Simulating Focal Plane Array Observations with MeqTrees

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Topics

- Overview of Measurement Equation

- Overview of MeqTrees

- Example of MeqTrees Configuration

- Correction for E-Jones effects

- Simulation Setup

- Examples of MeqTrees Simulations

  - Phase-Conjugate Weighting

  - Optimization for Gaussian beam shape

  - AzEl observation tracking a fixed offset position

- What's Next?

Measurement Equation - HBS
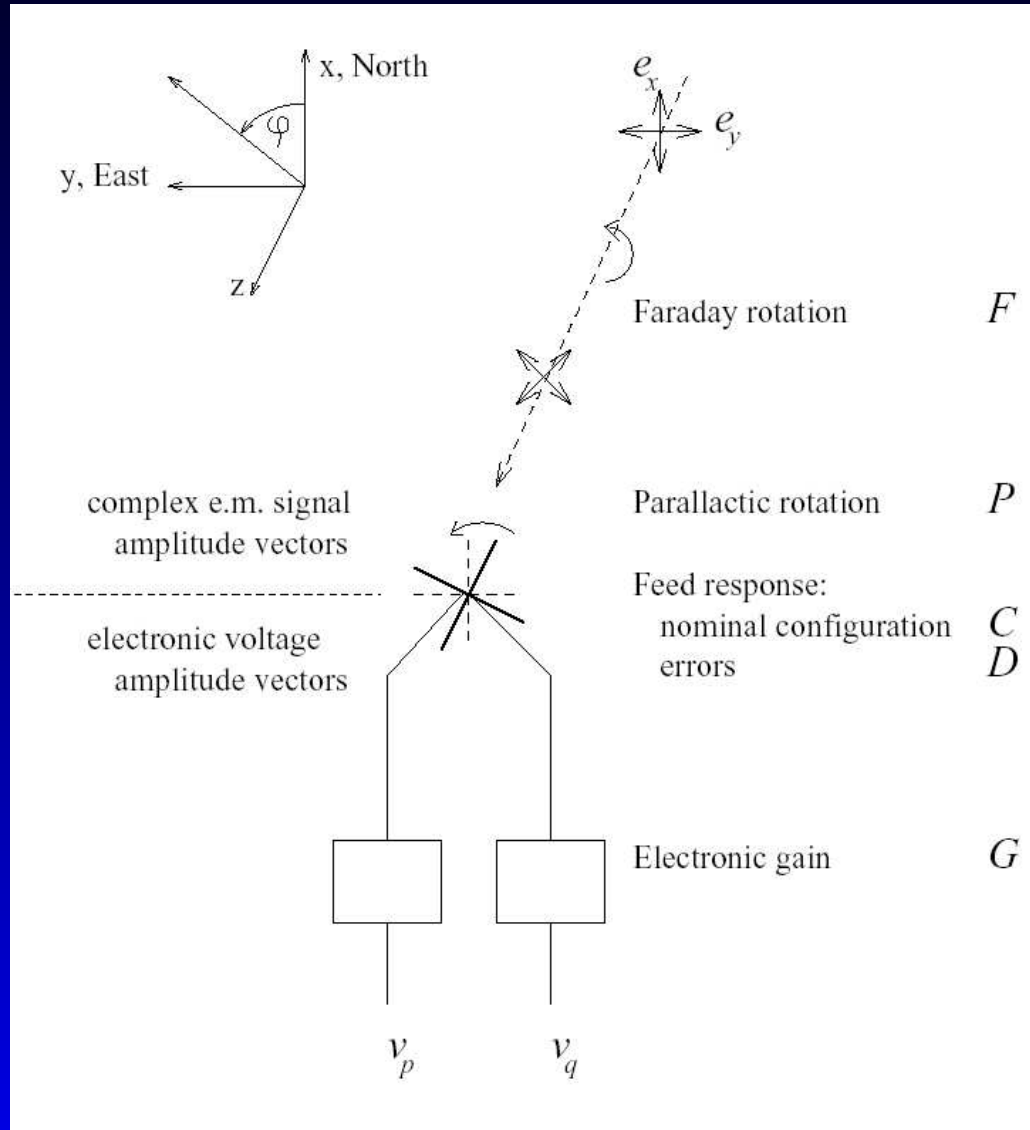
Jones Matrices

- The real heart of the Measurement Equation (M.E.) is composed of of two 2×2 *station*-based response matrices, called 'Jones matrices'.

- The 2×2 Jones matrix J_i for *station* i can be decomposed into a product of several 2×2 Jones matrices, each of which models a specific *station*-based instrumental effect in the signal path (see Hamaker, Bregman, Sault papers and aips++ notes from Noordam and Cornwell).

$$J_i \; = \; G_i \, [H_i] \, E_i \, P_i \, K_i \, T_i \, F_i$$

- The visibility for an interferometer composed of *station* i and *station* j with linearly polarized receptors is given by the following equation, where $\vec{V}_{ij}$ is the visibility, $\vec{I}$ is the incoming electromagnetic coherency matrix, and J_j^* is the complex conjugate of J_j.

$$\vec{V}_{ij} \; = \; J_i \, \vec{I} \, J_j^*$$

$$\vec{I} \; = \; 0.5 \begin{pmatrix} I + Q & U - iV \\ U + iV & I - Q \end{pmatrix}$$

Jones Matrix Definitions

$F_i(\vec{\rho}, \vec{r_i})$ ionospheric Faraday rotation

$T_i(\vec{\rho}, \vec{r_i})$ atmospheric complex gain

$K_i(\vec{\rho}.\vec{r_i})$ factored Fourier Transform kernel

P_i projected *receptor* orientation(s) w.r.t. the sky

$E_i(\vec{\rho}, \vec{r_i})$ voltage primary beam

$[H_i]$ hybrid (conversion to circular polarization coord)

G_i electronic complex gain (*station* contributions)

- E-Jones definition

$$E_i^+(\vec{\rho}, \vec{r_i}) \;=\; E_i^{\odot}(\vec{\rho}, \vec{r_i}) \;=\; E_i(\vec{\rho}, \vec{r_i}) \;=\; \begin{pmatrix} e_{iaa} & e_{iba} \\ e_{iab} & e_{ibb} \end{pmatrix}$$

- On axis diagonal terms describe position dependant primary beam attenuation

- Non-zero off-diagonal terms e_{iba} and e_{iab} describe 'leakage' between *receptor*s

MeqTrees Summary

- M.E. predicts data measured with a particular instrument.
    - Model the instrument and observed data
    - Use for both system calibration and extraction of data parameters
    - Work mostly with Fourier (Visibility) data
- Procedure
    - Implement model in software using tree structure
    - Use a priori guesses to set model parameters
    - Compare observed data with predicted values
    - Solver/Condeq nodes adjust model parameters for best fit
    - Can solve for many discrepant parameters at same time
        - Hubble constant not yet done
- Multi-threaded processing available
- In on-going development
- NOT an antenna / FPA design tool or a synthesis imaging tool

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Example E-Jones Calculation

- The voltage beam pattern, E, of a Large Aperture Reflector (LAR) measured at the position of a source whose direction coordinates L and M are defined with respect to the field centre in an AzEl reference frame can be given as:

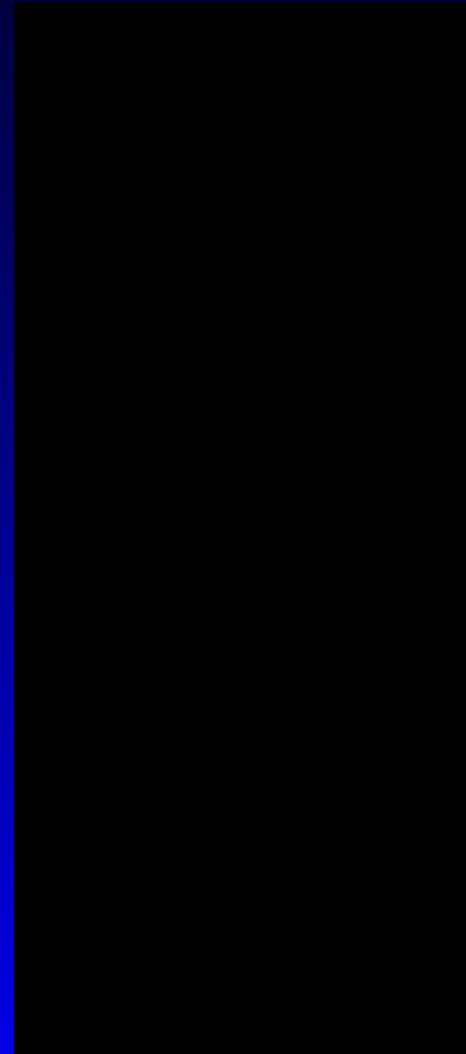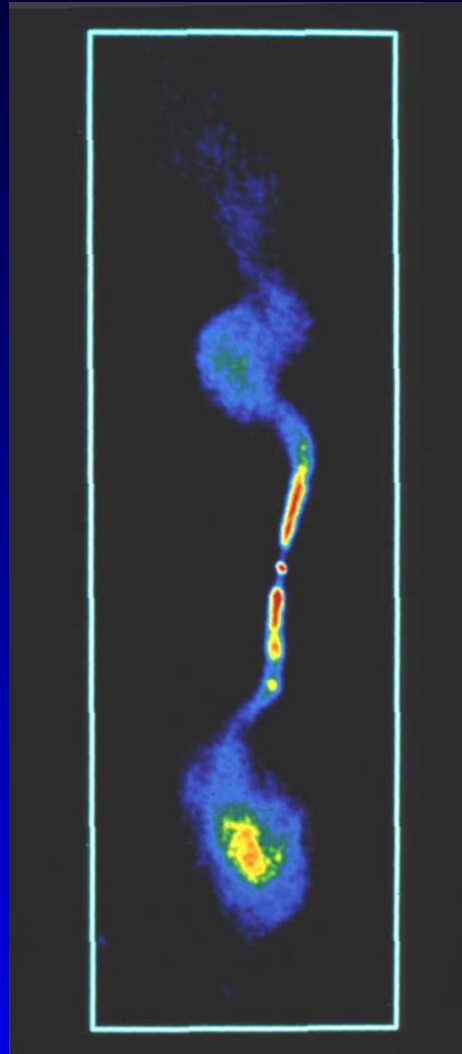$$E(L, M) = \sqrt{\exp(-\ln 16 \times (\frac{1}{HPBW})^2 (L^2 + (M \sin(El))^2))}$$

- HPBW = half power beam width at zenith

- El = elevation of field or tracking centre

The LAR Beam as a MeqTree
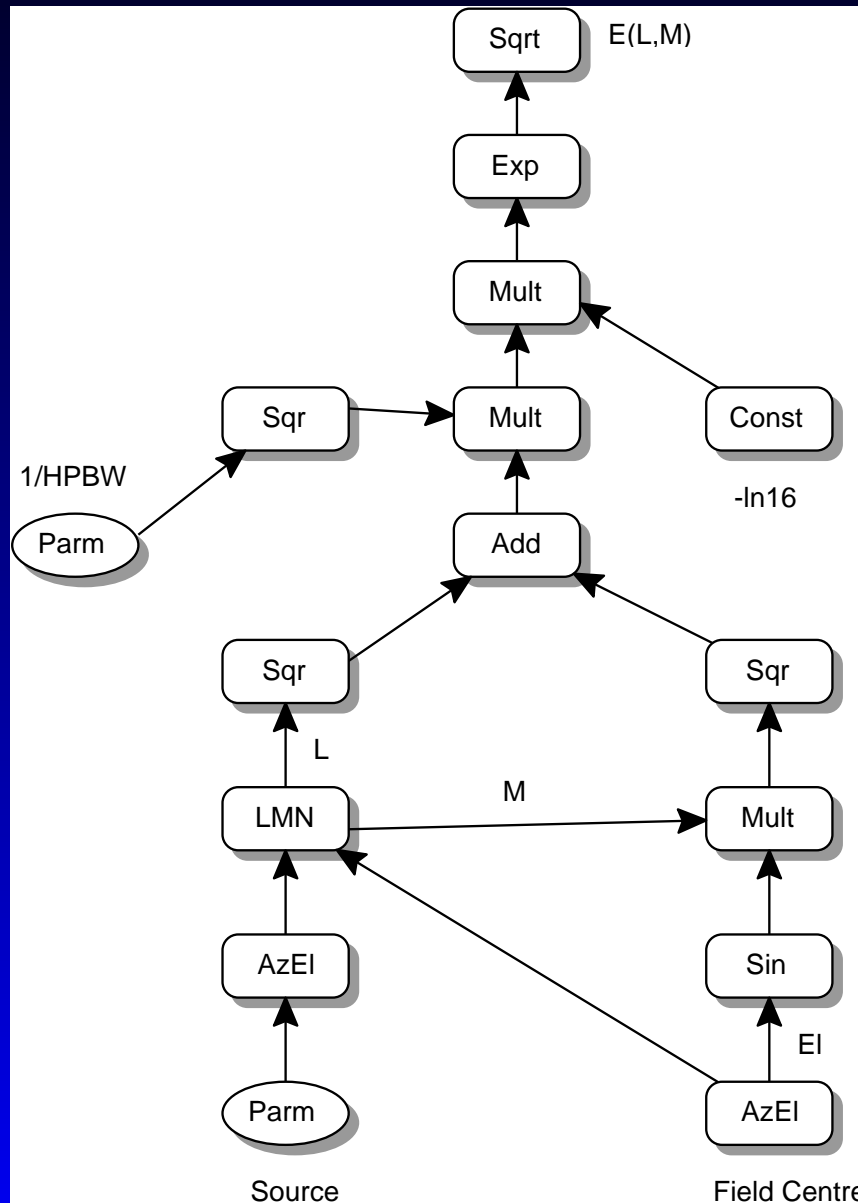
Reduction Goals

- Left - most reduction packages; Right - MeqTrees

Know Thy E-Jones

- No longer acceptable to model primary beams as simple Gaussians

- South Africa SKA Calibration and Imaging Workshop 2006
  - At least 4 or 5 presentations concerned with detailed measurements of telescope primary beams
  - Example - work of R. Reid et al. at DRAO on polarization leakage
    - Each telescope of DRAO SST has different E-Jones voltage pattern
    - Detailed measurements made of the pattern for each dish
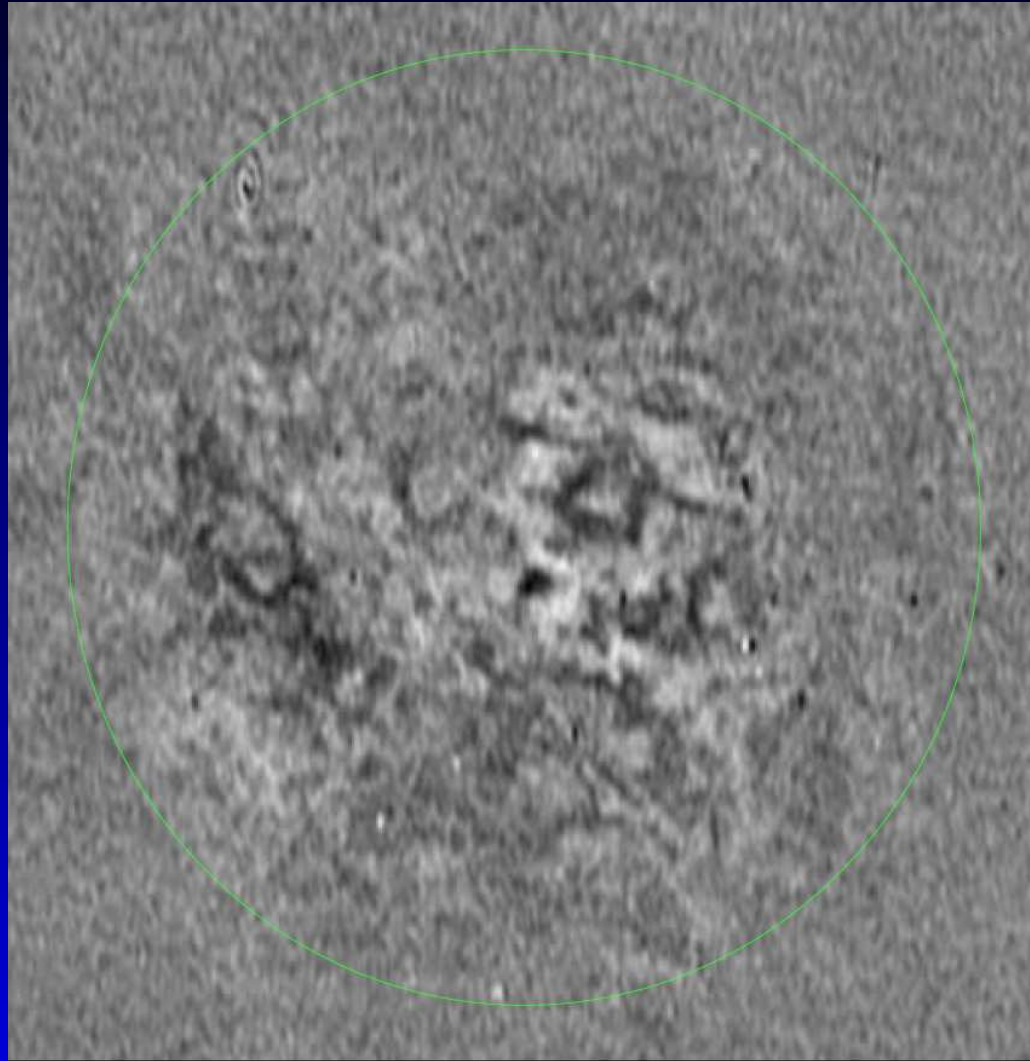   - Accurate correction for instrumental polarization now possible

DRAO Stokes I
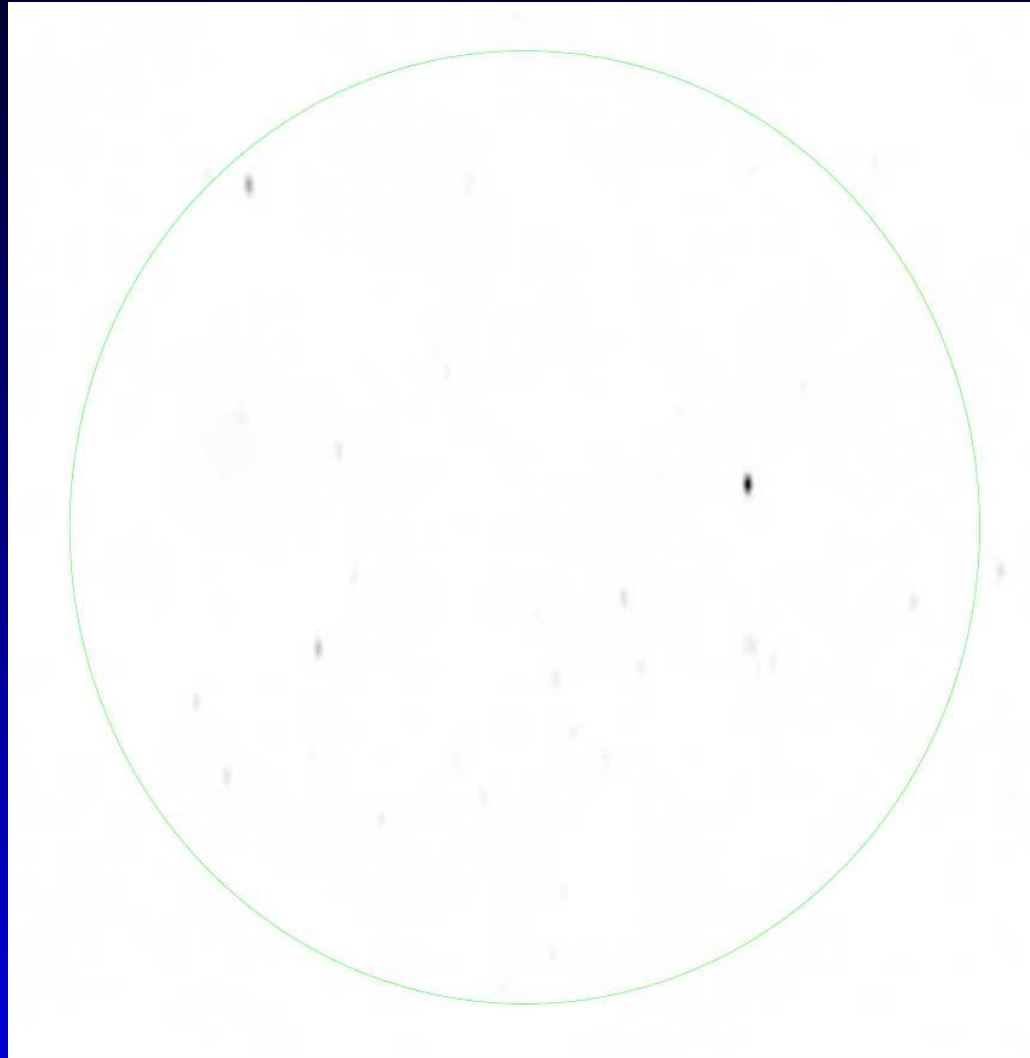
# Stokes U No Correction
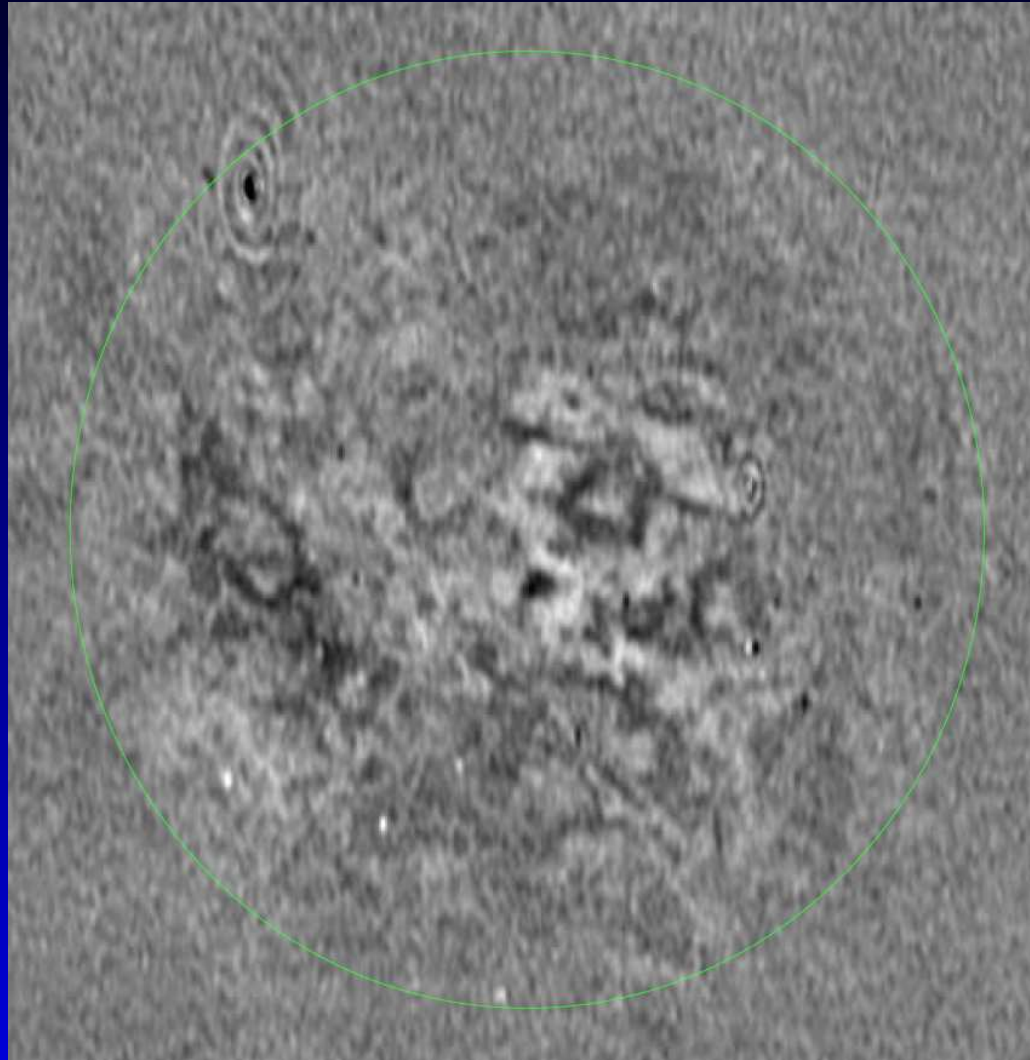
Stokes U Corrected

Know Thy FPA E-Jones

- Detailed knowledge of individual FPA voltage patterns allows accurate 'first order' prediction of phased array beam shapes
  - Resampling and interpolation tools allow extrapolation from coarse 'grid' measurements of actual FPA elements to finer grid for prediction of actual values associated with radio sources in the field
- Assuming MIRANdA / SKA dishes and receiver elements are stamped out of uniform molds, detailed measurements of FPA voltage patterns on 'representative' dishes should allow us to model entire array.
- GRASP calculations are the equivalent of the above activity for purposes of the simulations presented here.

Simulated FPA

- 30 dipole elements in each of X and Y directions

- Frequency = 1500 MHz; Spacing = lambda / 2

- Dish diameter = 10m; Focal length = 4.5m

- No coupling between elements; No feed struts in simulation

- Not meant as a 'realistic' final FPA design, but a good testbed for various aspects of software development and data processing
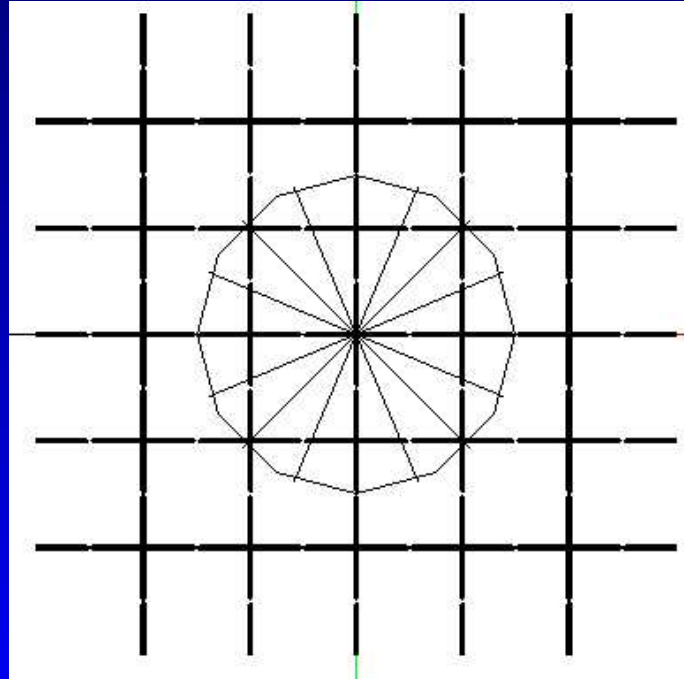
Simulation Procedure

- Do GRASP calculations of voltage radiation patterns for each of the X and Y dipoles used in this simulation

  - We get both co-polarization and cross-polarization leakage terms

- Convert GRASP 'grd' files to FITS images

- MeqTrees reads in radiation patterns from the FITS images

- Phase up X and Y radiation patterns, depending on optimization criteria, for requested observing position. In most of the simulations shown here we observe on a 5 x 5 grid centred on L=M=0, in steps of 82 arcmin (HPBW).

- Form E-Jones Matrix (fully complex) from weighted combinations

- Simulate observations of the 'visible' sky via our equation:

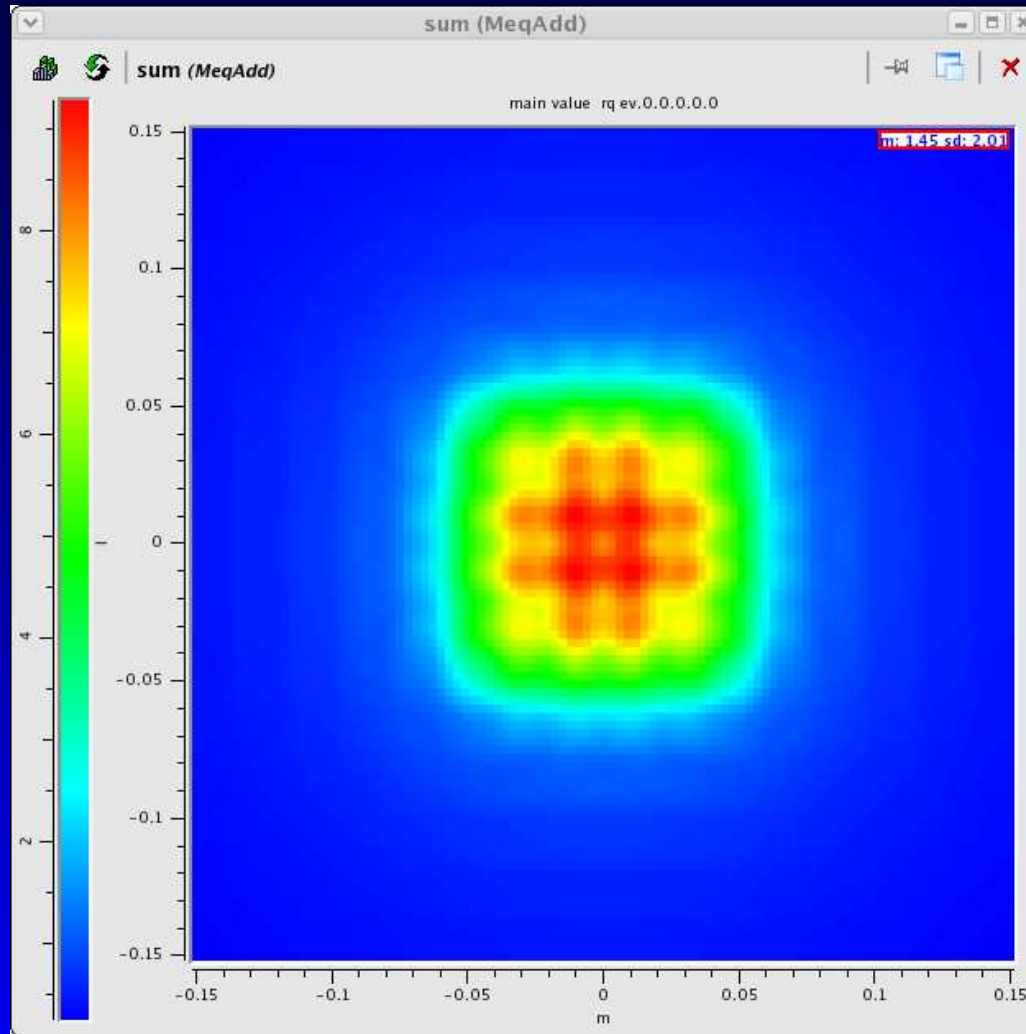$$\vec{V}_{ij} \;=\; \mathsf{E}_i \; \vec{I} \; \mathsf{E}_j^*$$

Typical GRASP Dipole Pattern

- In reality, we must measure these patterns in order to do accurate predicts, and thus compare with observations
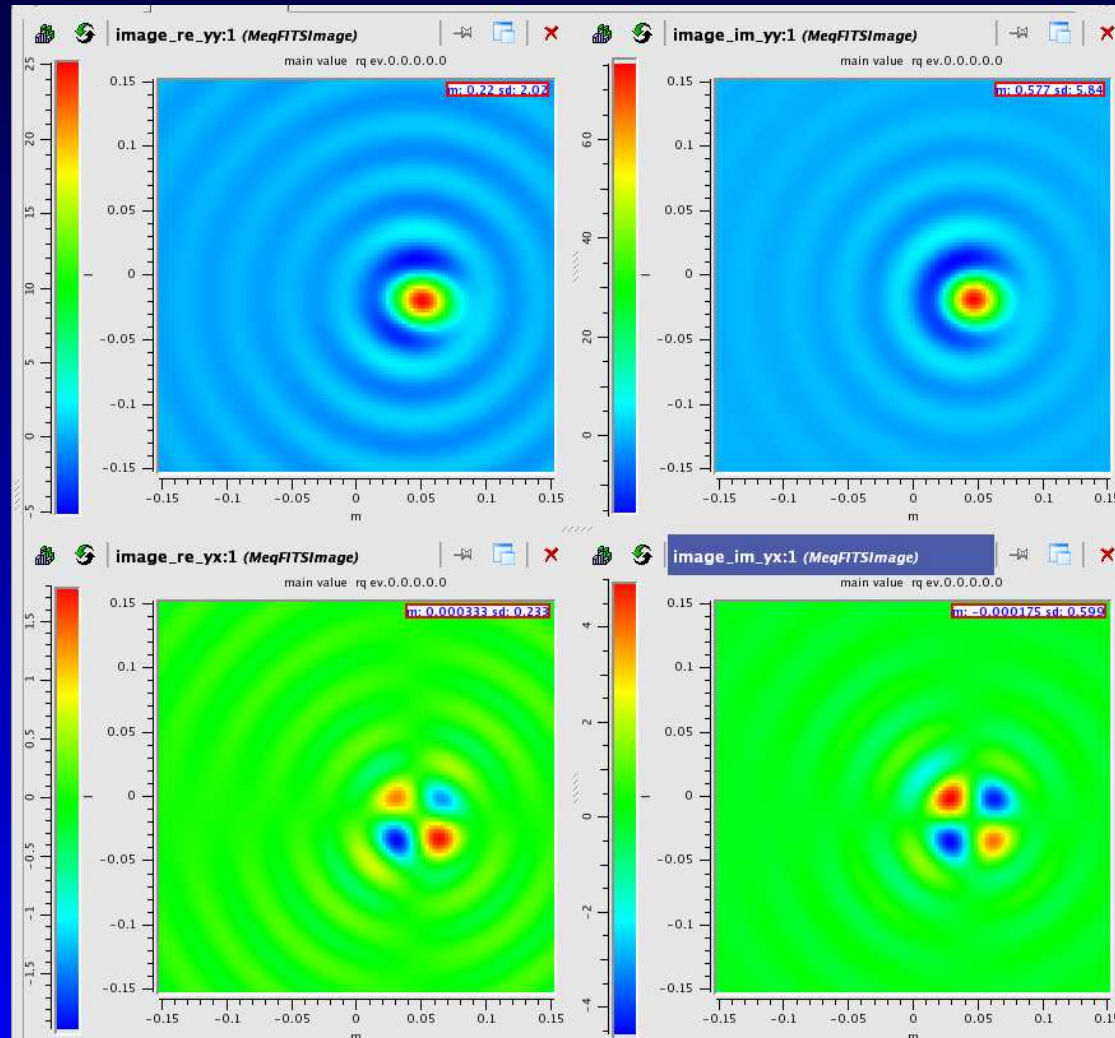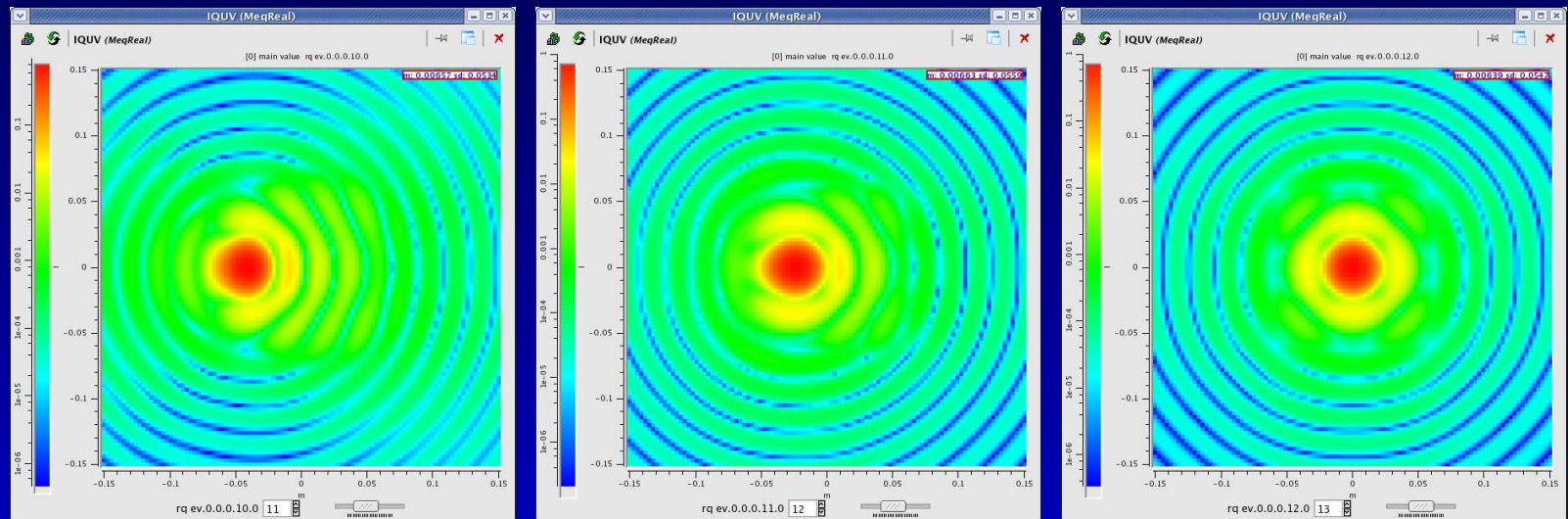
Sky Coverage

- Basically we can attempt to do beam-forming over the range -0.05 to 0.05 radians in L and M.
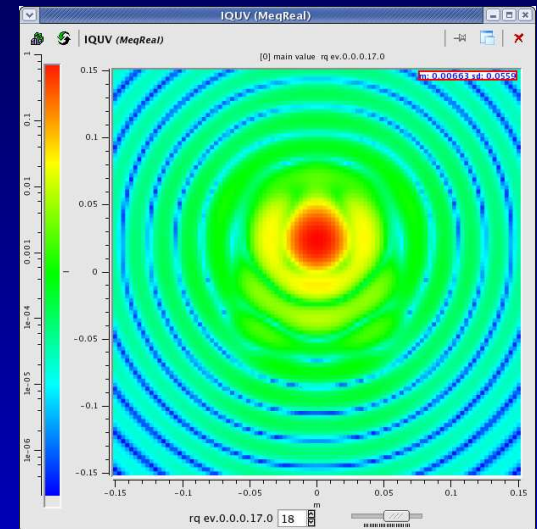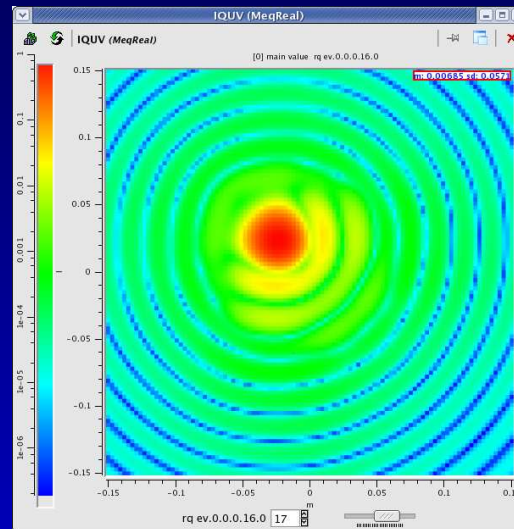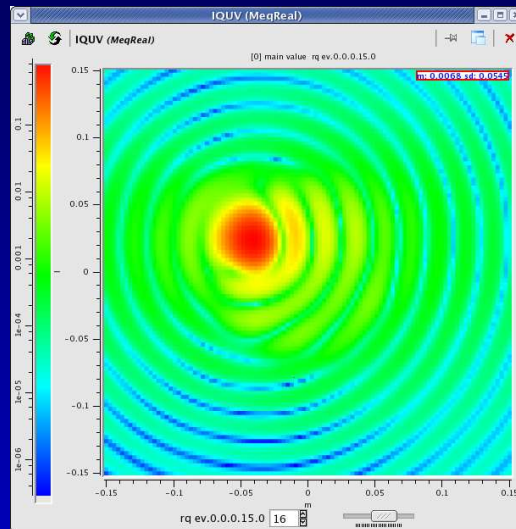
Phase Conjugate Weighting - I

- Phase conjugate weighting maximizes gain in observed direction, but does nothing particular for beam shape

- demo shows I beams for central row as we move from left edge toward centre of array in steps of 82 arcmin (HPBW)
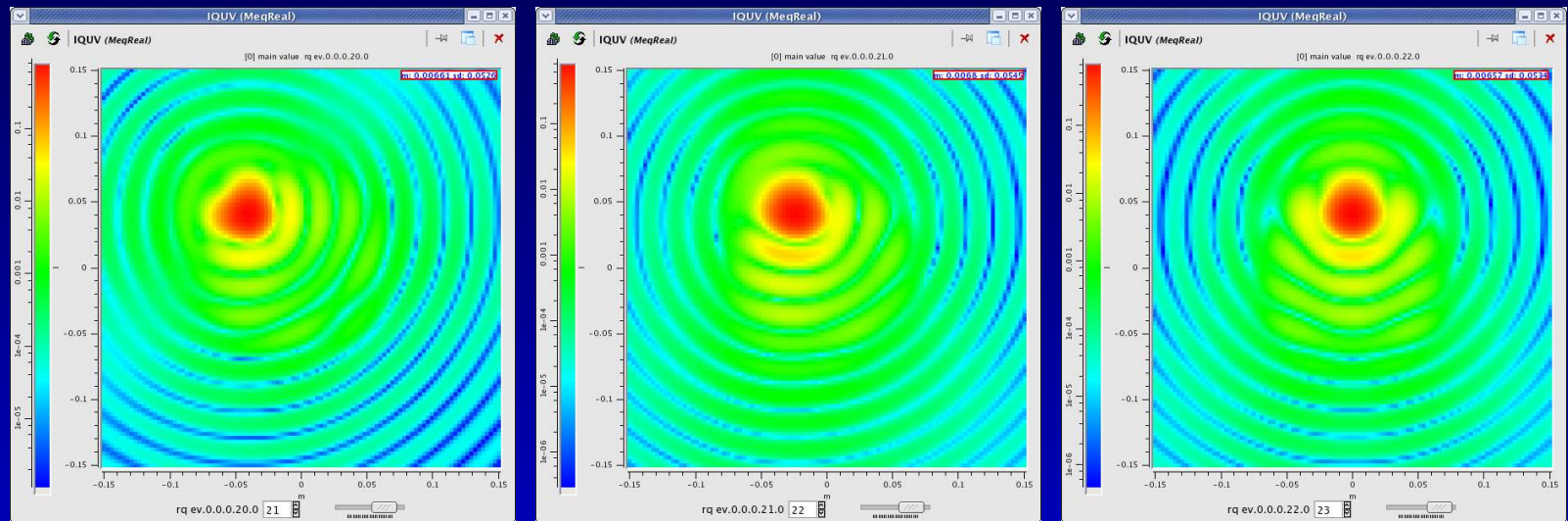
Phase Conjugate Weighting - I

- Phase conjugate weighting maximizes gain in observed direction, but does nothing particular for beam shape

- demo shows I beams for middle row as we move from left edge toward centre of array in steps of 82 arcmin (HPBW)
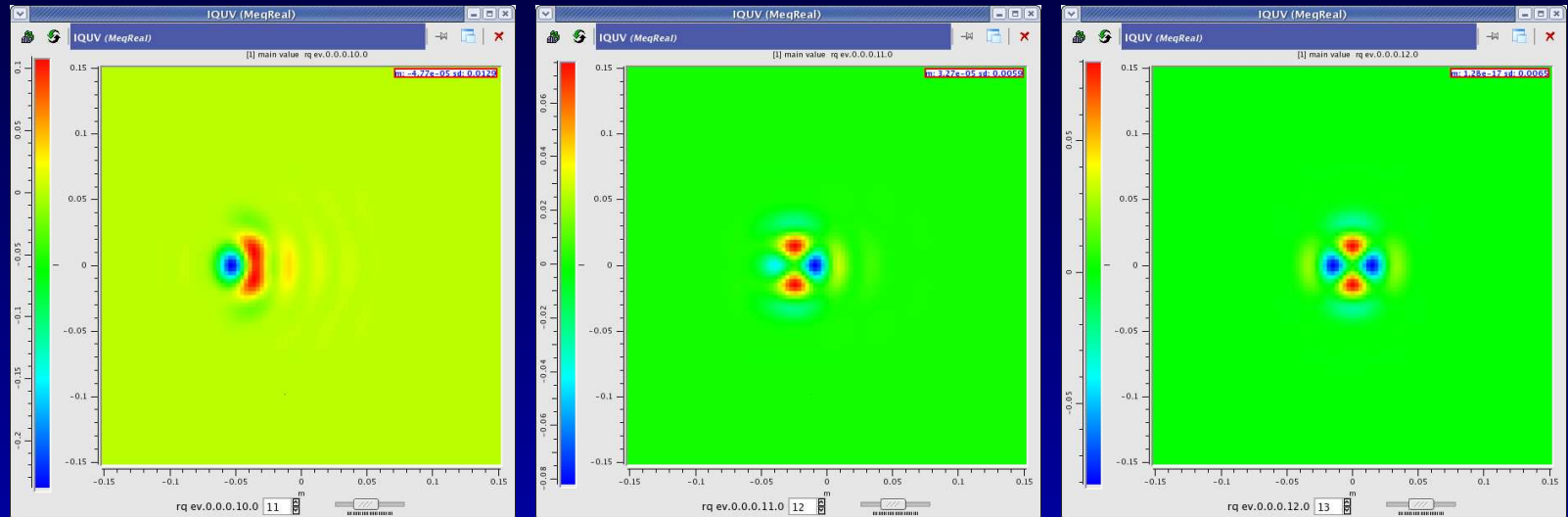
Phase Conjugate Weighting - I

- Phase conjugate weighting maximizes gain in observed direction, but does nothing particular for beam shape

- demo shows I beams as we move along top edge of array in steps of 82 arcmin (HPBW)
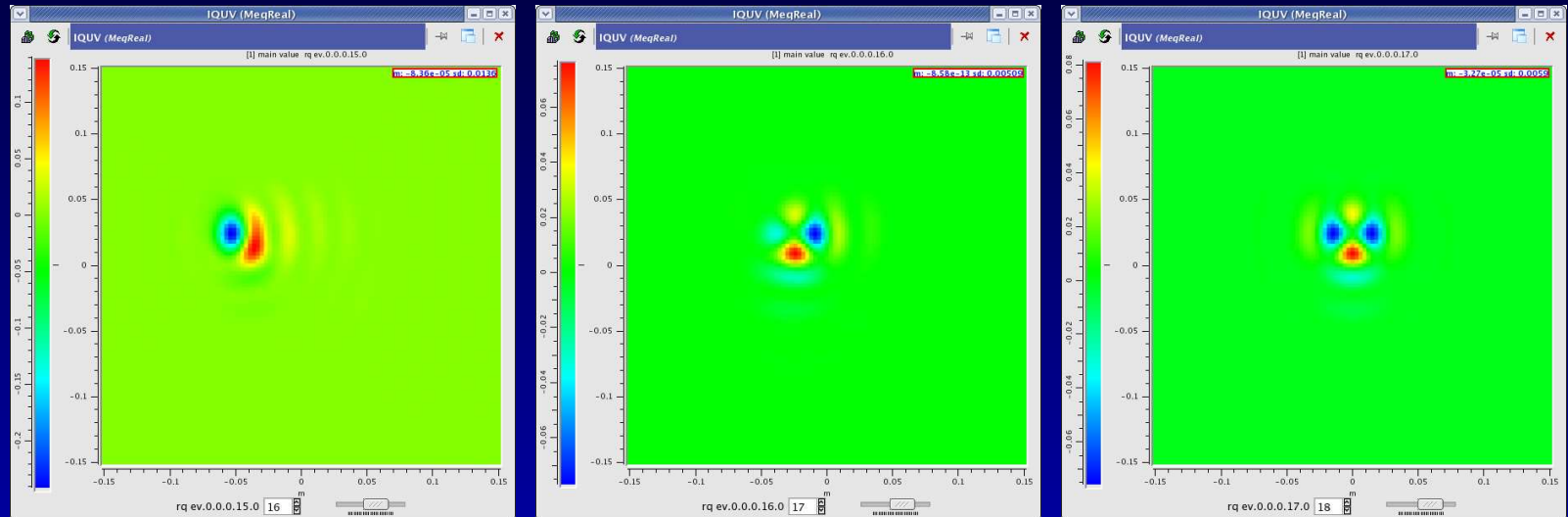
Phase Conjugate Weighting - Q

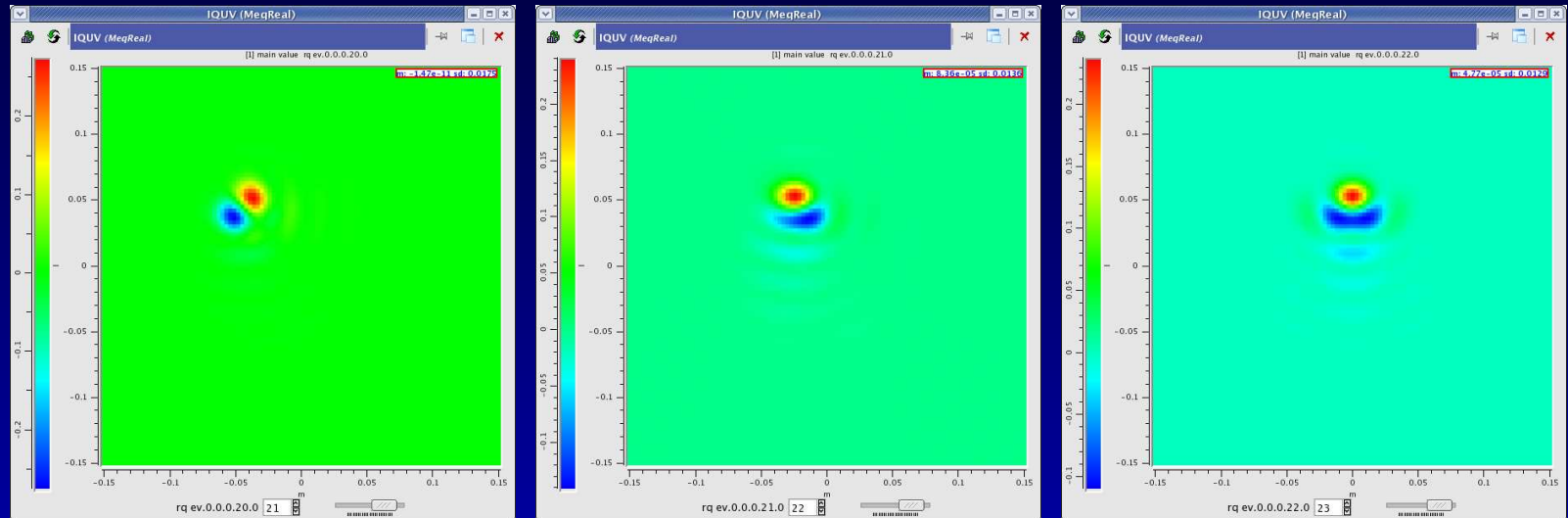- demo shows Q response for central row as we move from left edge toward centre of array in steps of 82 arcmin (HPBW)

Phase Conjugate Weighting - Q

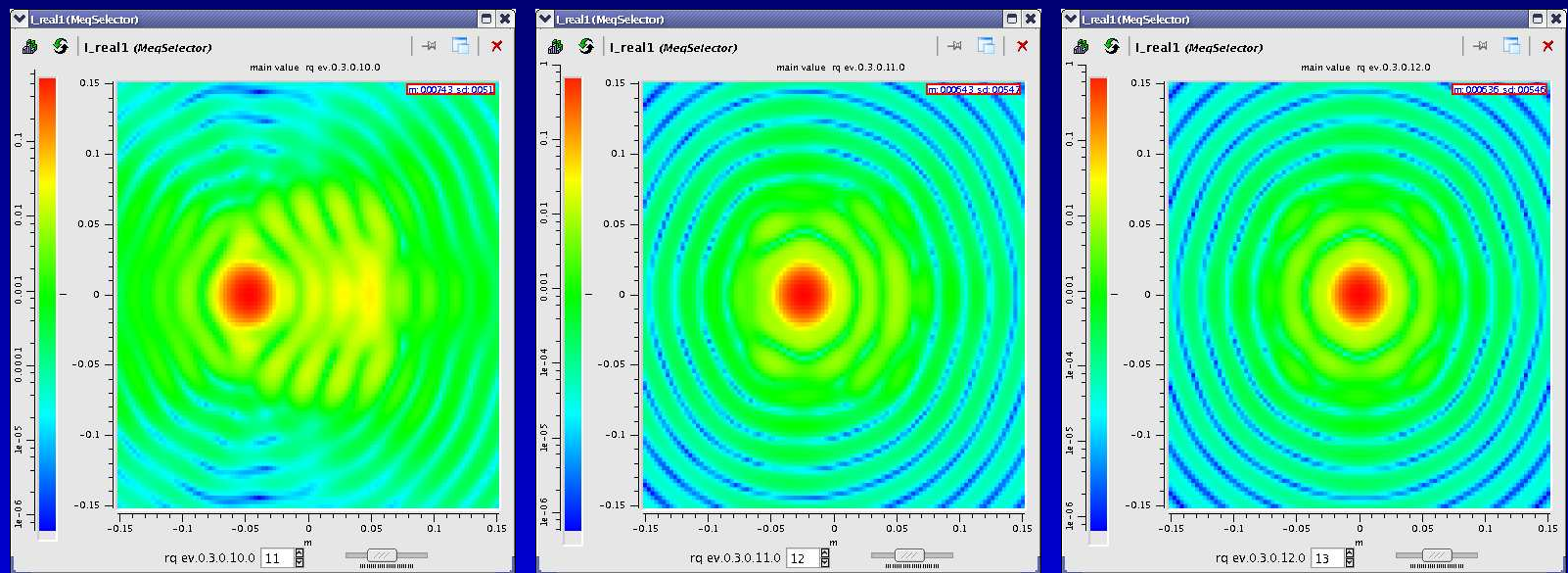- demo shows Q response for middle row as we move from left edge toward centre of array in steps of 82 arcmin (HPBW)

Phase Conjugate Weighting - Q

- demo shows Q response as we move along top edge of array in steps of 82 arcmin (HPBW)

Optimized Gaussian Beam - I

- Obtain values for phase-conjugate weighting in a particular direction

- Provide these values as initial guess for weights to MeqTrees solver

- Solver adjusts weights until phased beam has optimal gaussian shape

- demo shows I beams for central row as we move from left edge toward centre of array in steps of 82 arcmin (HPBW)
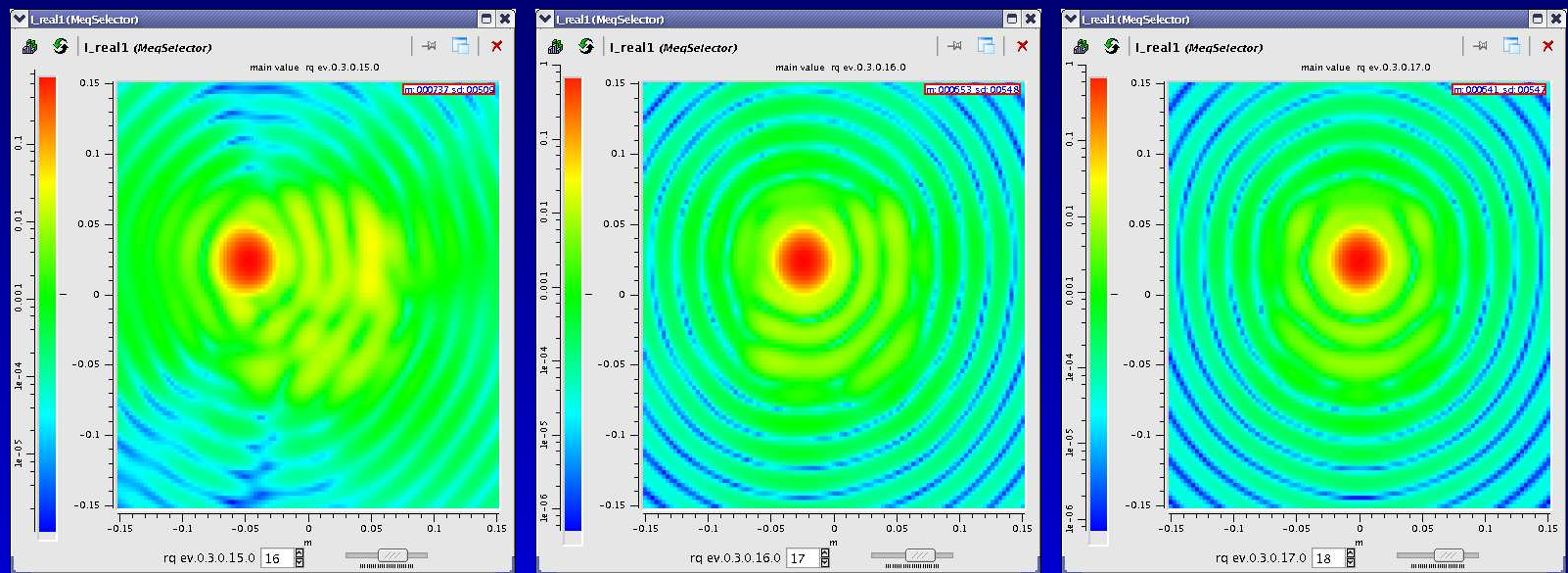
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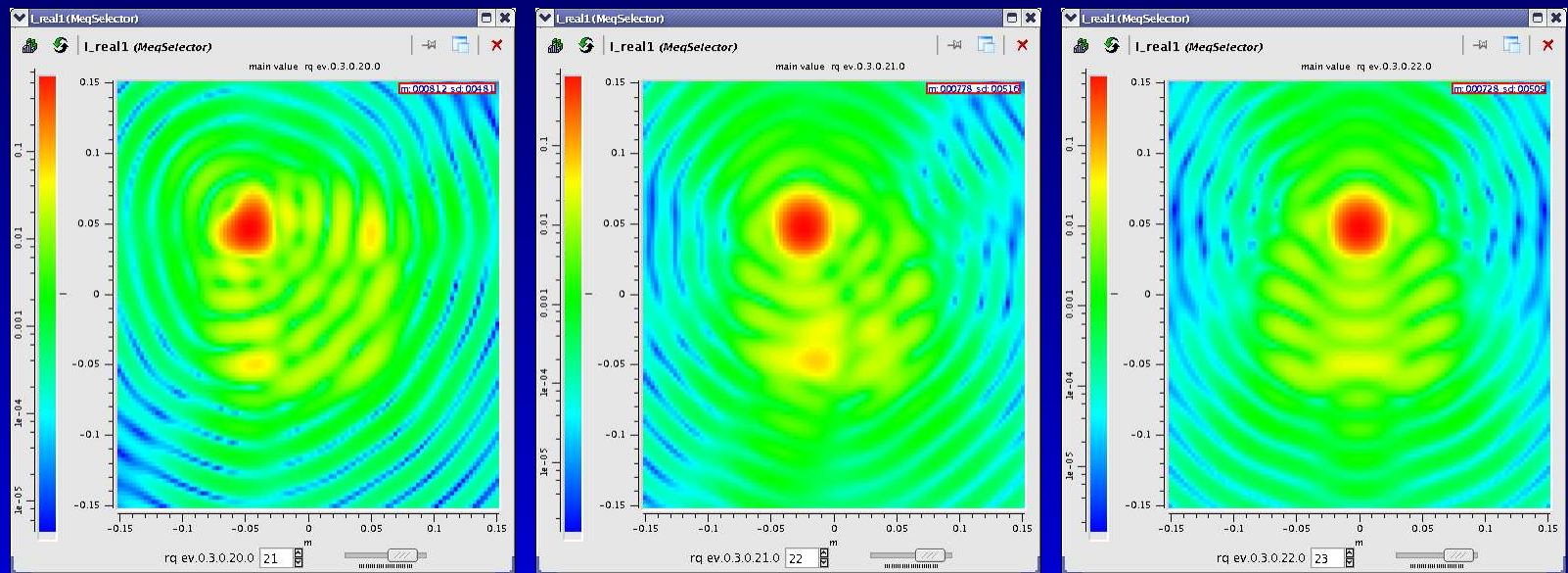
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- demo shows I beams as we move along top edge of array in steps of 82 arcmin (HPBW)

AzEl Telescope Simulation - I

- Calculate Parallactic Angle as a function of time for AzEl-mounted telescope stationed at VLA site which tracks position RA = 0 hr, Dec = 0 deg

- Phase up FPA at a position whose offset with respect to the tracking centre is -0.02 radians in both L and M when the Parallactic Angle is zero (transit)

- Adjust FPA phase conjugate weights to keep beam centred on this position.
  - 8 hour observation; calculate FPA beam every 10 minutes

- Total Intensity beam shown for start, middle and end of observation
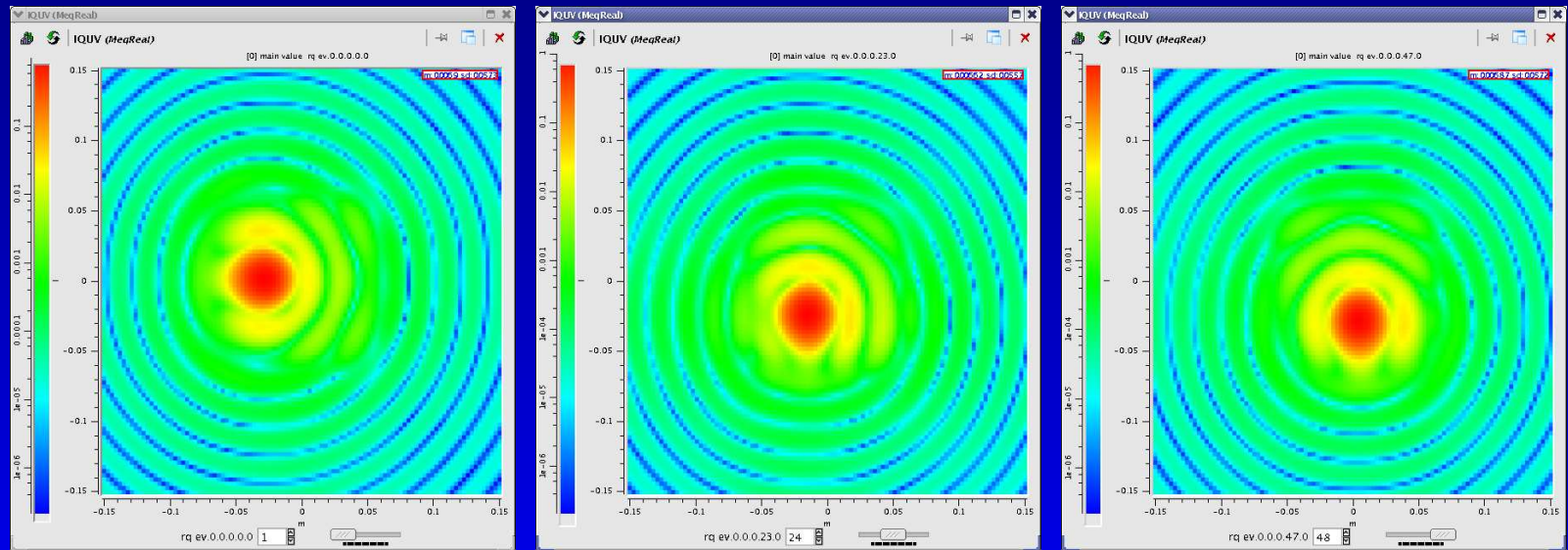
AzEl Telescope Simulation - Q

- Calculate Parallactic Angle as a function of time for AzEl-mounted telescope stationed at VLA site which tracks position RA = 0 hr, Dec = 0 deg

- Phase up FPA at a position whose offset with respect to the tracking centre is -0.02 radians in both L and M when the Parallactic Angle is zero (transit)

- Adjust FPA phase conjugate weights to keep beam centred on this position.
  - 8 hour observation; calculate FPA beam every 10 minutes

- Q response shown for start, middle and end of observation
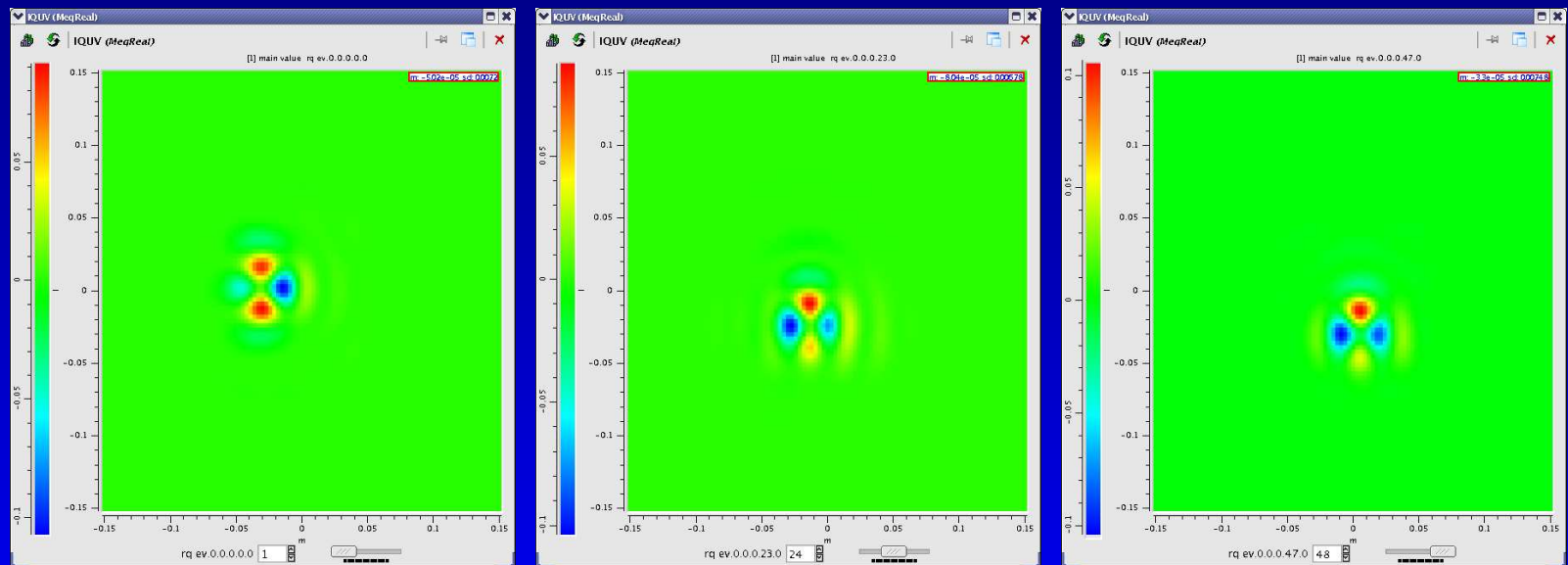
Modcal - Remove Anything

- Algorithm developed at DRAO to get rid of unwanted sources when you don't have a good understanding of your E-Jones.

- Baseline-based rather than antenna-based so not really part of the Jones Matrix formalism.

- Can be useful as a method of last resort.

- Only about 20 lines of python code with MeqTrees.

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Modcal - Example

- Right image shows source in sidelobe which does not clean properly; left image shows source vaporised by modcal algorithm.

Conclusion: Know Thy E-Jones

- Heuristics
- Learning

What's Next?

- Need Better Optimization than Gaussian Beam

  - Spheroids

  - Kaiser-Bessel

- Generate GRASP models of antennas more suitable for FPA such as Vivaldis and simulate observations with them.

- Look at effects of system gain variations on formed beams.

'Solving for the Hubble constant (say as a polc in time) should be possible too, but you need a machine big enough to model the universe on....'

- Oleg M Smirnov, Russian/Dutch computer scientist

Questions?

- Email: tony.willis@nrc.ca

Acknowledgements

- MeqTrees team, especially Oleg Smirnov, Maaijke Mevius and Sarod Yatawatta for assistance on MeqProblems related to focal plane arrays

- Jan Noordam for aips++ Note 185 on the Measurement Equation

- Bruce Veidt for GRASP calculations and advice on antenna-related issues

- 3C449 image made (a long time ago) at the VLA, operated by NRAO / AUI / NSF

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