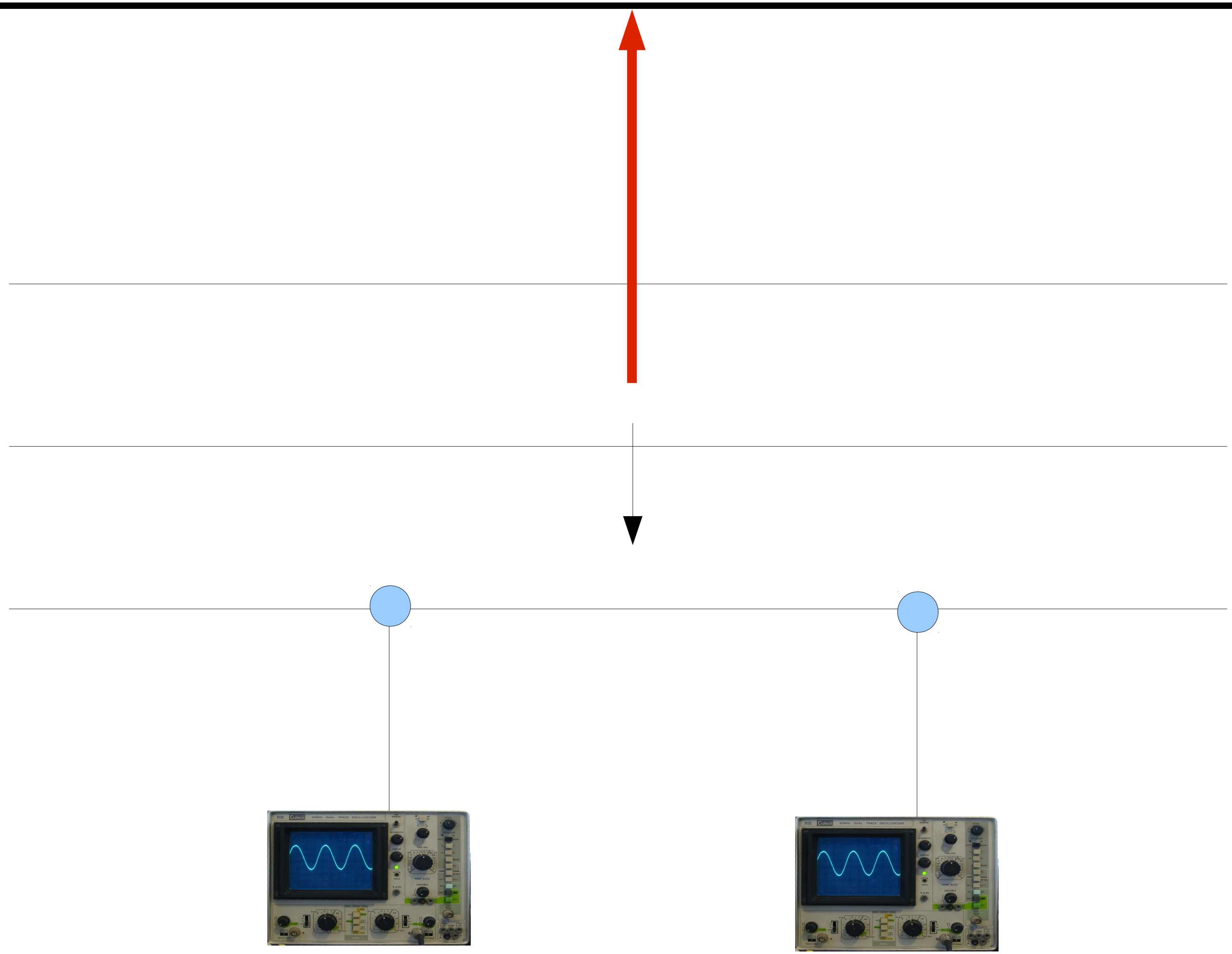
Sanjay Bhatnagar, Urvashi Rau, Kumar Golap

Outline

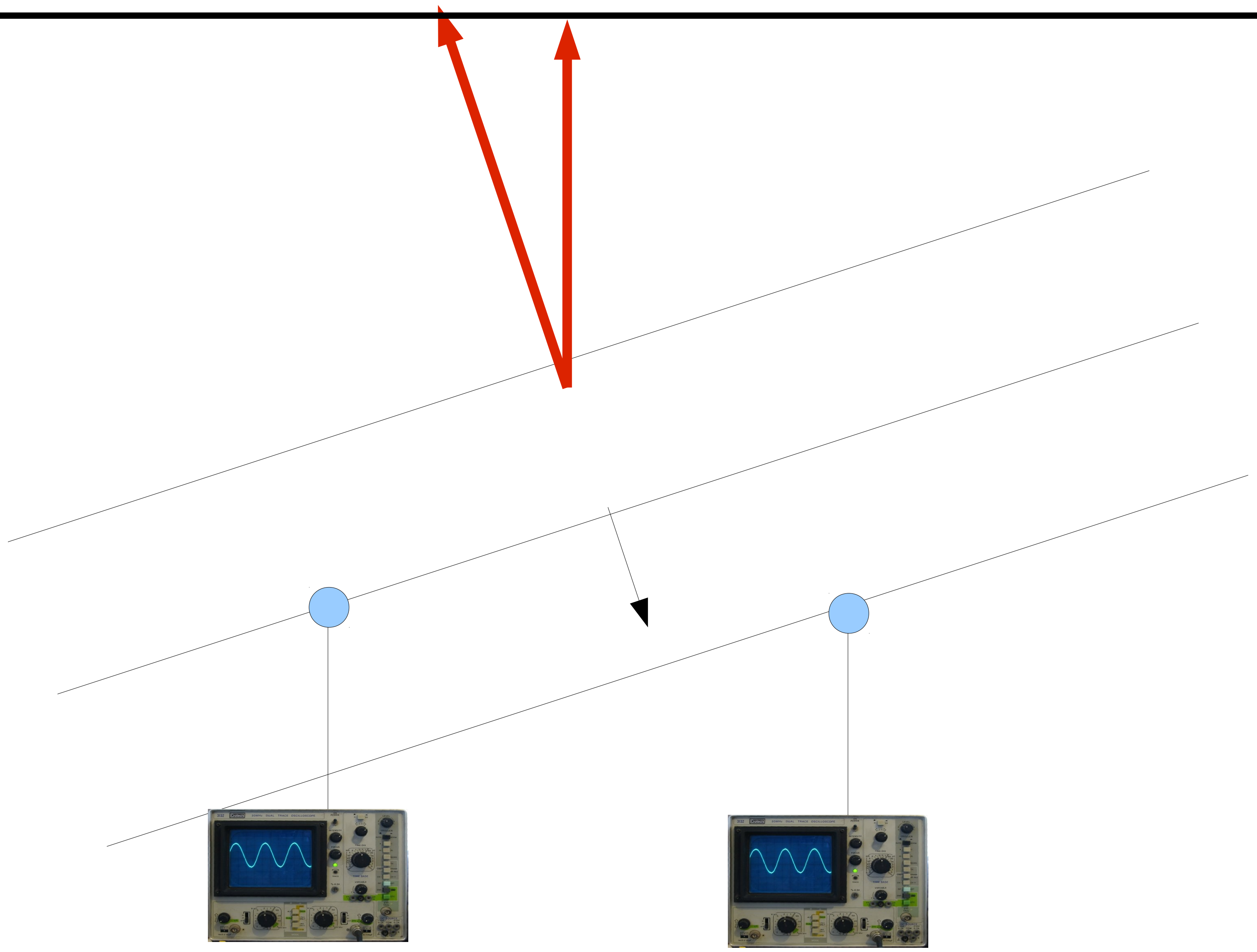
- Imaging for the dummies
- UV-Brick
- A-Projection

Principle

FOR DUMMIES



Principle



Principle

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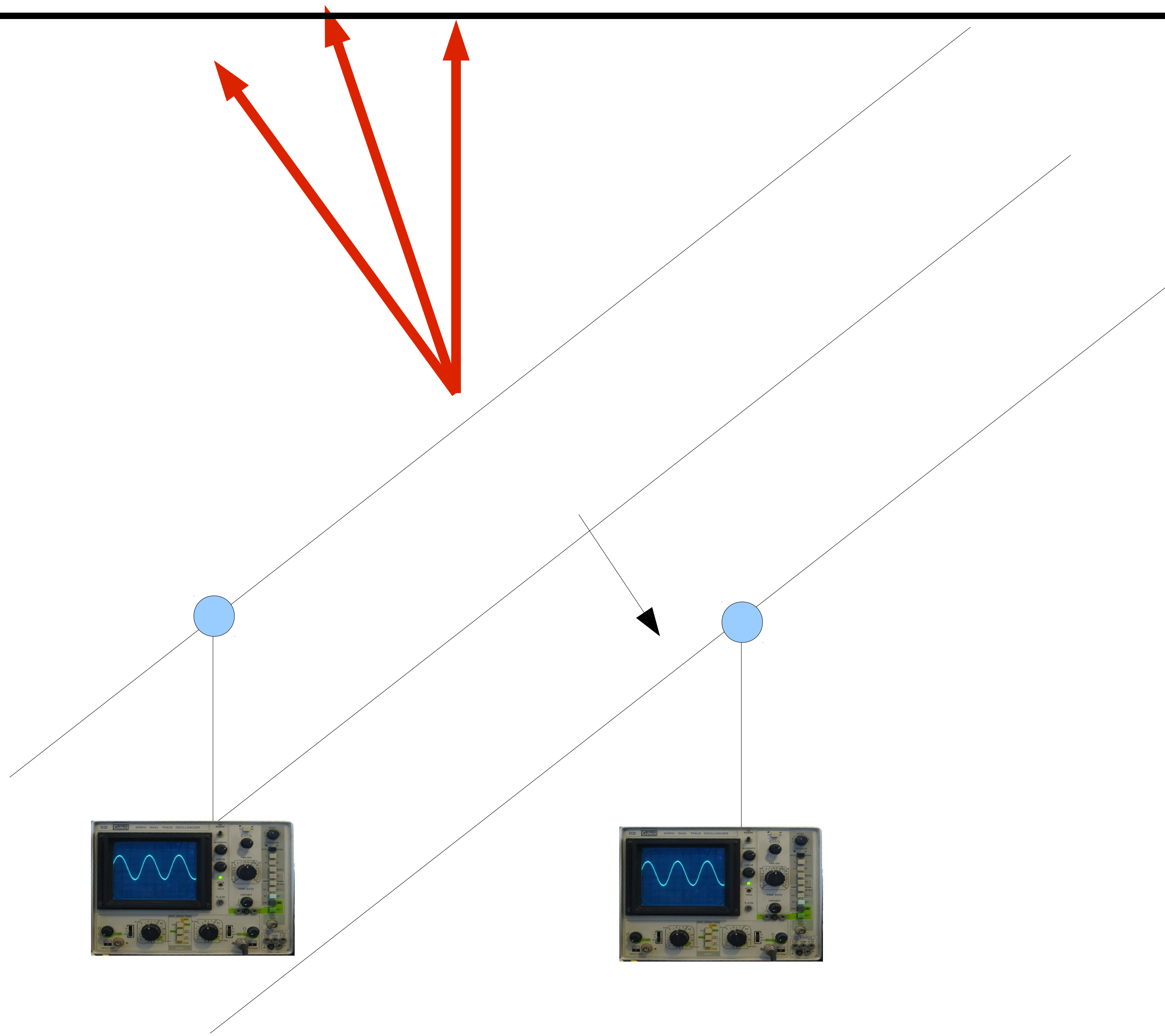

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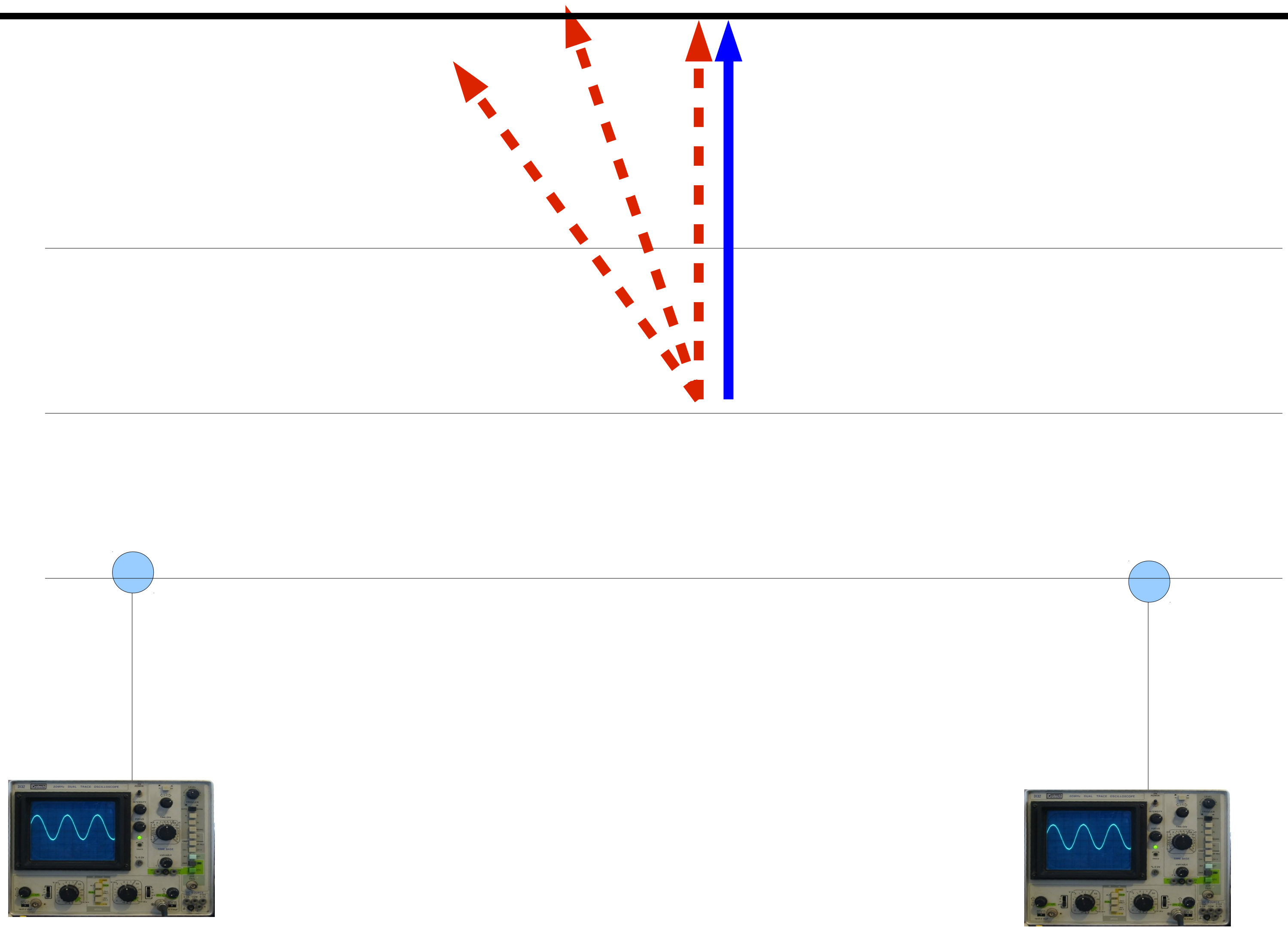

Principle

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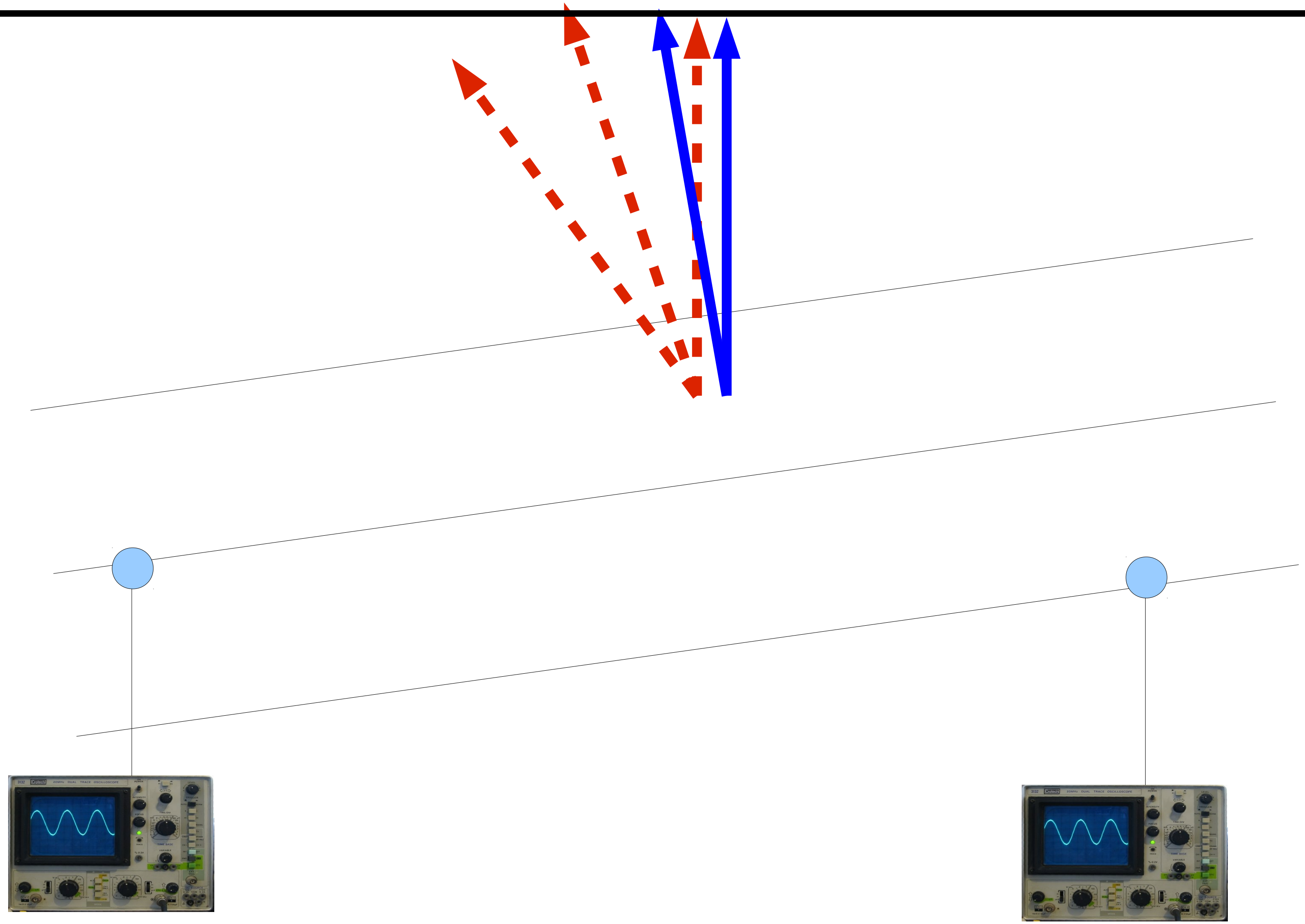

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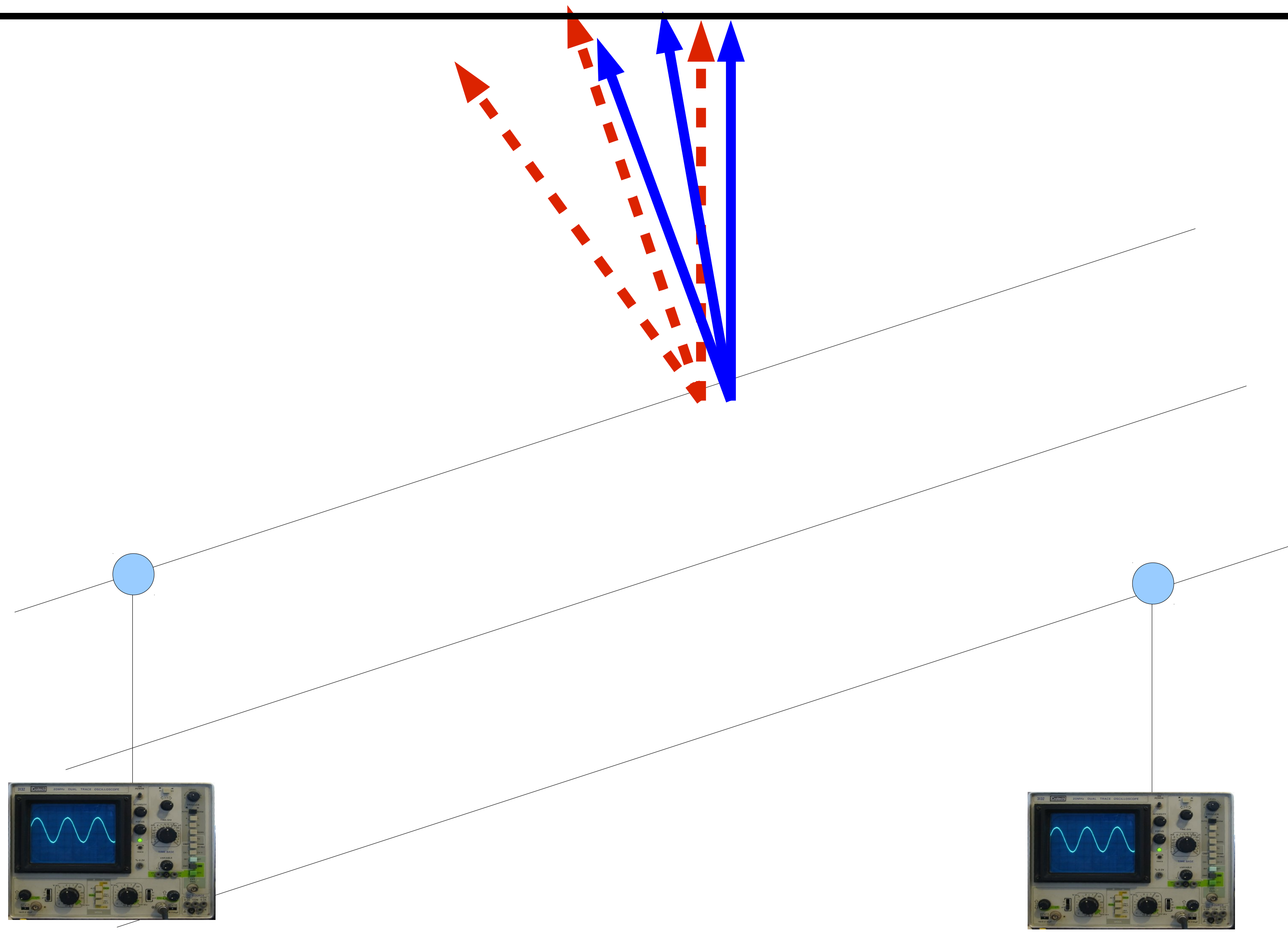

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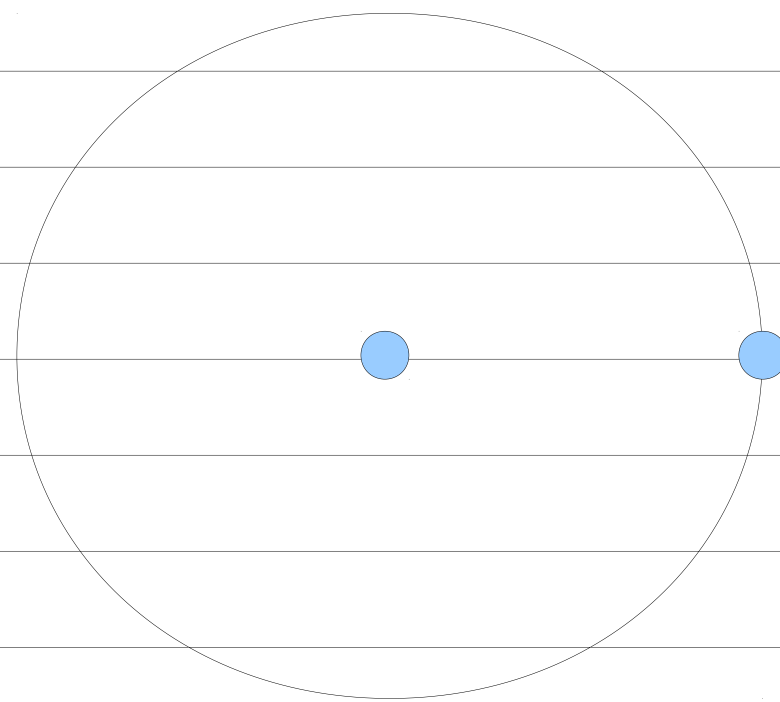

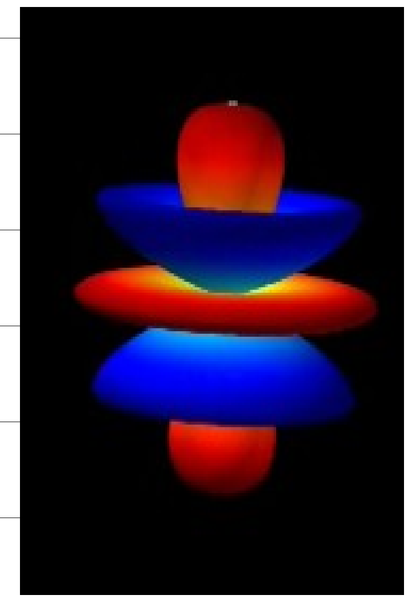

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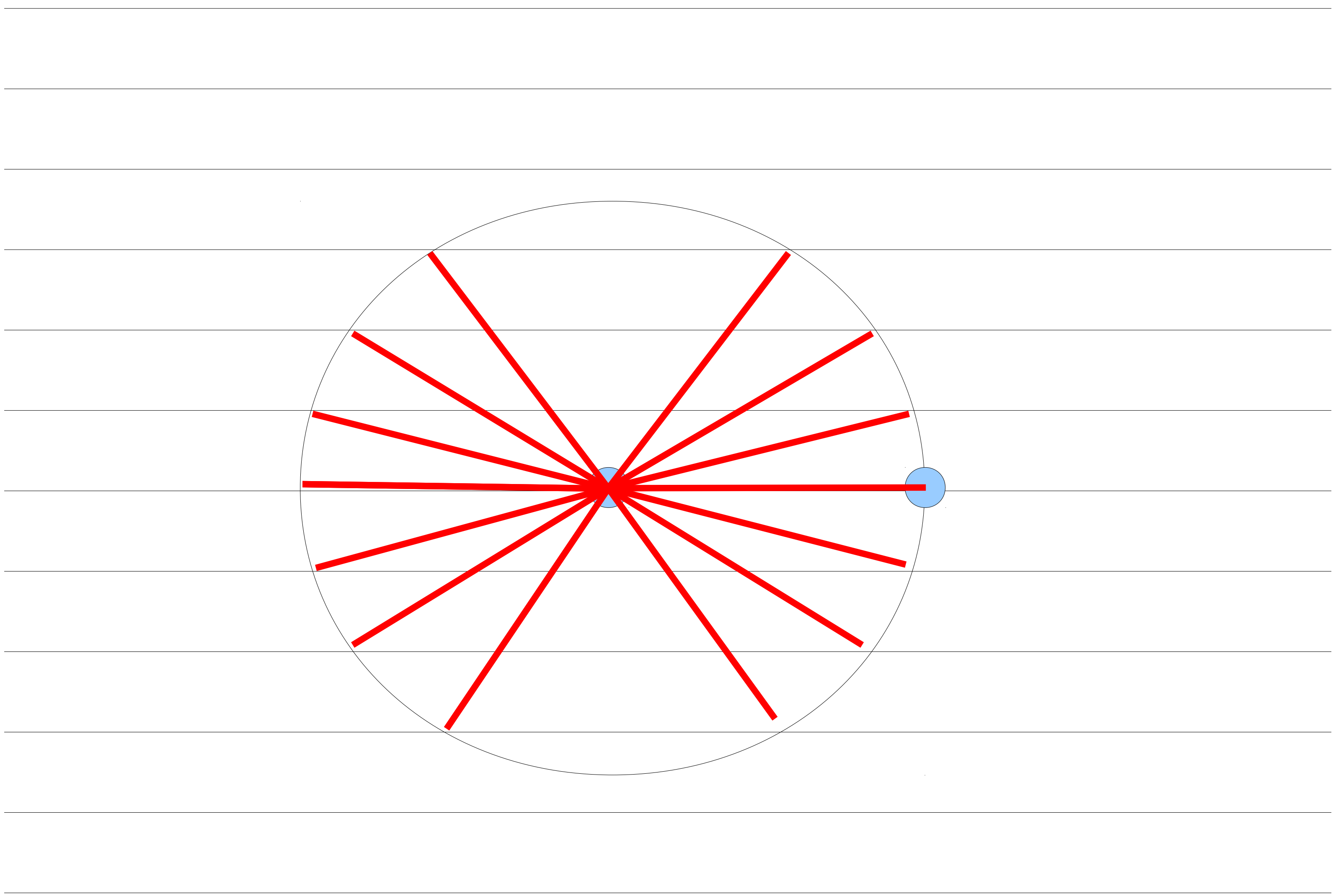

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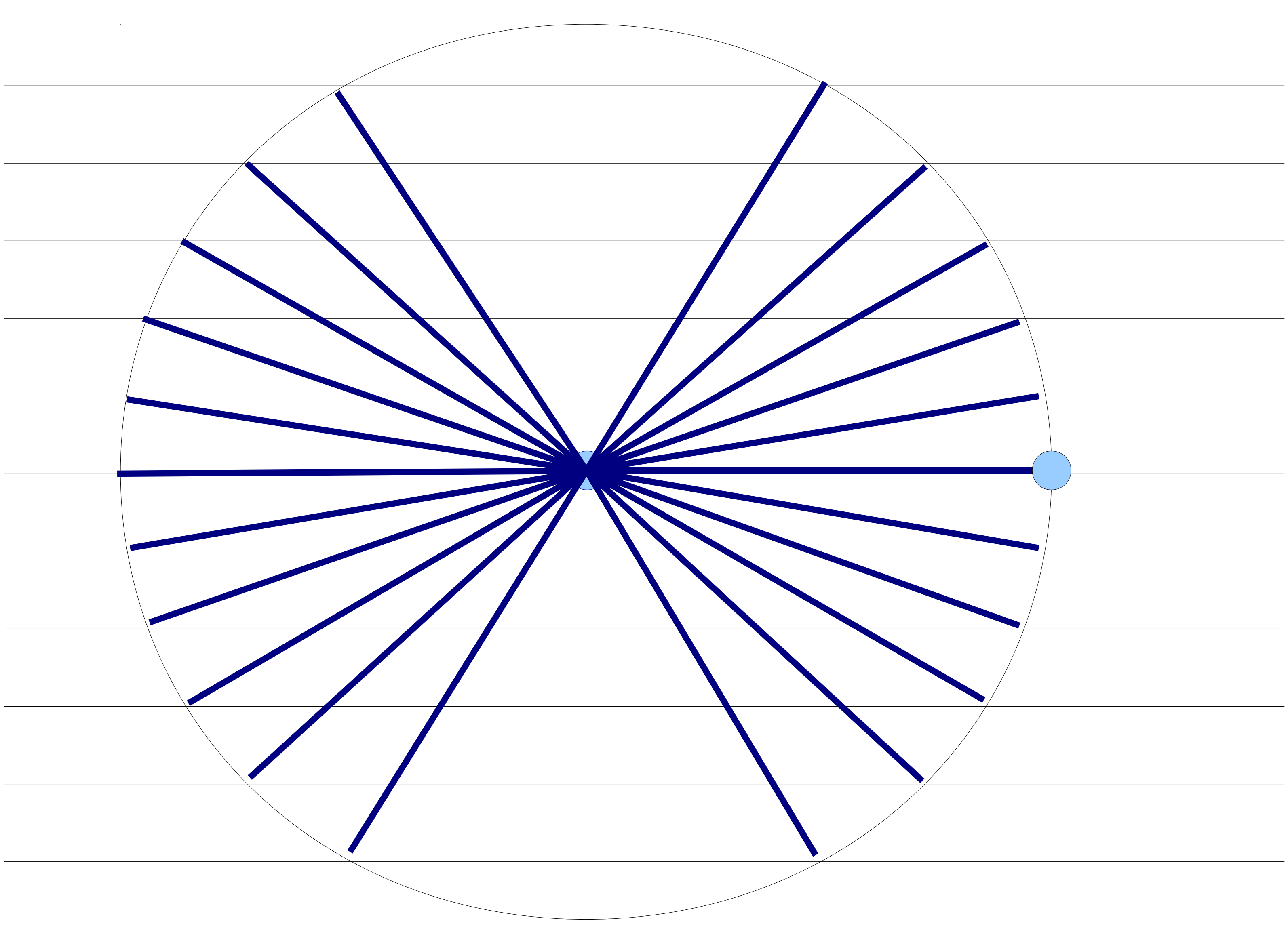
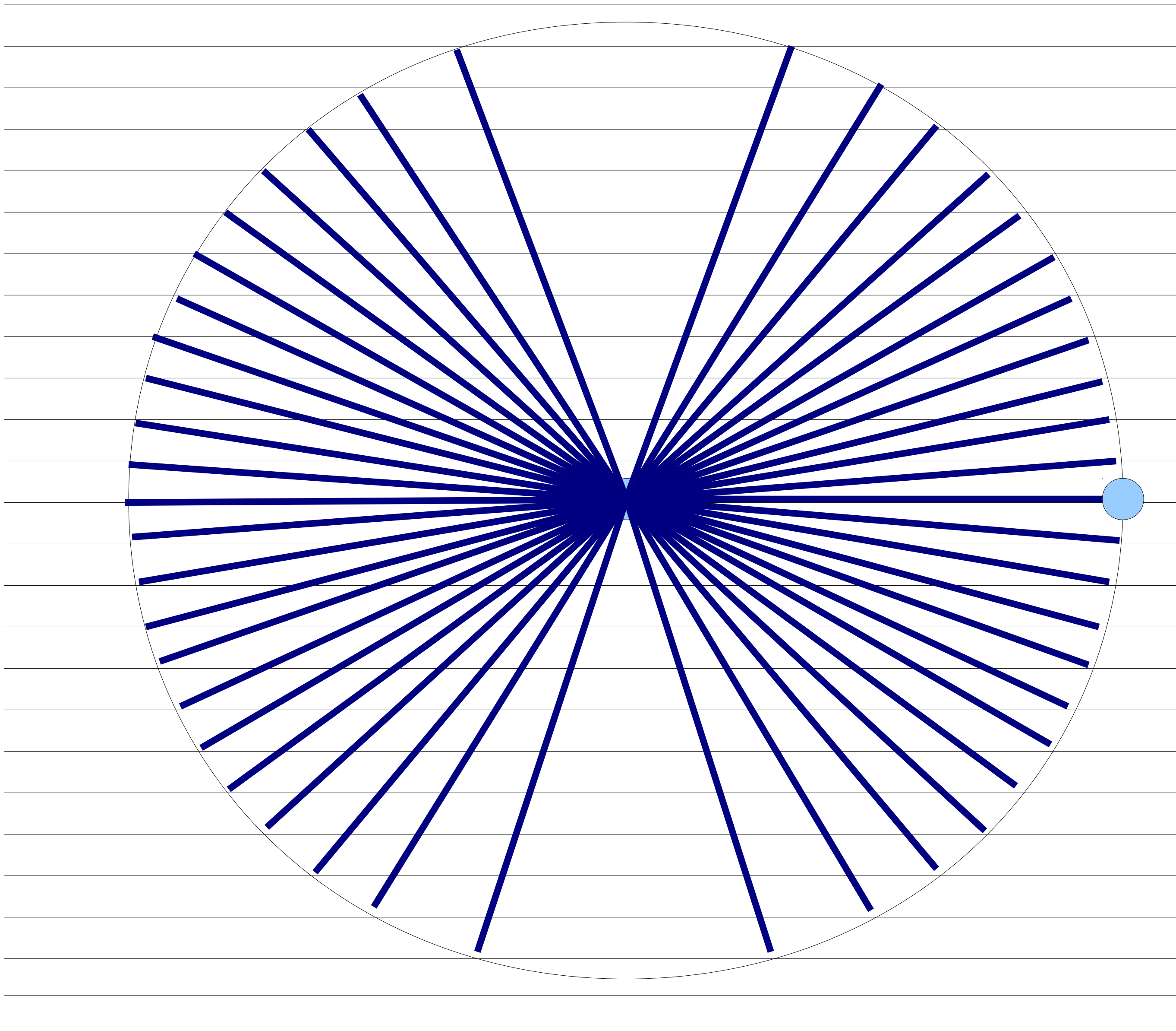
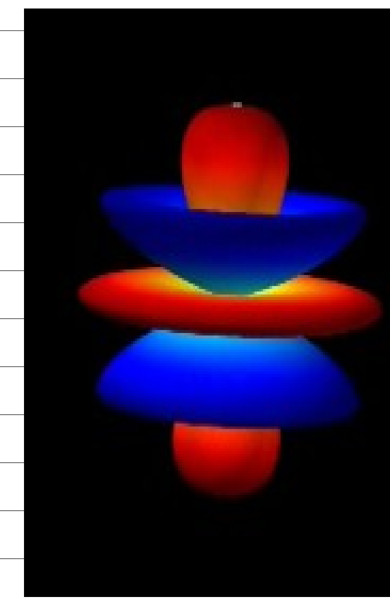


Principle

Resolution = Wavelength / Distance

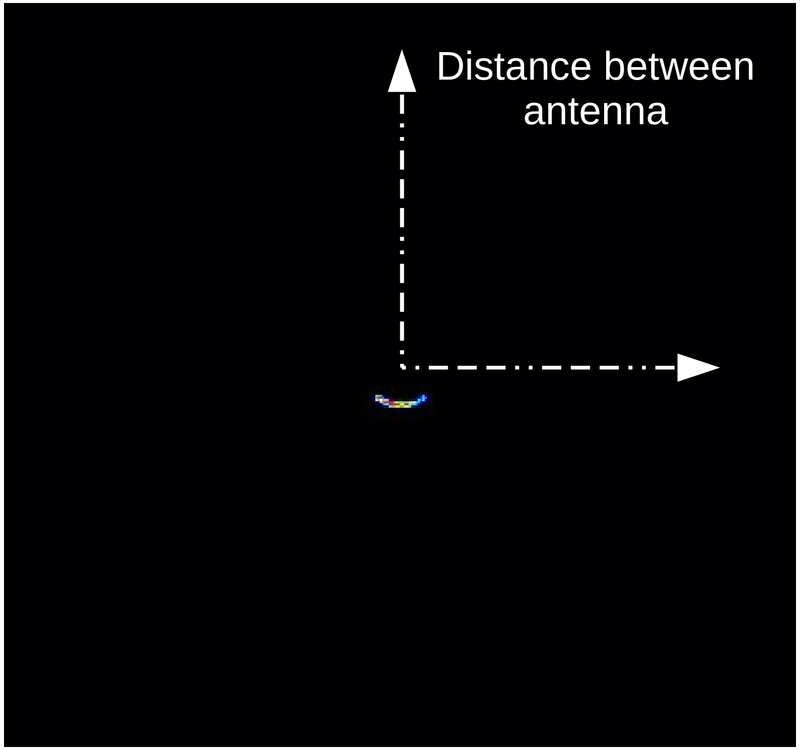


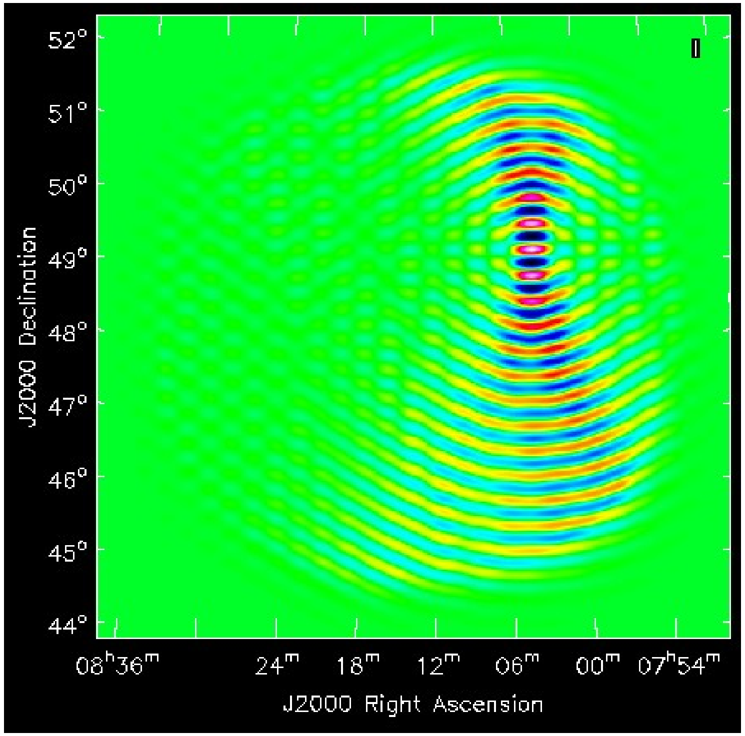


Principle

- Each baseline “draws” a fringe on the sky

- The superposition of the information of many baseline “draws” the image.





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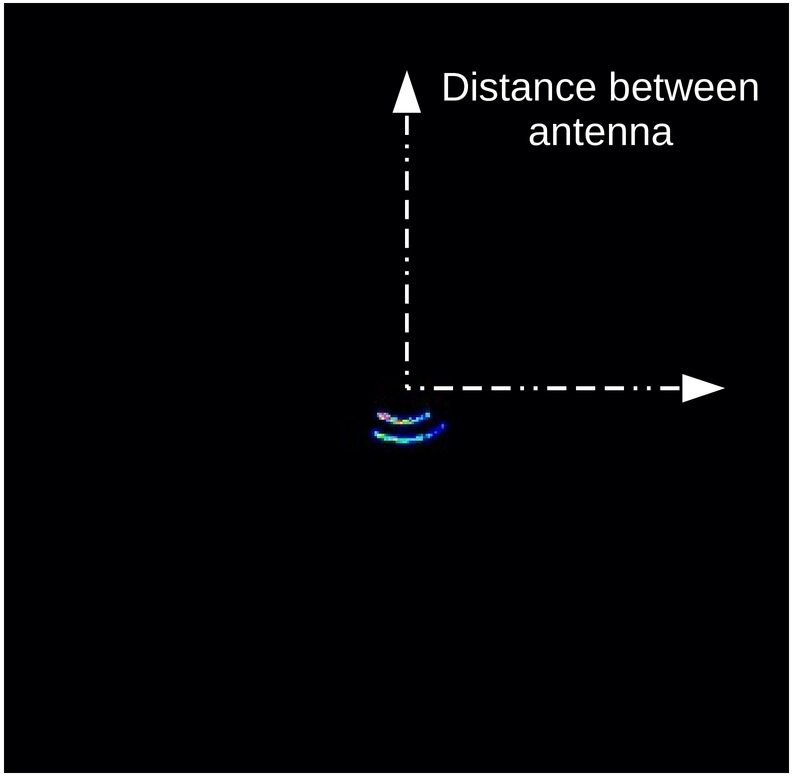

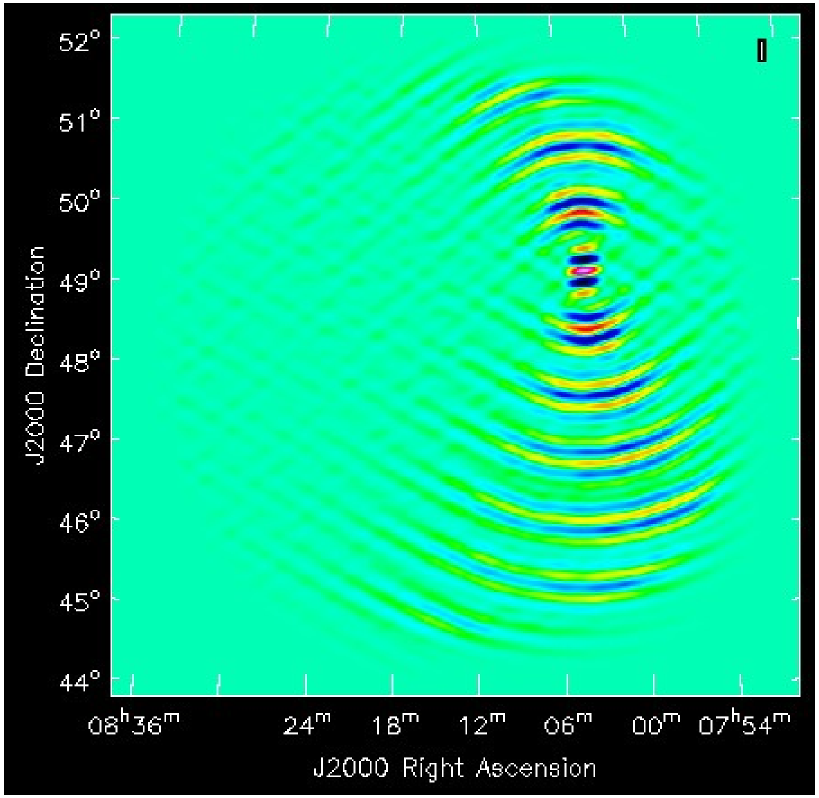

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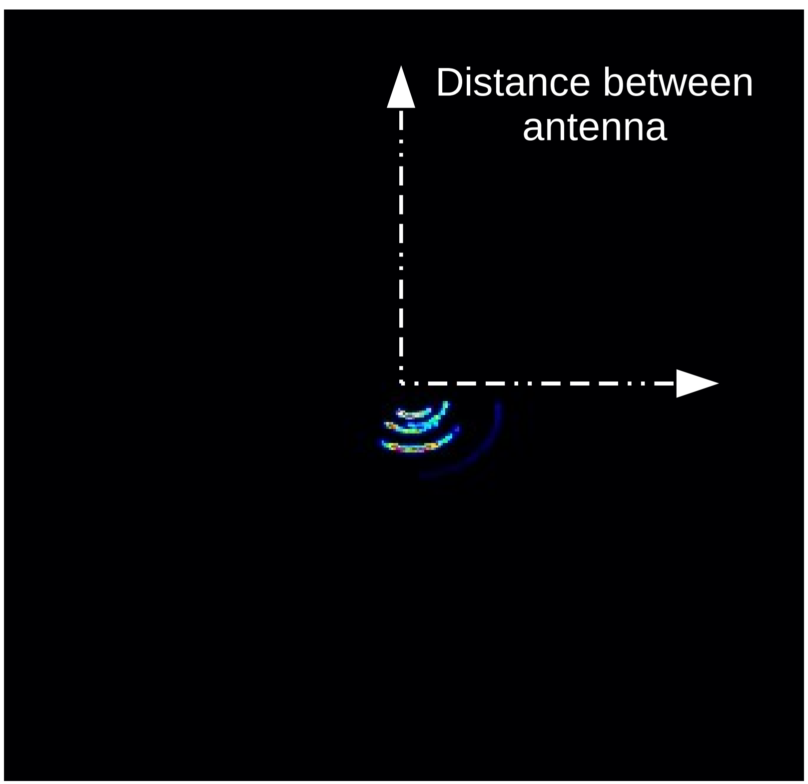

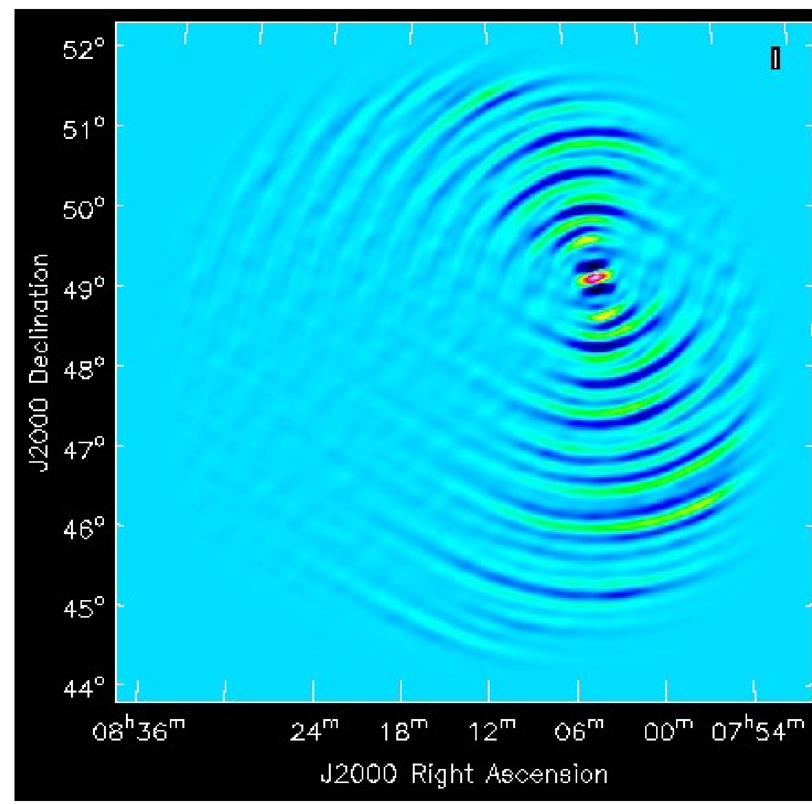

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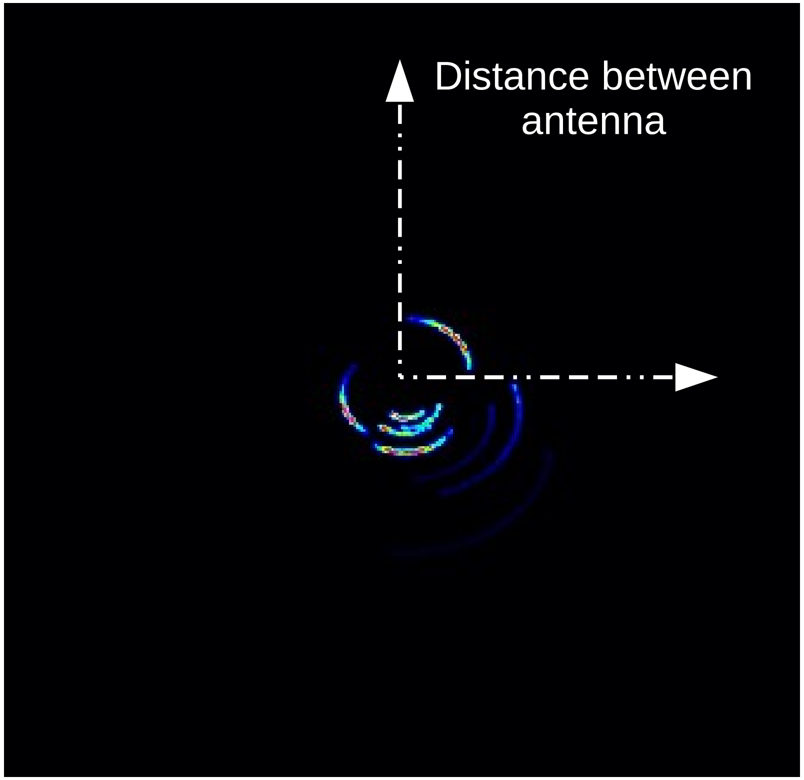
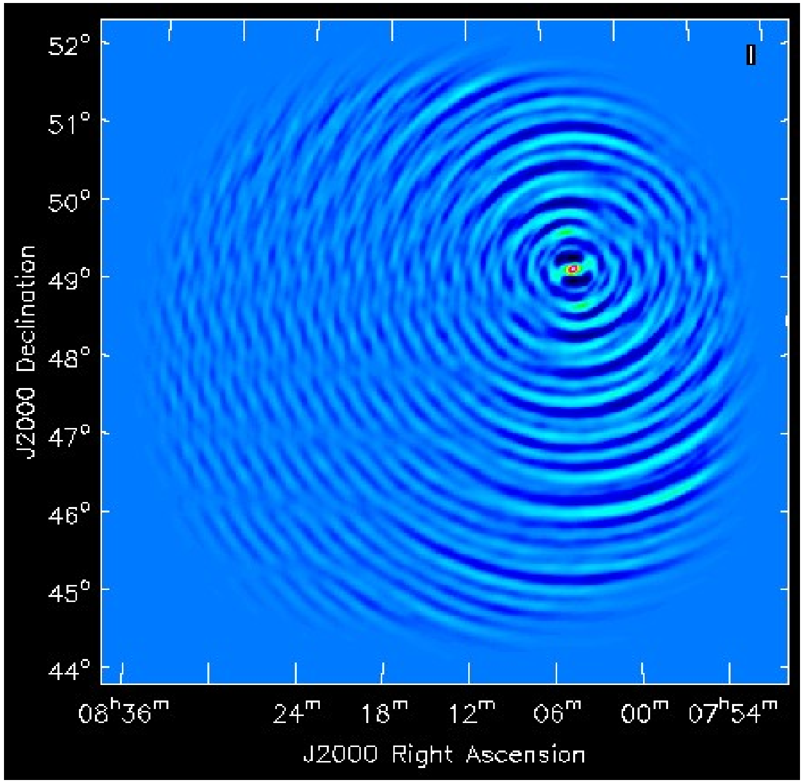
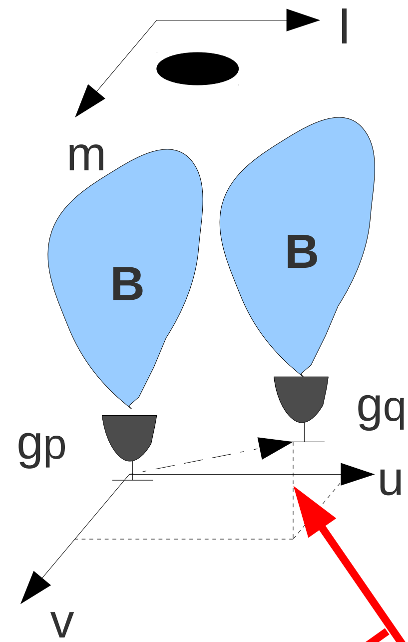




Traditional Calibration and imaging (scalar)



Correlator compensates for w

$$V_{pq} = (g_p . g_q^*) \int_S \mathrm{B(l,m).I(l,m)}$$
$$. \exp\left(-2\pi i(u_{pq} l + v_{pq} m + w_{pq}.(\sqrt{1 - l^2 - m^2} - 1))\right) dl.dm$$

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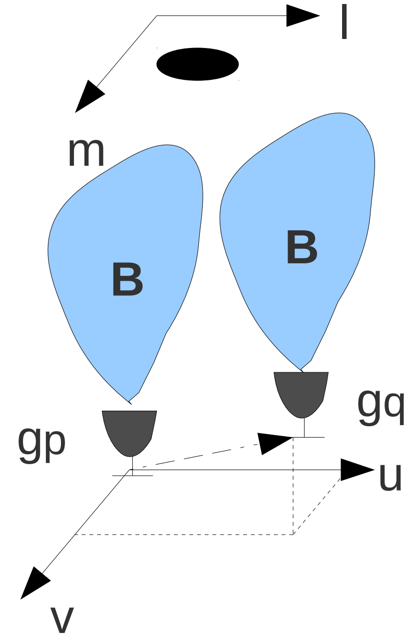

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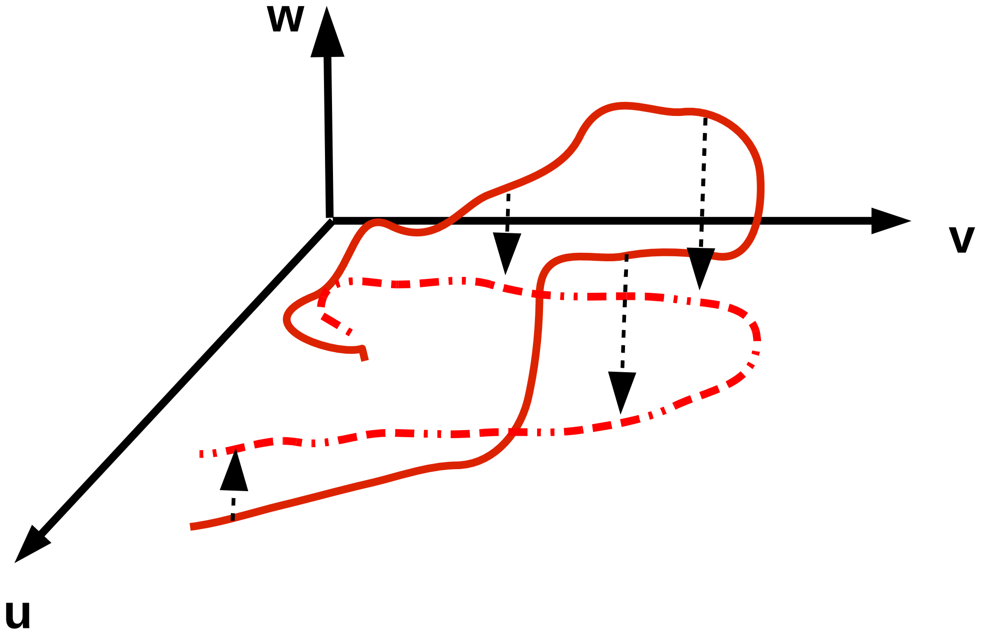
Small field of view

Gridding in practice?



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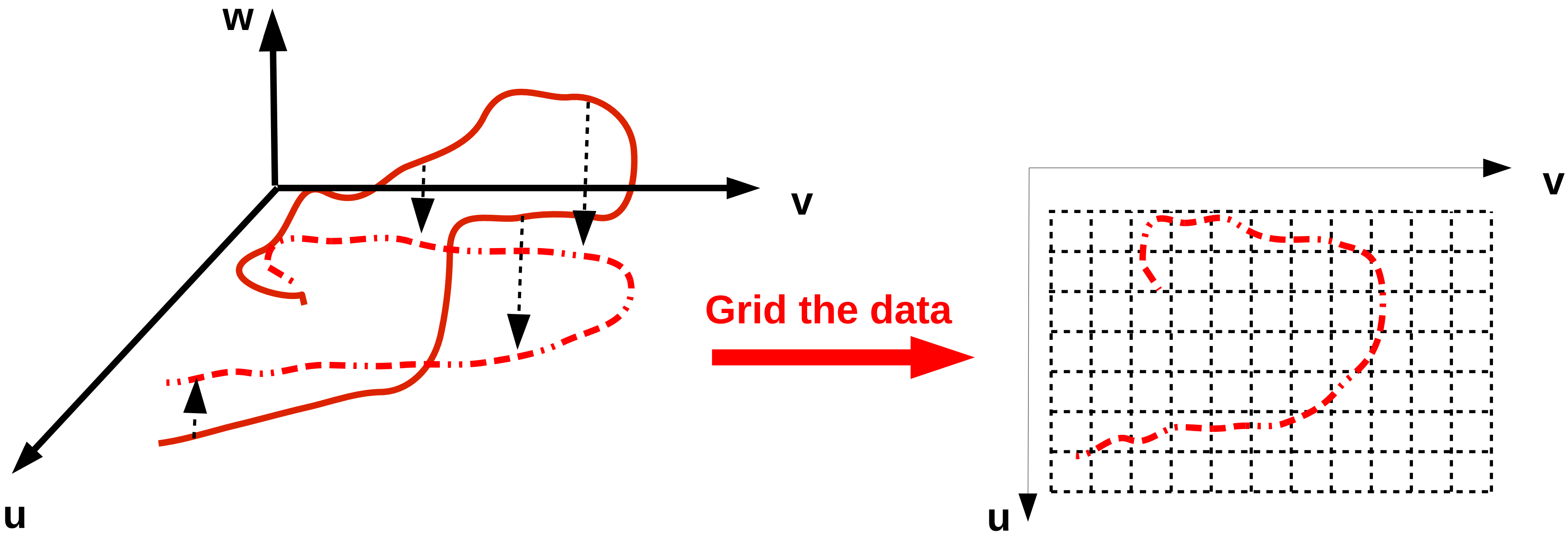

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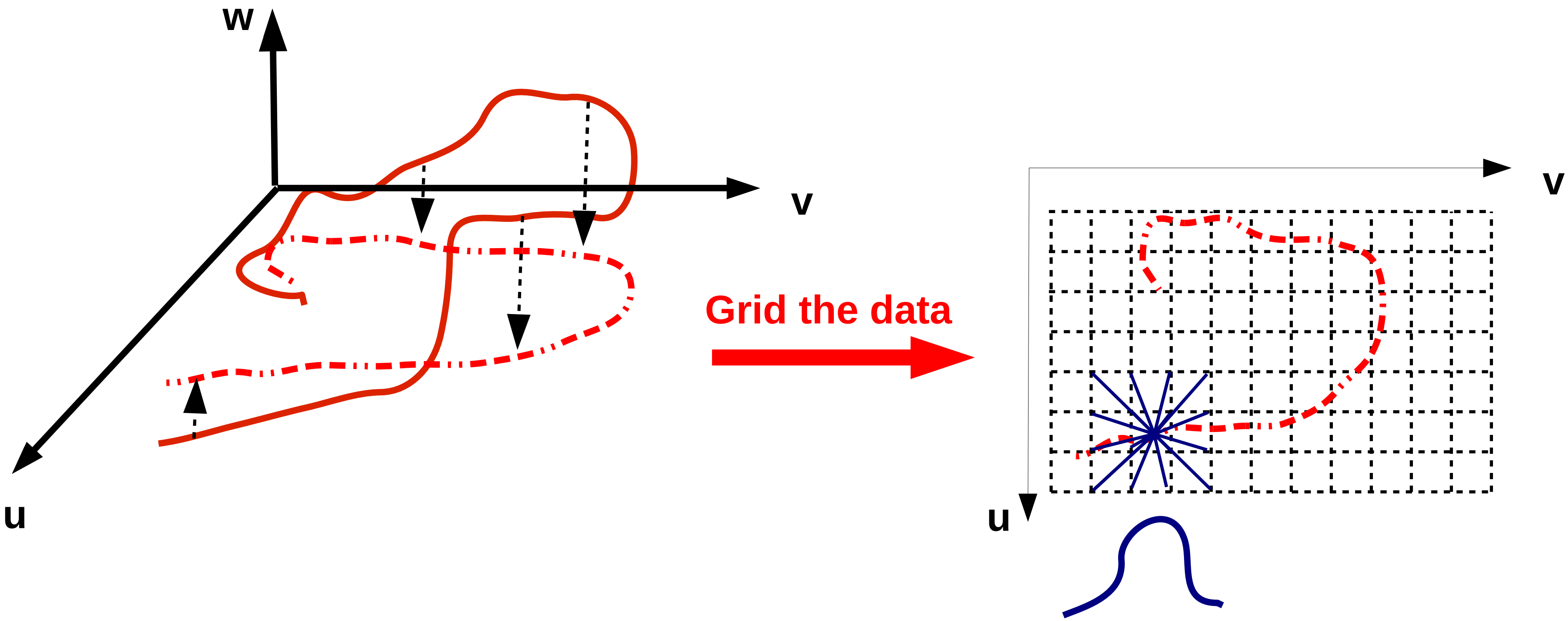

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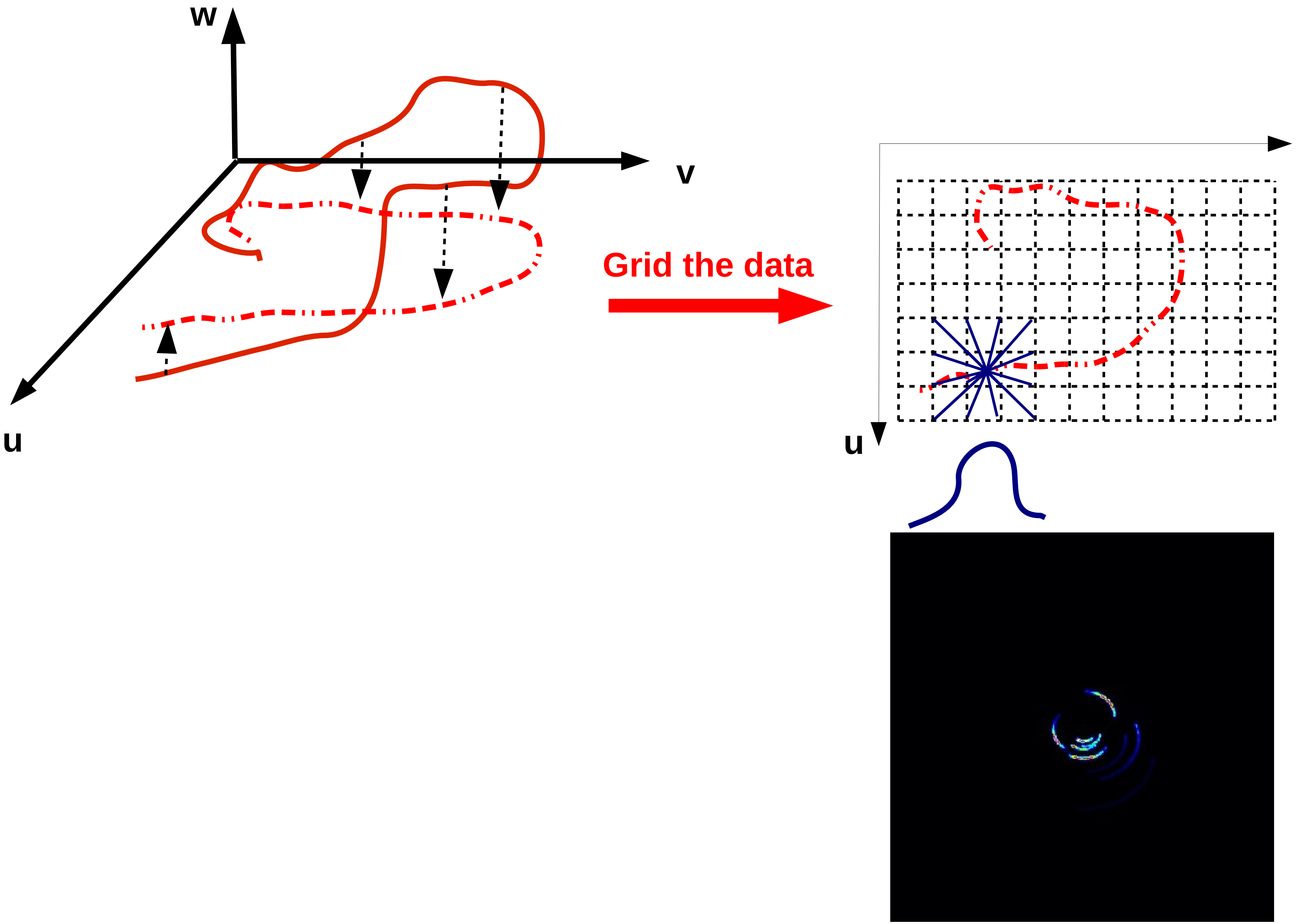

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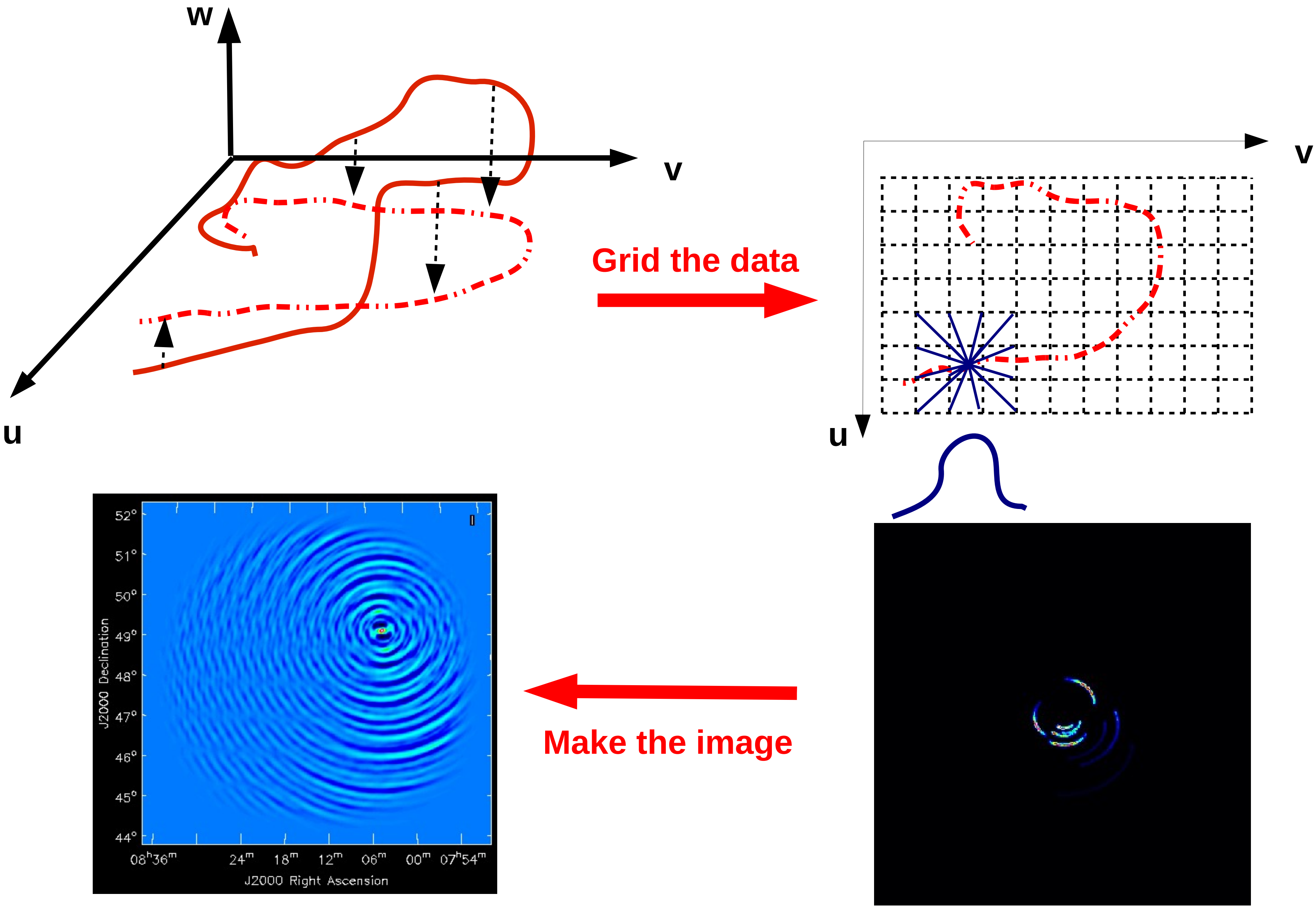
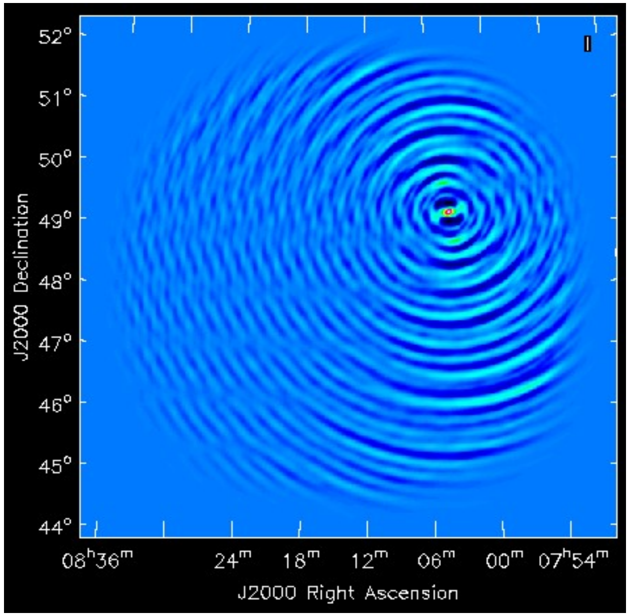
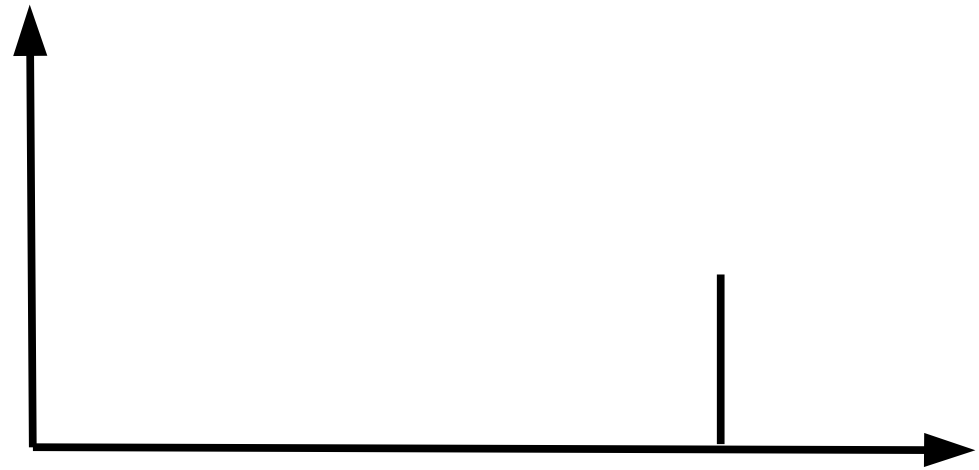


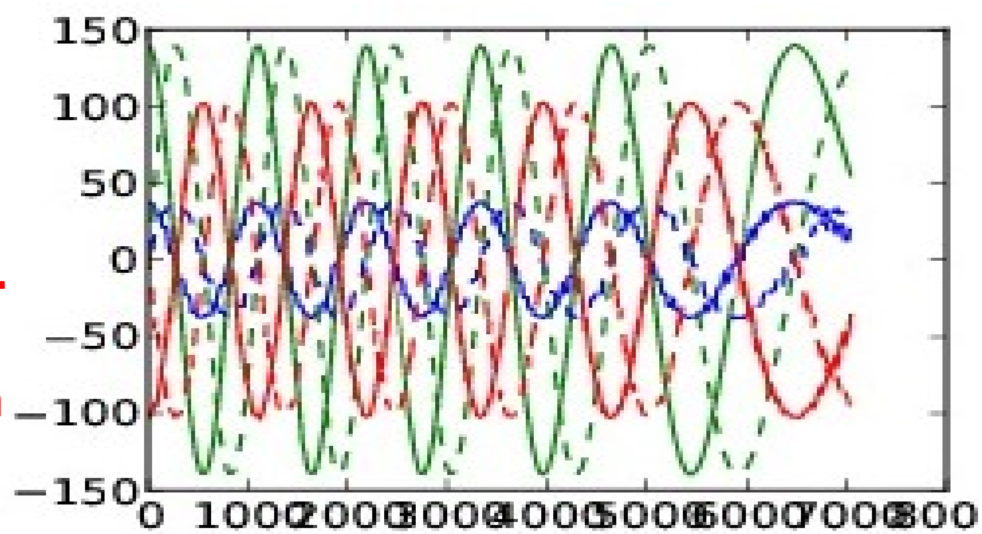
Deconvolution?



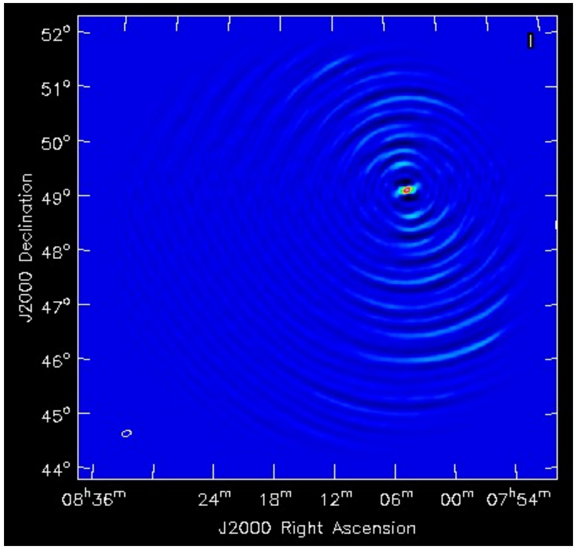
Minor Cycle



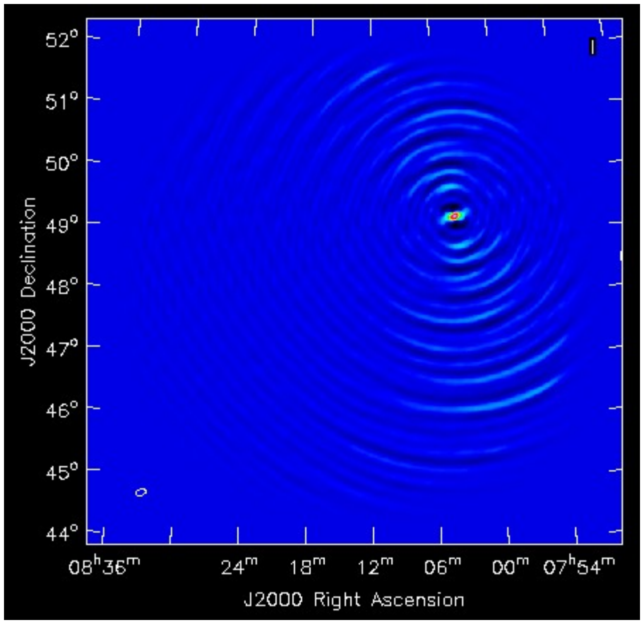
Fourier Transform



Fourier Transform the difference



FOR DUMMIES



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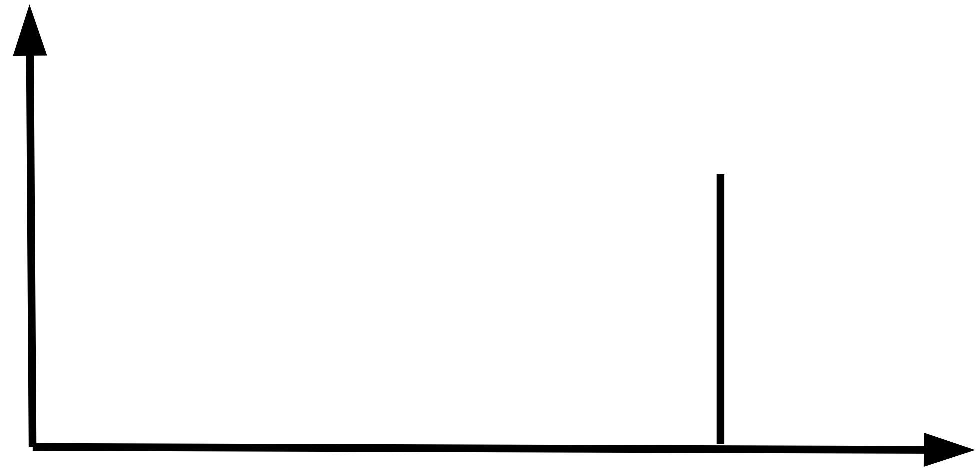

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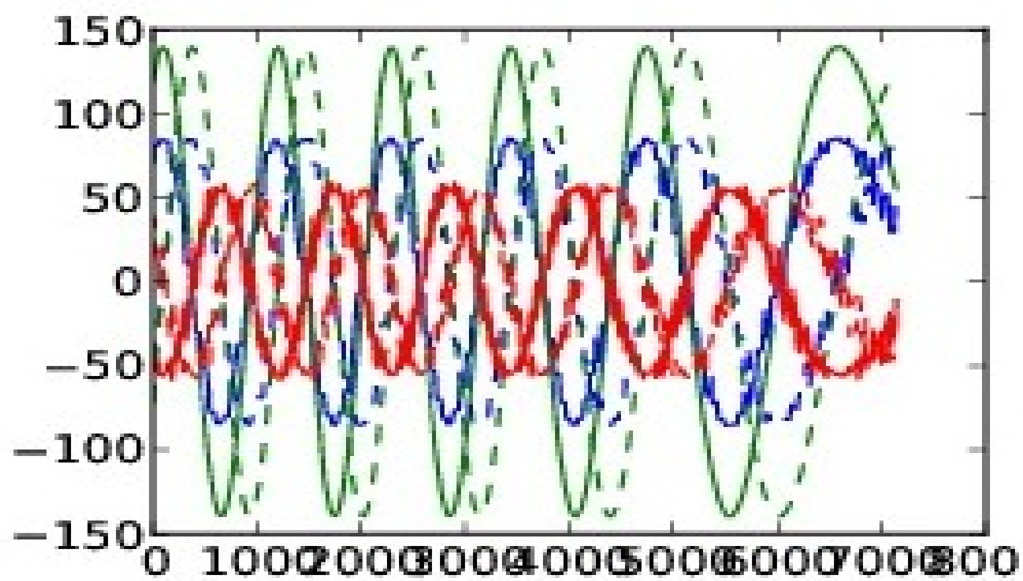
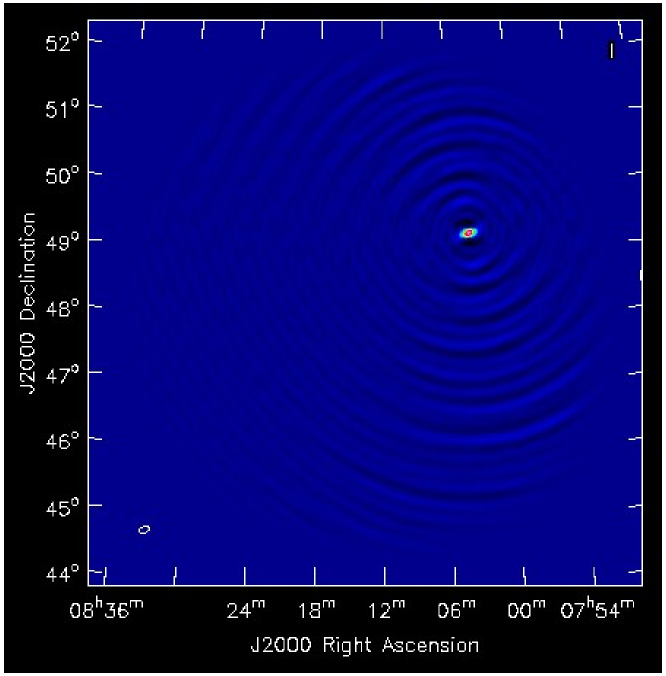


Deconvolution? FOR DUMMIES



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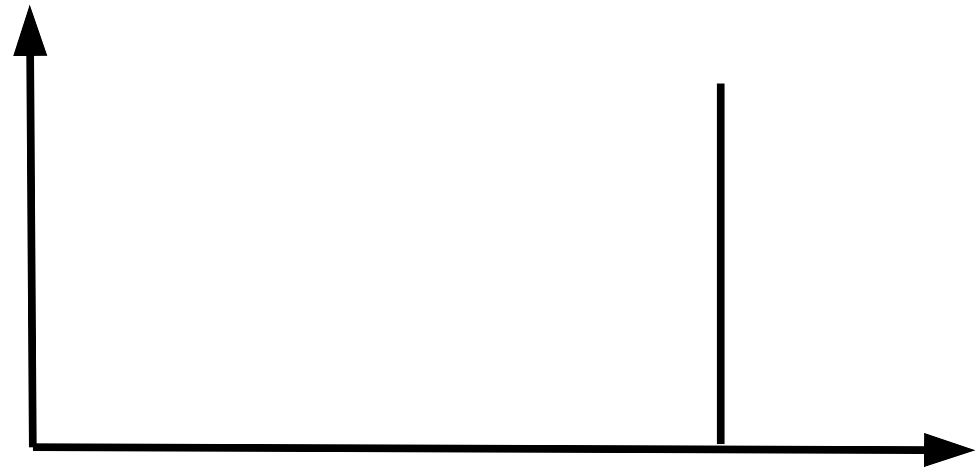

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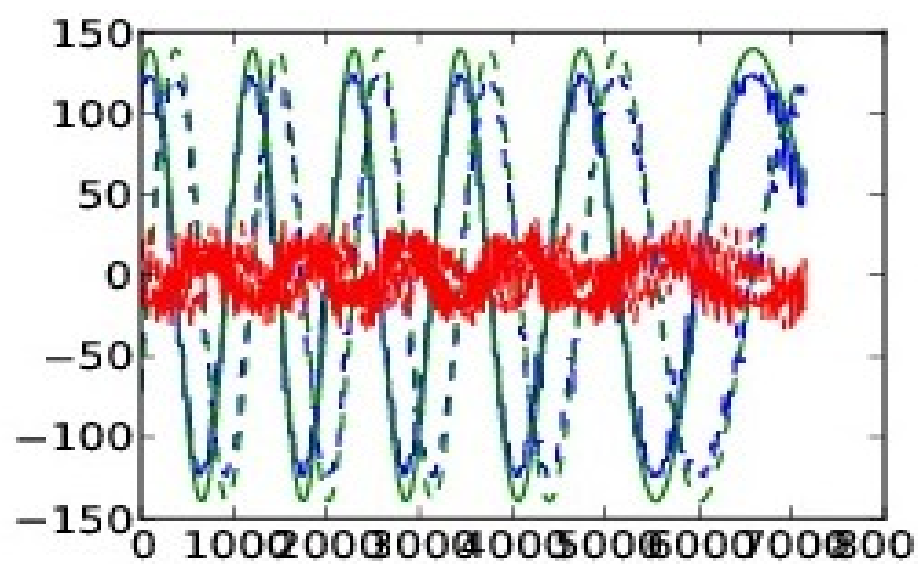

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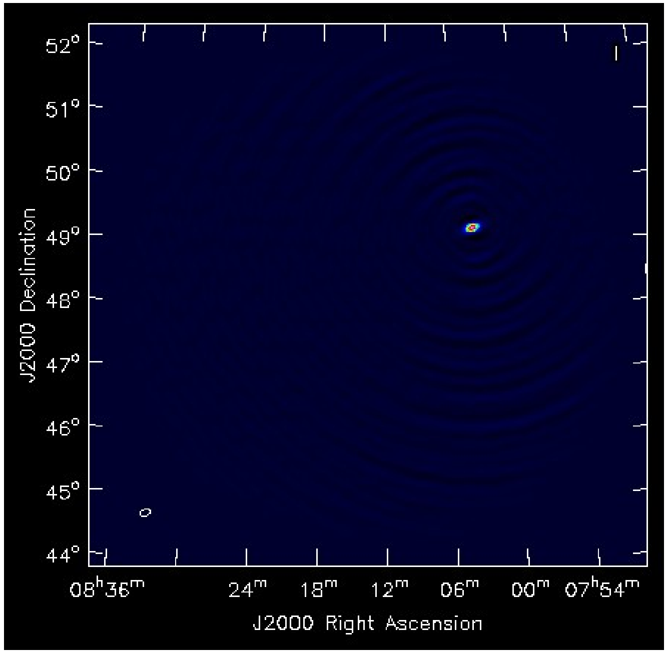

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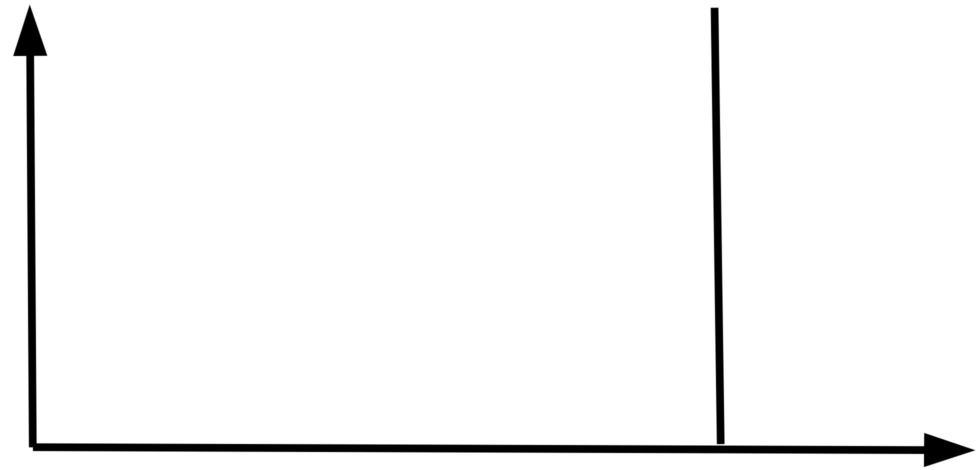

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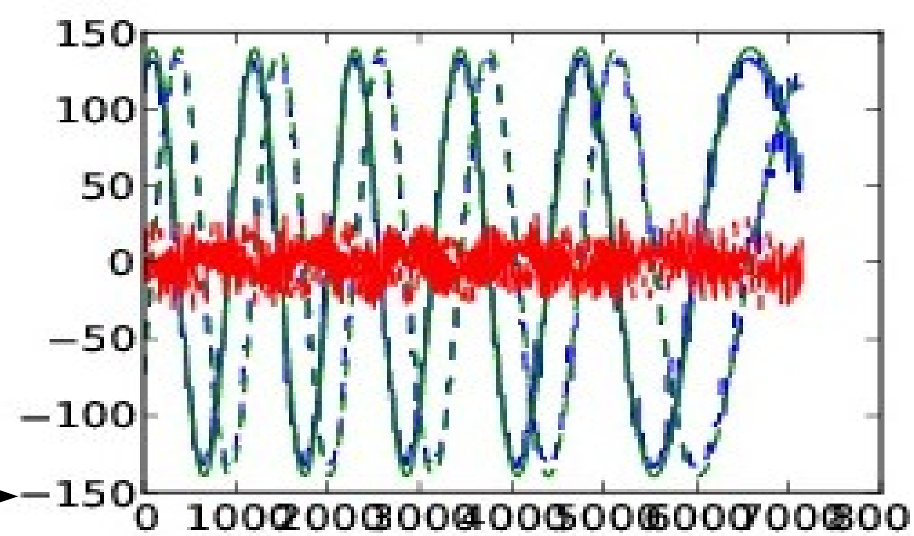
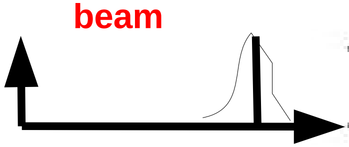
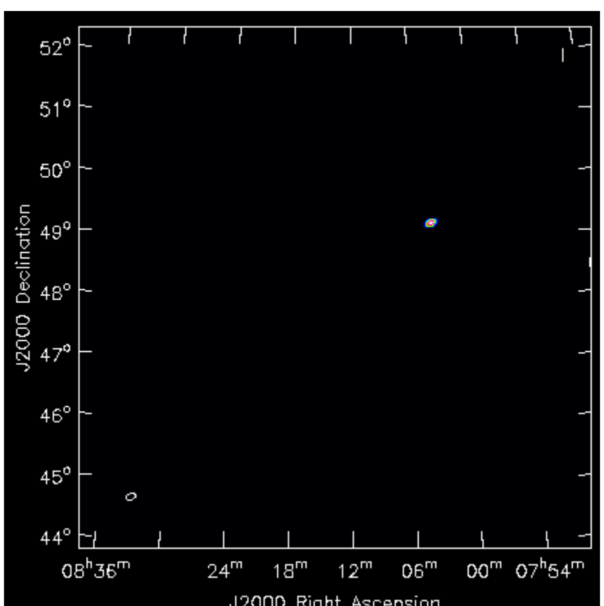


Fourier Transform the difference and convolve with restoring beam





Next Talk

Presentation of UV-Brick by Iniyan

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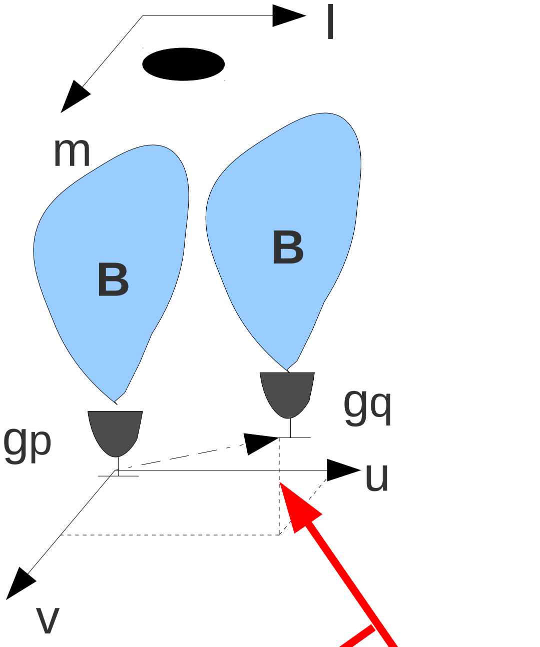

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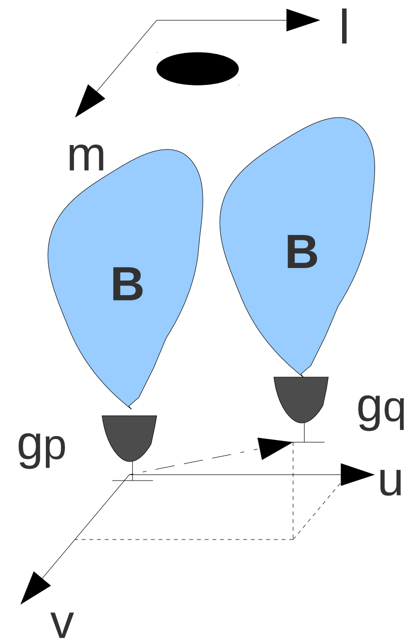

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Small field of view

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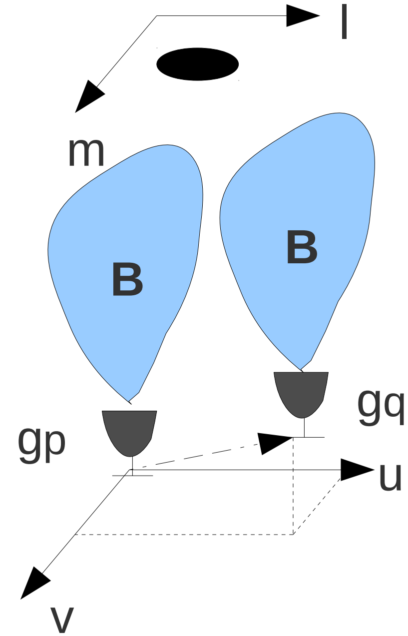

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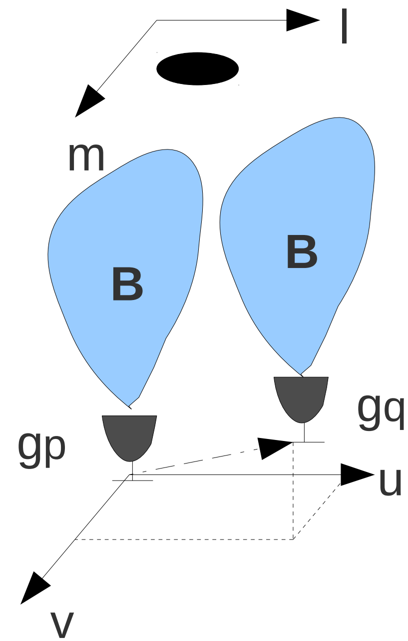

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Small field of view

- Imaging

$$\mathrm{I(l,m)} = \frac{1}{\mathrm{B(l,m)}} \mathrm{FT}\left(\frac{V(u,v)}{[g.g^*](u,v)}\right)$$

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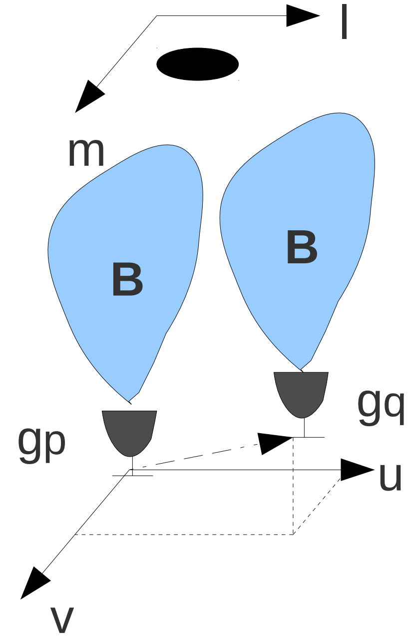

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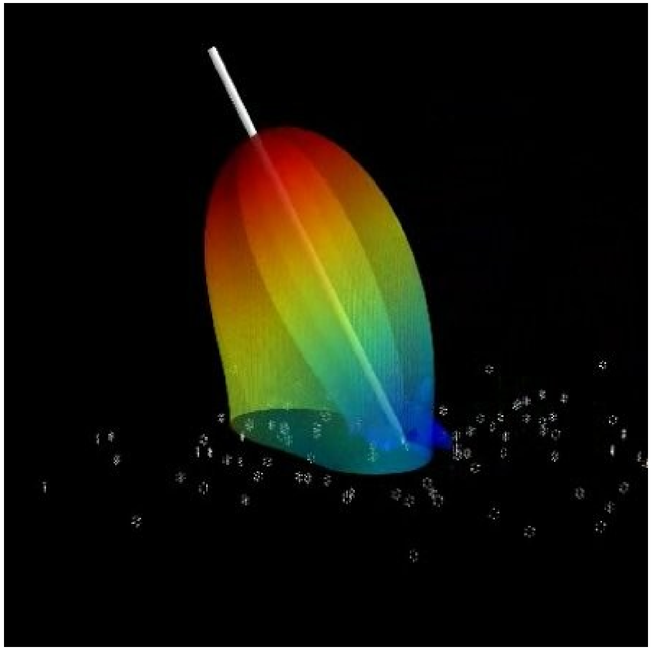
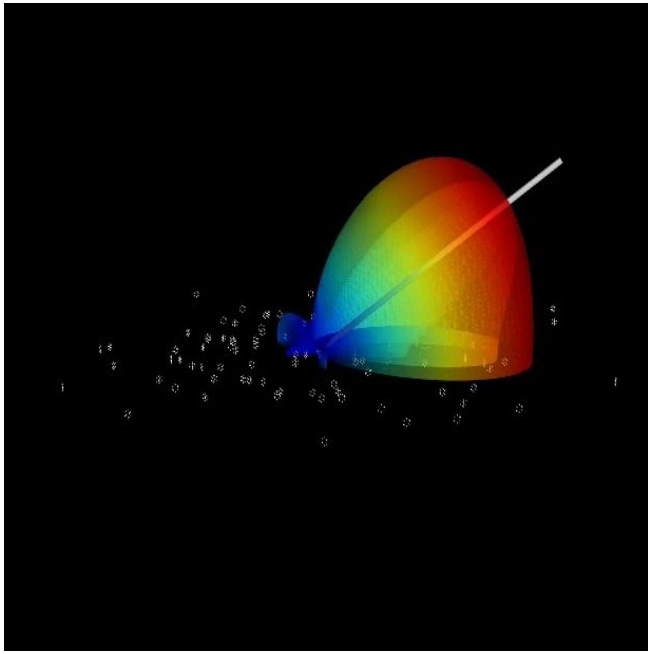
Beam correction in the image plane

… When Direction Dependent Effects (DDE) become a problem : Beam



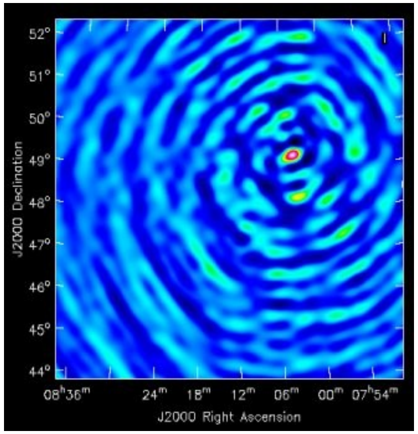


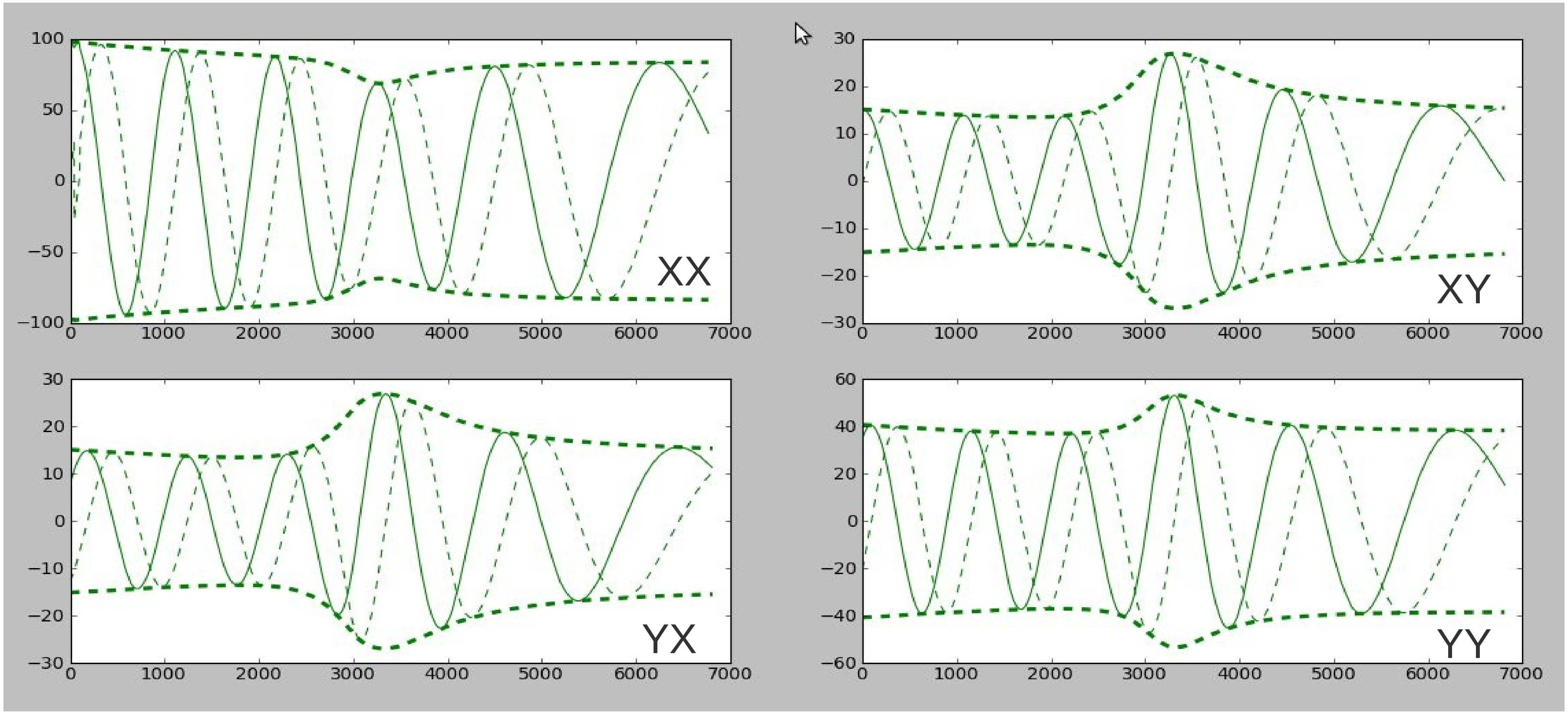
LOFAR stations are phased arrays

- Beam is variable in frequency and time
- Beam can be station-dependent

… When Direction Dependent Effects (DDE) become a problem : Beam

One off-axis source IQUV=(100, 40, 20 10)





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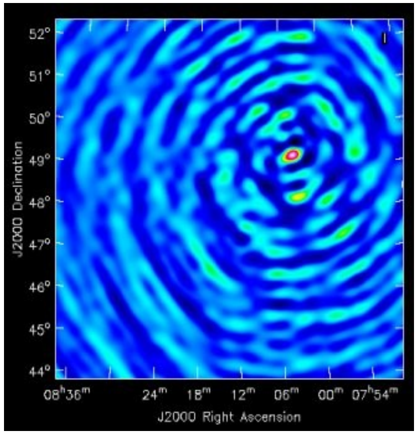
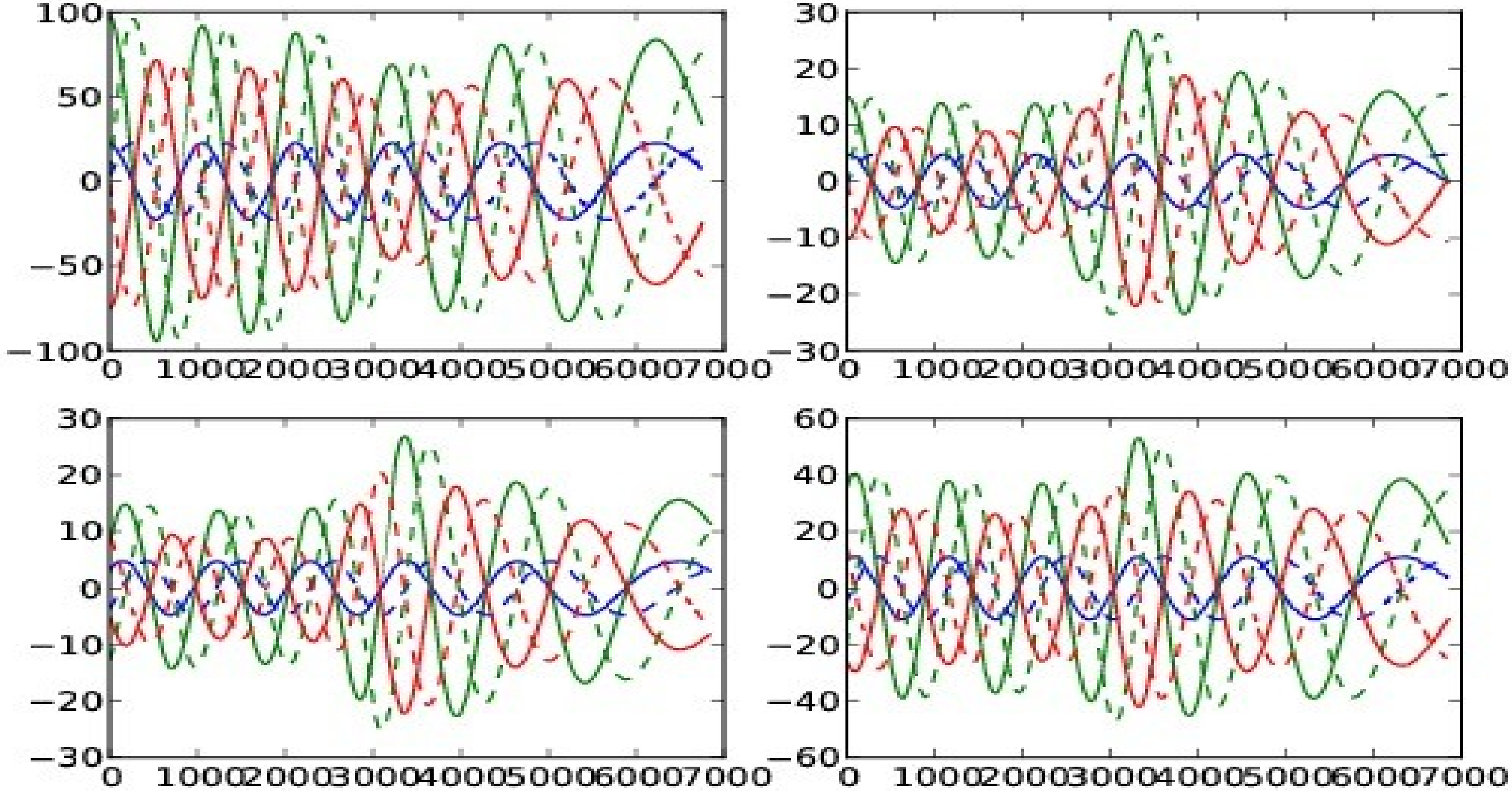


One off-axis source IQUV=(100, 40, 20 10)

“Traditional” imager removes visibility with constant amplitude



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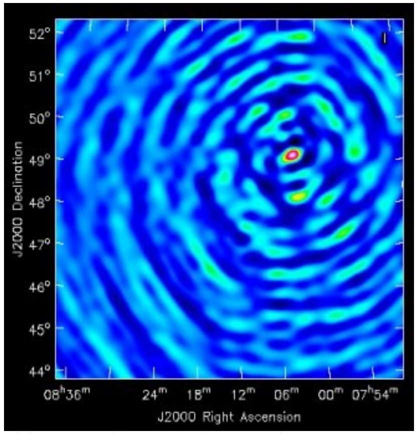

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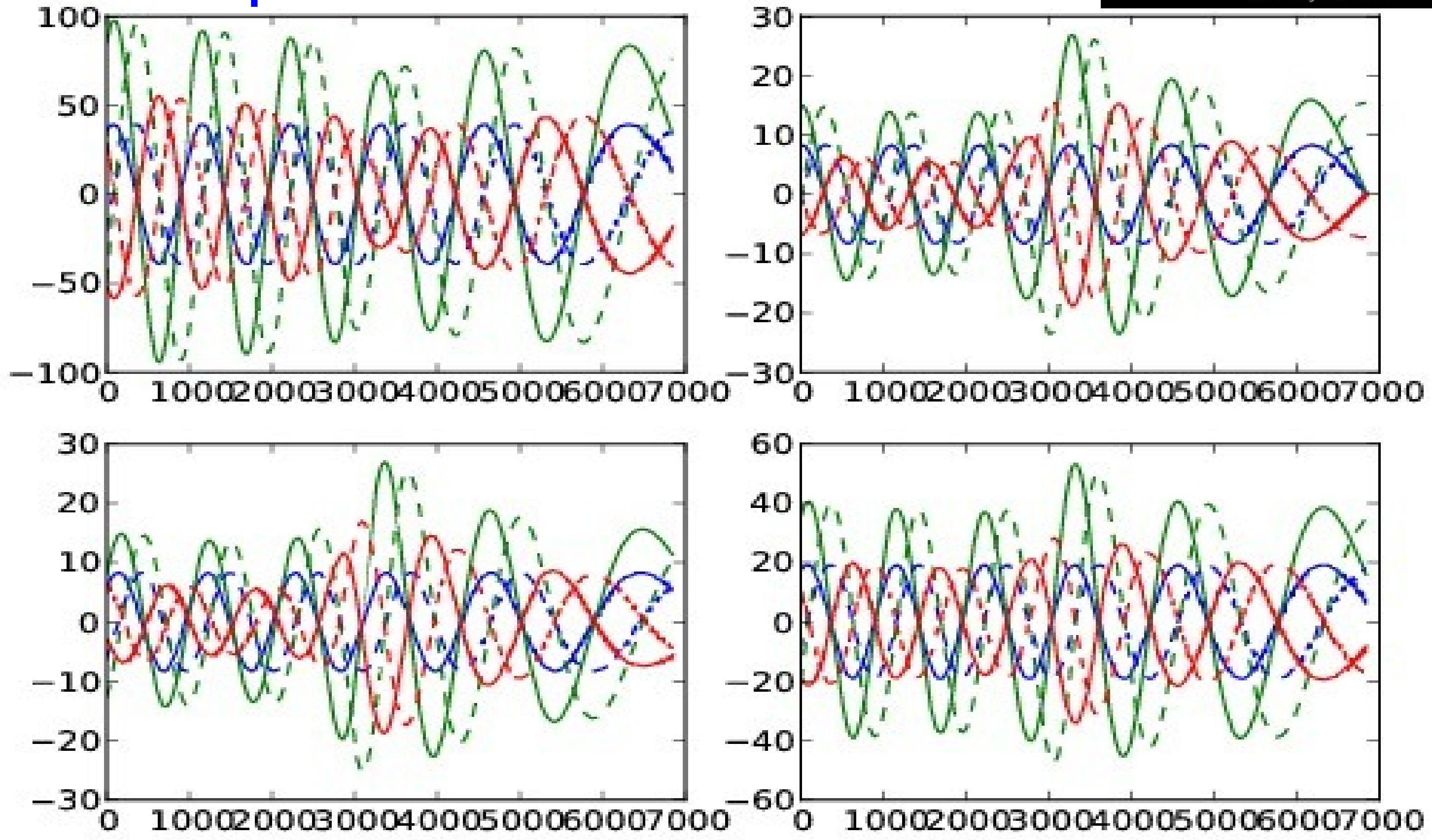

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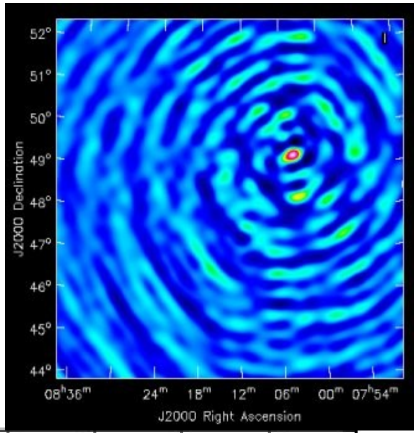

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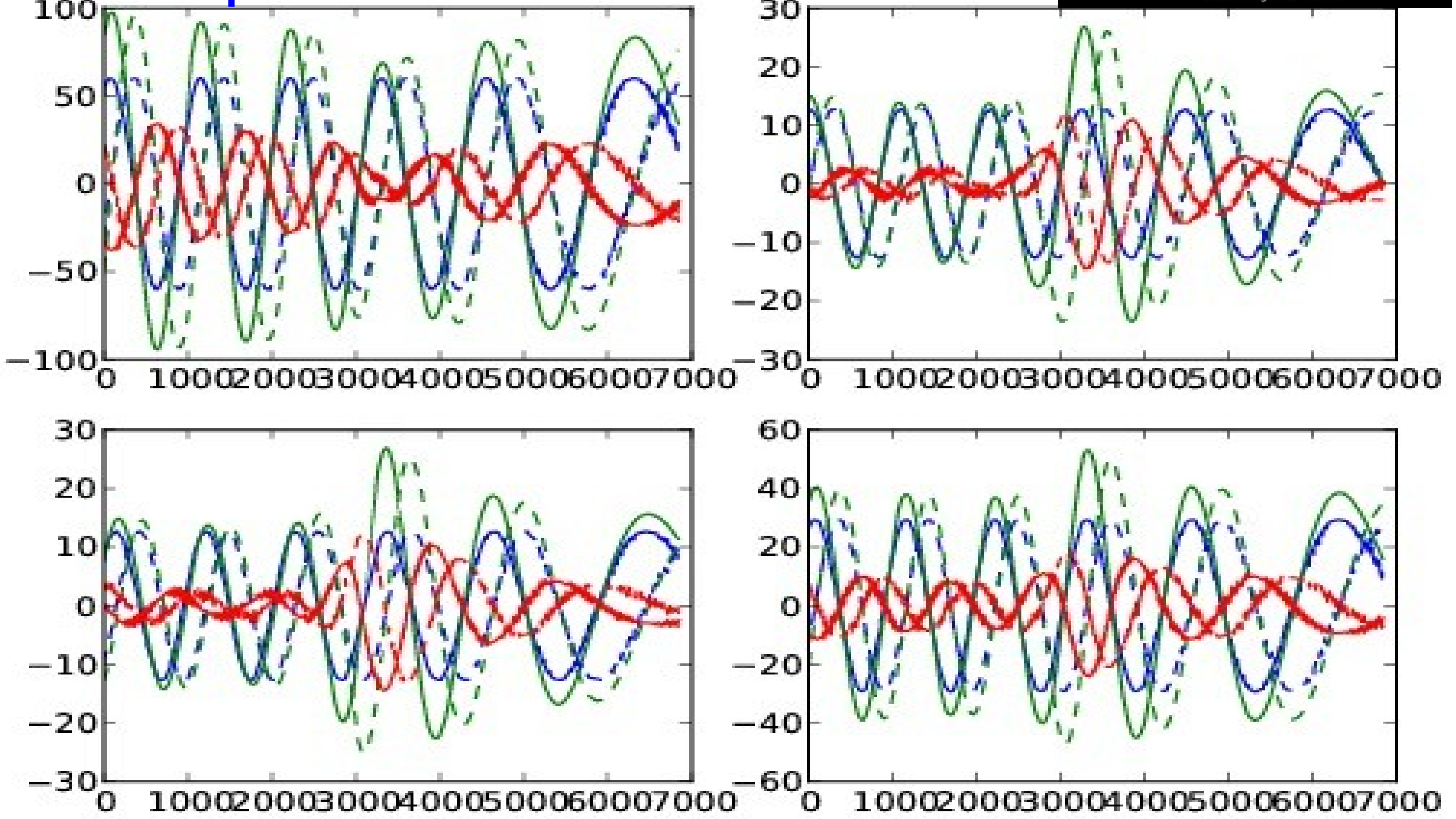

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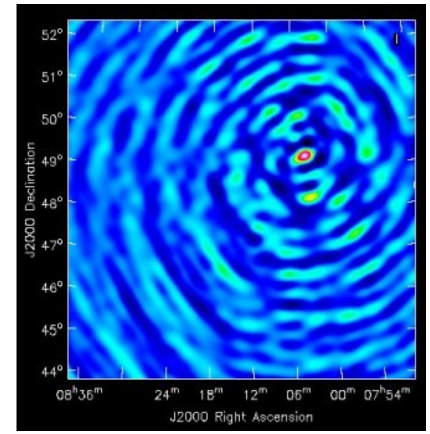

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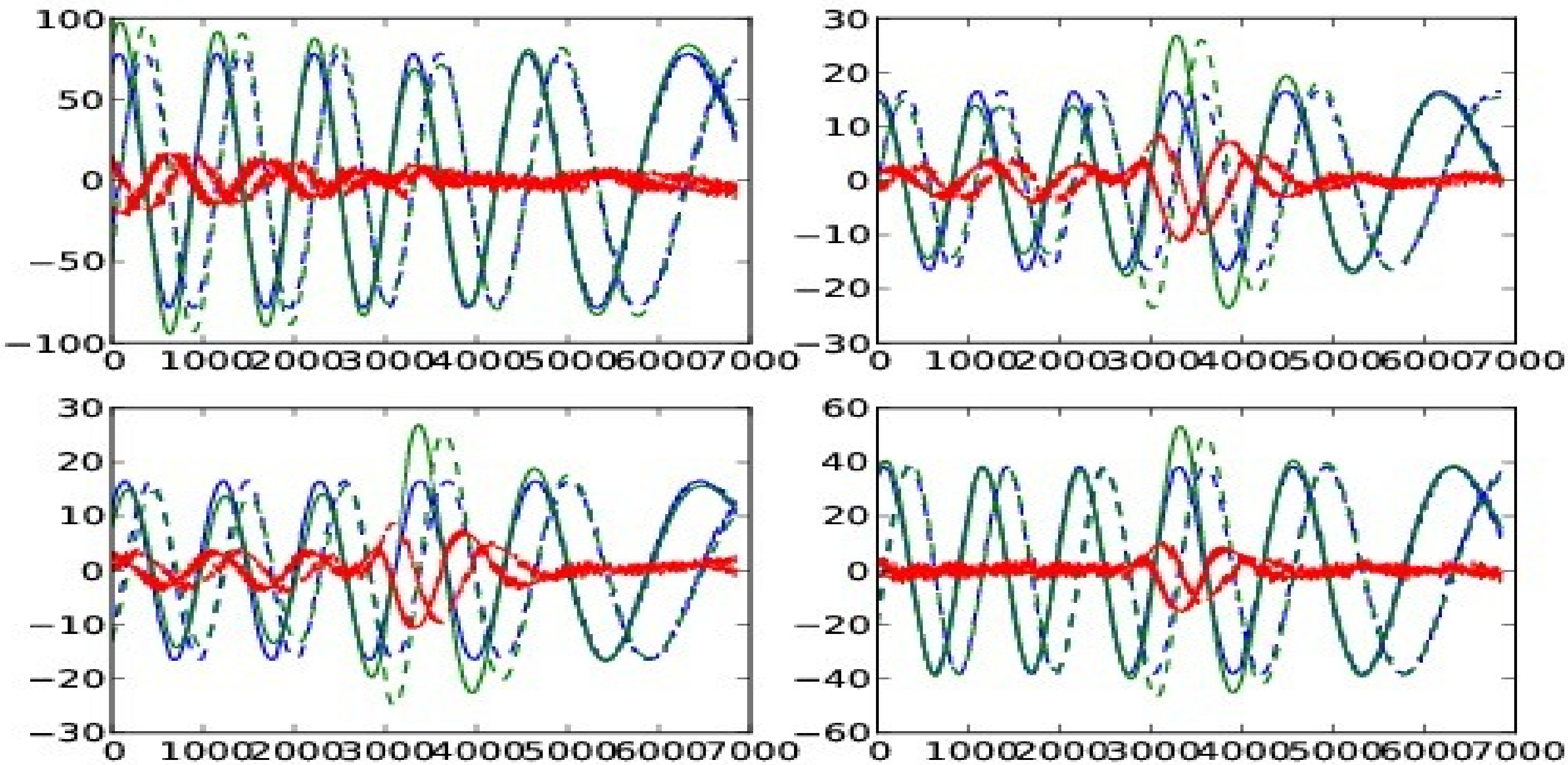
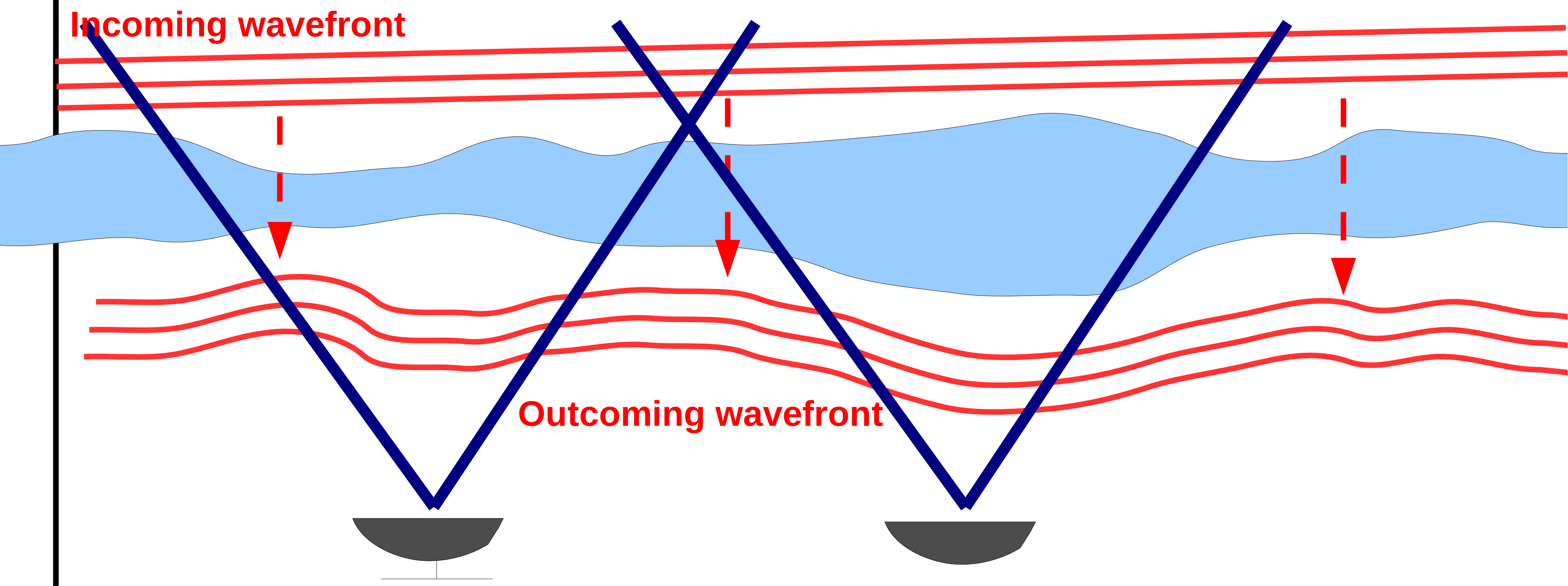


… When Direction Dependent Effects (DDE) become a problem : Ionosphere



Big field of view : station, direction, time and frequency dependent

Other direction dependent effects :

- Projection of the dipoles on the sky
- Faraday rotation

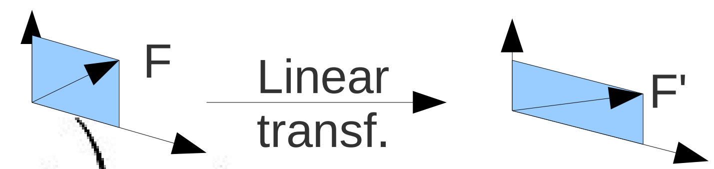
+ Effect on the polarisation

The Measurement Equation

Hamaker 1996

Direction independent — Direction dependent — Source coherency

$$V_{pq} = G_p \left(\sum_{i=1}^{N} \mathrm{B}_{pi}\mathrm{K}_{pi}\mathrm{I}_{pi}F_i \;.\; F_i^{+}\mathrm{I}_{qi}^{+}\mathrm{K}_{qi}^{+}\mathrm{B}_{qi}^{+} \right) G_q^{+}$$



[Voltage antenna p] x [Voltage antenna q]*

Beam — Geometrical delay +Correlator — Ionosphere — Electric field

$$\mathrm{K}_p\mathrm{K}_q^{+} = \exp(-2i\pi\phi_{pq}) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\phi_{pq} = u_{pq}l + v_{pq}m + w_{pq}(\sqrt{1 - l^2 - m^2} - 1)$$

The "*Vec*" Operator

If $$\boldsymbol{A} = [\boldsymbol{a}_1 \boldsymbol{a}_2 \cdots \boldsymbol{a}_n]$$

Columns of a Matrix

And $$\text{vec}(\boldsymbol{A}) = \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \vdots \\ \boldsymbol{a}_n \end{bmatrix}$$

then $$\text{vec}(\boldsymbol{AXB}) = (\boldsymbol{B}^T \otimes \boldsymbol{A})\, \text{vec}(\boldsymbol{X})$$

The "*Vec*" Operator

If $\boldsymbol{A} = [\boldsymbol{a}_1 \boldsymbol{a}_2 \cdots \boldsymbol{a}_n]$

Columns of a Matrix

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then $\text{vec}(\boldsymbol{AXB}) = (\boldsymbol{B}^T \otimes \boldsymbol{A})\,\text{vec}(\boldsymbol{X})$

Beam (4*4)

$$\text{Vec}(V_{pq}) = (G_q^* \otimes G_p) \int_S (E_{q,\vec{s}}^* \otimes E_{p,\vec{s}}).\text{Vec}(F_{\vec{s}}.F_{\vec{s}}^+).\exp\left(i\ \vec{b_{pq}}.\vec{s}\right)\ d\vec{s}$$

3D FT

A-Projection

Bhatnagar 08

Convolution function (4*4)

Beam (4*4)

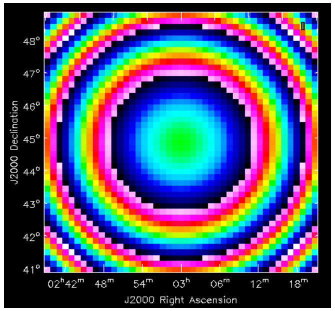
W term (scalar)

$$\mathrm{Vec}(V_{pq}) = (G_q^* \otimes G_p)\mathrm{FT}\left[\left(E^*_{q,\vec{s}} \otimes E_{p,\vec{s}}.\exp\left(-2\pi i w_{pq}.\left(\sqrt{1-l^2-m^2}-1\right)\right)\right)\right]$$

$$\star \int_S \mathrm{Vec}(X_{\vec{s}}).\exp\left(-2\pi i(u_{pq}l + v_{pq}m)\right)dl.dm$$

Convolution

2D FFT



This is an EXACT map from sky plane to the Visibilities in the UVW space!

A-Projection

Bhatnagar 08

The inverse map is approximative! (based on pseudo-inverse)

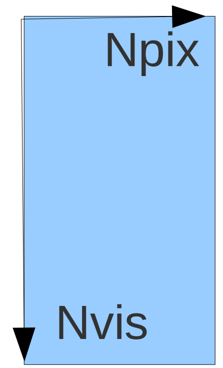
$$\mathrm{Vec}(V_{pq}) = (G_q^* \otimes G_p)\mathrm{FT}\left[\left(E_{q,\vec{s}}^* \otimes E_{p,\vec{s}}.\exp\left(-2\pi i w_{pq}.\left(\sqrt{1-l^2-m^2}-1\right)\right)\right)\right]$$

$$\star \int_S \mathrm{Vec}(X_{\vec{s}}).\exp\left(-2\pi i(u_{pq}l + v_{pq}m)\right)dl.dm$$

This equation is linear in Sky

$$\boldsymbol{V}^M = \mathbf{A}\boldsymbol{I}^M$$

A=S.AW.F =

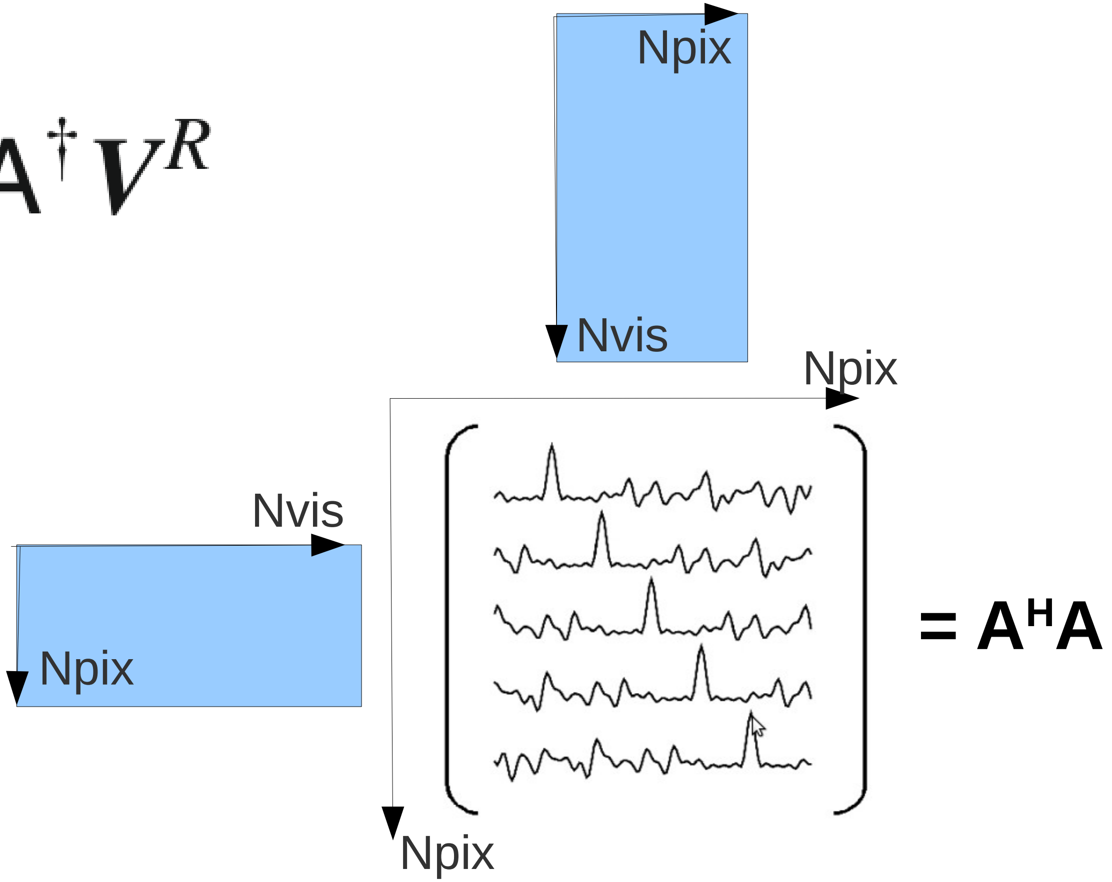


A-Projection

Bhatnagar 08

The inverse map is approximative! (based on pseudo-inverse)

$$I^R = \left[A^\dagger A\right]^{-1} A^\dagger V^R$$



See Urvashi Rau PhD thesis

A-Projection

Bhatnagar 08

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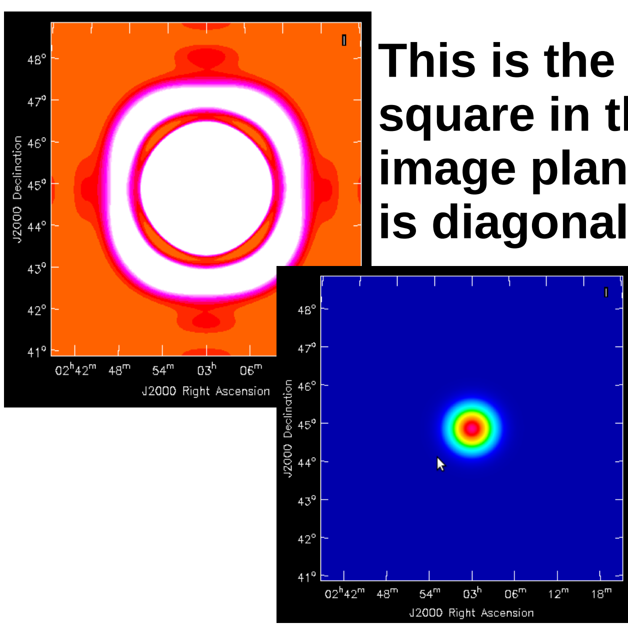
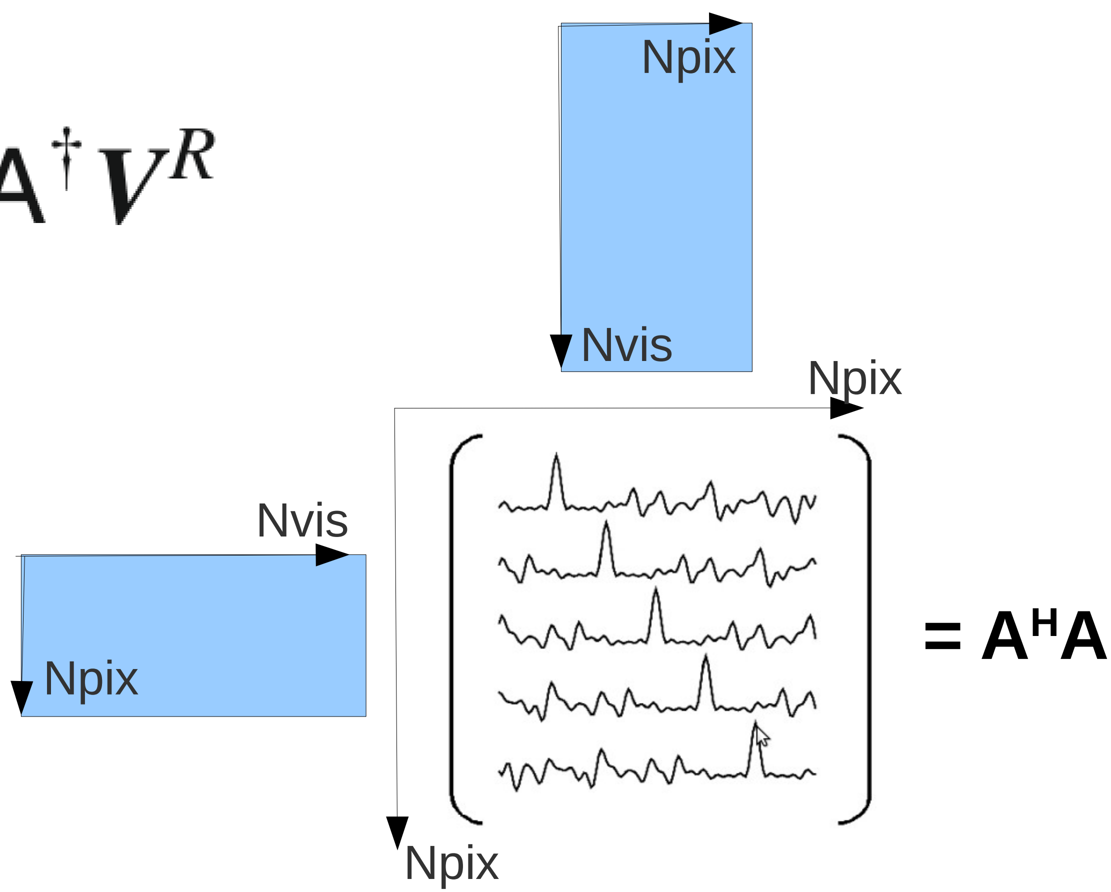


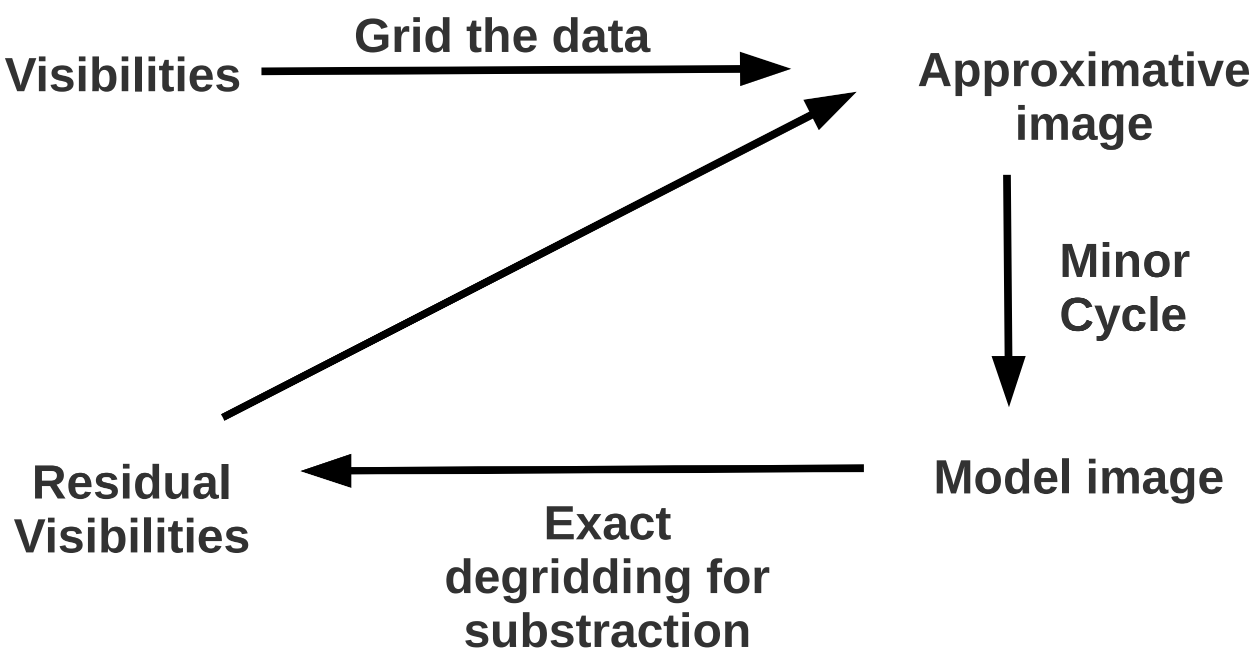
This is the beam square in the image plane if $A^H A$ is diagonal



See Urvashi Rau PhD thesis

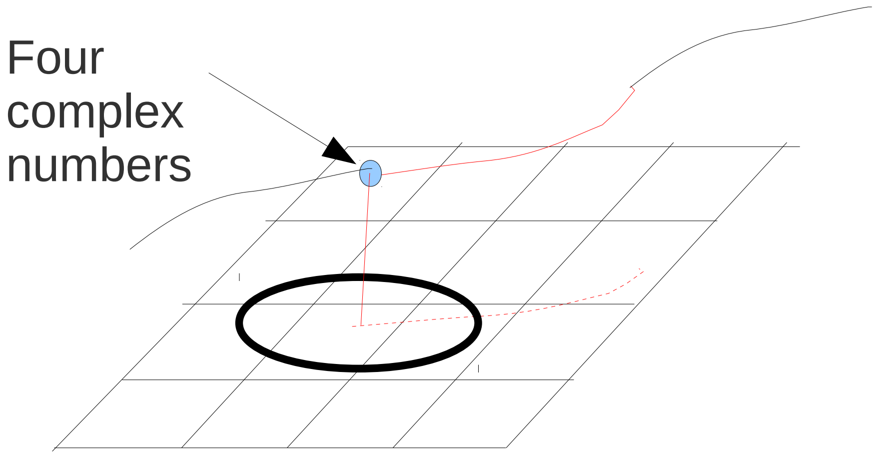
JAWS: the practice

- Plug in the casa architecture
- Full Polarization
- Convolution function is mapped by i,j,t, nu
- Ionosphere easy to plug in
- Will run in parallel

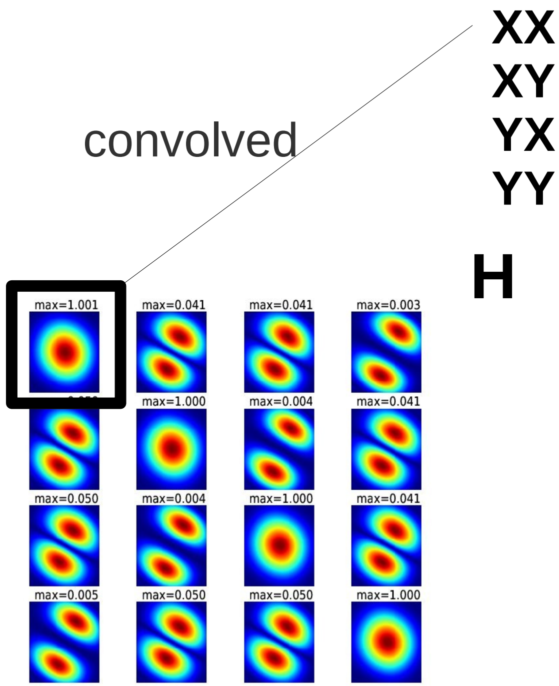


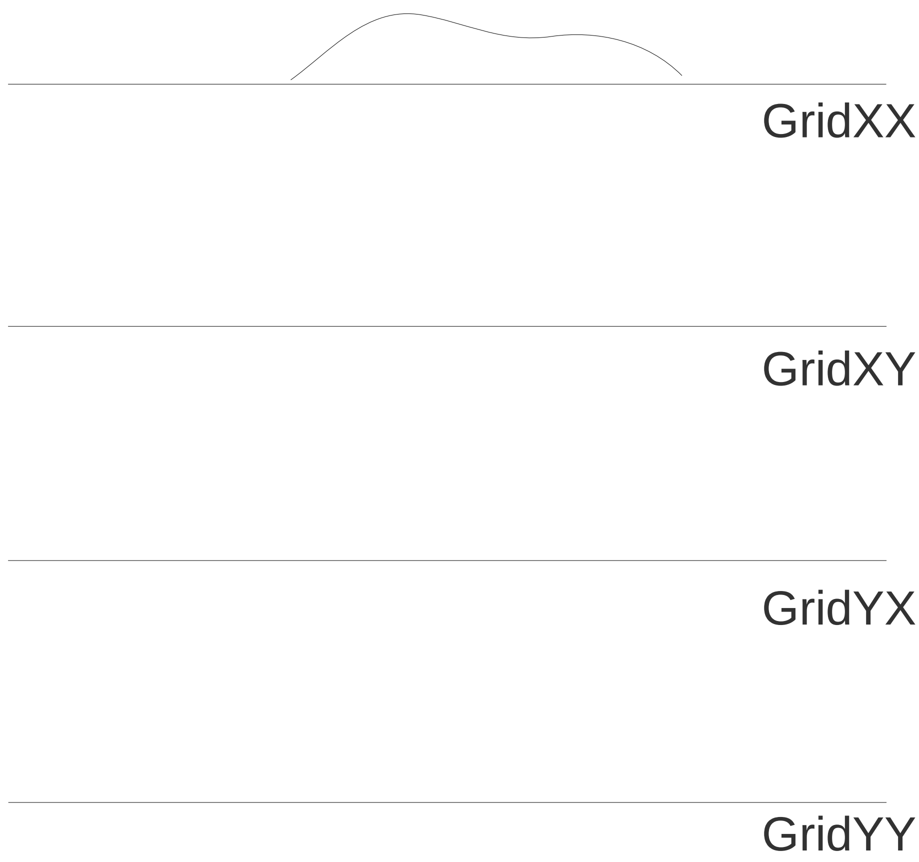
After a number of iteration, the flux in the clean component converges to the true values (to be studied)

Gridding



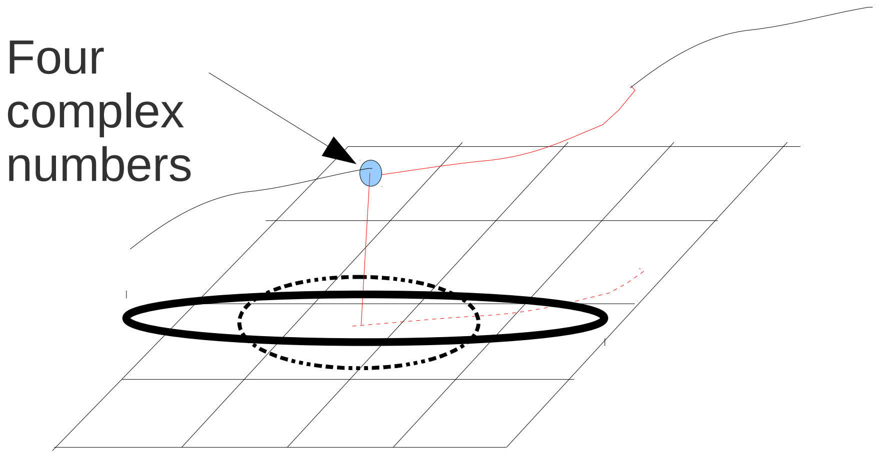
A given baseline
A given Timeslot
A given frequency slot

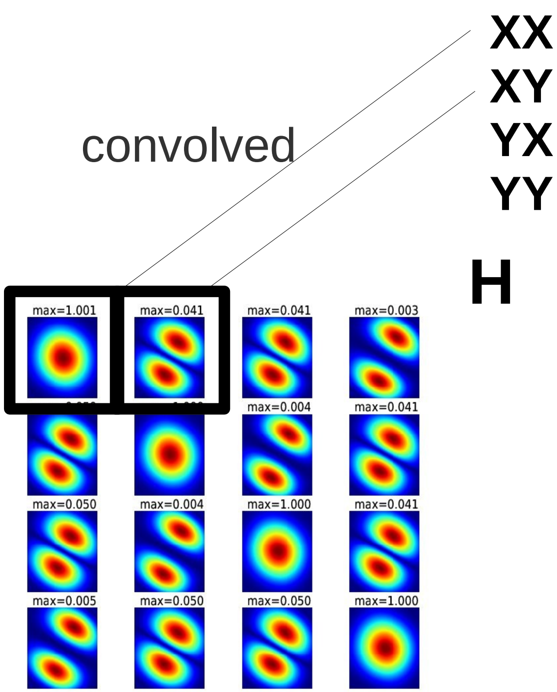


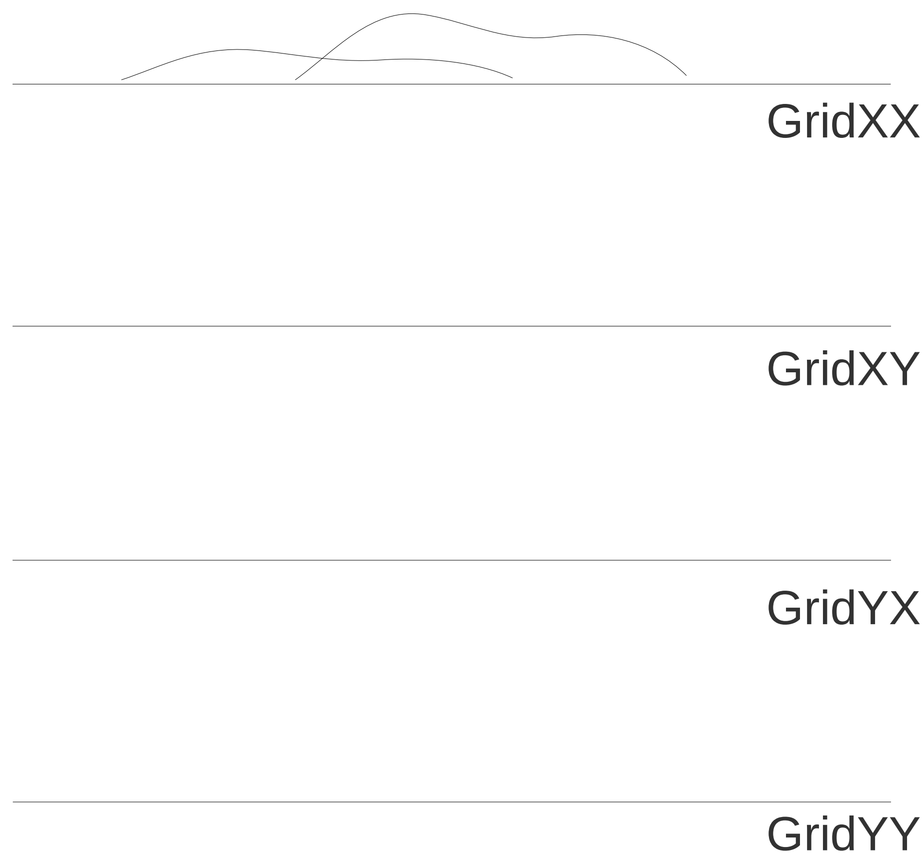


Gridding

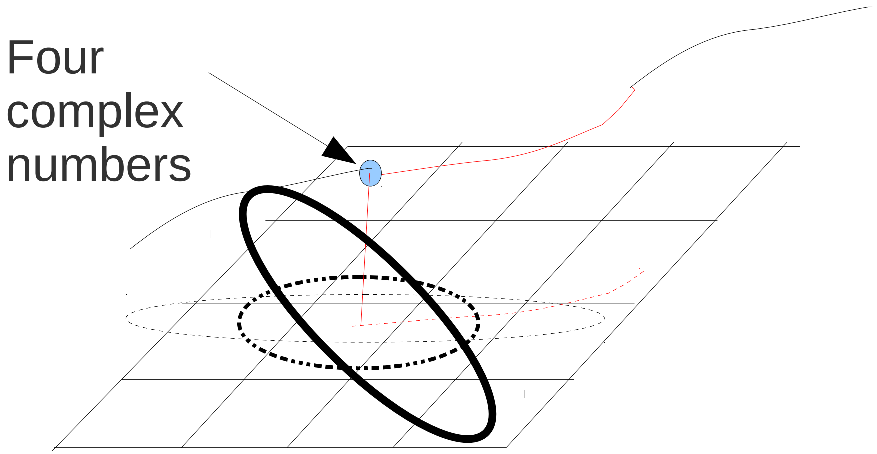
A given baseline
A given Timeslot
A given frequency slot



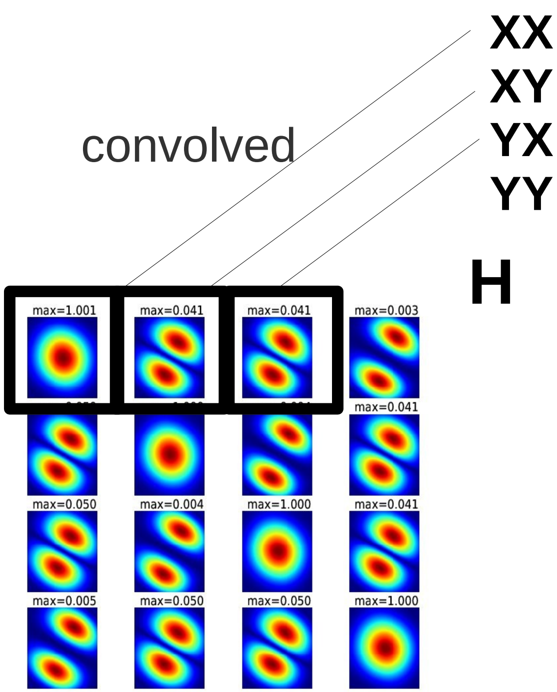


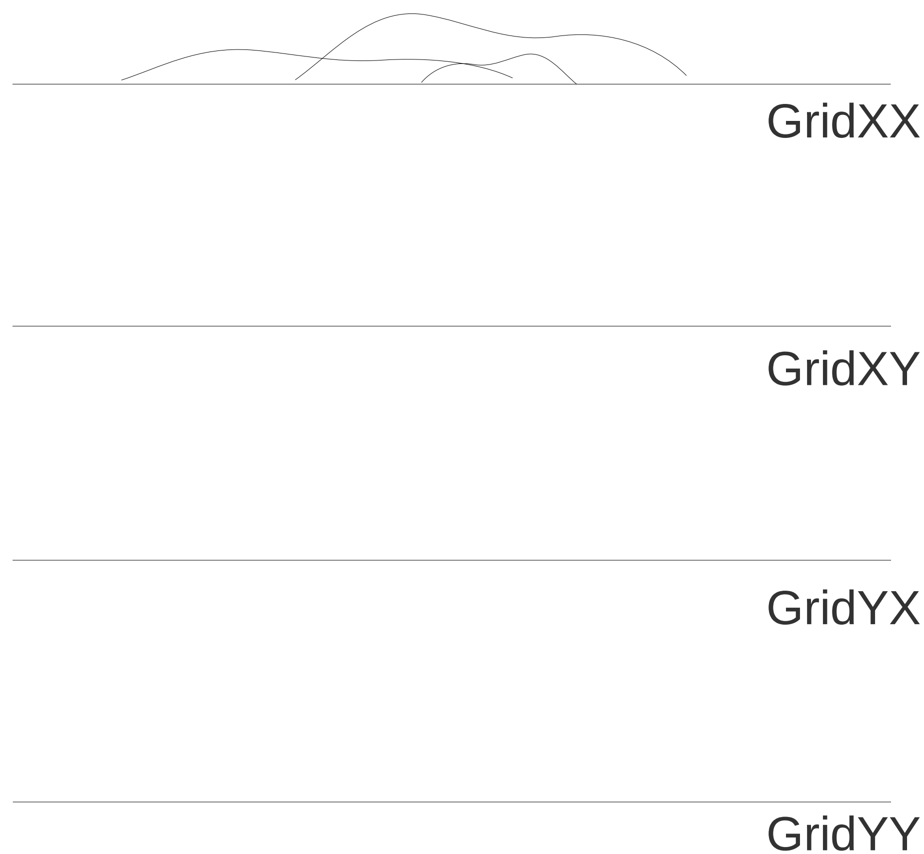


Gridding

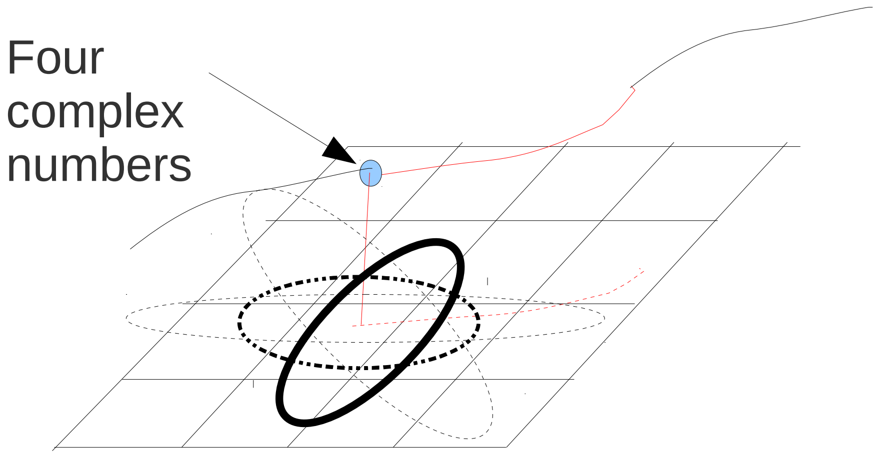


A given baseline
A given Timeslot
A given frequency slot

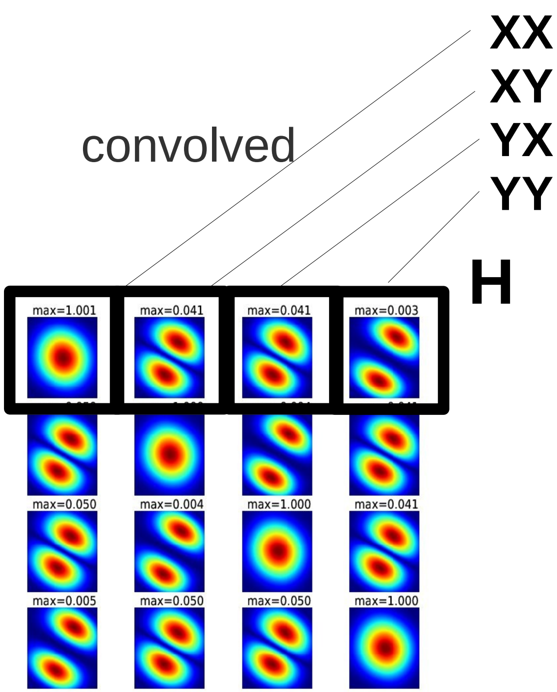


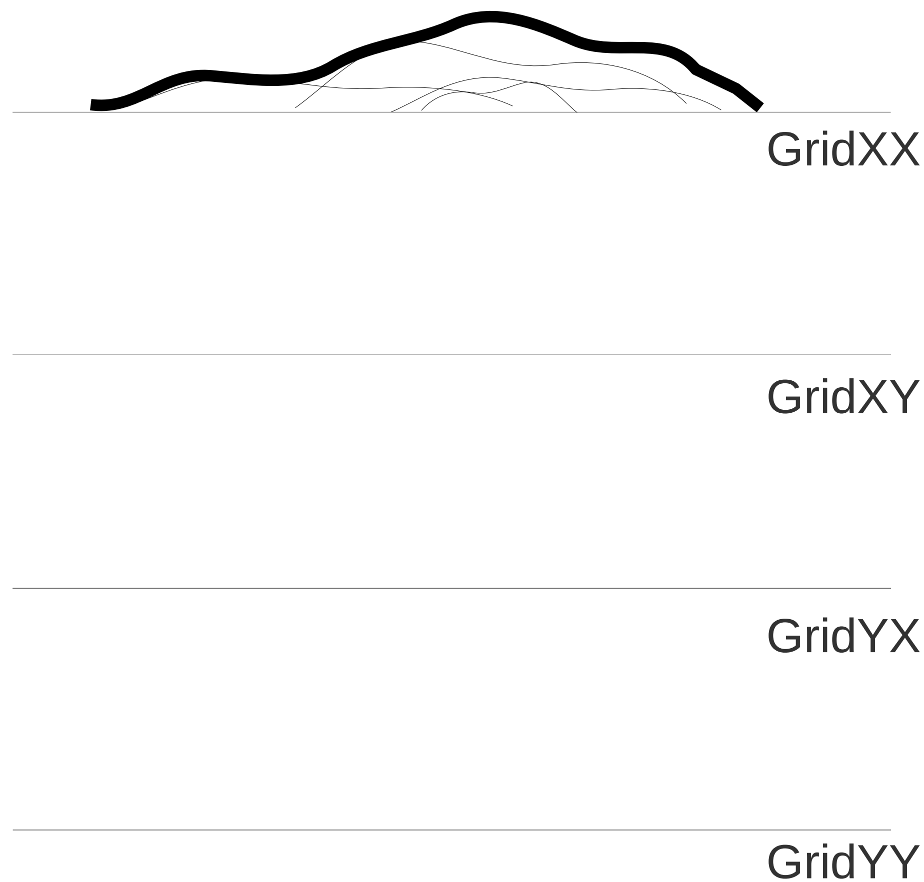


Gridding

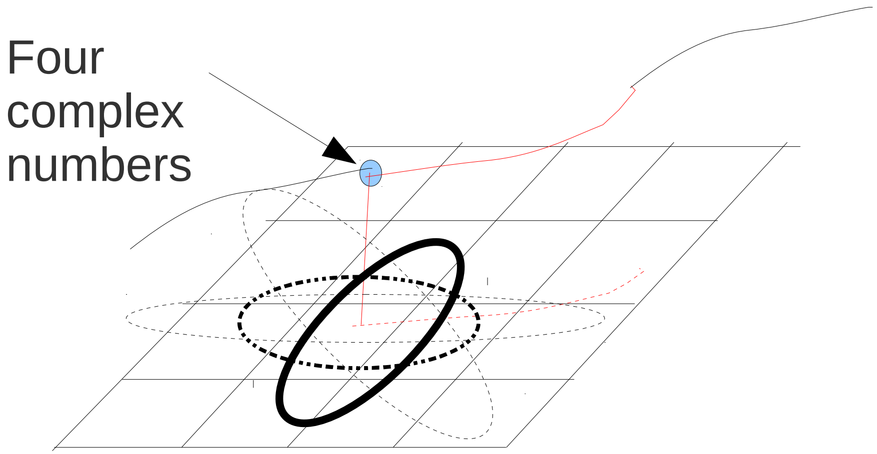


A given baseline
A given Timeslot
A given frequency slot

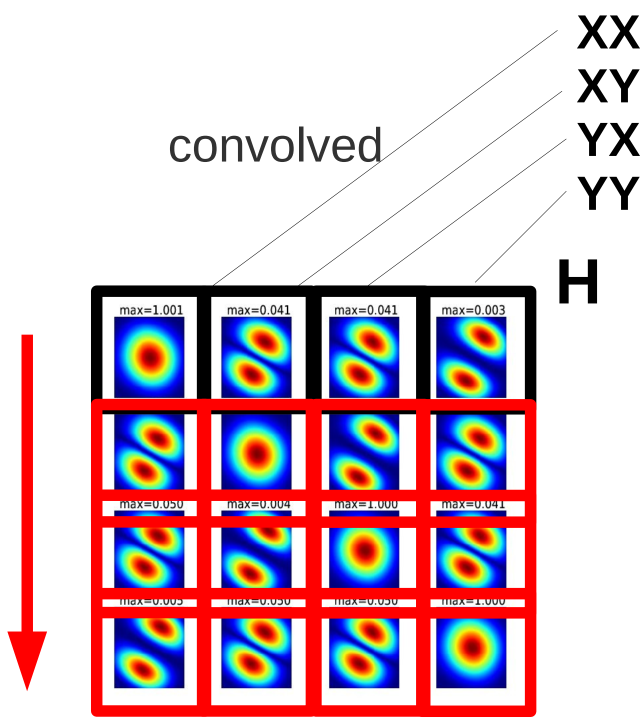


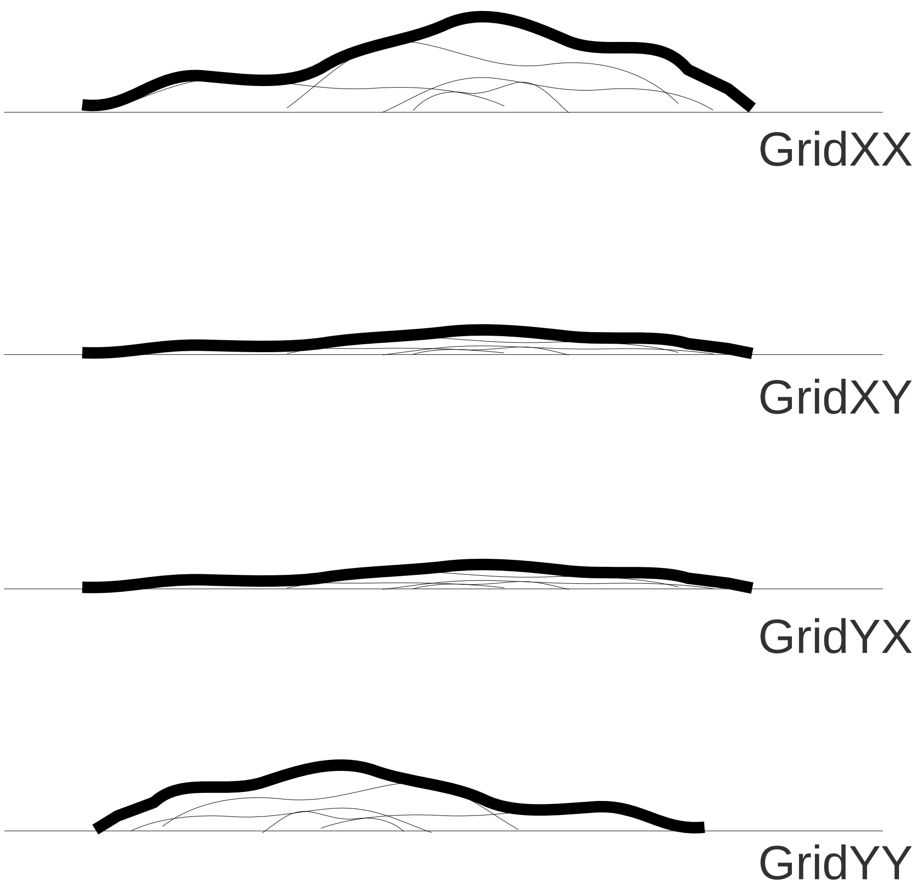


Gridding

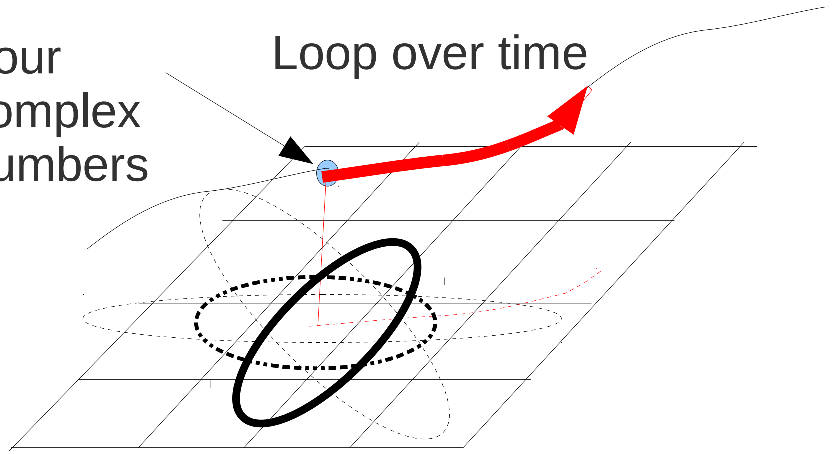


A given baseline
A given Timeslot
A given frequency slot

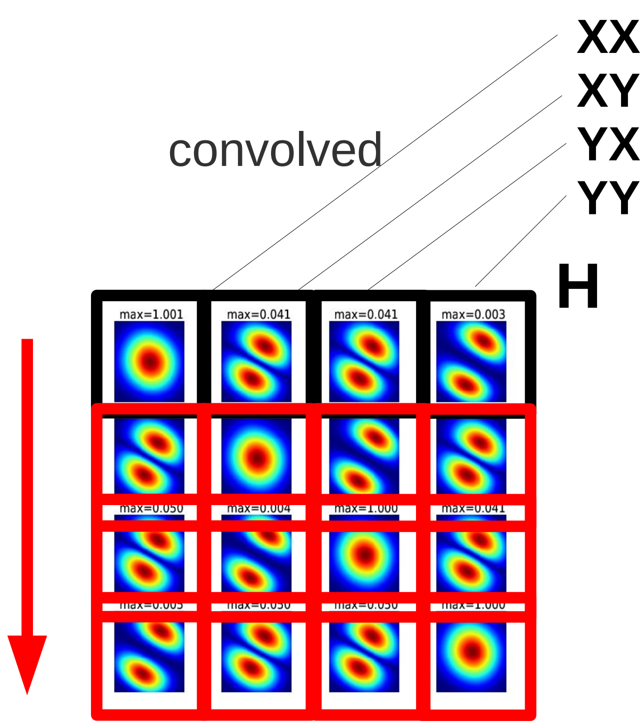


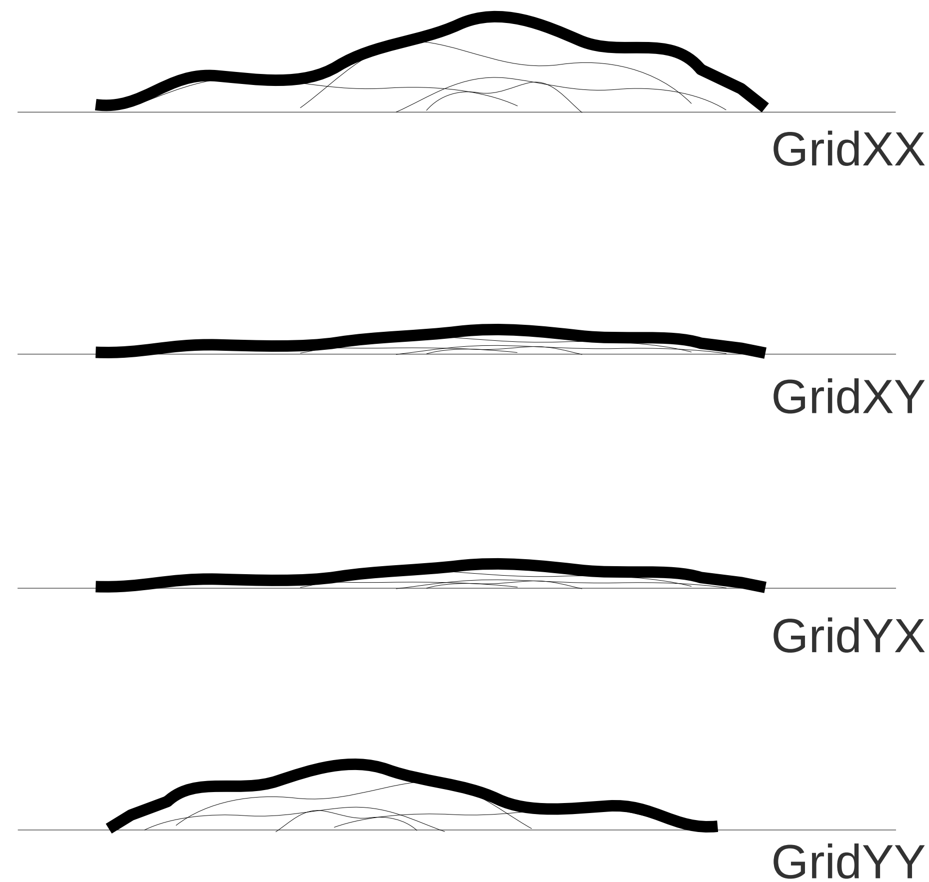


Gridding

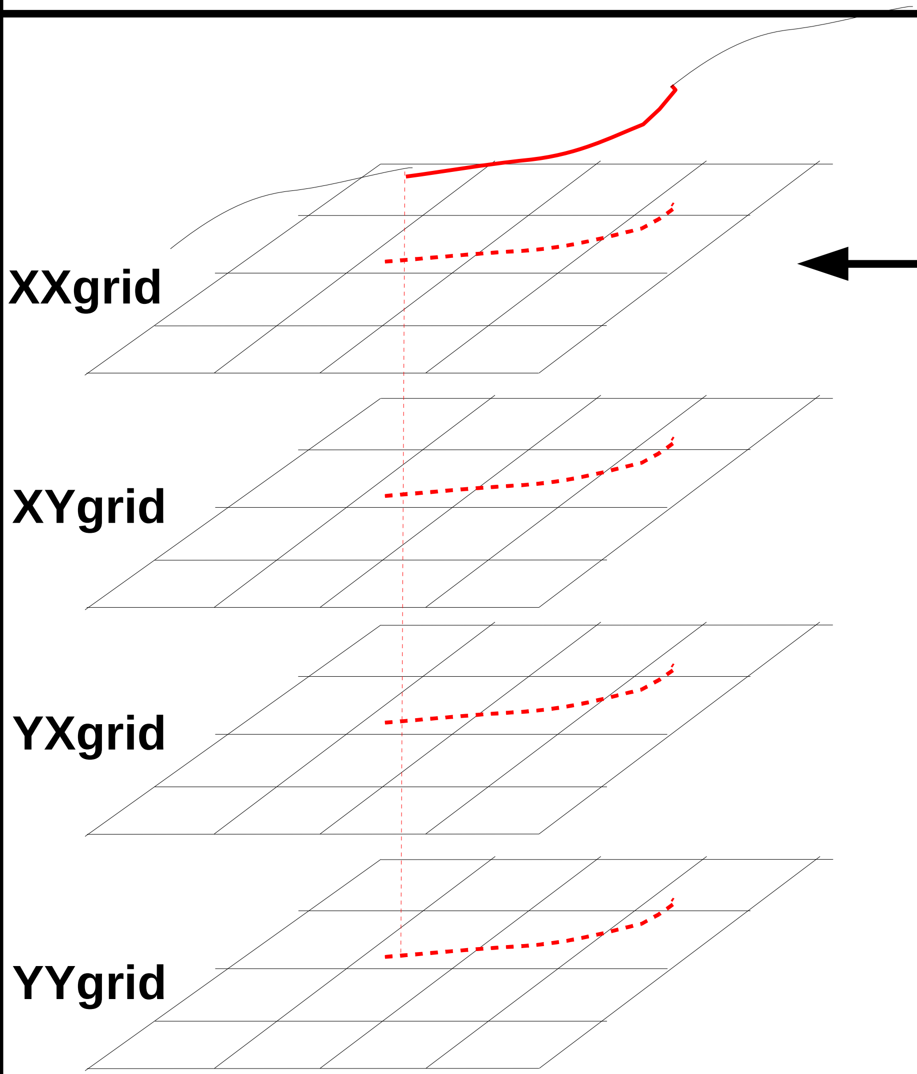


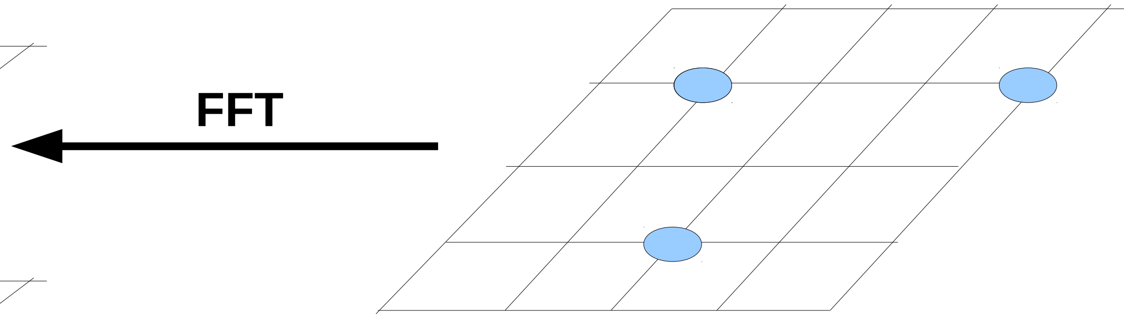
Loop over baseline
Loop over time
Loop over frequency





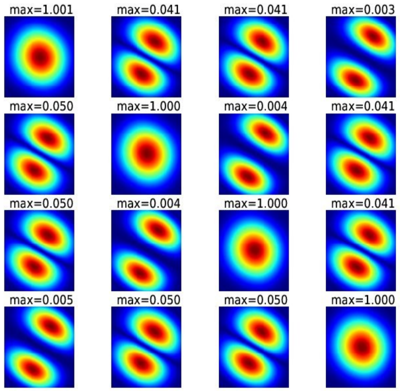
DeGridding



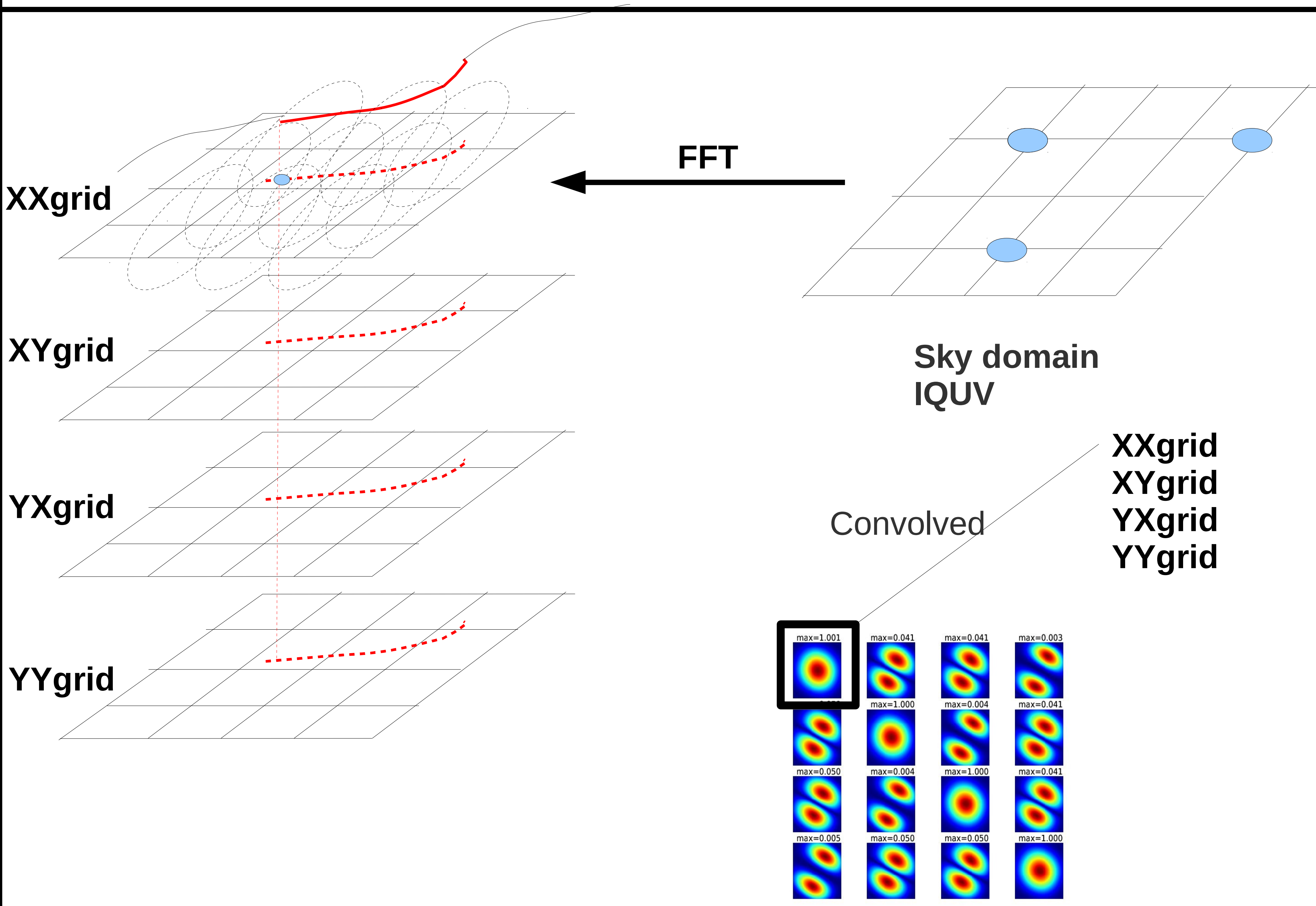


Sky domain
IQUV

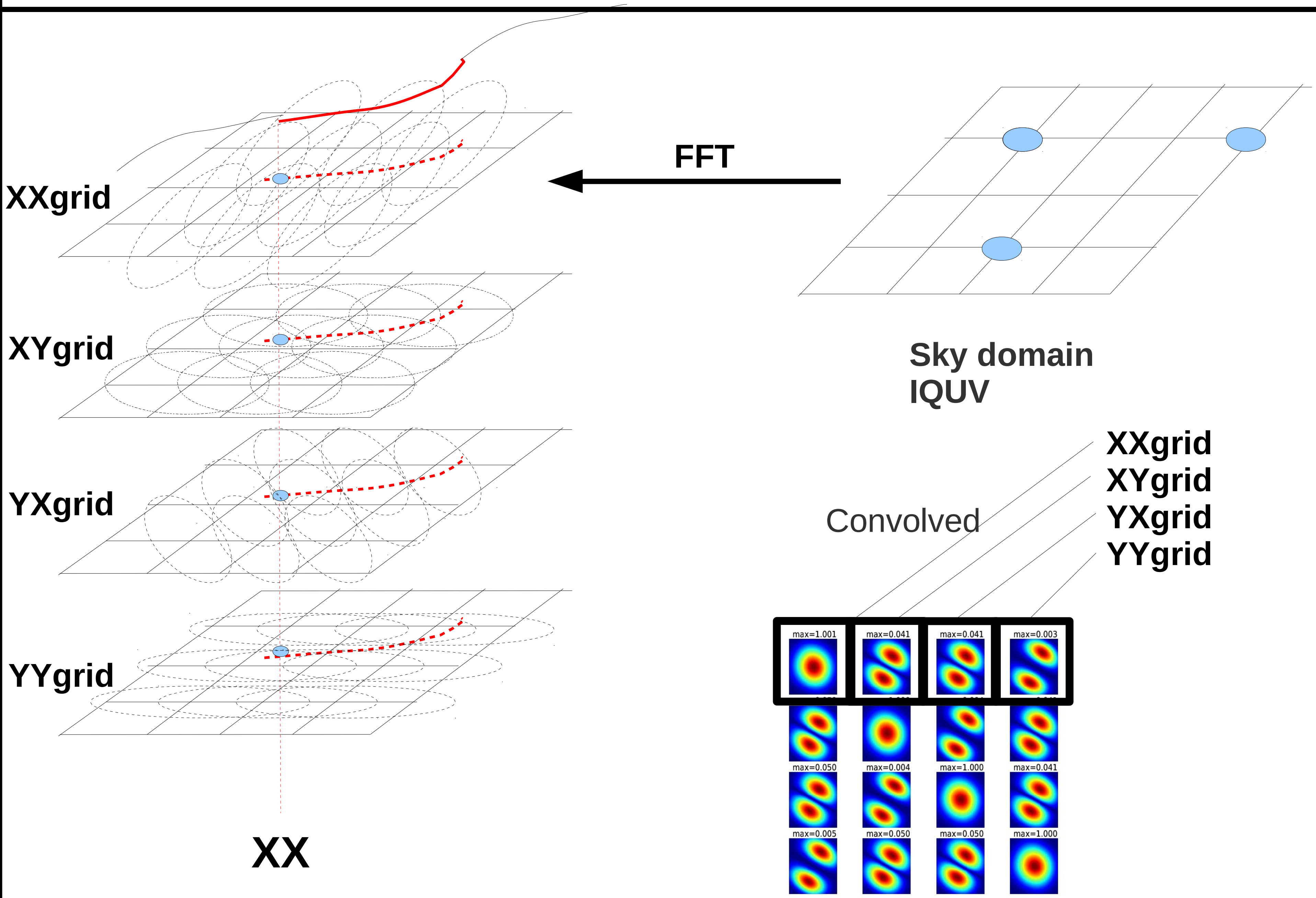
XXgrid
XYgrid
YXgrid
YYgrid



DeGridding


XXgrid
XYgrid
YXgrid
YYgrid
FFT
Sky domain
IQUV
Convolved
XXgrid
XYgrid
YXgrid
YYgrid
max=1.001
max=0.041
max=0.041
max=0.003
max=1.000
max=0.004
max=0.041
max=0.050
max=0.004
max=1.000
max=0.041
max=0.005
max=0.050
max=0.050
max=1.000

DeGridding



XXgrid
XYgrid
YXgrid
YYgrid
XX
FFT
Sky domain
IQUV
Convolved
XXgrid
XYgrid
YXgrid
YYgrid
max=1.001
max=0.041
max=0.041
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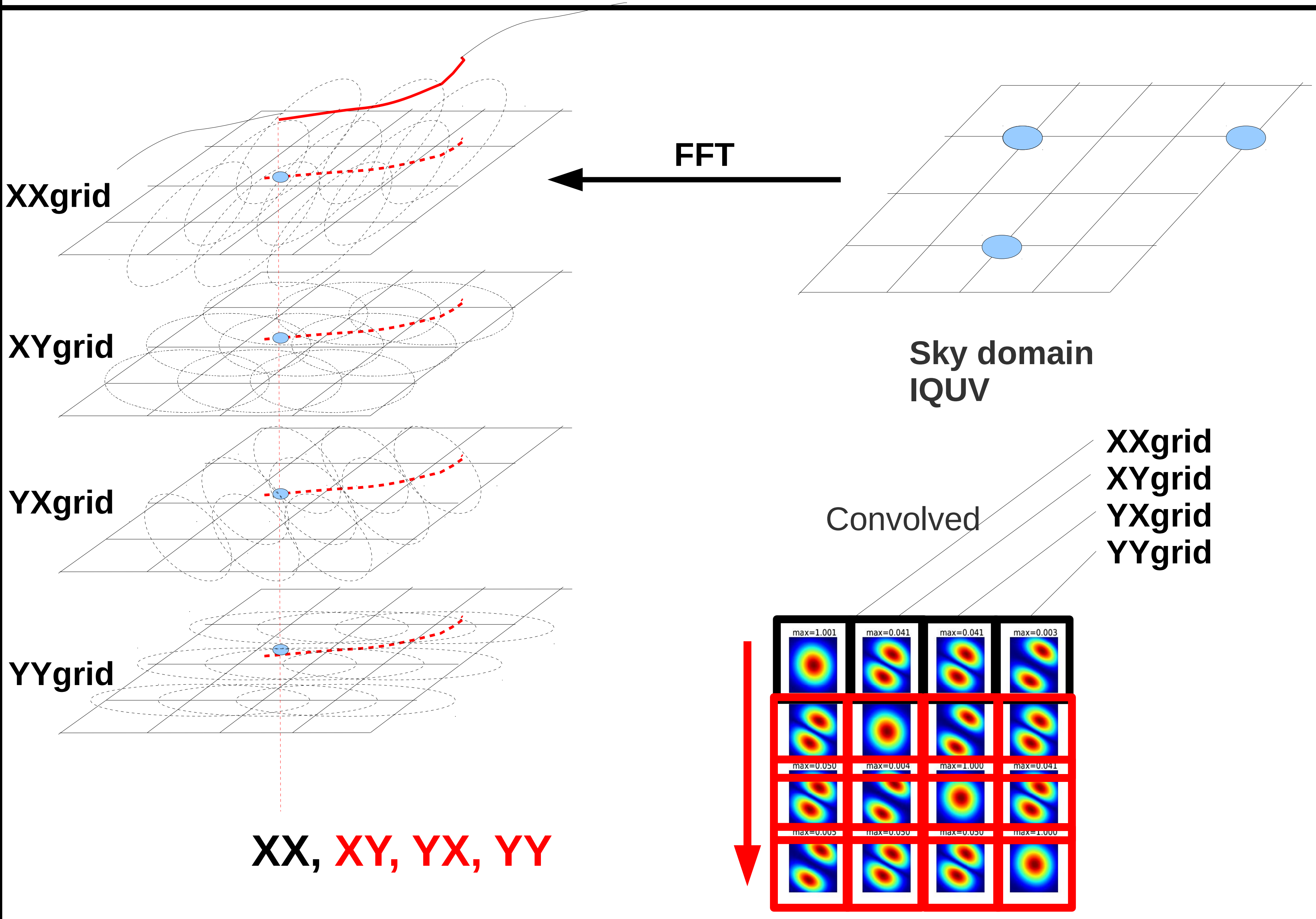
max=0.050
max=0.004
max=1.000
max=0.041
max=0.005
max=0.050
max=0.050
max=1.000

DeGridding


XXgrid
XYgrid
YXgrid
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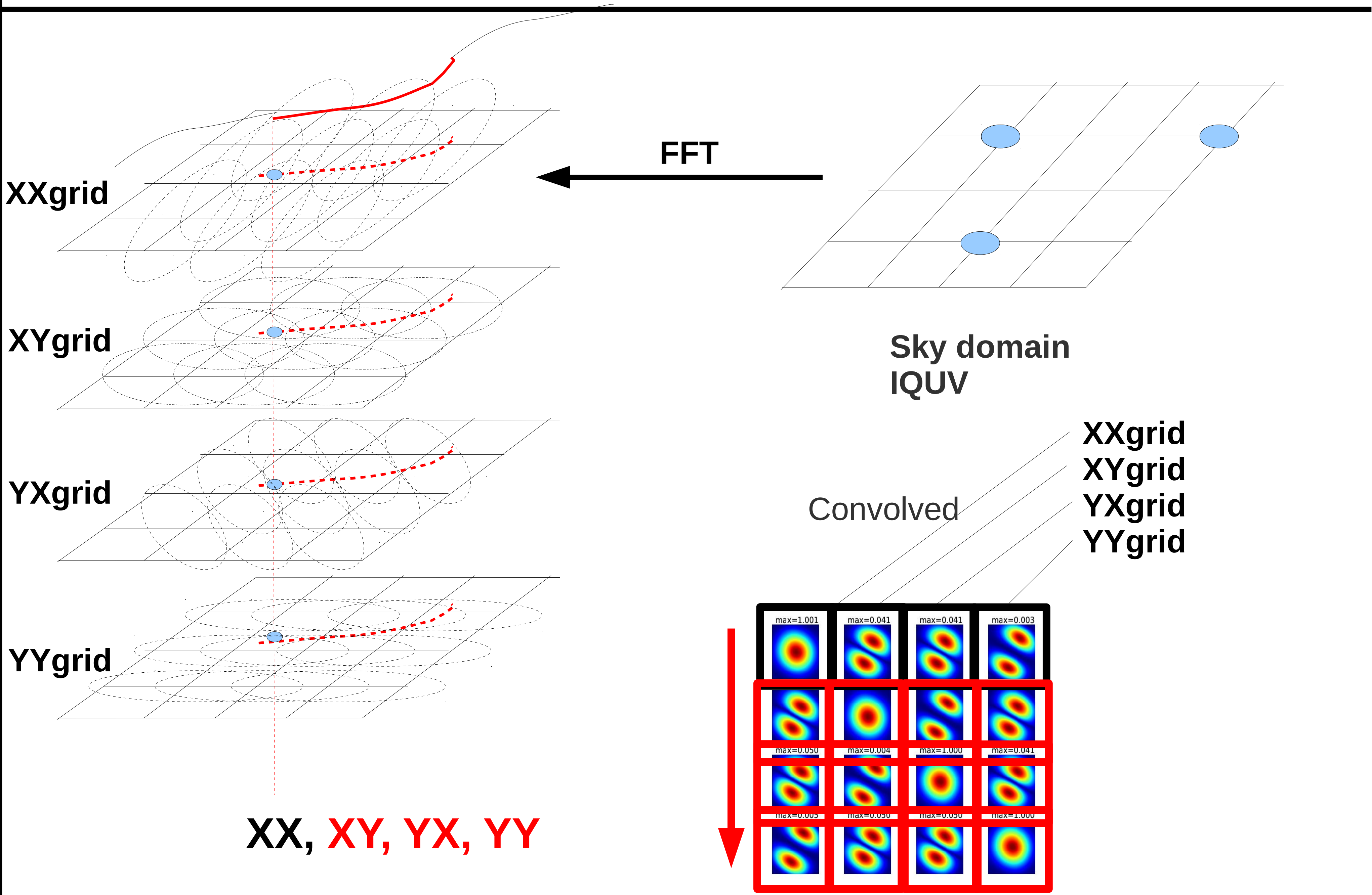
FFT
Sky domain
IQUV
Convolved
XXgrid
XYgrid
YXgrid
YYgrid
max=1.001
max=0.041
max=0.041
max=0.003
XX, XY, YX, YY

DeGridding


XXgrid
XYgrid
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Convolved
XXgrid
XYgrid
YXgrid
YYgrid
max=1.001
max=0.041
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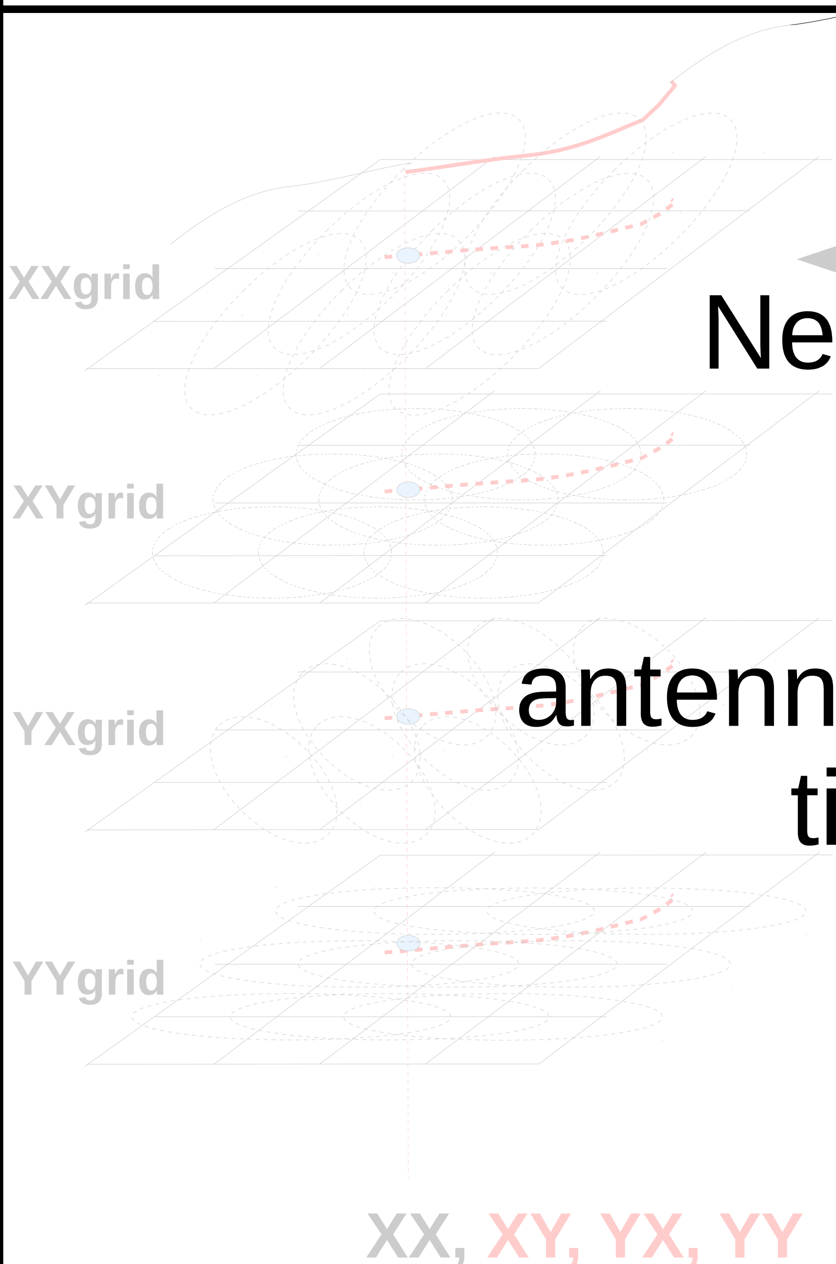
XX, XY, YX, YY

DeGridding

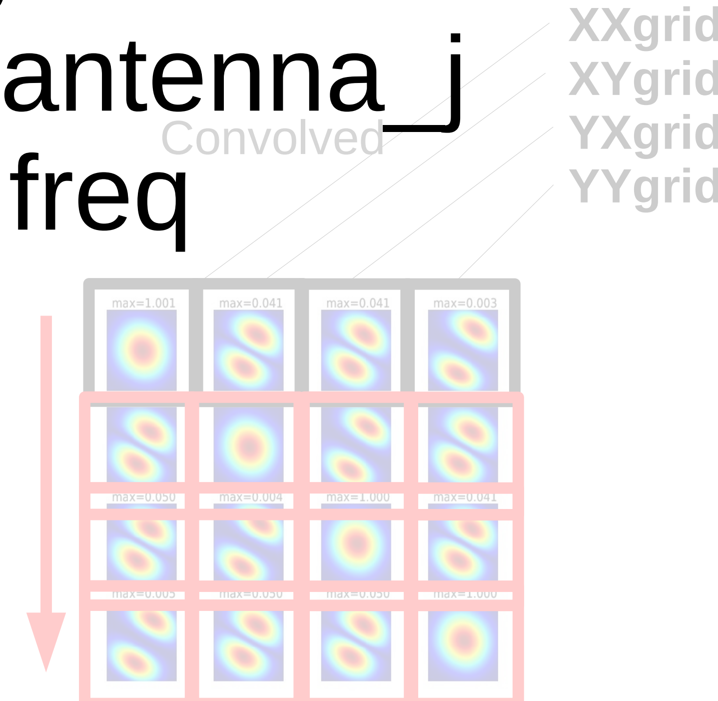


Next point in:

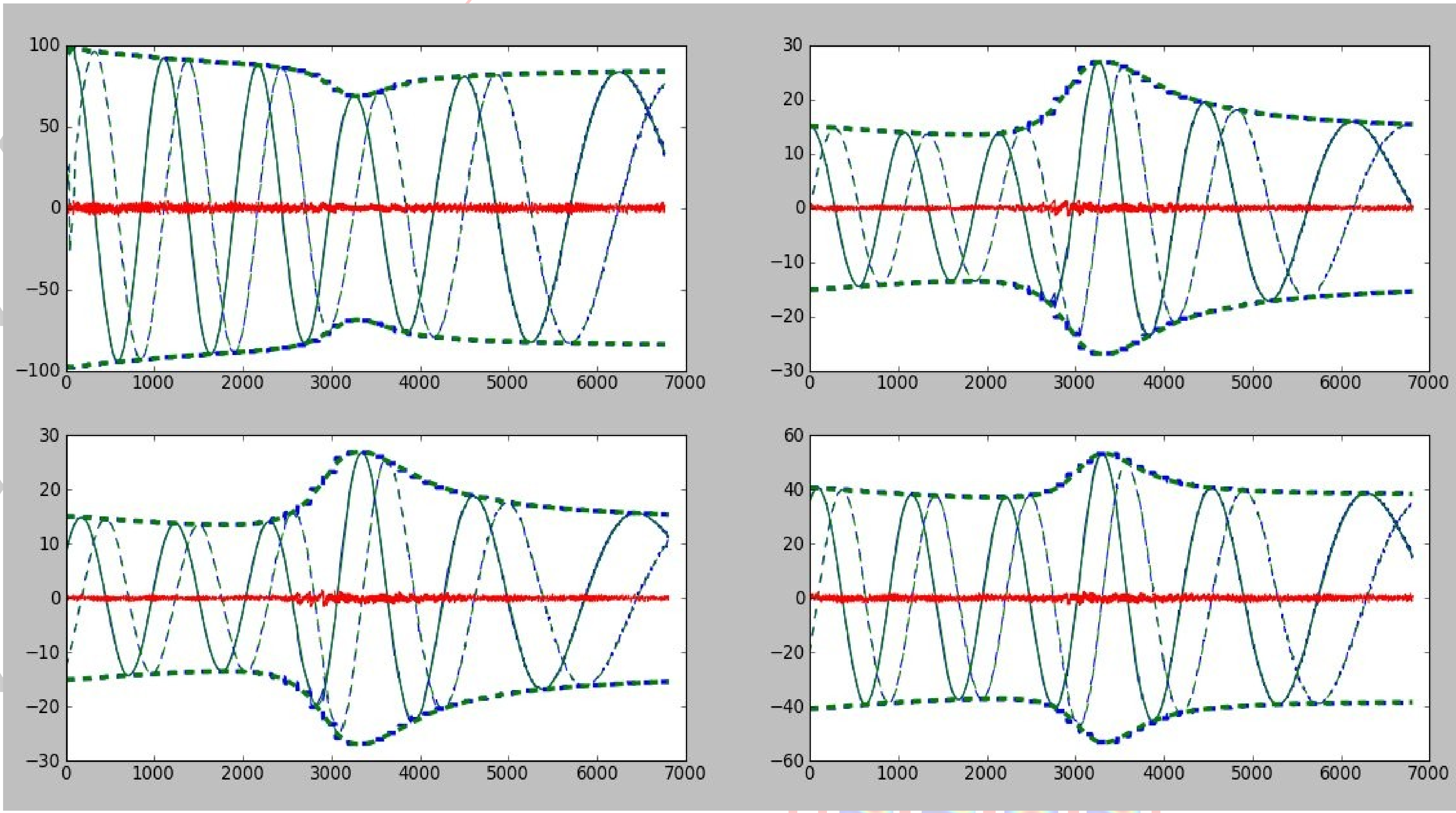
u, v, w

antenna_i, antenna_j

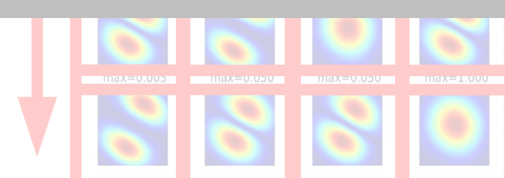
time, freq



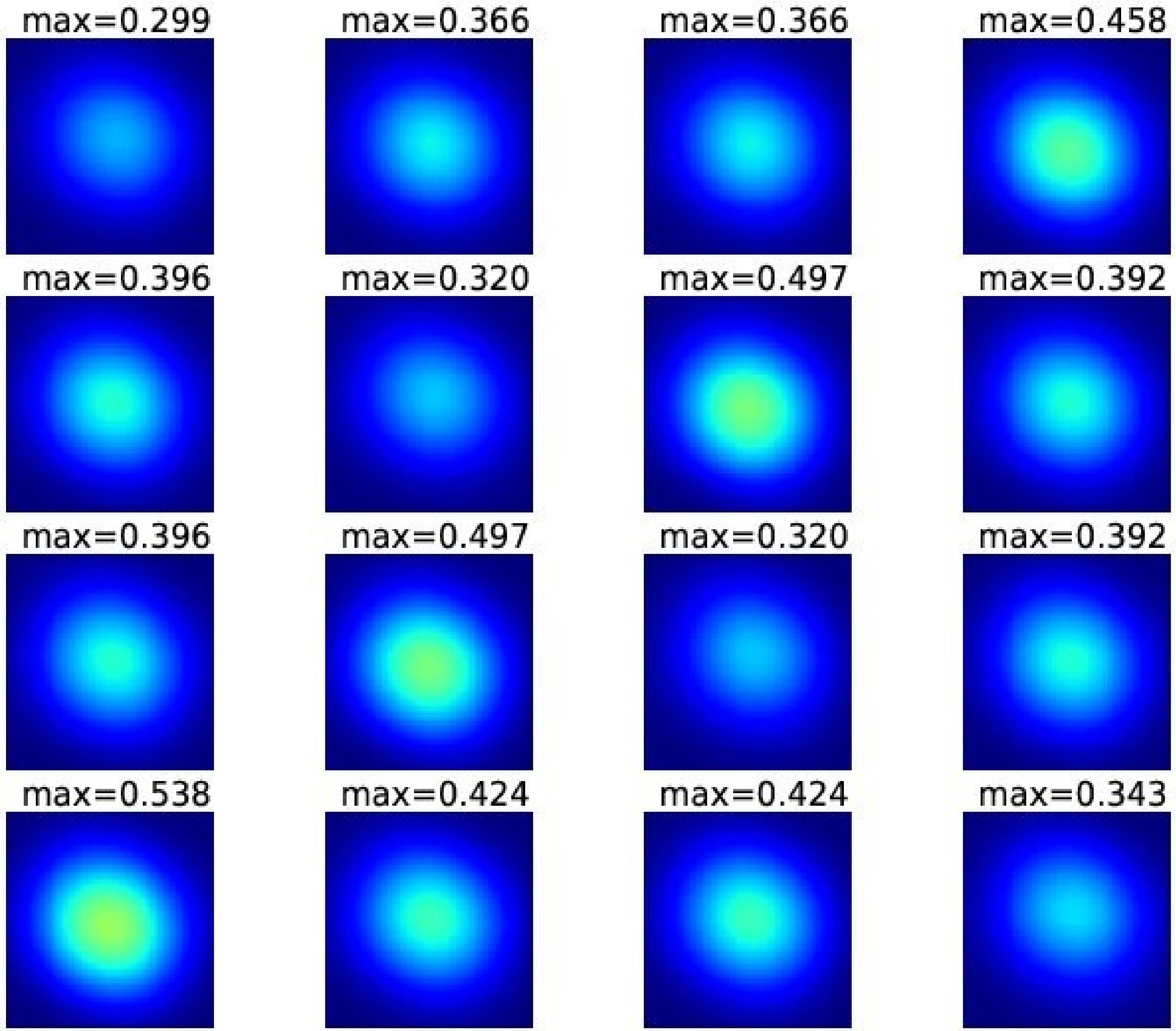
DeGridding



XX, XY, YX, YY



LOFAR Beam: The Mueller Matrix varying over the image plane

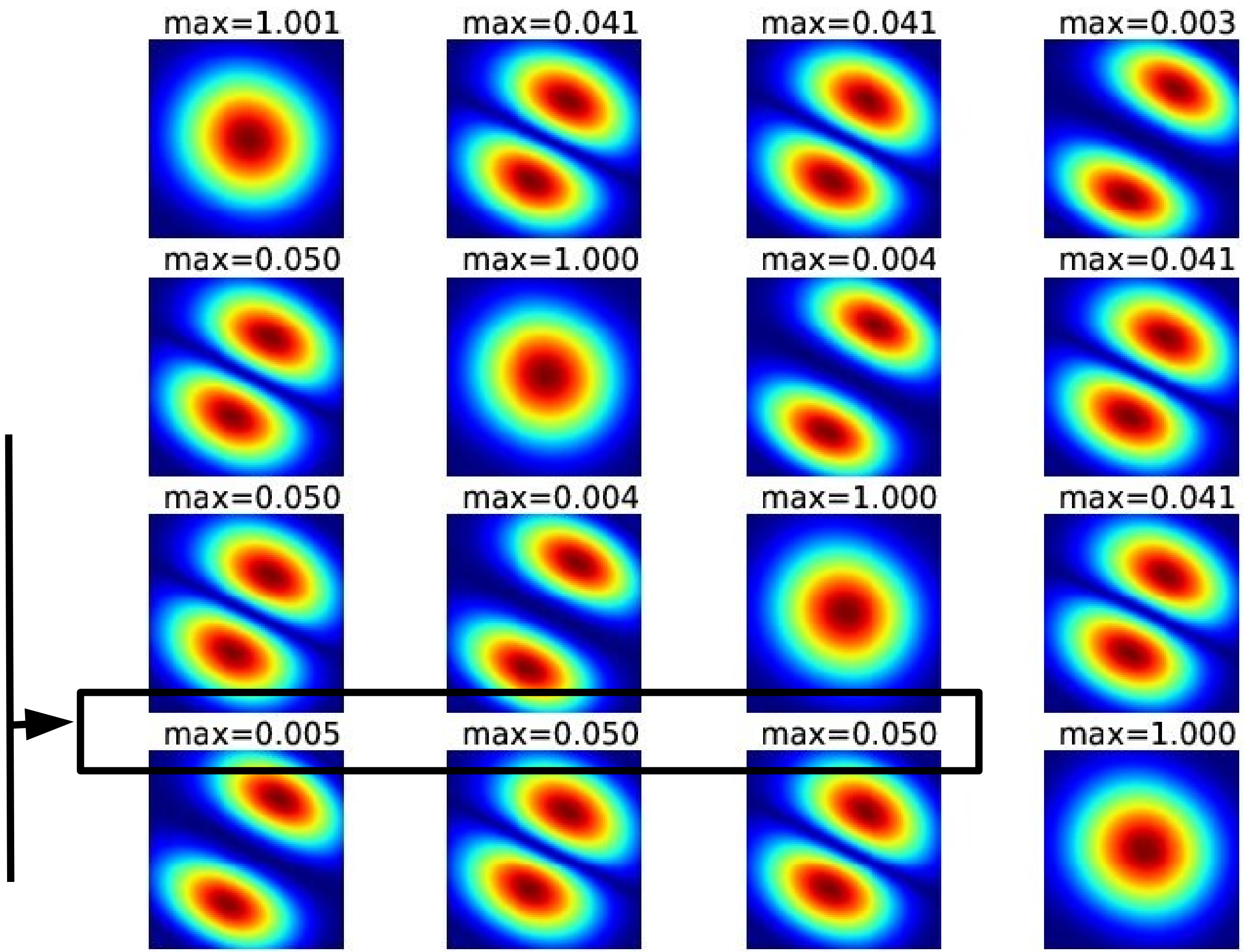


One pair of antennae, one time and frequency value

LOFAR Beam: The Mueller Matrix varying over the image plane

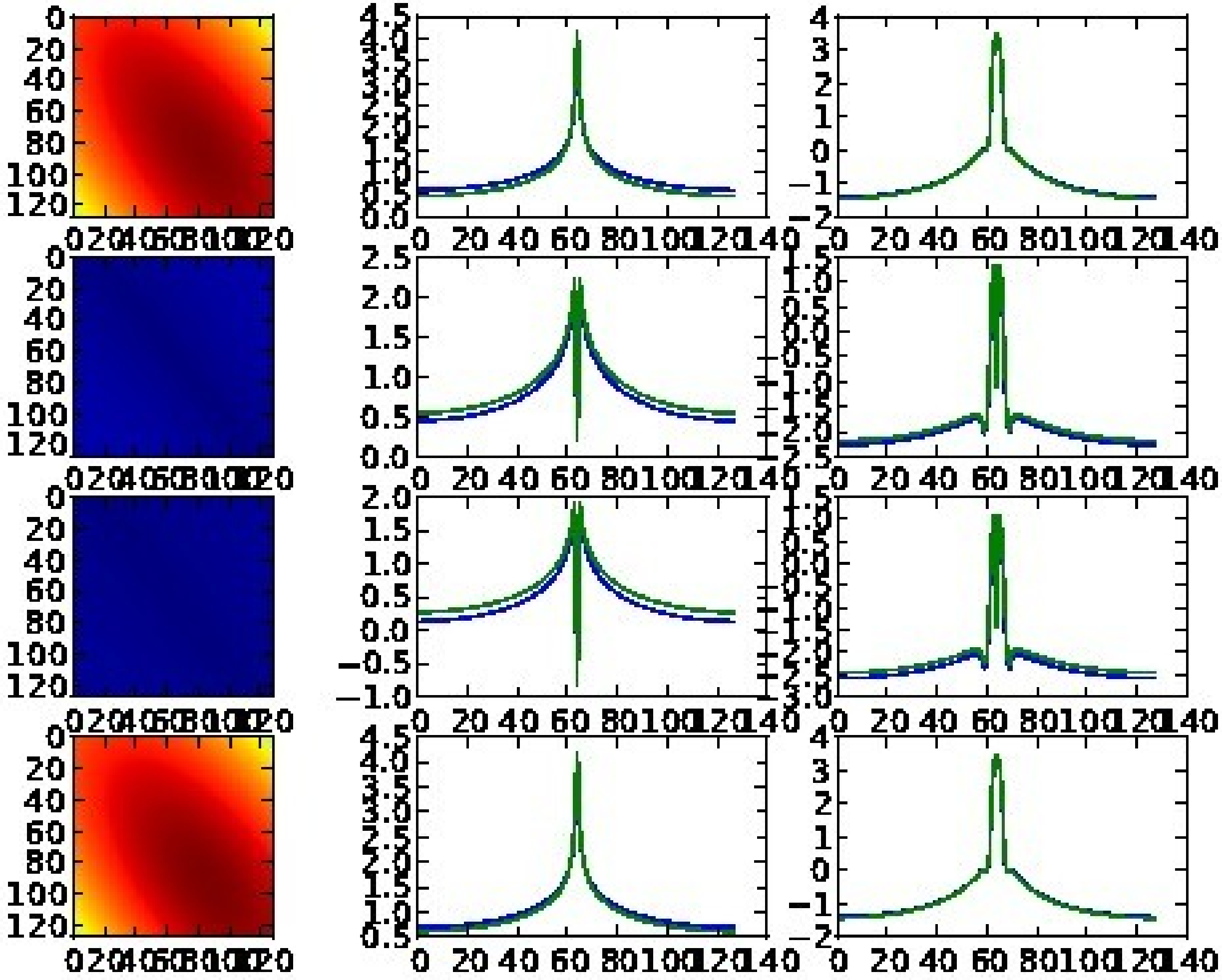
Beam bormalized by Beam Jones matrix at the center of the field (we correct the visibilities accordingly before the imaging)

Off axis small but perhaps not negligible for the degridding ?



!!! Color bar is adapted to the image here otherwise you don't see anything!!!

… When Direction Dependent Effects (DDE) become a problem : Beam



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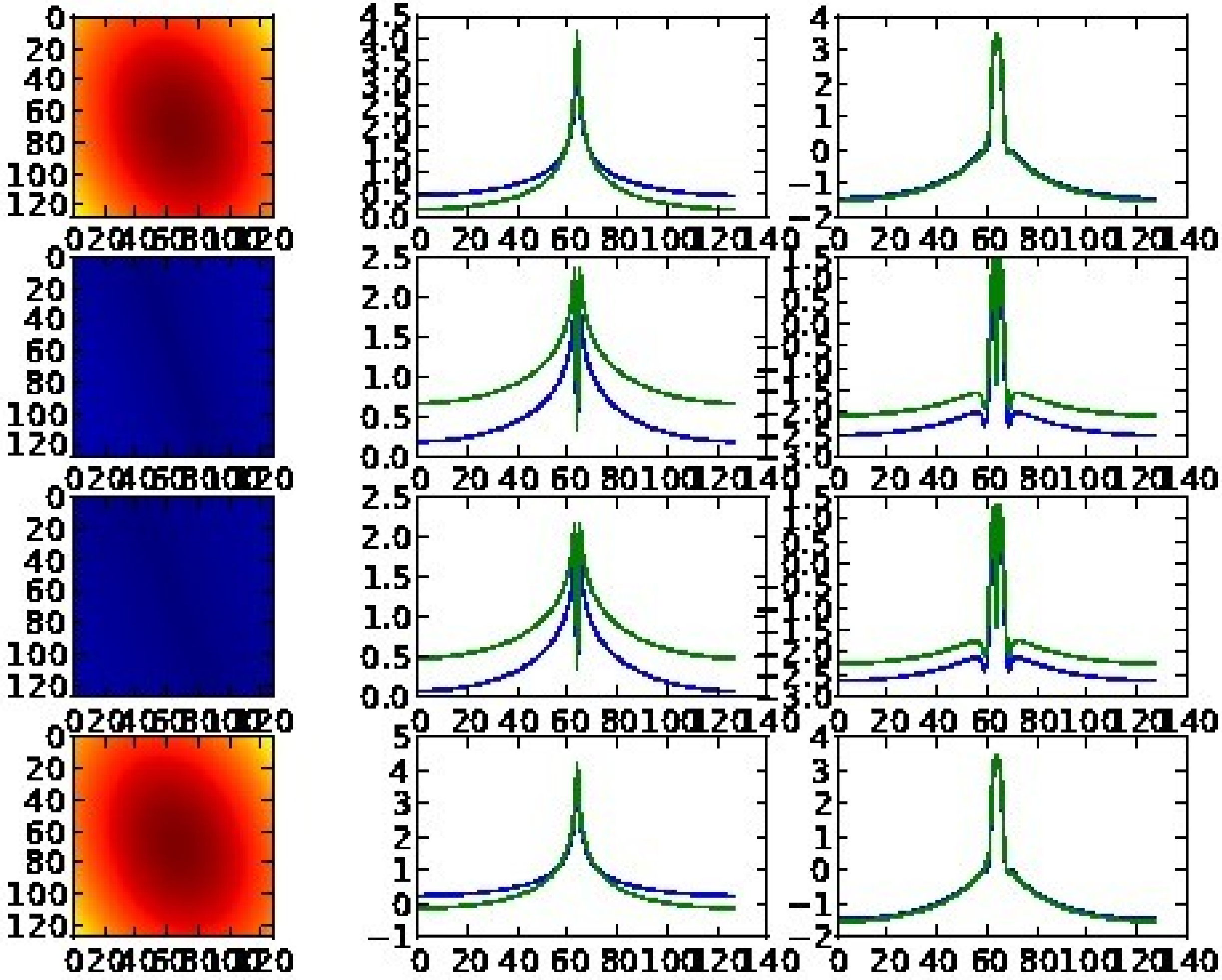

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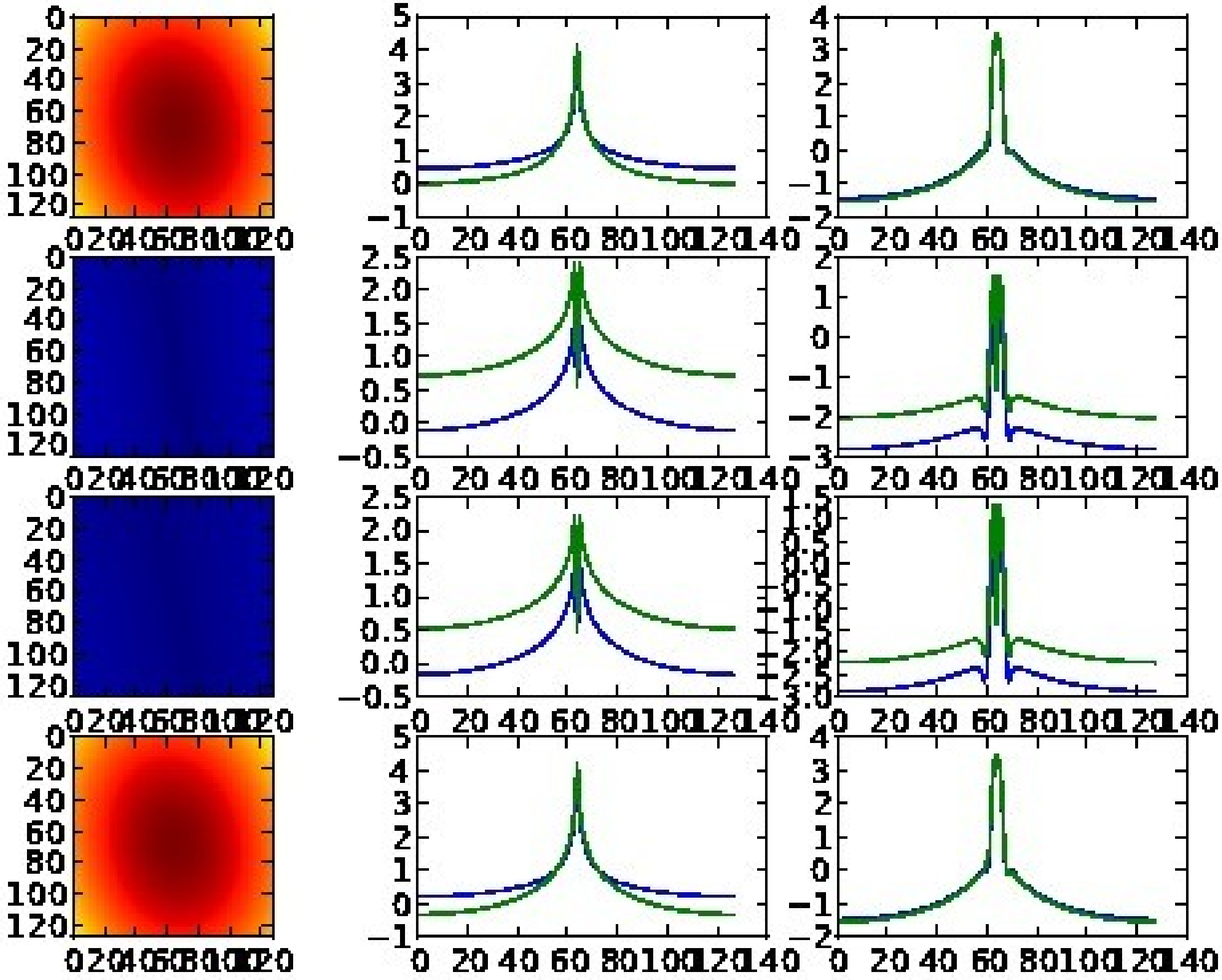

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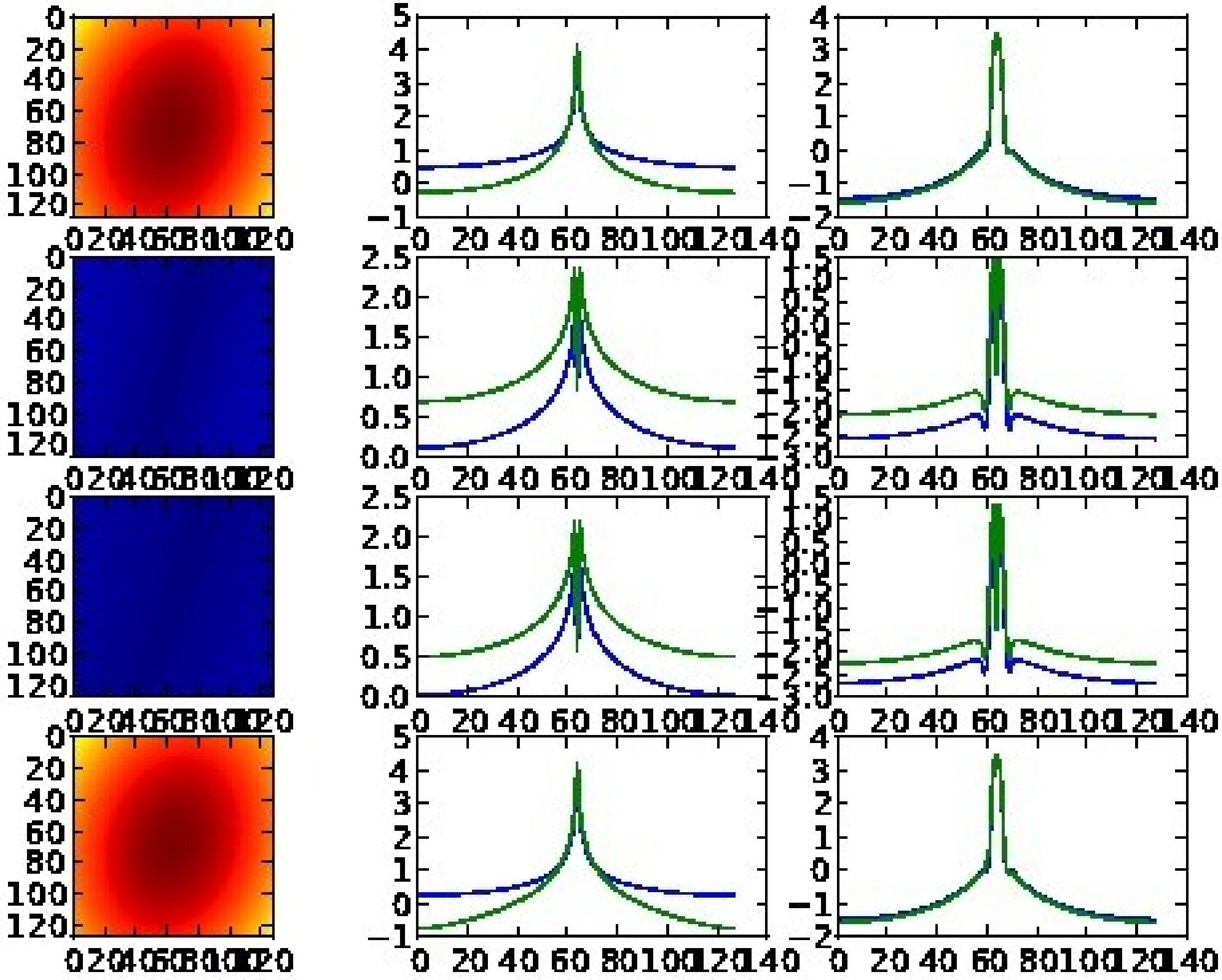

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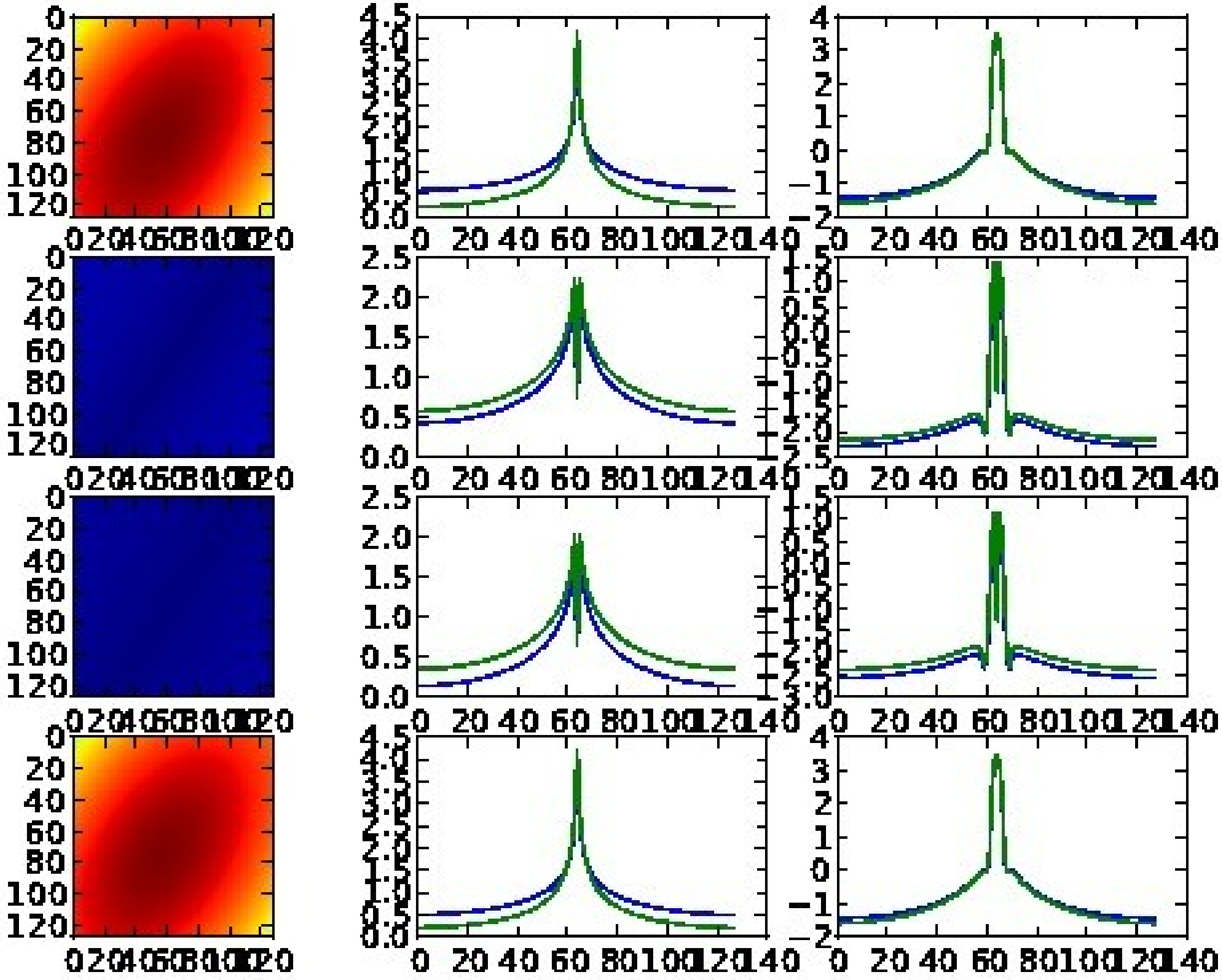

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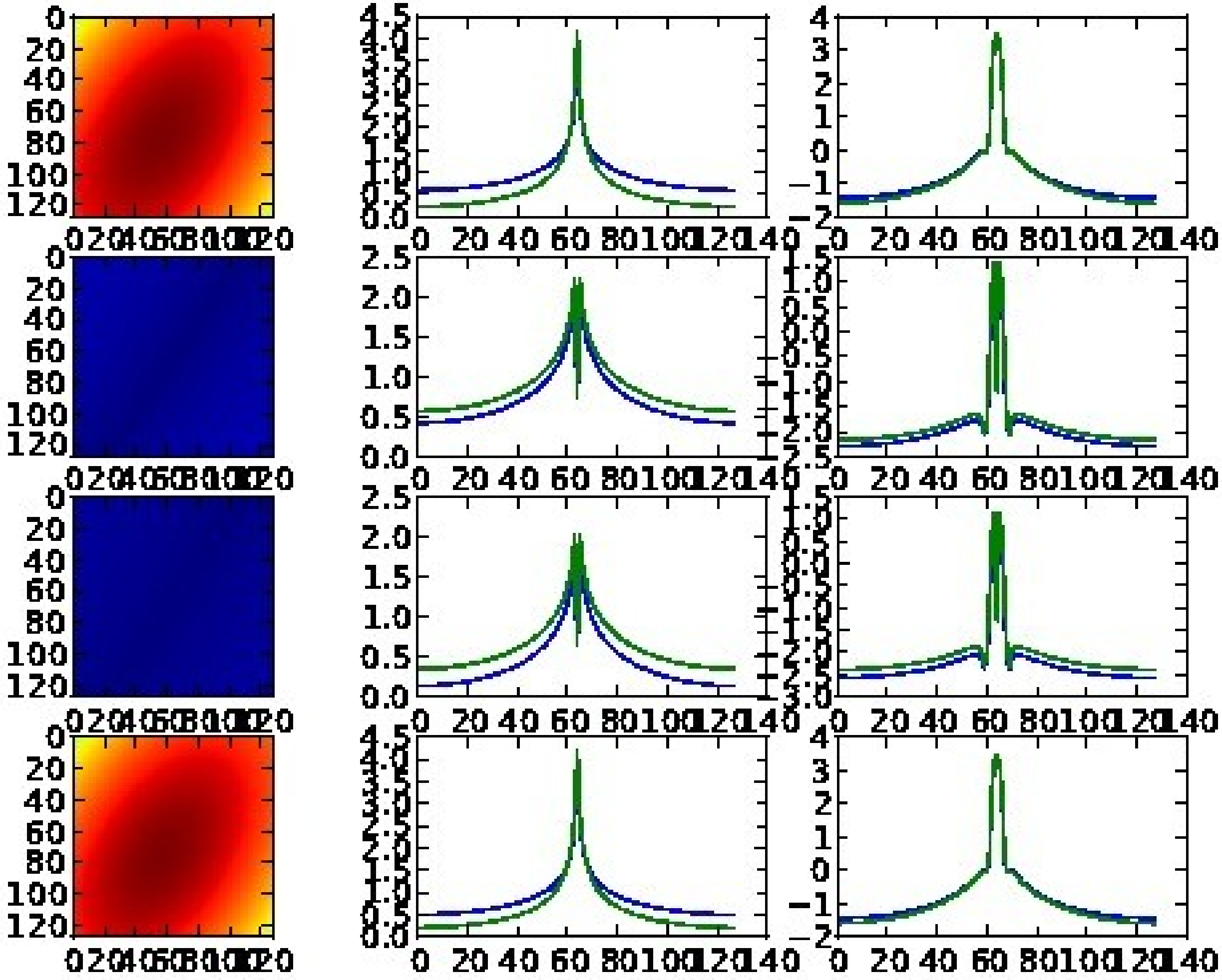

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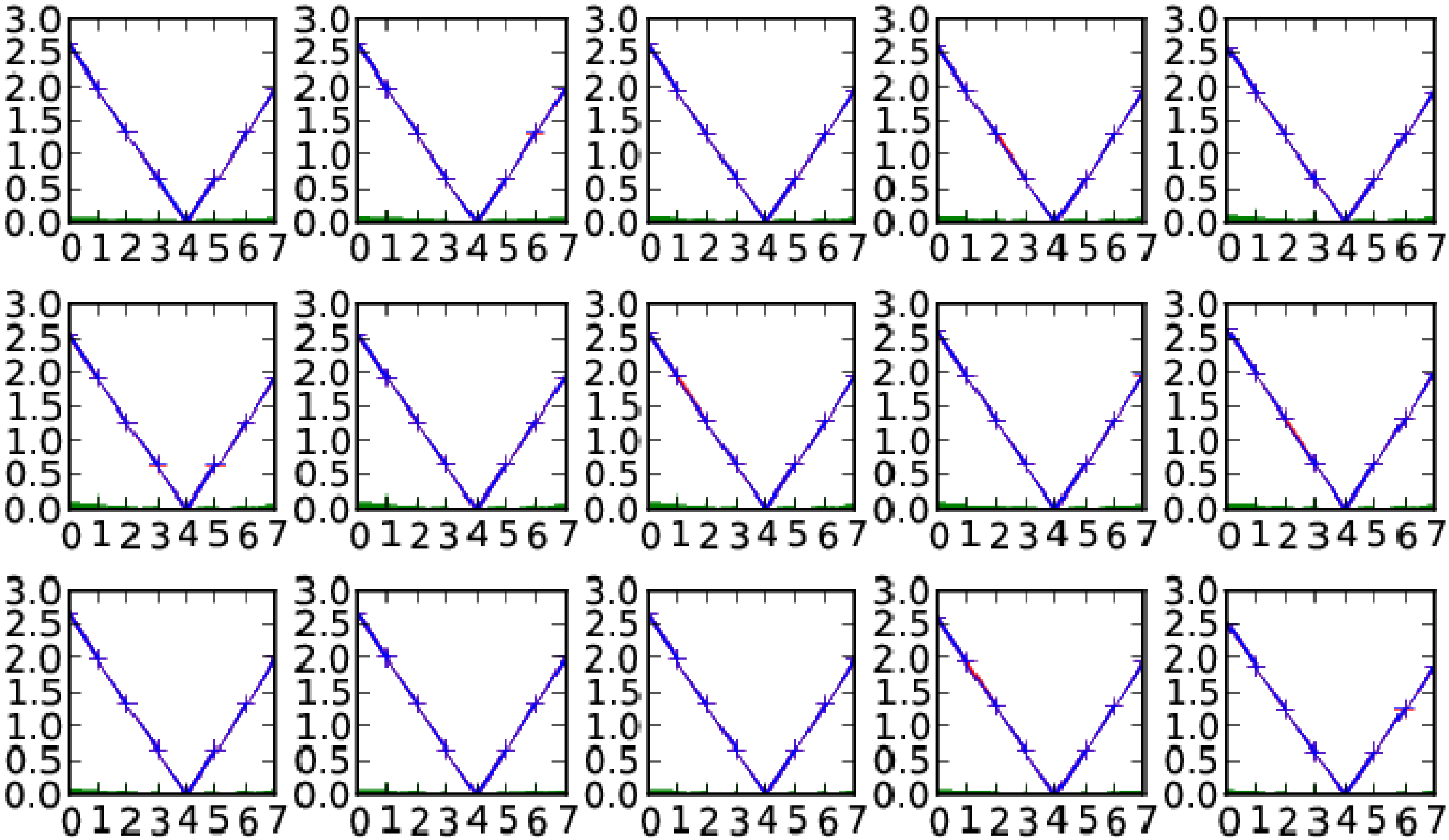
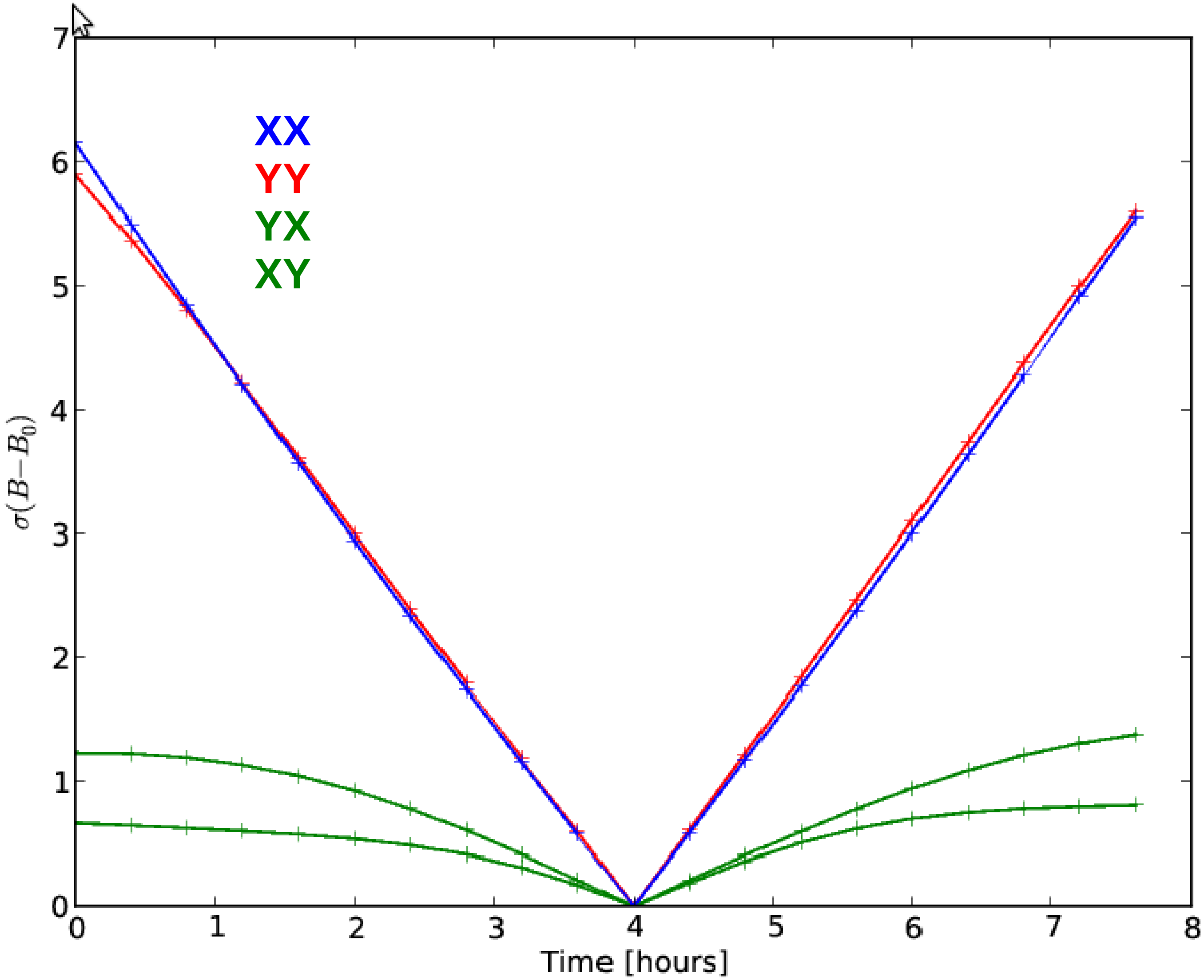


Beam variability across a subband during a 6 hours observation (ordinate in per thousand)

… When Direction Dependent Effects (DDE) become a problem : Beam



JAWS: the practice

How many convolution function?
- One convolution every 10 minutes
- 8 hour oberving run
- 45 antenna: 990 baselines
- 16 Mueller elements
- 1 complex number pert pixel
- Average size 30*30 pixel

= 1216 Tbytes

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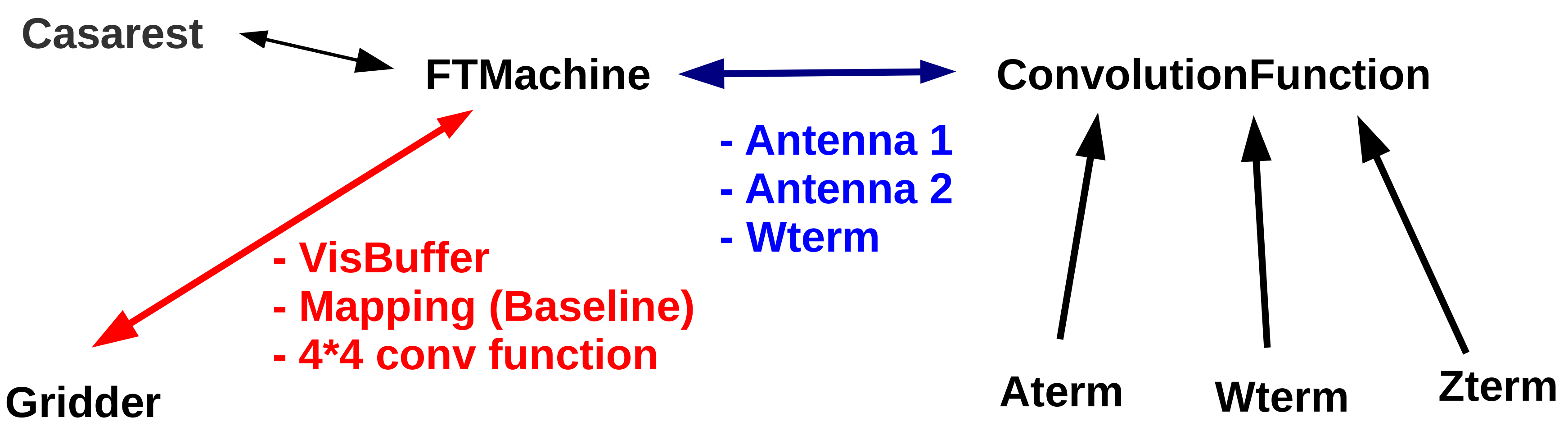
→ We compute the convolution functions on the fly

- We compute and store the Aterm and Wterm at the minimum resolution



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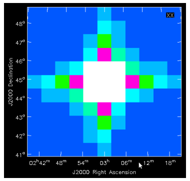
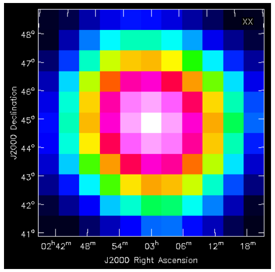
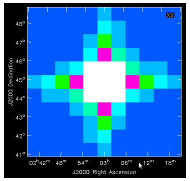
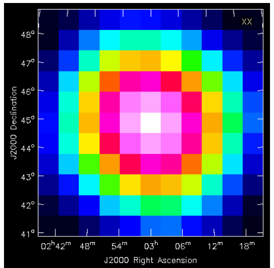
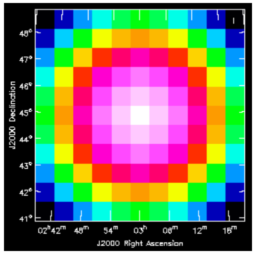






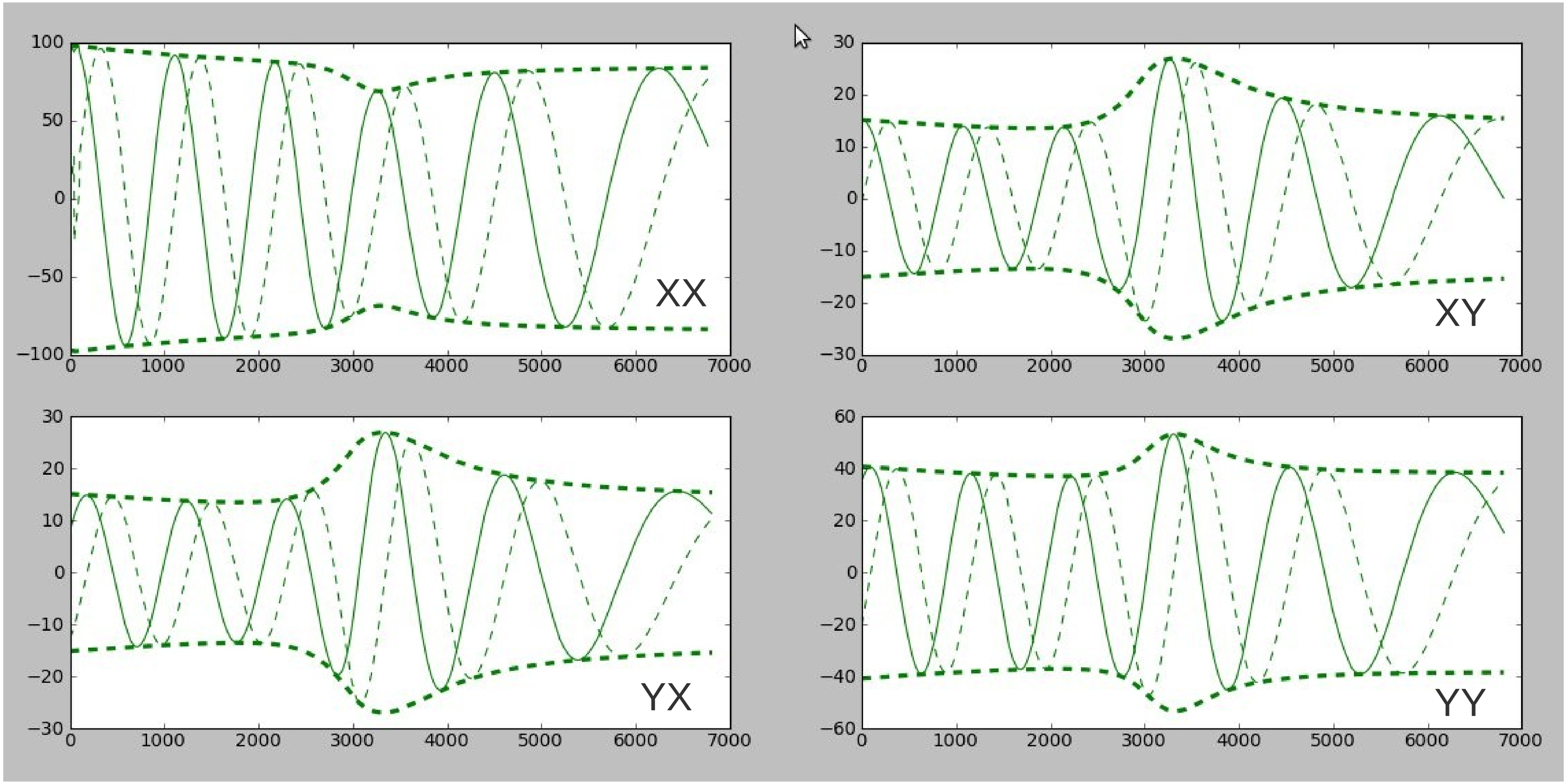




Mathematical framework-works

One off-axis source IQUV=(100, 40, 20 10)

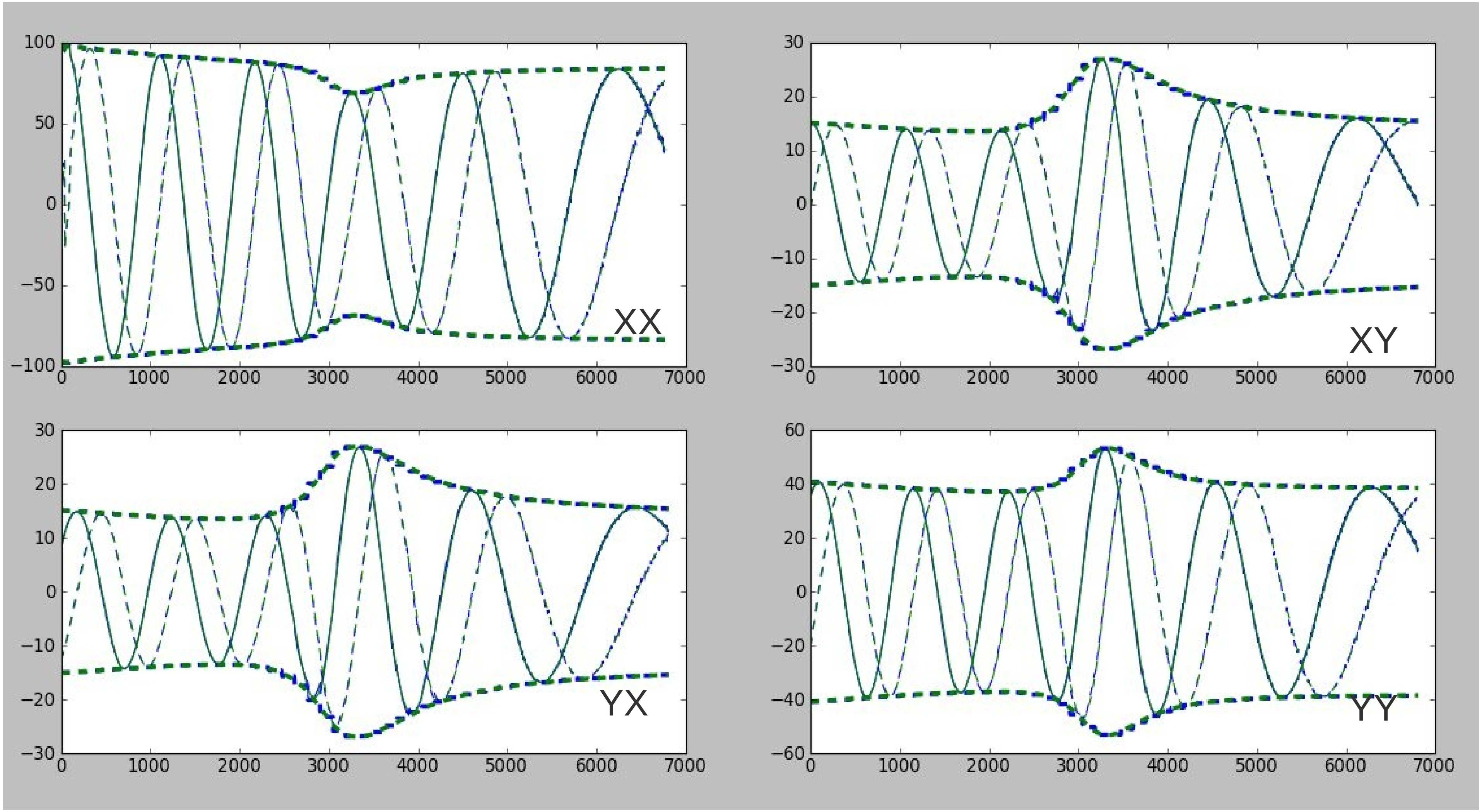
BBS predict (DFT)



Mathematical framework-works

BBS predict (DFT)

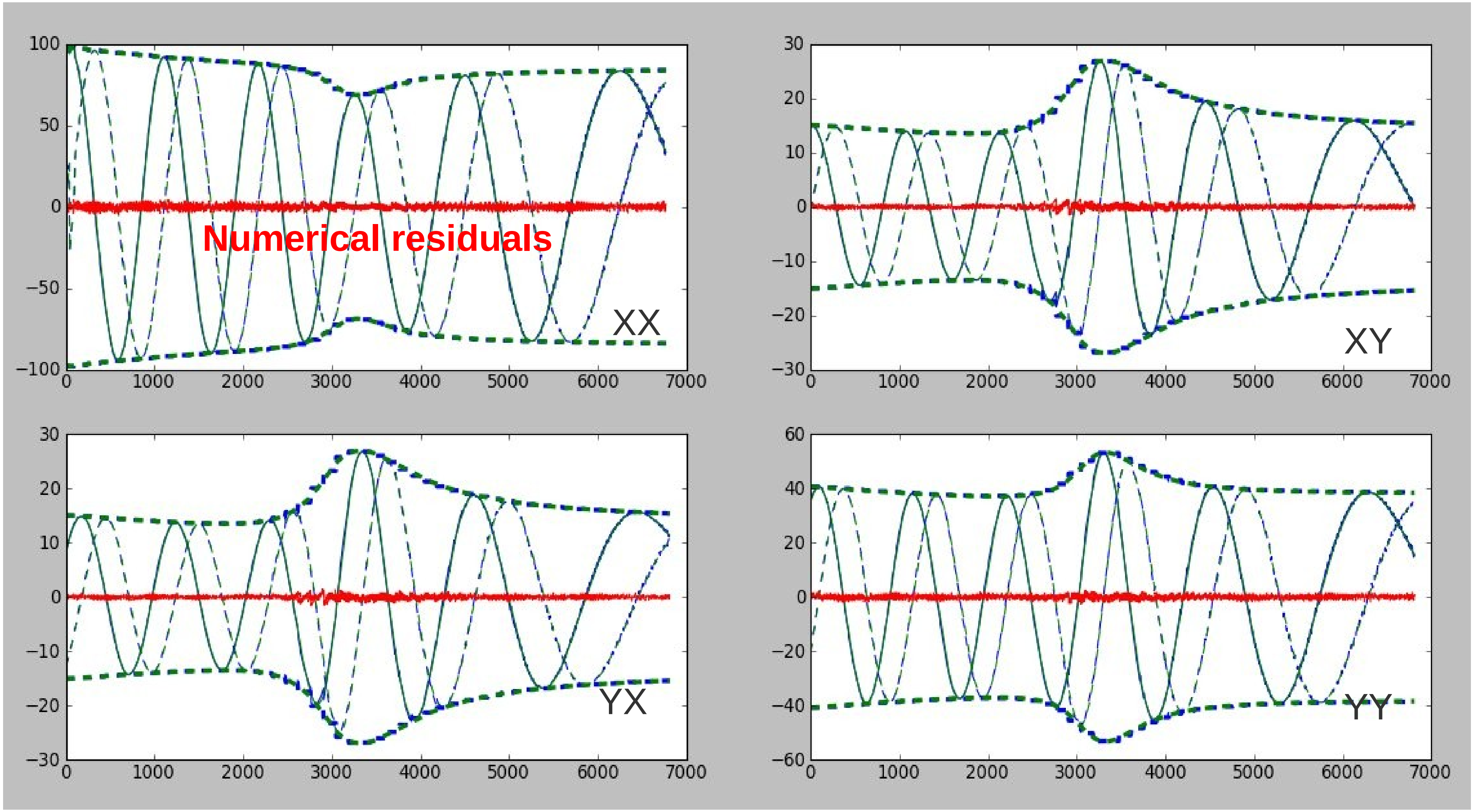
AW degridding (clean component put by hand)



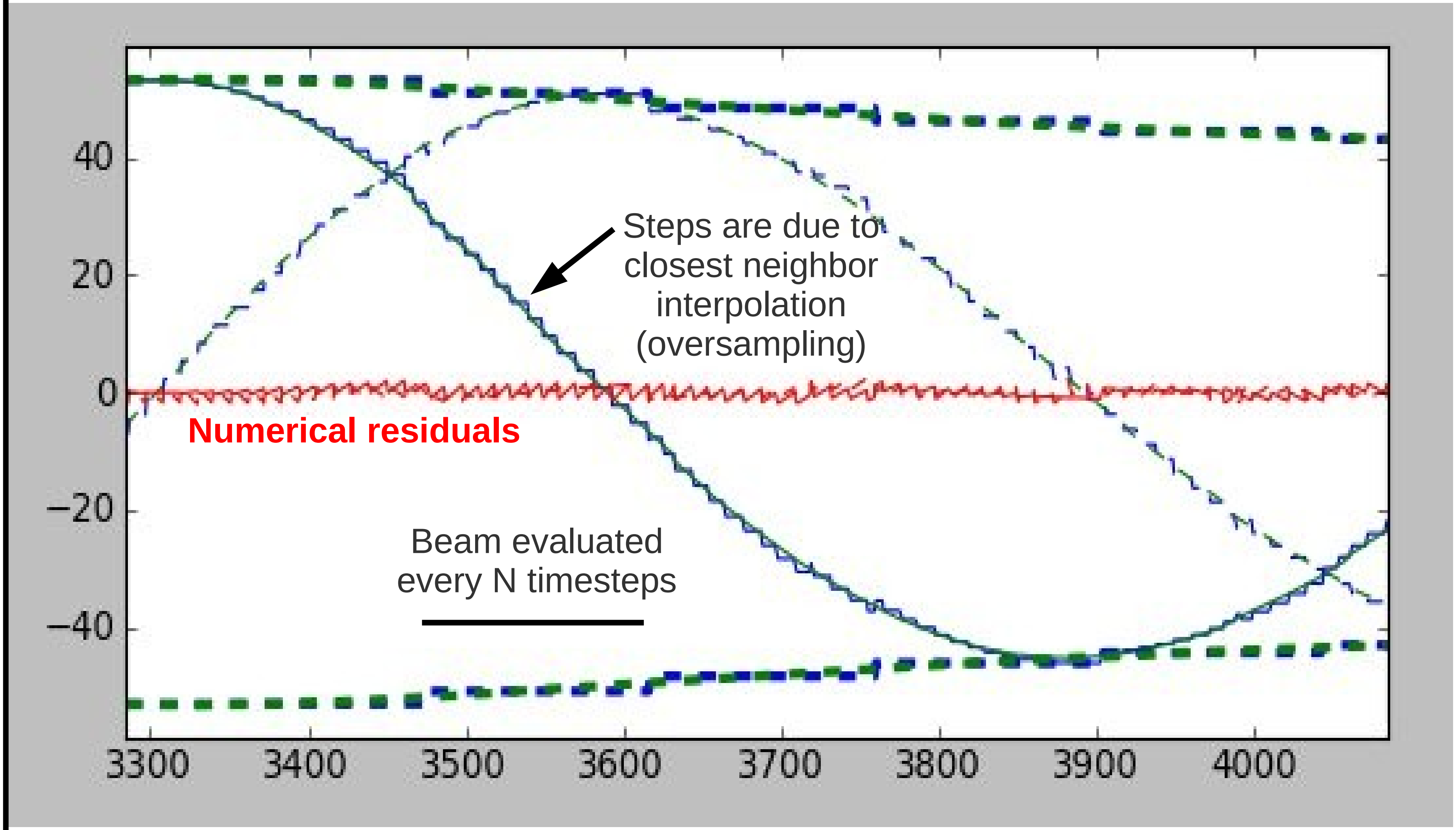
Mathematical framework-works

BBS predict (DFT)

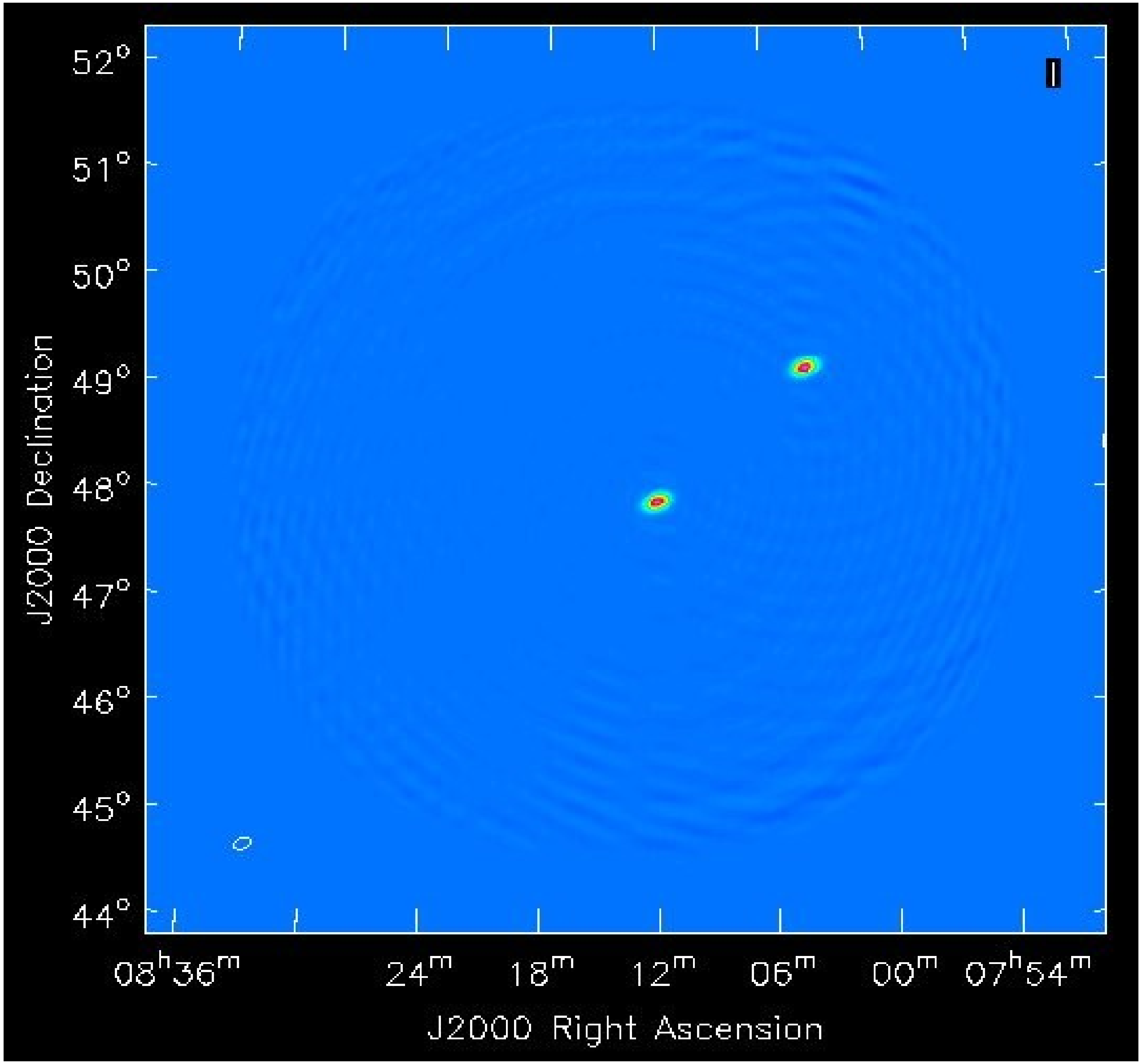
AW degridding (clean component put by hand)



Mathematical framework-works

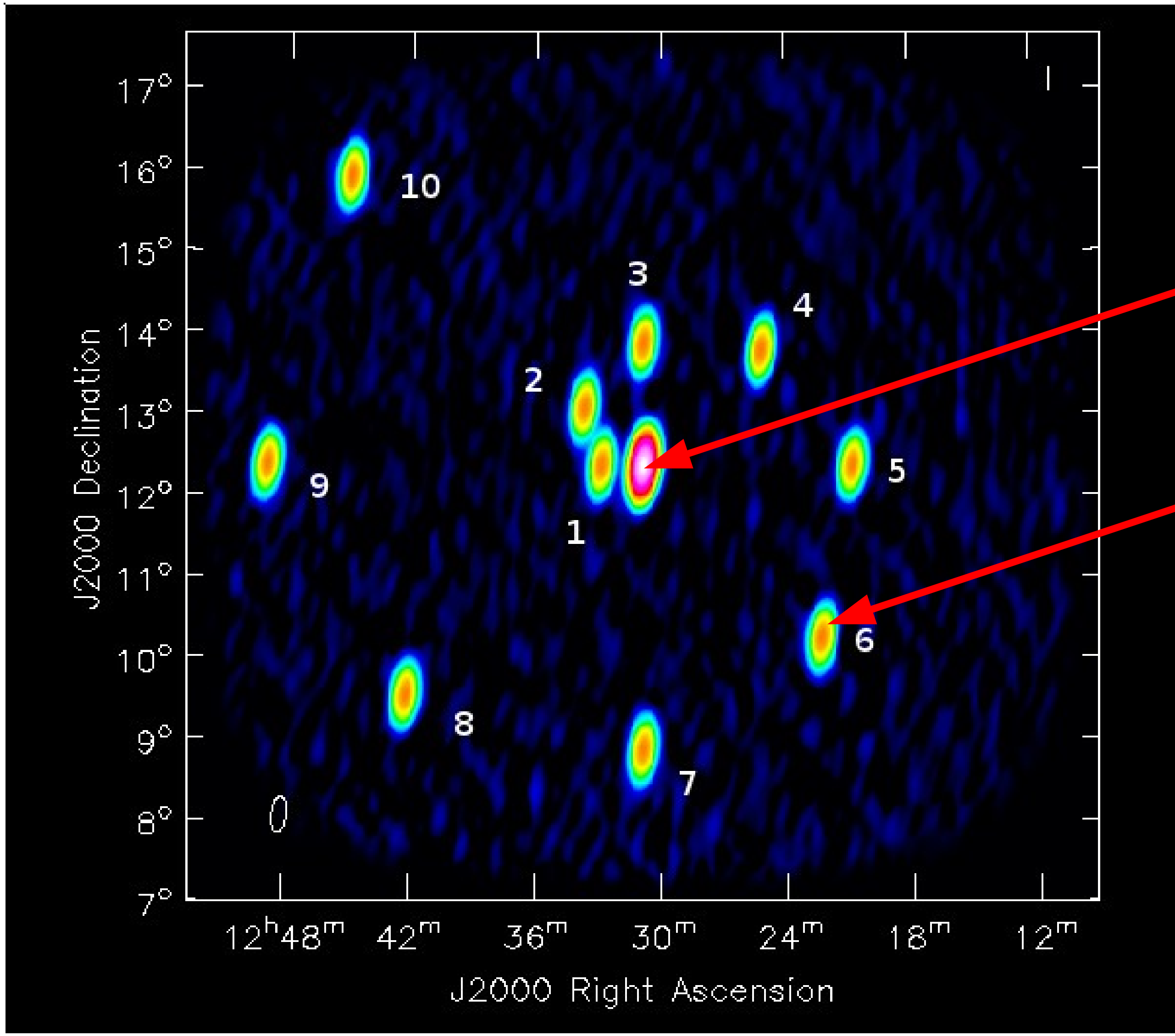


Mathematical framework-works



Recovered IQUV=(100, 40, 20 10) fluxes to better than 1%

Mathematical framework-works



10 Jy Source

1Jy source

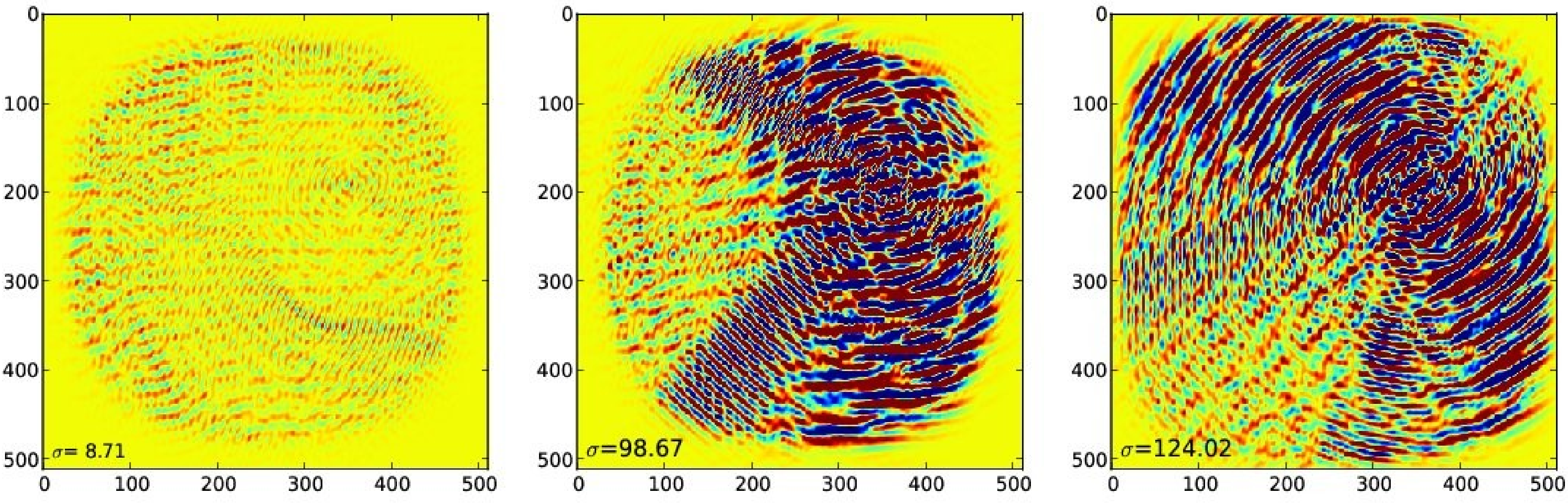
Recovered flux better than 1%

Francesco Da Gasperin

Mathematical framework-works

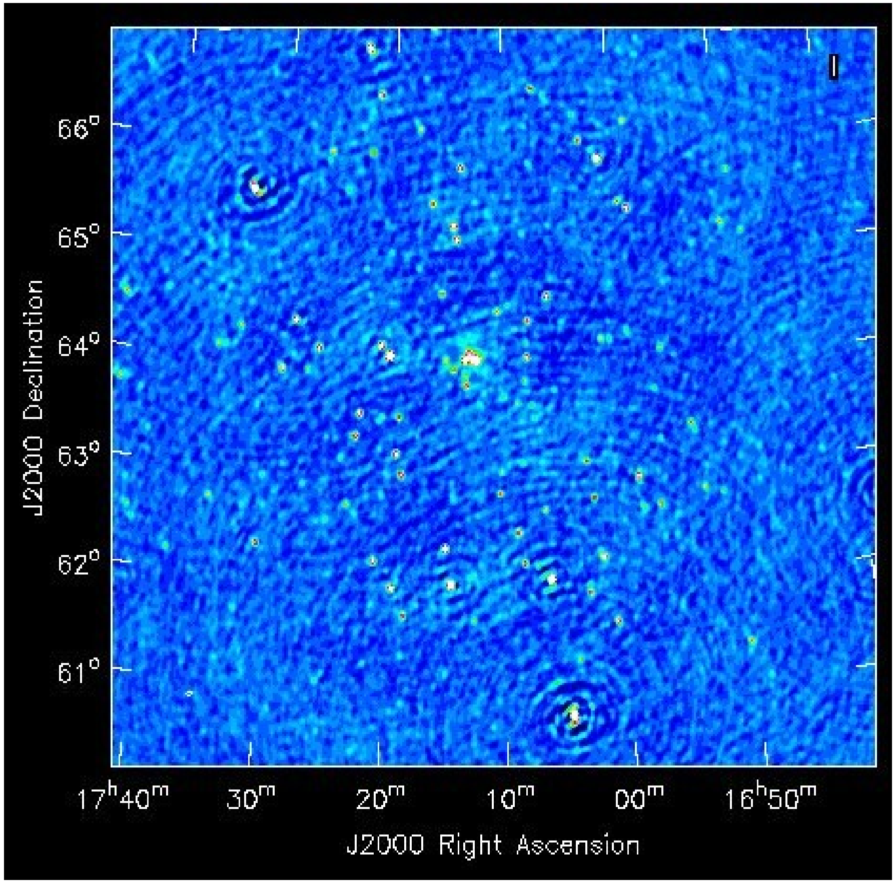
Same simulated dataset with one off-axis source and the beam (IQUV=100,40,20,10)

Residual images, Stokes I

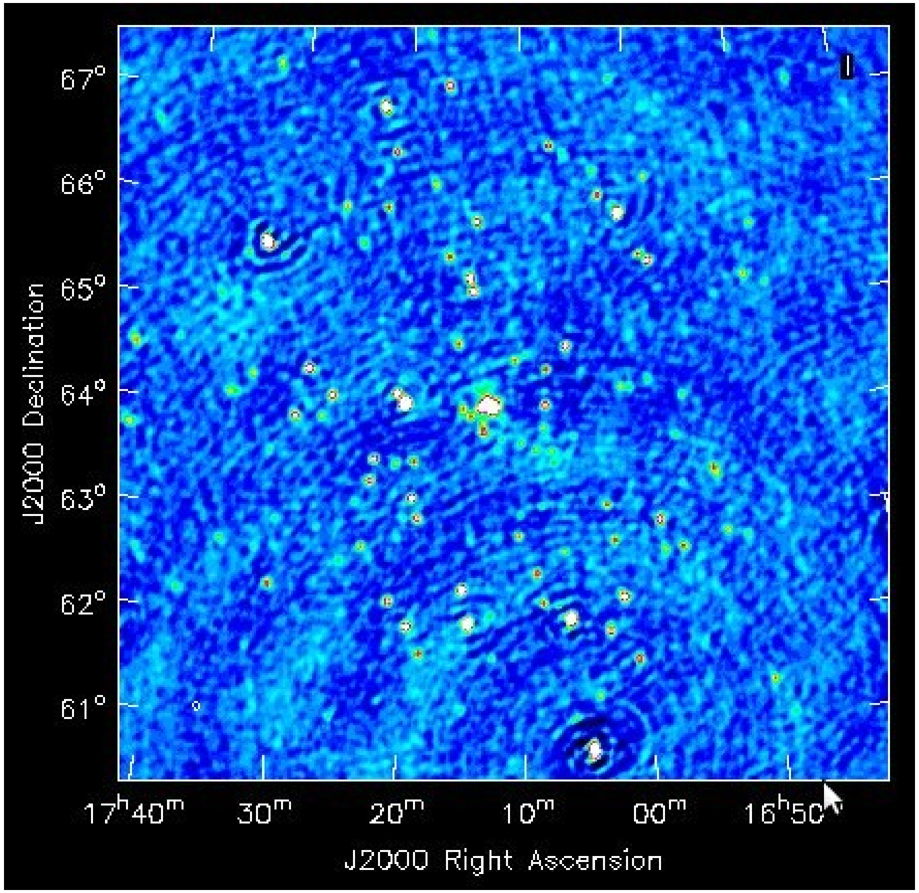


AW projection | **AW, only diag terms** | **W projection only**

On real data (A2255)



Casa



JAWS

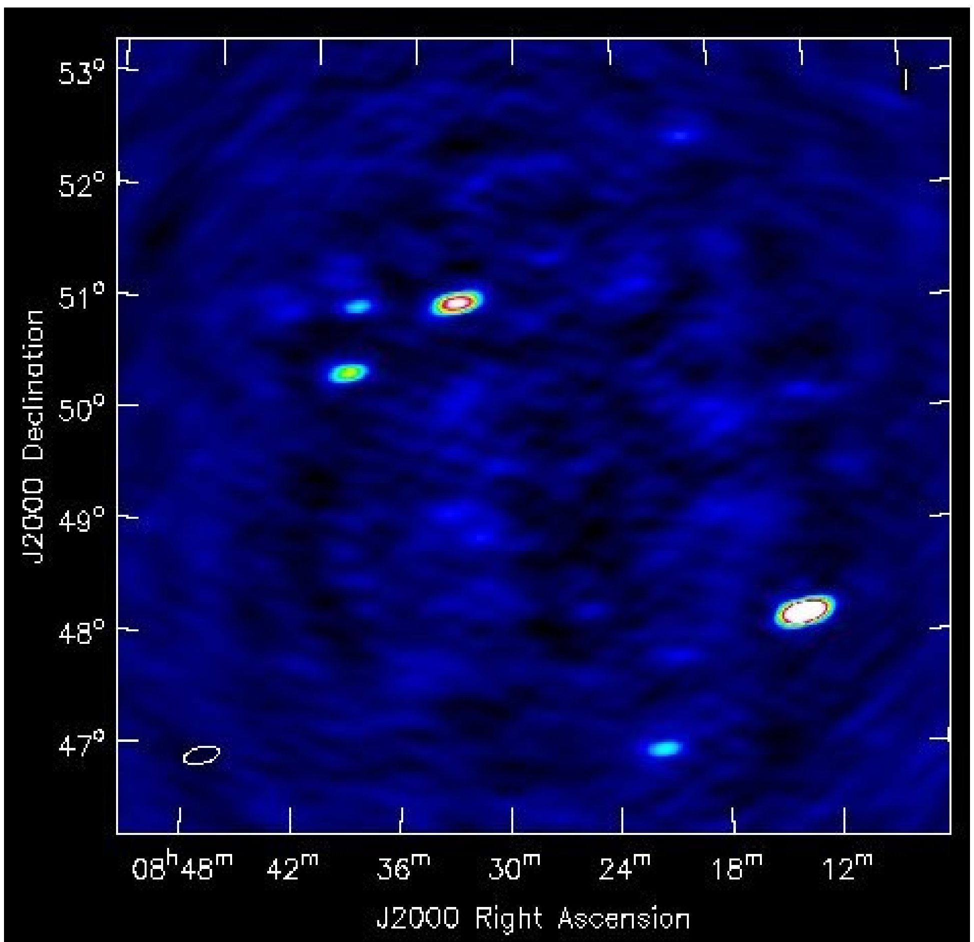
Roberto Pizzo

On real data (3C196)

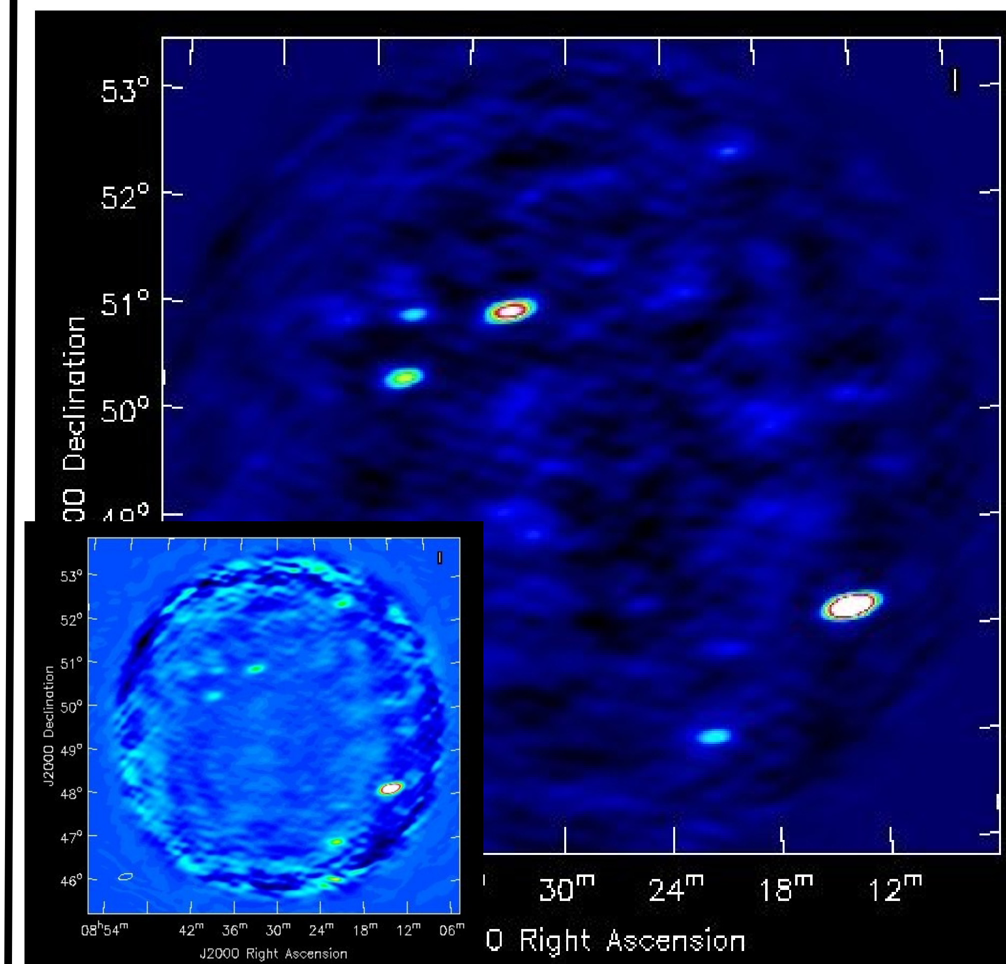
3C196 off axis ~150MHz

- Calibrated using 3C196+2 sources sources
- AW visibility estimates for those. Little difference?

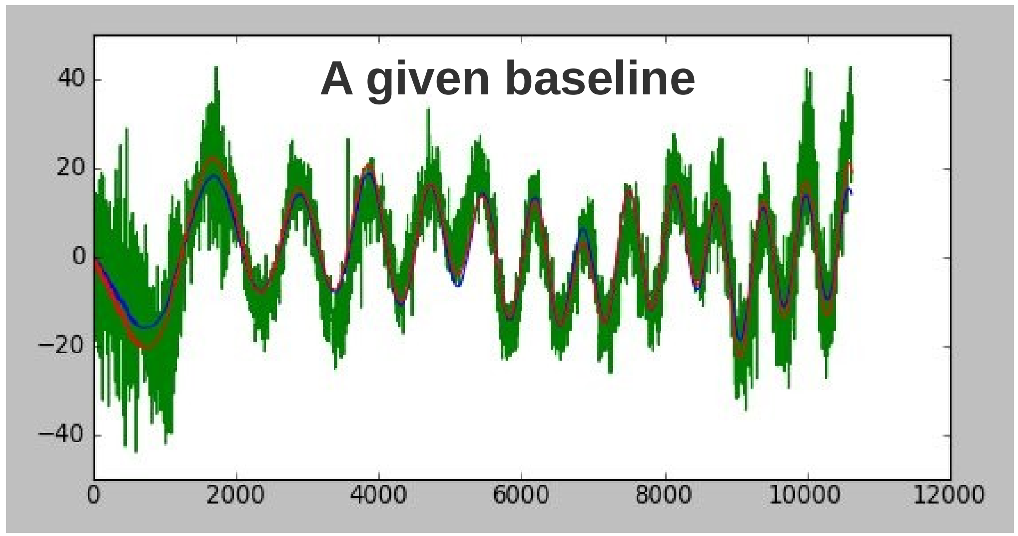
NOT Taking the beam into account



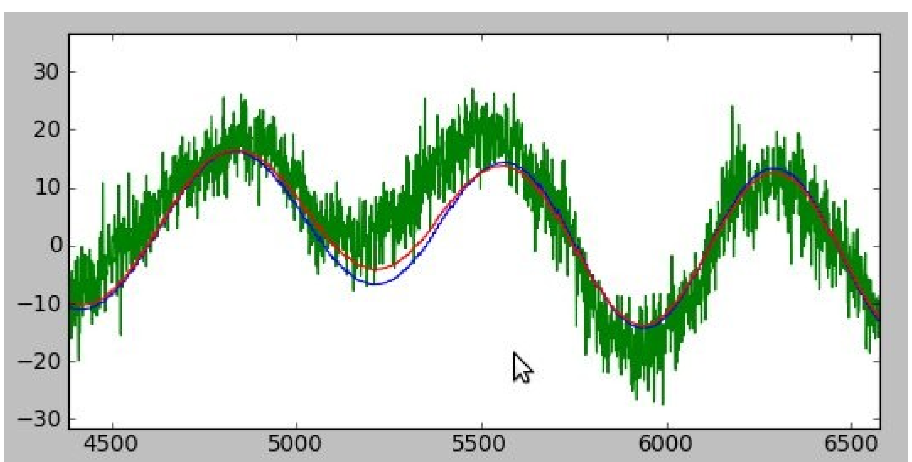
Taking the beam into account

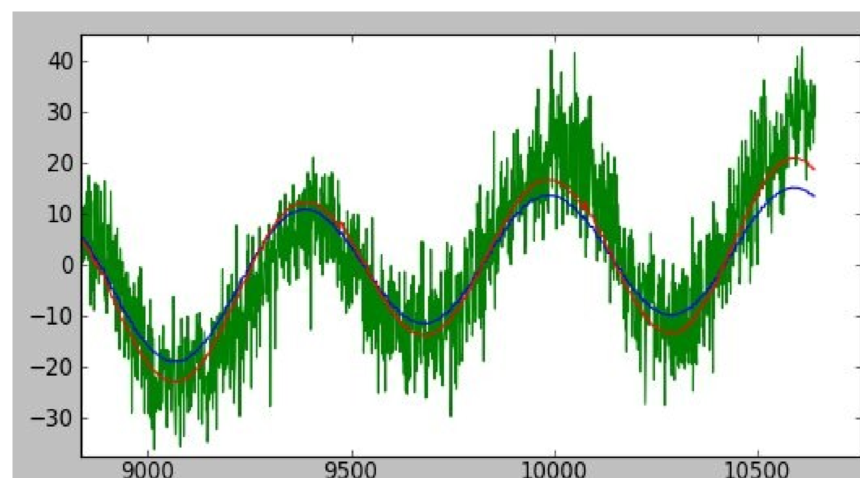


On real data (3C196)



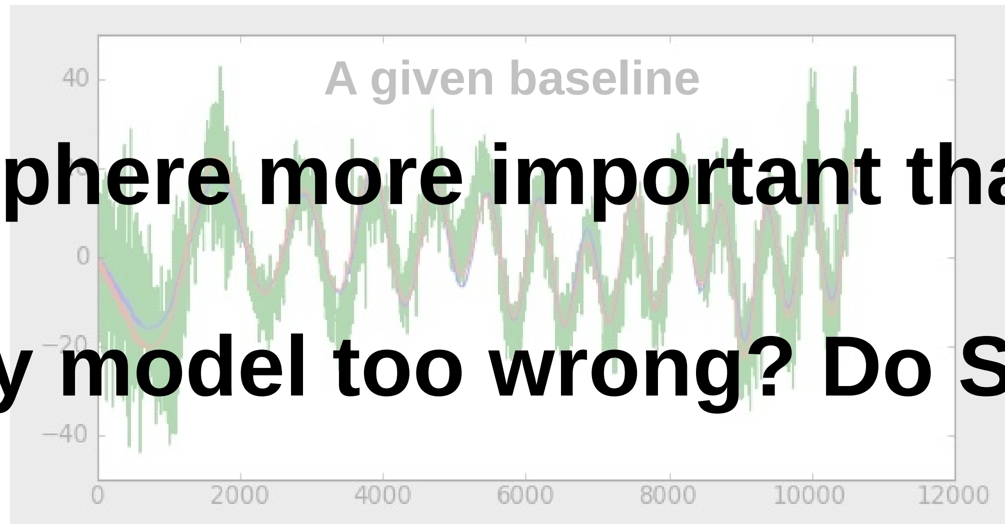
Beam taken into account





No Beam taken into account

On real data (3C196)



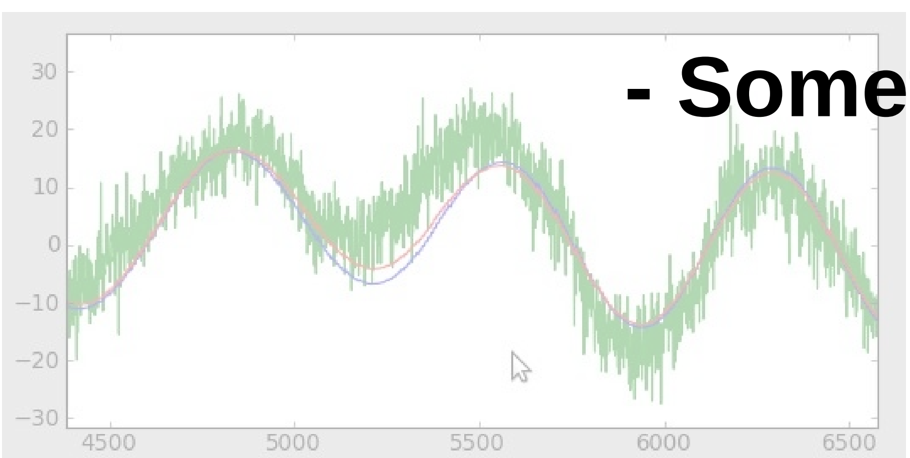
- Ionosphere more important than beam ?

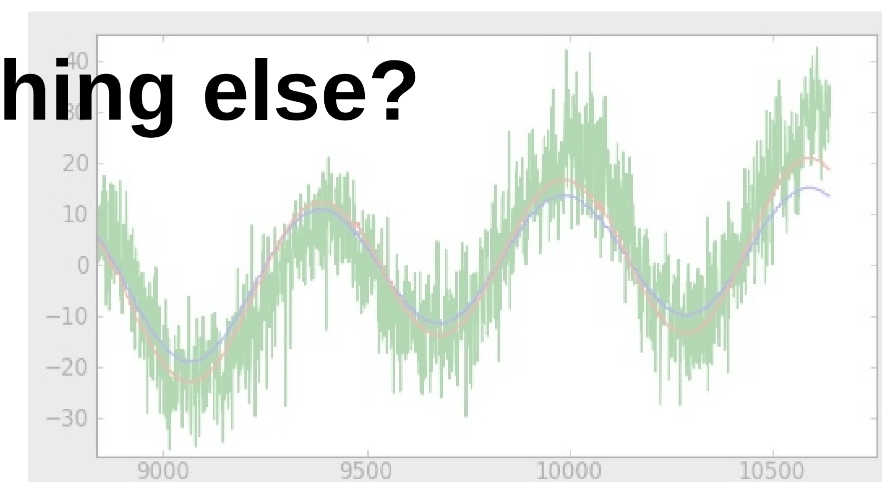
- Sky model too wrong? Do SelfCal?

- Beam model too wrong?

- Something else?

Beam taken into account

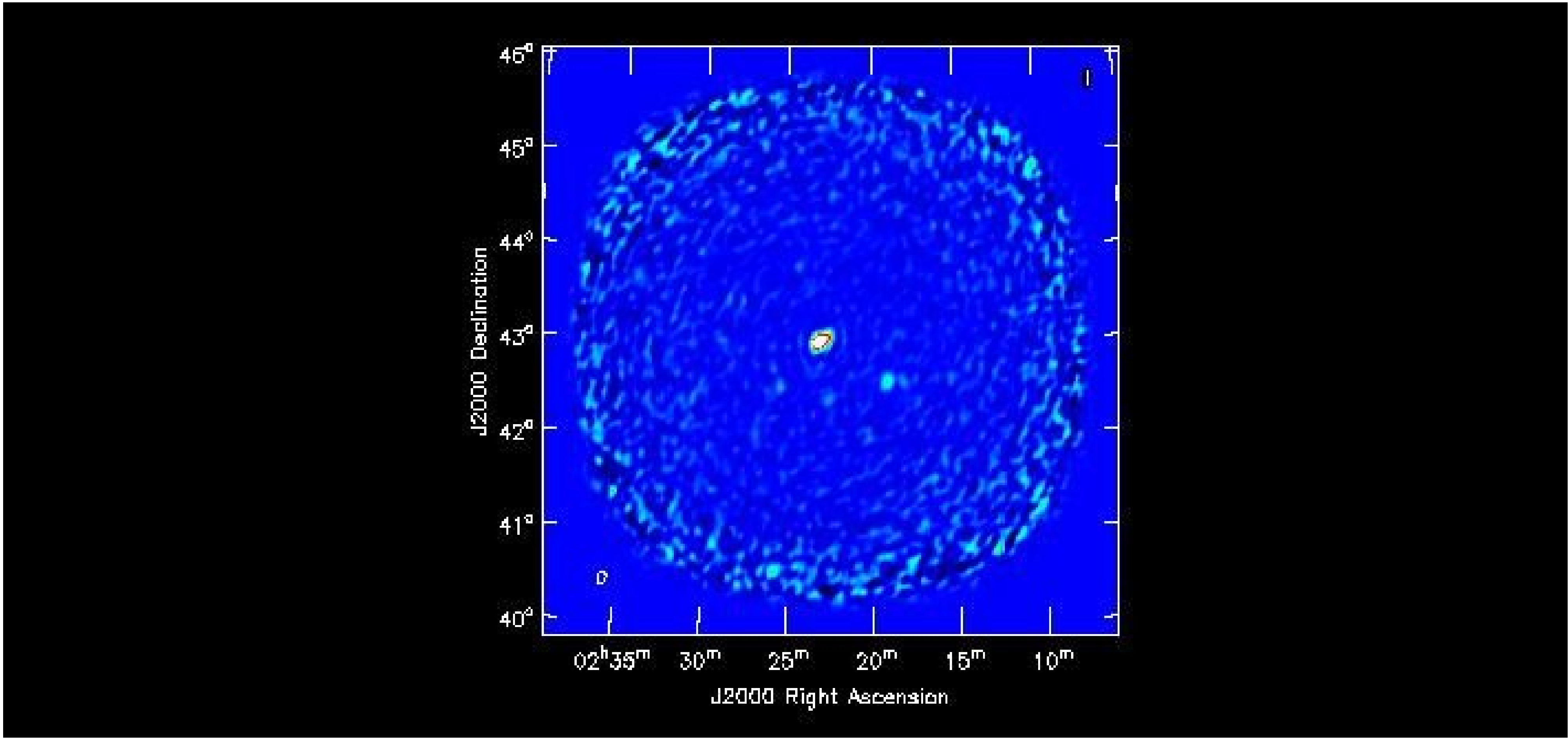




No Beam taken into account

JAWS: 3C66

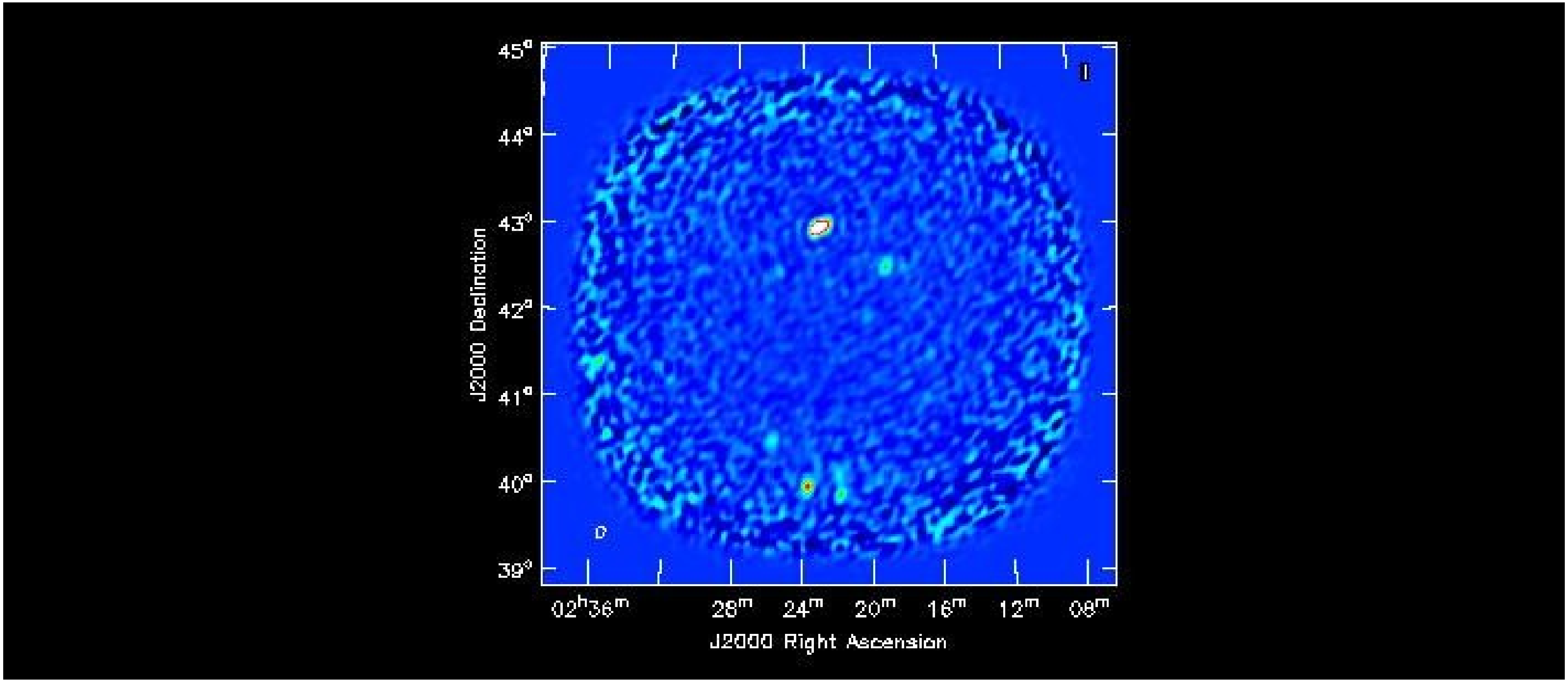
Flux = 63 Jy



Aleksandar Sulevski

JAWS: 3C66

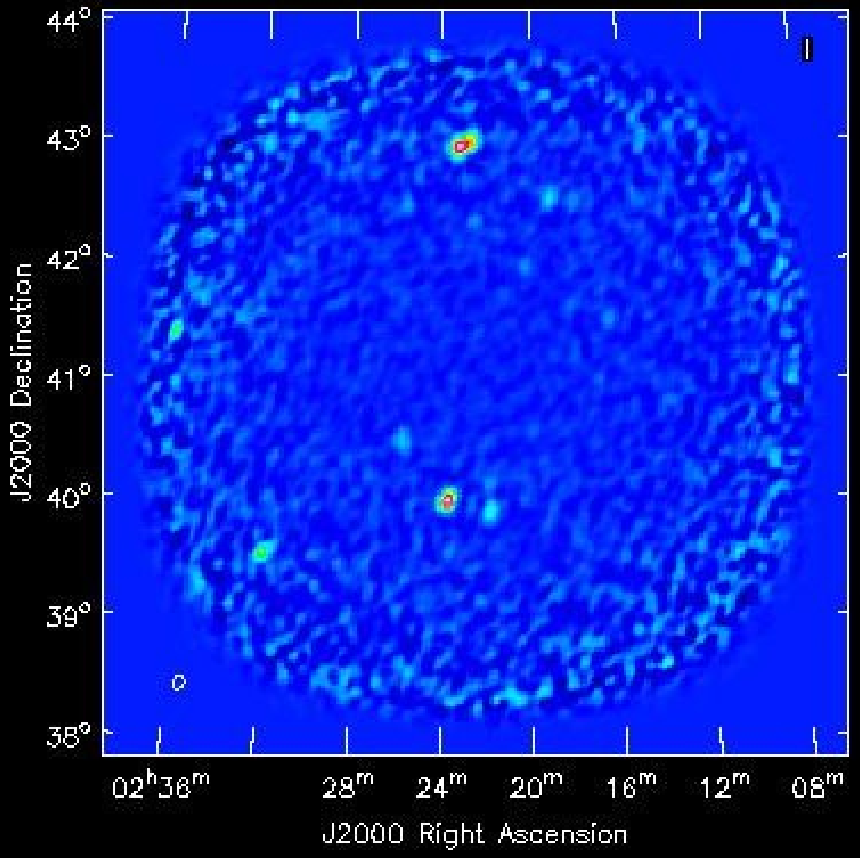
Flux = 65 Jy



Aleksandar Sulevski

JAWS: 3C66

Flux = 51 Jy



Aleksandar Sulevski

Conclusion and Next steps

Conclusion:

- Full Polarisation Framework based on Measurement Equation is working
- Very flexible
- Effect will be seen at higher dynamical range?

Next steps:

- Optimise code
- Study convergence major cycle & SelfCal
- Ionosphere phase screen model
- Full Multi-Frequency cleaning
- Faraday Rotation?

… Start doing serious survey science