

Fundamentals of radio interferometry I

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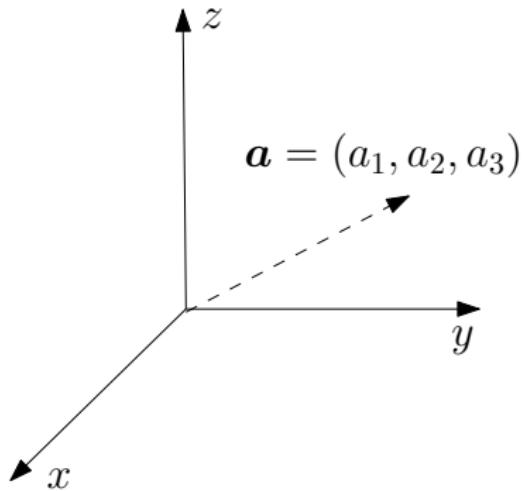
Overview

- 1 Background
- 2 Positional Astronomy (locating the stars)
- 3 Interferometry: The Fourier transform
- 4 uv -Coverage

References

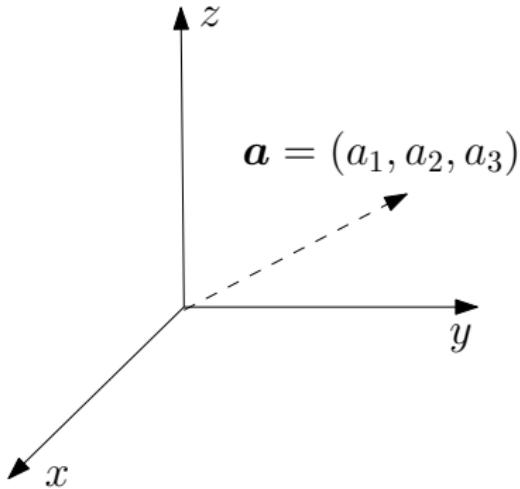
- ① **TCP:** G.B. Taylor et al., Synthesis imaging in radio astronomy II, 1998
- ② **TSM:** A. Thomson et al., Interferometry and synthesis in radio astronomy, second edition

vectors



- ① Vector length: $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.
- ② Vector addition: $\mathbf{a} + \mathbf{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.
- ③ Vector subtraction: $\mathbf{a} - \mathbf{b} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$.

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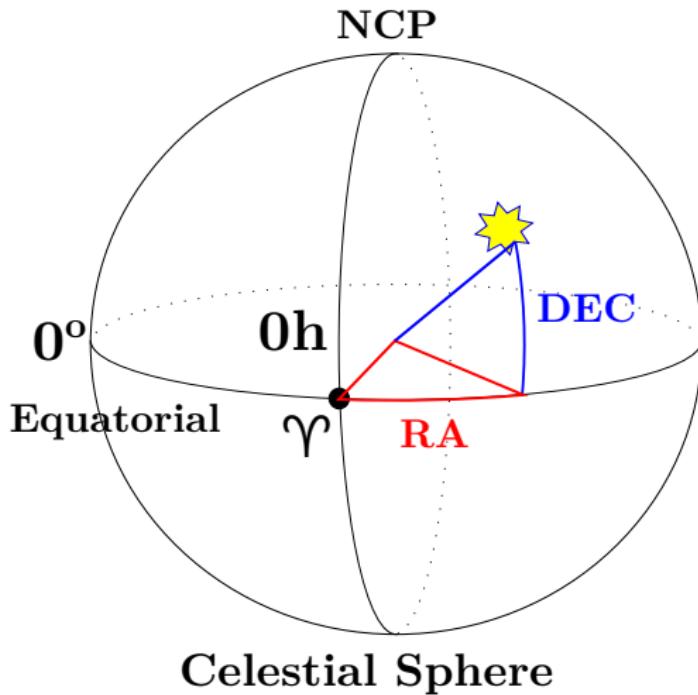
Simple conversions

- ① 1 degree ($^{\circ}$) = 60 arcminutes ($'$) = 3600 arcseconds ($''$), where 360° = circle.
- ② 1 radian (rad) = 57.296° = 206265 $''$, where 2π rad = circle.
- ③ 1 hour (h) = 60 minutes = 3600 seconds, where 24 h = circle.
- ④ 1 h = 15°

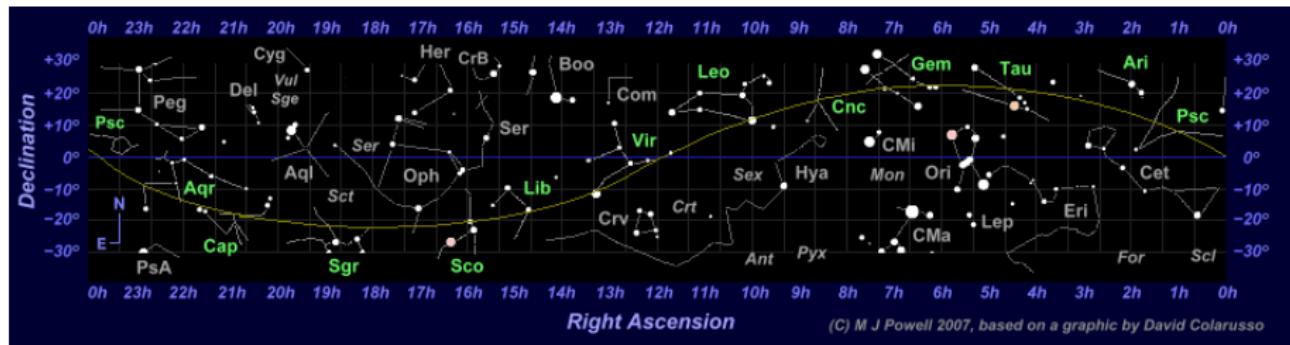
Notation

- ① δ - DEC/Declination
- ② α - RA/Right Ascension
- ③ H - Hour Angle
- ④ LST - Local Sidereal Time
- ⑤ L - Latitude
- ⑥ A - Azimuth
- ⑦ E - Elevation/Altitude

Equatorial Coordinates

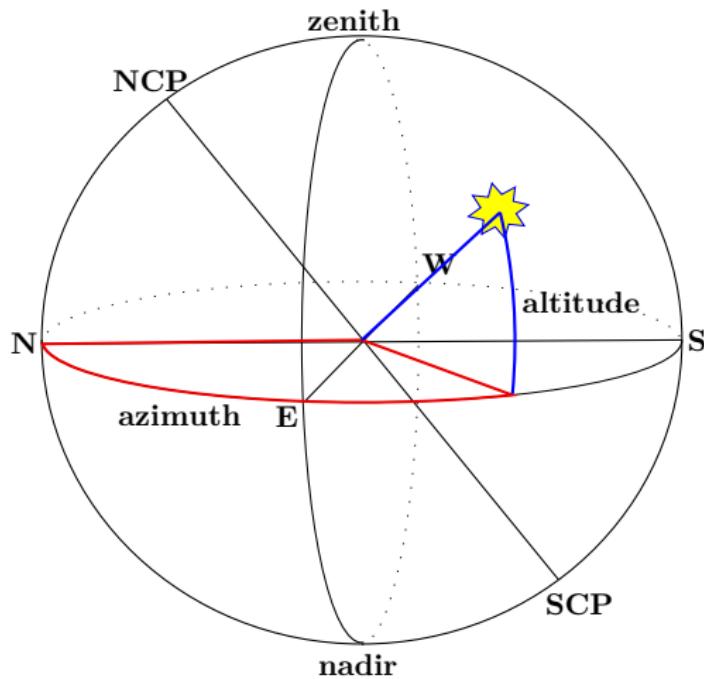


Vernal equinox (the first point of Aries)

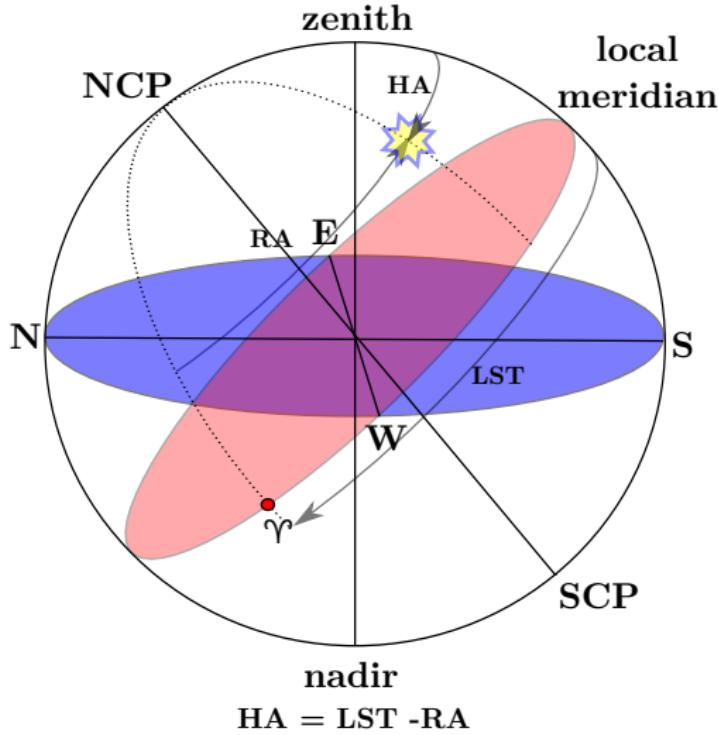


- The coordinate pair $(0h, 0^\circ)$ is the first point of Aries (the vernal equinox), the point where the sun crosses the celestial equator from south to north.
- It used to be in Aries, but due to precession (change in orientation of earth's rotation axis) it is now found in Pisces.

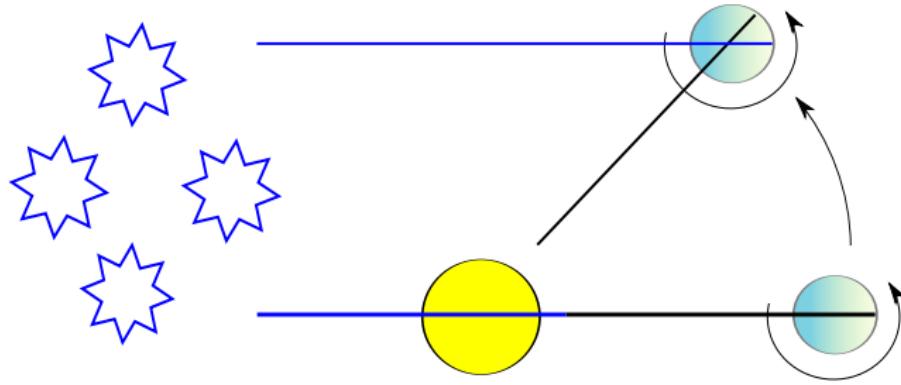
Horizontal Coordinates



Hour Angle

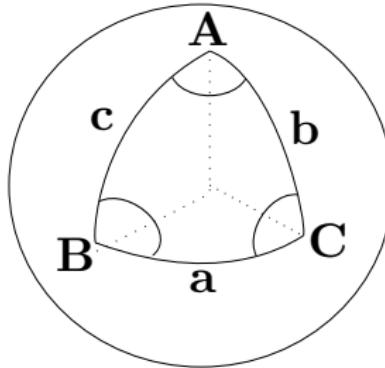


A sidereal day



The solar day is 4m longer than the sidereal day, due to the fact that the earth rotates around the sun.

Spherical Trigonometry

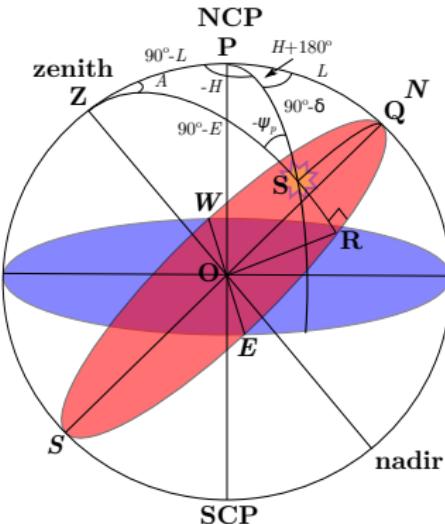


Cosine Rule: $\cos b = \cos a \cos c + \sin a \sin b \cos A$

Sine Rule: $\sin b \sin A = \sin B \sin a$

Five-part Rule: $\sin b \cos A = \cos a \sin c - \sin a \cos c \cos B$

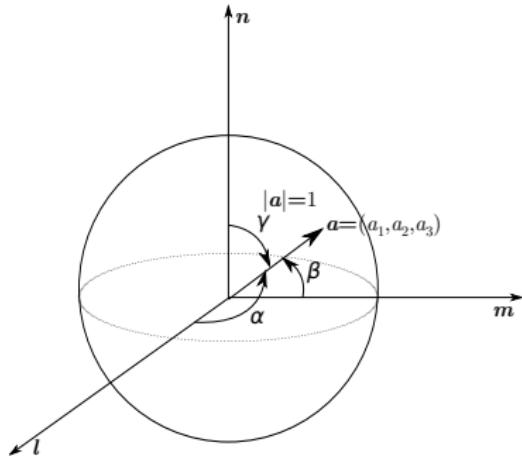
Conversion between Equatorial and Horizontal



$$\begin{aligned}\cos \delta \cos H &= \cos L \sin E - \sin L \cos E \cos A \\-\cos \delta \sin H &= \cos E \sin A \\ \sin \delta &= \sin L \sin E + \cos L \cos E \cos A\end{aligned}$$

TSM (p. 89 Eq. 4.4)

Direction Cosines



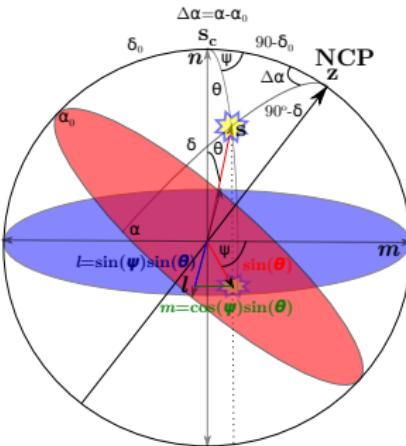
$$l = \cos \alpha = \frac{a_1}{|\mathbf{a}|}$$

$$m = \cos \beta = \frac{a_2}{|\mathbf{a}|}$$

$$n = \cos \gamma = \frac{a_3}{|\mathbf{a}|}$$

$$1 = l^2 + m^2 + n^2$$

Conversion between Equatorial and Direction Cosines



$$l = \sin \theta \sin \psi = \cos \delta \sin \Delta \alpha$$

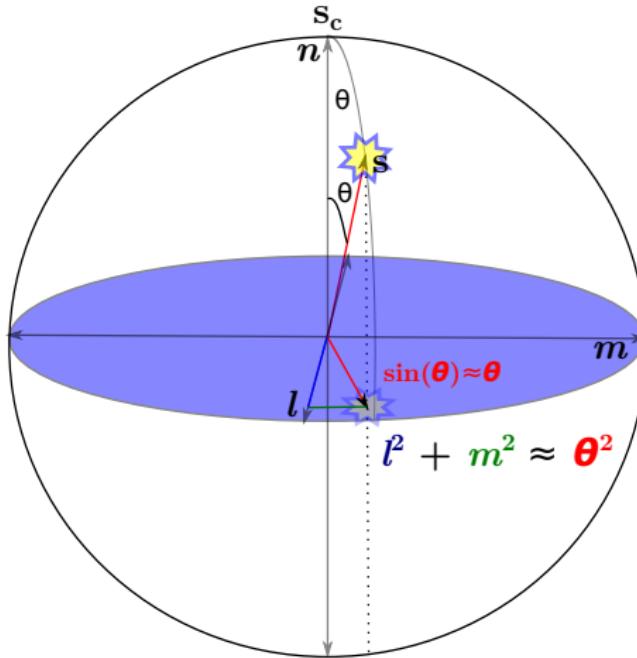
$$m = \sin \theta \cos \psi = \sin \delta \cos \delta_0 - \cos \delta \sin \delta_0 \cos \Delta \alpha$$

$$\delta = \sin^{-1} (m \cos \delta_0 + \sin \delta_0 \sqrt{1 - l^2 - m^2})$$

$$\alpha = \alpha_0 + \tan^{-1} \left(\frac{l}{\cos \delta_0 \sqrt{1 - l^2 - m^2} - m \sin \delta_0} \right)$$

TCP (p. 388 Eq. 19.8–19.12)

Understanding l and m



We therefore measure l and m in $^\circ$.

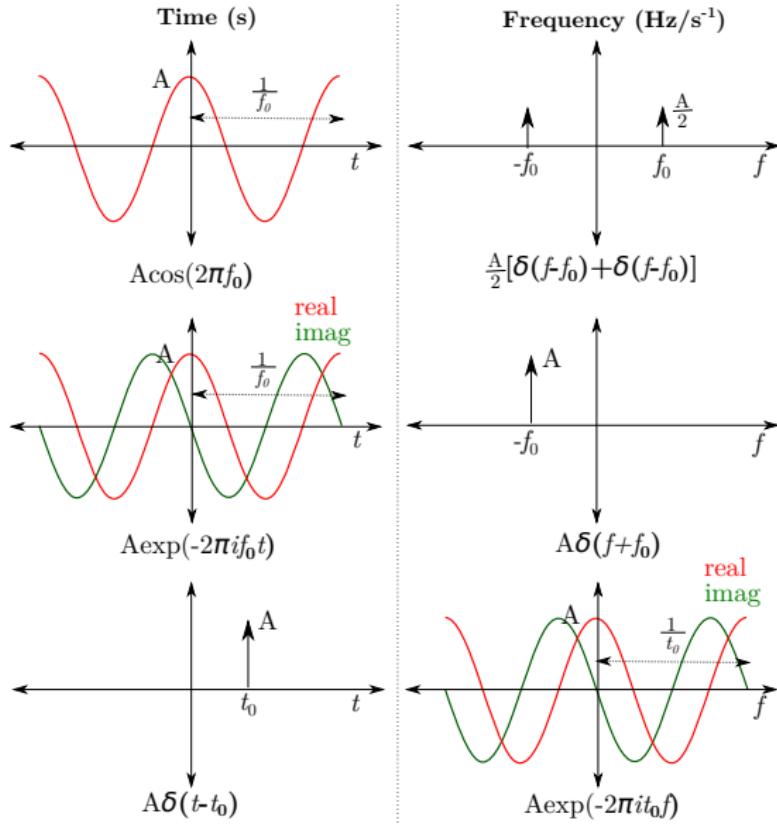
Fourier Transform



$$F(f) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-2\pi ift} dt$$

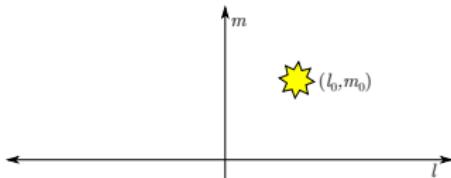
Fourier transform: Frequency decomposition of $f(t)$.

1D Fourier Transform Pairs

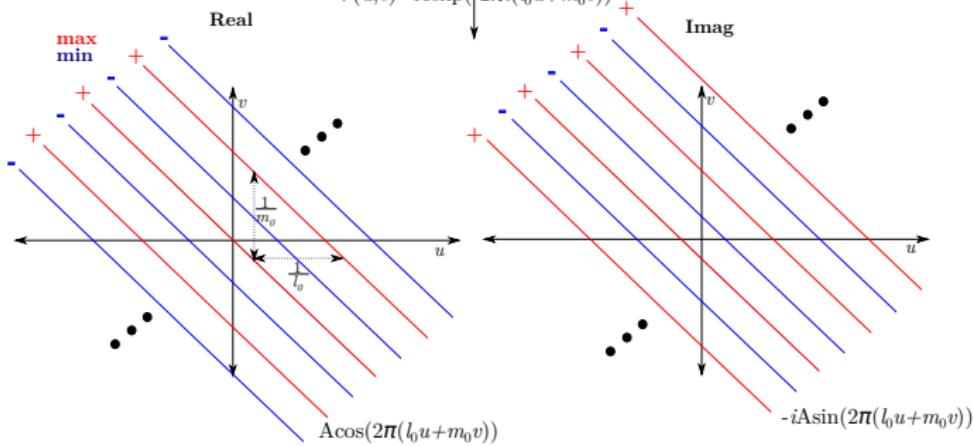


2D Fourier Transform Example

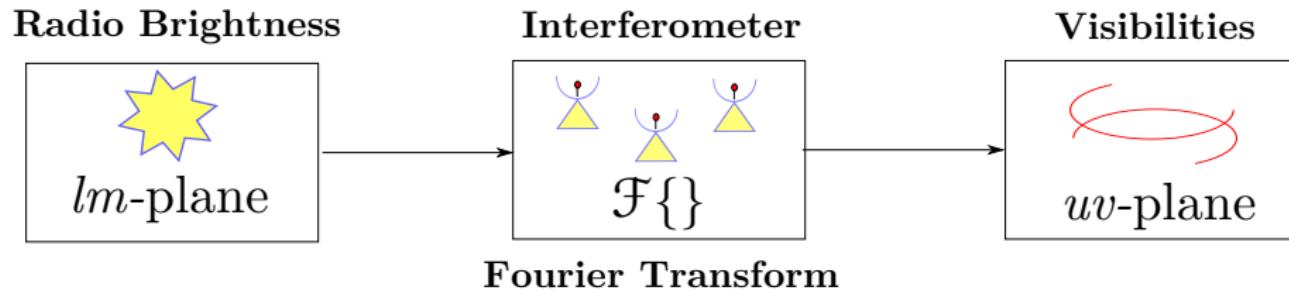
$$I(l,m) = A\delta(l-l_0, m-m_0)$$



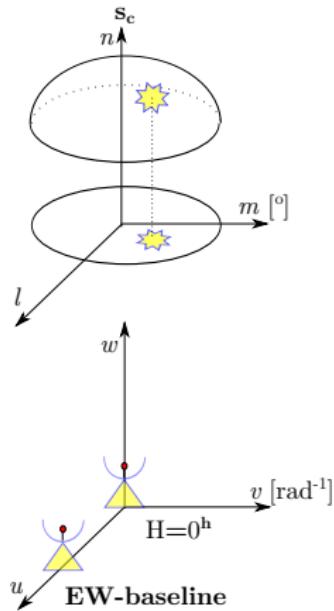
$$V(u,v) = A \exp(-2\pi i(l_0 u + m_0 v))$$



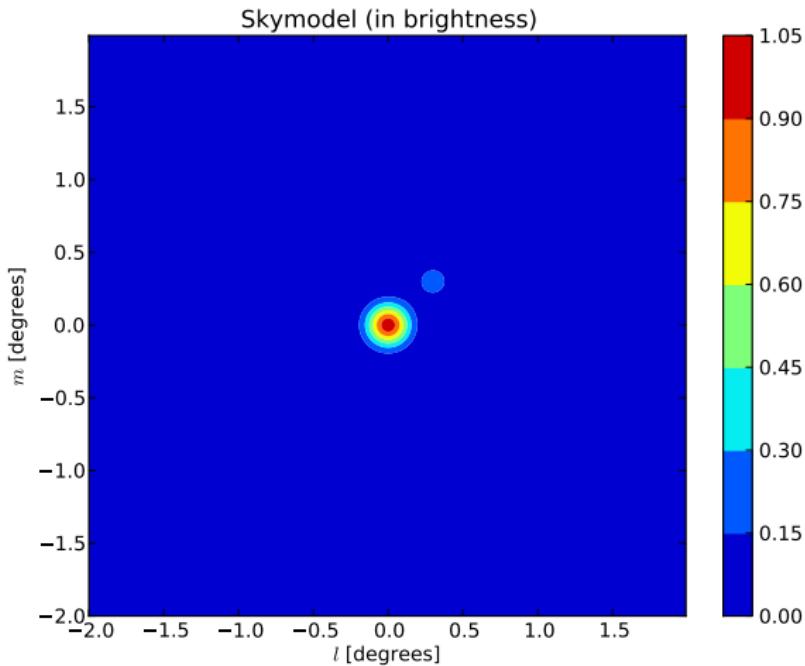
Basic Functional Diagram of Interferometer



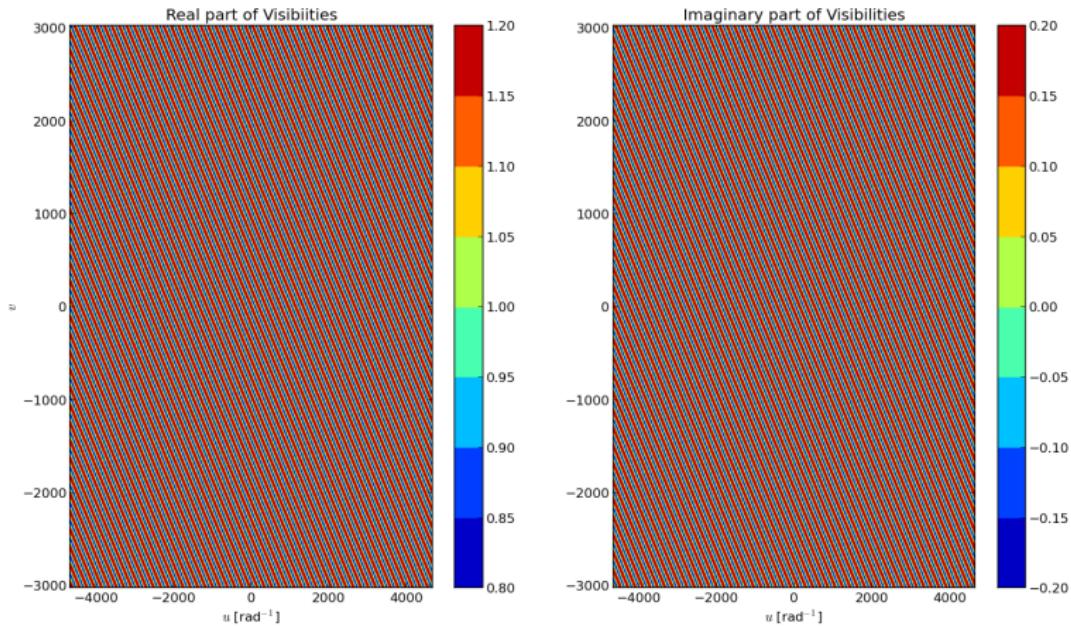
Interferometer: Coordinate systems



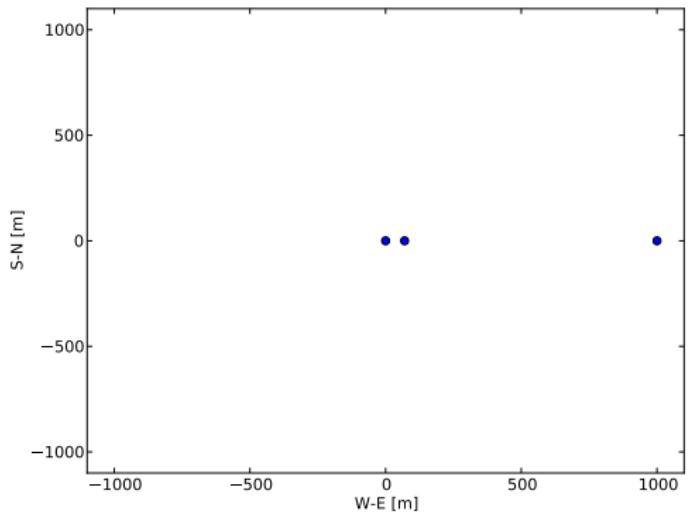
Example: Sky model



Example: Visibilities

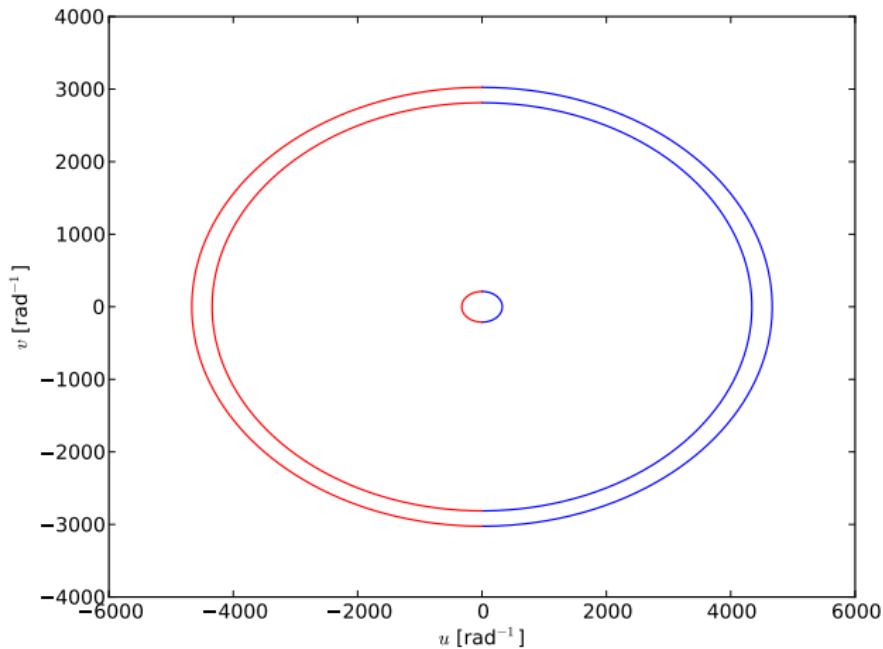


Example: ENU layout of array

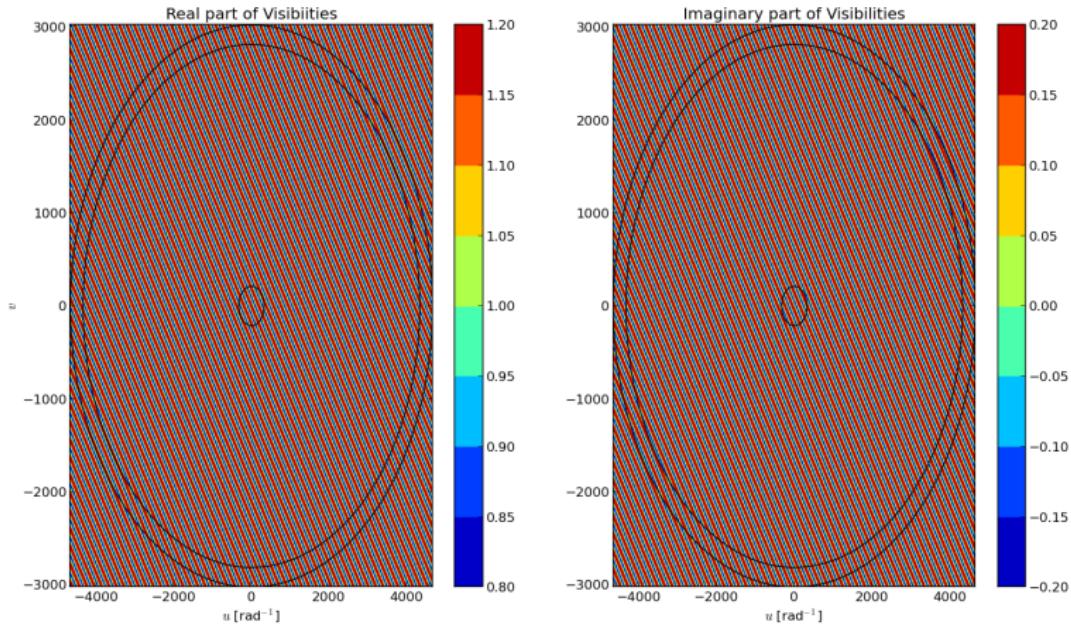


- ➊ 3 antennas, 3 baselines.
- ➋ A baseline is the vector that is formed by two antennas.

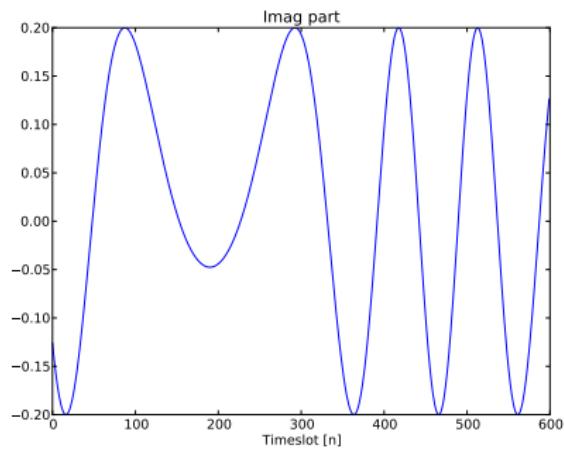
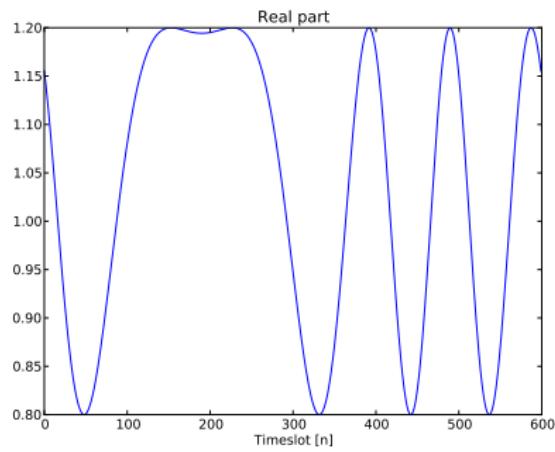
Example: *uv-tracks*



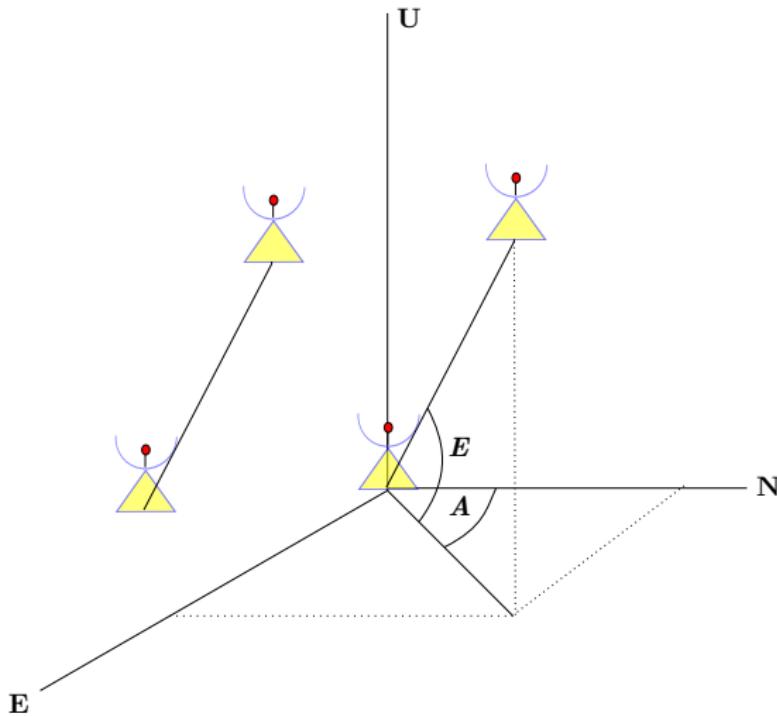
Example: Sampled visibilities



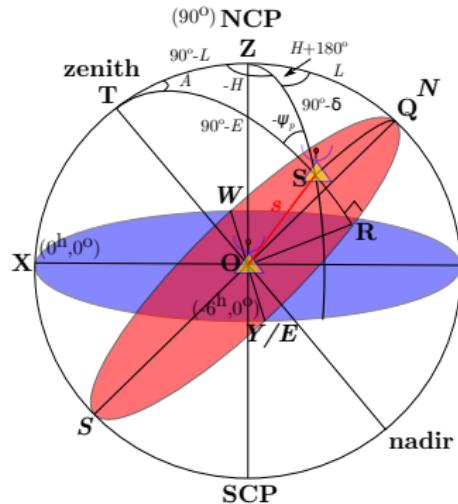
Example: Visibilities of shortest baseline



ENU (xyz) to horizontal (baseline)



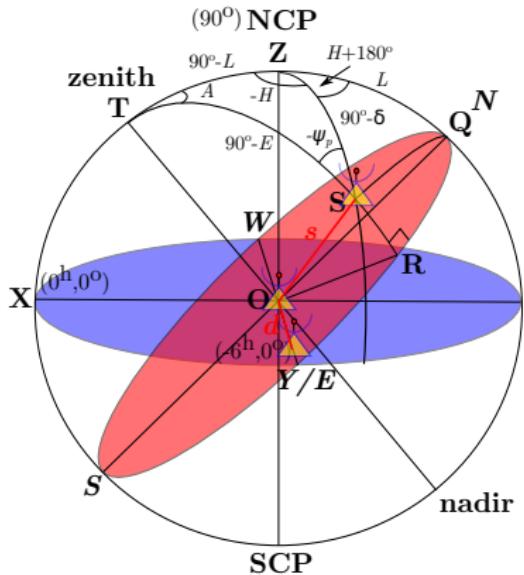
Conversion between Equatorial and Horizontal (baseline)



$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} s \cos \delta \cos H \\ -s \cos \delta \sin H \\ s \sin \delta \end{bmatrix} = s \begin{bmatrix} \cos L \sin E - \sin L \cos E \cos A \\ \cos E \sin A \\ \sin L \sin E + \cos L \cos E \cos A \end{bmatrix}$$

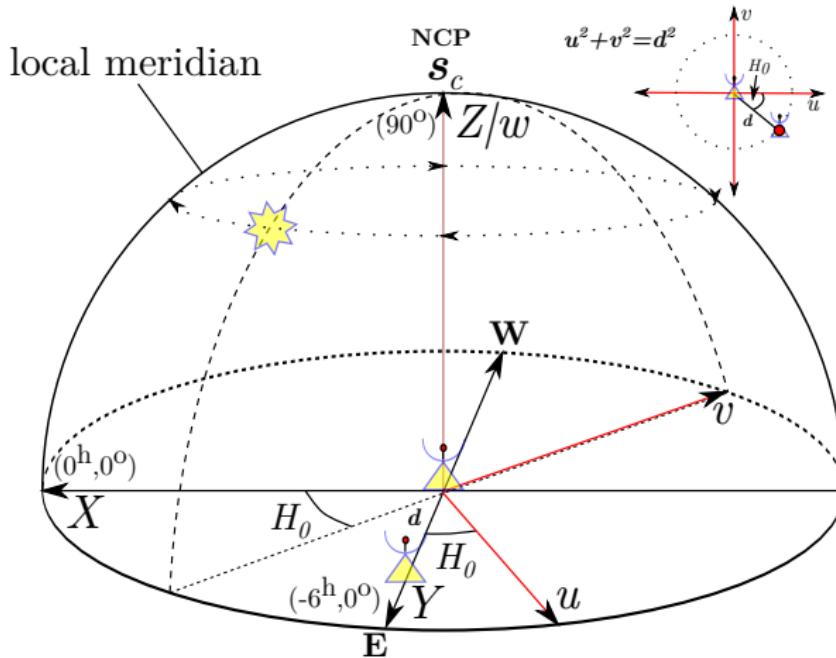
TSM (p. 89 Eq. 4.4)

XYZ coordinates of EW baseline

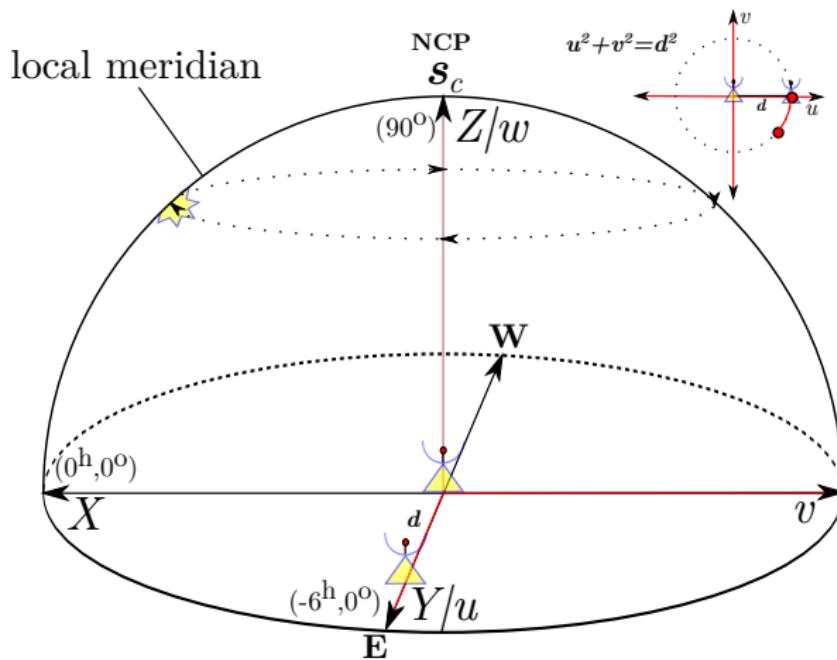


$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix} = d \begin{bmatrix} \cos L \sin 0^\circ - \sin L \cos 0^\circ \cos 90^\circ \\ \cos 0^\circ \sin 90^\circ \\ \sin L \sin 0^\circ + \cos L \cos 0^\circ \cos 90^\circ \end{bmatrix}$$

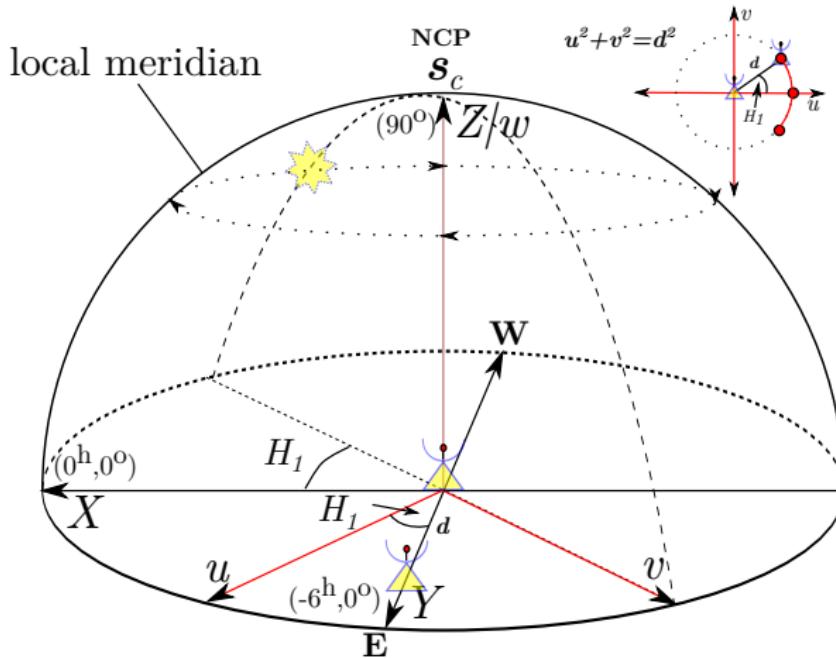
EW example I: $\delta = 90^\circ$



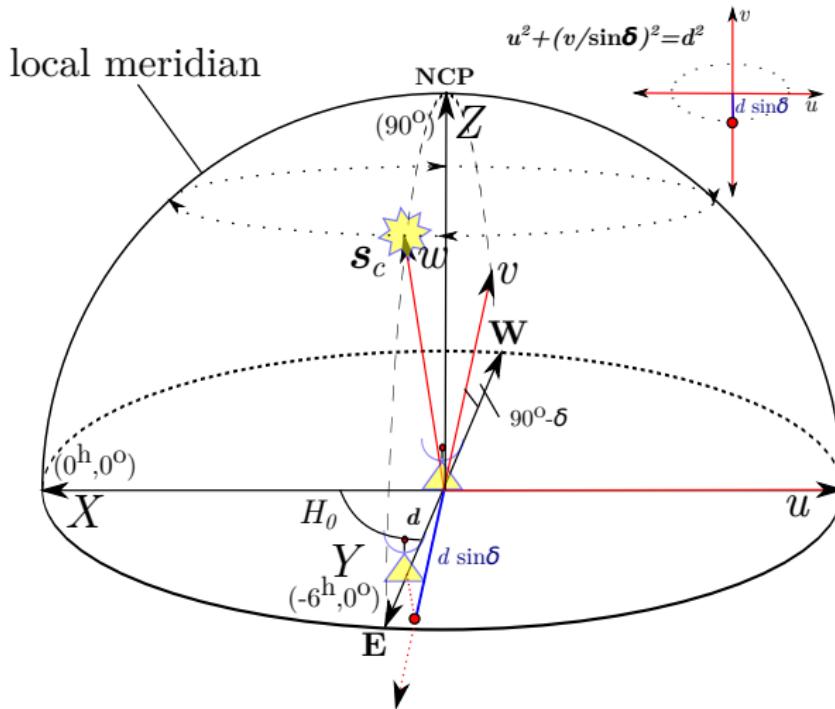
EW example II: $\delta = 90^\circ$



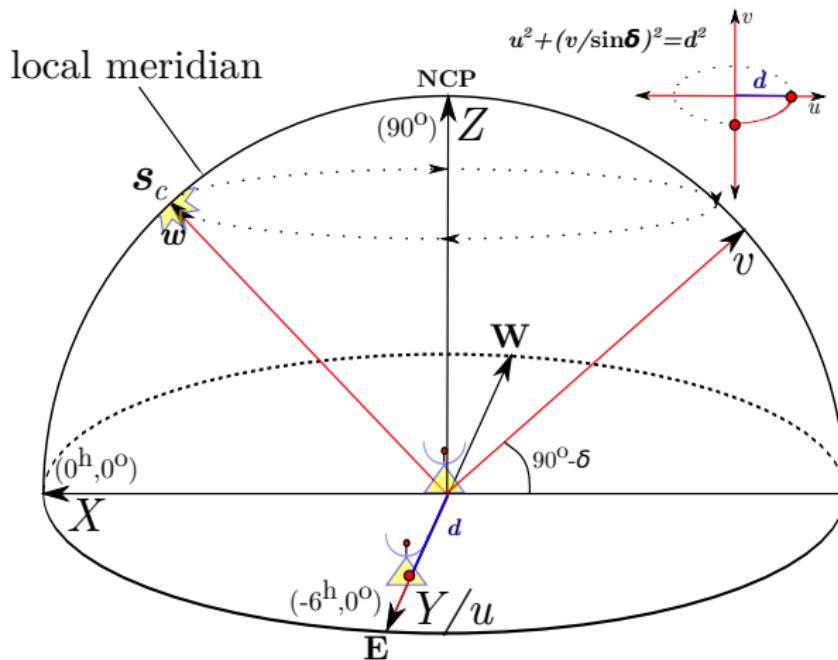
EW example III: $\delta = 90^\circ$



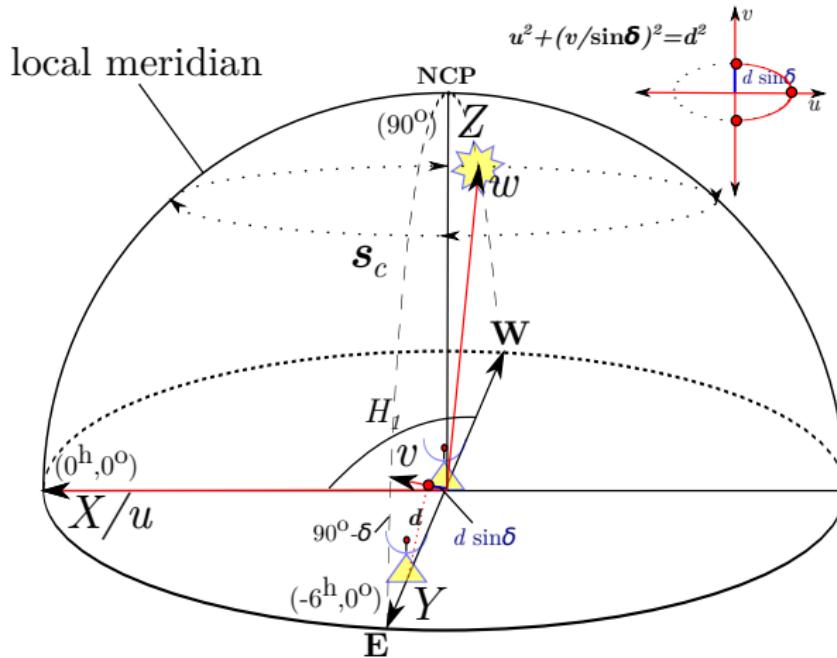
EW example I: General δ



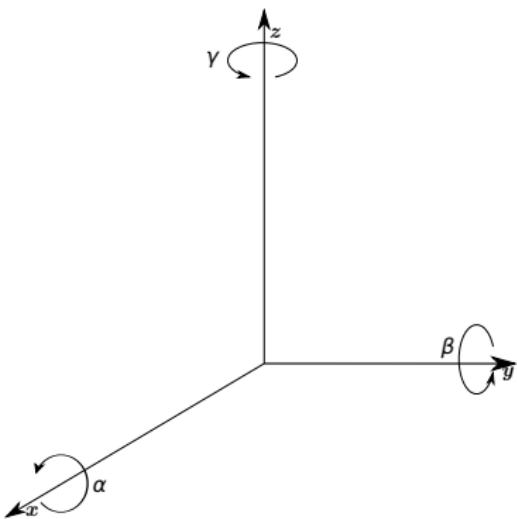
EW example II: General δ



EW example III: General δ

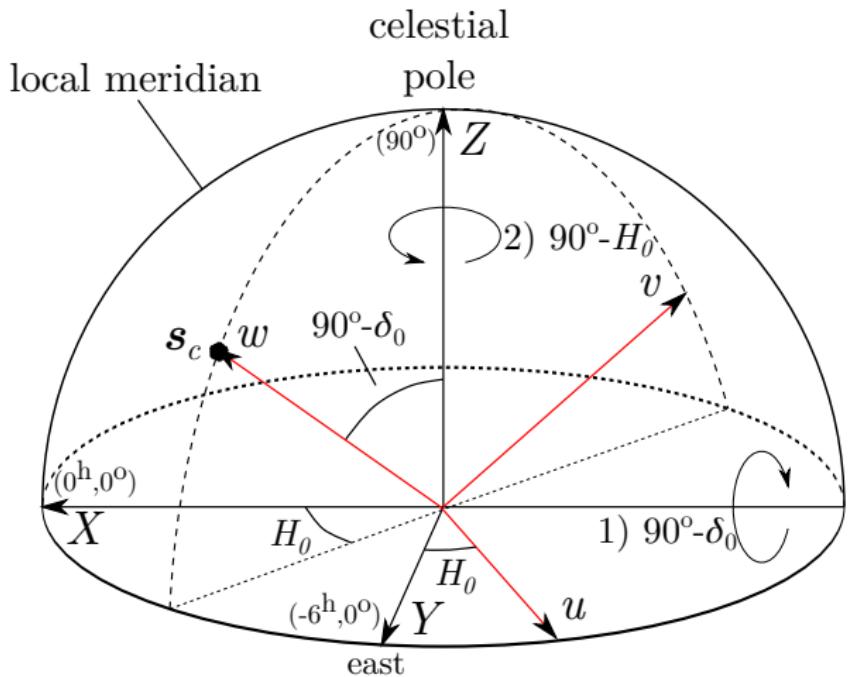


3D-Rotations



$$R_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha & 0 \\ 0 & -\sin \alpha & \cos \alpha & 0 \end{bmatrix} \quad R_3(\gamma) = \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Conversion between XYZ and uvw



Conversion matrix

$$R_1(90 - \delta_0)R_3(90 - H_0) = \begin{bmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin \delta_0 \cos H_0 & \sin \delta_0 \sin H_0 & \cos \delta_0 \\ \cos \delta_0 \cos H_0 & -\cos \delta_0 \sin H_0 & \sin \delta \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin \delta_0 \cos H_0 & \sin \delta_0 \sin H_0 & \cos \delta_0 \\ \cos \delta_0 \cos H_0 & -\cos \delta_0 \sin H_0 & \sin \delta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

If we incorporate wavelength then we use

$$\lambda^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

TCP (p. 25 Eq. 2.30)

EW example: General δ

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sin H & \cos H & 0 \\ -\sin \delta_0 \cos H & \sin \delta_0 \sin H & \cos \delta_0 \end{bmatrix} \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix}$$

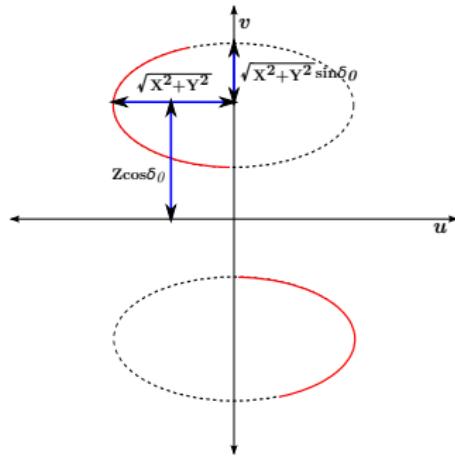
$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} d \cos H \\ d \sin \delta_0 \sin H \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ -d \sin \delta_0 \end{bmatrix} = \begin{bmatrix} d \cos -6^h \\ d \sin \delta_0 \sin -6^h \end{bmatrix}; \begin{bmatrix} d \\ 0 \end{bmatrix} = \begin{bmatrix} d \cos 0^h \\ d \sin \delta_0 \sin 0^h \end{bmatrix};$$

$$\begin{bmatrix} 0 \\ d \sin \delta_0 \end{bmatrix} = \begin{bmatrix} d \cos 6^h \\ d \sin \delta_0 \sin 6^h \end{bmatrix}$$

$$u^2 + \left(\frac{v}{\sin \delta_0} \right)^2 = d^2$$

The general case: A general uv -track



$$u^2 + \left(\frac{v - Z\lambda^{-1} \cos \delta_0}{\sin \delta_0} \right)^2 = \frac{X^2 + Y^2}{\lambda^2}$$

TCP (p. 25 Eq. 2.31)