Fundamentals of radio interferometry I

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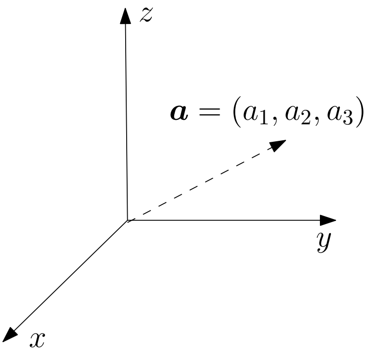
Overview

References

1. **TCP**: G.B. Taylor et al., Synthesis imaging in radio astronomy II, 1998
2. **TSM**: A. Thomson et al., Interferometry and synthesis in radio astronomy, second edition

Vectors



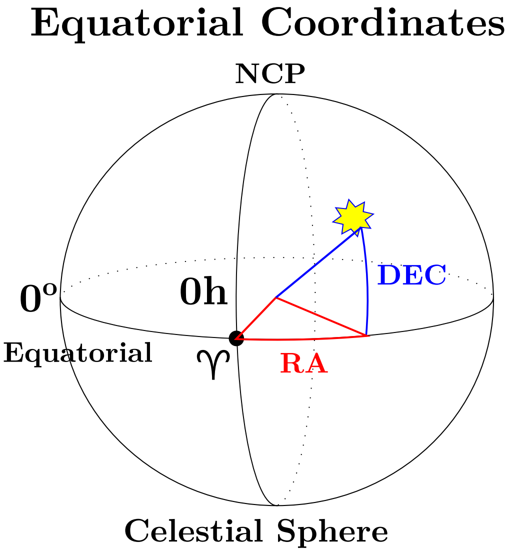
1. Vector length: $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.
2. Vector addition: $\mathbf{a} + \mathbf{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.
3. Vector subtraction: $\mathbf{a} - \mathbf{b} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$.

Simple conversions

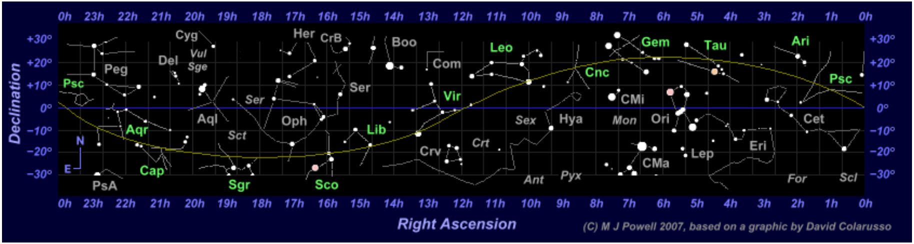
1. 1 degree ($^\circ$) $=$ 60 arcminutes ($'$) $=$ 3600 arcseconds ($''$), where 360° $=$ circle.
2. 1 radian (rad) $= 57.296^\circ = 206265''$, where 2π rad $=$ circle.
3. 1 hour (h) $=$ 60 minutes $=$ 3600 seconds, where 24 h $=$ circle.
4. 1 h $= 15^\circ$

Notation

1. δ - DEC/Declination
2. α - RA/Right Ascension
3. H - Hour Angle
4. LST - Local Sidereal Time
5. L - Latitude
6. A - Azimuth
7. E - Elevation/Altitude

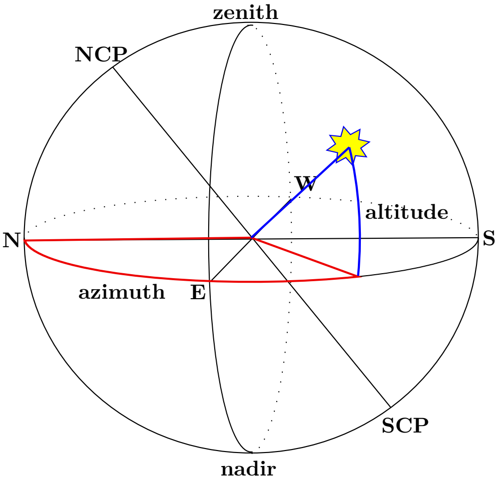

Equatorial Coordinates
NCP
0h
0°
DEC
Equatorial
♈
RA
Celestial Sphere

Vernal equinox (the first point of Aries)

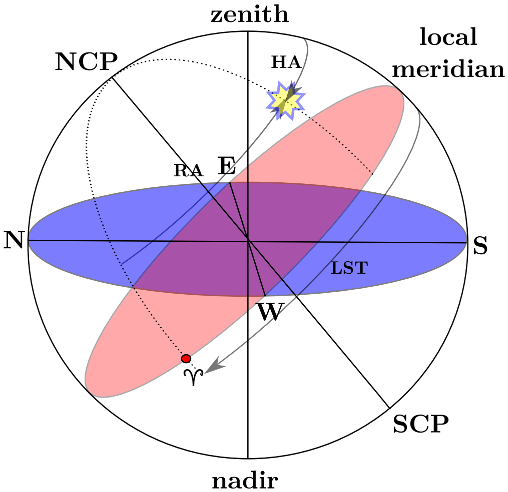


- The coordinate pair (0h,0°) is the first point of Aries (the vernal equinox), the point where the sun crosses the celestial equator from south to north.
- It used to be in Aries, but due to precession (change in orientation of earth's rotation axis) it is now found in Pisces.

Horizontal Coordinates

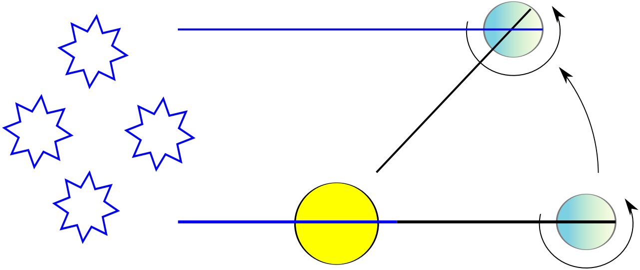


Hour Angle



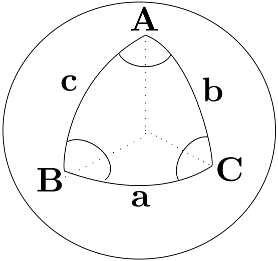
HA = LST -RA

A sidereal day



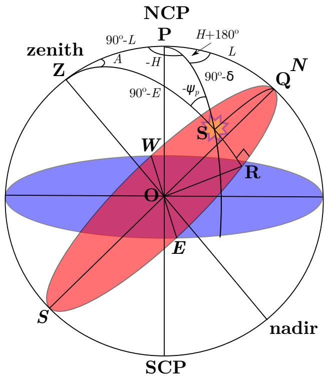
The solar day is 4m longar than the sidereal day, due to the fact that the earth rotates around the sun.

Spherical Trigonometry



Cosine Rule: $\cos b = \cos a \cos c + \sin a \sin b \cos A$
Sine Rule: $\sin b \sin A = \sin B \sin a$
Five-part Rule: $\sin b \cos A = \cos a \sin c - \sin a \cos c \cos B$

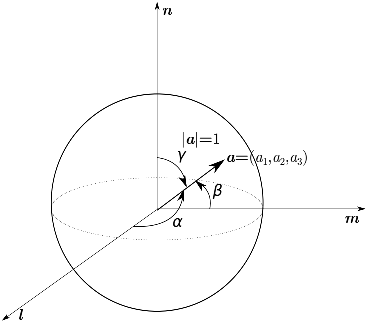
Conversion between Equatorial and Horizontal



$$\cos\delta\cos H = \cos L\sin E - \sin L\cos E\cos A$$
$$-\cos\delta\sin H = \cos E\sin A$$
$$\sin\delta = \sin L\sin E + \cos L\cos E\cos A$$

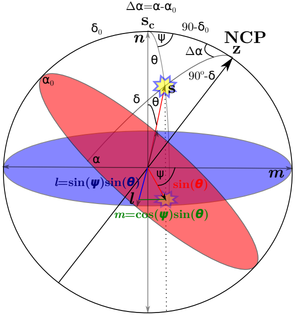
TSM (p. 89 Eq. 4.4)

Direction Cosines



$$
\begin{aligned}
l &= \cos\alpha = \frac{a_1}{|\mathbf{a}|} \\
m &= \cos\beta = \frac{a_2}{|\mathbf{a}|} \\
n &= \cos\gamma = \frac{a_2}{|\mathbf{a}|} \\
1 &= l^2 + m^2 + n^2
\end{aligned}
$$

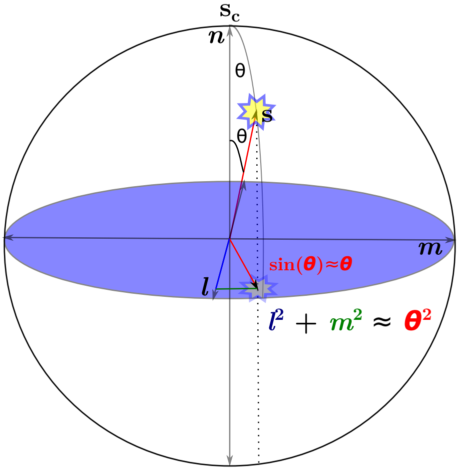
Conversion between Equatorial and Direction Cosines



$$
\begin{aligned}
l &= \sin\theta\sin\psi = \cos\delta\sin\Delta\alpha \\
m &= \sin\theta\cos\psi = \sin\delta\cos\delta_0 - \cos\delta\sin\delta_0\cos\Delta\alpha \\
\delta &= \sin^{-1}(m\cos\delta_0 + \sin\delta_0\sqrt{1-l^2-m^2}) \\
\alpha &= \alpha_0 + \tan^{-1}\left(\frac{l}{\cos\delta_0\sqrt{1-l^2-m^2} - m\sin\delta_0}\right)
\end{aligned}
$$

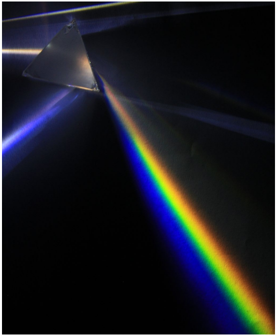
TCP (p. 388 Eq. 19.8–19.12)

Understanding l and m



We therefore measure l and m in $^\circ$.

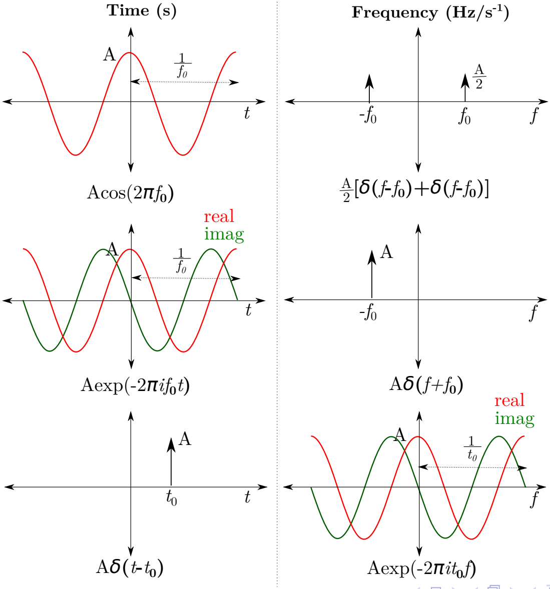
Fourier Transform



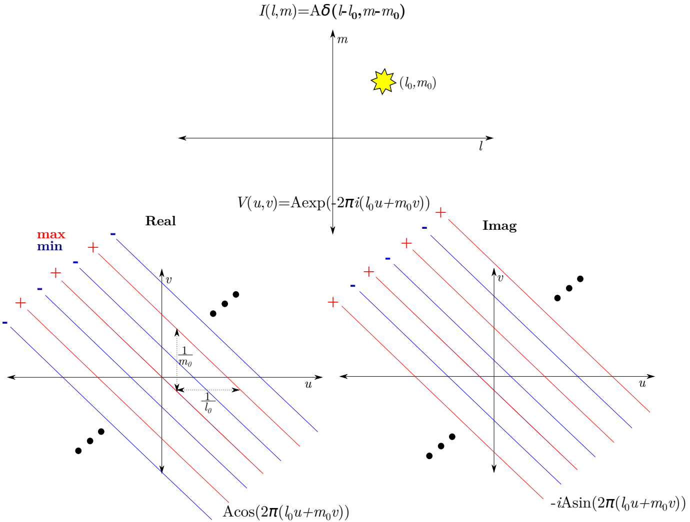
$$F(f) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-2\pi i f t}dt$$

Fourier transform: Frequency decomposition of $f(t)$.

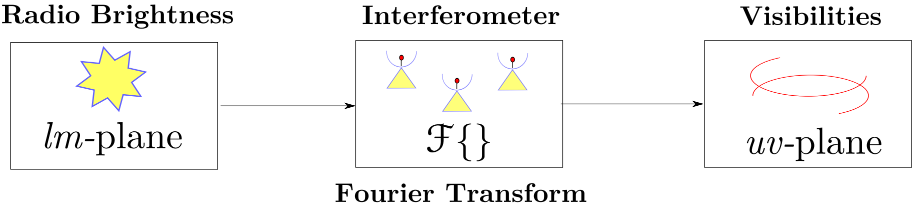
1D Fourier Transform Pairs



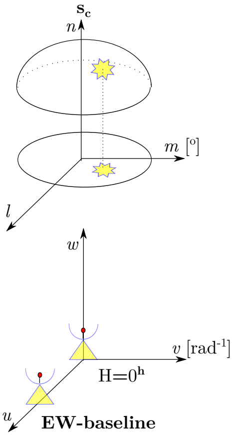
2D Fourier Transform Example



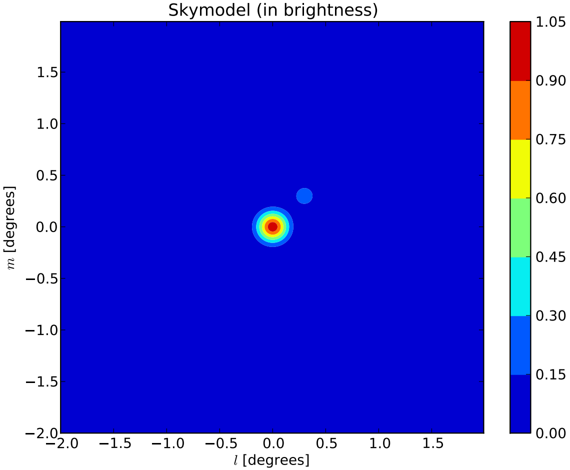
Basic Functional Diagram of Interferometer



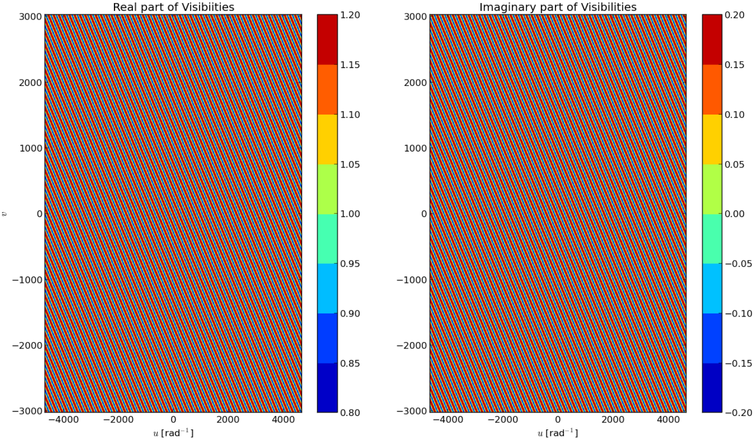
Interferometer: Coordinate systems



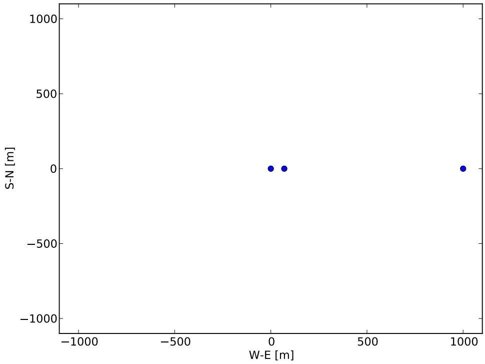
Example: Sky model



Example: Visibilities

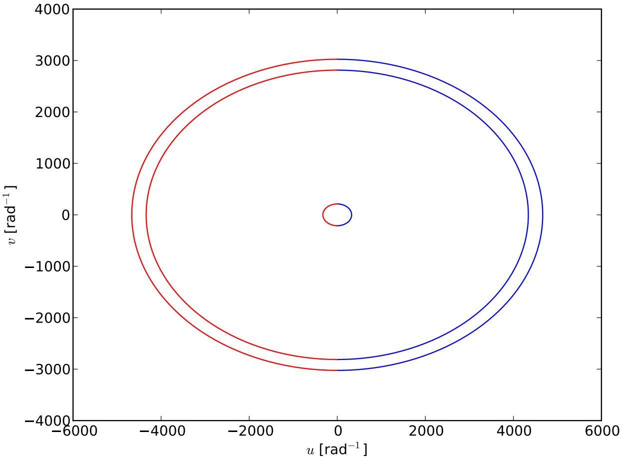


Example: ENU layout of array

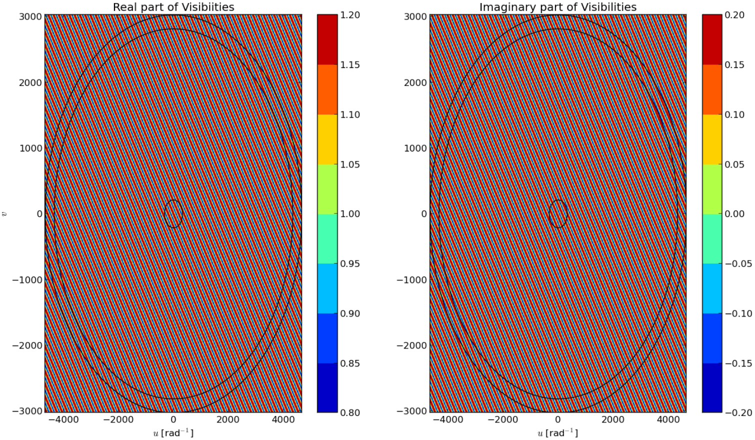


1. 3 antennas, 3 baselines.
2. A baseline is the vector that is formed by two antennas.

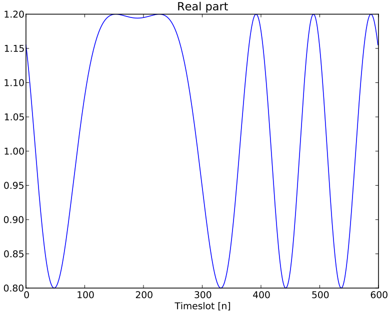
Example: *uv*-tracks

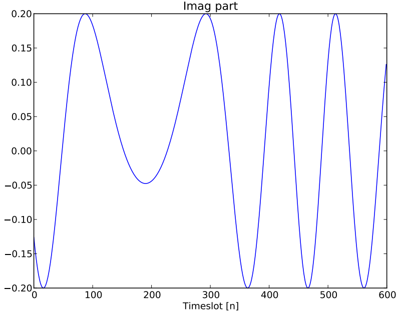


Example: Sampled visibilities

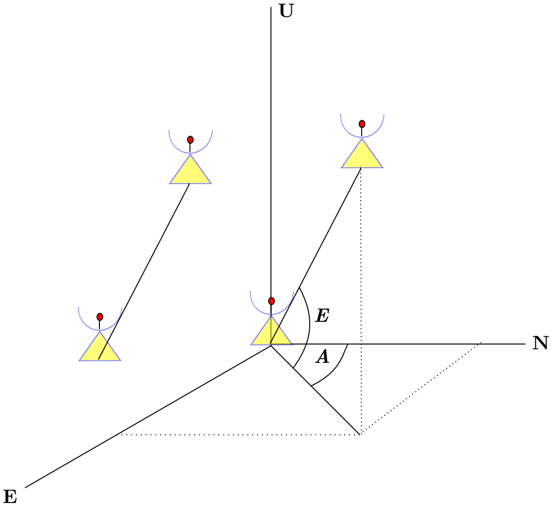


Example: Visibilities of shortest baseline

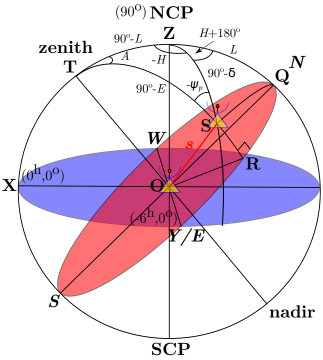




ENU (xyz) to horizontal (baseline)



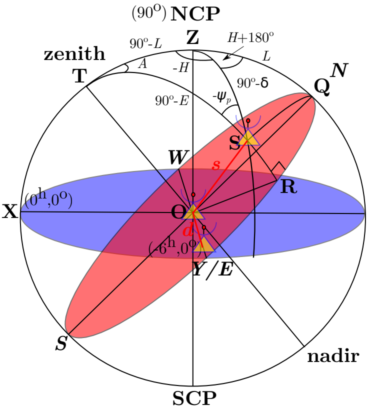
Conversion between Equatorial and Horizontal (baseline)



$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} s\cos\delta\cos H \\ -s\cos\delta\sin H \\ s\sin\delta \end{bmatrix} = s\begin{bmatrix} \cos L\sin E - \sin L\cos E\cos A \\ \cos E\sin A \\ \sin L\sin E + \cos L\cos E\cos A \end{bmatrix}$$

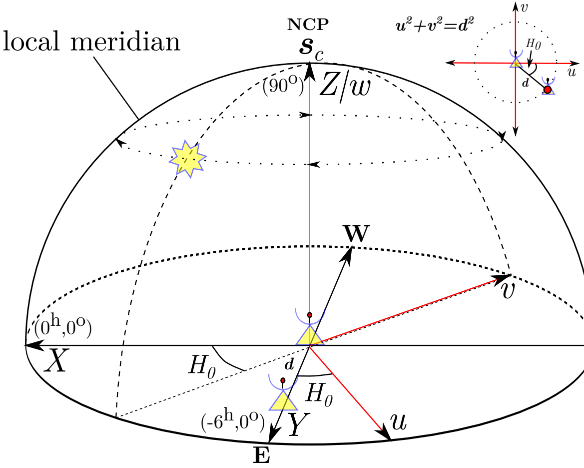
TSM (p. 89 Eq. 4.4)

XYZ coordinates of EW baseline

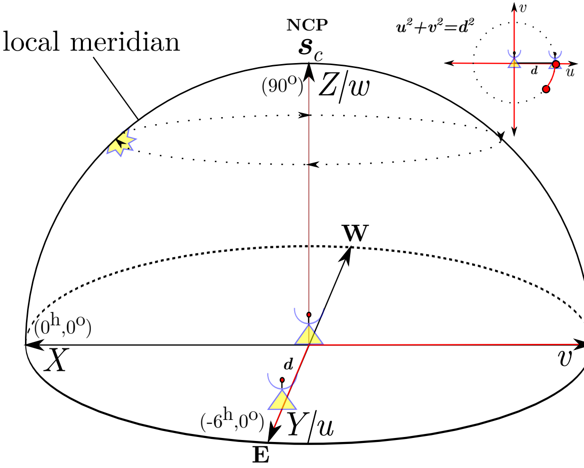


$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix} = d \begin{bmatrix} \cos L \sin 0^\circ - \sin L \cos 0^\circ \cos 90^\circ \\ \cos 0^\circ \sin 90^\circ \\ \sin L \sin 0^\circ + \cos L \cos 0^\circ \cos 90^\circ \end{bmatrix}$$

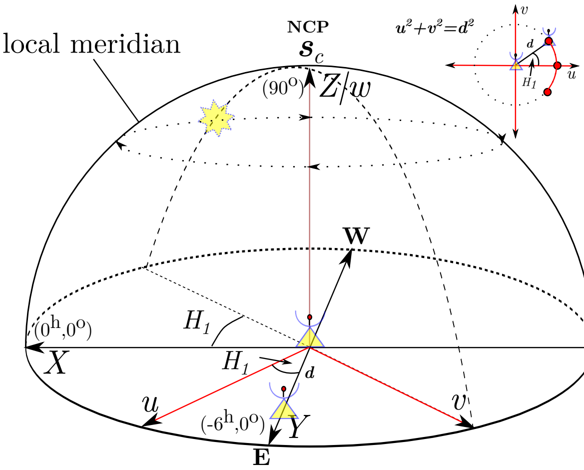
EW example I: $\delta = 90^\circ$



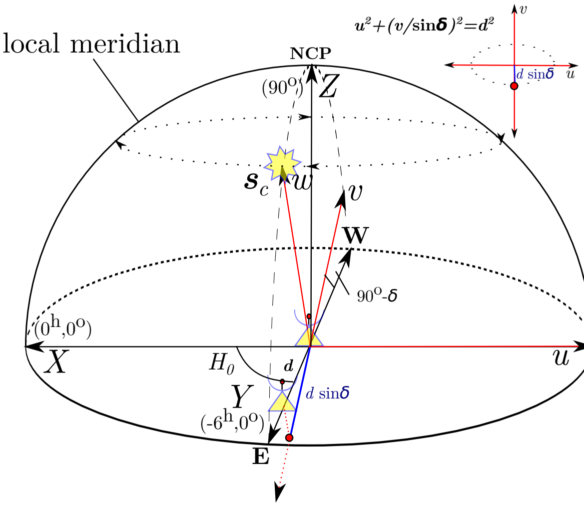
EW example II: $\delta = 90^\circ$



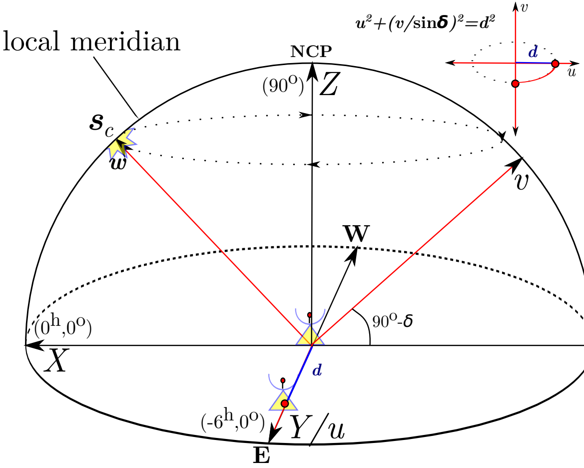
EW example III: $\delta = 90^\circ$



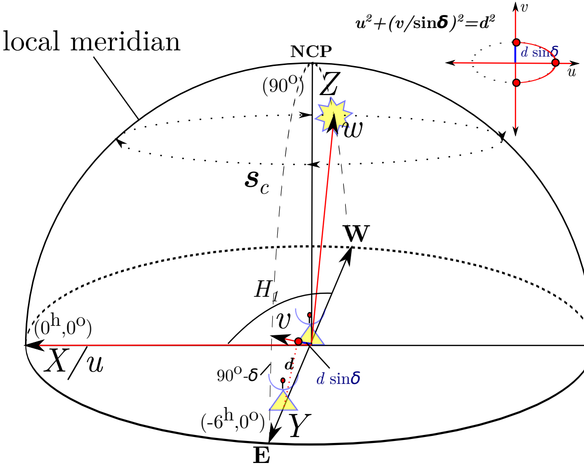
EW example I: General δ



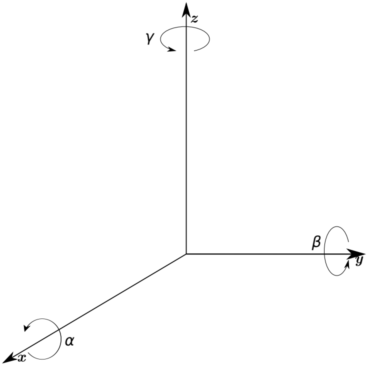
EW example II: General δ



EW example III: General δ

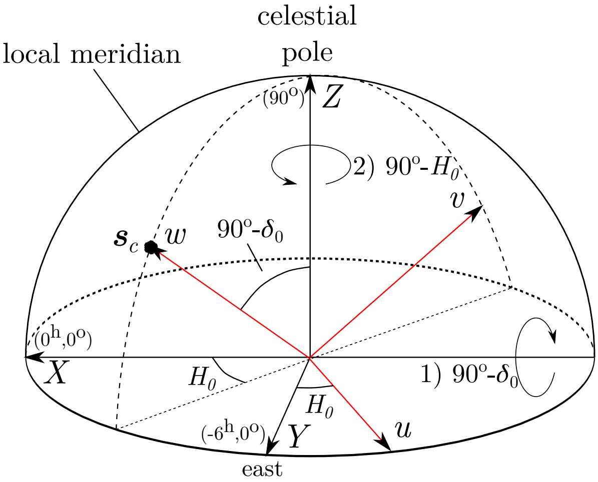


3D-Rotations



$$R_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & \\ 0 & -\sin\alpha & \cos\alpha & \end{bmatrix} \quad R_3(\gamma) = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Conversion between XYZ and *uvw*



Conversion matrix

$$R_1(90-\delta_0)R_3(90-H_0) = \begin{bmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0\cos H_0 & \sin\delta_0\sin H_0 & \cos\delta_0 \\ \cos\delta_0\cos H_0 & -\cos\delta_0\sin H_0 & \sin\delta \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin\delta_0\cos H_0 & \sin\delta_0\sin H_0 & \cos\delta_0 \\ \cos\delta_0\cos H_0 & -\cos\delta_0\sin H_0 & \sin\delta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

If we incorporate wavelength then we use

$$\lambda^{-1}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

TCP (p. 25 Eq. 2.30)

EW example: General δ

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sin H & \cos H & 0 \\ -\sin\delta_0 \cos H & \sin\delta_0 \sin H & \cos\delta_0 \end{bmatrix} \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix}$$

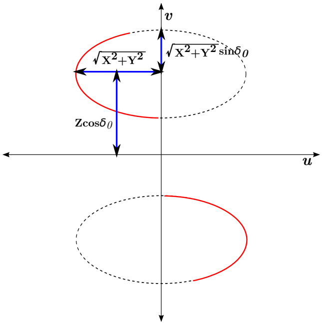
$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} d\cos H \\ d\sin\delta_0 \sin H \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ -d\sin\delta_0 \end{bmatrix} = \begin{bmatrix} d\cos -6^h \\ d\sin\delta_0 \sin -6^h \end{bmatrix}; \begin{bmatrix} d \\ 0 \end{bmatrix} = \begin{bmatrix} d\cos 0^h \\ d\sin\delta_0 \sin 0^h \end{bmatrix};$$

$$\begin{bmatrix} 0 \\ d\sin\delta_0 \end{bmatrix} = \begin{bmatrix} d\cos 6^h \\ d\sin\delta_0 \sin 6^h \end{bmatrix}$$

$$u^2 + \left(\frac{v}{\sin\delta_0}\right)^2 = d^2$$

The general case: A general *uv*-track



$$u^2 + \left(\frac{v - Z\lambda^{-1}\cos\delta_0}{\sin\delta_0}\right)^2 = \frac{X^2+Y^2}{\lambda^2}$$

TCP (p. 25 Eq. 2.31)