Required Instrumentation Information for Interferometrists and Radio Astronomers

Any measurement is a loss in information

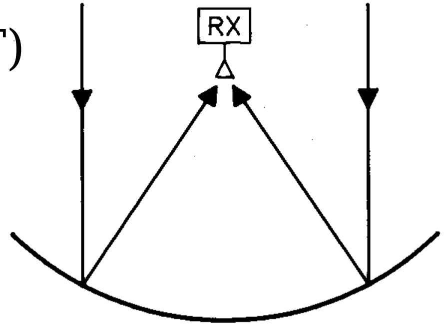
From the sky we can measure infinite frequency bandwidth, frequency resolution, time resolution, across a 4π area of the sky.

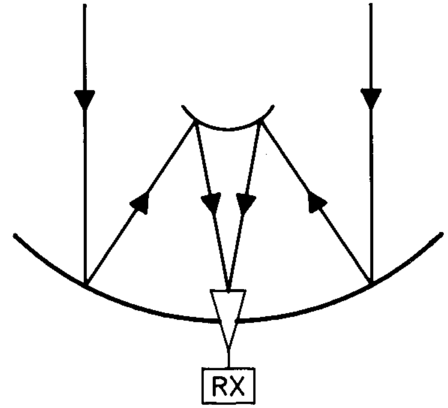
In reality, our instruments have limits on all these, so we must be selective about which information we retain based on the science goals and engineering limitations.

Primary Beams

focusing light

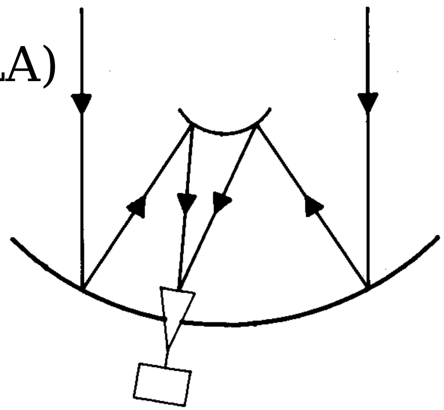
Prime Focus (GMRT)

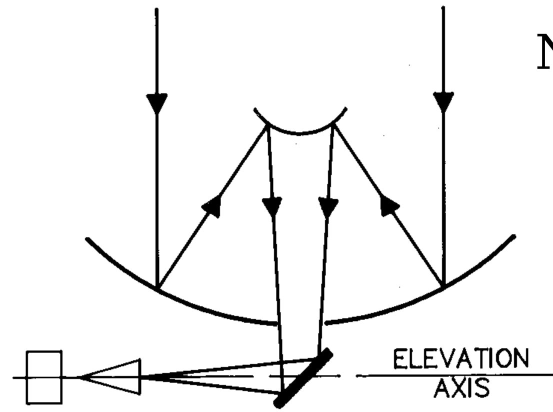




Cassegrain (ATCA)

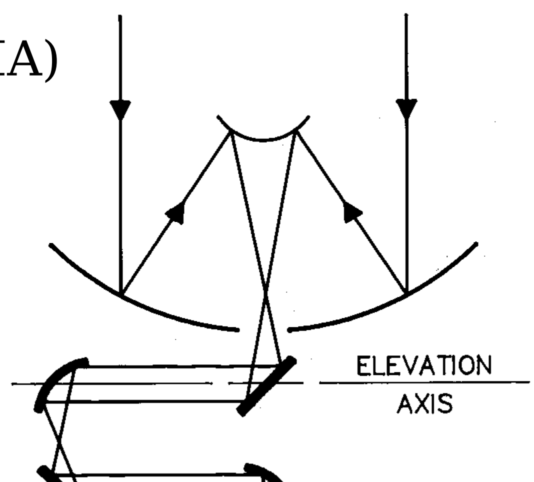
Offset Cassegrain (VLA)

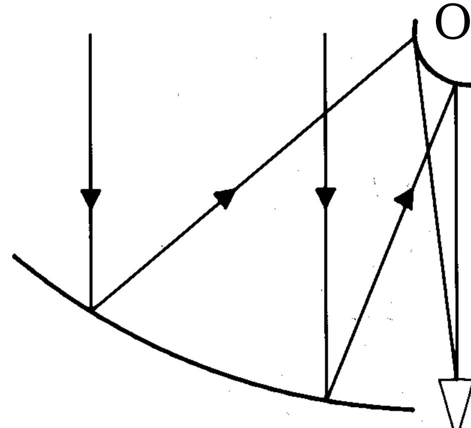




Nasmyth (CARMA)

Bent Nasmyth (SMA)





Offset Gregorian (GBT)

η : 'aperture' efficiency

$\eta=\eta_{surface}\eta_{blockage}\eta_{spillover}\eta_{taper}\ldots$

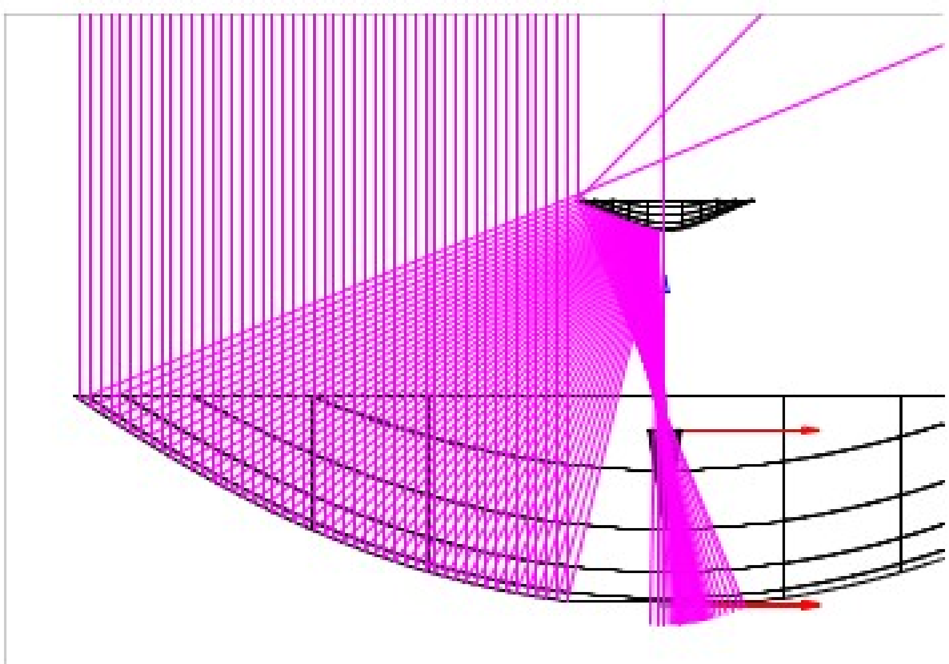
$\eta_{surface}$: any surface has reflective loss

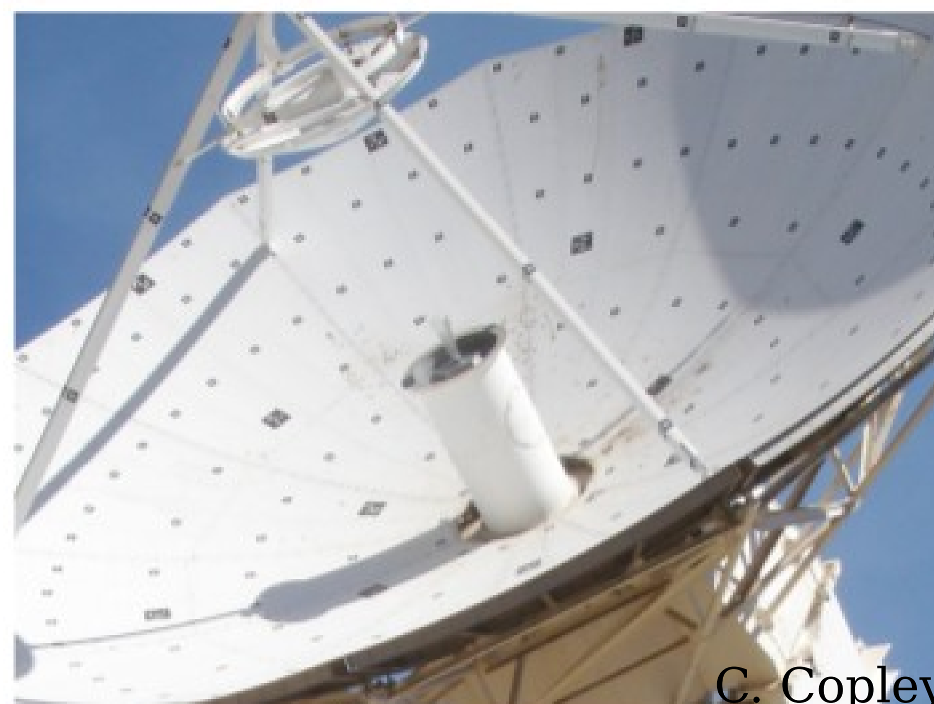
$\eta_{blockage}$: the structure above the dish block a portion of the light (to 0^{th} order)

$\eta_{spillover}$: loss due to the caustic illumination onto the receiver feed

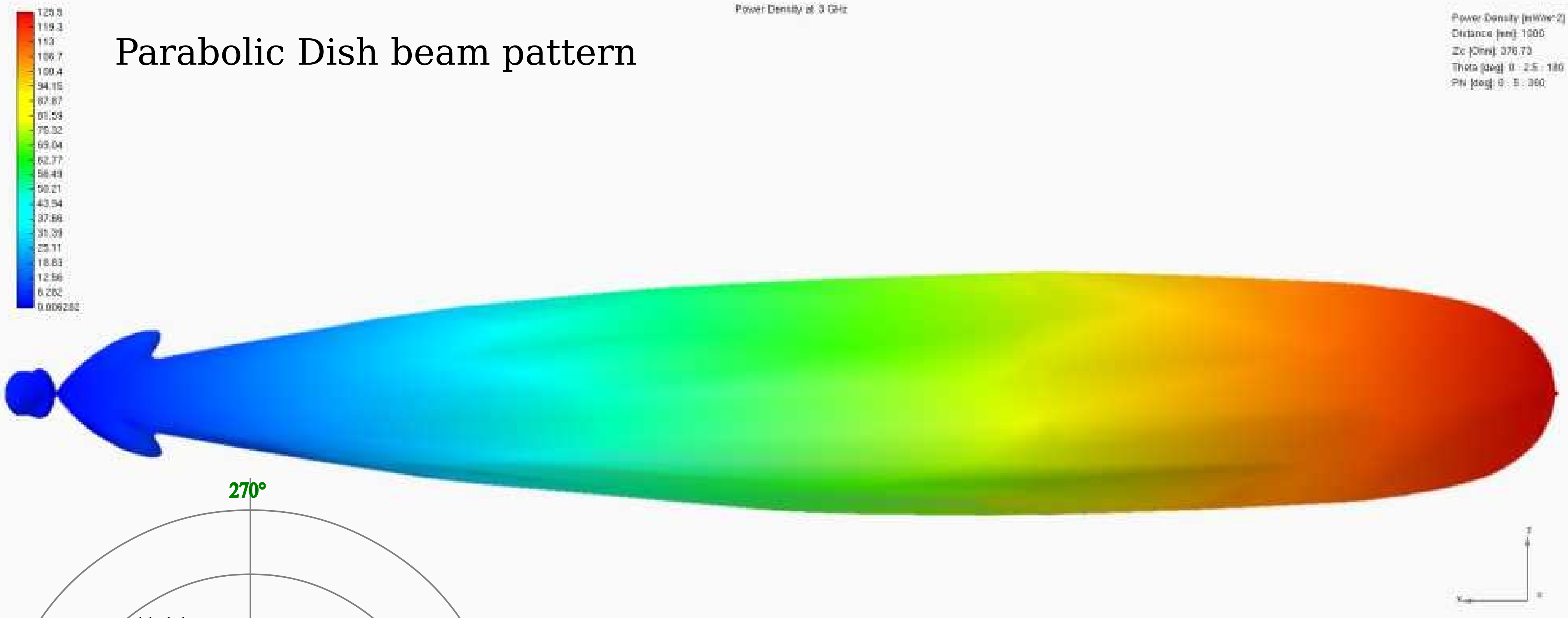
η_{taper} : there is a radius dependent loss with respect to illumination

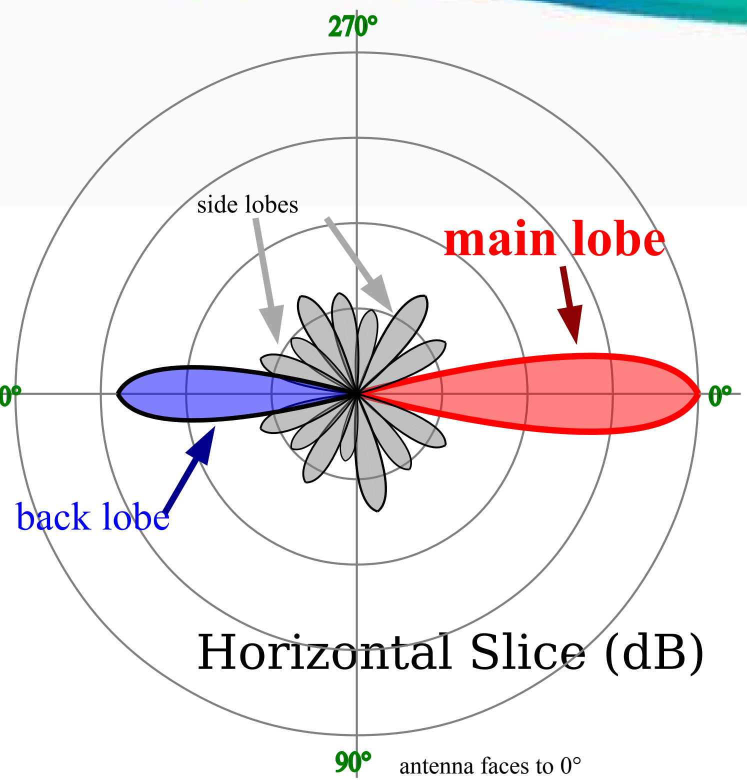
These efficiencies are approximate metrics, in reality, a electro-magnetic model of the primary beam provides a more complete description





Parabolic Dish beam pattern



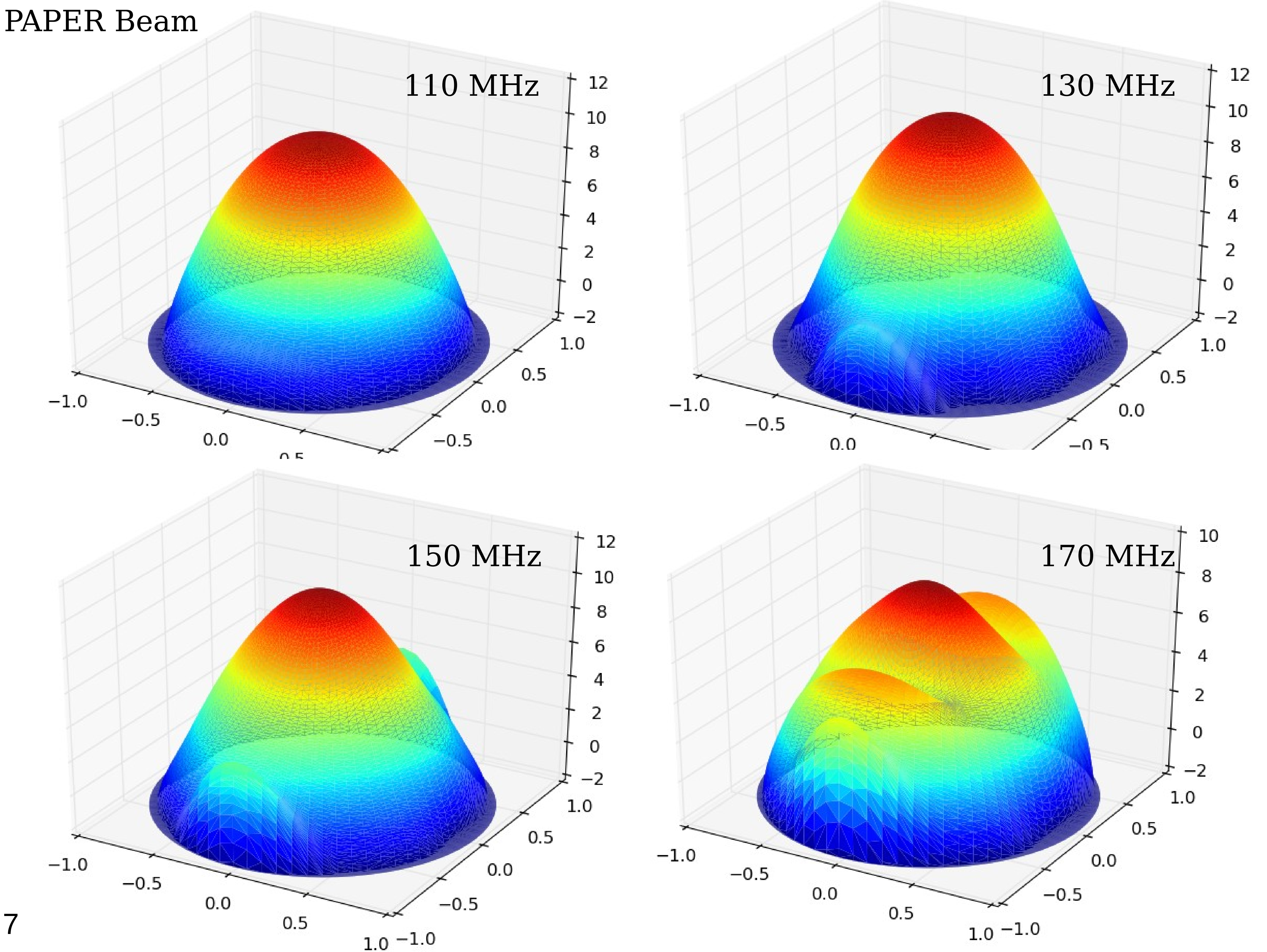


Directivity: a figure of merit which is a measurement of an antenna's power in the direction of strongest emission versus an isotropic (all-direction) antenna

Electrical efficiency: efficiency at which a receiving system converts radio power

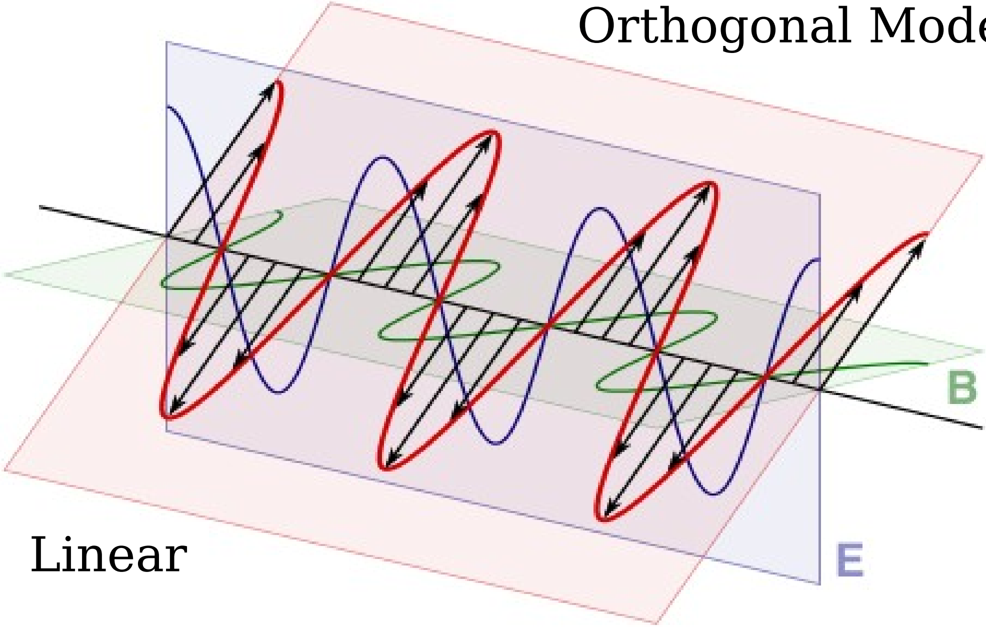
Gain: Combination of the antenna directivity and efficiency

Phased Array Antenna Handbook : Mailloux

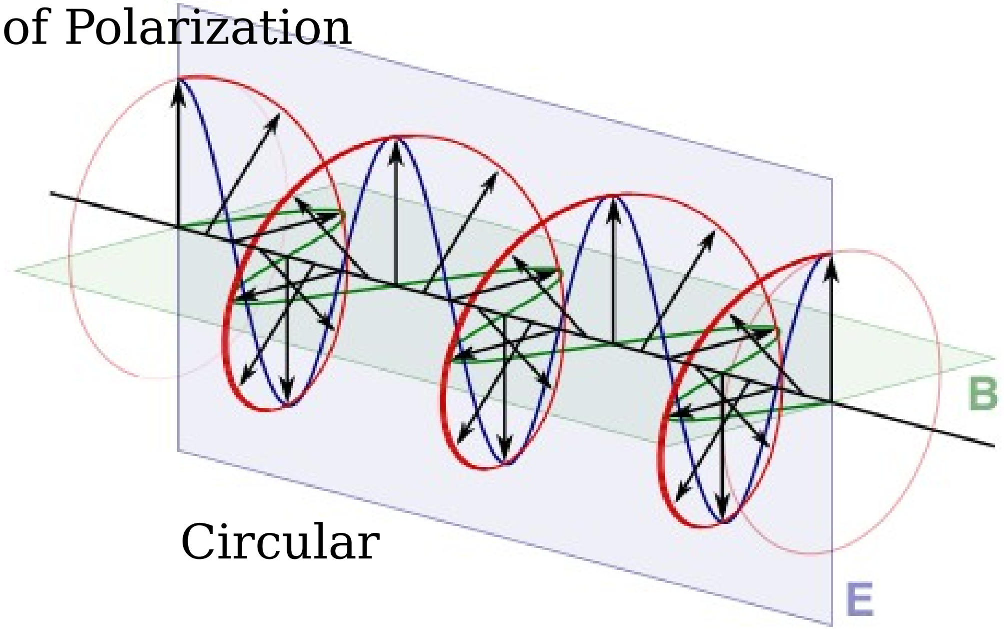


PAPER Beam
110 MHz
130 MHz
150 MHz
170 MHz

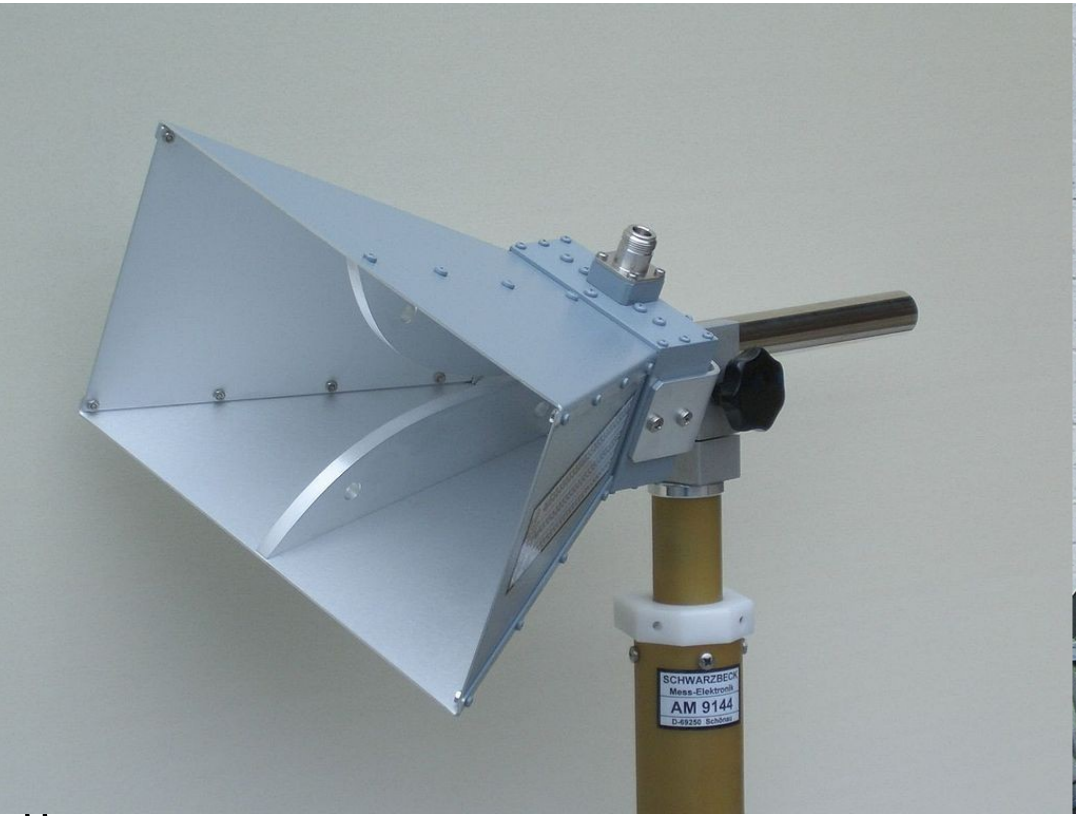
Orthogonal Modes of Polarization



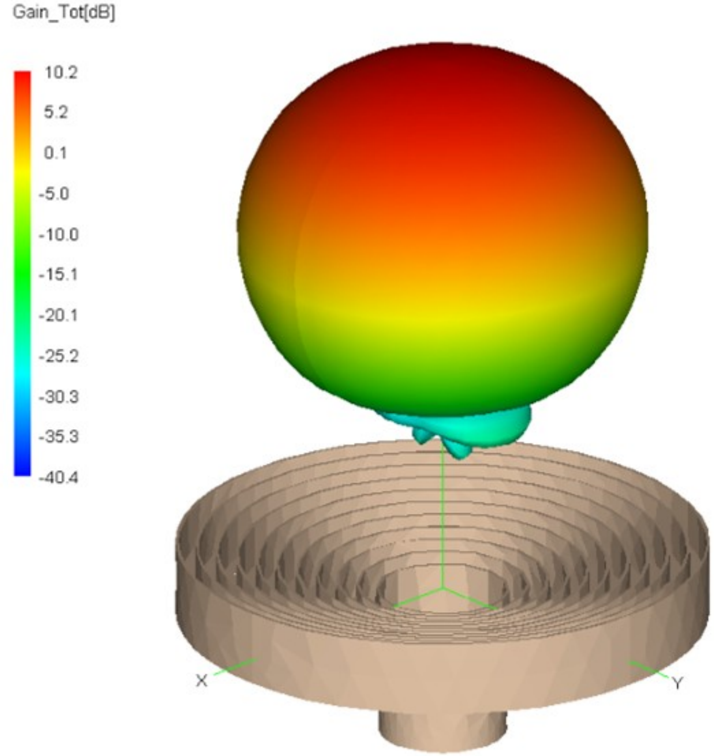
Linear



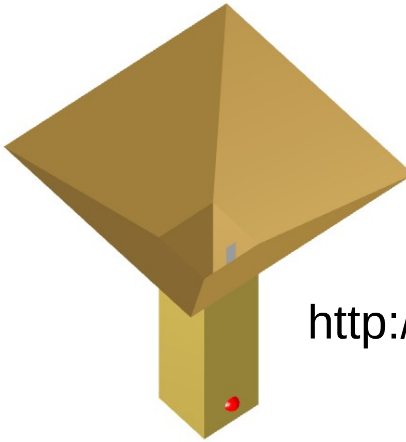
Circular

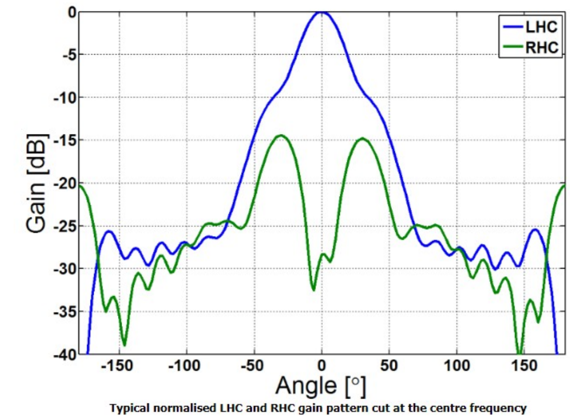


Conical Horn



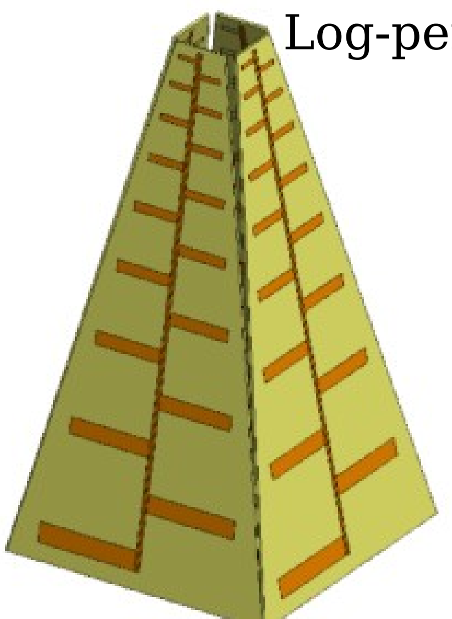
Square Horn

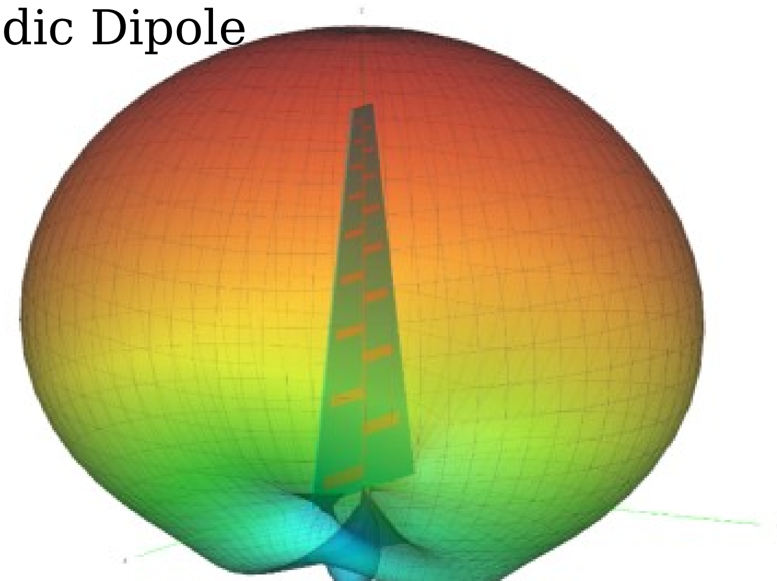


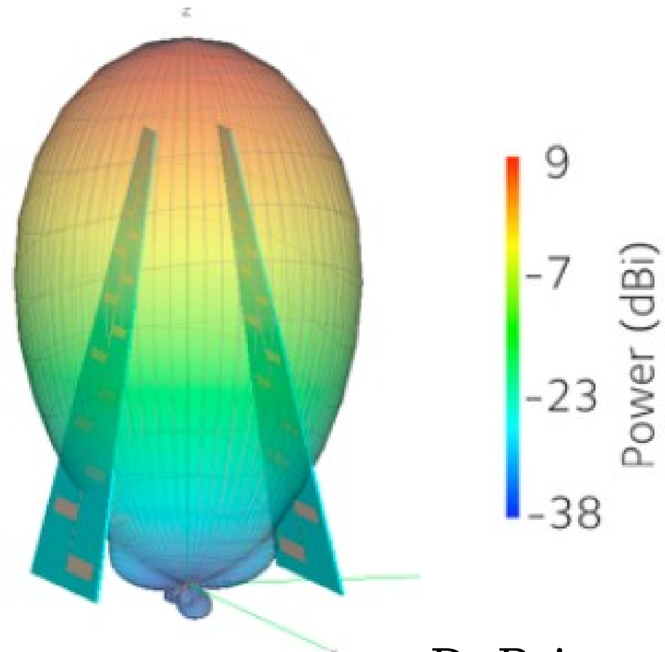


http://www.antennamagus.com/

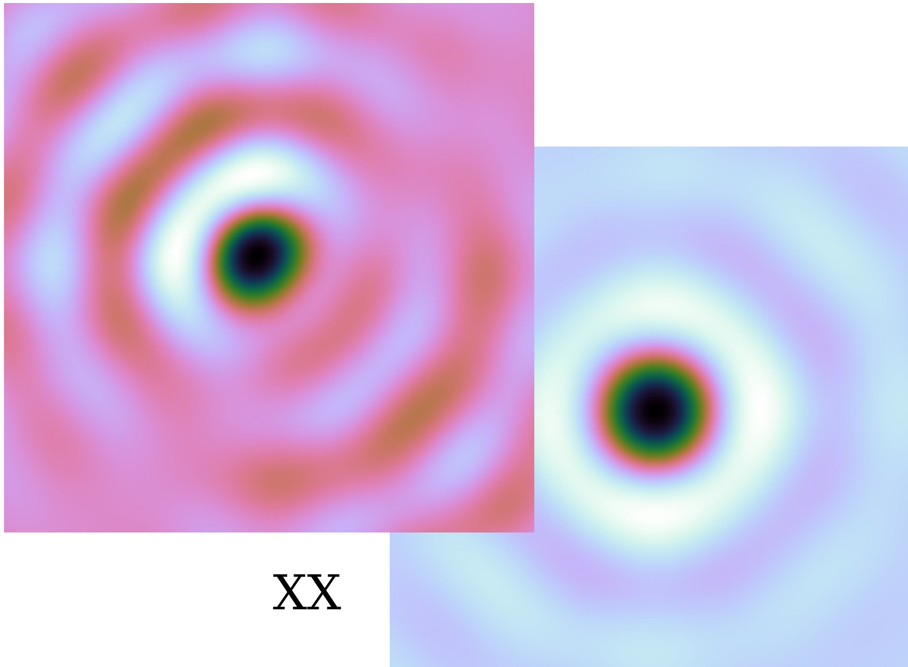
Log-periodic Dipole



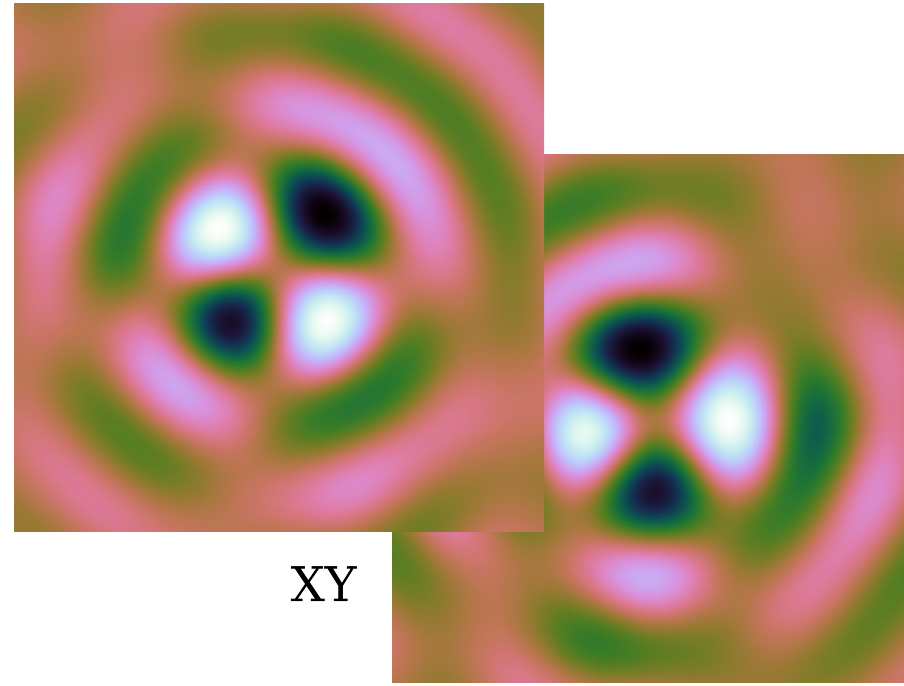




D. Price

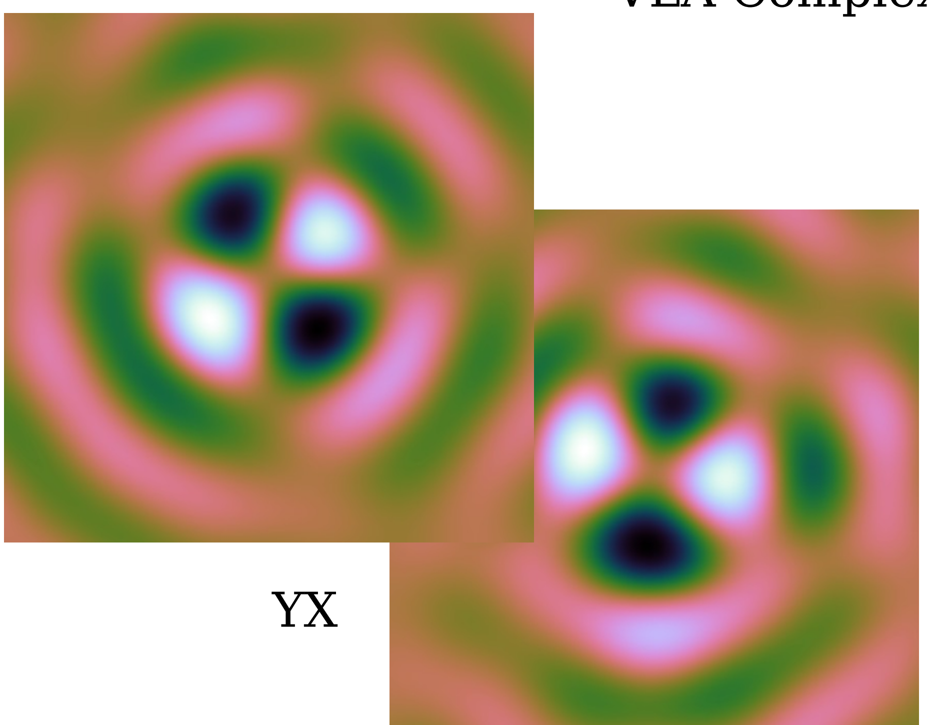


XX

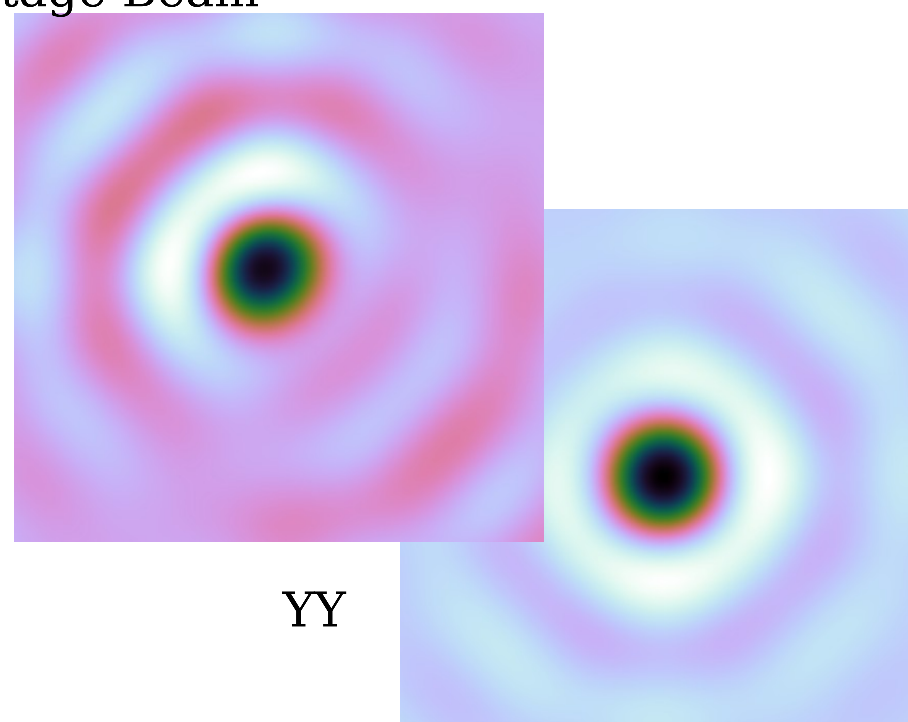


XY

VLA Complex Voltage Beam



YX



YY

Conversion between Jones and Mueller (Brightness and Stokes)

$$\mathbf{E} = \begin{pmatrix} E_{xx}(\nu,t,l,m) & E_{xy}(\nu,t,l,m) \\ E_{yx}(\nu,t,l,m) & E_{yy}(\nu,t,l,m) \end{pmatrix}$$

Jones representation of the primary beam,
Complex voltage value with is frequency and position dependent,
and potentially other effects (time, pointing angle, temperature...)

$$\mathbf{M_E} = \mathbf{S}^{-1}\left(\mathbf{E} \otimes \mathbf{E}^*\right)\mathbf{S}$$

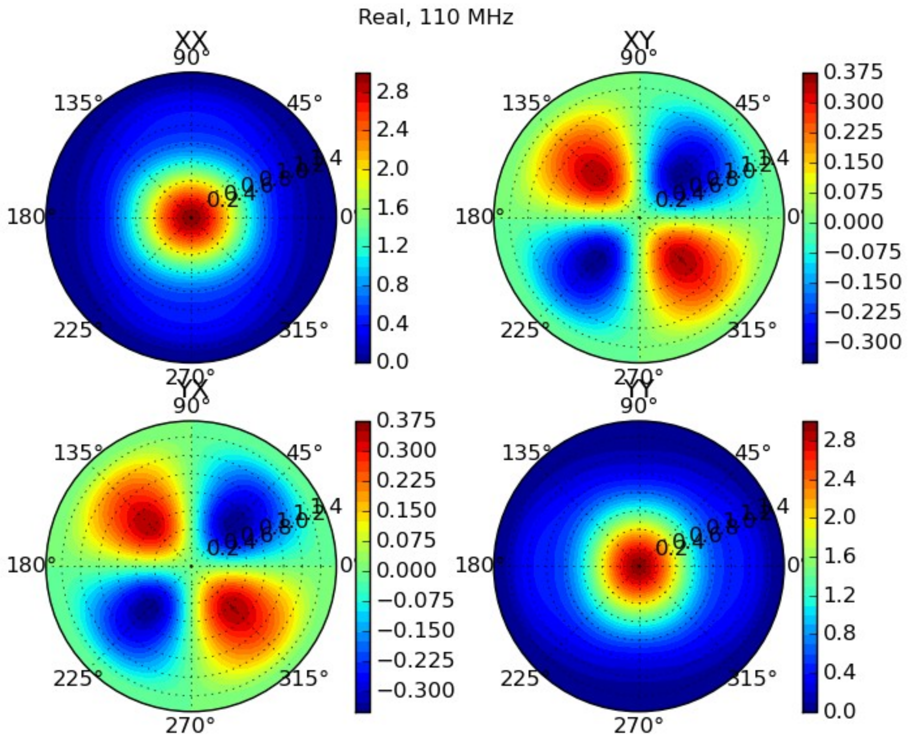
Stokes Mueller matrix representation of the primary beam

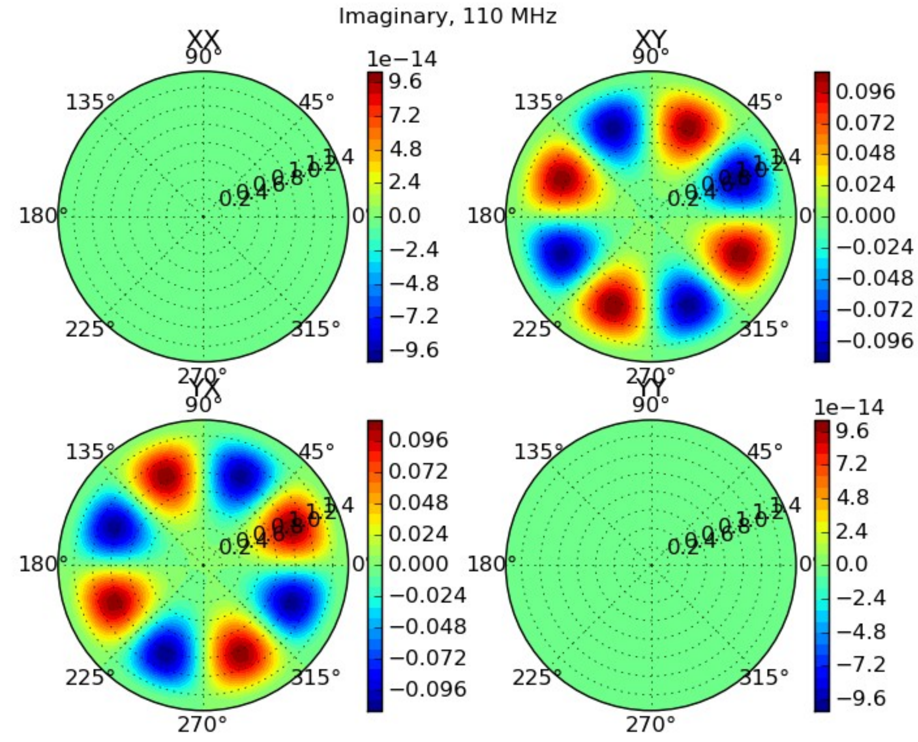
$$\mathbf{S} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

Stokes → Brightness Muller

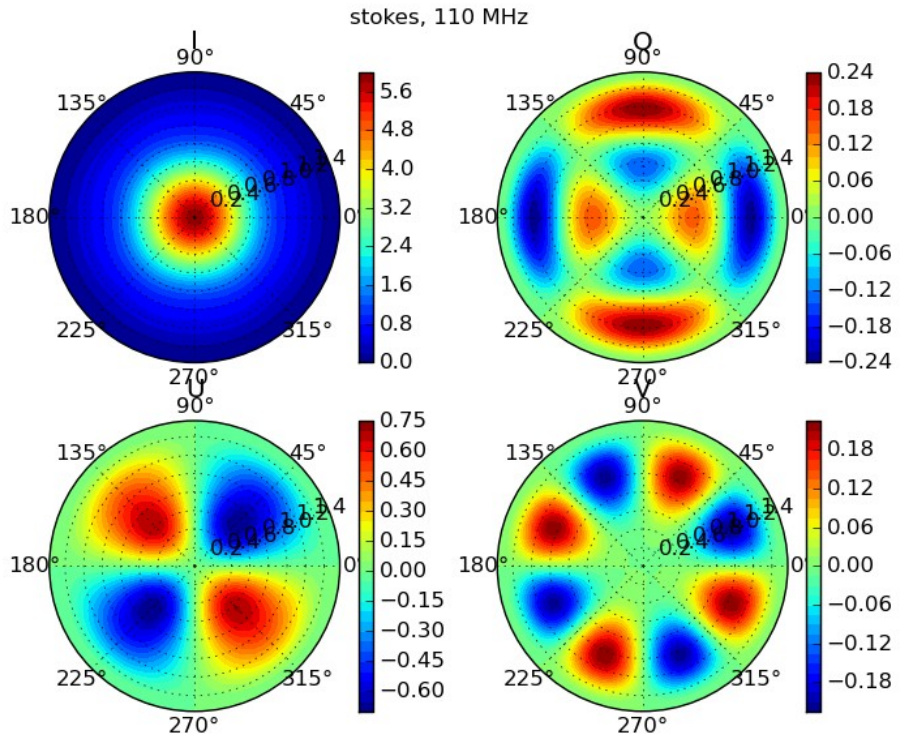
$$\mathbf{S}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -i & i & 0 \end{pmatrix}$$

Brightness → Stokes Muller





Jones representation conversion to Stokes Parameters

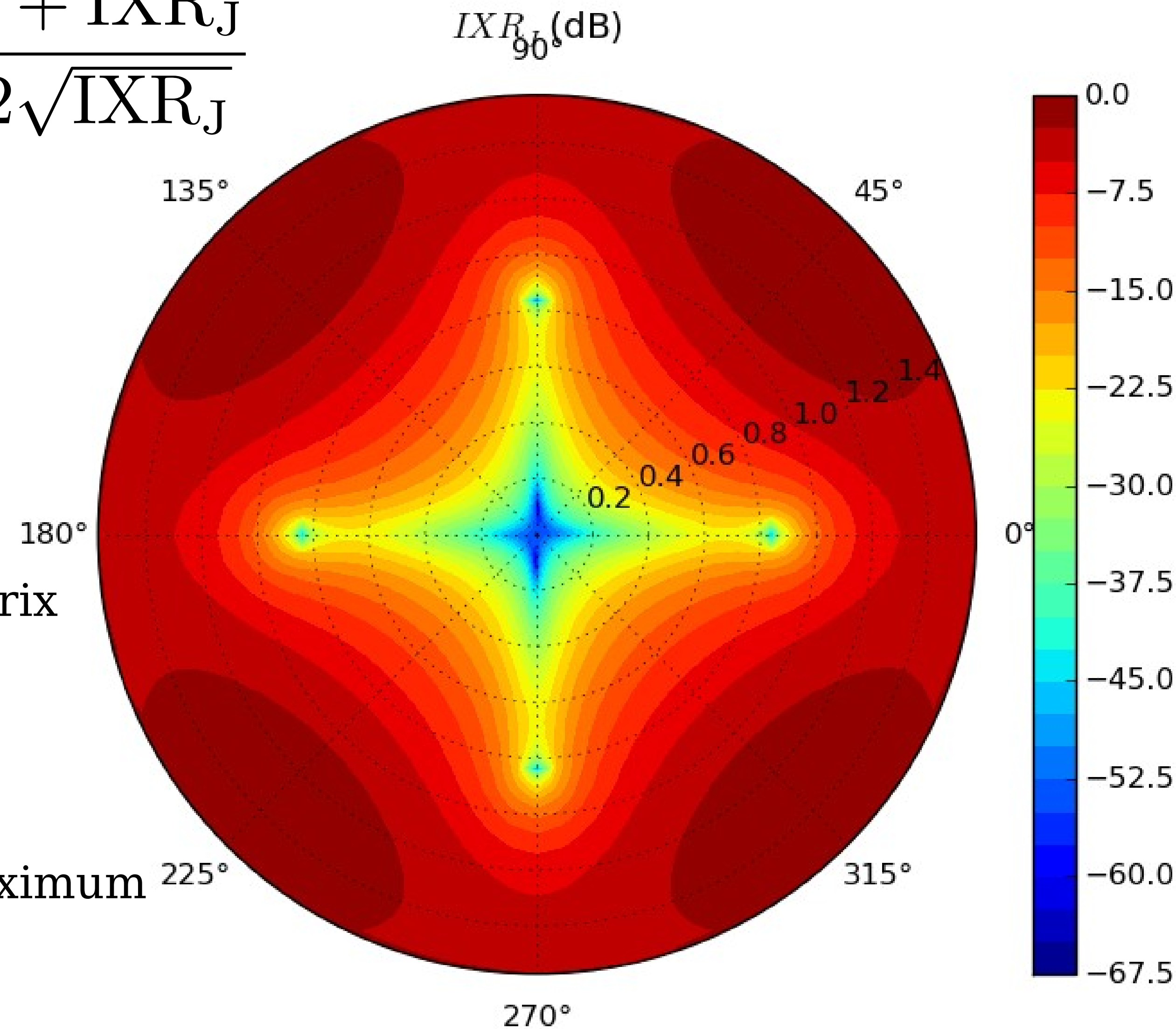


Polarization Leakage

Intrinsic Cross-Polarization Ratio (IXR) [Carozzi & Woan 2011]

$$\mathrm{IXR_M} = \frac{\kappa(M)+1}{\kappa(M)-1} = \frac{1+\mathrm{IXR_J}}{2\sqrt{\mathrm{IXR_J}}}$$

$$\mathrm{IXR_J} = \left(\frac{\kappa(J)+1}{\kappa(J)-1}\right)^2$$



$\kappa(M)$, $\kappa(J)$ Mueller and Jones matrix condition numbers

g_{min} and g_{max}: Minimum and maximum values when performing SVD

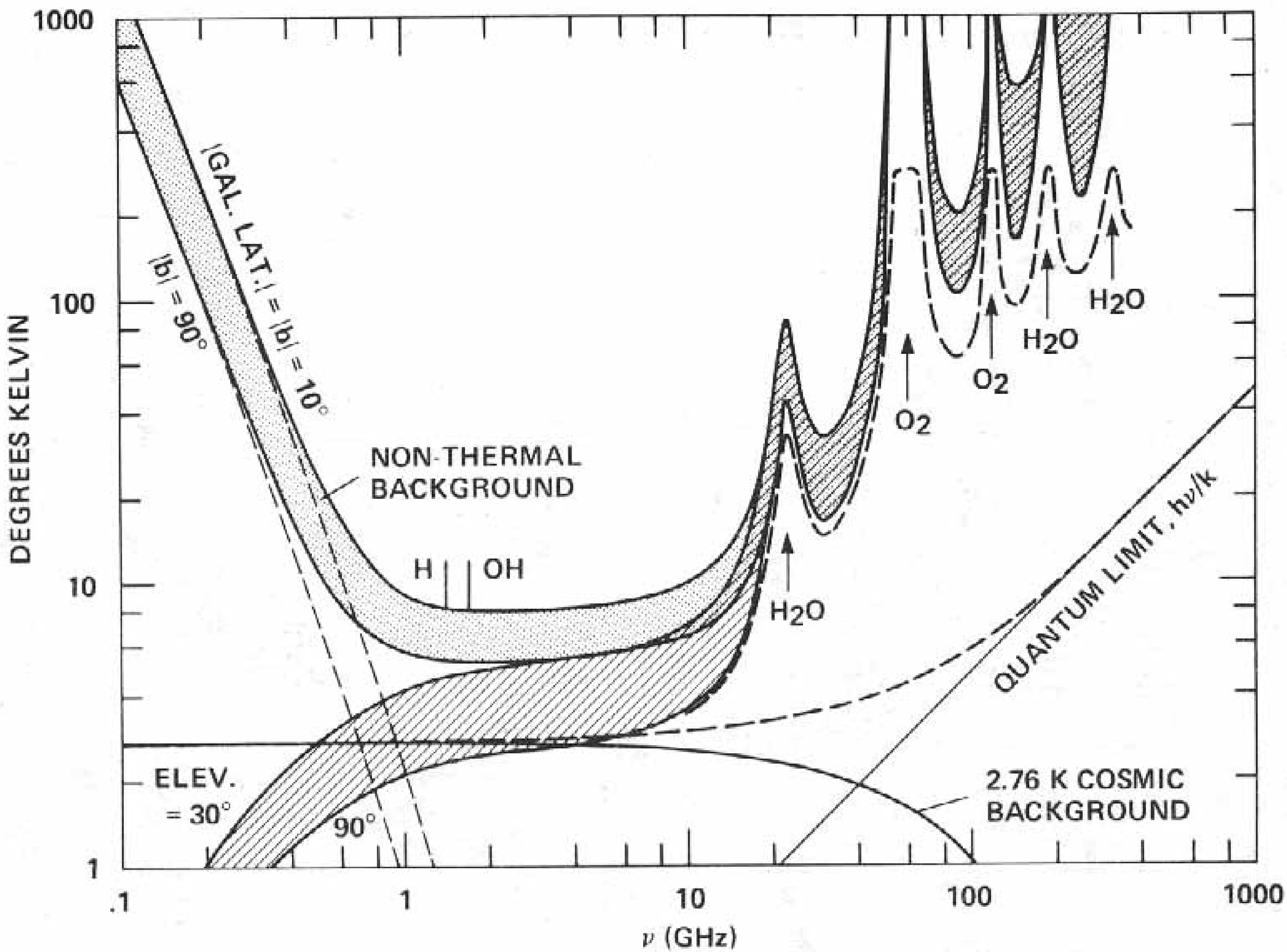
$$J = \frac{g_{\max}+g_{\min}}{2}\begin{pmatrix} 1 & 1/\sqrt{\mathrm{IXR_J}} \\ 1/\sqrt{\mathrm{IXR_J}} & 1 \end{pmatrix}$$

PAPER Beam @ 110 MHz

Analogue Electronics

(selectively) converting light to voltage

Simple model of the radio sky



System Temperature

$$T_{sys} = T_{sky} + T_{atmosphere} + T_{spillover} + T_{rx} + \ldots$$

T_{sky}: radio sky background (synchrotron, CMB (2.76K), ...)
$T_{atmosphere}$: atmospheric foregrounds (important at mm wavelengths)
$T_{spillover}$: pick-up of ~300K ground in the side and back-lobes
T_{rx}: receiver temperature from the Friis Cascade Noise Equation

$T_{passive}$: passive components (cables, connectors, OMT) before the LNA
T_{LNA}: Low-Noise Amplifier temperature
T_{amp}: secondary amplification/attenuation temperature

G_{LNA}: gain of the LNA
G_{feed}, $G_{passive}$: feed and passive 'gain' (related to efficiency)

$$T_{rx} = T_{feed} + \frac{T_{passive}}{G_{feed}} + \frac{T_{LNA}}{G_{feed}G_{passive}} + \frac{T_{amp}}{G_{feed}G_{passive}G_{LNA}} + \ldots$$

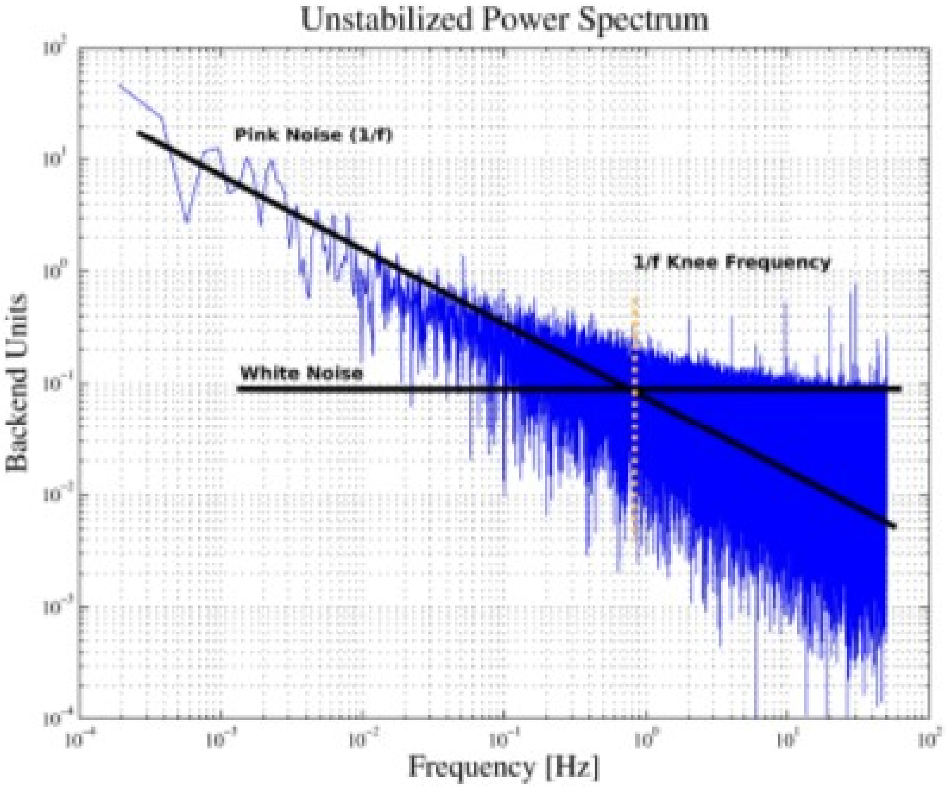
Radiometer Equation

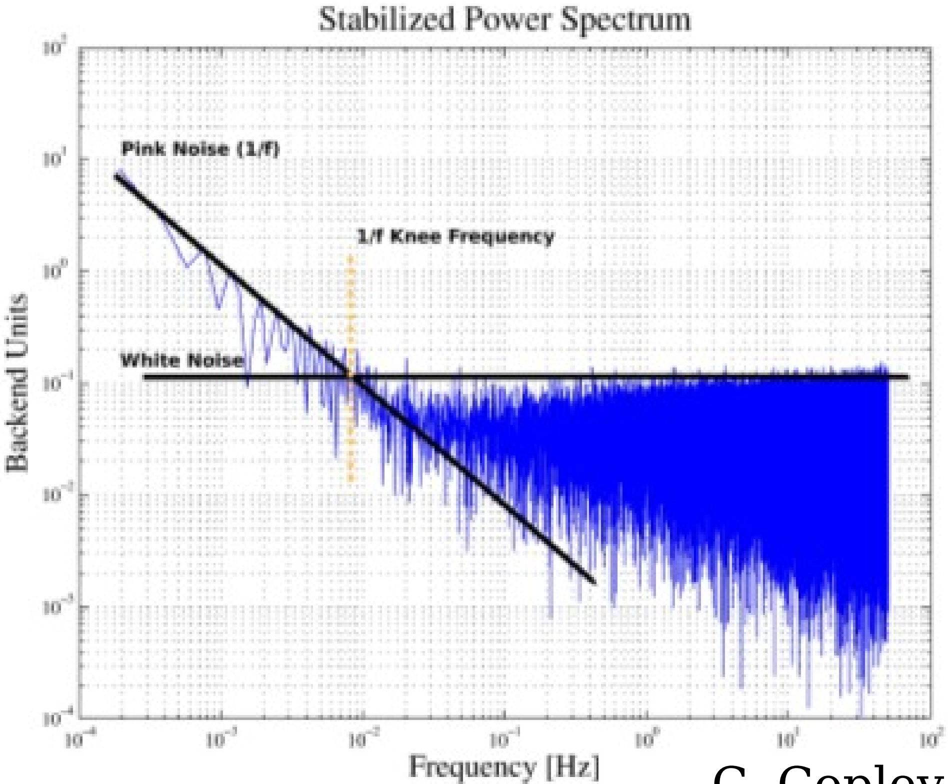
$$\sigma_T = \frac{T_{sys}}{\sqrt{\Delta\nu\tau}}$$

σ_T: residual (root-mean-square) in the measurement

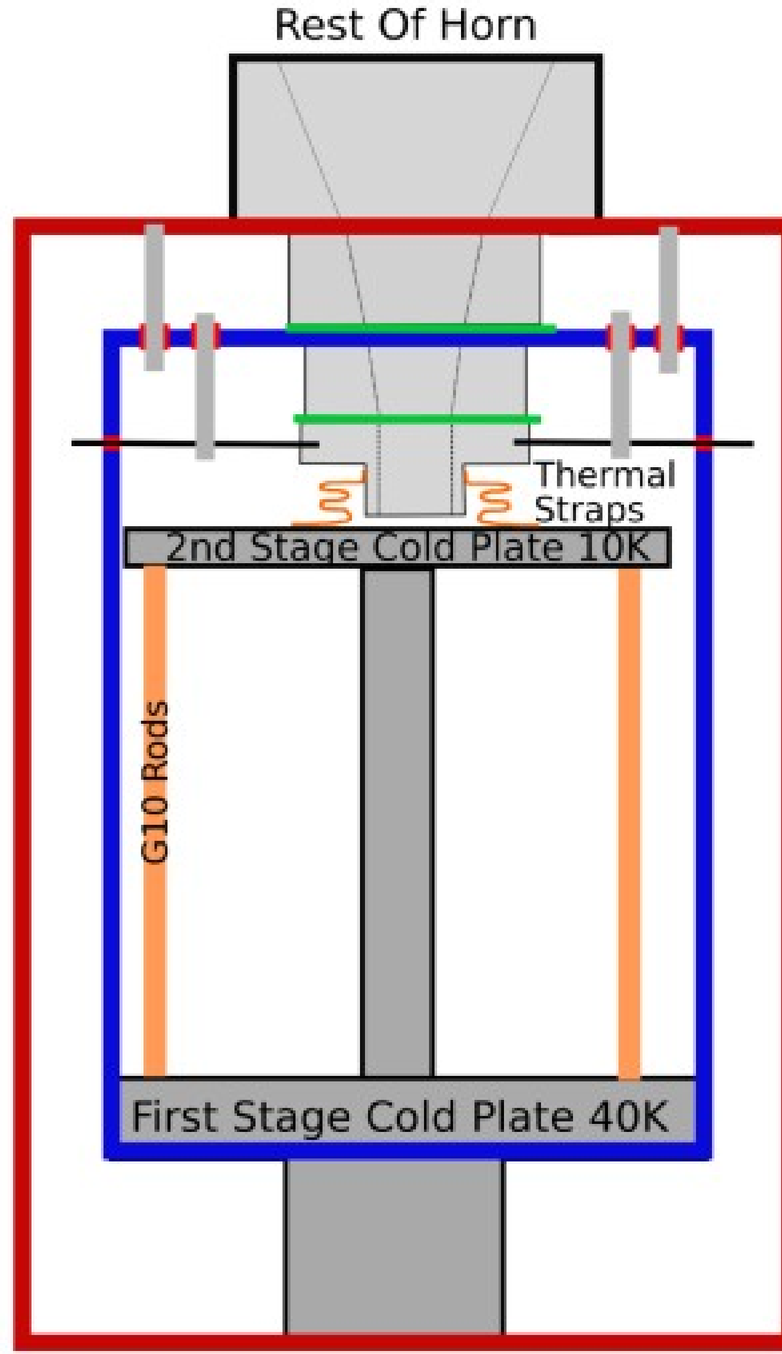
$\Delta\nu$: bandwidth of observation (Hz)
τ: integration time (seconds)

→ the smaller the T_{sys} the shorter the required observation time



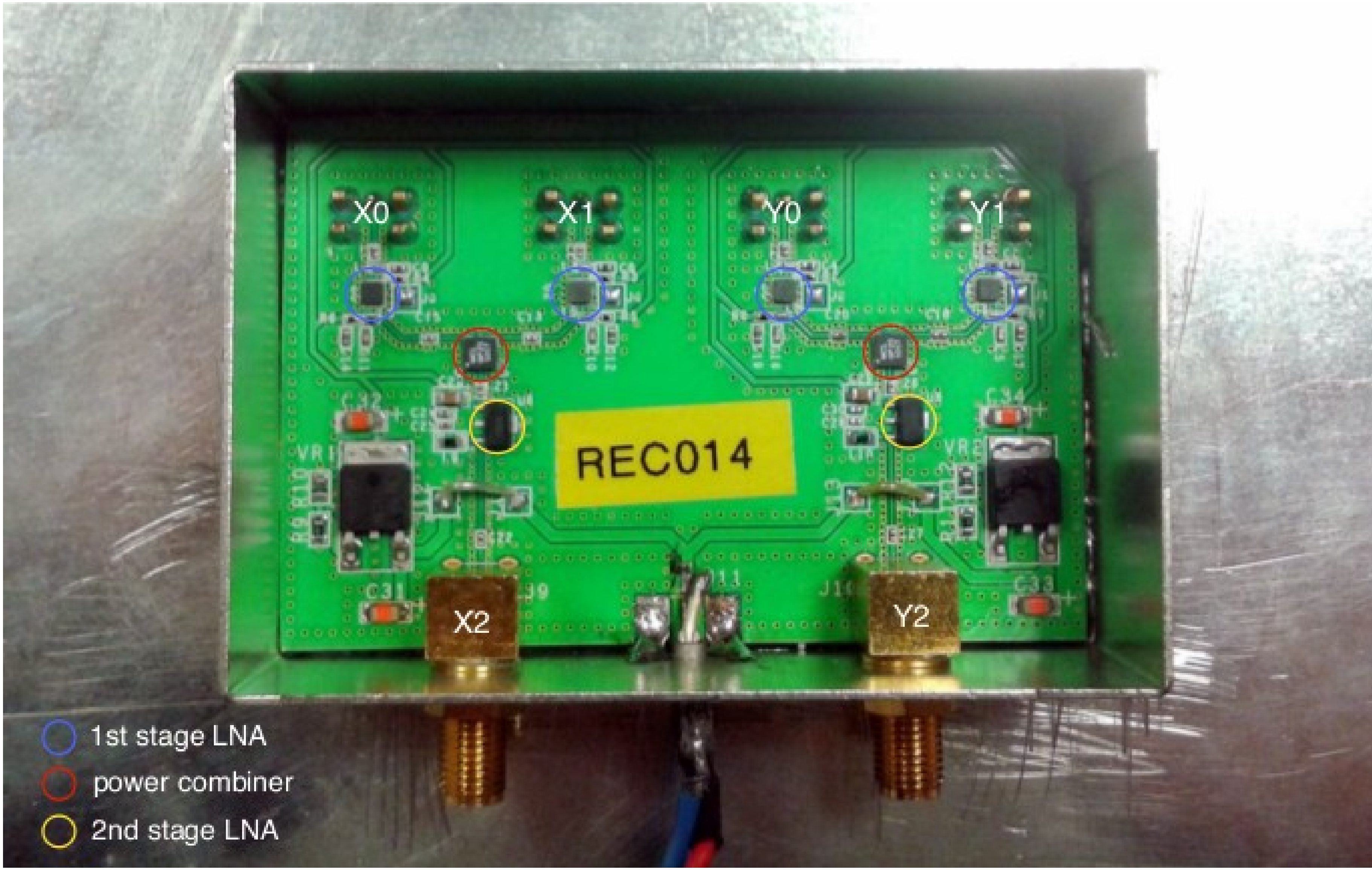


C. Copley

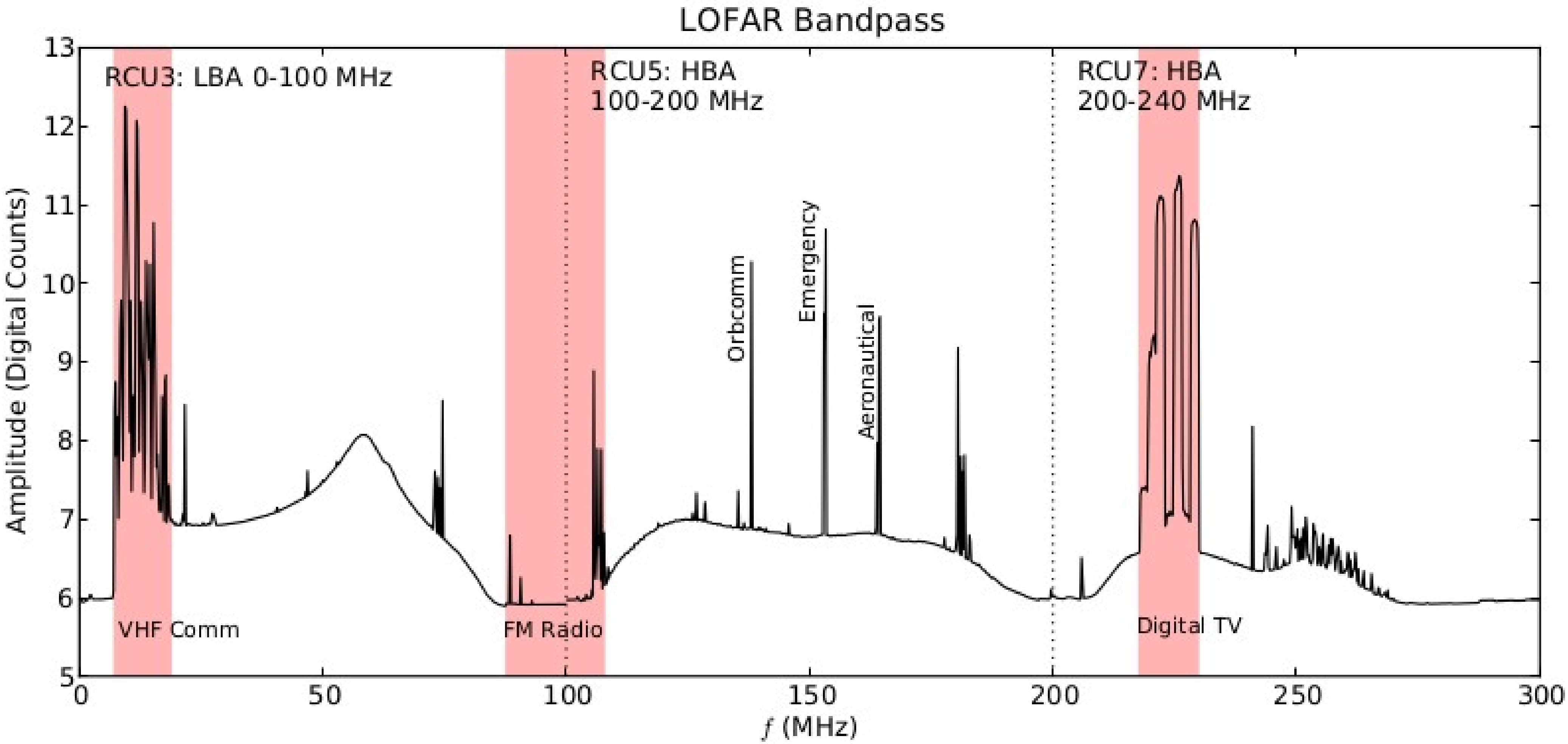


Rest Of Horn
Thermal
Straps
2nd Stage Cold Plate 10K
G10 Rods
First Stage Cold Plate 40K

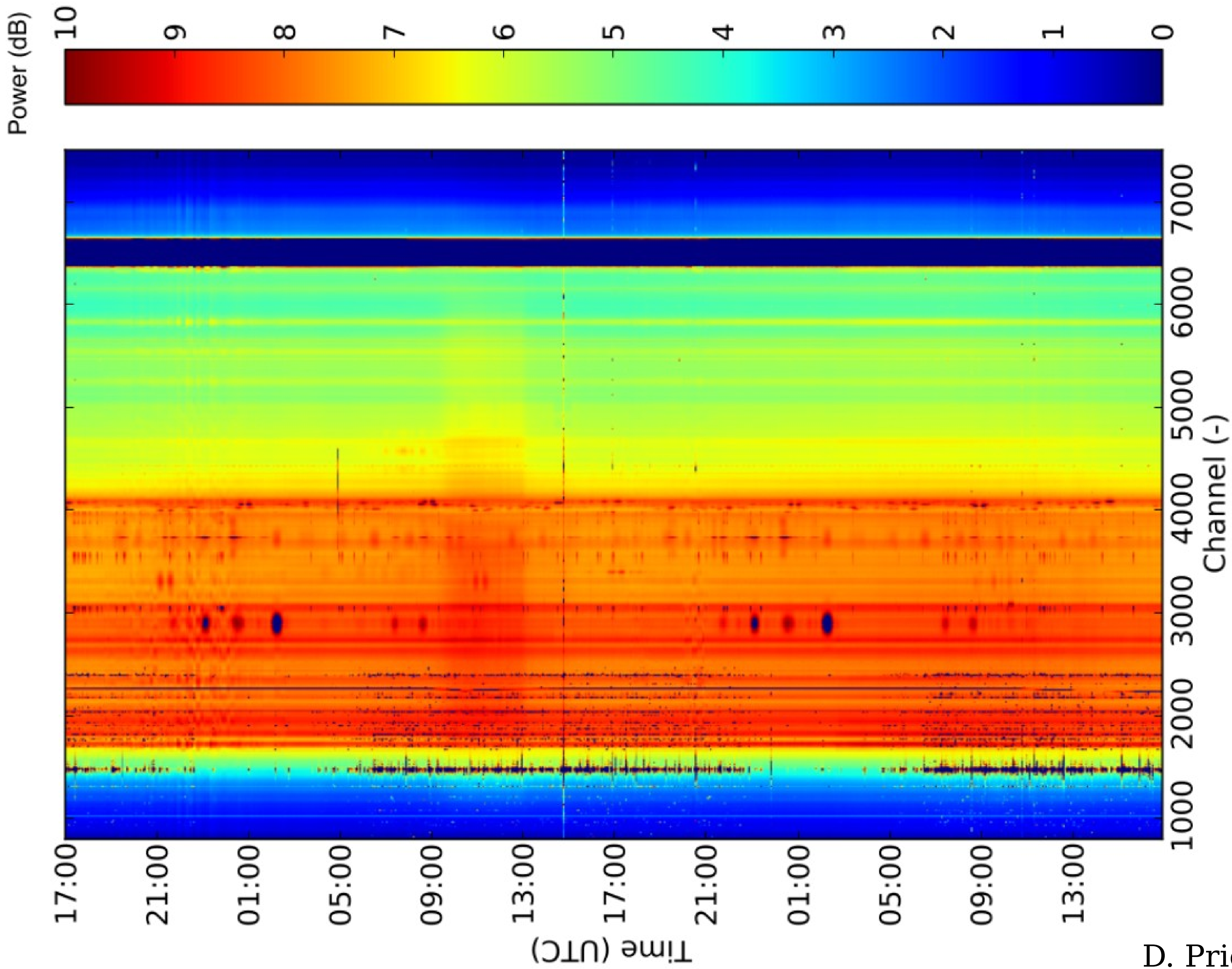
C. Copley


X0
X1
Y0
Y1
REC014
X2
Y2
1st stage LNA
power combiner
2nd stage LNA

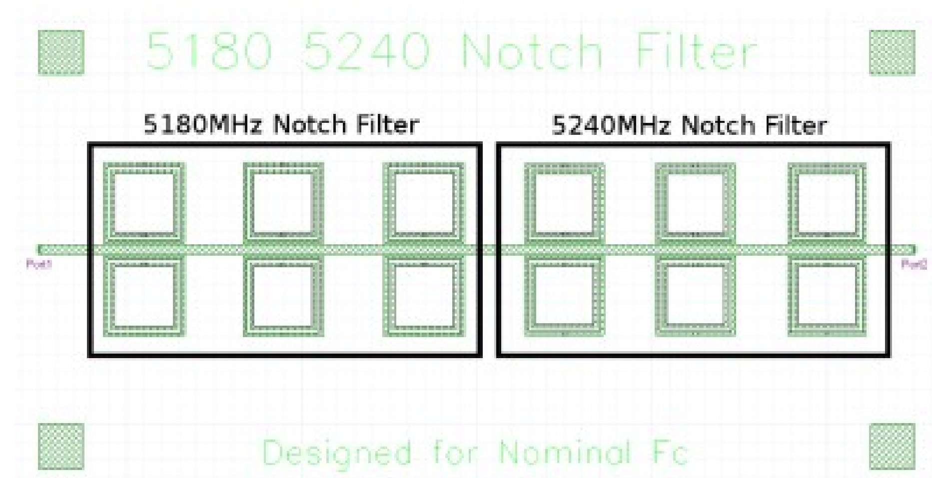
Radio Frequency Interference (RFI)



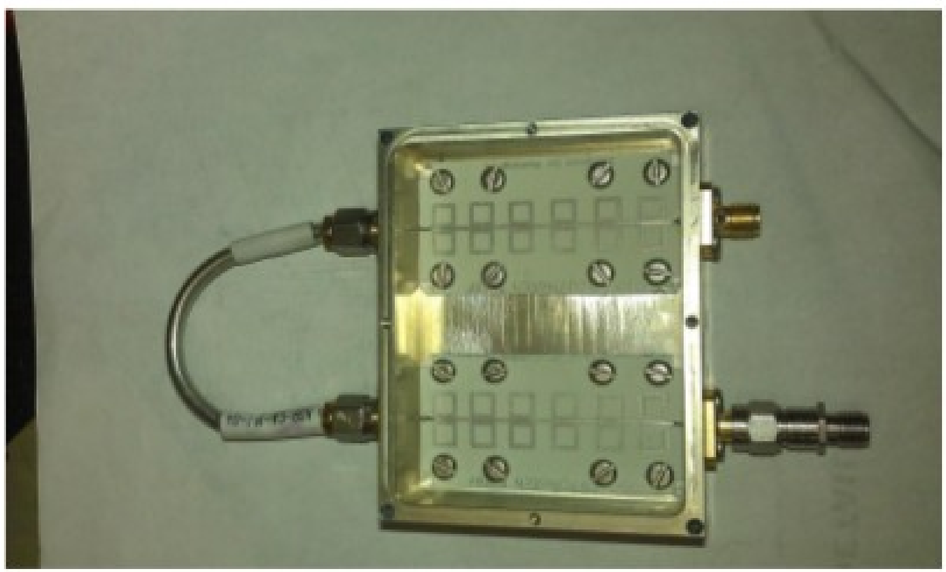
G. Foster

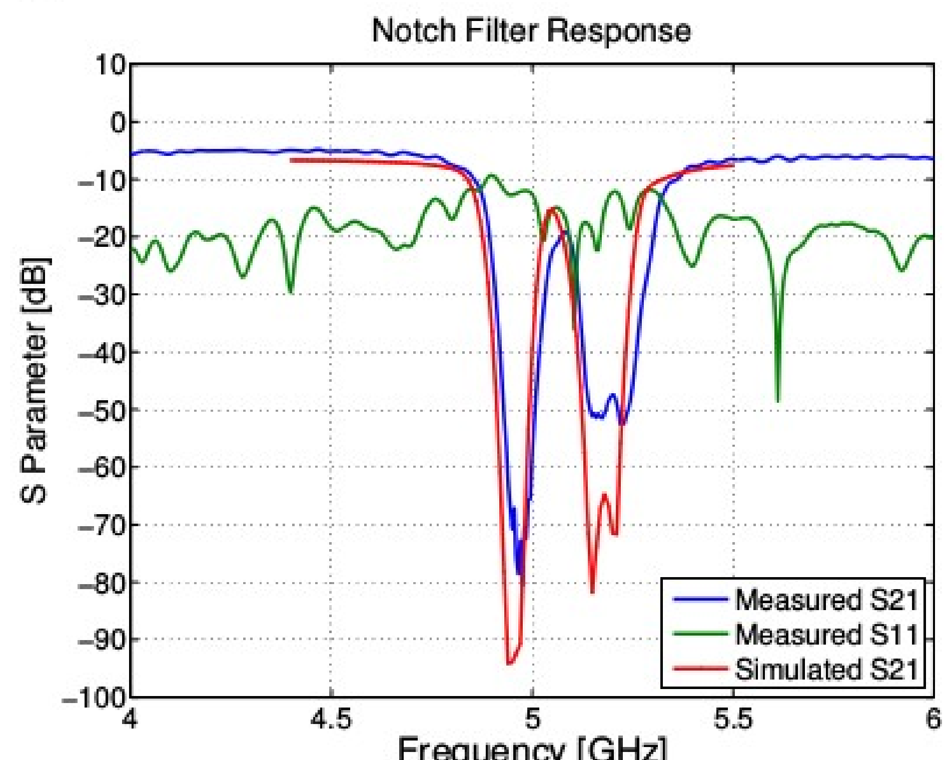

Power (dB)
10
9
8
7
6
5
4
3
2
1
0
1000
2000
3000
4000
5000
6000
7000
Channel (-)
17:00
21:00
01:00
05:00
09:00
13:00
17:00
21:00
01:00
05:00
09:00
13:00
Time (UTC)

(a) The 4.79 GHz and 4.92 GHz notch filter

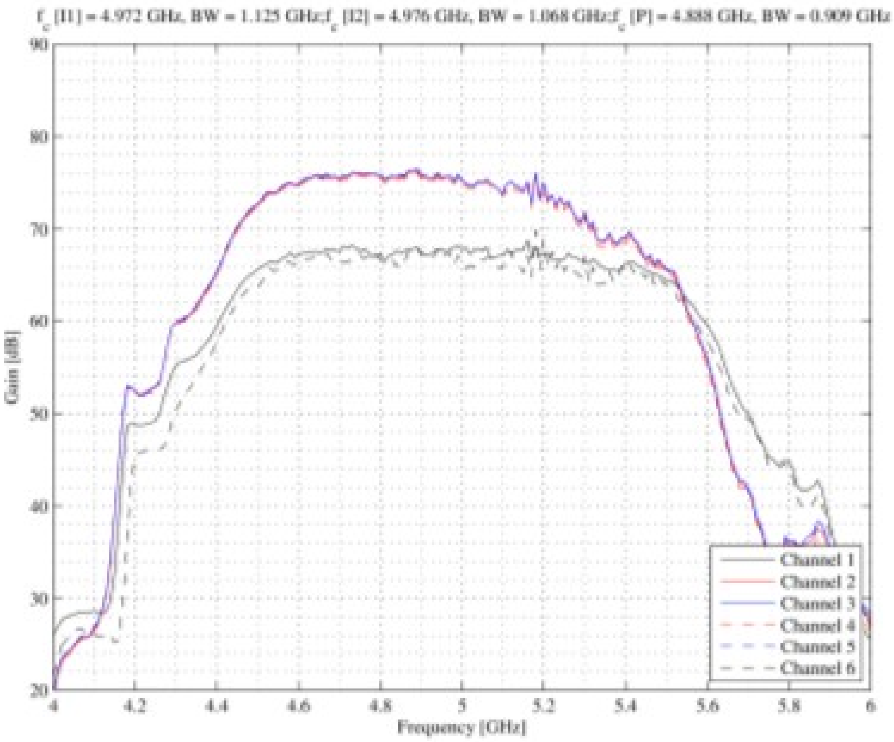


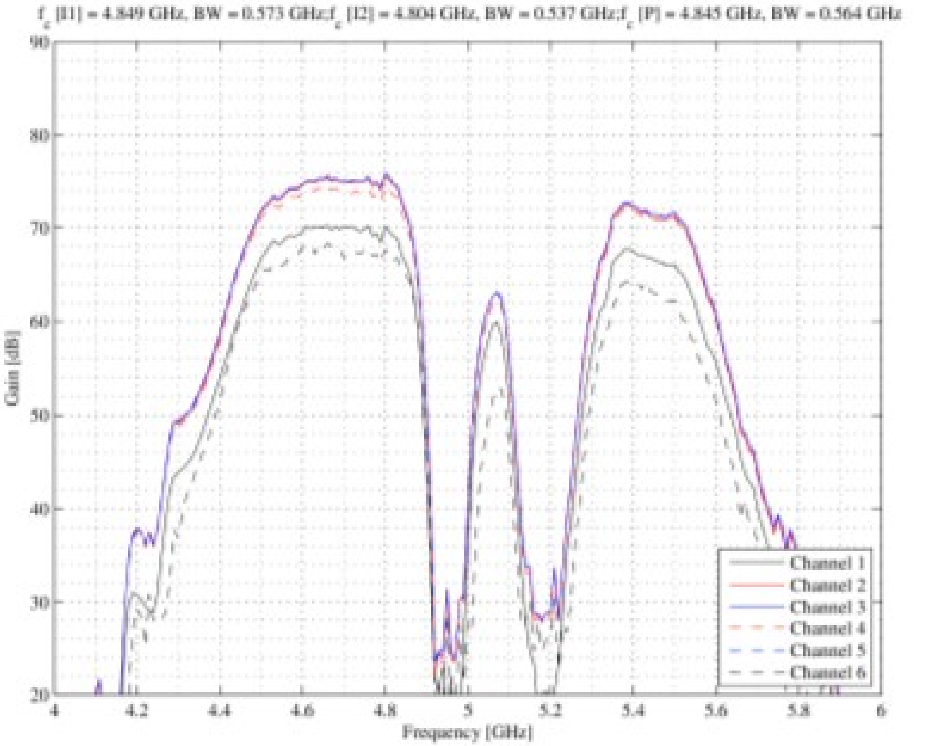
(b) The 5.18 GHz and 5.24 GHz notch filter


(c) Manufactured notch filter (with 6 dB attenuator to improve input match)



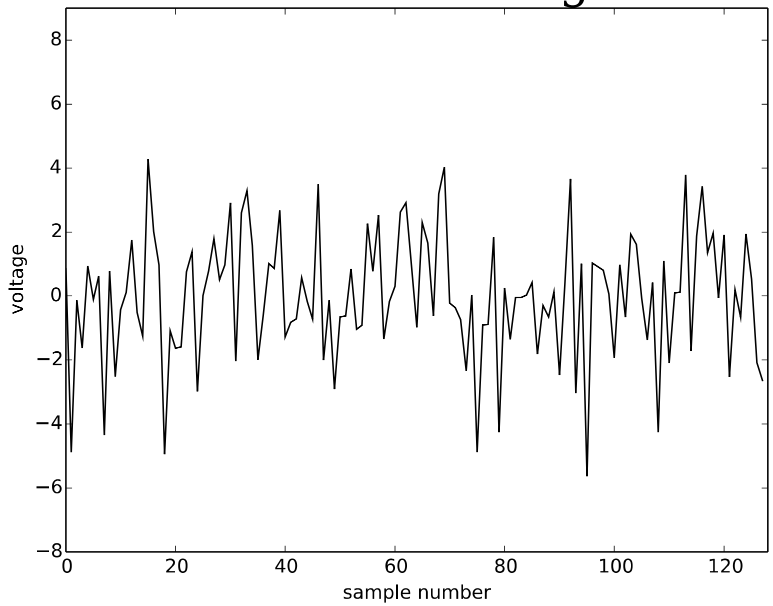
(d) Measured vs Simulated Responses

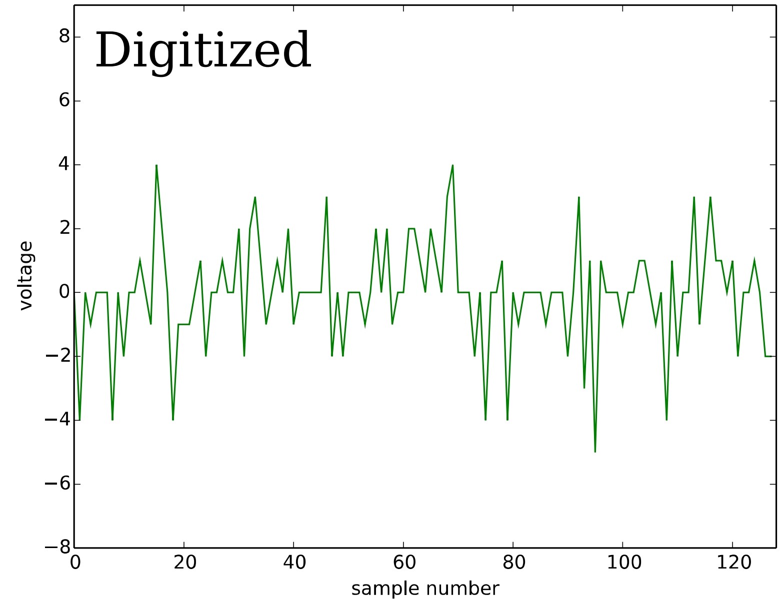

f_c [I1] = 4.972 GHz, BW = 1.125 GHz;f_c [I2] = 4.976 GHz, BW = 1.068 GHz;f_c [P] = 4.888 GHz, BW = 0.909 GHz
Gain [dB]
Frequency [GHz]
Channel 1
Channel 2
Channel 3
Channel 4
Channel 5
Channel 6


f_c [I1] = 4.849 GHz, BW = 0.573 GHz;f_c [I2] = 4.804 GHz, BW = 0.537 GHz;f_c [P] = 4.845 GHz, BW = 0.564 GHz
Gain [dB]
Frequency [GHz]
Channel 1
Channel 2
Channel 3
Channel 4
Channel 5
Channel 6

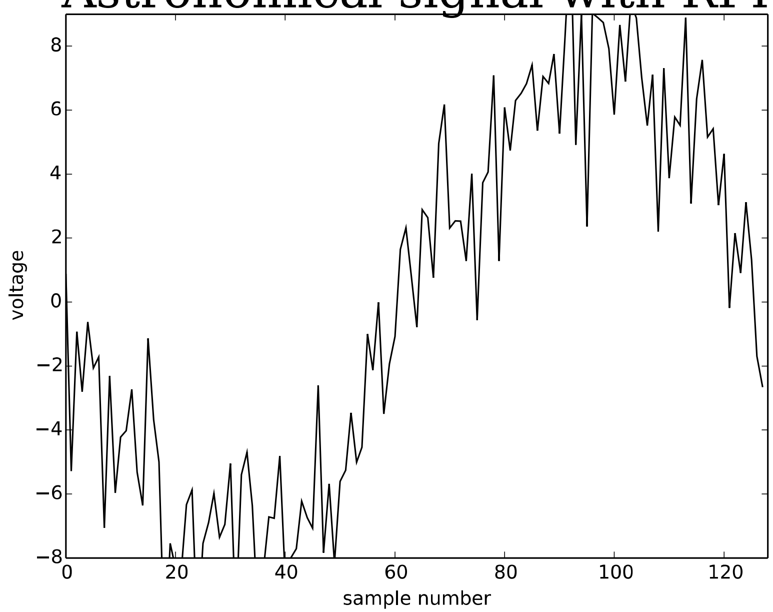
Analogue to Digital Converter (ADC)
the last analogue component

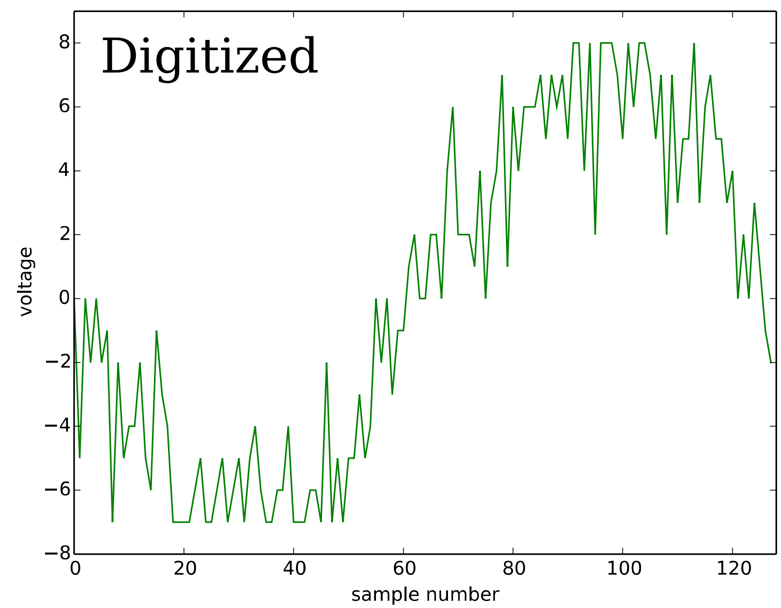
Astronomical signal





Astronomical signal with RFI





Digital Electronics

Computing the visibilities

Ambiguity in the Frequency Domain and the Nyquist Criterion

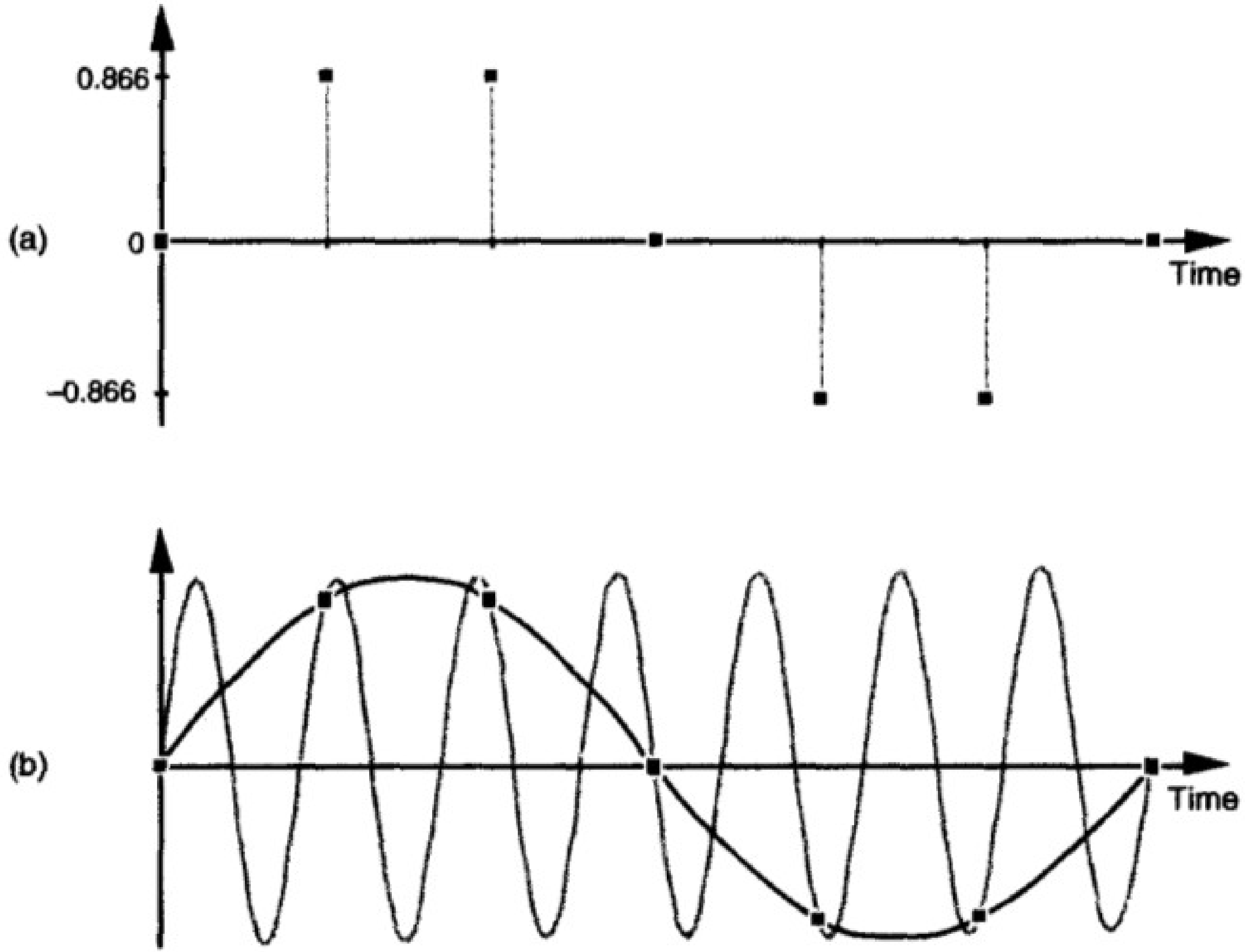


Figure 2-1 Frequency ambiguity: (a) discrete-time sequence of values; (b) two different sinewaves that pass through the points of the discrete sequence.

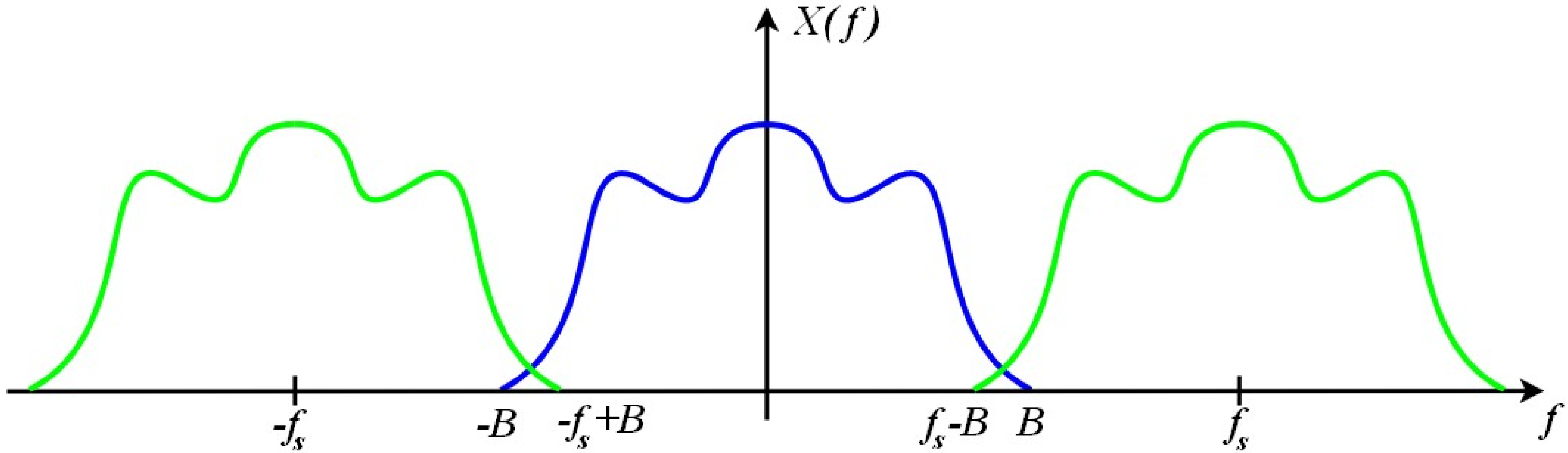
Ambiguity in the Frequency Domain and the Nyquist Criterion

f_s: frequency sampling rate

Ambiguity: When sampling at a rate of f_s, for k any integer, we cannot distinguish between sampled values of a sine wave with frequency f_0 and sine wave of frequency f_0+kf_s

This puts a limit on the maximum frequency:

$f_{nyquist} = 0.5\ f_s$, Nyquist frequency, the maximum frequency which can be resolved before aliasing

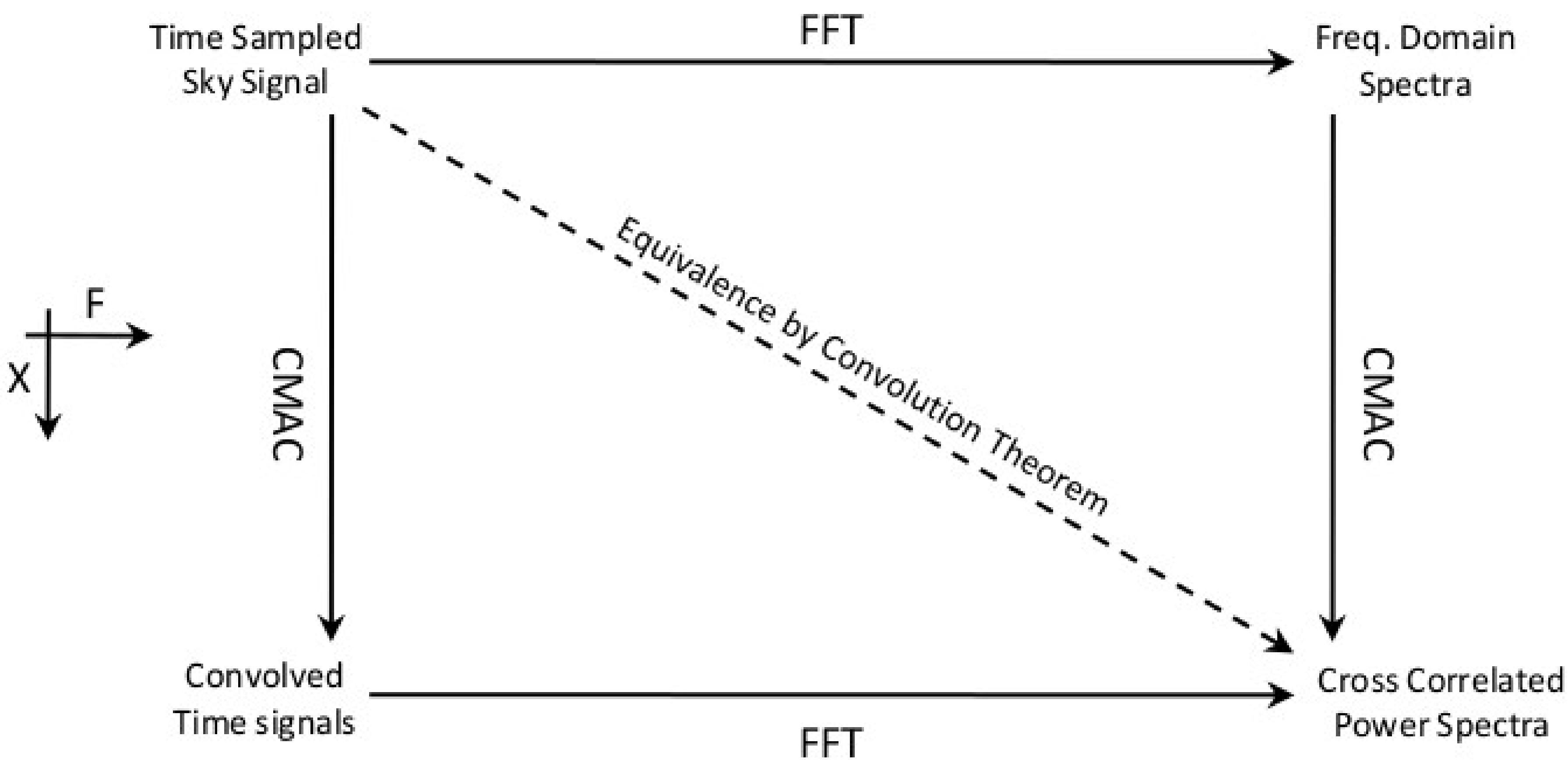


Convolution Theorem and Correlators

To compute visibilities, we would like to correlate (convolve) for each antenna pair (f,g)

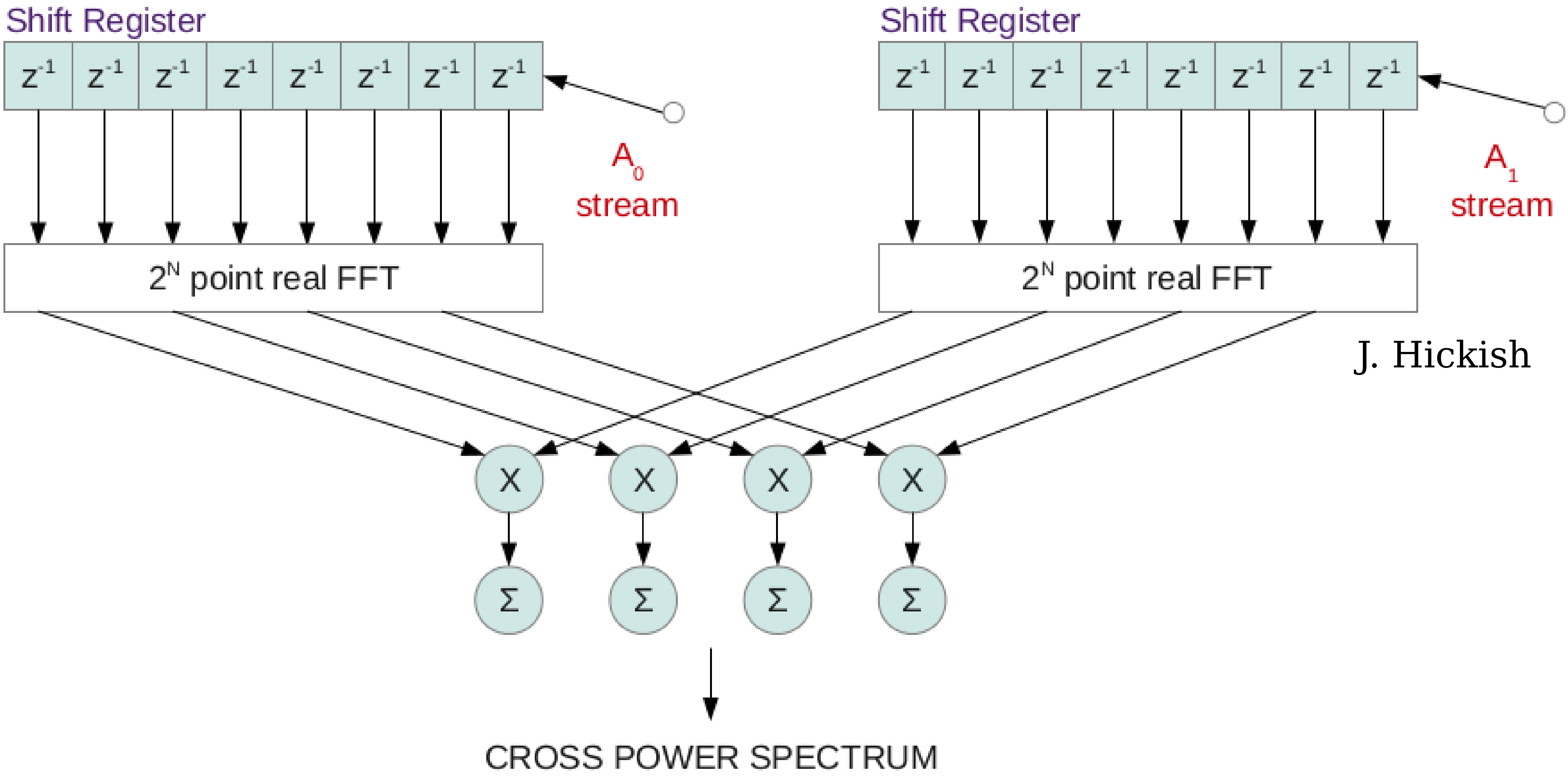
Convolution Theorem: $\mathcal{F}\{f * g\} = \mathcal{F}\{f\}\,\mathcal{F}\{g\}$

Where the convolution symbol is defined as: $f * g = \int f(x)g(z-x)\mathrm{d}x$



G. Foster

FX Correlator Design



J. Hickish

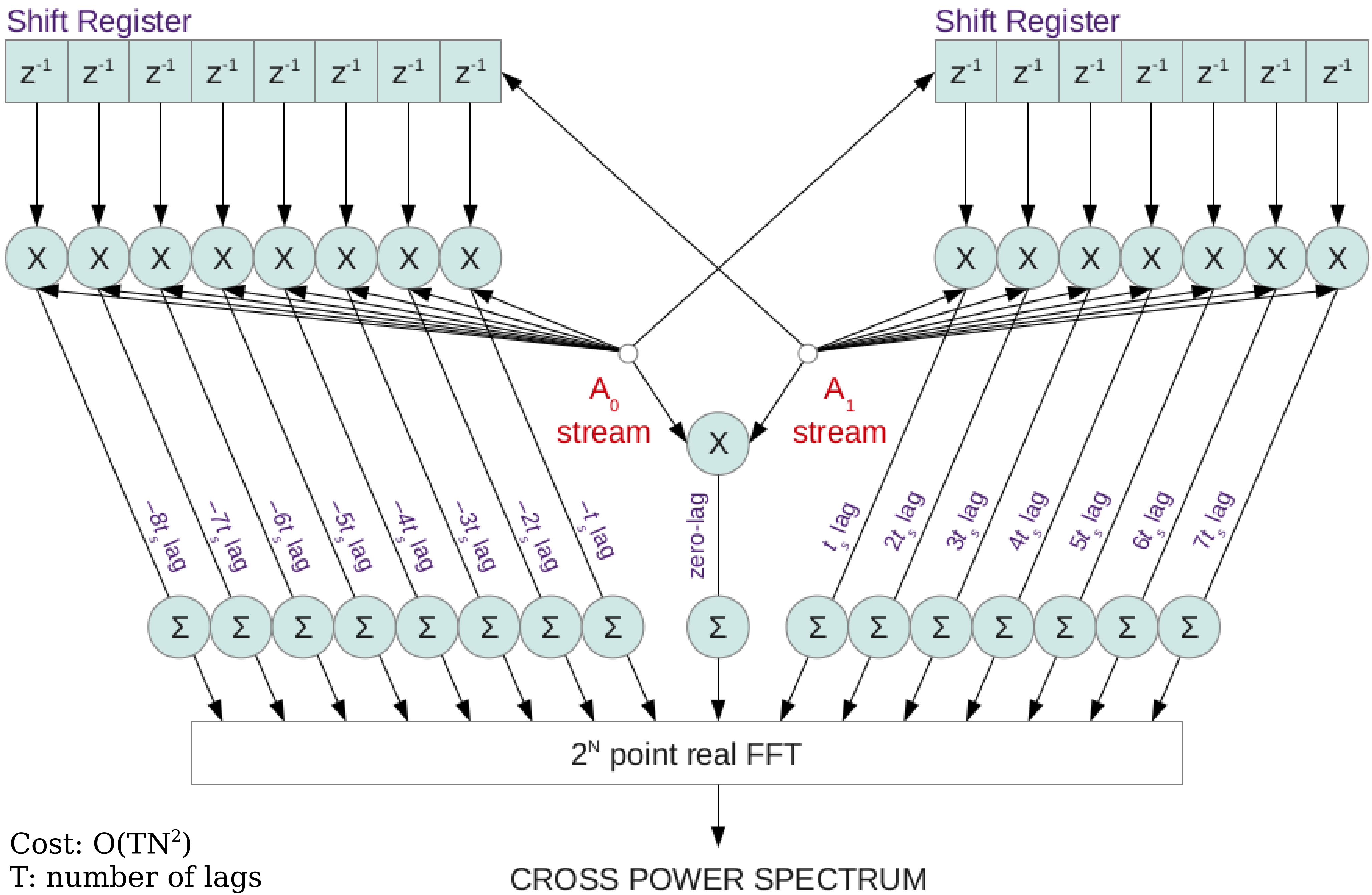
$$N_{bls} = N_{ant} + \frac{N_{ant}(N_{ant}-1)}{2}$$

Auto-correlations

Cross-correlations

Cost: $O(NM \log(M) + MN^2)$
M: frequency channels
N: number of antennas

XF Correlator Design



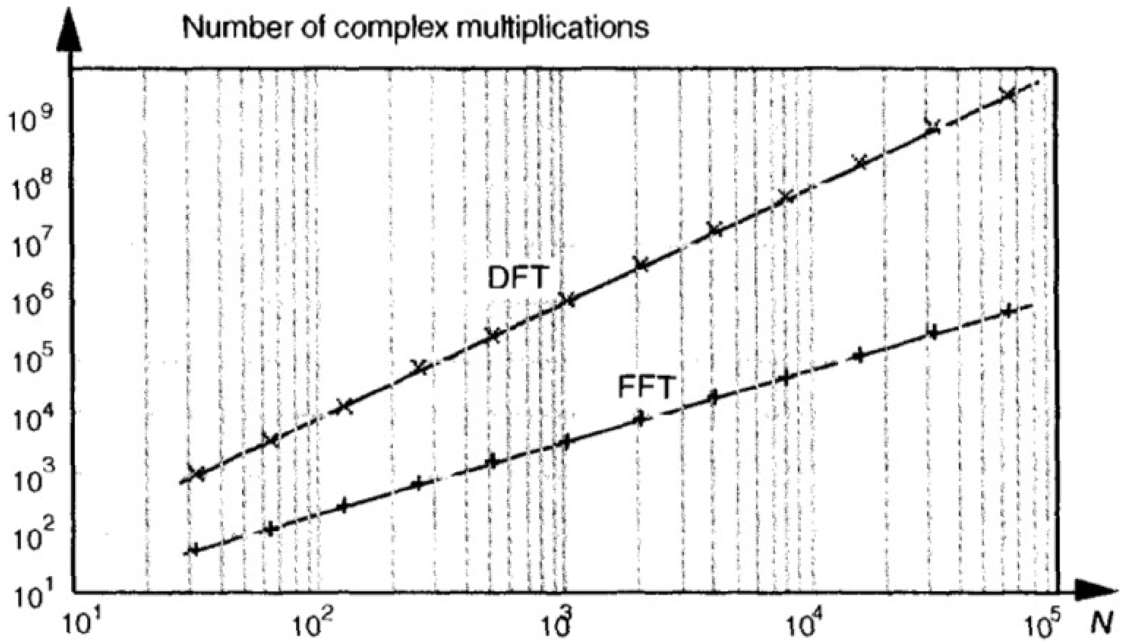
Cost: $O(TN^2)$
T: number of lags
N: number of antennas

J. Hickish

Fast Fourier Transform (FFT)

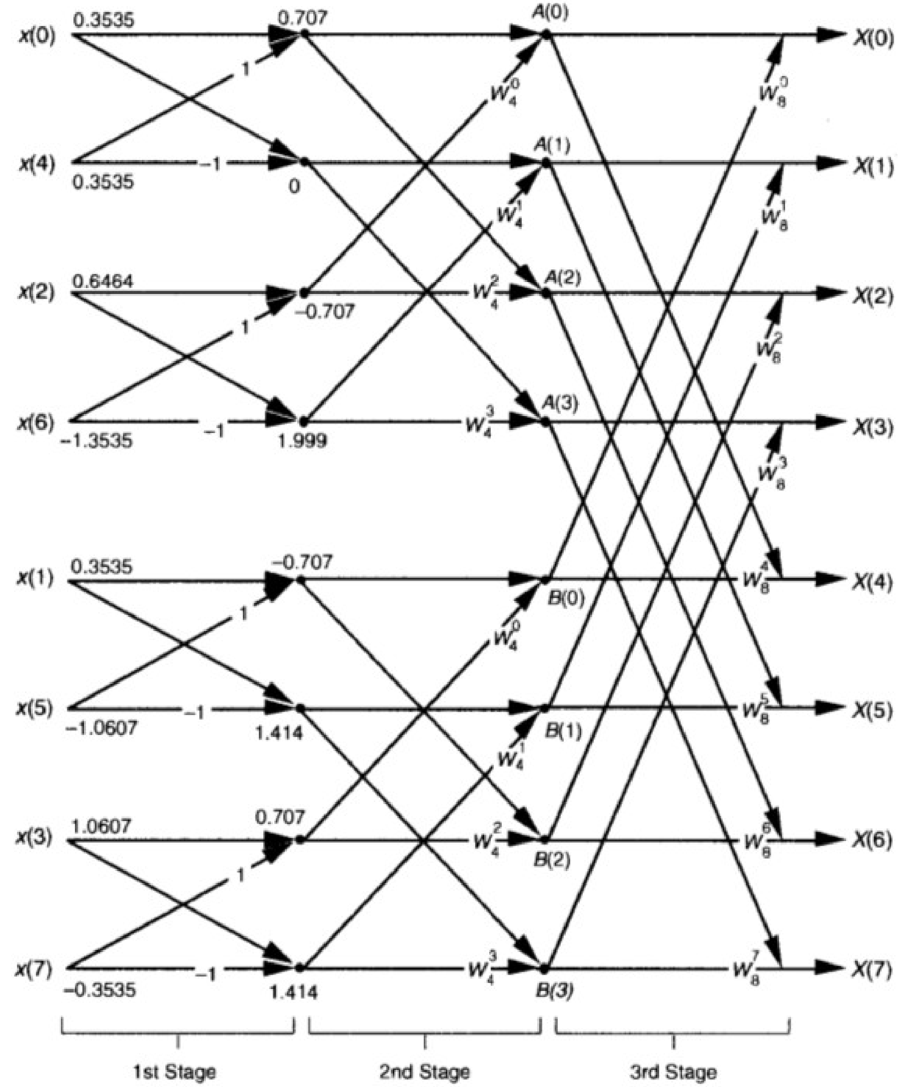
$$X_k = \sum_{n=0}^{N-1} x_n e^{\frac{-i2\pi}{N} nk}$$

Discrete Fourier Transform (DFT)



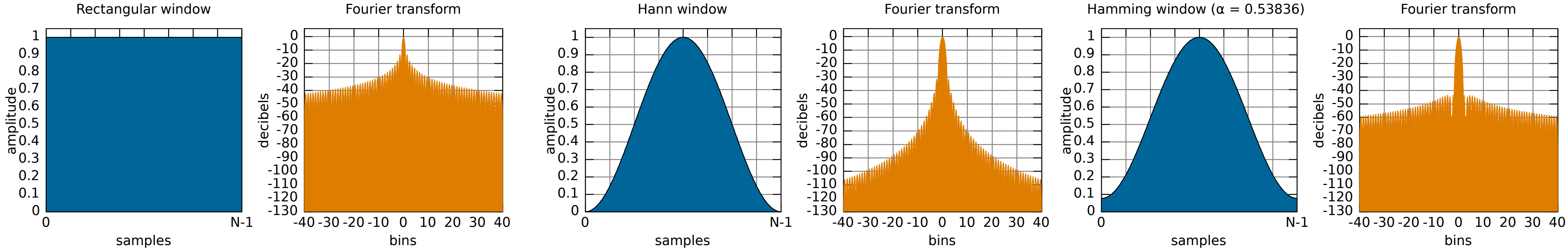
DFT Cost: $O(N^2)$
FFT Cost: $O(N \log N)$

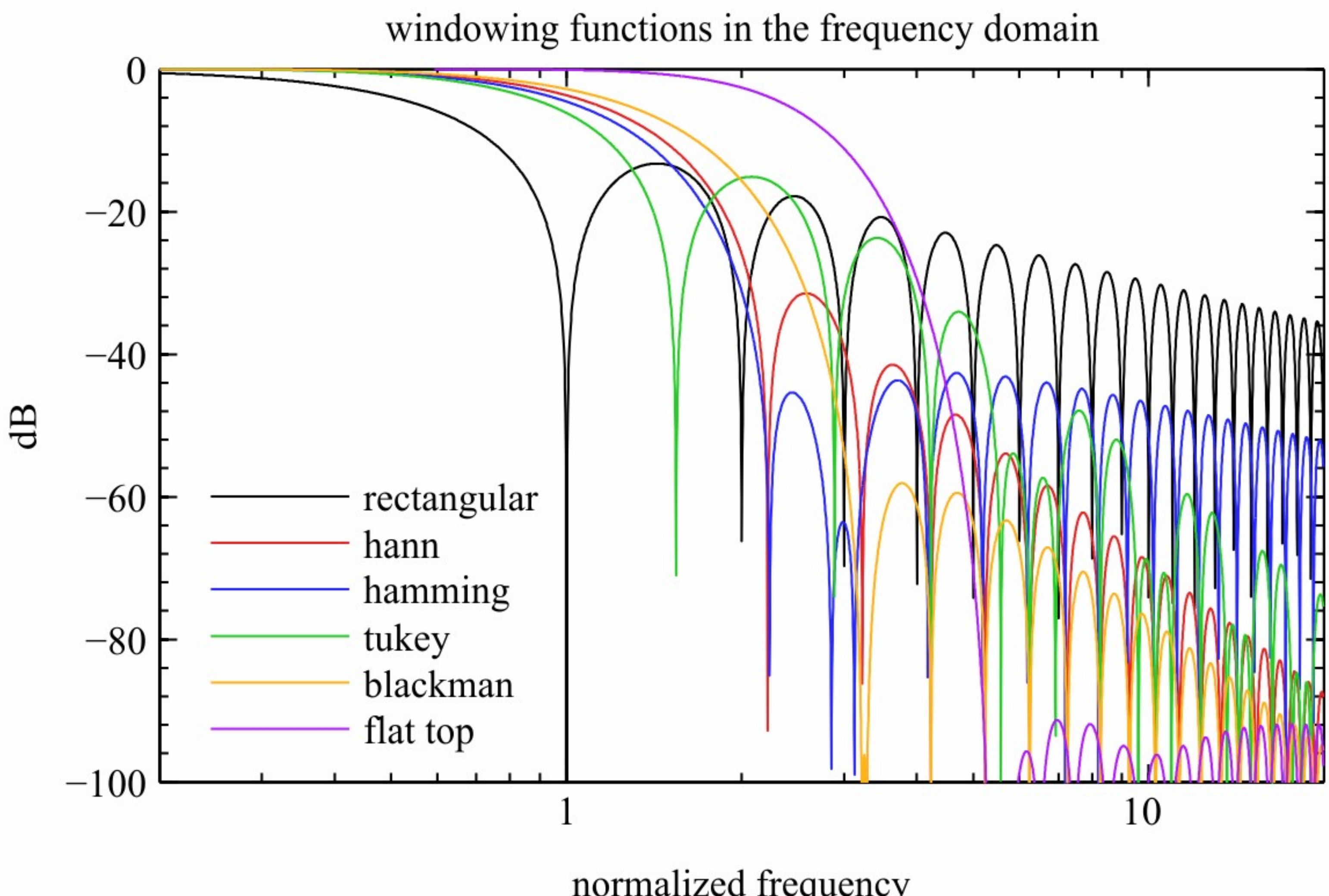
Radix-2 DIT 8-point
Cooley-Tukey FFT



Almost all FFT implementations use a radix-2 system, so FFT of size 2^N are ideal. Try to avoid Fourier transforms of prime number size.

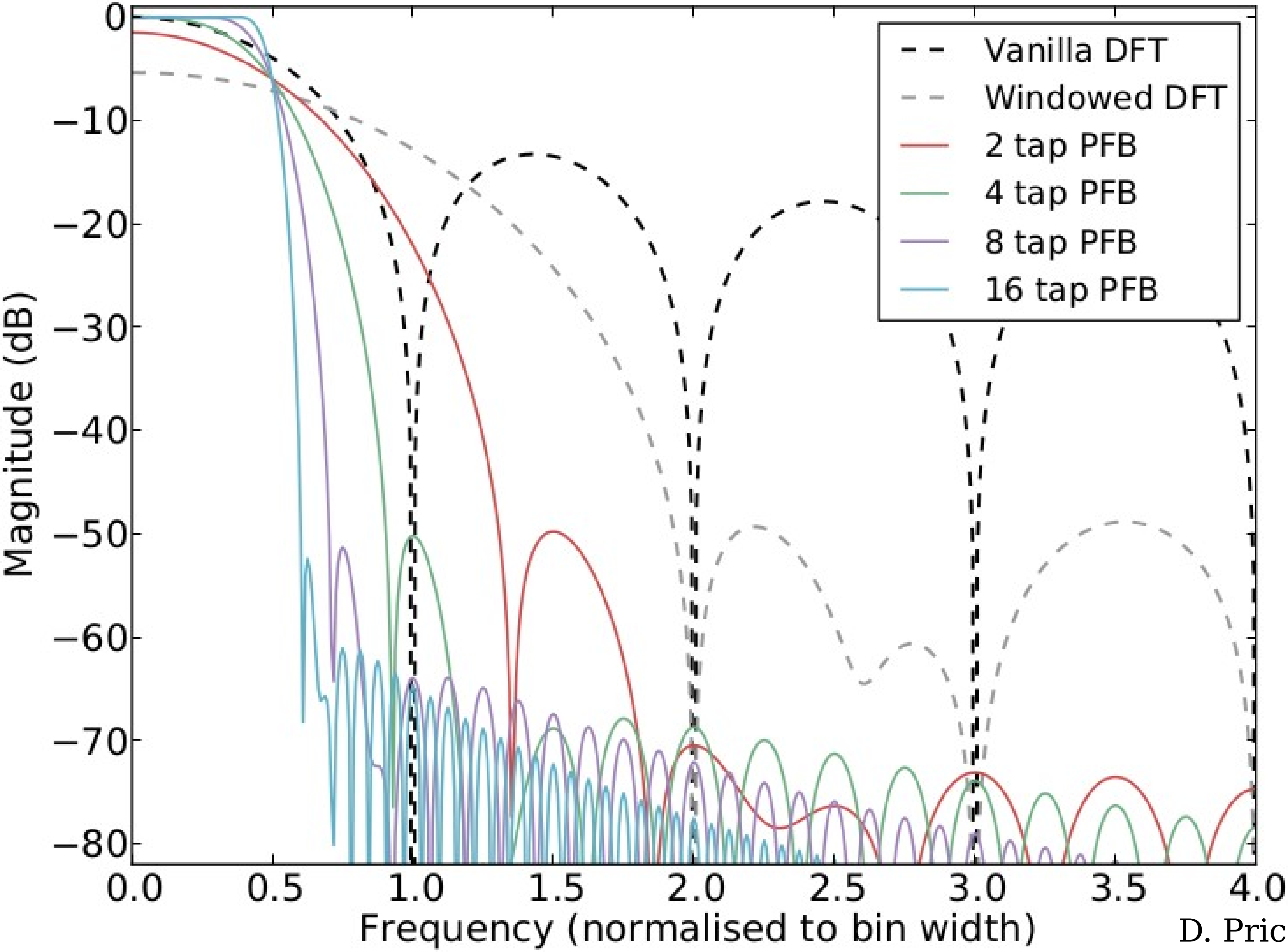
Windowing Functions



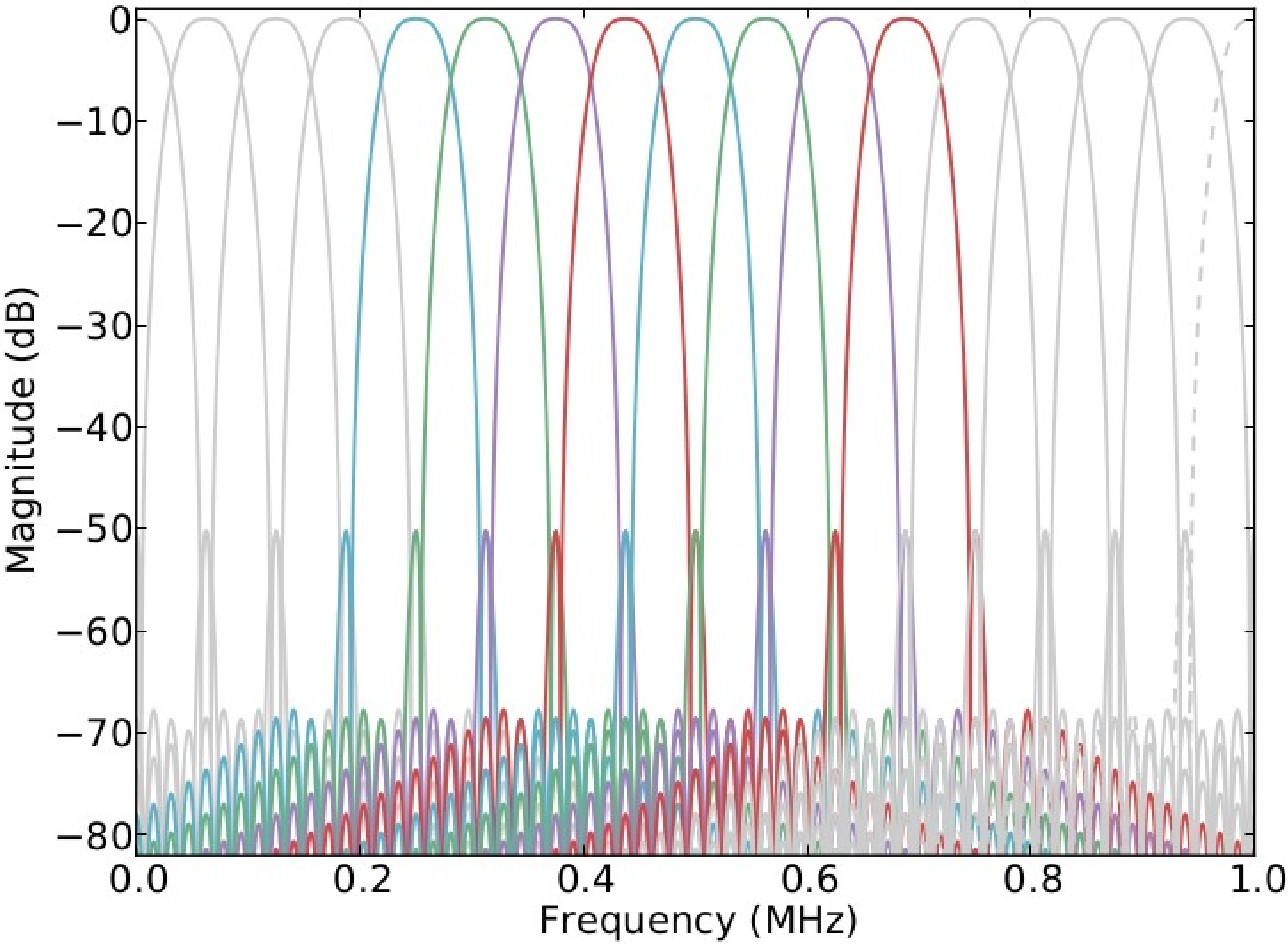


http://en.wikipedia.org/wiki/Window_function

Polyphase Filter Bank (PFB)



D. Price

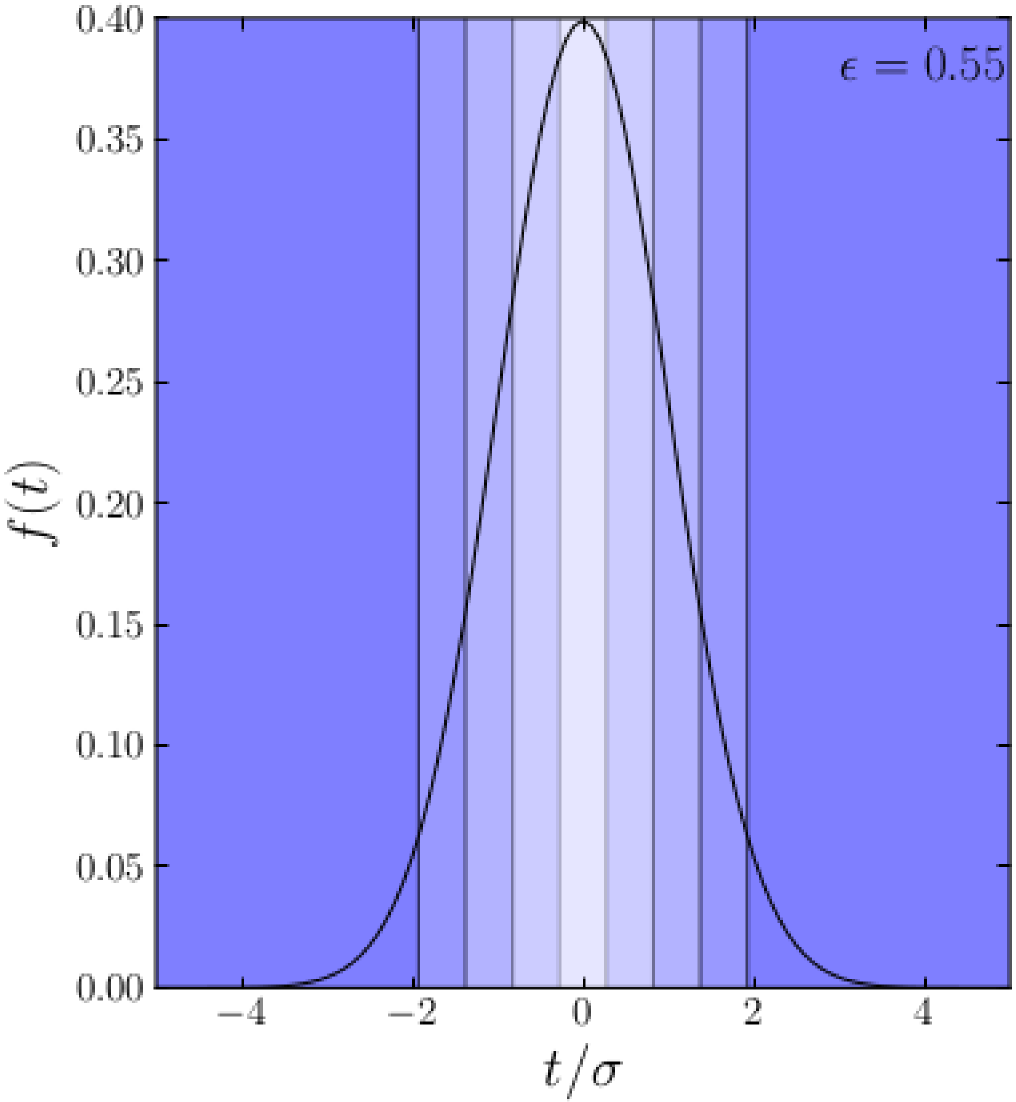

0
−10
−20
−30
−40
−50
−60
−70
−80
Magnitude (dB)
0.0
0.2
0.4
0.6
0.8
1.0
Frequency (MHz)

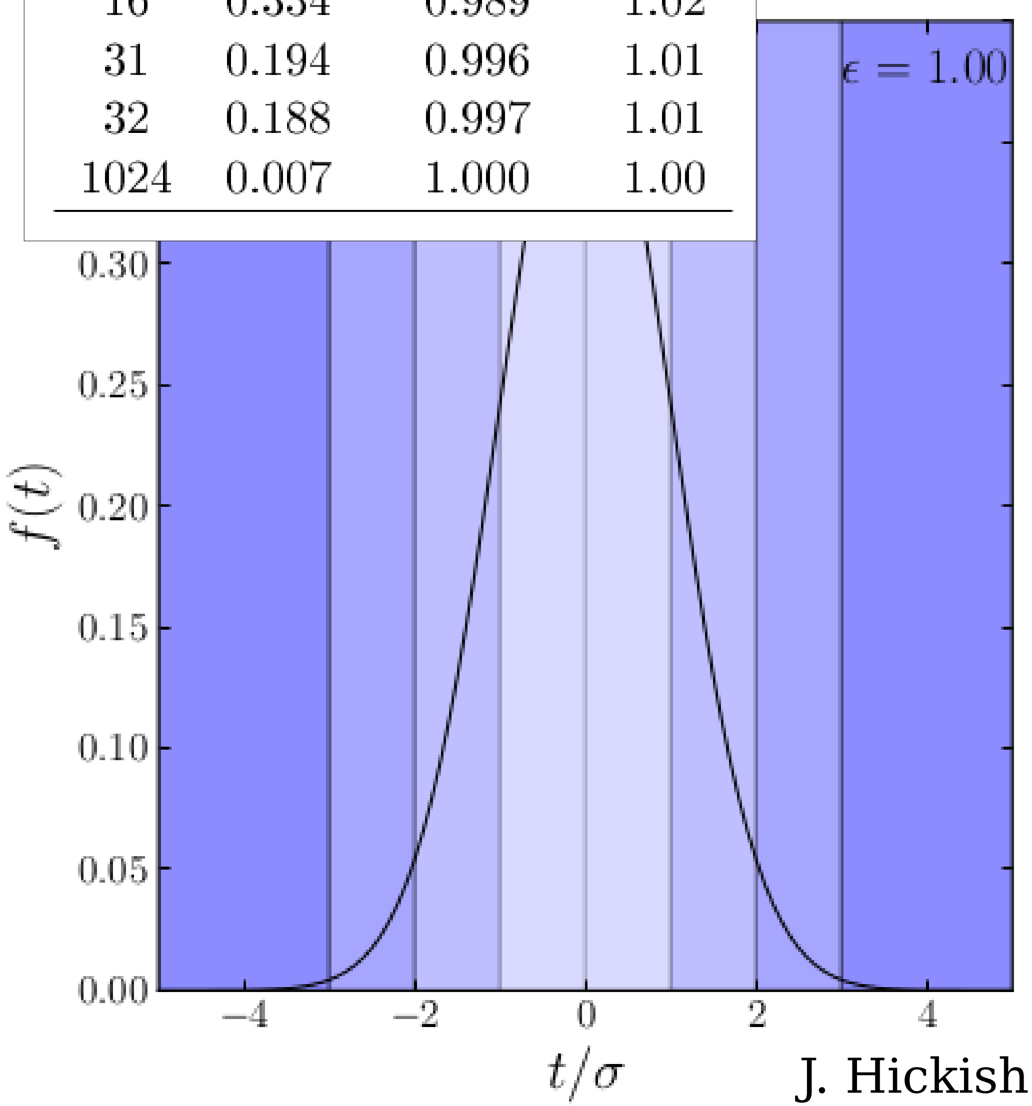
D. Price

Low bit resolution quantization and efficiency

Key point: assuming a Gaussian signal (i.e. Astronomical signals), we only need 4 bits to get 98.9% efficiency

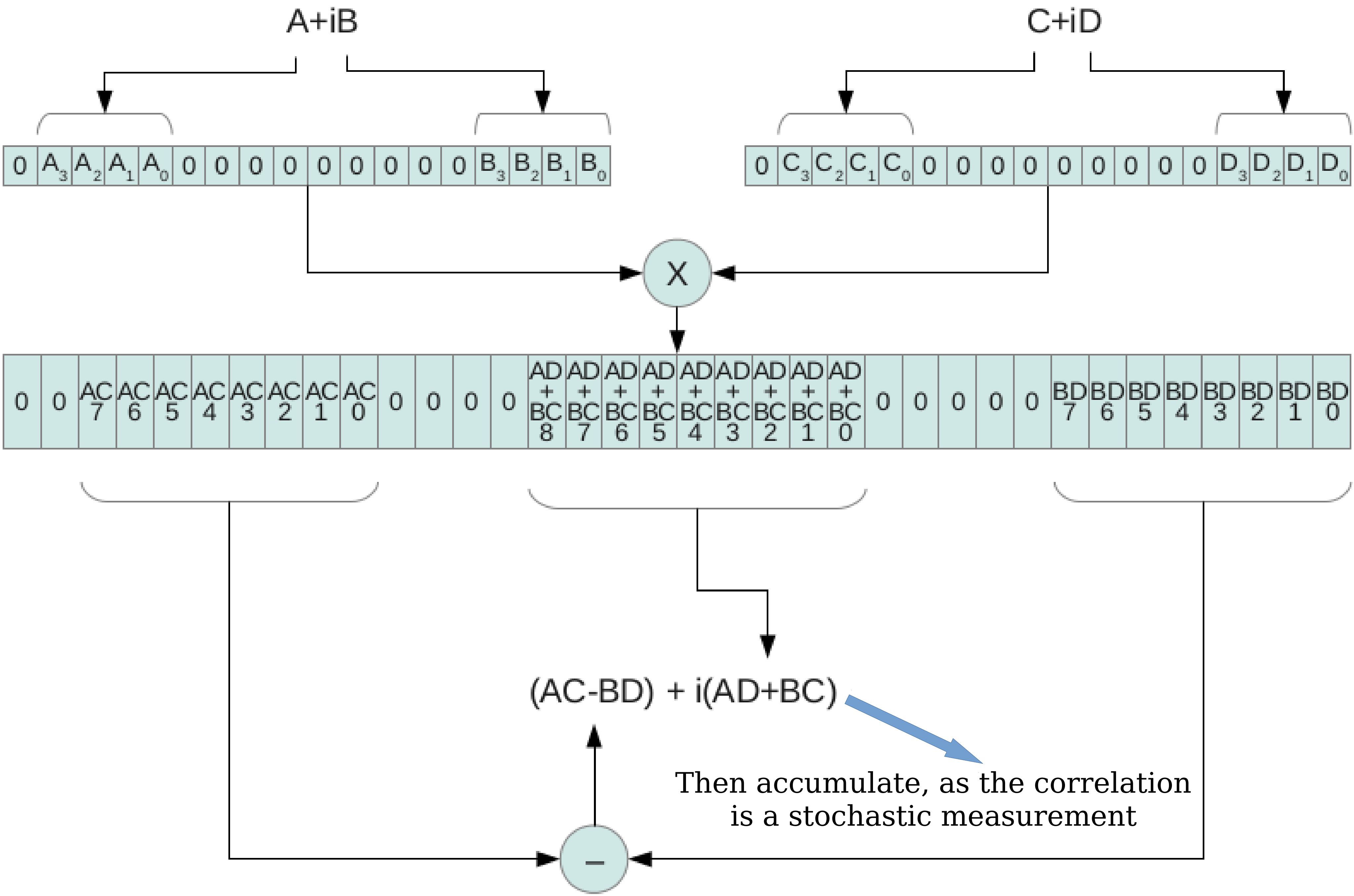
L	ϵ_{opt}	$\eta(L, \epsilon_{opt})$	Δt
2	-	0.637	2.46
3	0.612	0.810	1.52
7	0.674	0.950	1.11
8	0.575	0.966	1.07
15	0.358	0.987	1.03
16	0.334	0.989	1.02
31	0.194	0.996	1.01
32	0.188	0.997	1.01
1024	0.007	1.000	1.00



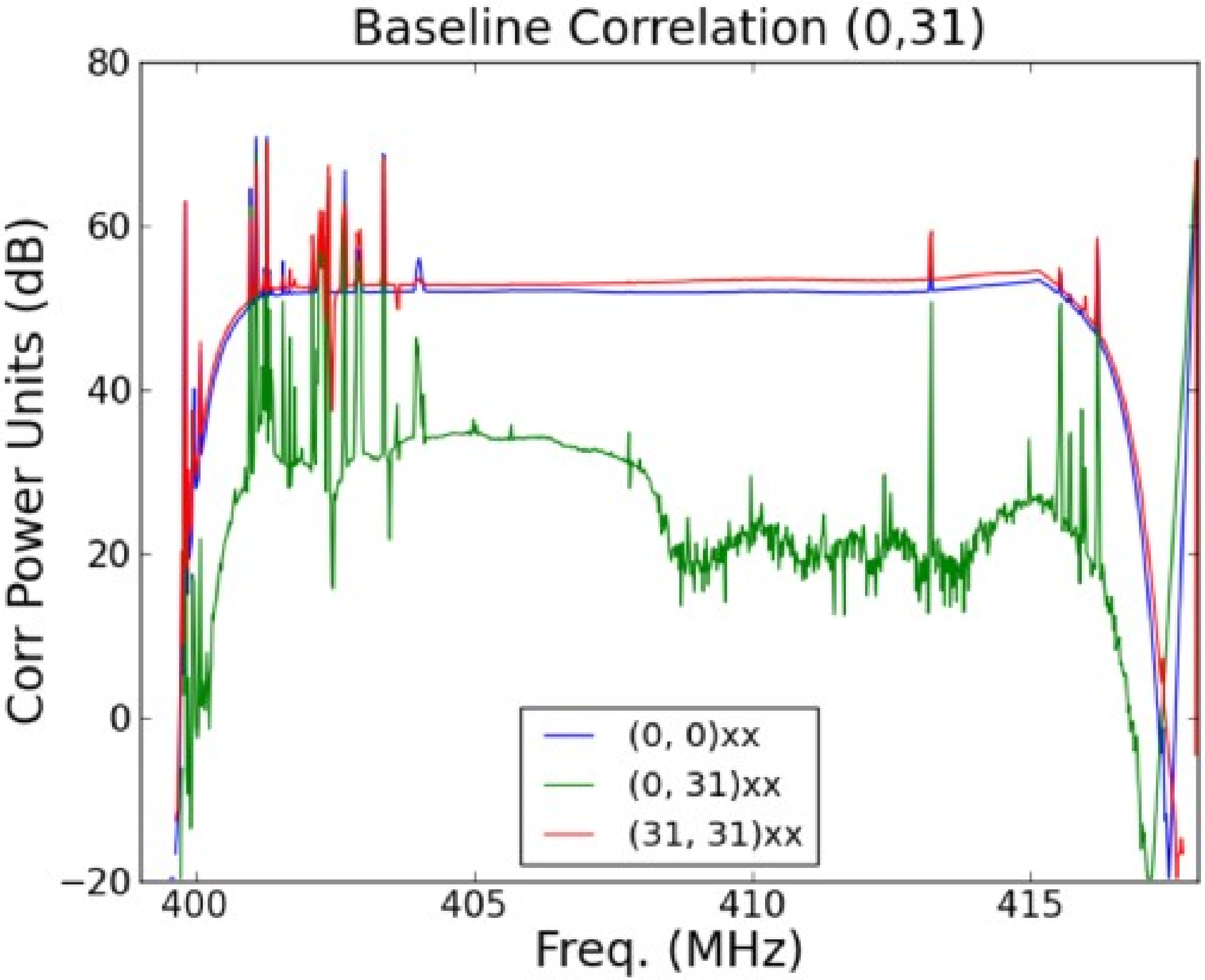


J. Hickish

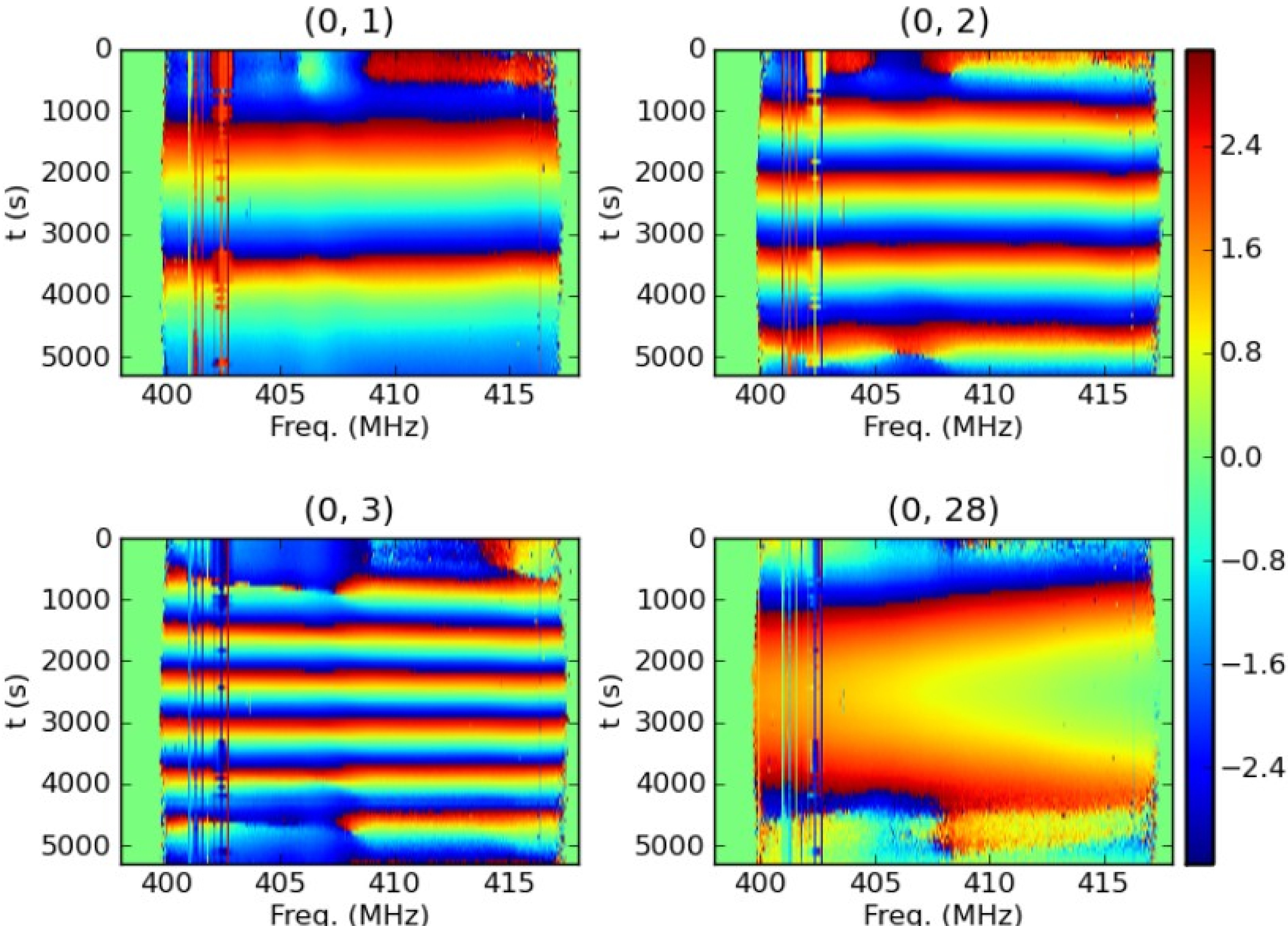
Efficient 4-bit Multiplication in an FPGA



Then accumulate, as the correlation
is a stochastic measurement


Baseline Correlation (0,31)
Corr Power Units (dB)
Freq. (MHz)
80
60
40
20
0
−20
400
405
410
415
(0, 0)xx
(0, 31)xx
(31, 31)xx

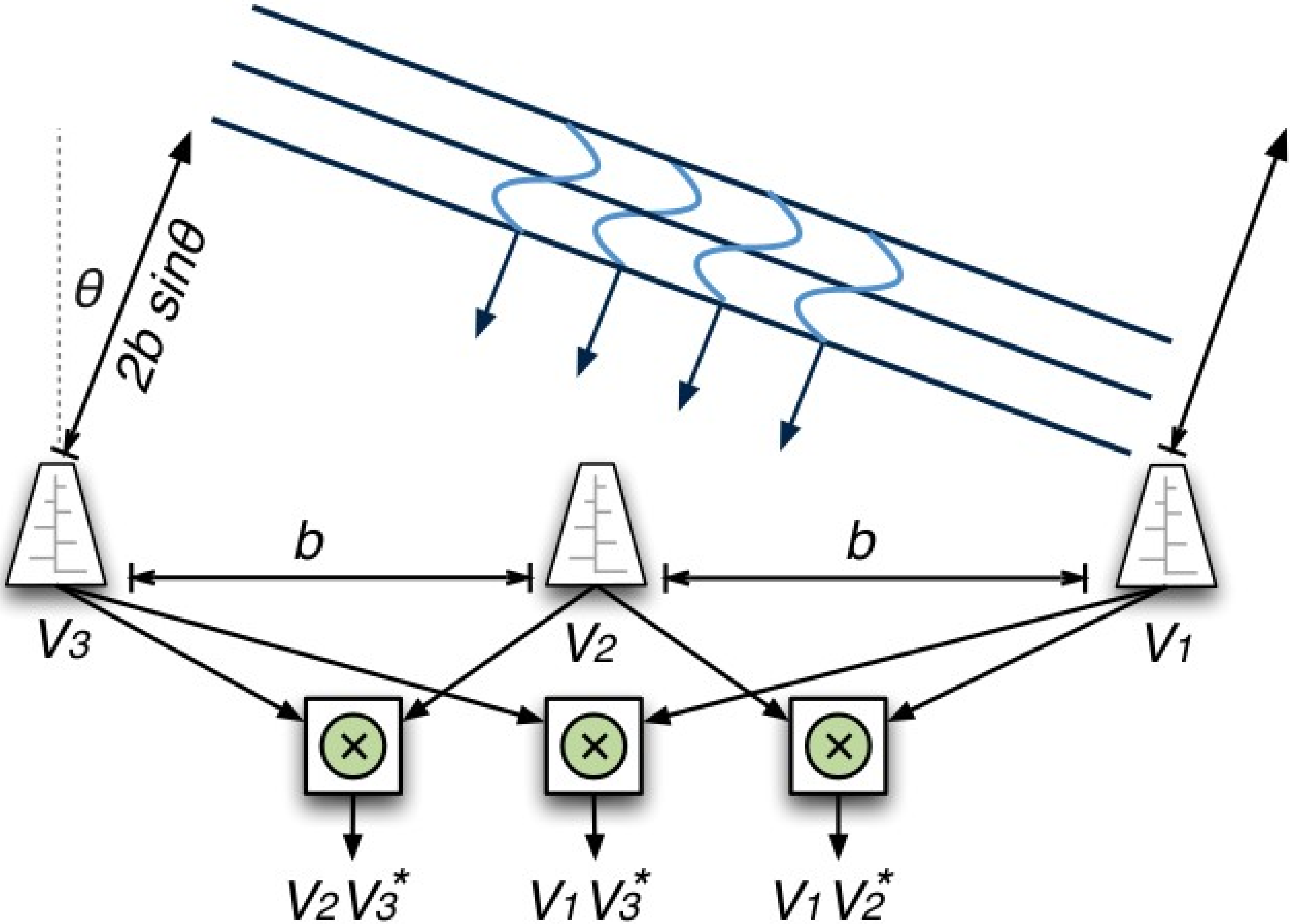
G. Foster


(0, 1)
(0, 2)
(0, 3)
(0, 28)
t (s)
Freq. (MHz)
0
1000
2000
3000
4000
5000
400
405
410
415
2.4
1.6
0.8
0.0
−0.8
−1.6
−2.4

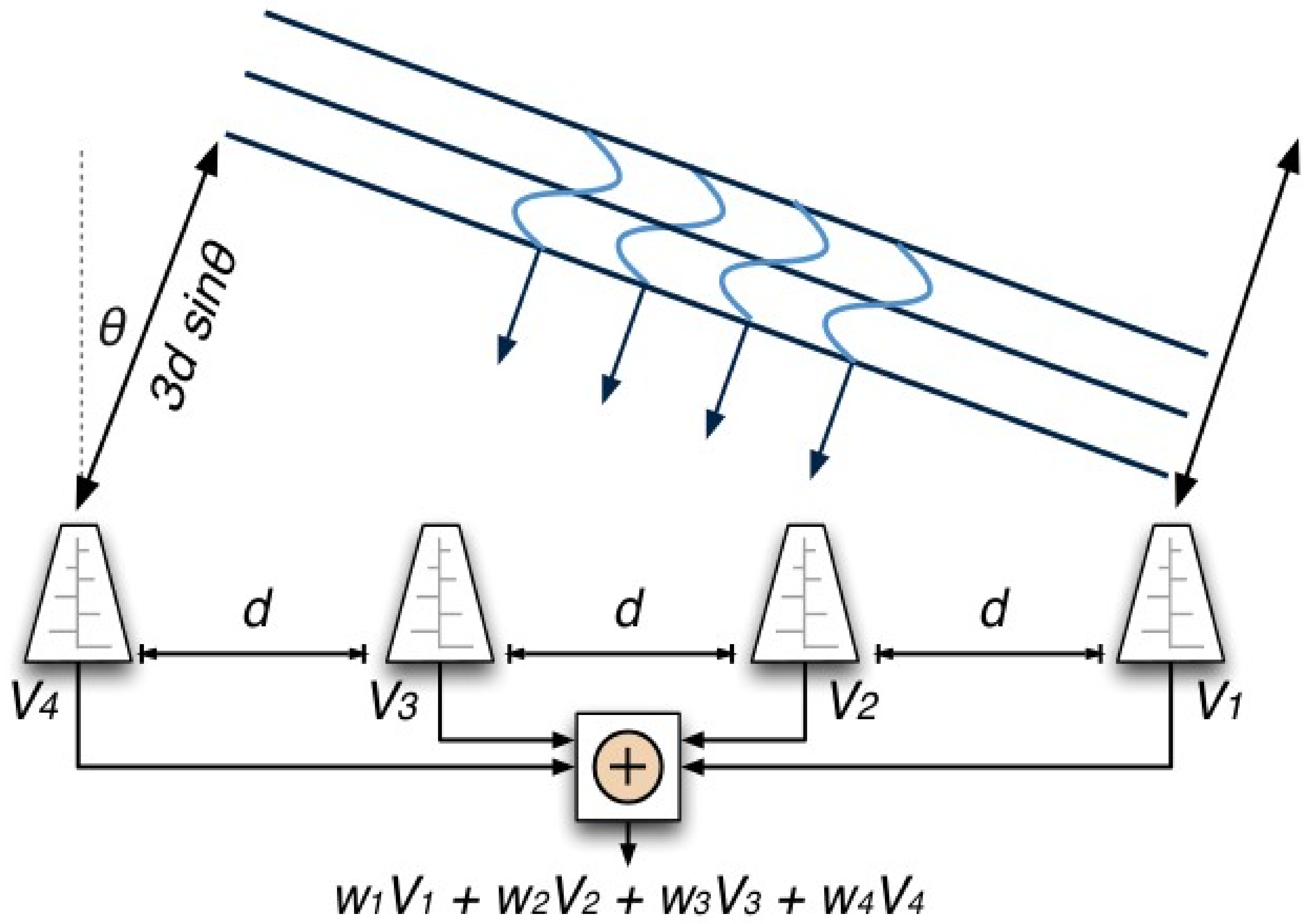
G. Foster

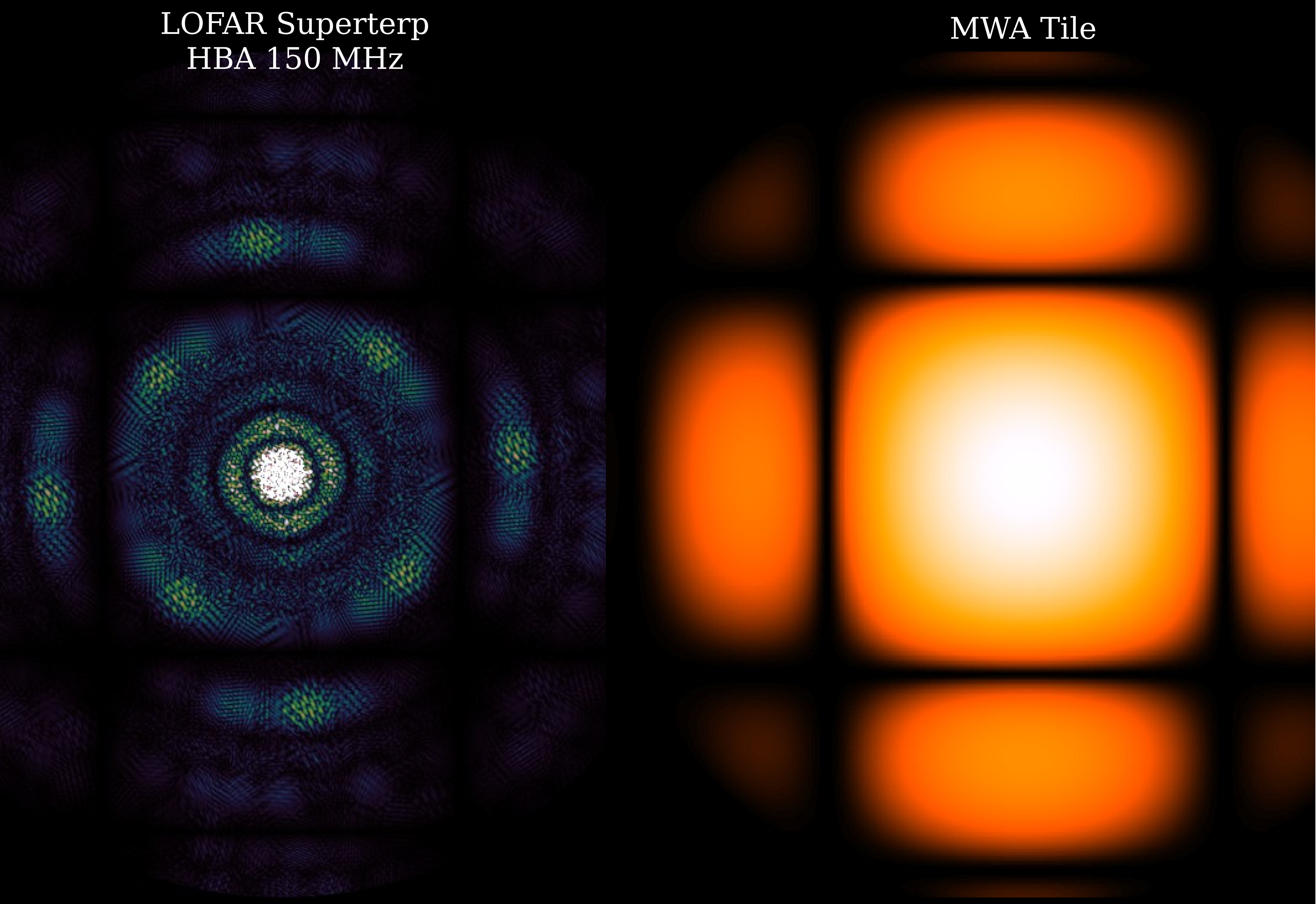
Transiting Arrays,
Aperture Arrays,
and Phased Array Feeds

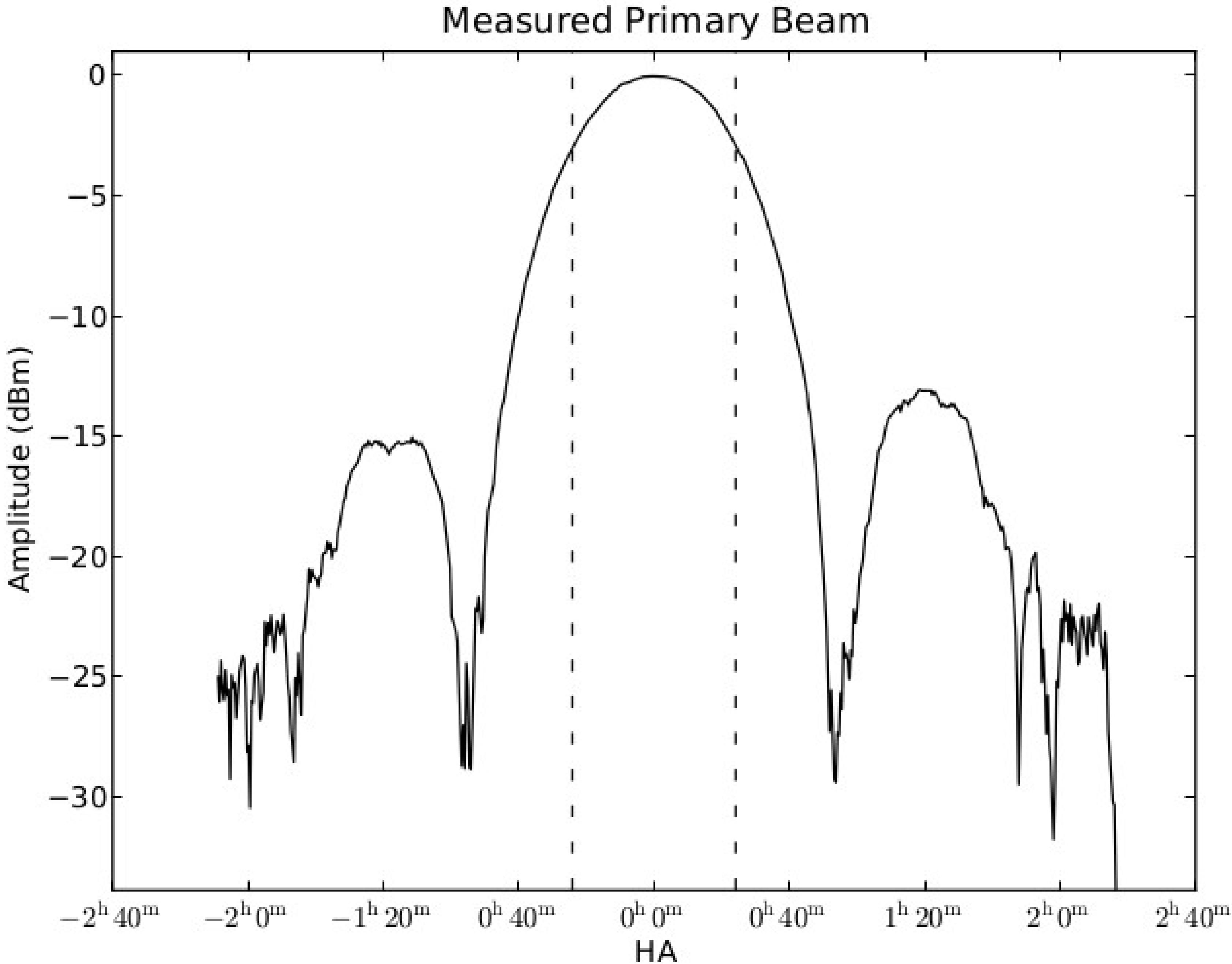
Simple Interferometer Model

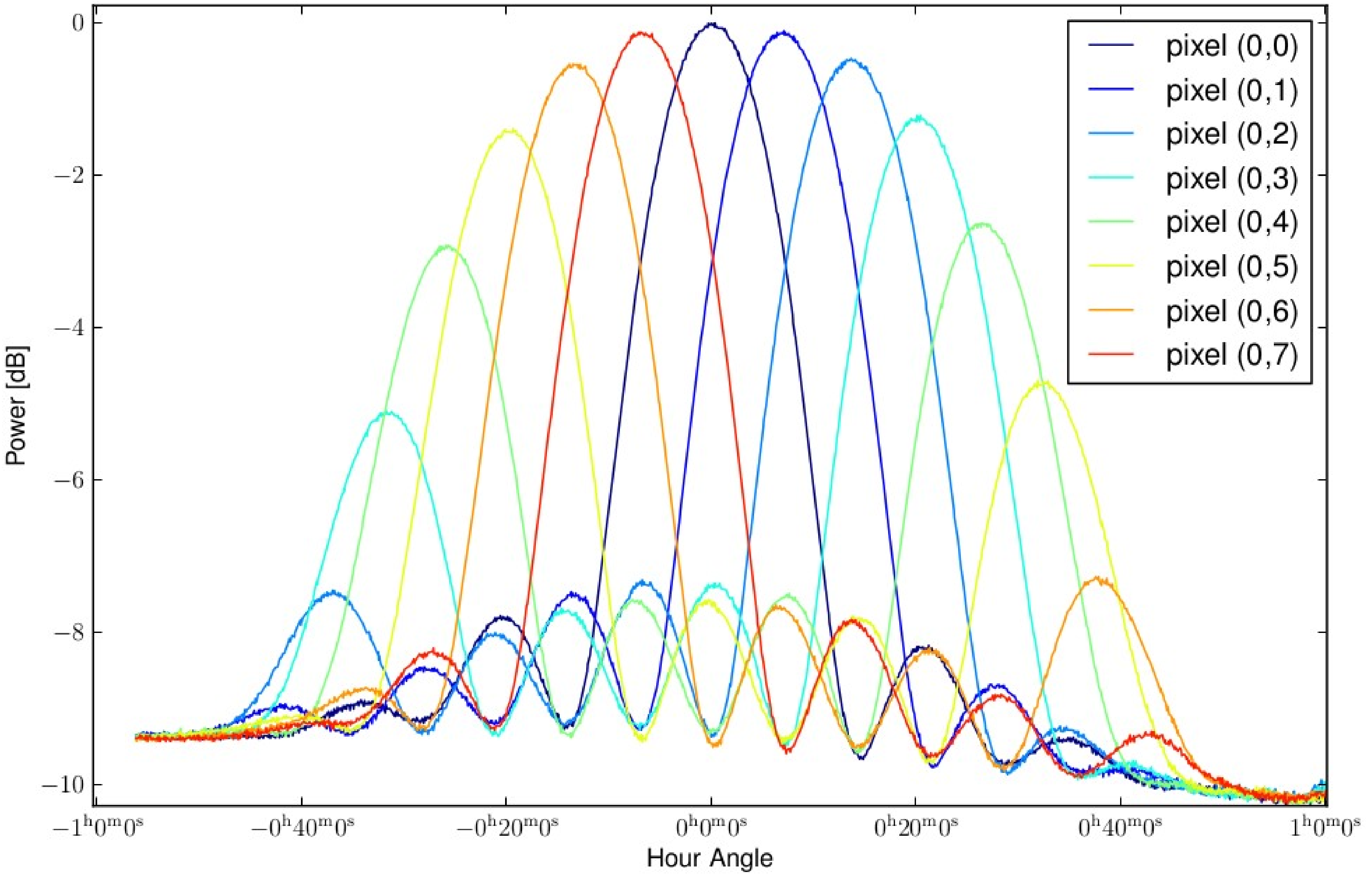


Simple Beamformer Model




LOFAR Superterp
HBA 150 MHz
MWA Tile

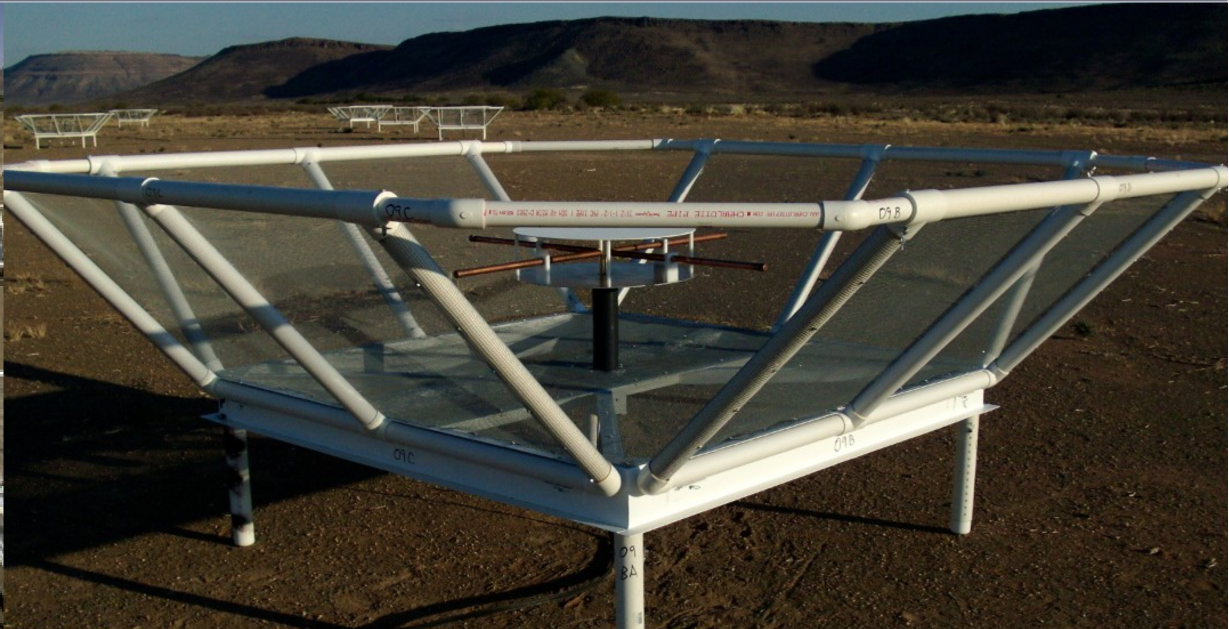

Measured Primary Beam
Amplitude (dBm)
0
−5
−10
−15
−20
−25
−30
HA
$-2^{\mathrm{h}}\,40^{\mathrm{m}}$
$-2^{\mathrm{h}}\,0^{\mathrm{m}}$
$-1^{\mathrm{h}}\,20^{\mathrm{m}}$
$0^{\mathrm{h}}\,40^{\mathrm{m}}$
$0^{\mathrm{h}}\,0^{\mathrm{m}}$
$0^{\mathrm{h}}\,40^{\mathrm{m}}$
$1^{\mathrm{h}}\,20^{\mathrm{m}}$
$2^{\mathrm{h}}\,0^{\mathrm{m}}$
$2^{\mathrm{h}}\,40^{\mathrm{m}}$


Power [dB]
0
−2
−4
−6
−8
−10
−1h0m0s
−0h40m0s
−0h20m0s
0h0m0s
0h20m0s
0h40m0s
1h0m0s
Hour Angle
pixel (0,0)
pixel (0,1)
pixel (0,2)
pixel (0,3)
pixel (0,4)
pixel (0,5)
pixel (0,6)
pixel (0,7)

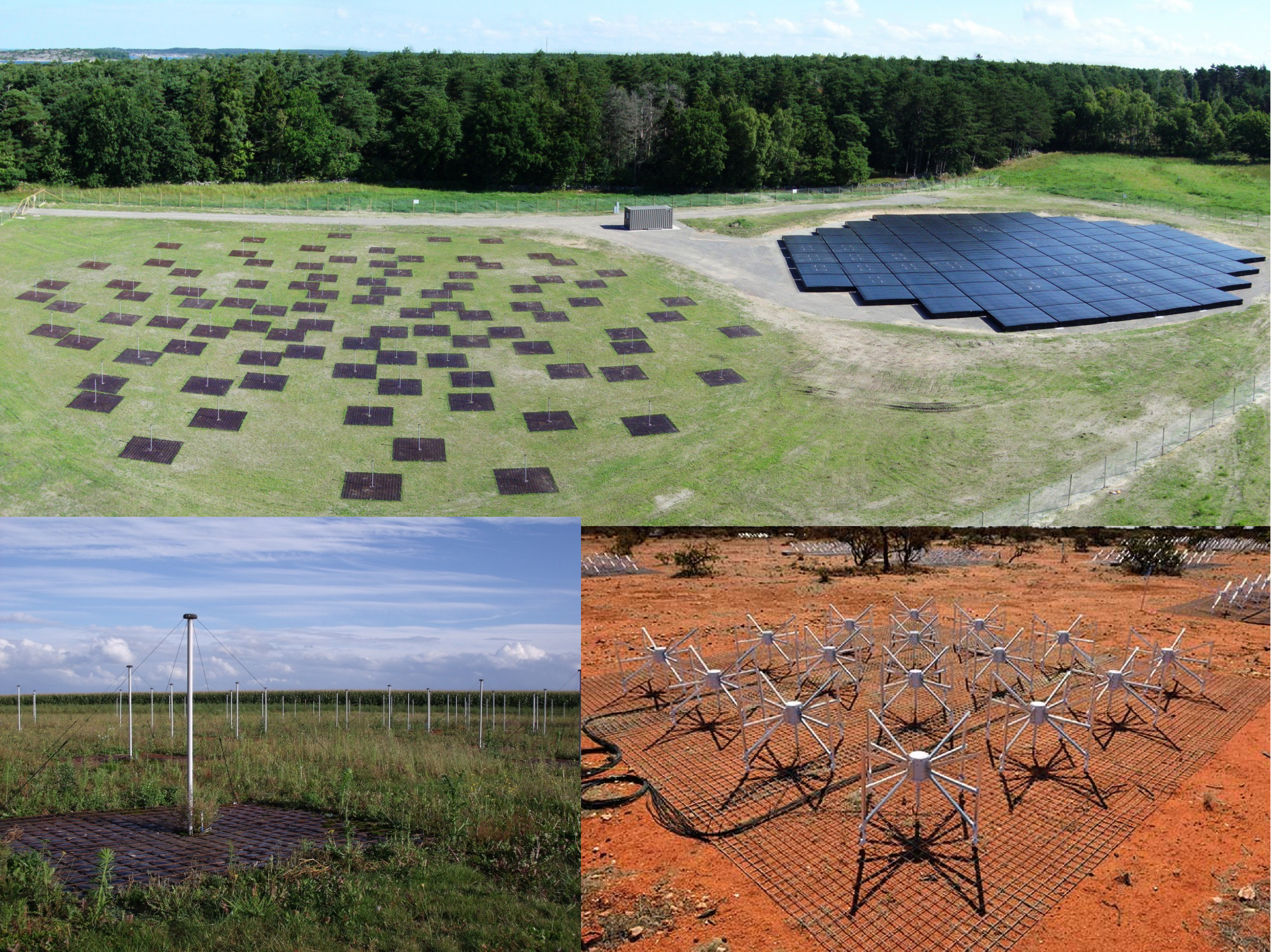
J. Hickish

Phased Array Feeds (PAFs)

Transiting Arrays




Aperture Arrays
LOFAR Superterp



References

Antennas and Aperture Arrays:

Phased Array Antenna Handbook : Mailloux
Antenna Theory : Balanis

Analogue Electronics:

Microwave Engineering : Pozar

Digital Signal Processing:

Understanding Digital Signal Processing : Lyons
FFT and Its Applications : Brigham