Using MeqTrees to Simulate an SKA Composed of LARs

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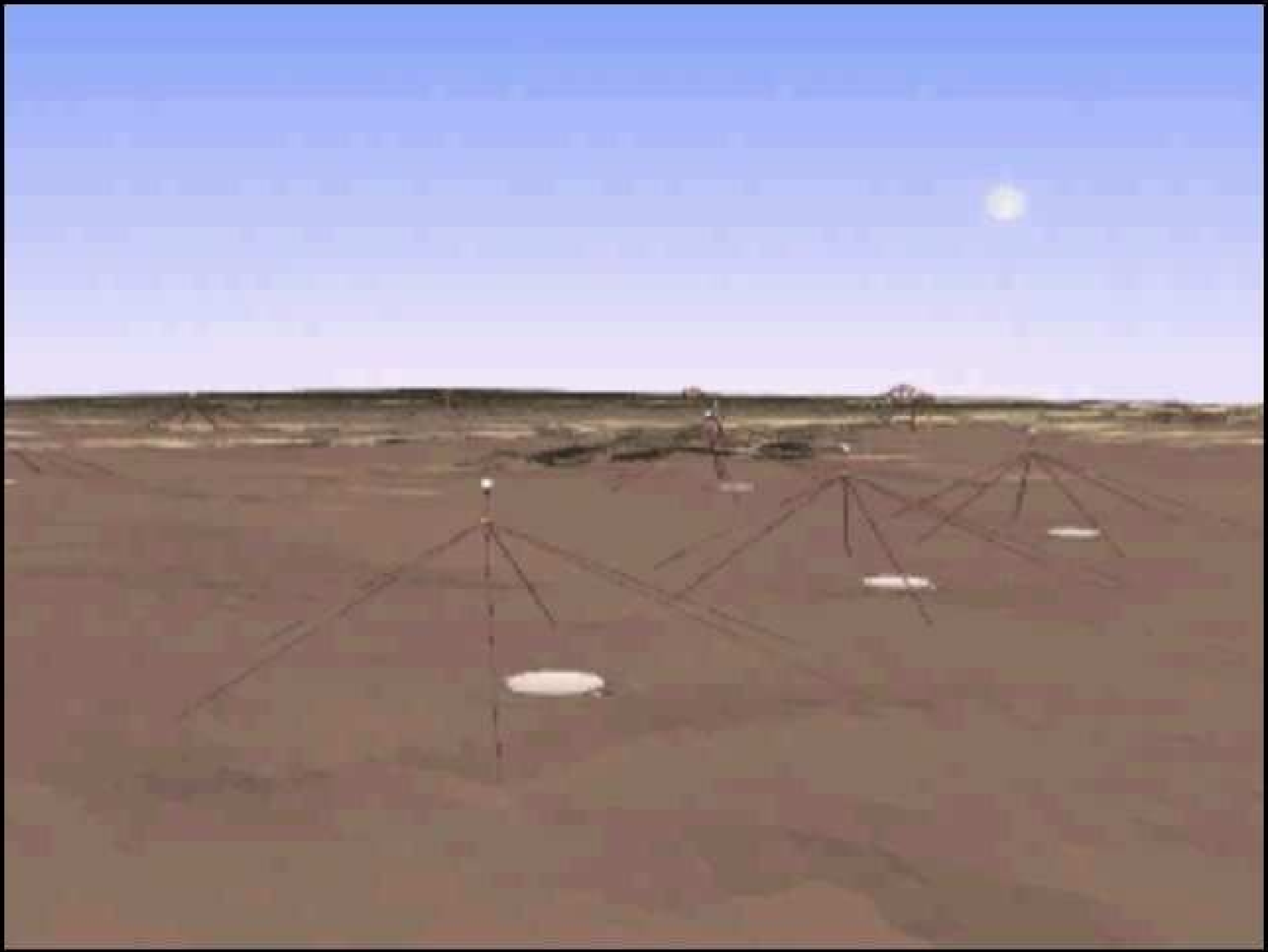
Outline of Talk

- SKA and LAR background
- LAR primary beam
- Measurement Equation
- Introduction to MeqTrees
- The LAR primary beam as a MeqTree
- Experimental setup
- Results
- Conclusions and Experience

The SKA Composed of LARs



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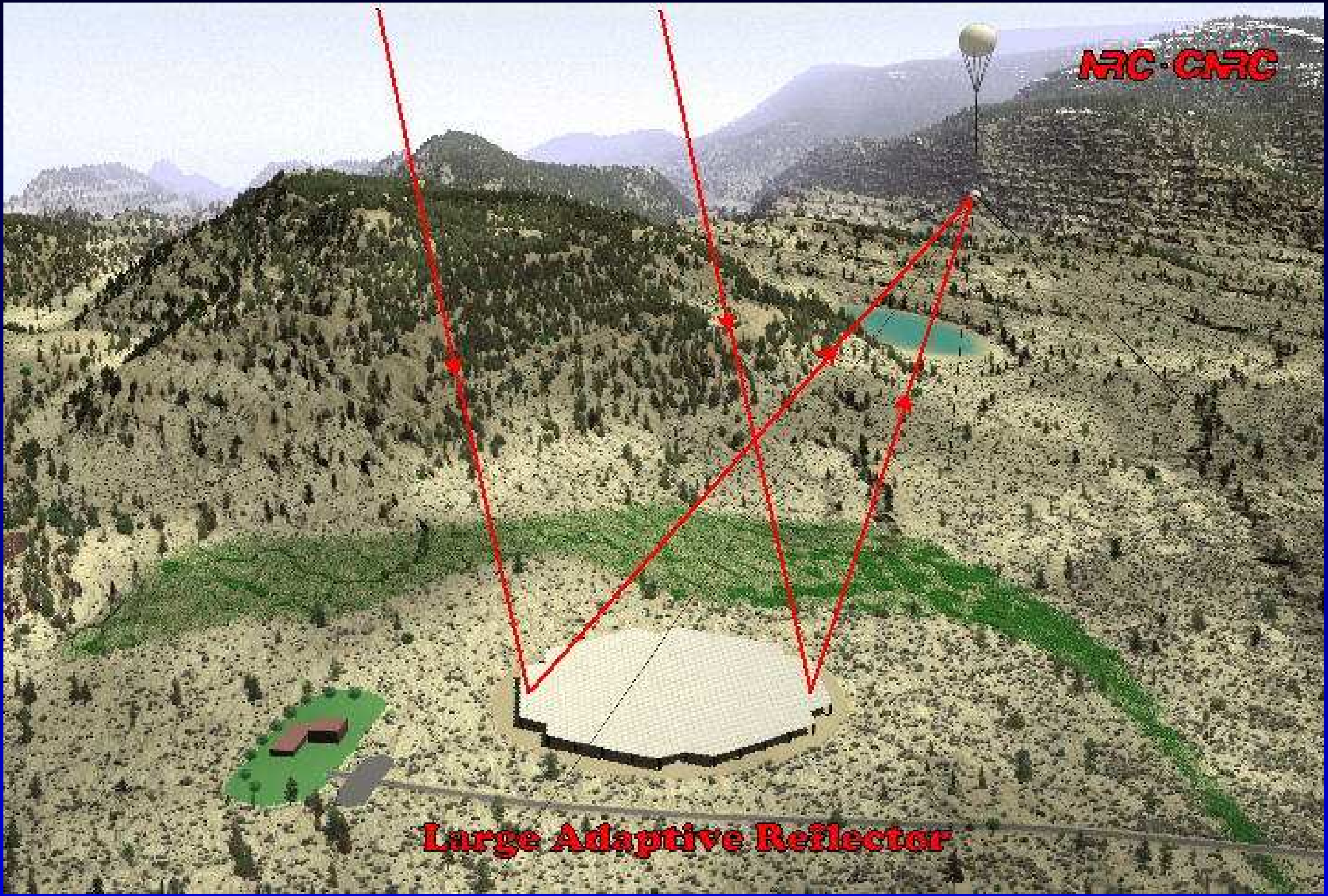
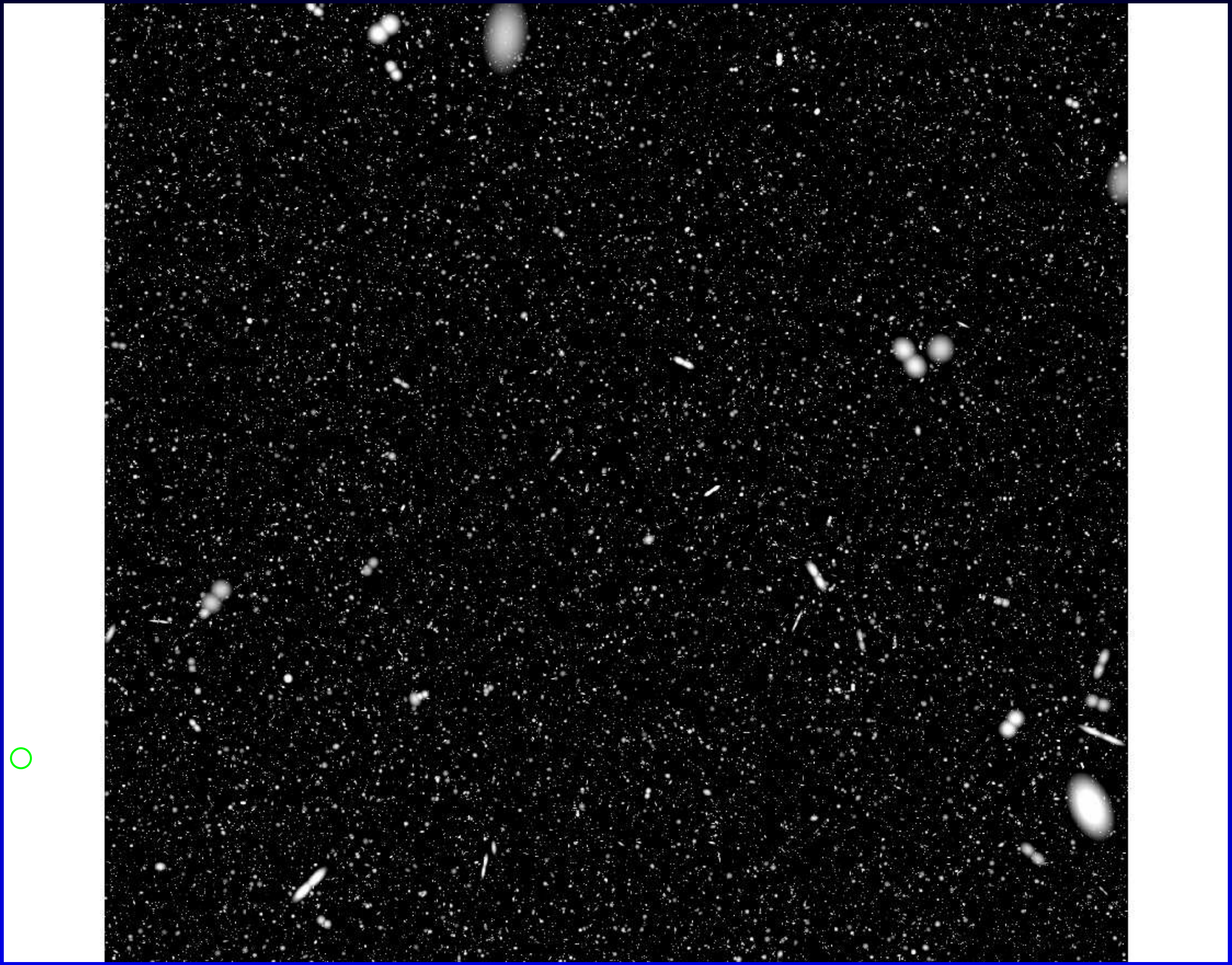
The CLAR



LAR Aerostat

Ultimate Goal: nanoJansky Sky



Time Variable Beam

- Will be a problem for a number of new or proposed instruments.
- In an SKA composed of LARs, at zenith beam is symmetrical.
- As the LAR tracks away from the zenith, the primary beam becomes elliptical with the major axis along a line that runs from the zenith to the horizon. The factor that it is stretched is $1/sin(\mathrm{elevation})$.
- The voltage pattern (E) will depend on both the distance from the field centre ($\vec{\rho}$) and the antenna position ($\vec{r}$). Antennas many hundreds of kilometres apart will observe the same source at slightly different elevations.

The Impact on Images

- Time variable primary beam gives a time variable gain that is a function of position within the field of view
- Uncorrected time variable gain generates artificial source structure
  - How to separate time variable visibility changes (due to source structure) from time variable gain effects?
  - Important for long integrations to nJy level
  - Standard ‘Clean’ does not work
- ‘Snapshots’ will give wrong flux density
- We can investigate, and solve for, these effects with the help of the Measurement Equation.

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Measurement Equation

- For k 'real' incoherent sources, observed with a 'real' telescope, we have:

$$\vec{V}_{\mathsf{ij}} = \frac{1}{\Delta \mathsf{t} \Delta \mathsf{f}} \int d\mathsf{t} \int d\mathsf{f} \sum_k \frac{1}{\Delta \mathsf{l} \Delta \mathsf{m}} \int d\mathsf{l}\, d\mathsf{m}\ \mathsf{J_i} \otimes \mathsf{J_j}^*\ \mathsf{S}\ \vec{I}(\mathsf{l}, \mathsf{m})$$

- where the observed visibility vector $\vec{V}_{\mathsf{ij}}$ is obtained from integration over the extent of the sources ($\int d\mathsf{l}\, d\mathsf{m}$), over the integration time ($\int d\mathsf{t}$) and over the channel bandwidth ($\int d\mathsf{f}$). The 'Stokes matrix' S is a constant 4×4 coordinate transformation matrix, while $\vec{I}(\mathsf{l}, \mathsf{m})$ is the 'Stokes vector'.

Measurement Equation Detail

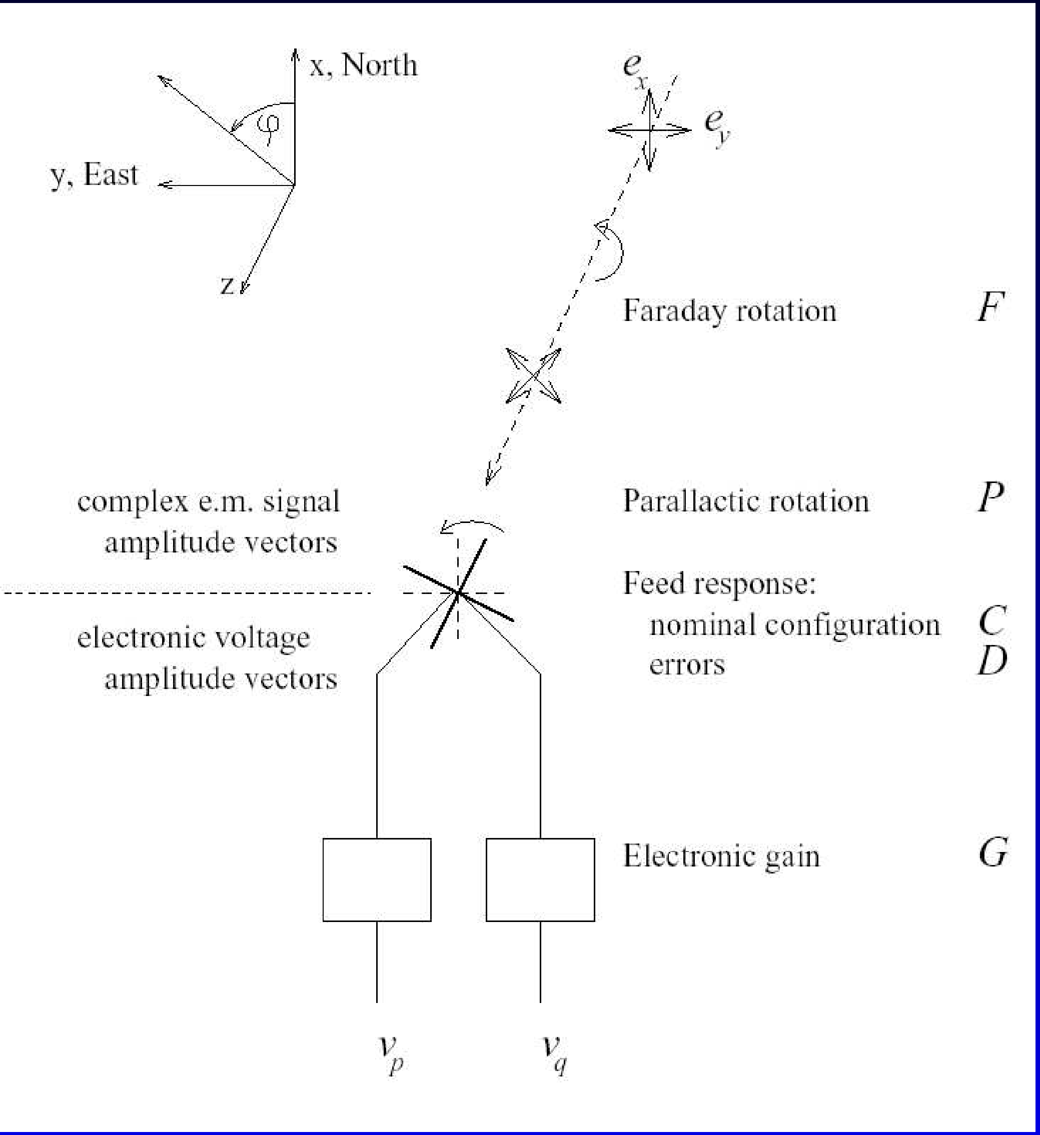
- The real heart of the Measurement Equation (M.E.) is the 'direct matrix product' $\mathsf{J_i} \otimes \mathsf{J_j}^*$ of two 2×2 *station*-based response matrices, called 'Jones matrices'.
- The 2×2 Jones matrix $\mathsf{J_i}$ for *station* i can be decomposed into a product of several 2×2 Jones matrices, each of which models a specific *station*-based instrumental effect in the signal path (see Hamaker, Bregman, Sault papers and aips++ notes from Noordam and Cornwell).

$$\mathsf{J_i} = \mathsf{G_i}\,[\mathsf{H_i}]\,[\mathsf{Y_i}]\,\mathsf{B_i}\,\mathsf{K_i}\,\mathsf{T_i}\,\mathsf{F_i} = \mathsf{G_i}\,[\mathsf{H_i}]\,[\mathsf{Y_i}]\,(\mathsf{D_i}\,\mathsf{E_i}\,\mathsf{P_i})\,\mathsf{K_i}\,\mathsf{T_i}\,\mathsf{F_i}$$

Jones Matrix Definitions

$\mathsf{F_i}(\vec{\rho}, \vec{r}_\mathsf{i})$	ionospheric Faraday rotation
$\mathsf{T_i}(\vec{\rho}, \vec{r}_\mathsf{i})$	atmospheric complex gain
$\mathsf{K_i}(\vec{\rho}.\vec{r}_\mathsf{i})$	factored Fourier Transform kernel
$\mathsf{P_i}$	projected *receptor* orientation(s) w.r.t. the sky
$\mathsf{E_i}(\vec{\rho}, \vec{r}_\mathsf{i})$	voltage primary beam
$\mathsf{D_i}$	position-independent *receptor* cross-leakage
$[\mathsf{Y_i}]$	commutation of *IF-channels*
$[\mathsf{H_i}]$	hybrid (conversion to circular polarisation coord)
$\mathsf{G_i}$	electronic complex gain (*station* contributions)

Hamaker et al. View



Rationale for MeqTrees

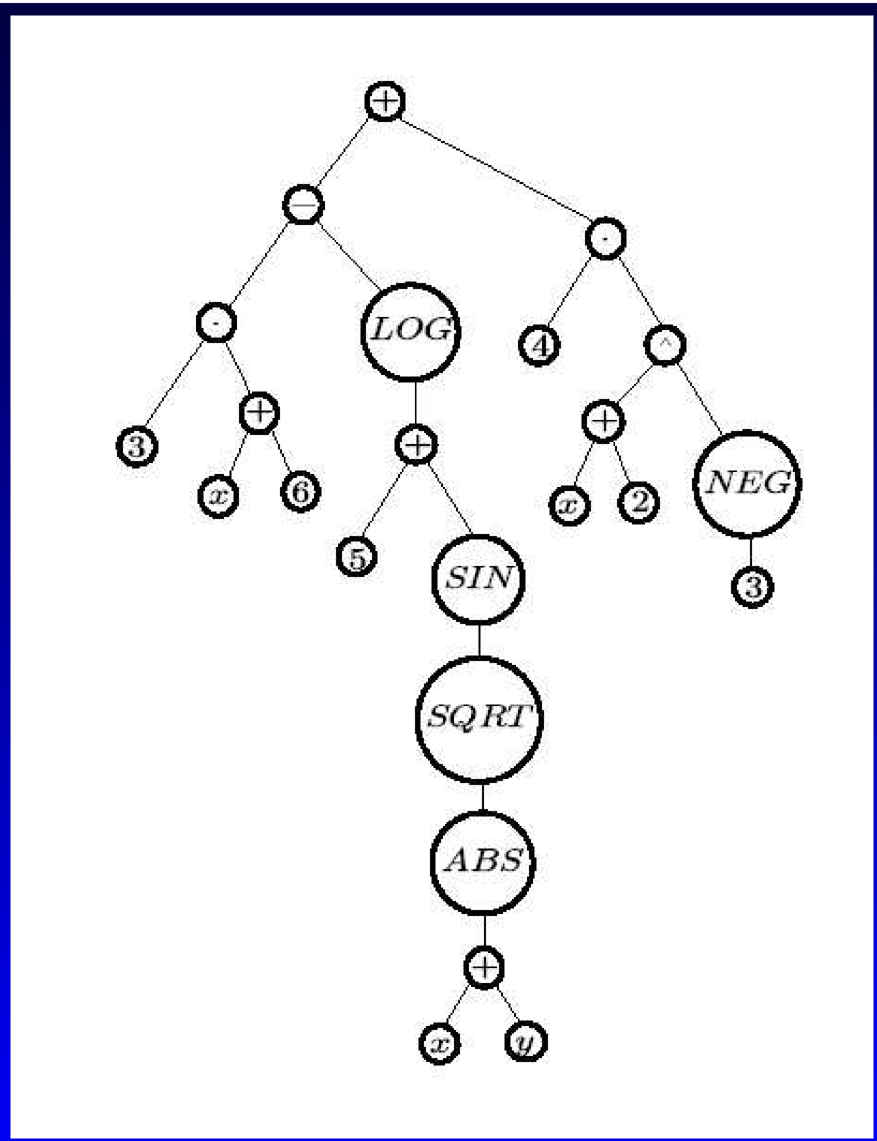
- See ADASS 2004 paper by Smirnov and Noordam.
- Why Another software module?
  - Current packages may not adequately describe M.E. for new instruments (LOFAR, SKA).
  - Current packages may be difficult to understand, modify or extend.
- Alternative - create a M.E. specific to a particular situation or for a new instrument from basic mathematical components (MeqTree nodes).
- Models of any complexity can be constructed.
- Can solve for arbitrary subsets of parameters.

MeqTrees in General

- M.E. predicts data measured with a particular instrument.
  - Model the instrument and observed data
  - Use for both system calibration and extraction of data parameters
  - Work mostly with Fourier (Visibility) data
- Procedure
  - Implement model in software using tree structure
  - Use apriori guesses to set model parameters
  - Compare observed data with predicted values
  - Adjust model parameters for best fit
- Trees have some similarity to Reverse Polish Notation

Math Expression as Tree

$$3(x+6) - log(5 + sin(\sqrt{|x+y|})) + 4(x+2)^{-3}$$



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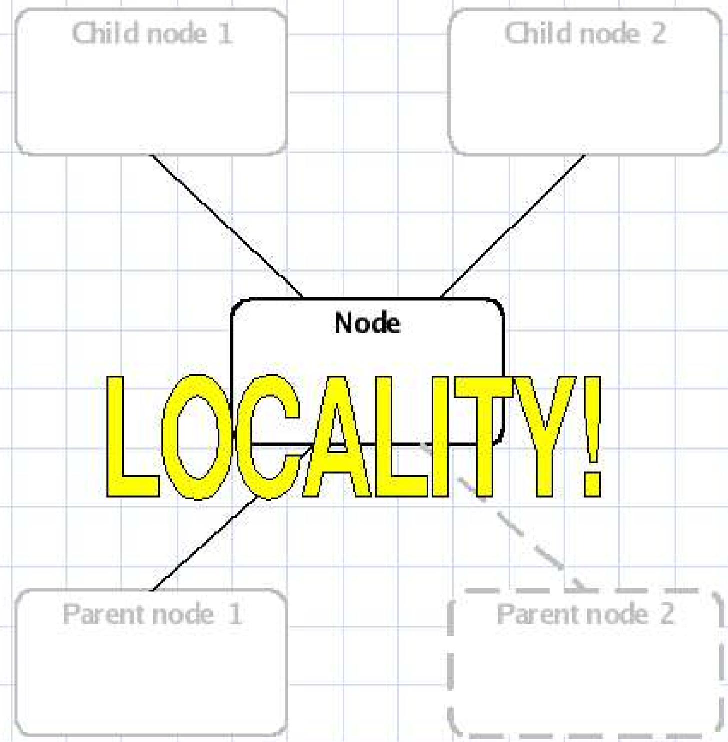
MeqTree Basics

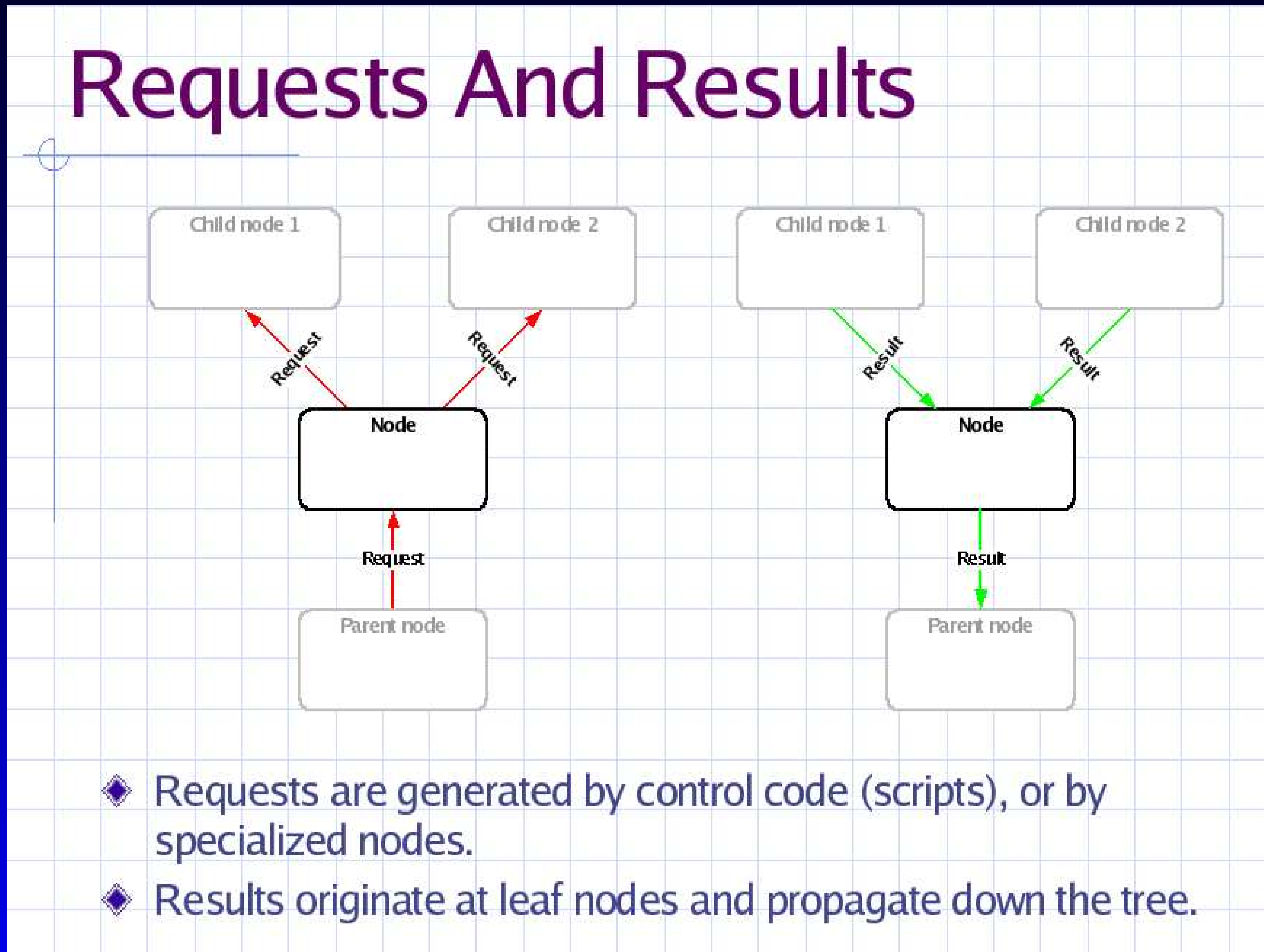
MeqTree Basics

- A MeqTree* is built out of MeqNodes.
- A Node performs some calculation based on it's children's results.
- Has no direct knowledge of parent(s).
- Abstract interface, has no direct knowledge of child *type*.

(*) *technically, a directed acyclic graph*



Request - Reply



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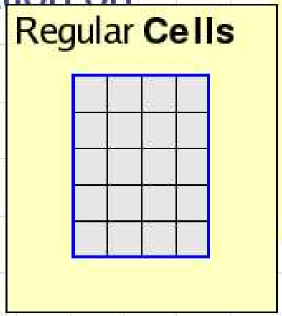
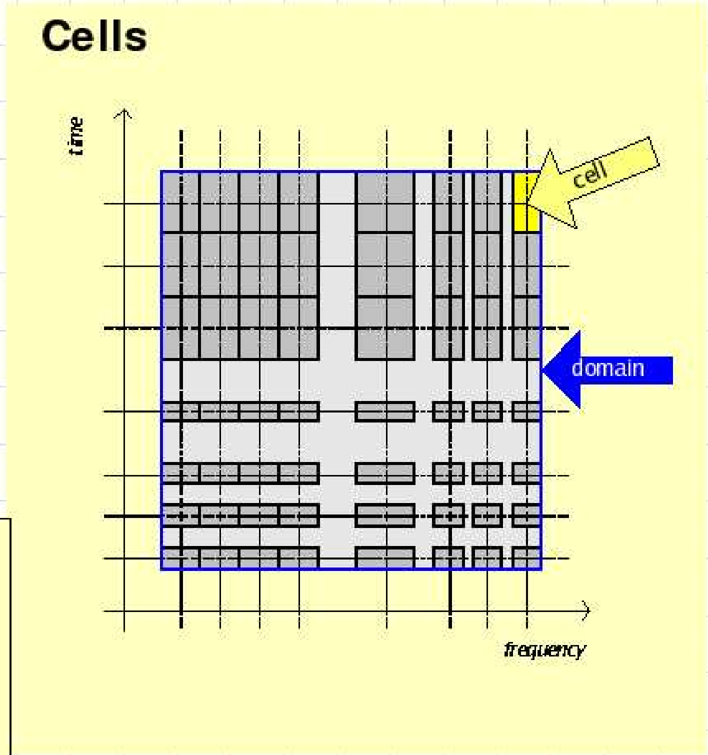
MeqTree Cell

Cells

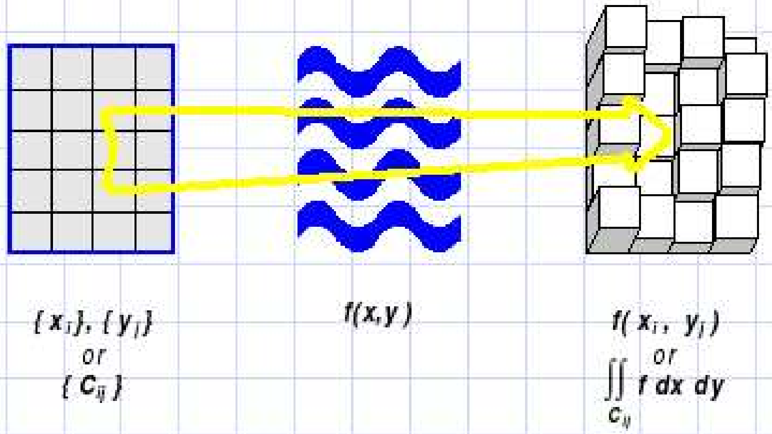
- Represents a gridding in the *x-y* (freq-time) domain
- Four vectors
  - Cell centers: $\{x_i\}, \{y_j\}$
  - Cell sizes: $\{\Delta x_i\}, \{\Delta y_j\}$
- Can be a complete tiling or partial coverage
- Maintains information on regular segments
- Simplest case:





MeqTree Vells

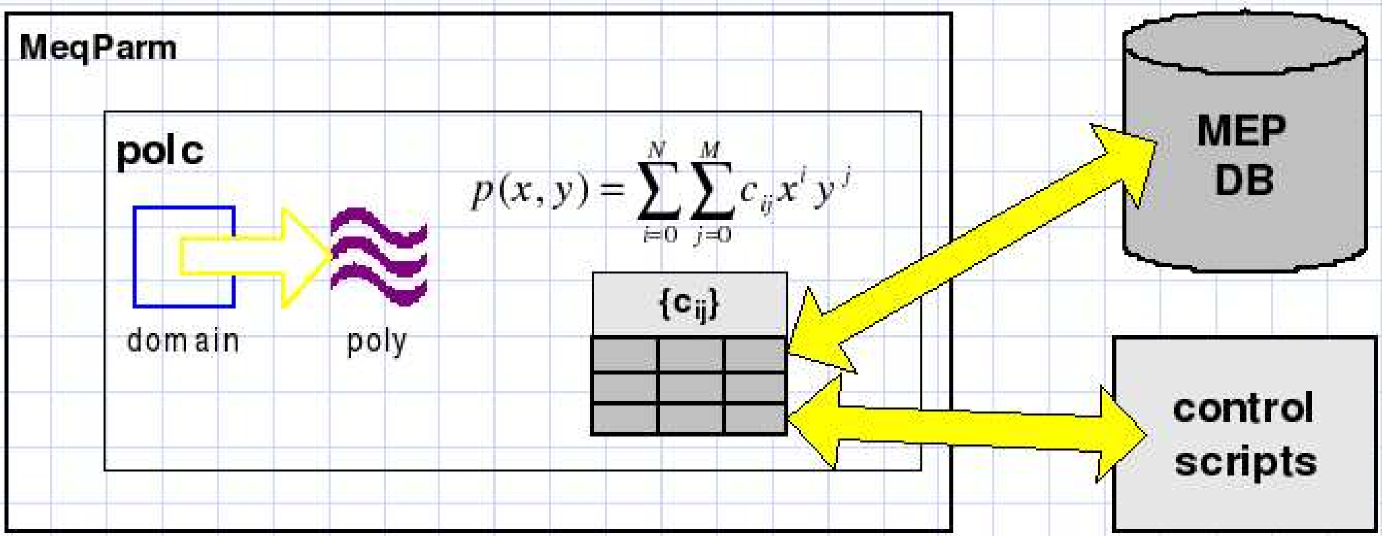
Vells



- Vells: a set of values for a given Cells, representing a measurement or a predict of a function.
- double or complex<double> values.
- Sampling at cell centre, or integration over each cell.
- Internal representation: $N_t \times N_f$ array, or a single scalar (constant Vells).

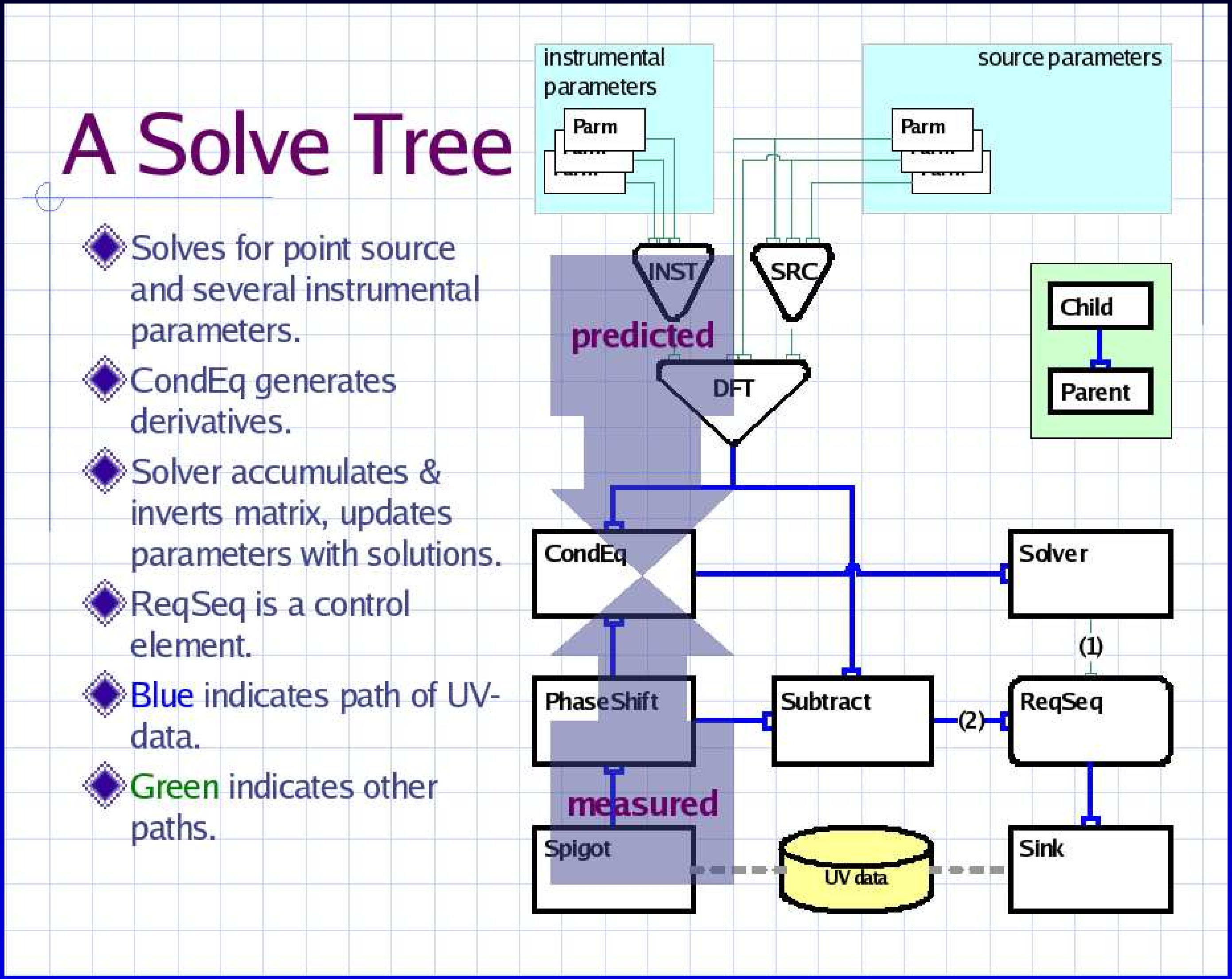
Meq Parms

MeqParms



- MeqParm: a leaf node representing a ME parameter.
- ME parameters are smooth functions of freq/time.
- Current implementation uses polynomials, other functions can be subclassed.

A Solve Tree


A Solve Tree
Solves for point source and several instrumental parameters.
CondEq generates derivatives.
Solver accumulates & inverts matrix, updates parameters with solutions.
ReqSeq is a control element.
Blue indicates path of UV-data.
Green indicates other paths.
instrumental parameters
source parameters
Parm
Parm
INST
SRC
predicted
DFT
Child
Parent
CondEq
Solver
(1)
PhaseShift
Subtract
(2)
ReqSeq
measured
Spigot
UV data
Sink

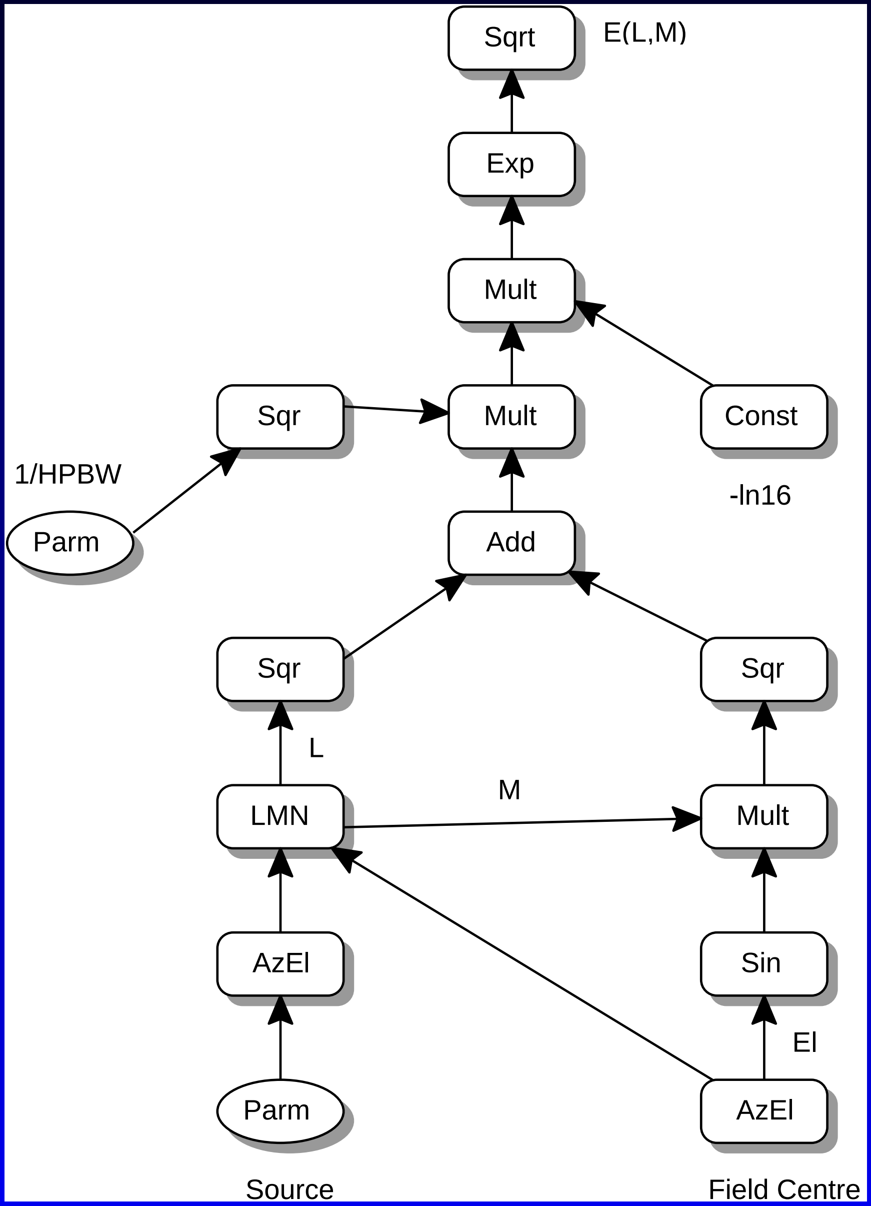
LAR Primary Beam Equation

- The voltage beam pattern, E, of an LAR measured at the position of a source whose direction coordinates L and M are defined with respect to the field centre in an AzEl reference frame can be given as:

$$\mathrm{E(L,M)} = \sqrt{\exp(-\ln 16 \times (\frac{1}{\mathrm{HPBW}})^2(\mathrm{L}^2 + (\mathrm{M}\sin(\mathrm{El}))^2))}$$

- HPBW = half power beam width at zenith
- El = elevation of field or tracking centre

The LAR Beam as a MeqTree



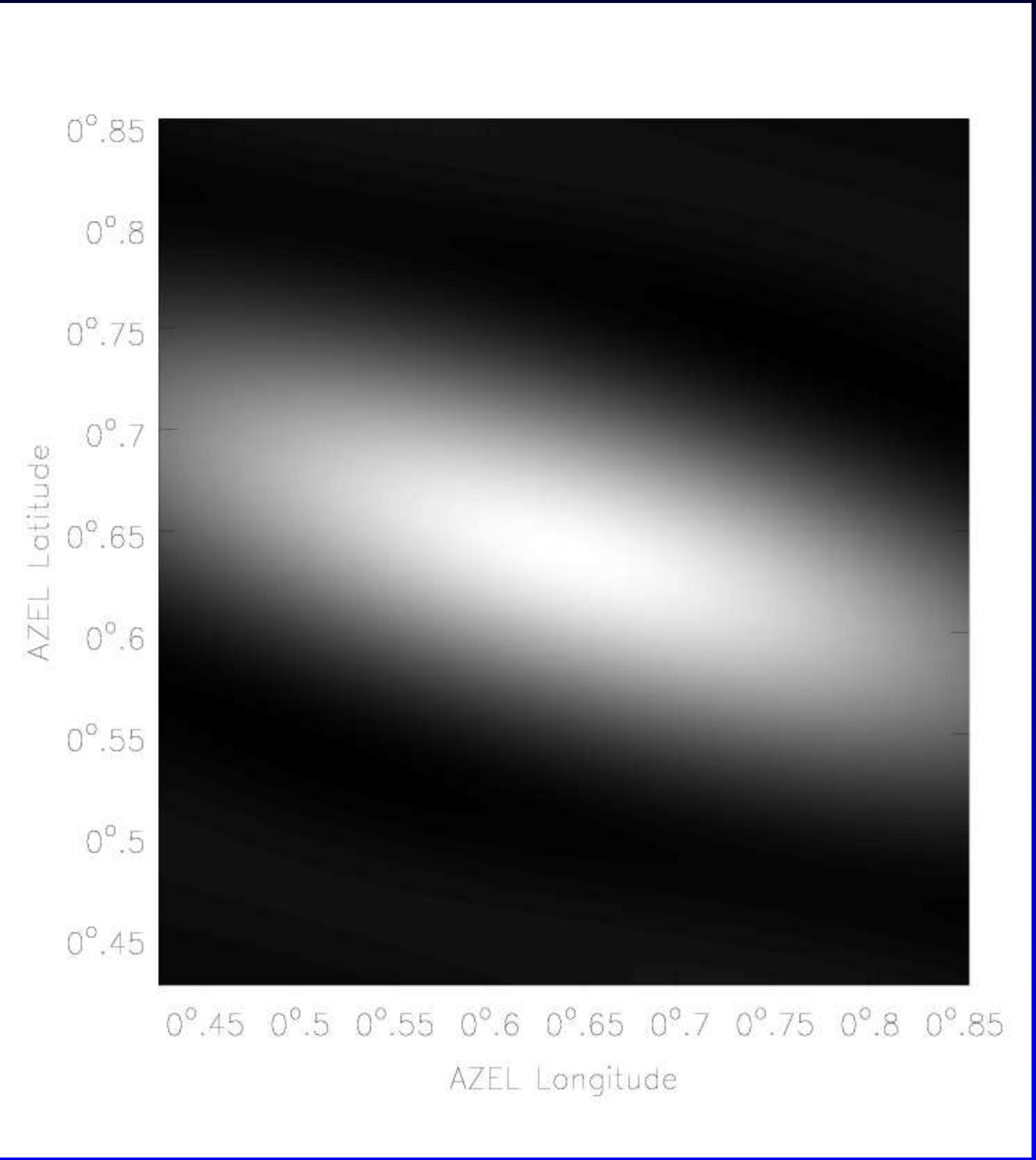
Simple Experimental Model

- Use every second station in VLA 'C' array configuration and multiply relative station coordinates by factor of 10.
- Make dish diameters 250 metres.
- Put ten 1 Jy sources at random positions inside 3 arcmin field of view with field centre at 33 degree declination.
- Observe field from -4 hrs hour angle to +4 hrs hour angle with 6 sec integration.
- Data set: 4800 integrations x 91 baselines (one frequency).
- Solve for HPBW and source flux densities by adapting an earlier MeqTrees script written by Michiel Brentjens (thanks Michiel!)

‘Control’ Observation

- Use aips++ Newsimulator and Vpmanager
  - Obtain basic aips++ simulation script (thanks Sanjay!)
  - Adapt ‘observation’ above to aips++ ‘newsimulator’.
  - Calculate average LAR beam every 20 minutes.
  - Multiply theoretical sky by average LAR beam in this time slice (after appropriate coordinate transformations).
  - Invert result into UV plane.
  - Over 8 hours we have 24 time slices.
  - Thanks to Tim Cornwell!
- Can compare output of this method to MeqTrees simulation.

Starting SKA Beam



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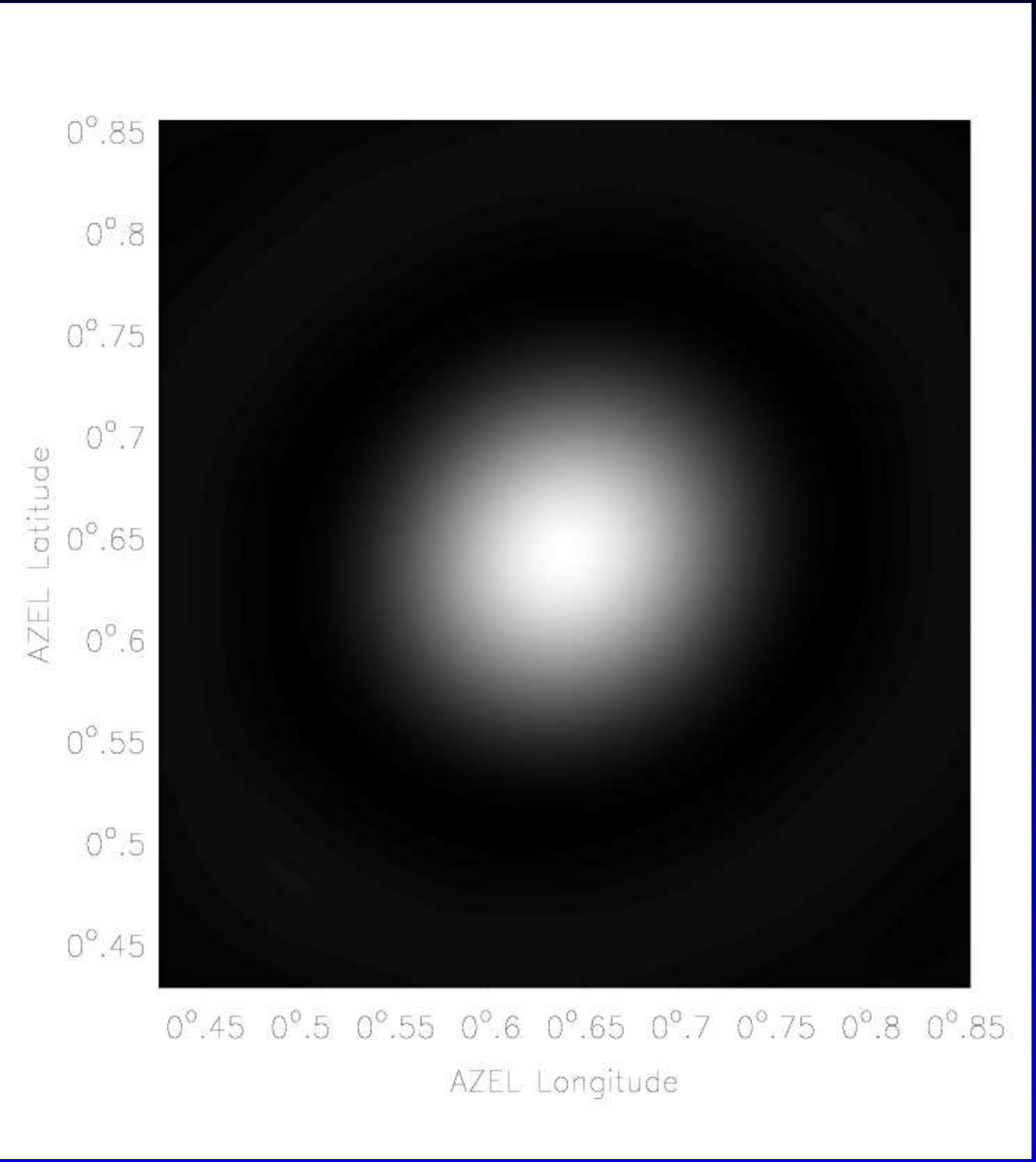
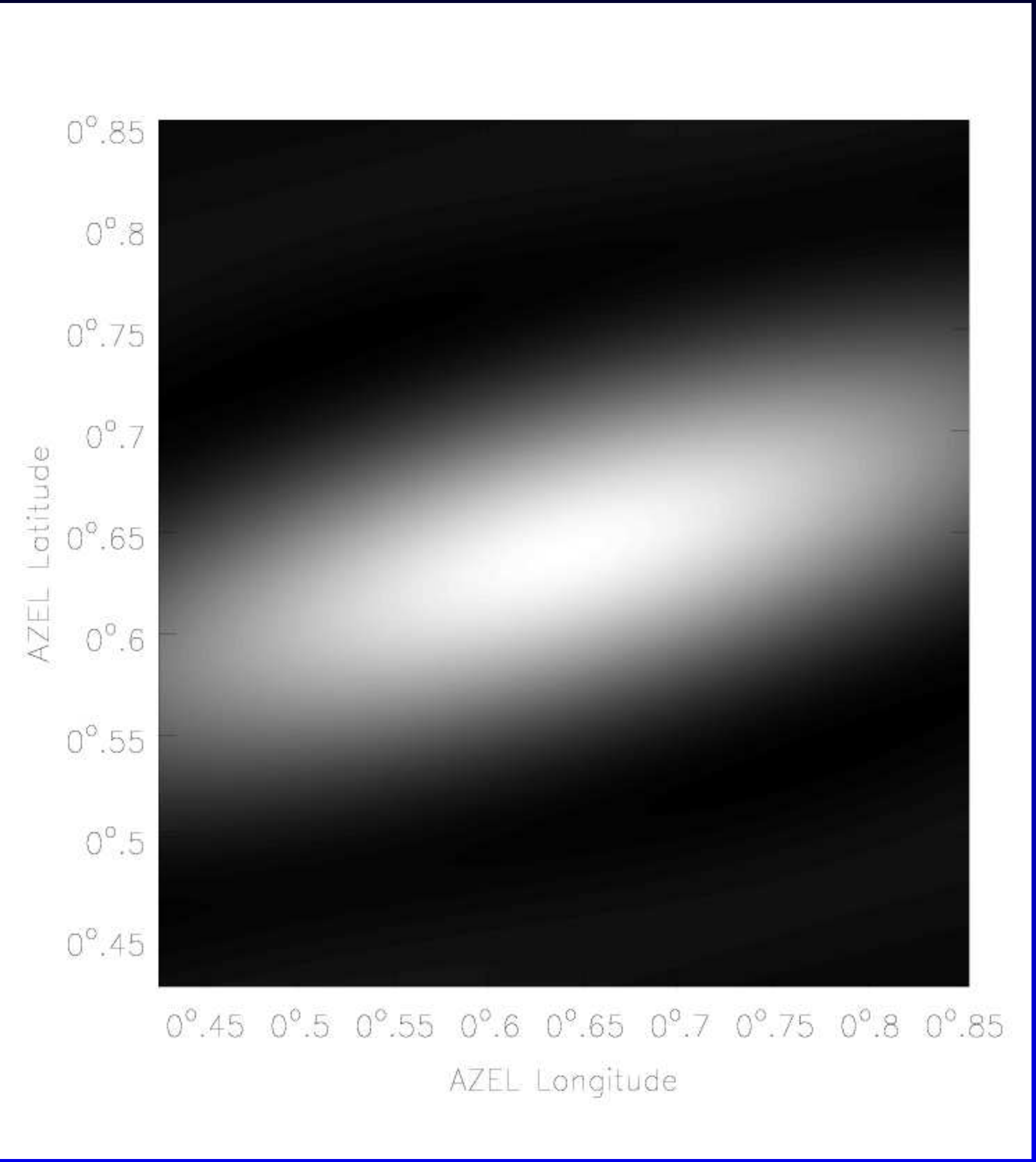
Source Near Zenith



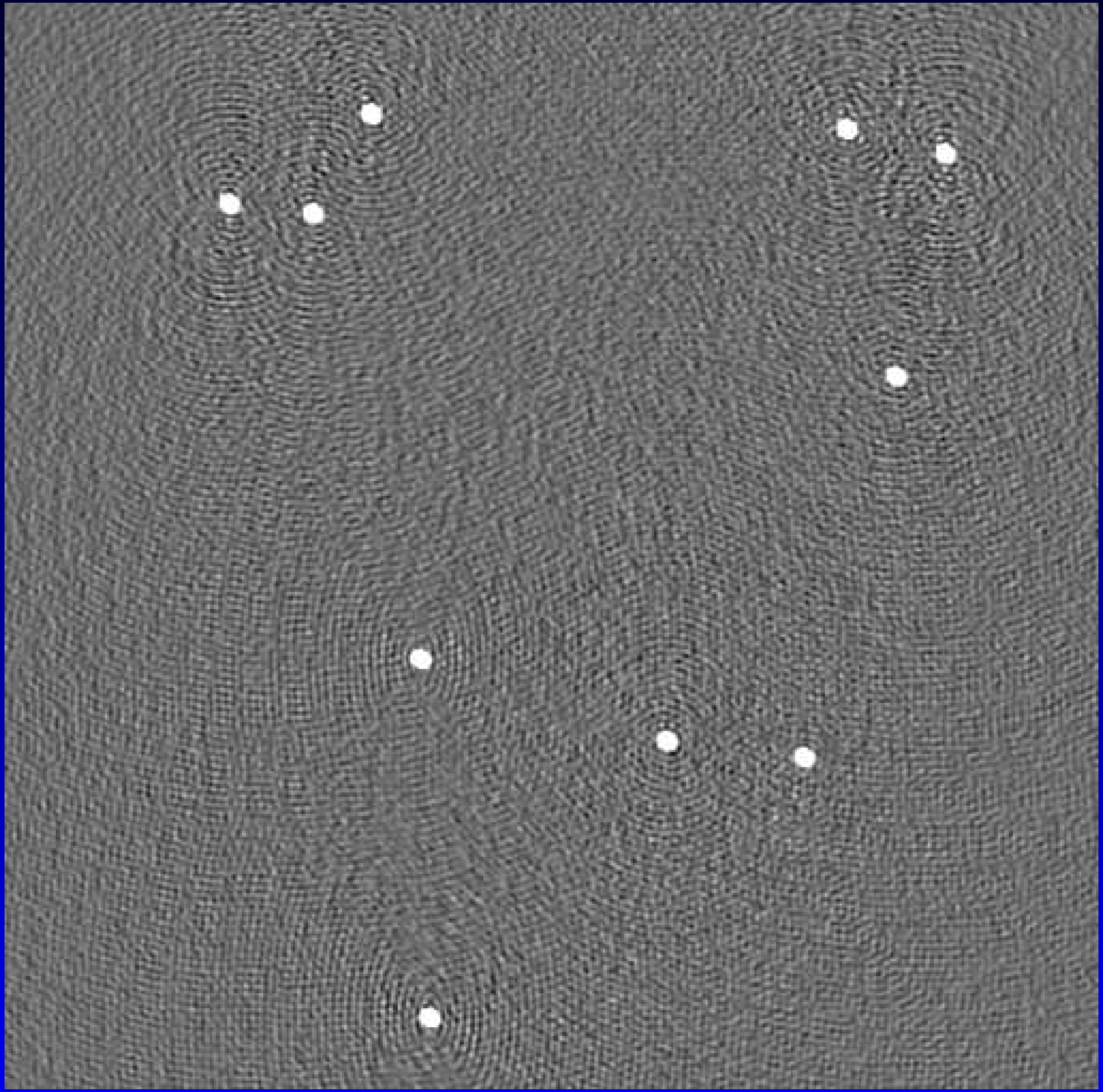
Ending SKA Beam



Movie of LAR Beam

http://www.atnf.csiro.au/people/Tim.Cornwell/clarmovie.gif

Test Field - 3 arcmin FOV



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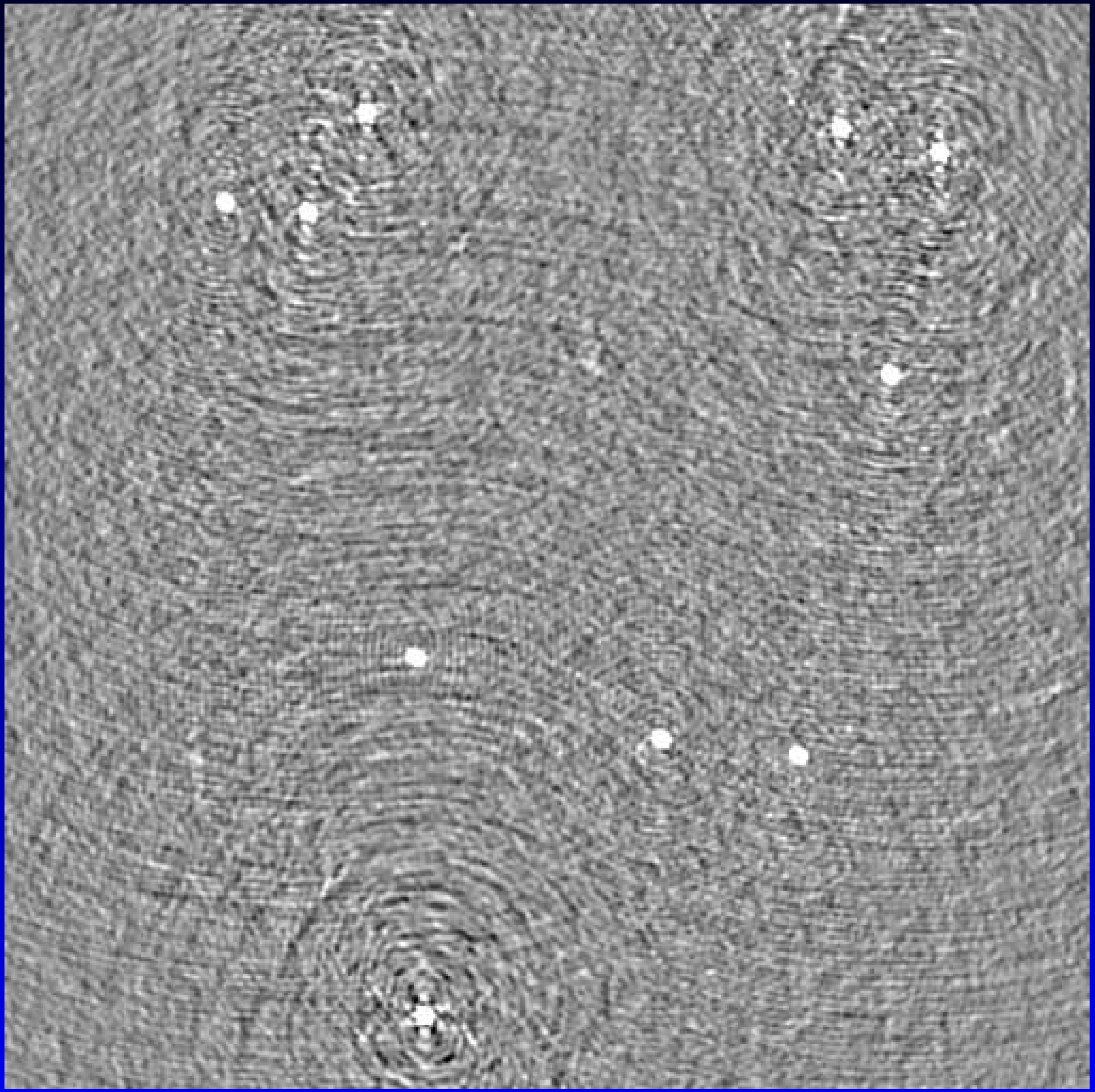
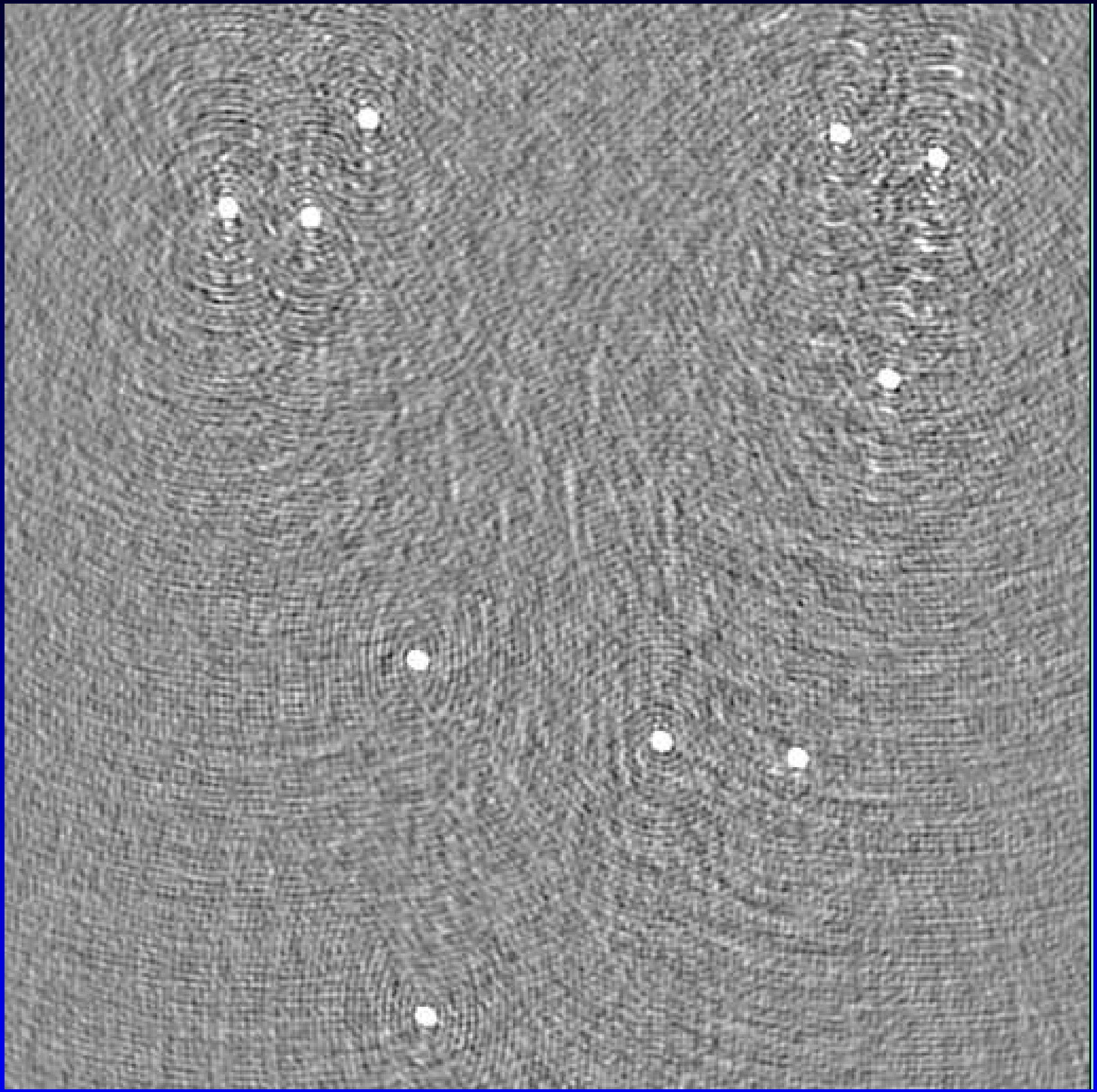
Newsimulator Field as Cleaned



MeqTree Field as Cleaned

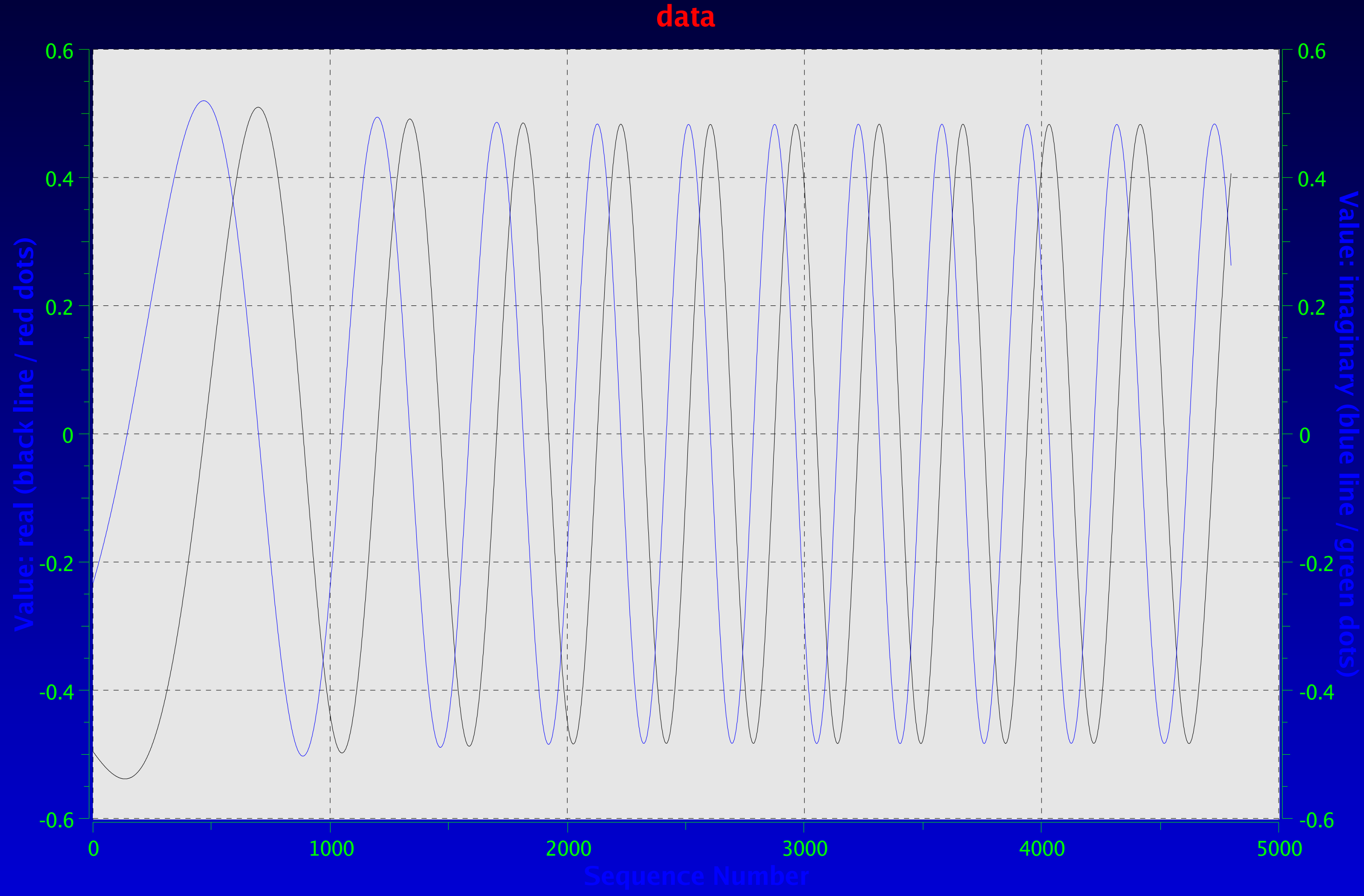


Southern Source in L

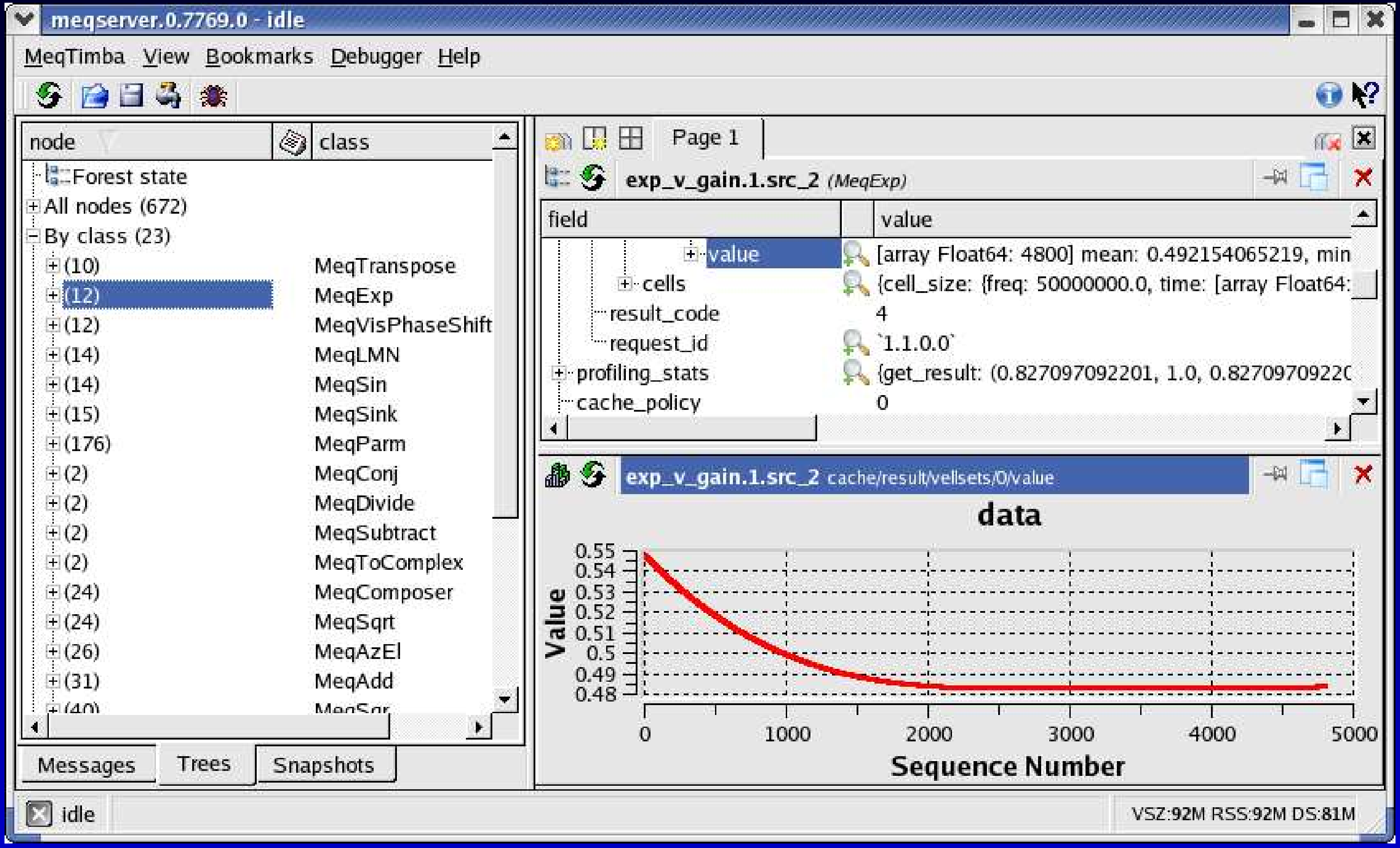
Southern Source in M

Resulting Power Pattern

Southern Source Fringes



The MeqBrowser



Apriori Guesses

- 3 arcmin for HPBW
- Observed flux densities for sources

Comparison Fluxes: 4.3 arcmin HPBW

Newsimulator		*MeqTrees*	
Observed	Fitted	Observed	Fitted
0.89	1.012	0.88	1.002
0.85	1.006	0.84	0.998
0.83	0.971	0.85	0.998
0.78	1.104	0.70	0.999
0.75	1.139	0.66	1.000
0.74	1.007	0.73	1.004
0.72	1.008	0.72	1.002
0.68	0.921	0.74	0.999
0.67	1.042	0.64	0.994
0.61	0.952	0.64	1.001

Conclusions and Experience

- MeqTrees system can be used to accurately model and derive LAR parameters!
- Direct use of visibility data
  - Clean beam is normally calculated on the basis of UV sampling
  - Not valid for case of variable beam; gain is position and time dependant
- Lots of nodes - this small test system required 6000 nodes
- Tree must be constructed for specific imaging task
  - Gives greatest accuracy
  - But academic astronomers want easy to use system

What's Next?

- Handle extended sources
- Need 600 MHz bandwidth split into 300 channels to get to nanoJansky level at 1400 MHz
  - So model fi elds having sources with different spectra

'Astronomy is terrifying. It describes a hell in which we seem to be the only inhabitants.'

Louis Dudek, Canadian poet

acknowledgements

- MeqTrees team, and especially
  - Jan Noordam and Oleg Smirnov for advice, assistance, and slides
  - Michiel Brentjens for prototype solutions script
- Sanjay Bhatnagar and Tim Cornwell for aips++ simulation scripts
- John Kennedy for math tree example

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