Calculating Beam Pattern Inaccuracies and Their Implications

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Context

SKA: ambition to achieve >70 dB DR

Possible limiting factors

- PAF compound beam / AA station beam accuracy
- PAF compound beam / AA station beam stability
- ionospheric modeling accuracy

Pivotal issues

- How do we specify DDE modeling accuracy?
- What accuracy is required?

Answers needed for rigorous system design!

Example: Aperture Tile in Focus
Van Cappellen and Bakker, PAST, 2010

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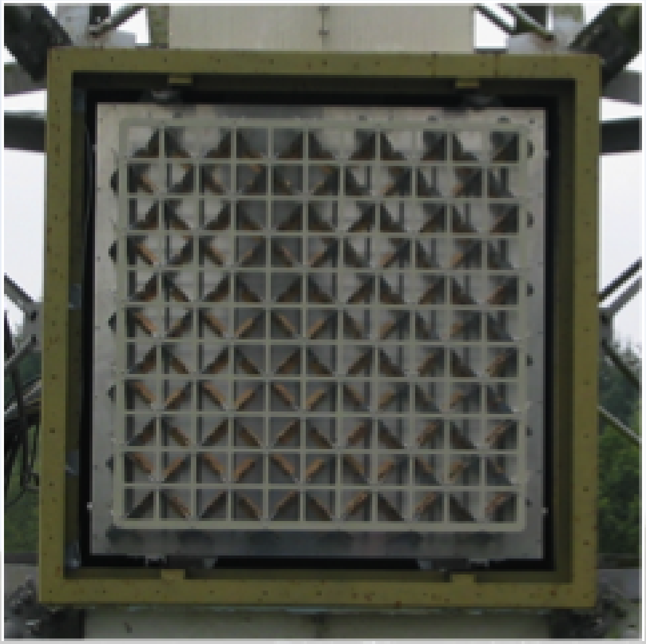
- PAF for WSRT, increases survey speed 25x
- **key specs**

Frequency range	1000 – 1750 MHz
Instantaneous bandwidth	300 MHz
System temperature	< 55 K
Aperture efficiency	75%
Polarization	dual linear
Simultaneous beams	37 dual pol
Field of view	8 deg^2
Reflectors	12 x 25 m



- **Beam spec: 1% error at HPBW rel. to main beam**

Error propagation in beamforming

Stefan J. Wijnholds, CalIm, July 2011

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- Beamformer equation: $y(t) = \mathbf{w}^H(\boldsymbol{\theta})\ \mathbf{v}(t)$

  $\mathbf{w}^H(\boldsymbol{\theta})$ weight vectors parameterized by $\boldsymbol{\theta}$

  $\mathbf{v}(t)$ receiving element output voltages

  $y(t)$ beamformer output voltage

- $\boldsymbol{\theta}$ depends on element response and noise covariance
- assumed parameter covariance models:
  - for calibration: Cramer-Rao bound
  - for drift: independent parameter variation
- standard error propagation formula

$$\mathrm{var}(y) = (\partial y / \partial \boldsymbol{\theta}^T)\ \mathrm{cov}(\boldsymbol{\theta})\ (\partial y / \partial \boldsymbol{\theta}^T)^T$$

Propagation of calibration errors

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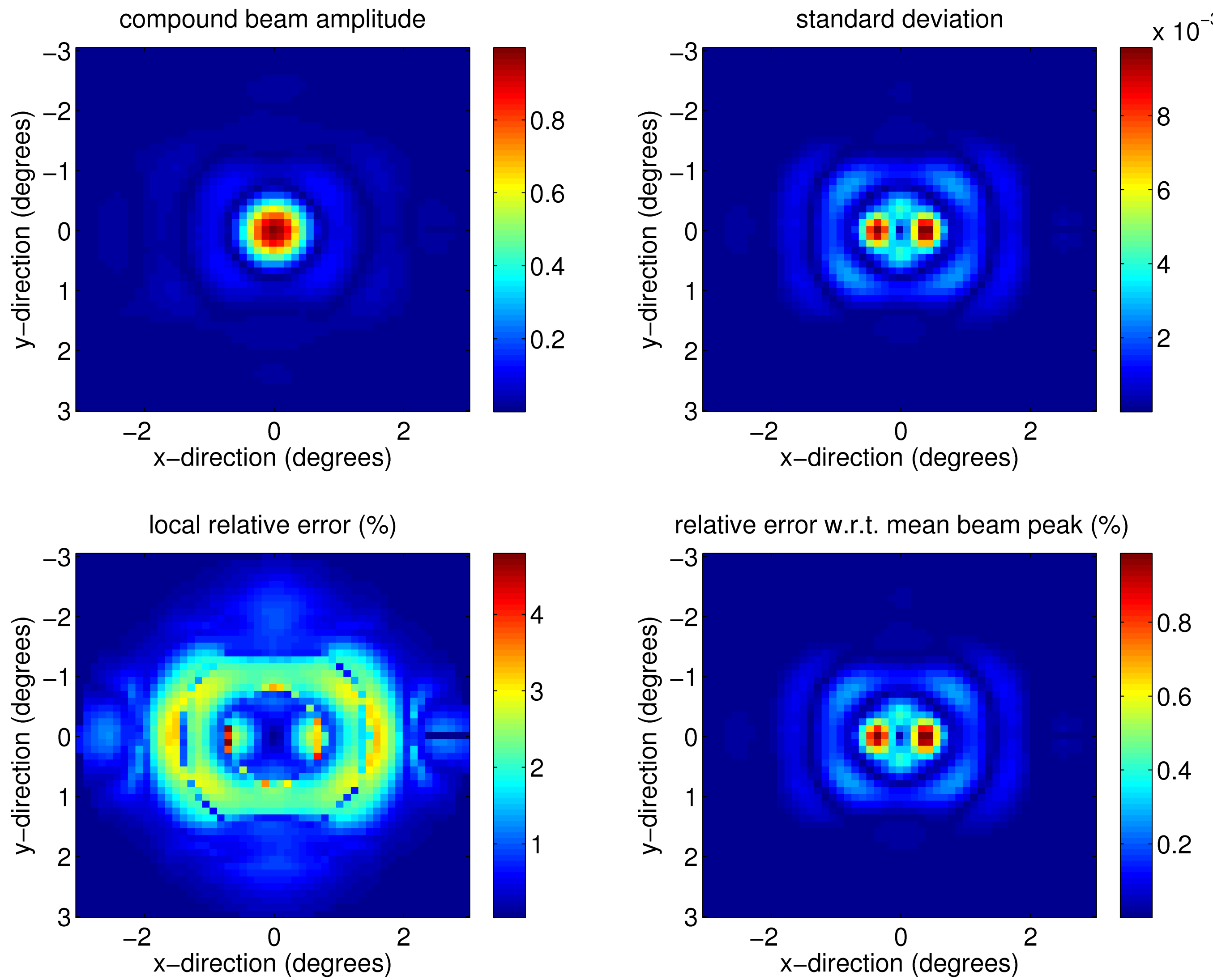
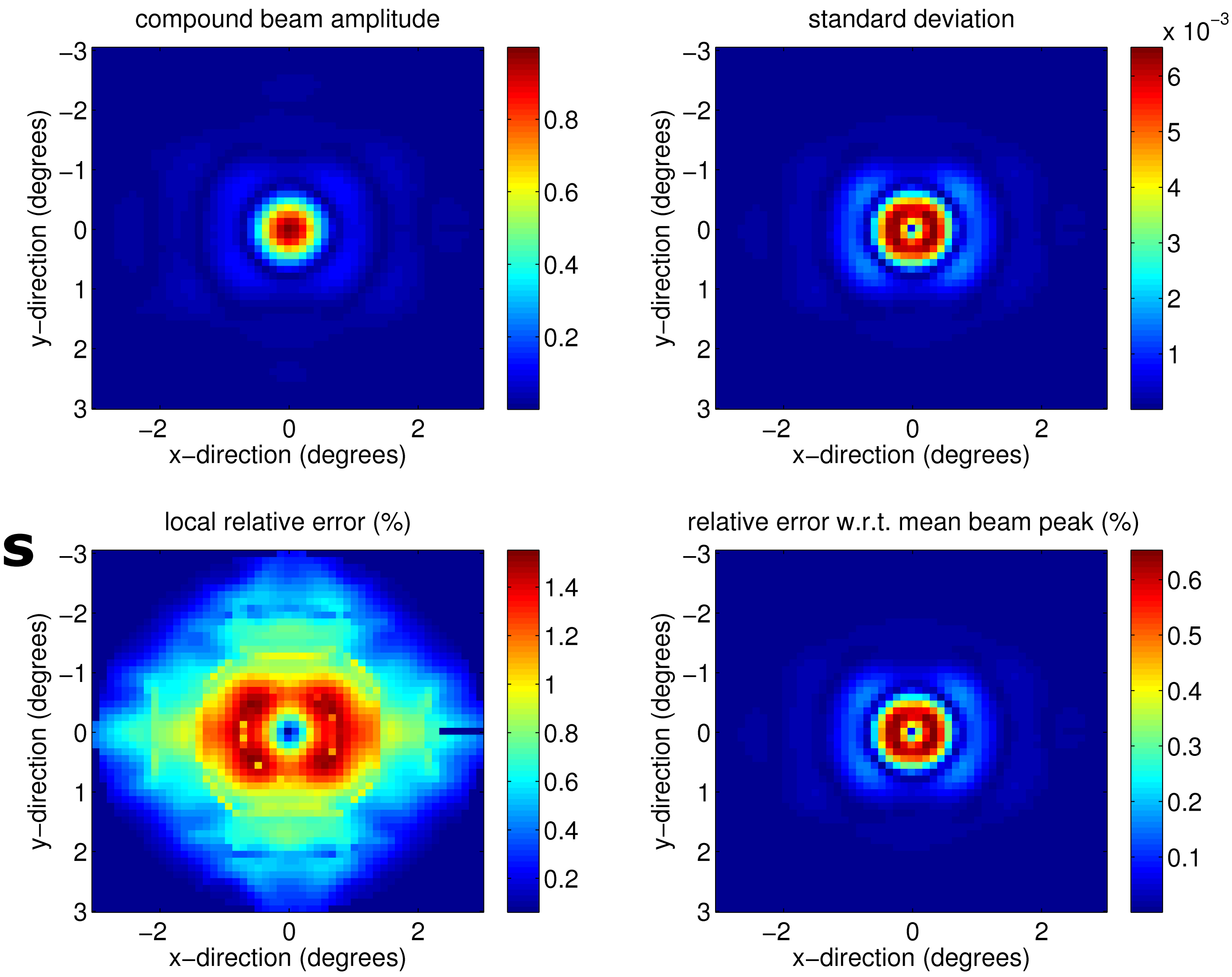
- SNR = 200
- bi-scalar BF
- constraint: beam peak fixed (selfcal)
- **SNR of 200 needed to satisfy beam requirement for APERTIF**



Propagation of drift errors (on axis)

Stefan J. Wijnholds, CalIm, July 2011

- 2% rel. error
- bi-scalar BF
- constraint: beam peak fixed (selfcal)
- **2% variations well within acceptable tolerances**



Element patterns on the sky

Van Cappellen, AJDI, 27 Mar 2008

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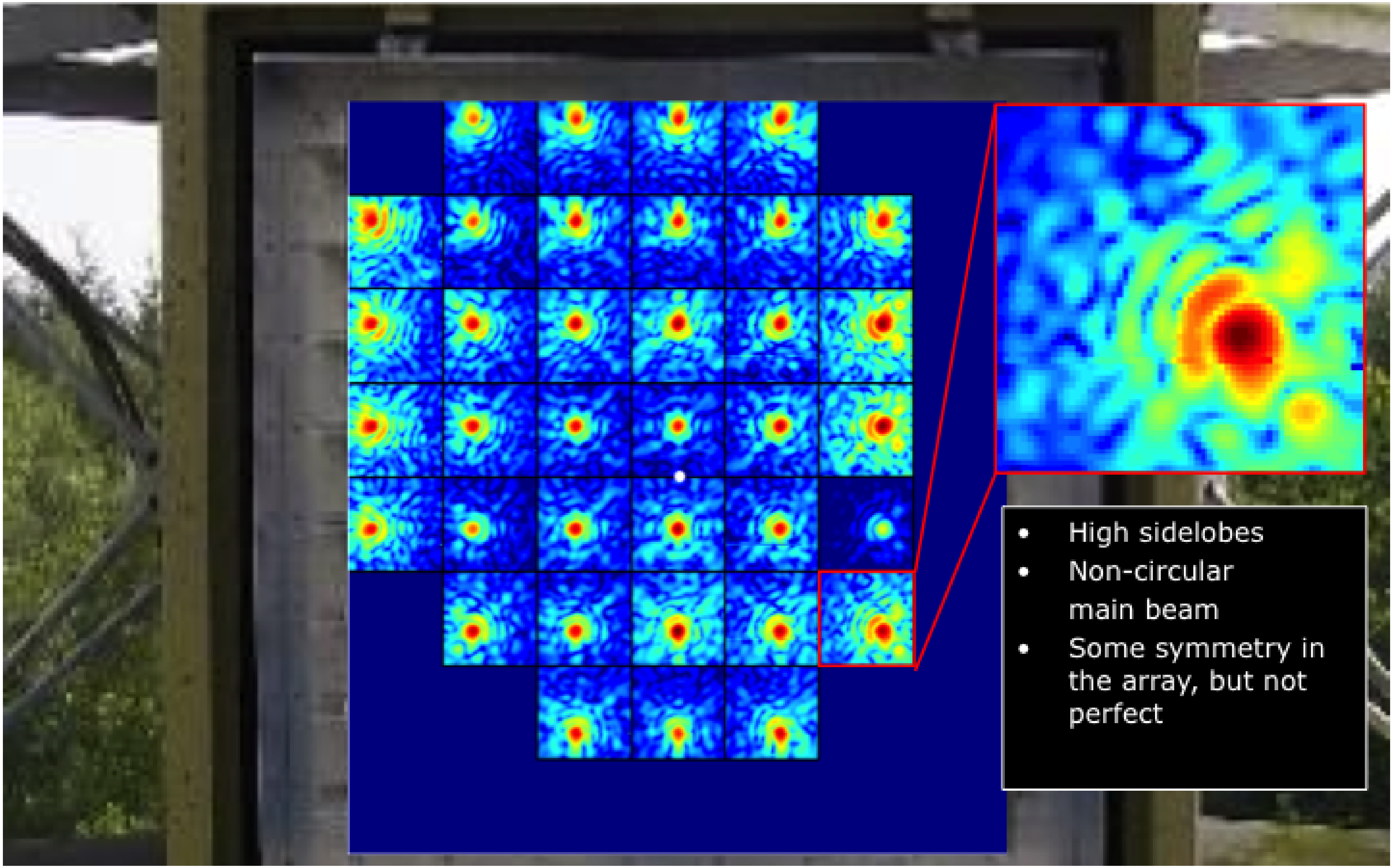

- High sidelobes
- Non-circular main beam
- Some symmetry in the array, but not perfect

Propagation of drift errors (off axis)

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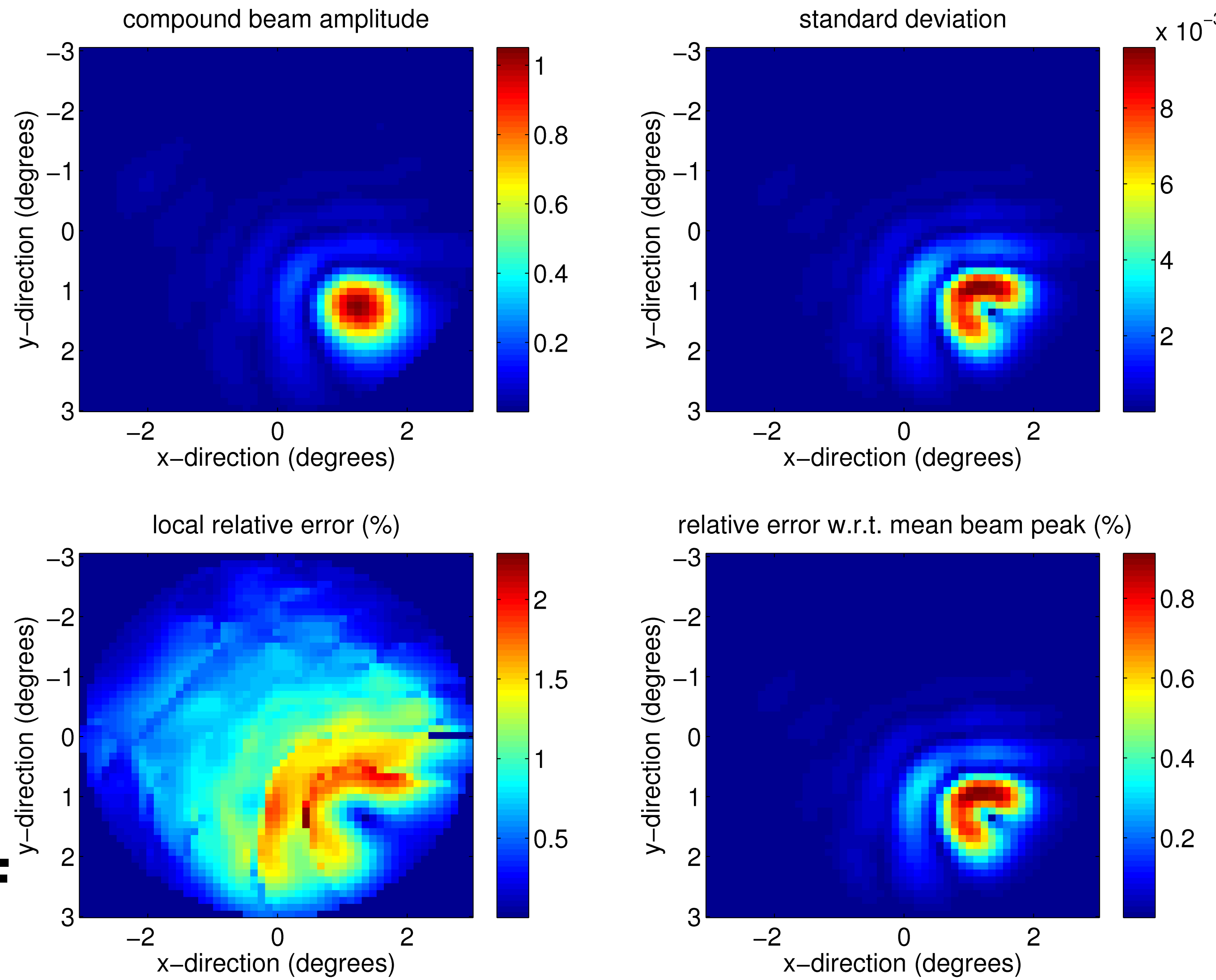
- 2% rel. error
- bi-scalar BF
- constraint: beam peak fixed (selfcal)
- **max 2% variation acceptable to satisfy beam spec APERTIF**



Measured drift using apex-source

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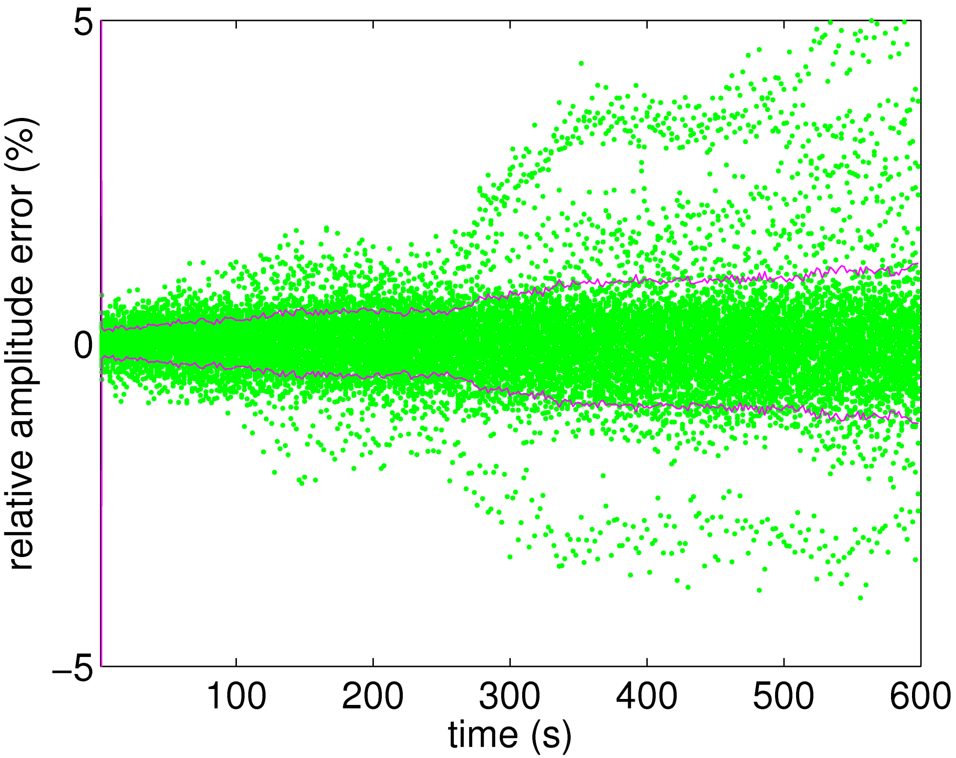
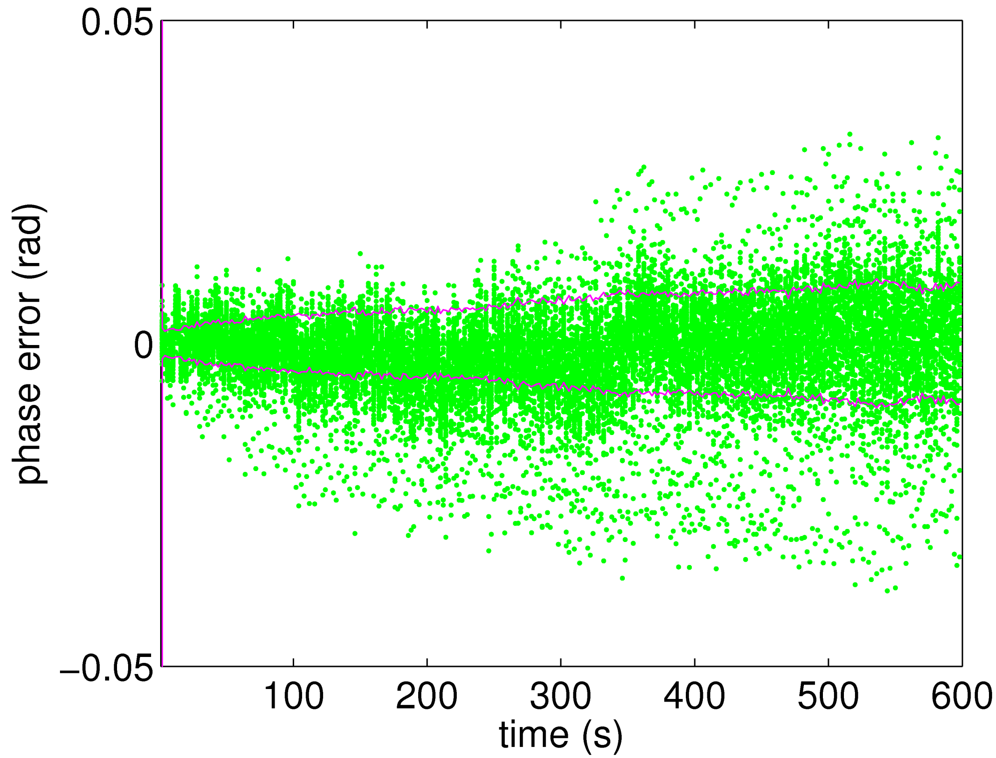
- 5 min observation at 1441.5 MHz
- gain calibrated using first 10 s
- < 1% variation after 5 min → 10 – 15 min update rate?





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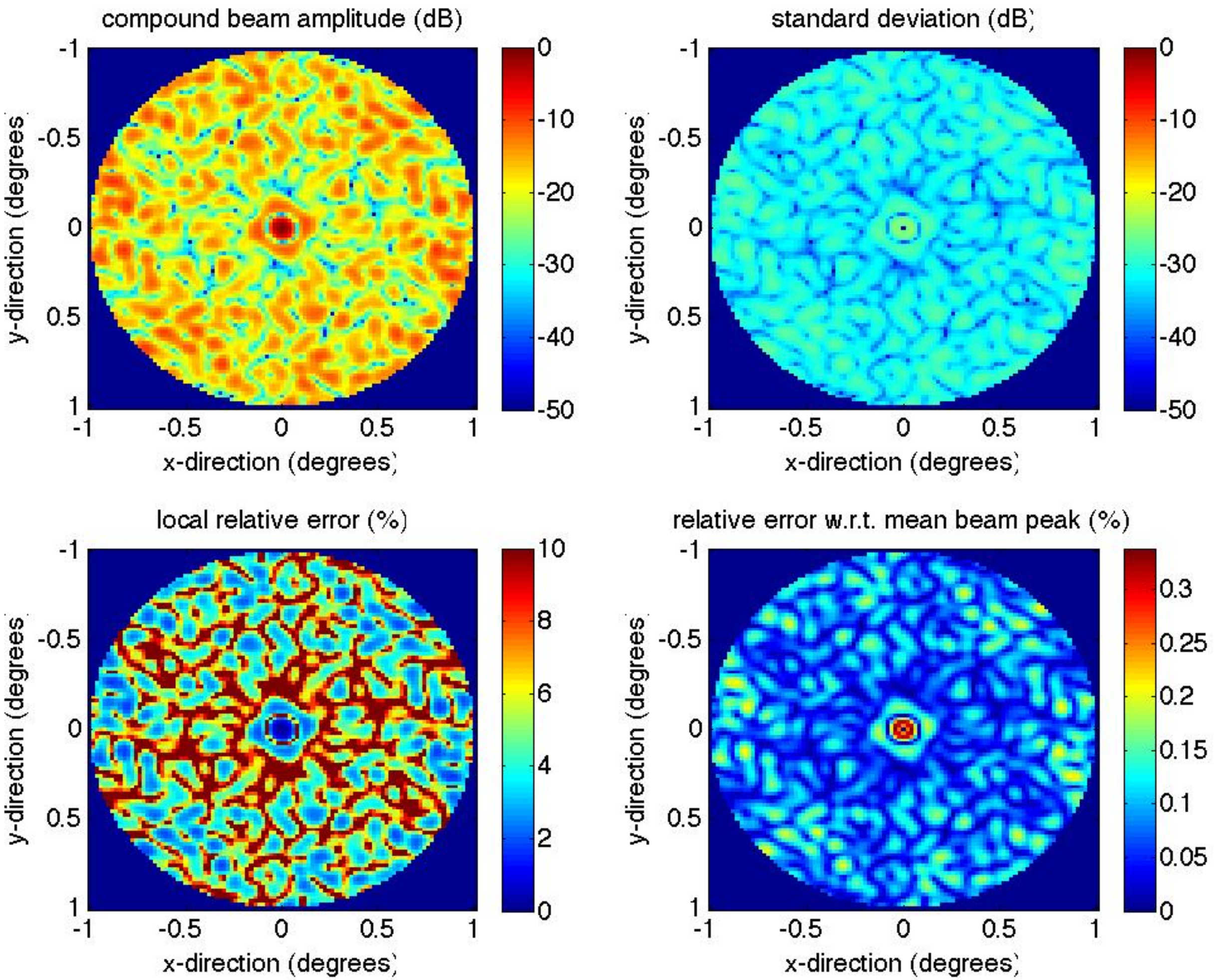
Impact of station cal. errors on LOFAR LBA station beam

Assumptions

- LBA_OUTER, CS302
- 4-9-'11, 15:00 UTC
- 1 s, 195 kHz
- @ 50 MHz
- calibration errors from CRB
- SNR_{max} = 0.01



Take away points

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Rigorous PAF and AA station error propagation

Pivotal for translation top level → hardware level specs

APERTIF example: 1% rel. error at HPBW

- SNR > 200 in calibration measurement
- calibration update at most every 10 minutes

Key questions

- How do we specify beam pattern accuracy?
- What beam pattern accuracy is required?

Fundamental approach

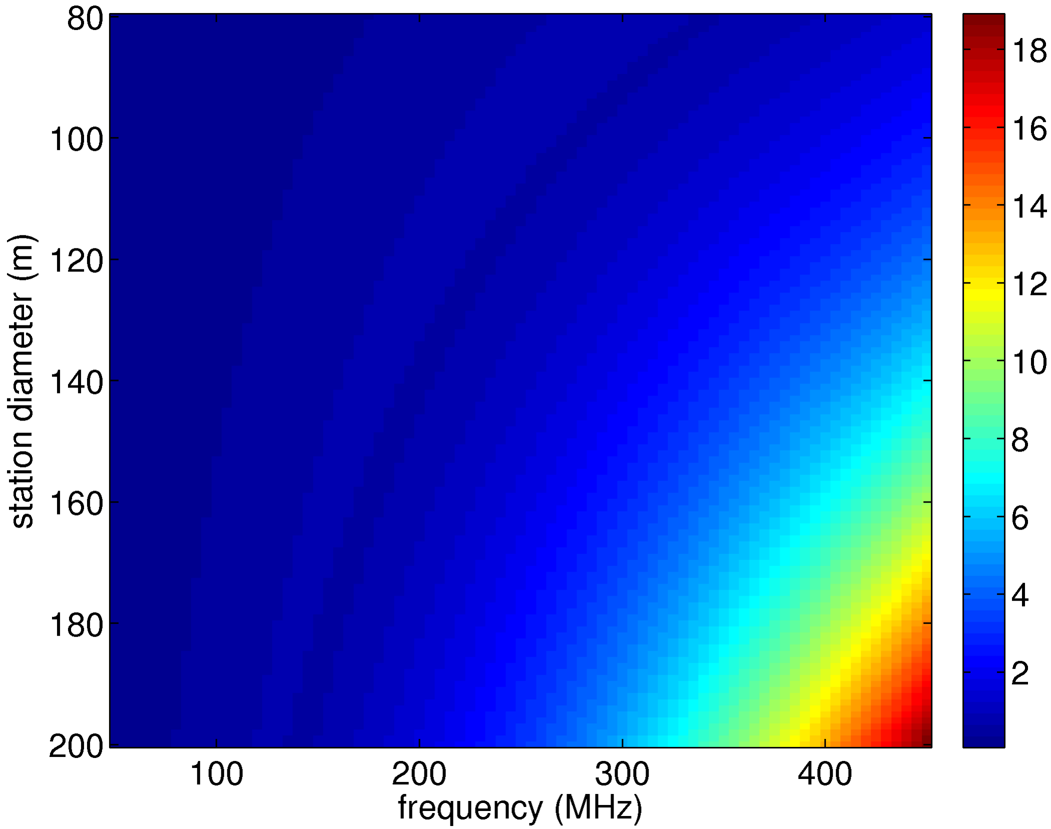
Basic principle: beam errors should stay below noise

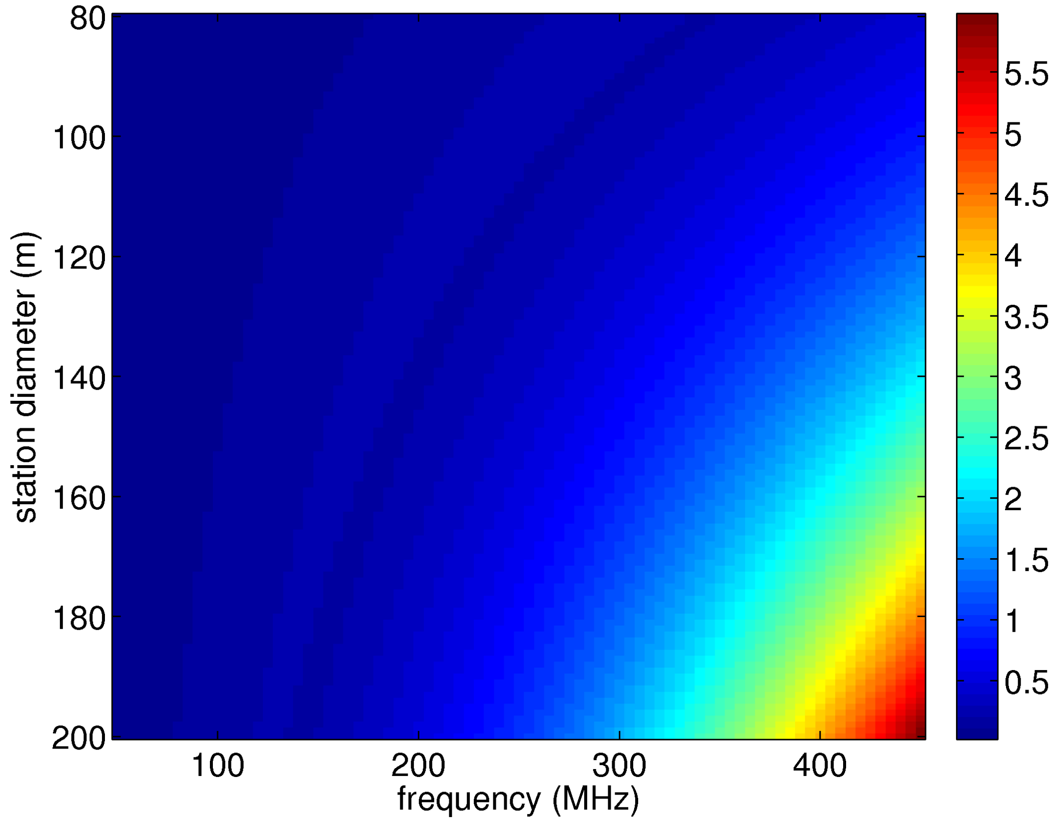
Implications (example)

- “random” beam errors every 5 minutes
- station sensitivity 20 m^2/K (from AA-low specs)
- 1σ (60 MHz, 300 s): 0.51 mJy
- FoV (180-m station, 300 MHz): 2.42e-5 sr
- strongest source (typical field): 40 mJy
- required accuracy: 0.51 / 40 = 1.3 % (w.r.t. peak)
- for 90-m station: 0.18 % (w.r.t. peak)

Implications

beam accuracy (%) at time scales of 30 s (l) and 300 s (r)





Practical approach

Balancing against other errors (e.g. ionosphere)

- snapshot calibration with ~3 – 5 in FoV
- second order ionospheric phase screen
- interpolation errors due to higher order terms
- small scale variations between calibration sources

Beam modeling and measurement limitations

- Craeye (CalIm): fit difference with modeled pattern
- Maaskant et al.: use CBFPs (modeling accuracy ~1%)

Current state of the art at this workshop!

Conclusions

Specification of beam pattern accuracy is pivotal

- translation from top level to hardware level specs
- **Fundamental approach**
  - keep errors due to beam inaccuracy below noise
- **Practical approach**
  - balance beam errors against other errors
  - limitations of state-of-the-art models

We can gain crucial insight from this workshop!