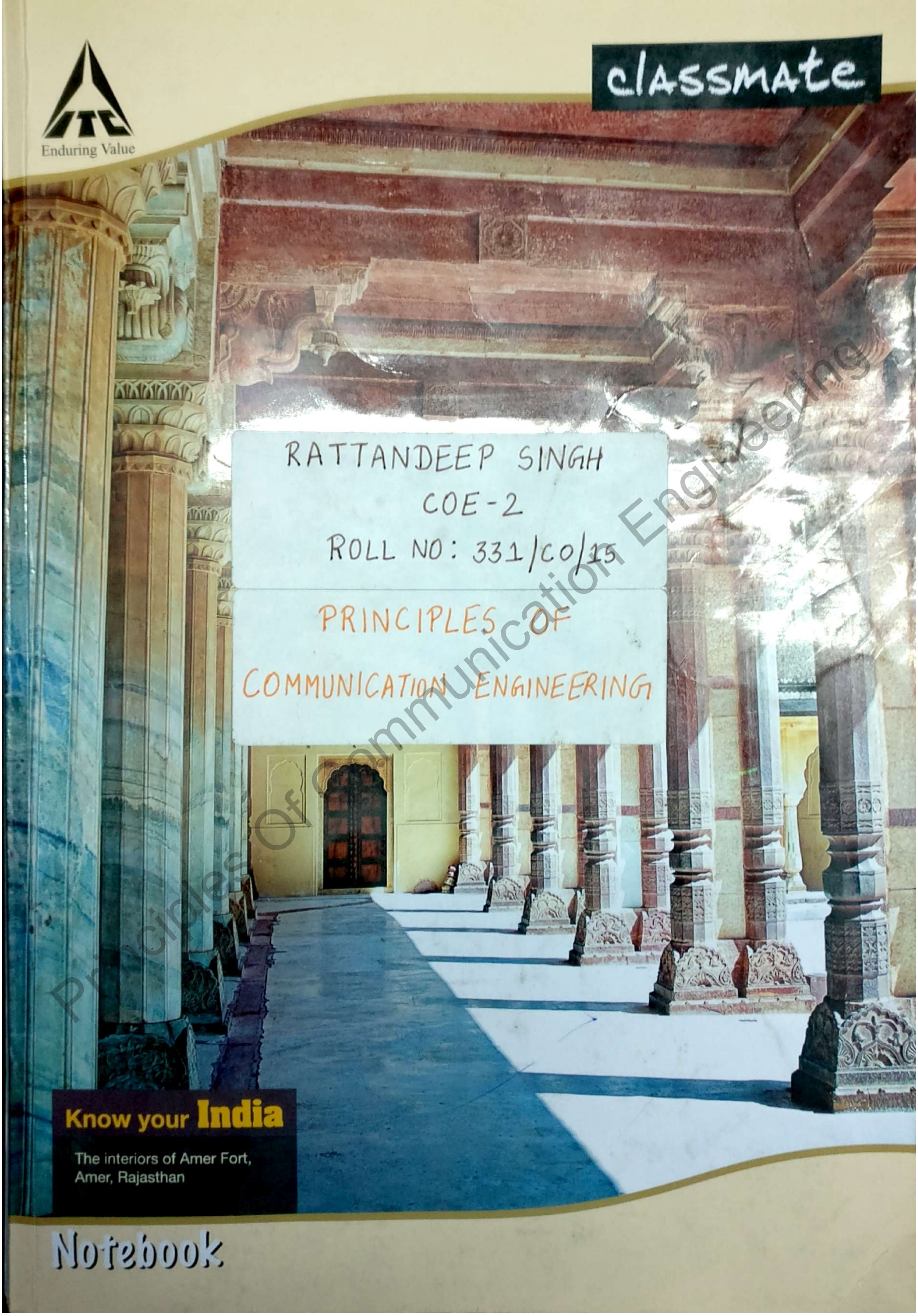

classmate
ITC
Enduring Value
RATTANDEEP SINGH
COE-2
ROLL NO: 331/CO/15
PRINCIPLES OF
COMMUNICATION ENGINEERING
Know your India
The interiors of Amer Fort,
Amer, Rajasthan
Notebook

PRINCIPALS OF COMMUNICATIONS

block diagram of communication system.

Sender → Source transducer → Receiver transducer → Receiver

Channel

Sender and receiver are imp for communication.

Source transducer converts physical form to electrical signal.

Channel or medium can be wired or wireless.

Receiver transducer converts electrical signal to physical signal.

Channel (environment) has noise which corrupts the message, corruption depends upon channel and frequency of source.

$$\text{Energy} \propto \frac{1}{\text{distance}}$$

ya toh max. energy bhejyo ya fir modulate karke bhejo.

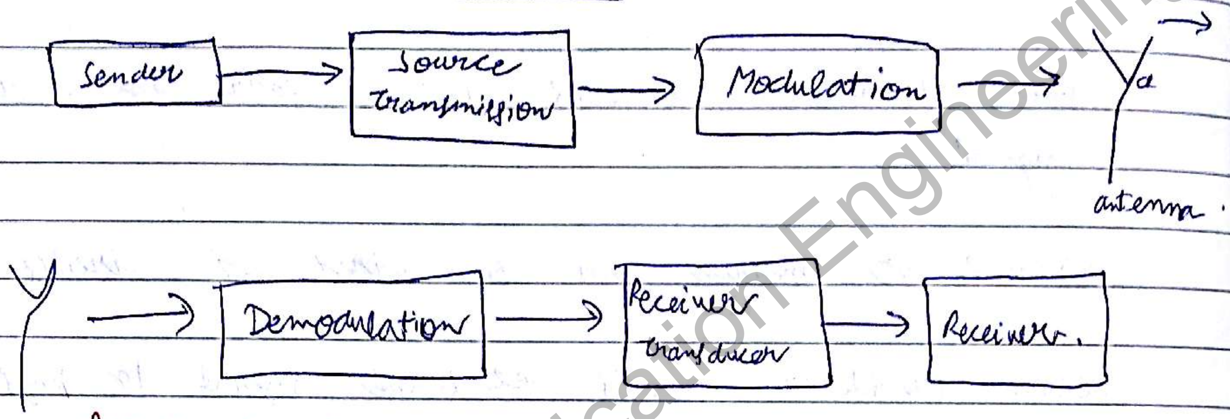
Radio signals (e.g. 98.3 MHz) are un modulated (energy = hc/λ). It is limited for a city.

All India Radio (AIR) works through satellite, hence, whole country.

All India Radio (AIR) works through satellite, hence, whole country.

Modulation use needs some more equipments which are very costly (e.g - antenna, satellite).

WIRELESS COMMUNICATION



BLOCK DIAGRAM OF WIRELESS COMMUNICATION

Ranges	Voice signal	300 Hz to 3.5 kHz
	Audio Signal	20 Hz to 20 kHz
	Vedio signal	0 to 4.5 MHz.

NEED FOR MODULATION

- height of antenna is taken to be $\lambda/4$ to transmit the maximum signal.

$$ht = \frac{\lambda}{4} \quad \text{as} \quad \lambda = \frac{hc}{E} = \frac{hc}{h\nu} = \frac{c}{\nu} = \frac{3\times10^8}{\nu}$$

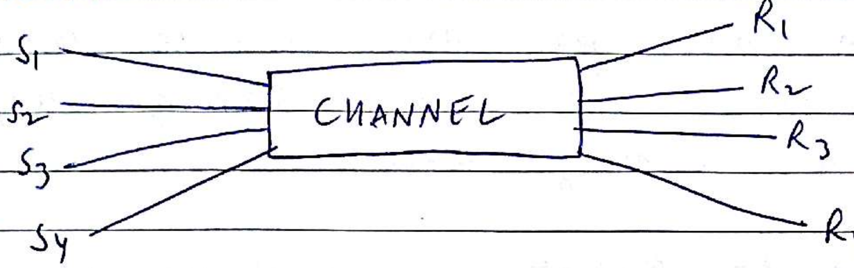
for $\nu = 15\,kHz$ we have

$$ht \approx \frac{2\times10^4}{4} = \frac{20\ km}{4}$$

$\therefore$ $ht = 5\ km$ which is impractical.

∴ we need modulation so that frequency reaches gigahertz or megahertz, so that the height of antenna is in the practical range.

MULTIPLEXING

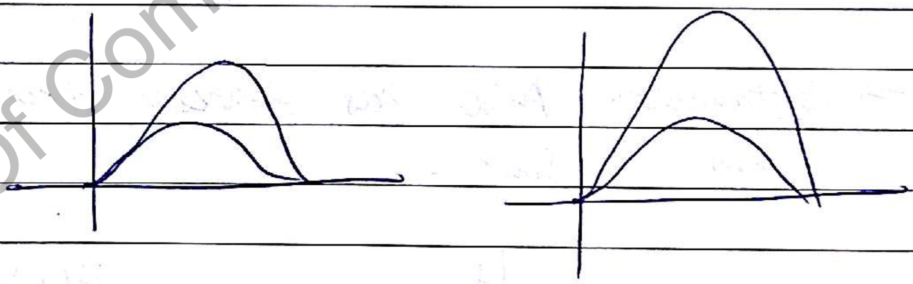


as channel is same for all the senders.

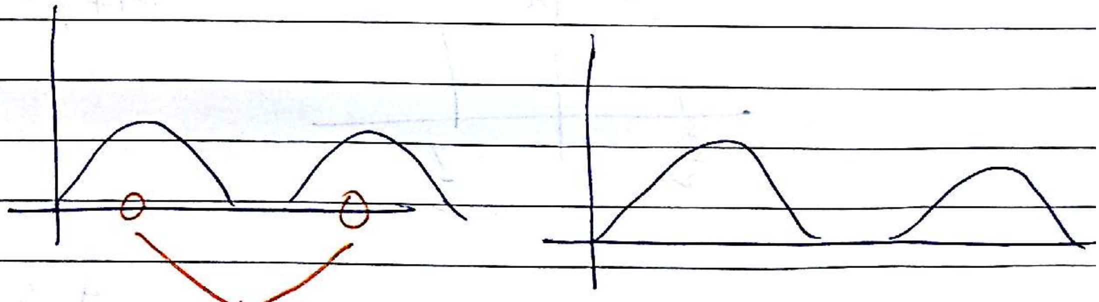
∴ optimal range of frequencies (frequency band) is same.

Therefore, overlapping will happen. and receivers will get distorted unmodulated signals

without modulation



with modulation



frequency is changed.

we can receive the signal by doing demodulation at receiver's end.

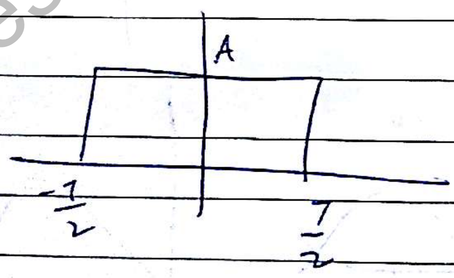
FOURIER TRANSFORM :

Rise time and fall time can be handled in time domain, but for frequency range, we need frequency domain.

We need to convert a signal to its fourier transform and then get information about signals bandwidth.

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

⇒ Rectangular pulse has fourier transform in terms of Sinc.

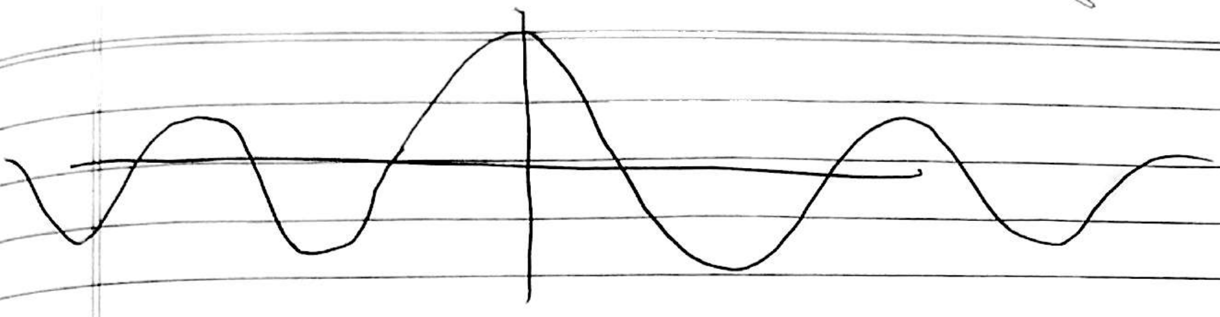


$$f(t) = A \,\mathrm{rect}(\tau).$$

$$\updownarrow$$

$A\tau \,\mathrm{sinc}(\pi f \tau)$

$$\underbrace{A\tau}_{\text{area of pulse}} \,\mathrm{sinc}\underbrace{(f\tau)}_{\text{equal to time duration.}}$$



frequency is only +ve not -ve, but we show -ve curve.

To visualise the signal we use spectrum analyser, it generates frequency response.

Mathematical analysis of full signal is easy as compared to only positive part of it.

$$X(\omega) = \int_{-\infty}^{\infty} e^{-j2\pi f t} f(t)\, dt$$

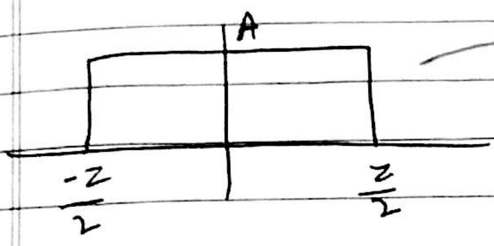
(for rect. pulse)

$$= \int_{-\tau/2}^{\tau/2} e^{-j2\pi f t}\, dt$$

$$= \left. \frac{e^{-j2\pi f t}}{-j2\pi f} \right|_{-\frac{\tau}{2}}^{\tau/2}$$

$$= \frac{1}{\pi f} \left(\frac{e^{j\pi f \tau} - e^{-j\pi f \tau}}{2j} \right)$$

$$= \frac{\sin \pi f \tau}{\pi f} \times \frac{\tau}{\tau} = \tau \operatorname{sinc}(\tau \pi f).$$



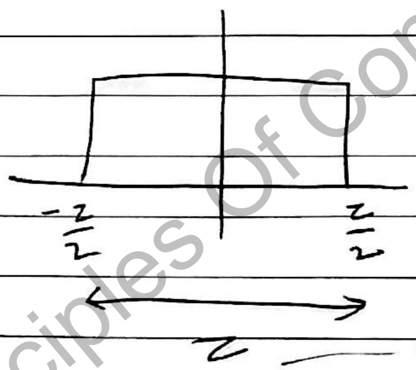
→ Rectangular Pulse.

$$f(t) = A \, rect(\tau).$$

τ is the time period symmetrically.

$$f(t) = A \, rect(\tau) \xleftrightarrow{F.T.} A\tau \, Sinc(f\tau)$$

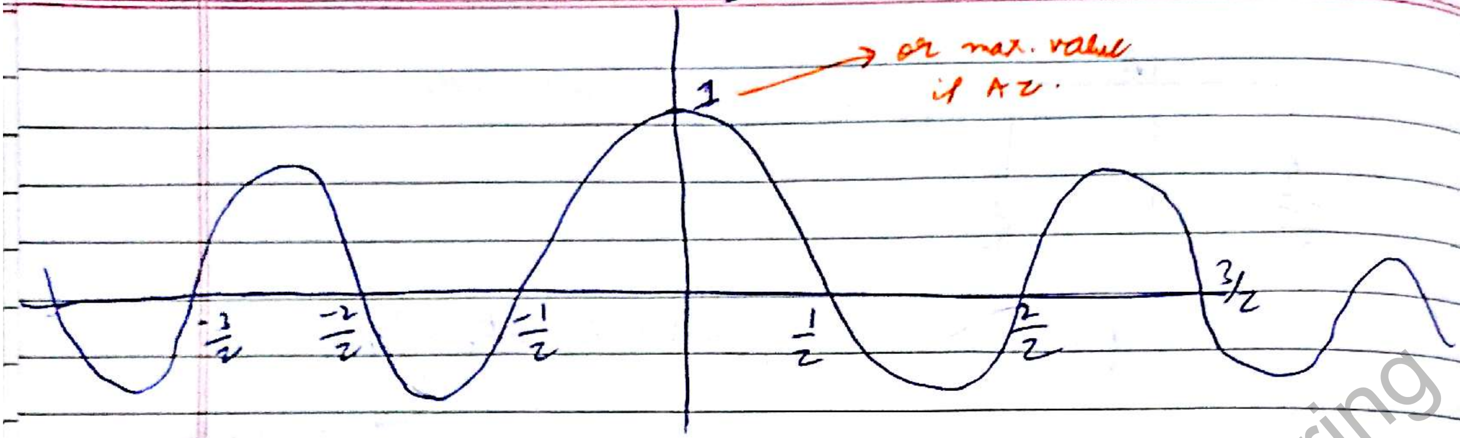
$A\tau$ → amplitude of fourier transform is area under rec. pulse.



⇓

its freq. is equal to the total time period of the wave.

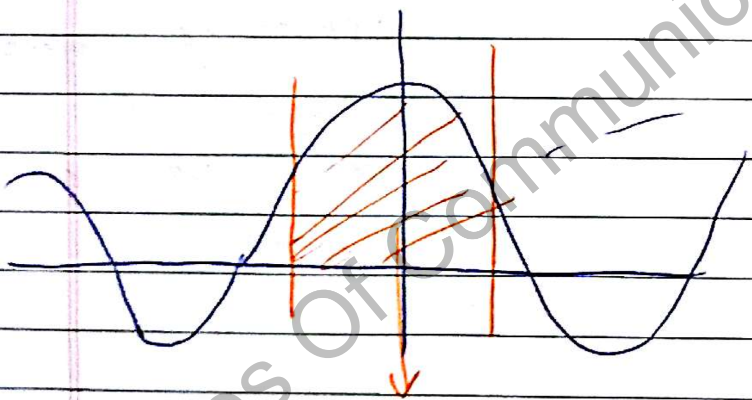
Sinc :



Sinc $\frac{\sin(\pi f \tau)}{\pi f \tau}$.

this signal has infinite bandwidth.

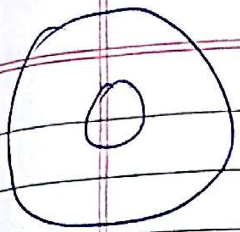
but our channel is of finite bandwidth.



this portion contains max. information of the given signal, so only significant portion is sent.

but majority of info. is not lost.

so, signal also recieves max. amount of information.



in 700 mB in CD,
only significant info. is stored in CD,
and only some bandwidth is passed.

as bandwidth increases, the channel price also increases, since more components are required.

we want to pass only necessary information in significant bandwidth.

→ Significant Energy ⇒ 95 − 98 %

↓

redundant data,
only this much is required.

bandwidth of the signal should be less than the bandwidth of the channel

Channel Bandwidth (CBW) > Signal band width (SBW).

sum of bandwidths of all the signals.

classmate
Date
Page

PROPERTIES OF FOURIER TRANSFORM

1) LINEARITY

2 signals $x_1(t)$ $x_2(t)$

$$x_3(t) = a x_1(t) + b x_2(t)$$

$$y(t) = x_3(t)$$

$$= a y_1(t) + b y_2(t)$$

$$x_1 \xleftrightarrow{F.T.} X_1(f)$$

$$x_2 \xleftrightarrow{F.T.} X_2(f)$$

$$x_3(t) \xleftrightarrow{F.T.} a x_1(f) + b X_2(f)$$

2) TIME SHIFTING

If $x(t) \xleftrightarrow{F.T.} X(f)$

$$x(t - t_0) \xleftrightarrow{F.T.} e^{-j\omega t_0} X(f)$$

3) FREQUENCY SHIFTING

If $x(t) \xleftrightarrow{F.T.} X(\omega)$

$$x(t) e^{j\omega_0 t} \longleftrightarrow X(\omega - \omega_0)$$

classmate
Date
Page

4) SCALING PROPERTY

If $x(t) \xleftrightarrow{F.T.} X(f)$

$$x(at) \xleftrightarrow{F.T.} \frac{1}{|a|} X\left(\frac{f}{a}\right).$$

If $a > 1$,

time domain ~~expansion~~ compression and hence, frequency domain expansion.

If $a < 1$,

time domain expansion and hence, frequency domain compression.

5) TIME REVERSAL

If $x(t) \longleftrightarrow X(f)$.

$$x(-t) \longleftrightarrow X(-f).$$

(this is time scaling with $a = -1$).

6) DUALITY PROPERTY

Value of Sinc cannot be defined with the help of fourier tranform, directly.

$\because$

$$\text{Sinc} = \frac{\sin \pi f \tau}{\pi f \tau} \approx 1$$

at $\tau \to 0$,
this value comes out to be infinity without approx.

So, we use duality principle here.

↓

Used where we have problem in defining fourier transform directly.

7) TIME DIFFERENTIATION

$$x'(t) \longleftrightarrow j\omega \frac{d\,X(\omega)}{d\omega}$$

8) TIME INTEGRATION

$$\int_{-\infty}^{t} x(t)\,dt \xleftrightarrow{F.T.} \frac{X(\omega)}{j\omega}$$

9) CONVOLUTION

$$x_1(t) * x_2(t) \longleftrightarrow X_1(f) \cdot X_2(f).$$

Proof:

$$F.T.\left(x_1(t) * x_2(t)\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\,d\tau\, e^{-j\omega t}\,dt$$

10) MULTIPLICATION..

$$x_1(t) \cdot x_2(t) \xleftrightarrow{F.T.} X_1(f) * X_2(f).$$

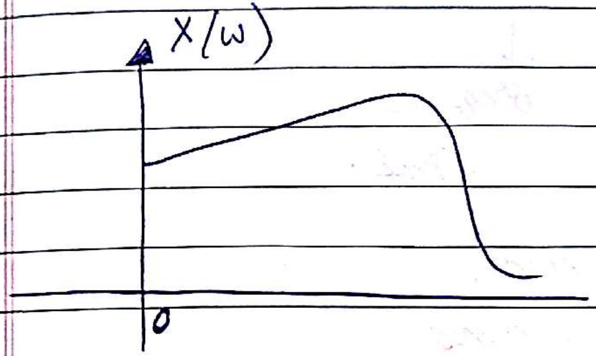
11) MODULATION

$$x(t) \cos 2\pi f_0 t \longleftrightarrow \frac{X(f-f_0) + X(f+f_0)}{2}$$

carrier signal
freq. f_0.

freq. of carrier signal (f_0) $\gg$ freq. of signal $x(t)$.

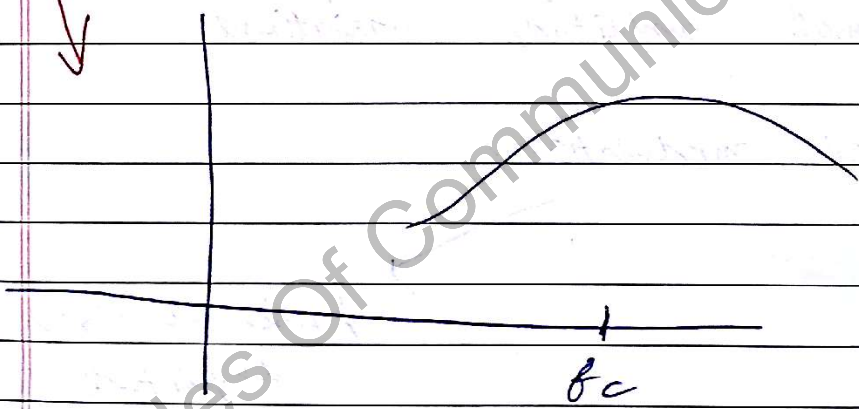
LOW PASS SIGNALS AND HIGH PASS SIGNALS



LOW PASS SIGNALS.

as frequency is near zero, so it is low pass signals.

Signal modulated to higher freq.



HIGH PASS SIGNALS.

(centre frequency comes at fc).

translated signal at frequency fc.

when actual signal is modulated.

message signals are low pass. → so, its freq. is modulated to higher freq.

$$h_t \propto \frac{1}{f}$$

h_t → height of antenna

f → freq. of signal.

freq. increased, so height decreased.

MODULATION

It is the process in which one of the characteristic parameter i.e. amplitude, frequency or phase of carrier signal vary according to message signal amplitude variations.

2 types of modulation

→ amplitude modulation

→ phase or angle amplitude.

$$c \cos(w_c t + \theta)$$

c → amplitude varies with amplitude of message sig.

w_c → freq. varies with amplitude of message signal.

1) AMPLITUDE MODULATION

$c(t) \longrightarrow$ carrier signal.

$m(t) \longrightarrow$ message signal.

$$c(t) = A_c \cos 2\pi f_c t$$

** $$\underbrace{S_{Am}(t)}_{\text{modulated signal}} = \underbrace{A_c[1 + K_a m(t)]}_{\text{amplitude of modulated signal varies with amplitude of } m(t)} \cos(2\pi f_c t)$$

~~modulated~~ signal

$$= \underbrace{A_c \cos 2\pi f_c t}_{\text{carrier signal}} + \underbrace{A_c K_a m(t) \cos 2\pi f_c t}_{\text{modulated signal}}$$

DISADVANTAGE:

carrier signal is sent alongwith modulated signal, so more power is required to send both the signals.

($\because$ power ↑ to be sent or transmit).

ADVANTAGE :

demodulation of this signal is easier.

this signal is multiplied by $\cos 2\pi f_c(t)$

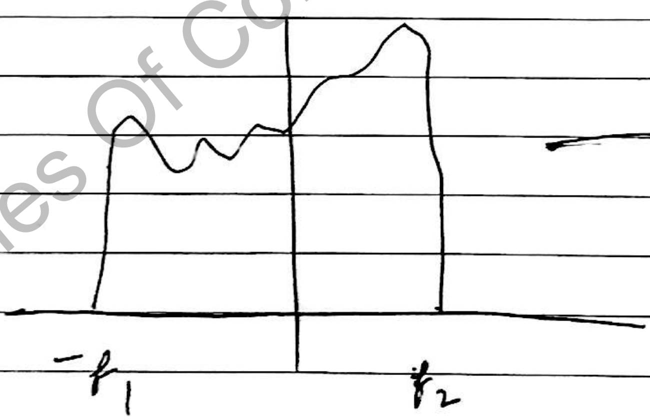
and ~~then~~ using formula of

$\cos^2 f(t)$

$$\cos^2 ft = \frac{1+\cos 2ft}{2}$$

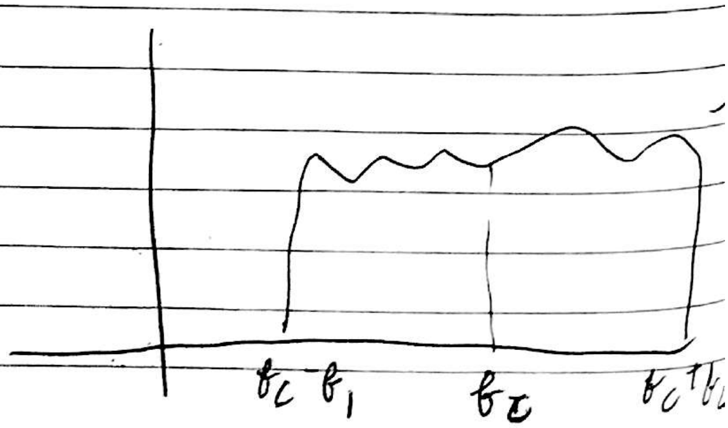
converted into linear form with freq. $2f$.

BAND WIDTH



message signal.

modulation



bandwidth is only defined for +ve frequencies.

B.W. of message signal $= f_2 - 0$

$= f_2$ Hz.

The -ve part of freq. is used for calculation & spectrum analysis.

So, on modulation, frequency of the modulated signal increases. from f_2 Hz to $(f_1 + f_2)$ Hz.

→ now, for the modulated signal bandwidth is equal to highest freq. − lowest freq.

$= (f_c + f_2) - (f_c - f_1)$

$= (f_2 + f_1)$ Hz

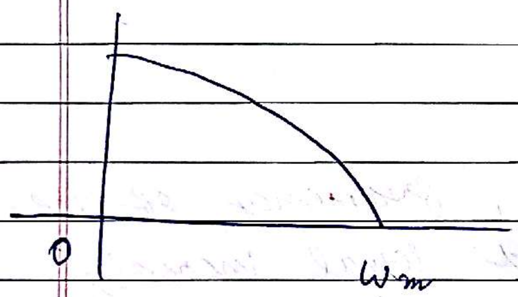
TYPES OF MODULATION

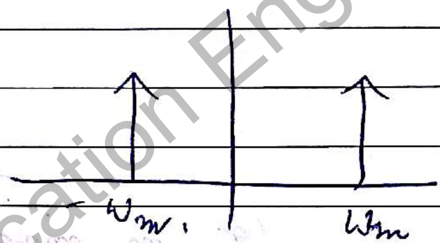
1) SINGLE TONE (single freq.) → if message signal contains single frequency.

2) MULTI TONE (more than one freq. components).

$m(t) = A_m \cos \omega_m t$

e.g.



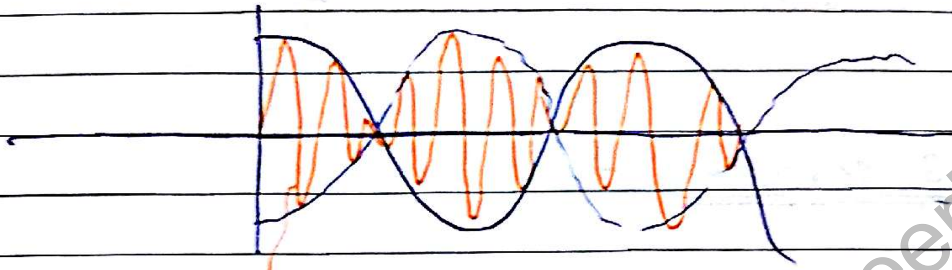


it contain all freq. signals from 0 to ω_m.

$$S_{AM}(t) = \underbrace{A_c\left(1 + K_a m(t)\right)}_{\Downarrow} \cos 2\pi f_c t$$

this amplitude is termed as envelope.

Max. and min. values.



cos fun t.

message signal.

If value of $|K_a m(t)| < 1$,

Cross over does not takes place

and there is better recovery.

If $|K_a m(t)| > 1$

then there is distortion in the recovered message.

$$\mu = \max(K_a m(t)).$$

→ MODULATION INDEX (Depth of modulation).

Over modulation under modulation or critical modulation

Critical modulation $(\mu=1) \Rightarrow$ no distortion, same signal is obtained.

we prefer under modulation.

3 conditions :-

1) $\mu < 1$ under modulation

2) $\mu > 1$ over modulation.

3) $\mu = 1$ critical modulation.

If $m(t) = A_m \cos(2\pi f_m t)$,

$$\mu = \max \left(k_a m(t) \right)$$

$$\boxed{\mu = k_a A_m}$$

modulation index can be varied with k_a.

($\because A_m$ is fixed, message signal is same)

If value of K_a is not given, then

$$\boxed{K_a = \frac{1}{A_c}}$$

where A_c is amplitude of carrier signal.

$$S_{AM}(t) = A_c\left(1 + K_a\, m(t)\right)\cos 2\pi f_c(t)$$

$$= A_c\left(1 + K_a A_m \cos 2\pi f_m t\right)\cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + A_c \underbrace{K_a A_m}_{\mu} \cos 2\pi f_m(t) \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + A_c\, \mu \cos 2\pi f_m t \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + \frac{A_c \mu}{2}\left[\cos 2\pi(f_m - f_c)t + \cos 2\pi(f_c + f_m)t\right]$$

$$= A_c \cos 2\pi f_c t + \frac{A_c \mu}{2}\cos 2\pi\underbrace{(f_c - f_m)}t$$

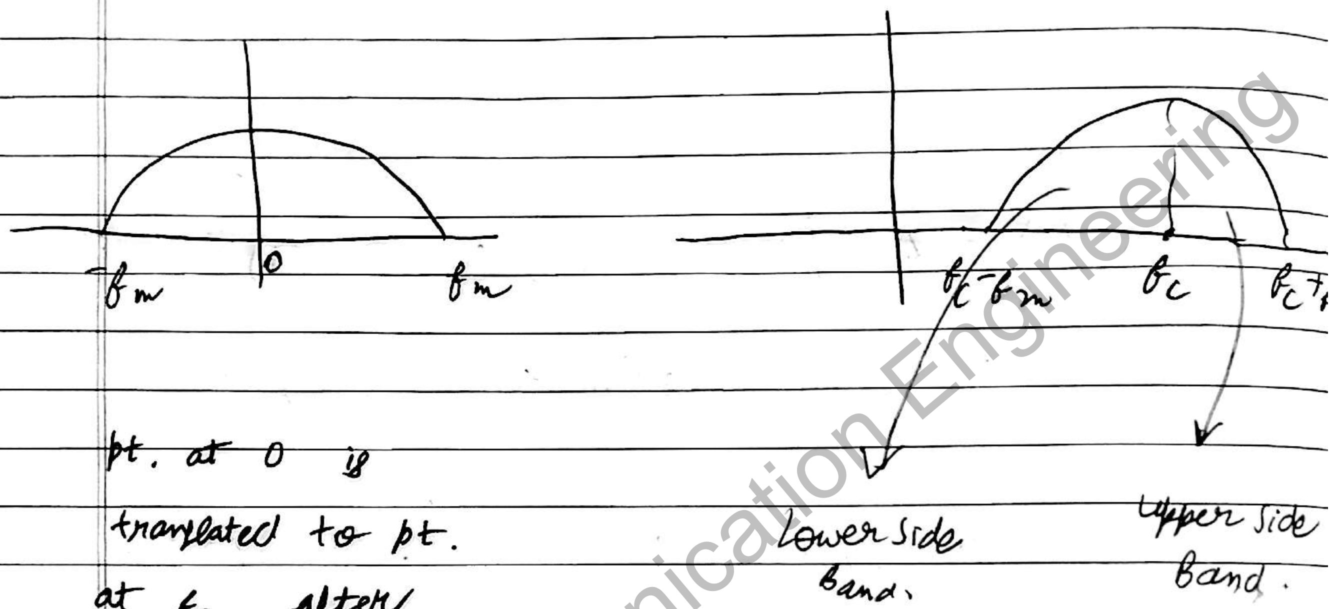
Lower Side Band. $(f_c - f_m)$

$$+ \frac{A_c \mu}{2}\cos 2\pi\underbrace{(f_c + f_m)}t$$

Upper side band $(f_c + f_m)$.

$f_c - f_m \rightarrow$ Lower Side Band

$f_c + f_m \rightarrow$ Upper Side Band.



pt. at 0 is translated to pt. at f_c, after modulation.

$A_c \cos 2\pi f_c t$

$\downarrow$

power $= \frac{A_c^2}{2}$

$\frac{A_c \mu}{2} \cos 2\pi (f_c - f_m) t$

$\downarrow$

power $= \frac{\left(\frac{A_c \mu}{2}\right)^2}{2}$.

$$\boxed{P_c = \text{Power of carrier} = \frac{A_c^2}{2}}$$

$$\boxed{P_{SB} = P_{LSB} + P_{USB}}$$

side band — lower side band — upper side band.

$$= \frac{\frac{A_c^2 \mu^2}{4}}{2} + \frac{\frac{A_c^2 \mu^2}{4}}{2}$$

$$= \frac{A_c^2 \mu^2}{4}$$

$$= \frac{\mu^2}{2} P_c$$

$$\boxed{P_{SB} = \frac{\mu^2}{2} P_c}$$

Total power, $P_T = P_c + P_{SB}$

$$\boxed{P_T = \left(1 + \frac{\mu^2}{2}\right) P_c}$$

classmate
Date
Page

⇒ P_c is independent of μ.

carrier power

modulation index.

⇒ If μ ↑ses, then P_{SB} ↑ses and therefore P_T ↑ses.

side band power.

total power.

$$\boxed{0 < \mu < 1}$$

$\mu = 0 \longrightarrow$ no modulation.

μ b/w 0 & 1 $\longrightarrow$ then critical modulation.

$\mu > 1 \longrightarrow$ over modulation.

$\mu = 1 \longrightarrow$ critical modulation.

$P_t = P_c \qquad \mu = 0$

$P_t = \frac{3}{2} P_c \qquad \mu = 1.$

when we increase μ from 0 to 1, then power increased by 50%.

At $\mu = 1$, $P_{SB} = \frac{\mu^2}{2} P_c = \frac{1}{2} P_c$.

Side band power is half of carrier power at $\mu = 1$.

POWER EFFICIENCY

(how efficient is amplitude modulated signal).

$$\eta = \frac{\text{Side Band Power}}{\text{Total Power}}.$$

$$= \frac{\frac{\mu^2}{2} \cancel{P_c}}{\left(1 + \frac{\mu^2}{2}\right) \cancel{P_c}} = \frac{\mu^2}{\mu^2 + 2}.$$

$$\boxed{\eta = \frac{\mu^2}{\mu^2+2}}$$

At $\mu = 1$, max. efficiency,

$\eta = \frac{1}{3}$ i.e. 33% efficiency on choosing amplitude modulation.

Q) An unmodulated power of a signal is 500W. Find the A.M. power, with $\mu = 200\%$.

$\underbrace{\text{A.M.}}_{\text{amplitude modulated}}$

$$P_T = \left(1 + \frac{\mu^2}{2}\right) P_C$$

$\boxed{\mu = 2}$

$\boxed{P_c = 500\ W}$

$$= 3P_C = 1500\ W$$

Q) For an AM, total side band power is given by 100 W with $\mu = 0.5$. Find total power and carrier power.

$$P_{SB} = \frac{\mu^2}{2} P_C$$

$$P_C = \frac{100 \times 2}{(0.5)^2} = \frac{2000}{0.5 \times 0.5} = \frac{4000}{5} = 800\ W.$$

$P_T = 100W + 800W$

$\boxed{P_T = 900\ W}$

Q) For an AM, each of side band power is given by 2 kW and carrier power is given by 8 kW, find power efficiency.

$$12 = \left(1 + \frac{\mu^2}{2}\right) 8 \qquad P_T = 8 + 2 + 2 = 12\ kW.$$

$$\frac{3}{2} - 1 \qquad \boxed{\mu = 1}$$

$$\eta = \frac{\mu^2}{\mu^2 + 2} = \frac{1}{3} = 33\% \quad \text{— Ans.}$$

Q) A carrier signal of $10 \cos 2\pi \times 10^6 t$ is amplitude modulated by a message signal, of $6 \cos(4\pi \times 10^4 t)$

Find all possible parameters of Am.

μ, power (all).

$$c(t) = 10 \cos(2\pi \times 10^6 t)$$

$$m(t) = 6 \cos(4\pi \times 10^4 t)$$

$$\left(K_a = \frac{1}{A_c}\right)$$

$$\mu = K_a A_m = \frac{A_m}{A_c} = \frac{6}{10} = 0.6$$

$$P_c = \frac{A_c^2}{2}$$

$$= \frac{(10)^2}{2} = 50W$$

$$P_{SB} = \frac{\mu^2}{2} P_c$$

$$= \frac{(0.6)^2}{2} P_c = 9W$$

$P_T = 59$ W

$$\eta = \frac{9}{59} = 0.15.$$

Q) A carrier signal $c(t) = 10\cos(4\pi \times 10^5)t$.
is amplitude modulated by a message signal of $m(t) = 4\cos(\pi \times 10^4)t$.

with $\mu = 0.5$ and $r = 5\Omega$.
↓
resistance.

we consider resistance to be 1Ω for all cases.

We are given voltage signals, and to cal. power, we will use $\dfrac{V^2}{R}$.

so, now, power of carrier signal will be $\dfrac{A_c^2}{2R}$.

$$P_c = \frac{A_c^2}{2R}$$

$$= \frac{(10)^2}{2 \times 5} = 10W.$$

$$P_{SB} = \frac{\mu^2}{2} P_c = \frac{(0.5)^2}{2} \times 10 = \frac{0.25 \times 10}{2}$$

$$= \frac{2.5}{2} = 1.25W.$$

$$P_T = P_c + P_{SB} = 10 + 1.25$$

$$= 11.25W.$$

$$\eta = \frac{1.25}{11.25} = 11\%$$

Also, calculate the frequency contained by A_m.

$$f_c = 2\times10^5 = 20\times10^4$$

$$f_m = \frac{10^4}{2} = 0.5\times10^4$$

$$\boxed{\text{Bandwidth of AM (single tone)} = 2 \times f_{\text{message signal}}}$$

Bandwidth of AM (for multi tone) $= 2 \times (f_{max})_{\text{message}}$.

~~freque~~ bandwidth of AM $= 10\ \text{kHz} = 10^4\ \text{Hz}$.

Frequencies contained will be.

$f_c = 200\ \text{kHz}$ $f_c,\ f_c + f_m,\ f_c - f_m$

$f_m = 5\ \text{kHz}$

$f_c + f_m = 205\ \text{kHz}$

$f_c - f_m = 195\ \text{kHz}$

classmate
Date
Page

d) An AM signal is given by

$$s(t) = 4\cos 3200\pi t + 10\cos 4000\pi t + 4\cos 4800\pi t$$

1) Find the parameters of Am.

2) Find $\frac{P_c}{P_T}$.

lower freq. upper freq.

$$s(t) = 4\cos 3200\pi t + 10\cos 4000\pi t + 4\cos 4800\pi t.$$

~~$A_c = 4$~~

this will be power signal. this is signal has ~~more than one~~ freq.

$\boxed{f_c = 2000\text{ Hz}}$

$f_c + f_m = 2400$

$\boxed{f_m = 400\text{ Hz}}$

$f_c - f_m = 1600$

$c(t) = 10\cos 4000\pi t$ $A_c = 10.$

$\frac{\mu A_c}{2} = 4$

$\frac{\mu A_c}{2} = 4$

$\mu = \frac{8}{10} = 0.8$

$\boxed{\mu = 0.8}$

classmate
Date
Page

$$P_c = \frac{A_c^2}{2} = \frac{(10)^2}{2} = \frac{100}{2} = 50W$$

$$P_{SB} = \frac{\mu^2}{2} P_c$$

$$= \frac{(0.8)^2}{2} P_c$$

$$= 25 \times 0.64$$

$$= 1600W$$

2.64
25
320
$128 \times$
1600

$$\boxed{P_T = 1650\,W}$$

$$\frac{P_c}{P_T} = \frac{5\cancel{0}}{165\cancel{0}} = \frac{1}{33} = 3.03\%$$

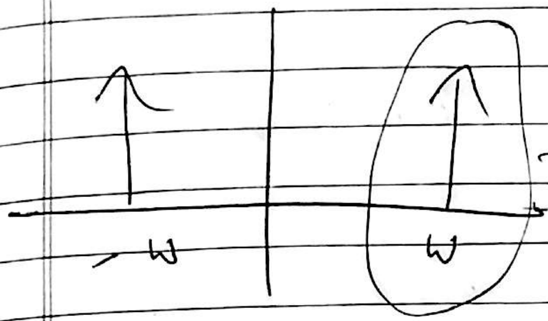
Bandwidth $= 2 * f_m$

$= 2 \times 400 = 800\ Hz.$

$$s(t) = \frac{A_c \mu}{2} \cos 2\pi (f_c - f_m) t + A_c \cos 2\pi f_c t$$

$$+ \frac{A_c \mu}{2} \cos 2\pi (f_c + f_m) t.$$

cos ωt is a single tone signal,



we see it as a single tone signal only, considering only the half of the frequencies.

classmate
Date
Page

Q) An AM power is given by 100 W, modulation index $\mu = \frac{1}{\sqrt{2}}$.

i) Find power of carrier & side band frequency component.

ii) Find power of carrier signal before and after modulation.

iii) Find peak amplitude of carrier before and after modulation.

$$P_T = \left(1 + \frac{\mu^2}{2}\right) P_c$$

$$100 = \left(1 + \frac{1}{4}\right) P_c$$

$$\frac{\overset{20}{\cancel{100}} \times 4}{\cancel{5}} = P_c \qquad \boxed{P_c = 80\ W}$$

$$P_{SB} = \frac{\mu^2}{2} P_c$$

$$= \frac{1}{4} \times 80 = 20W$$

$$\boxed{P_{SB} = 20W}$$

ii) $P_c = \frac{A_c^2}{2R} \longrightarrow$ here is no factor of μ.

So, before & after modulation P_c remains ...

classmate
Date
Page

P_c remains same before & after modulation.

iii) Amplitude of carrier signal.

$$P_c = \frac{A_c^2}{2}$$

$$A_c = \sqrt{P_c \times 2}$$

$$= \sqrt{80 \times 2} = \sqrt{160}$$

After modulation,
carrier signal & message signal combines.
⇓
so, amplitude changes

After modulation,

$$\boxed{A_c(1 + k_a\, m(t))} \cos 2\pi f_c t.$$

⇓

this is the amplitude after modulation & depends on the message signal.

Before modulation, $A_c \cos 2\pi f_c t$

$$m(t) = A_m \cos 2\pi f_m t$$

$\Rightarrow$

After modulation, $A_c\left(1 + K_a\, m(t)\right) \cos 2\pi f_c t.$

$$A_c\left(1 + K_a A_m \cos 2\pi f_m t\right) \cos 2\pi f_c t$$

$$\underbrace{A_c\left(1 + \mu \cos 2\pi f_m t\right)}_{} \cos 2\pi f_c t$$

$\Downarrow$

amplitude varies from

$A_c(1+\mu)$ to

$A_c(1-\mu)$.

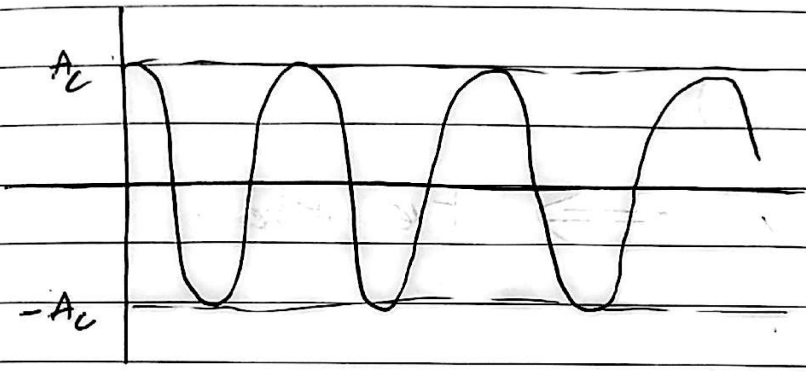
After modulation, amplitude of carrier,

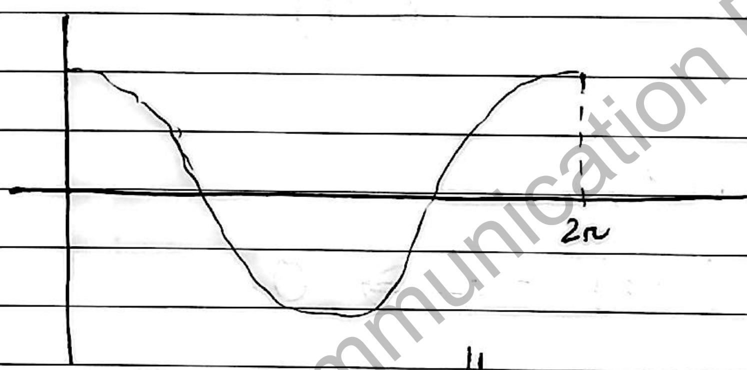
maximum $= A_c(1+\mu)$

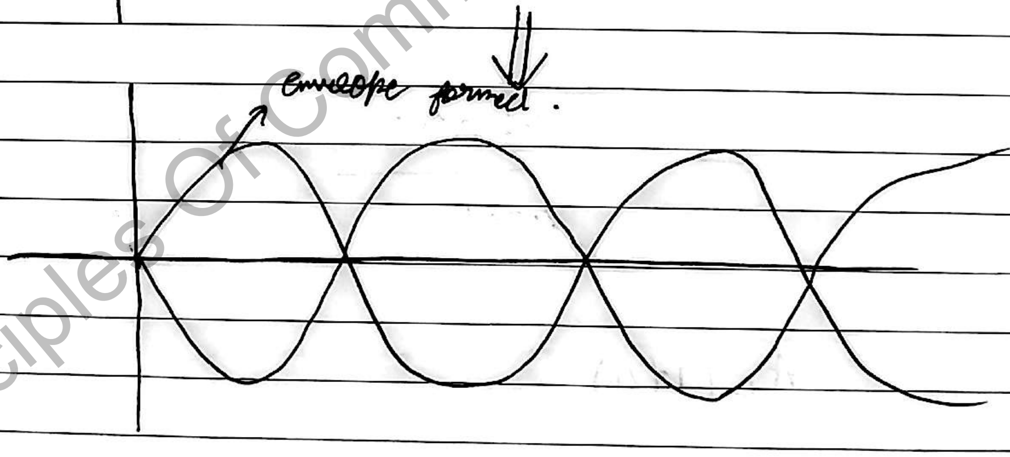
minimum. $= A_c(1-\mu)$

PEAK AMPLITUDE

$$A_c\,(1+\mu\cos 2\pi f_m t).$$







curve of modulated signal.

$$A_c\,(1+\mu)$$

$$A_c\,(1-\mu).$$

In case of under modulation,
($\mu < 1$)

$A_c(1+\mu)$

$A_c(1-\mu) \Longrightarrow$ +ve value.

critical modulation
($\mu = 1$)

$A_c(1+\mu)$

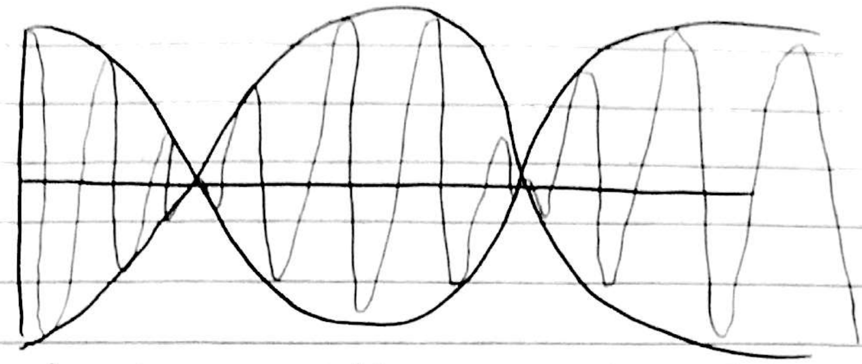
$A_c(1-\mu) \Longrightarrow 0$.

over modulation
$\mu > 1$

$A_c(1+\mu)$

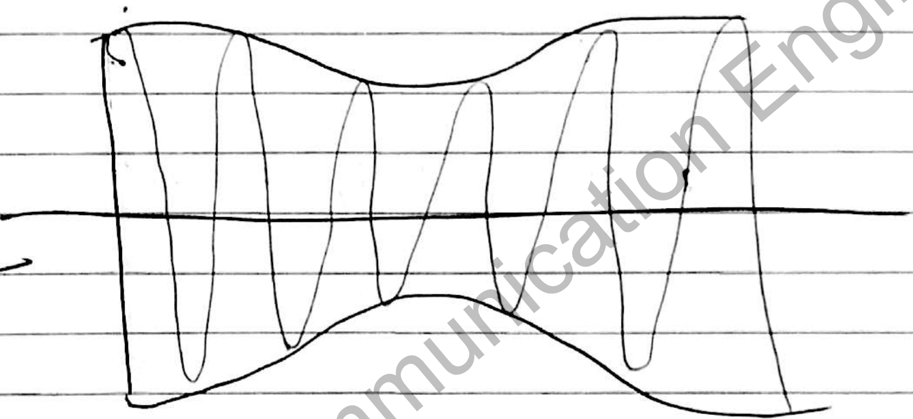
$A_c(1-\mu) \Longrightarrow$ -ve.

OVER MODULATION

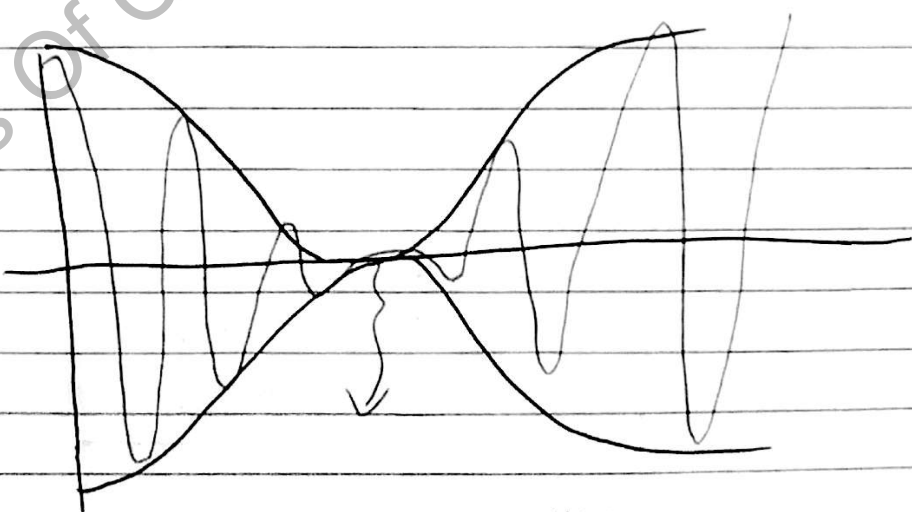


∴ min value is -ve this curve is for over modulation.

UNDER MODULATION



CRITICAL MODULATION



180° phase shift is there in critical modulation.

NOTE: Message signal should be stored in positive envelope.
Envelope detector gives the line touching the +ve peaks without addition of carrier signal.

It is not possible to detect message using envelope detector, so, there we have to use COHERENT DETECTOR, but coherent detector is highly complex.

this is part of demodulation.

In case of $\mu > 1$, coherent detector is to be used.
↓
to get the message.

$$(A)_{max} = A_c(1+\mu)$$

$$(A)_{min} = A_c(1-\mu)$$

$$\boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

$$\boxed{A_c = \frac{A_{max} + A_{min}}{2}}$$

Q). The peak amplitude of AM signal varies b/w 2V and 10V.

Find μ, total power P_T & efficiency η.

$A_{max} = 10V$

$A_{min} = 2V$

$$\mu = \frac{10-2}{10+2} = \frac{\cancel{8}\,2}{\cancel{12}\,3} = 0.666$$

$$A_c = \frac{10+2}{2} = 6$$

Total power, $P_T = \left(1+\frac{\mu^2}{2}\right)P_c$

$$P_T = \left(1+\frac{4}{9\times 2}\right)(18)$$

$$P_c = \frac{A_c^2}{2}$$

$$= \left(\frac{22}{18}\right)(18) = 22W$$

$$= \frac{36}{2} = 18$$

$$\boxed{P_T = 22W}$$

$$\eta = \frac{\mu^2}{\mu^2+2} = \frac{\frac{4}{9}}{\frac{4}{9}+2} = \frac{\cancel{4}\,2}{\cancel{22}\,11} \times 100$$

$$= \frac{200}{11}\ \%$$

CURRENT RELATION

$$P_T = \left(1 + \frac{\mu^2}{2}\right) P_c$$

$$I_t^2 R = I_c^2 R \left(1 + \frac{\mu^2}{2}\right)$$

$$I_t = I_c \sqrt{1 + \frac{\mu^2}{2}}$$

$\boxed{I_t = \left(\sqrt{1 + \frac{\mu^2}{2}}\right) I_c}$

$I_t \rightarrow$ Total current

$I_c \rightarrow$ carrier current

Similarly, for voltage,

$$\boxed{V_t = \left(\sqrt{1 + \frac{\mu^2}{2}}\right) V_c}$$

classmate
Date
Page

Q) An unmodulated AM current is given by 5A Find AM current with 50% ~~of~~ modulation.

$I_c = 5A \qquad \mu = 0.5$

$I_t = \sqrt{1+\frac{\mu^2}{2}}\, I_c = \left(\sqrt{1+\frac{0.25}{2}}\right) 5$

$= \left(\sqrt{1.125}\right) 5$

$= 5.3A.$

Q) AM current is given by 10A, with $\mu = 0.4$ Find AM current with $\mu = 0.8$.

$\frac{10}{\sqrt{1+\frac{(0.4)^2}{2}}} = I_c$

$I_t = \sqrt{1+\frac{(0.8)^2}{2}} \; \frac{10}{\sqrt{1+\frac{(0.4)^2}{2}}}$

64

0.16

$= \sqrt{1.32} \; \frac{\times 10}{\sqrt{1.08}}$

$= 11.04 A.$

MULTI TONE AM

$$s_{AM}(t) = A_c \left(1 + K_a \, m(t)\right) \cos 2\pi f_c t.$$

Assume,

$$m(t) = A_{m_1} \cos 2\pi f_{m_1} t + A_{m_2} \cos 2\pi f_{m_2} t$$

$$s_{AM}(t) = A_c \Big[1 + K_a A_{m_1} \cos 2\pi f_{m_1} t + K_a A_{m_2} \cos 2\pi f_{m_2} t \Big] \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + \frac{A_c \mu_1}{\cancel{2}} \cos 2\pi f_{m_1} t \cos 2\pi f_c t + \frac{A_c \mu_2}{\cancel{2}} \cos 2\pi f_{m_2} t \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + \frac{A_c \mu_1}{2} \Big[\cos 2\pi (f_{m_1} - f_c) t + \cos 2\pi (f_{m_1} + f_c) t \Big]$$

$$+ \frac{A_c \mu_2}{2} \Big[\cos 2\pi (f_{m_2} - f_c) t + \cos 2\pi (f_{m_2} + f_c) t \Big]$$

lower side Band of 1

upper side Band of 1

$$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu_1}{2}\left[\cos 2\pi (f_{m_1} - f_c)t + \cos 2\pi (f_{m_1} + f_c)t\right]$$

$$+ \frac{A_c \mu_2}{2}\left[\cos 2\pi (f_{m_2} - f_c)t + \cos 2\pi (f_{m_2} + f_c)t\right].$$

Lower side Band of 2

upper side band of 2.

Assume,

$\mu_2 > \mu_1$ and $f_{m_2} > f_{m_1}$

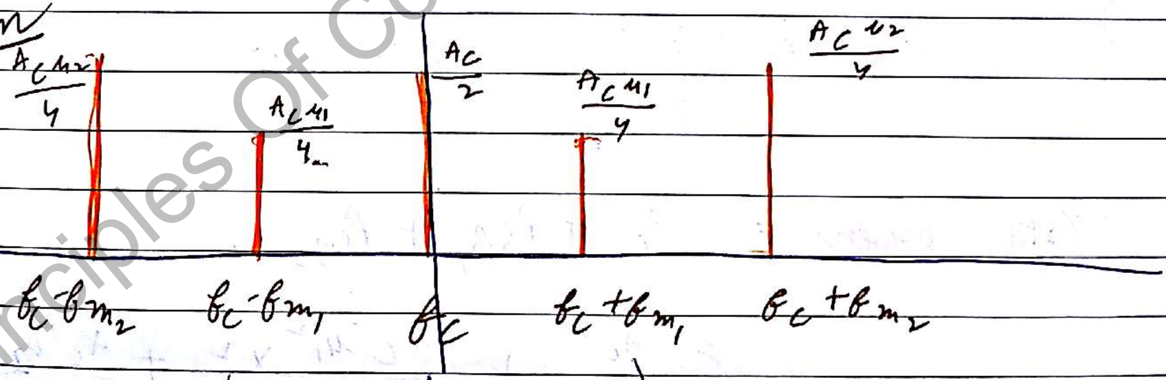
Bandwidth of AM $= 2 \times (f_{max})_{message}$

$$\boxed{\text{Bandwidth} = 2 \times f_{m_2}}$$

Magnitude Spectrum

↓

avg. values are represented in mag. spectrum. (so divide by 2).



$\frac{A_c\mu_1}{2}\cos 2\pi (f_c - f_{m_1})t$ $\quad A_c \cos 2\pi f_c t$ $\quad \frac{A_c\mu_1}{2}\cos 2\pi (f_c + f_{m_1})t$

↓

$\frac{A_c\mu_1}{4}$ $\quad \frac{A_c}{2}$ → mag. spectra. $\left(\frac{1}{2}\right)$ due to avg. value of $\cos 2\pi f_c t$.

\# $P_c = \frac{A_c^2}{2}$

$P_{USB_1} = \frac{A_c^2 \mu_1^2}{8}$ $P_{USB_2} = \frac{A_c^2 \mu_2^2}{8}$

$P_{LSB_1} = \frac{A_c^2 \mu_1^2}{8}$ $P_{LSB_2} = \frac{A_c^2 \mu_2^2}{8}$

while finding power, we have assumed resistance, $R = 1$.

If resistance is also given, then divide by R in formulas,

i.e. $P_c = \frac{A_c^2}{2R}$ and so on.

Total power $= P_c + P_{SB_1} + P_{SB_2}$

$$= \frac{A_c^2}{2} + \frac{A_c^2 \mu_1^2}{8} \times 2 + \frac{A_c^2 \mu_2^2}{8} \times 2$$

$$= \frac{A_c^2}{2} + \frac{A_c^2 \mu_1^2}{4} + \frac{A_c^2 \mu_2^2}{4}$$

$$= P_c \left(1 + \frac{\mu_1^2 + \mu_2^2}{2} \right)$$

$$P_T = P_C\left(1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right)$$

$$\boxed{P_T = P_C\left(1 + \frac{\mu_m^2}{2}\right)}.$$

$$\mu_m^2 = \mu_1^2 + \mu_2^2$$

$$\boxed{\mu_m = \sqrt{\mu_1^2 + \mu_2^2}}$$

Generalising this for n message signals,

$$\boxed{\mu_m = \sqrt{\mu_1^2 + \mu_2^2 + \dots + \mu_n^2}}$$

all other formulas will remain same.

$$P_T = \left(1 + \frac{\mu_m^2}{2}\right) P_C$$

$$P_{SB} = \frac{\mu_m^2}{2} P_C$$

$$\eta\,\% = \frac{\mu_m^2}{\mu_m^2 + 2} \times 100\,\%.$$

Q) An unmodulated AM power is given by 10 kW when the carrier is modulated by first message signal, AM power is increased to 13.5 kW, find power if carrier is simultaneously modulated with 2nd message signal with 60% of modulation.

$$P_t = \left(1+\frac{\mu^2}{2}\right) P_c$$

$$\frac{13.5}{10} = 1 + \frac{\mu^2}{2}$$

$$\frac{3.5}{10} \times 2 = \mu^2 \qquad \mu_1^2 = \frac{3.5}{5} = 0.7$$

$$\mu_1 = \sqrt{0.7}$$

$$\mu_m = \sqrt{0.7 + 0.36}$$

$$= \sqrt{1.06}$$

$$P_t = \left(1 + \frac{1.06}{2}\right)(10)$$

$$= (1 + 0.53)(10) = 15.3\,kW \text{ — Ans.}$$

Q) An AM current is given by 10A corresponds to carrier is modulated by 1st message signal with 40% modulation then carrier is simultaneously modulated with 2nd m(t). AM current is ↑se to 10.5 A. Find % of modulation due to modulation due to 2nd message signal.

$$10 = I_c \sqrt{1 + \frac{0.16}{2}} \qquad I_c = \frac{10}{\sqrt{1.08}}$$

$$10.5 = \frac{10}{\sqrt{1.08}} \sqrt{1 + \frac{0.16 + \mu^2}{2}}$$

$$\frac{10.5\sqrt{1.08}}{10} = \sqrt{1 + \frac{0.16 + \mu^2}{2}}$$

$\mu = 0.47$ or 47%

Q) An AM signal is given by

$$s(t) = \left(20 + 12 \cos 2\pi 10^4 t + 60 \cos 4\pi \times 10^4 t\right) \cos 2\pi 10$$

Find bandwidth, total power & efficiency η ??

$$B.W. = 2 \max f$$

$$= 2 \left(2 \times 10^4\right) Hz$$

$$= 4 \times 10^4 \; Hz$$

$$= 40 \; kHz.$$

Total power, $P_T = \left(1 + \frac{\mu^2}{2}\right) P_c$

$P_c = \frac{A_c^2}{2}$

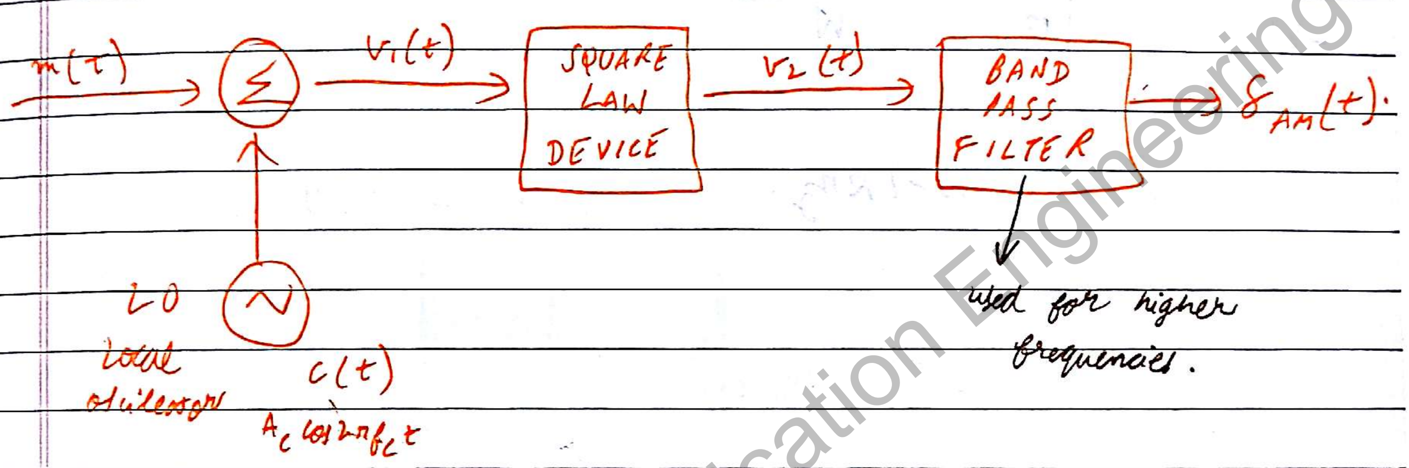
$$20 \left(1 + \frac{12}{20} \cos 2\pi 10^4 t + \frac{60}{20} \cos 4\pi \times 10^4 t\right)$$

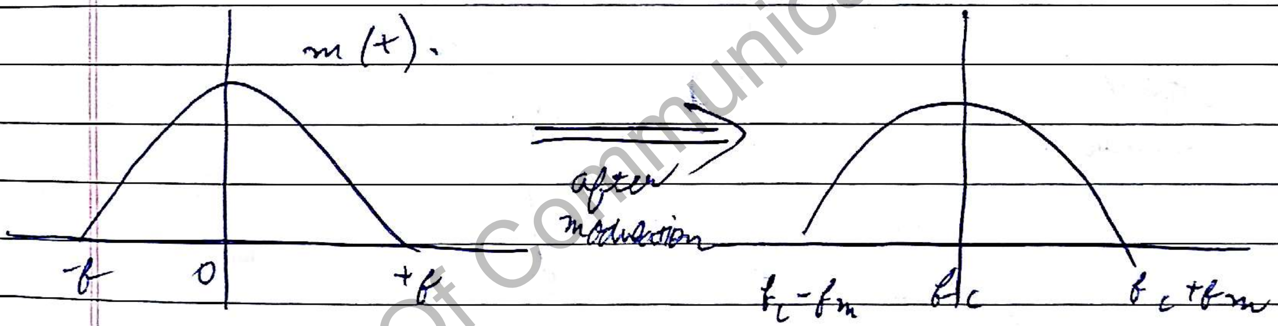
$\cos 2\pi \times 10$

~~Compare to~~

$$A_c \left(1 + K_a \, m(t)\right) \cos 2\pi f_c t.$$

SQUARE LAW MODULATION



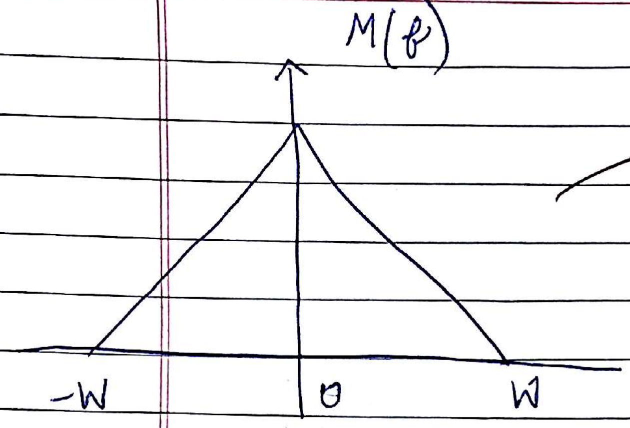


$$v_1(t) = m(t) + c(t)$$

$$= m(t) + A_c \cos 2\pi f_c t.$$

$$(SLD)_{o/p} = v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) + a_3 v_1^3(t) + \ldots$$

square law device.

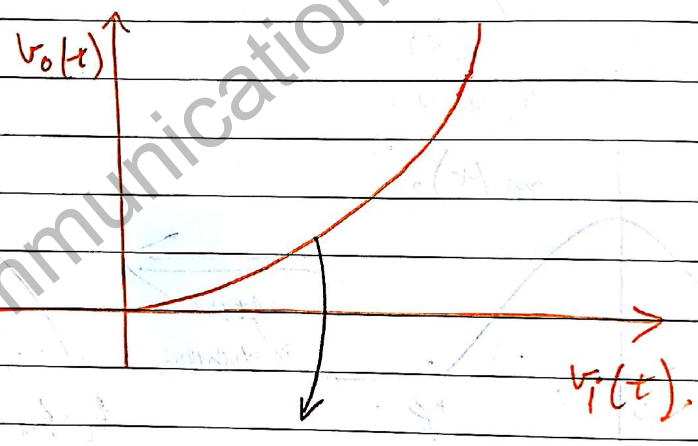


Bandwidth = W.

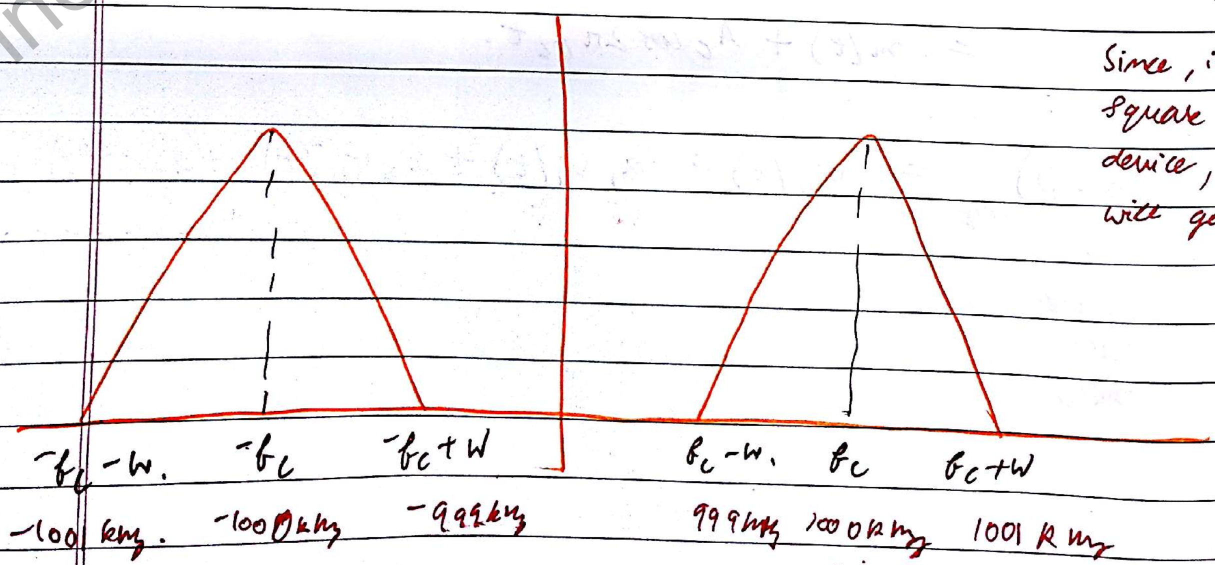
$W = 1\,kHz$.

Take $f_c = 1\,MHz$.

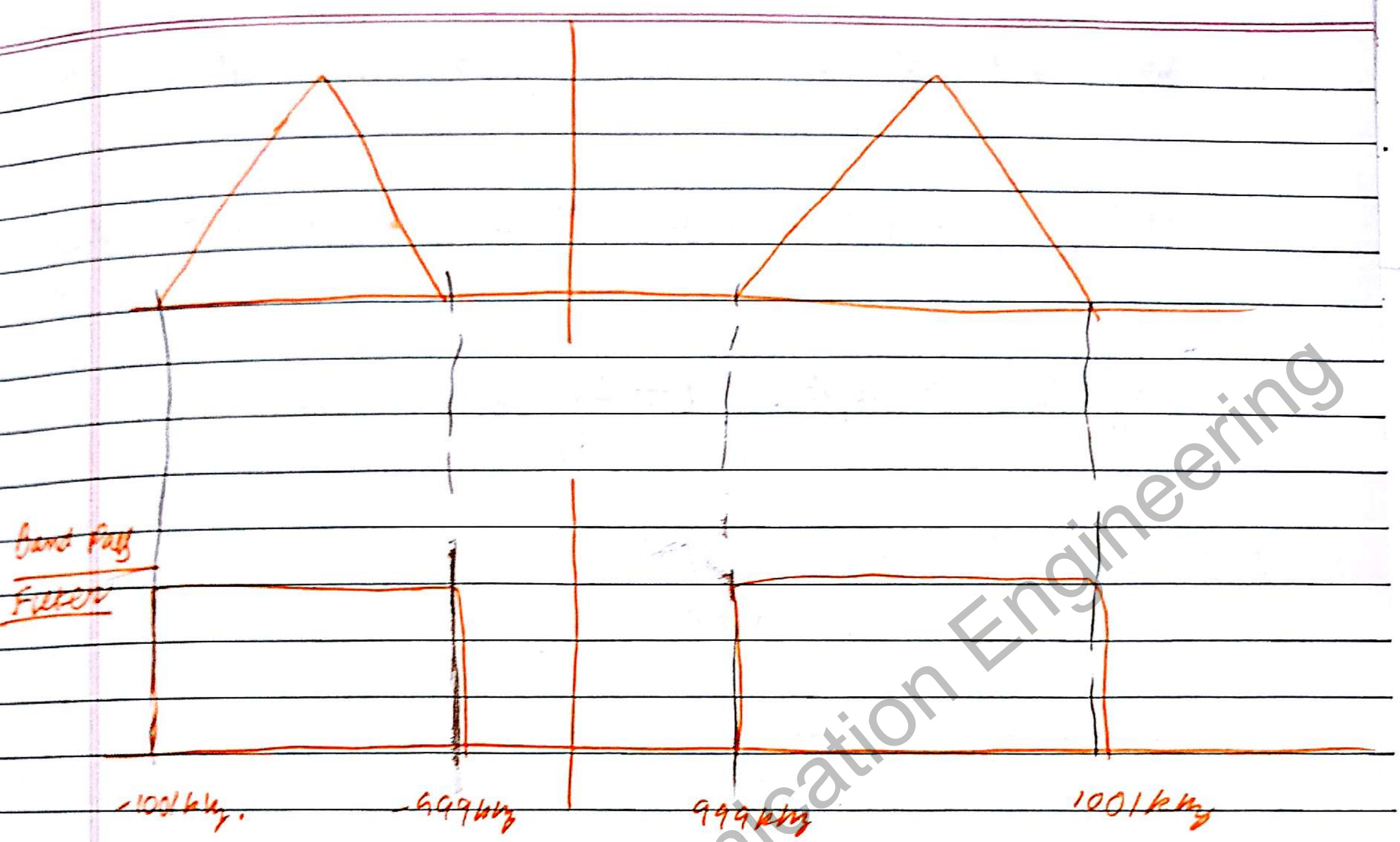
After modulation.



SQUARE LAW CHARACTERISTICS.



Since, it is square law device, values will get squared



since the freq. f_c is present. so it will be present.

$$(SLD)_{output} = a_1\left(m(t) + A_c \cos 2\pi f_c t\right) + a_2\left(m^2(t) + A_c^2 \cos^2 2\pi f_c t + 2A_c\, m(t) \cos 2\pi f_c t\right).$$

it will be present (f_c term)

$a_1 v_1(t)$
$a_2 v_2^2(t)$
$a_3 v_3^3(t)$
and so on.

message freq. is from 0 to W so this will not be there.

this will also not be there. Multiplication in time & there will be convolution in freq.

On applying trigo formula, its freq. will be $2f_c$, so it will not be present.

After this, all cube terms will become zero & not be present.

So, output of band pass filter will be,

$$\left(BPF\right)_{output} = a_1 A_c \cos 2\pi f_c t + 2 a_2 A_c \cos 2\pi f_c t \; m(t)$$

$$s_{AM}(t) = a_1 A_c \left(1 + \frac{2a_2}{a_1} m(t)\right) \cos 2\pi f_c t.$$

Here, ~~freq~~ amplitude of carrier signal $= a_1 A_c$.

Comparing this eq^n with standard form,

$$A_c \left(1 + k_a m(t)\right) \cos 2\pi f_c t.$$

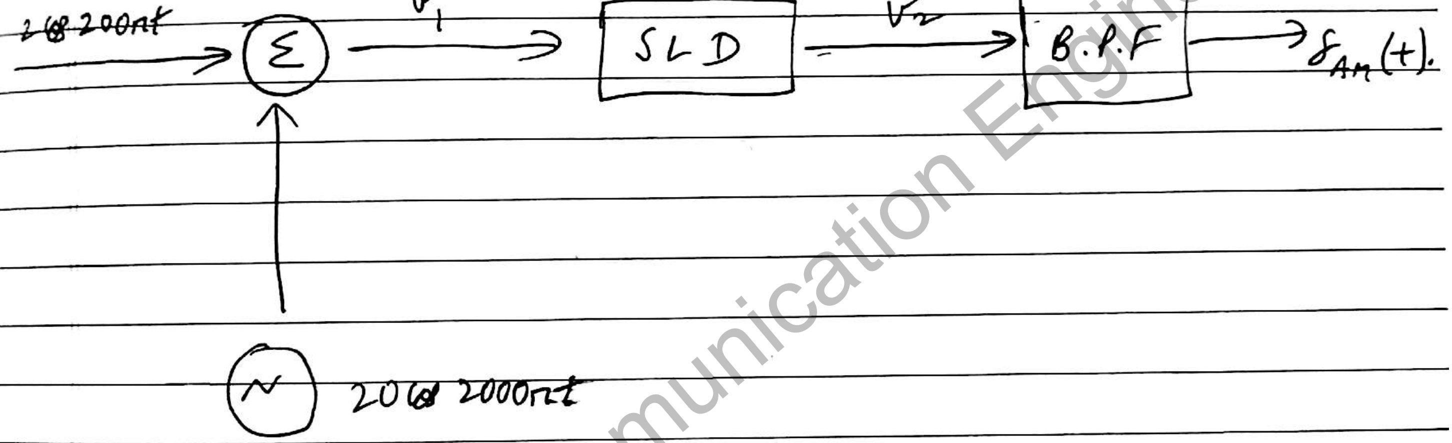
$\boxed{A_c' = a_1 A_c}$ $\boxed{k_a = \frac{2a_2}{a_1}}$

AMPLITUDE SENSITIVITY

Q) For the following square law device characterise by.

$$V_o = V_i + 0.1 V_i^2$$

pass band of band pass filter extends from 800 to 1200 Hz. Find parameters of resultant AM signal.



$$V_o = V_i + 0.1 V_i^2$$ K_a & μ.

$A_m = 2$ $\quad K_a = \frac{2 \times 0.1}{1} = 0.2$

$$V_i = 2\cos 200\pi t + 20\cos 2000\pi t$$

$$V_o = V_i + 0.1\, v_i^2$$

$$V_o = \left[\cancel{2\cos 200\pi t} + 20\cos 2000\pi t\right] + 0.1\left[\cancel{4\cos^2 200\pi t} + \cancel{400\cos^2 2000\pi t} + 80\cos 200\pi t \cos 2000\pi t\right].$$

1000 f

$$\left(BPF\right)_{output} = 20\cos 2000\pi t + 8\cos 200\pi t \cos 2000\pi t$$

$$= \underbrace{20}_{A_c}\left[1 + \underbrace{\frac{8}{20}\cos 200\pi t}_{K_a m(t)}\right]\cos 2000\pi t$$

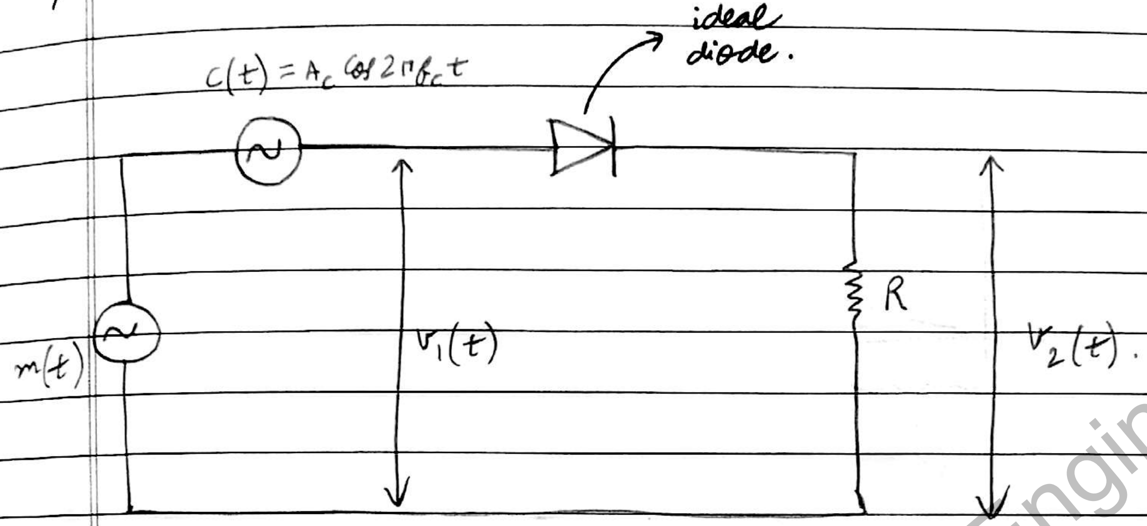
$$\boxed{K_a = \frac{4}{20}}$$

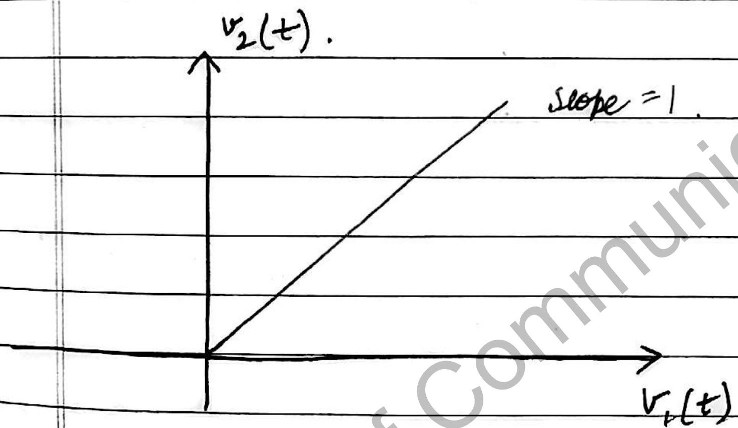
$$\mu = K_a A_m$$

$$= \frac{4}{20} \times 2 = \frac{8}{20}$$

$$\boxed{\mu = \frac{8}{20}}$$

2) SWITCHING MODULATION





$$v_1(t) = m(t) + c(t)$$

$$= m(t) + A_c \cos 2\pi f_c t$$

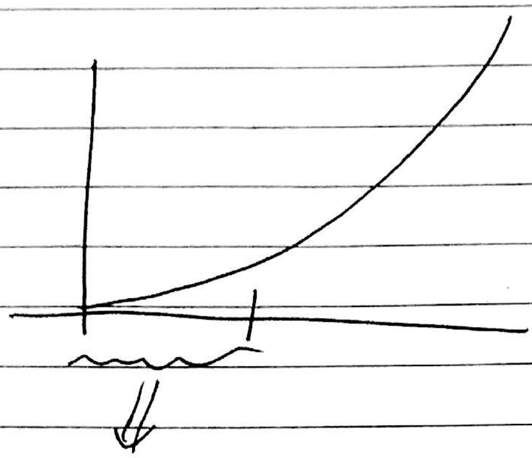
When $|m(t)| << A_c$

amplitude of message << amplitude of carrier.

$$v_2(t) = v_1(t) \quad c(t) > 0$$

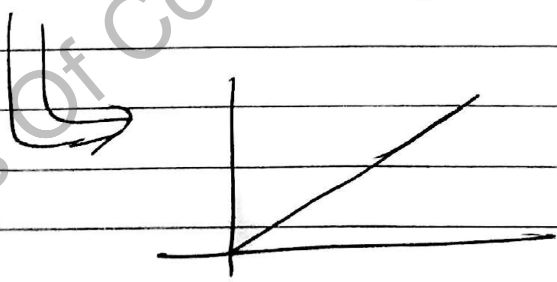
$$= 0 \quad c(t) < 0.$$

In the ideal characteristics of diode,



this region won't exist
because value of A_c is very high
so, low frequencies will not exist.

and therefore, this curve is approximated to be a straight line.

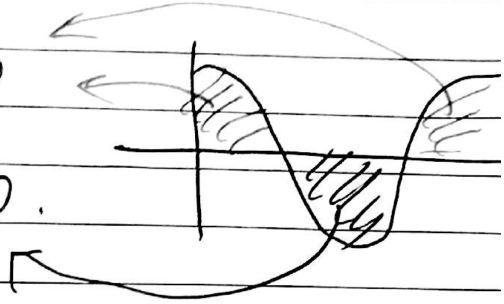


Diode is f. biased. $\quad v_1(t) > v_2(t).$
(short circuited)

diode is forward biased.

$v_2(t) = v_1(t) \qquad c(t) > 0$

$= 0 \qquad c(t) < 0.$



diode is reverse biased.

message signal in the form of cos wave.

$v_2(t) = \boxed{m(t) + c_1(t)}\, g(T_0).$

rectangular train wave.

when $c(t)$ will be +ve

$g(T_0)$ should be +ve.

when $c(t)$ is -ve

the $g(T_0)$ should be 0.

$c(t) = A_c \cos 2\pi f_c t.$

$T = \frac{1}{f_c}.$

both $c(t)$ & $g(\tau)$ should have the same frequency (or time period).

$$g_{T_0} = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left(2\pi f_c t (2n-1)\right)$$

eqⁿ of fourier series representation of ~~rectangular wave train.~~

$$v_2(t) = \left(m(t) + A_c \cos 2\pi f_c t\right) g_{T_0}$$

2 components :

~~One will be required &~~ other will be unwanted.

1st component (desired).

$$\frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right] \cos 2\pi f_c t .$$

~~this desired~~ signal is ~~passed through~~ band pass filter.

the freq. of this band pass ~~filter should~~ be equal to freq. of the signal.

$$v_2(t) = (m(t) + A_c \cos 2\pi f_c t)\, g_{T_0}.$$

$f_c \;,\; 2f_c.$

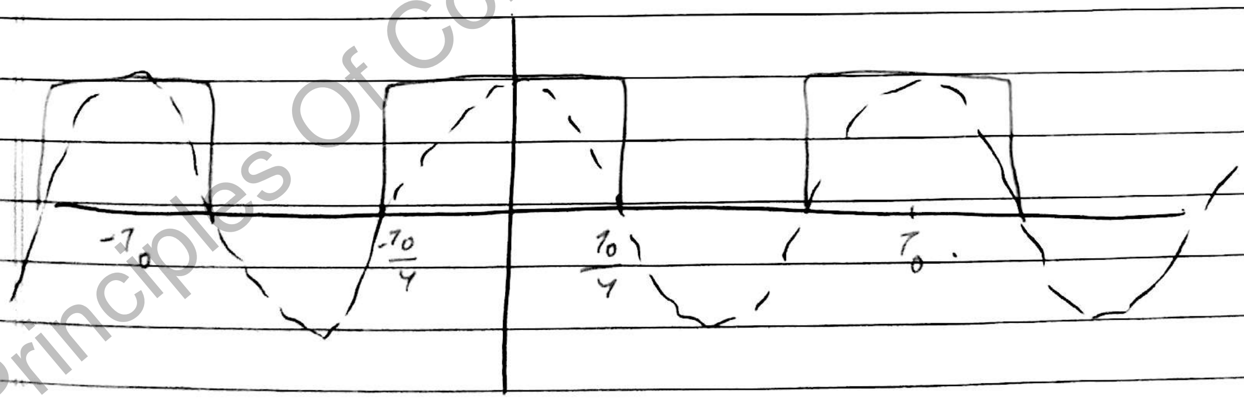
when $g_{T_0} \times \cos 2\pi f_c t$ $\quad \pm 2f_c \quad \pm 3f_c$

(doubled.

when $g_{T_0} \times m(t)$ $\quad f_c \pm W$

→ carrier +ve

whenever cos is +ve, then only rectangular pulse will be +ve, otherwise it will be 0.



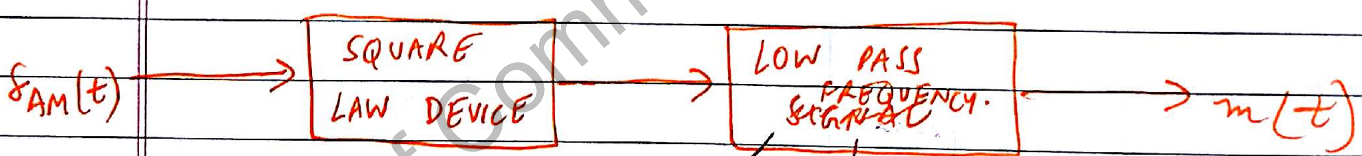
DEMODULATION OF AM

It is done in 3 ways :-

1) SQUARE LAW DETECTOR
2) ENVELOPE DETECTOR
3) SYNCHRONOUS / COHERENT DETECTOR.

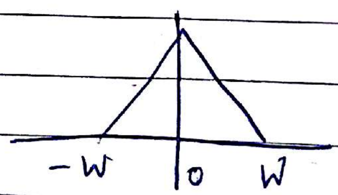
SQUARE LAW DETECTOR.

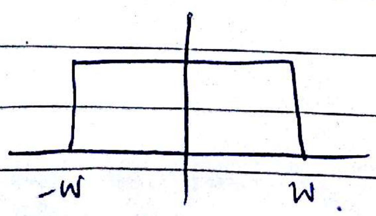
detection
↳ recovery of message.



it's cut off frequency should be equal to that of message signal.

∵ message will be present in lower side band.





classmate

Date ____

Page ____

$$S_{AM}(t) = A_c \left[\cancel{1 + K_a \cos} \right.$$

$$= A_c \left[1 + K_a \, m(t) \right] \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + A_c K_a \, m(t) \cos 2\pi f_c t$$

$$(SLD)_{o/p} \Rightarrow v_0(t) = a_1 S_{AM}(t) + a_2 S_{AM}^2(t) + \ldots$$

$$= a_1 \left[A_c \cos 2\pi f_c t + A_c K_a \, m(t) \cos 2\pi f_c t \right]$$

$$+ a_2 \left[A_c^2 \cos^2 2\pi f_c t + A_c^2 K_a^2 m^2(t) \cos^2 2\pi f_c t \right.$$

$$\left. + 2 A_c^2 K_a \, m(t) \cos^2 2\pi f_c t \right)$$

$$+ \ldots$$

neglected f_c neglected

$$= a_1 \left[A_c \cos 2\pi f_c t + A_c K_a \, m(t) \cos 2\pi f_c t \right]$$

neglected.

$$+ a_2 \left[\frac{A_c^2}{2} \left(1 + \cos 4\pi f_c t \right) + \frac{A_c^2 K_a^2 m^2(t)}{2} \left(1 + \cos 4\pi f_c t \right) \right.$$

neglected.

$$\left. + A_c^2 K_a \, m(t) \left(1 + \cos 4\pi f_c t \right) \right].$$

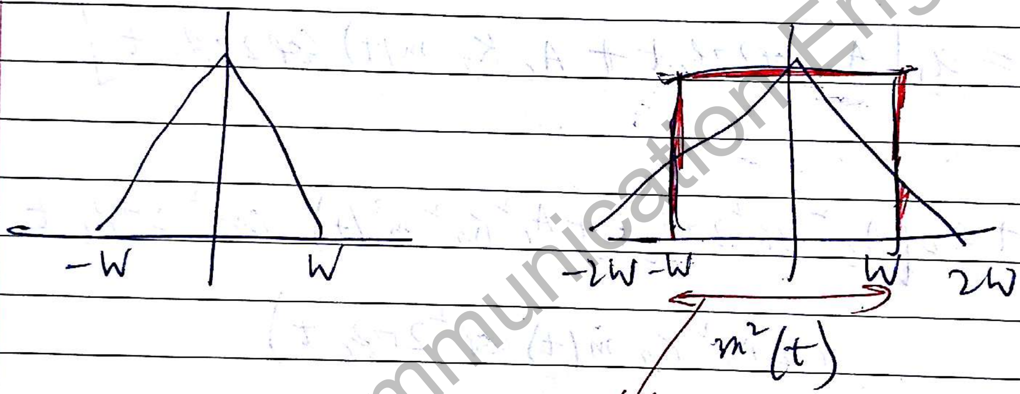
$$+ \ldots$$

neglected.

NOTE: At low pass filter output, required $m(t)$, message signal, will be interferenced with higher order harmonics i.e.

$m^2(t)$, $m^3(t)$, $m^4(t)$ .... so that $m(t)$ will be lost. this is called as

HARMONIC DISTORTION.



only this region is left.

this common part adds up for all the powers i.e. $m^2(t)$, $m^3(t)$

& original signal is lost.

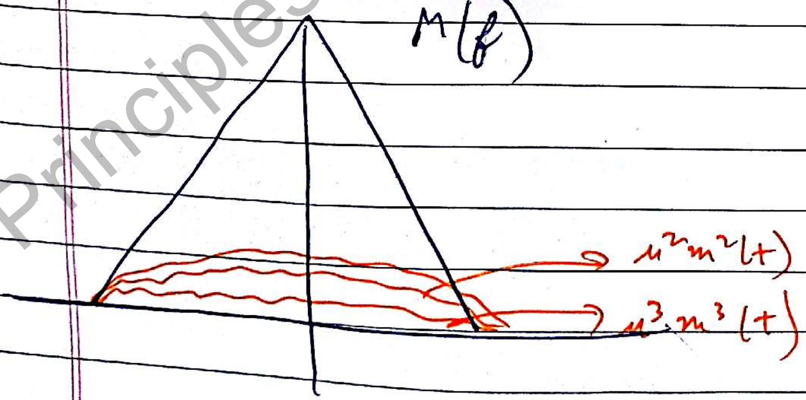
If modulation index, μ, is very very small, then strength of higher order harmonics will be negligibly small, so that $m(t)$ can be recovered.

But small values of modulation index μ are practically not acceptable.

$$\frac{A_c^2 k_a^2}{2} m^2(t)$$

if value of μ is small, then here $m^2(t)$ term coeff. will be small

So, these can be rejected.



value of μ is small, so message can be recovered easily.

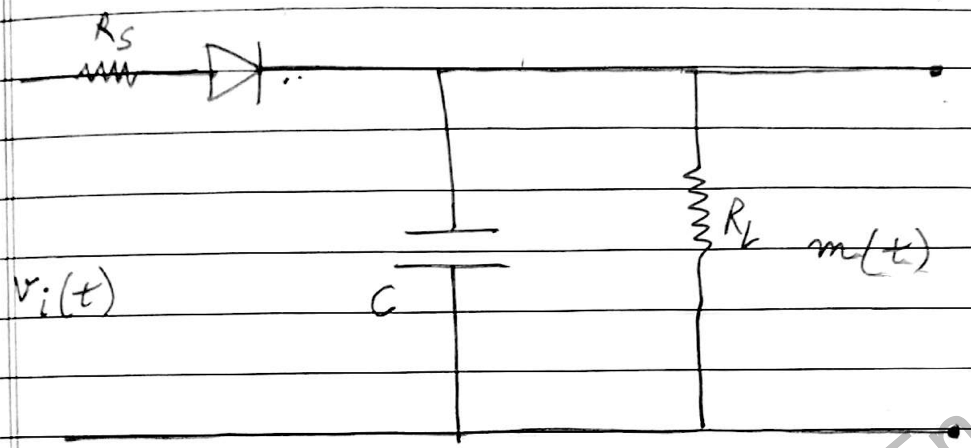
NOTE: Synchronous detector is complex and square law detector having drawbacks, so that for AM demodulation, envelope detector is preferred, which is simple and cheaper.

When

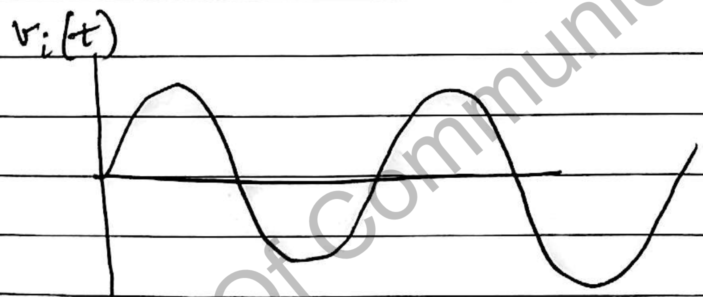
$\mu \leq 1$ Square Law Detector
Envelope Detector.

at any other value of μ Coherent Detector is used.

ENVELOPE DETECTOR.



1) For $t = 0^+$ to t_1



R_S & C values should be small.

$\Downarrow$

so that less time is taken in charging of capacitor & output is obtained for.

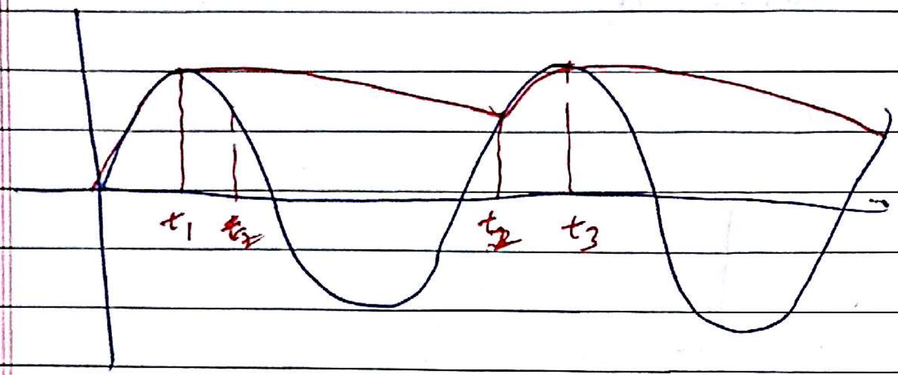
R_L should be of large value.

$\Downarrow$

so that discharge time of capacitor is high.

$R_S C <<$

$R_L C >>$



Value of t_1 is small

↓

capacitor will charge fastly.

Value at capacitor becomes V_m peak, and now input starts decreasing & diode is reverse bias.

Capacitor starts discharging through R_L.

* Envelope detector is very simple & cheap.

* Envelope detector extracts +ve envelope of applied signal and produces in the output. for m

* For modulation index $\mu \le 1$, message signal is stored in the form of +ve envelope so that envelope detection is possible.

1) For $t = 0^+$ to t_1

capacitor doesn't allow sudden change in voltage across it.
Time constant $R_s C$ should be small, to charge fast i.e. capacitor C rapidly charges to input.

2) For $t = t_1^+$ to t_2.

Potential of P < Potential of N $\Rightarrow$ Reverse Bias $\rightarrow$ Open circuit.

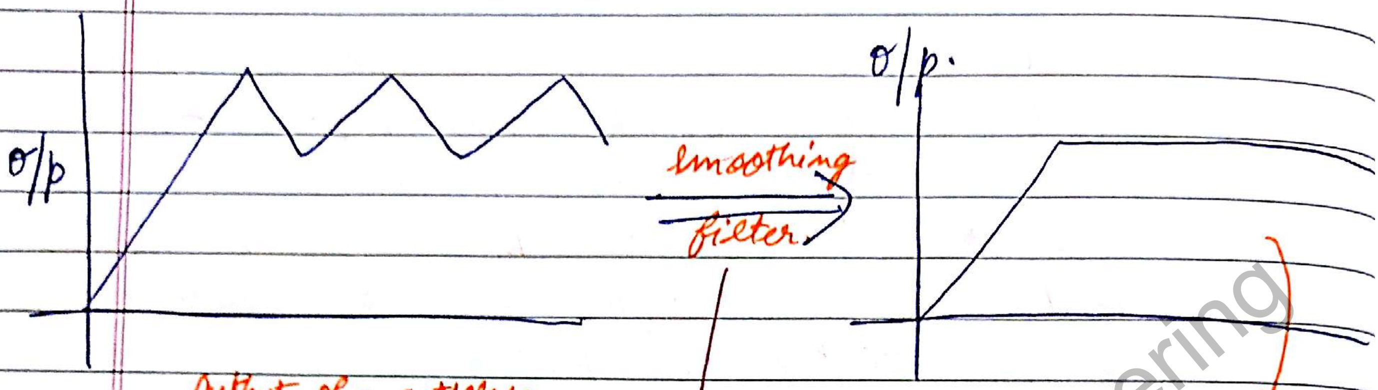
$R_L C$ should be high, so that capacitor C slowly discharges.

3) For $t = t_2^+$ to t_3.

Potential of P > Potential of N $\Rightarrow$ Forward Bias $\Rightarrow$ Short circuit.

capacitor charges.

Envelope Detector



Output of envelope detector

replace the edges with average value.

takes average of these points.

+ve envelope.

Envelope detector catches +ve envelope only

DIAGONAL CLIPPING

$R_L C > \frac{1}{f_m}$

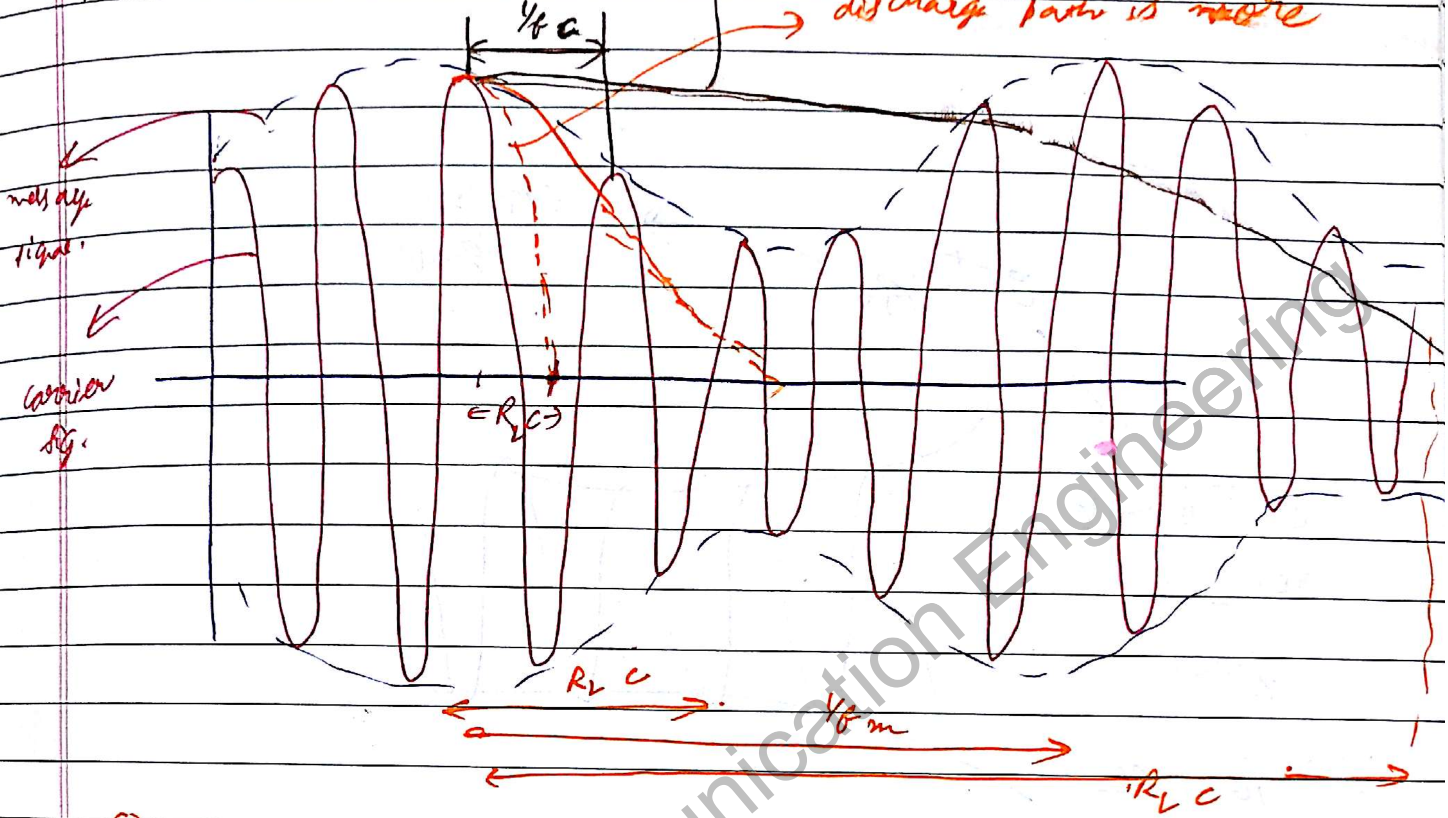
there is loss of msg in this case also.

$R_L C \ll \frac{1}{f_c}$

so there will be ~~loss~~ of msg. while ~~recovering~~.

PROPER CHOICE OF RLC

discharge path is more



① If lowest peak is +ve, then it is under modulation → ~~detection is easy.~~

② If ~~lowest~~ peak is −ve, then it is over modulation → msg signal cannot be detected easily.

① $R_L C \ll \frac{1}{f_c}$

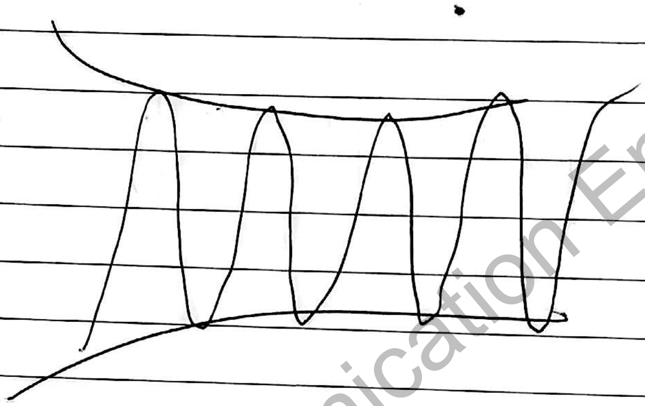
② $\frac{1}{f_c} < R_L C < \frac{1}{f_m}$ → ideal condition.

(3) $R_L C > \frac{1}{f_m}$ (Diagonal Clipping).

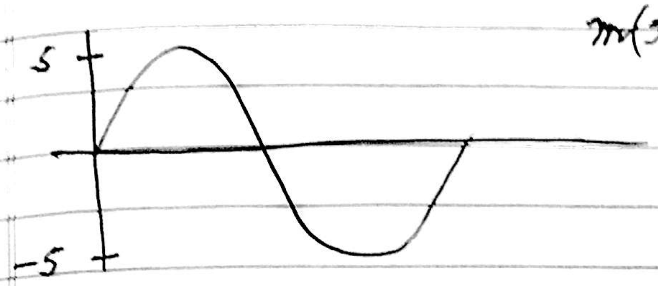
$e^{-t} \cos t$ → carrier signal.

e^{-t} is made message signal

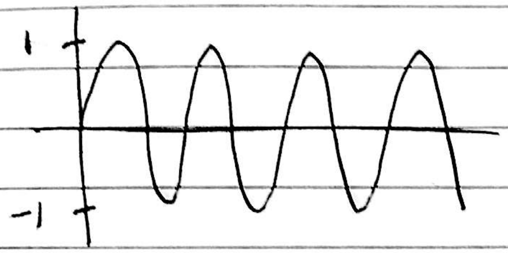
∴ it forms the envelope.

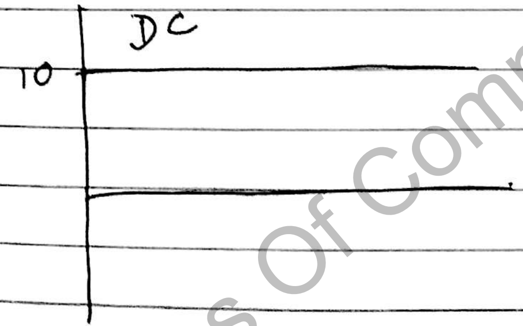


$m(t)$



$$c(t) = A_c \cos 2\pi f_c t$$
$$= 1 \cdot \cos 2\pi f_c t$$

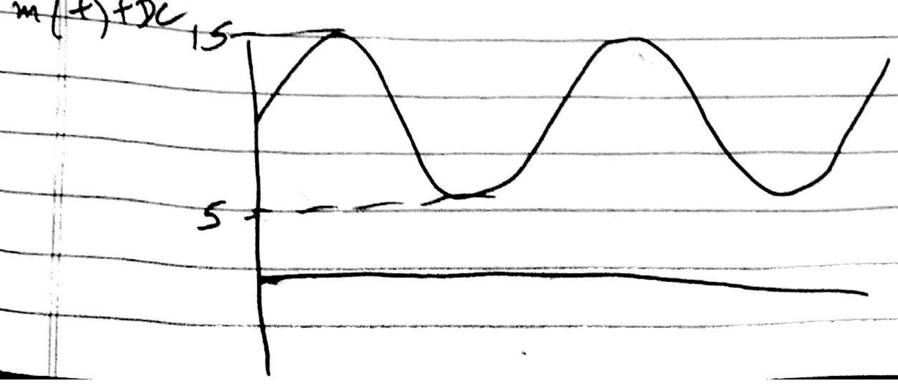


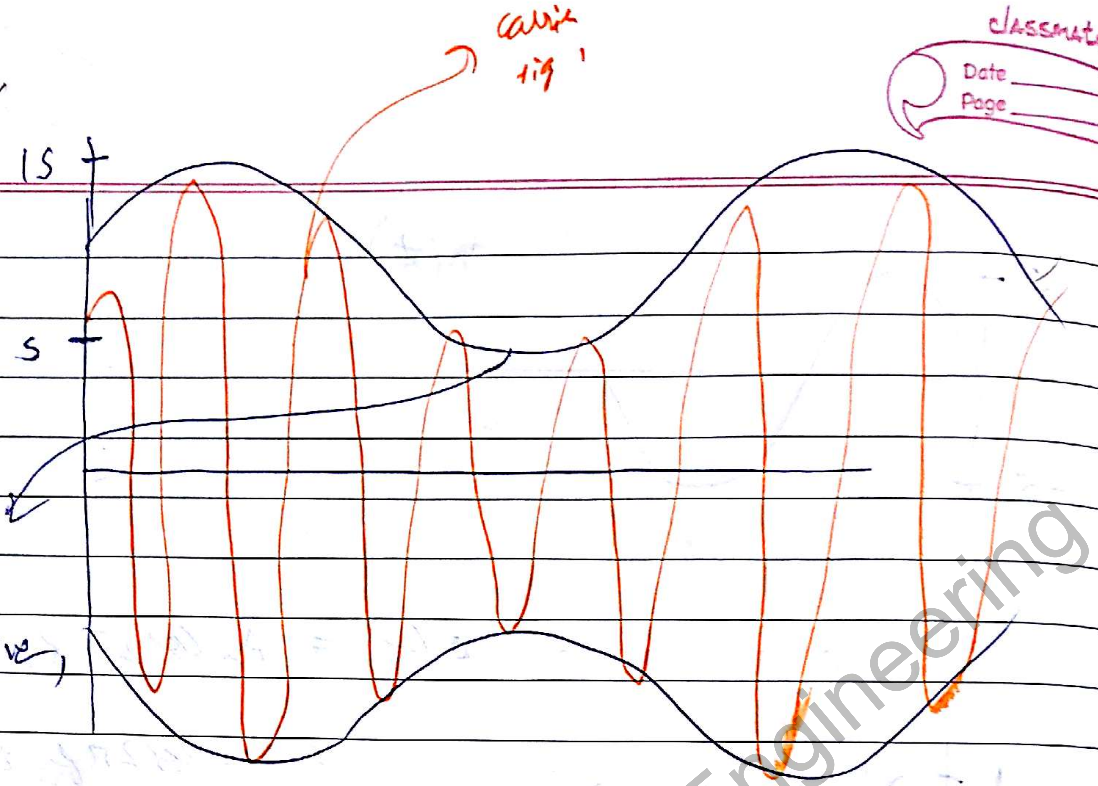


dependency on DC.

$$S_{AM}(t) = (m(t) + DC)\, c(t)$$
$$= (m(t) + DC) \cos 2\pi f_c t$$
$$= m(t) \cos 2\pi f_c t + DC \cos 2\pi f_c t$$

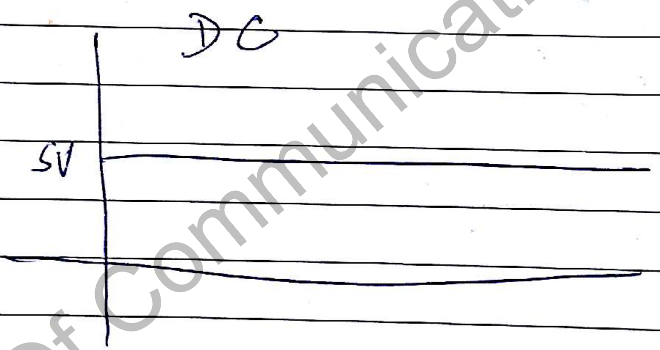
amplitude modulated wave.





this min. value is +ve, so, it is under modulation

Case 2

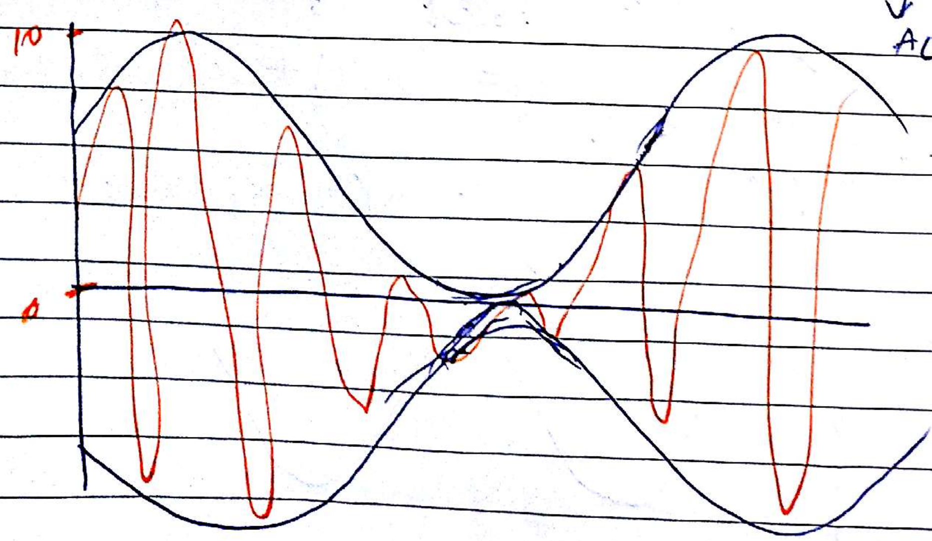


Now, DC value is taken = 5V.

$-5 + 5 = 0$

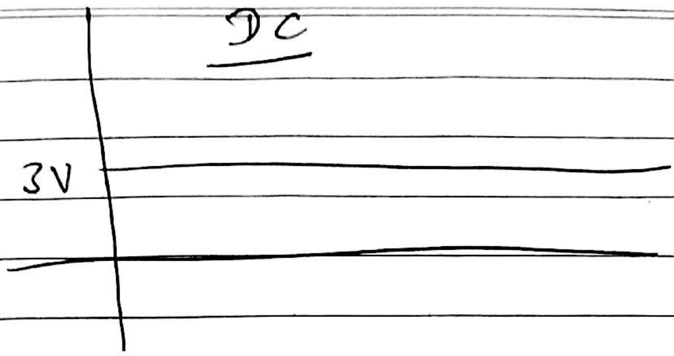
$5 + 5 = 10$

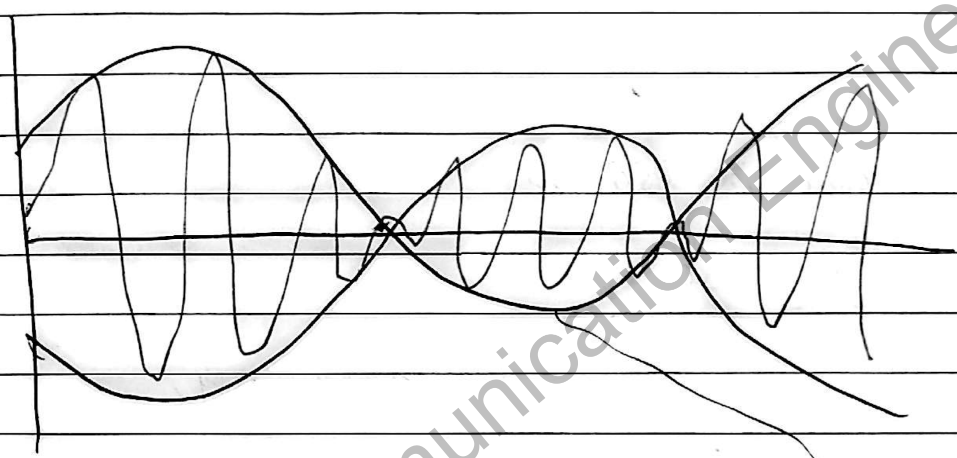
$\downarrow$ AC $\quad$ $\downarrow$ DC



It is case of critical modulation.

Case 3



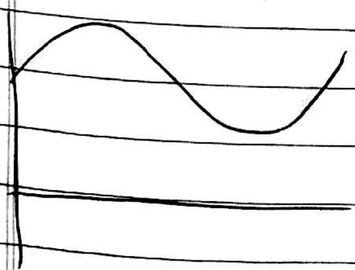


this is the case of
OVER MODULATION.

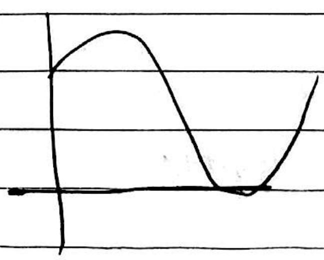
lowest value is -ve.

OUTPUT FROM ENVELOPE DETECTOR
IN ALL THESE CASES

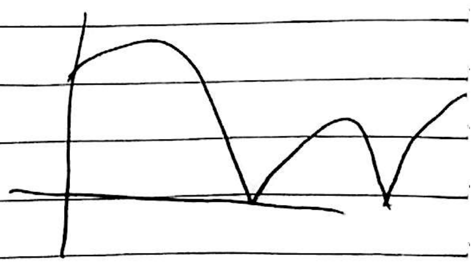
Case 1



Case 2



Case 3



When $m(t) + DC$ is completely +ve.

$$s_m(t) = \left(m(t) + DC\right) c(t)$$

$$= m(t)c(t) + DC\, c(t)$$

DSB - FC

double side band with full carrier.

Supressed in DSB-SC.

DSB - SC

is the new signal formed and in that only $m(t)\, c(t)$ portion is supressed.

double side band with supressed carrier.

DSB-SC can be detected only by using coherent detection.

Advantage:

~~carrier power is not to be sent alongwith~~

only message power is required.

classmate

Date ______
Page ______

SIGNIFICANCE OF K_a.

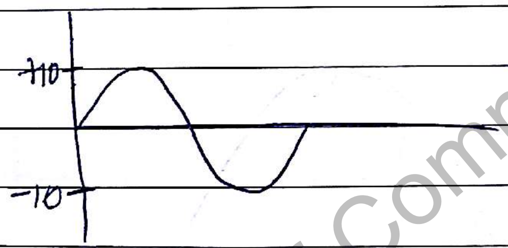
$$s_{AM} = A_c\left(1 + K_a\, m(t)\right)\cos 2\pi f_c t$$

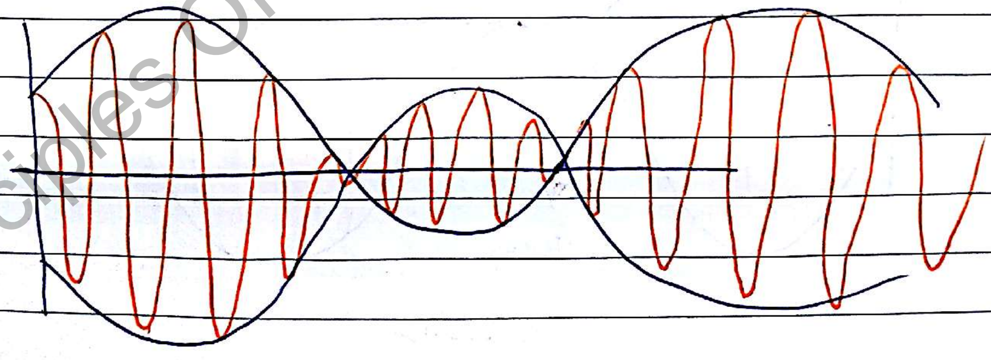
For envelope detection,

$A_c\left(1 + K_a\, m(t)\right)$ should be completely +ve.

If $A_c = 1 \Rightarrow \left(1 + K_a\, m(t)\right) \geq 0$

Case 1





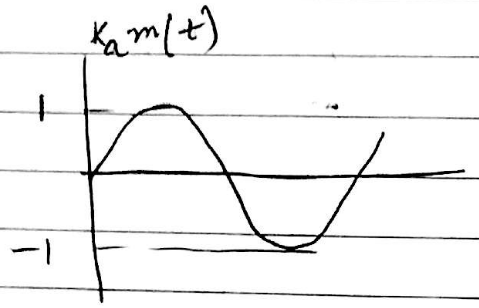
$\mu > 1$ (Over Modulation).

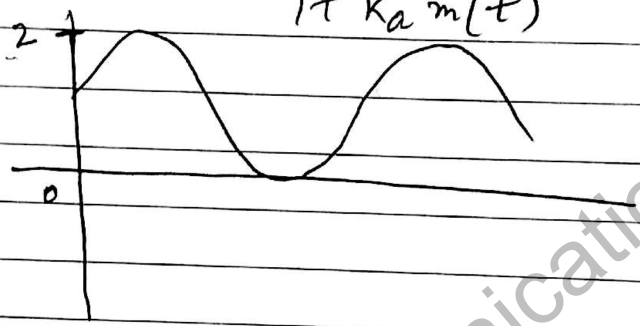
$K_a A_m > 1$. $K_a > \frac{1}{A_m}$ → Over modulation.

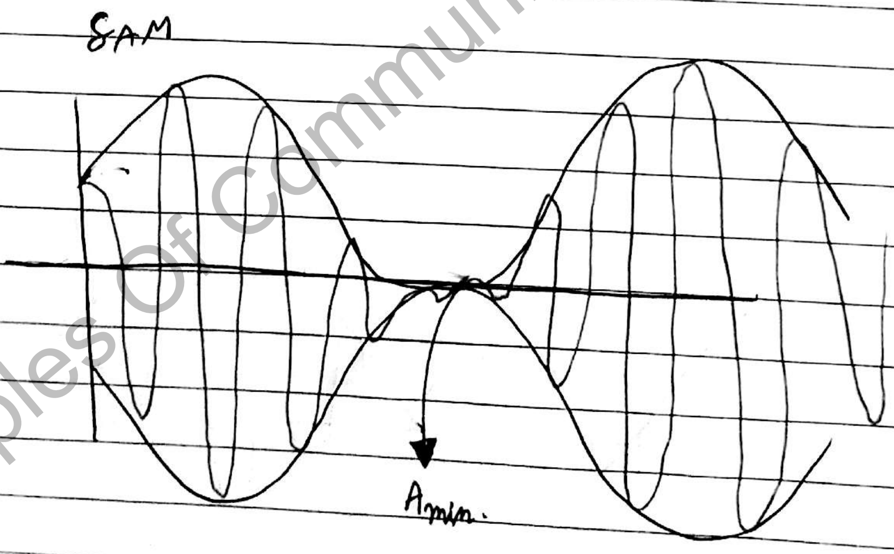
value of K_a is dependent on amplitude of $m(t)$

Case 2

$$K_a = \frac{1}{10}$$

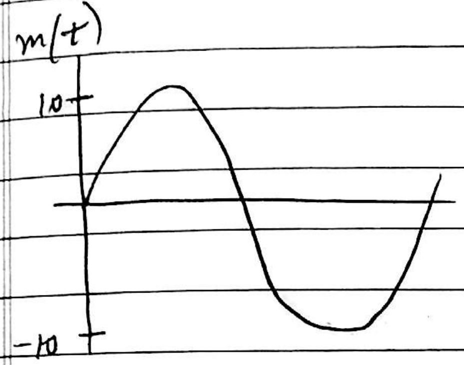


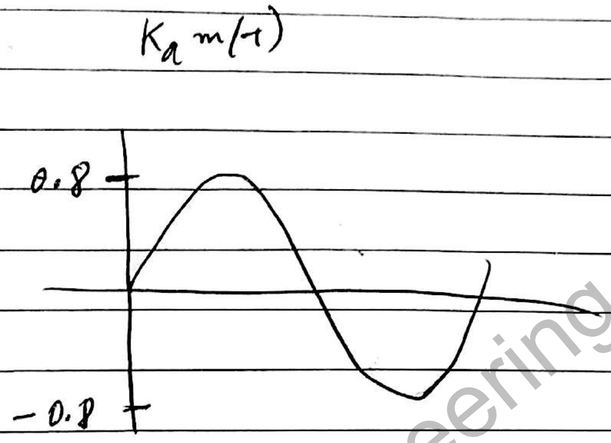


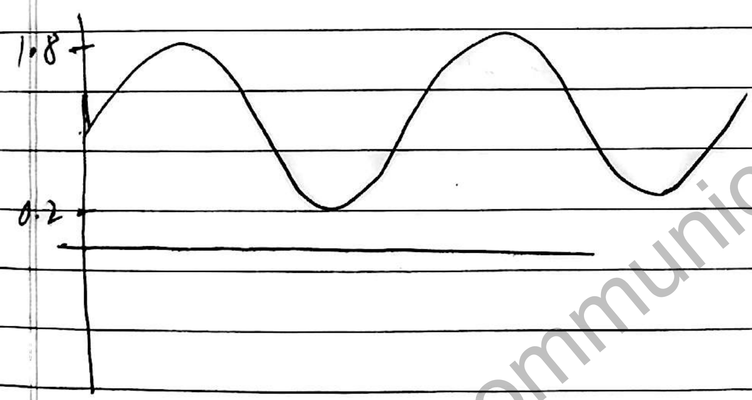


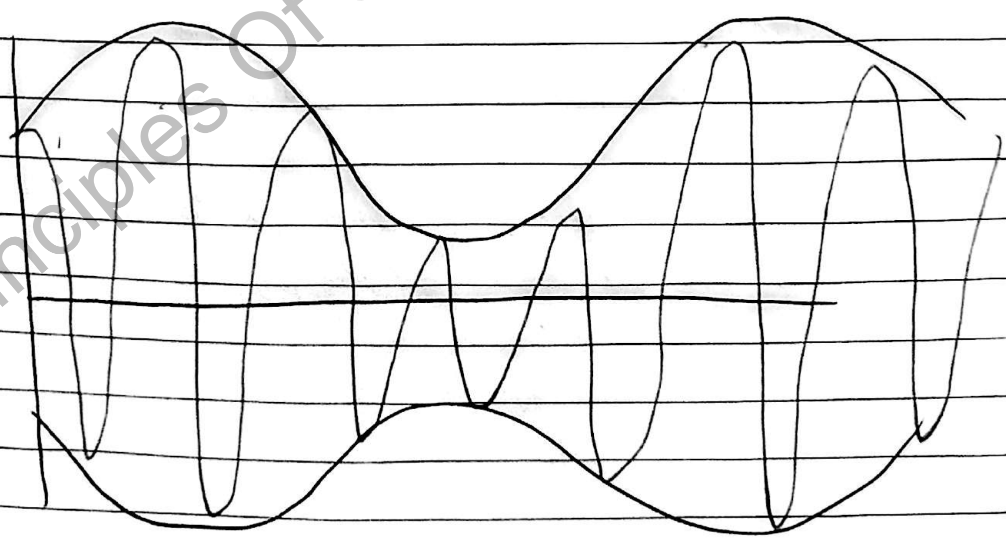
CRITICAL MODULATION

Q3 $K_a = \frac{1}{12}$









UNDER MODULATION.

K_a specifies normalization of message signal for proper envelope detection.

K_a value depends on the percentage of modulation expected.

value of K_a is inversely dependent on A_m.

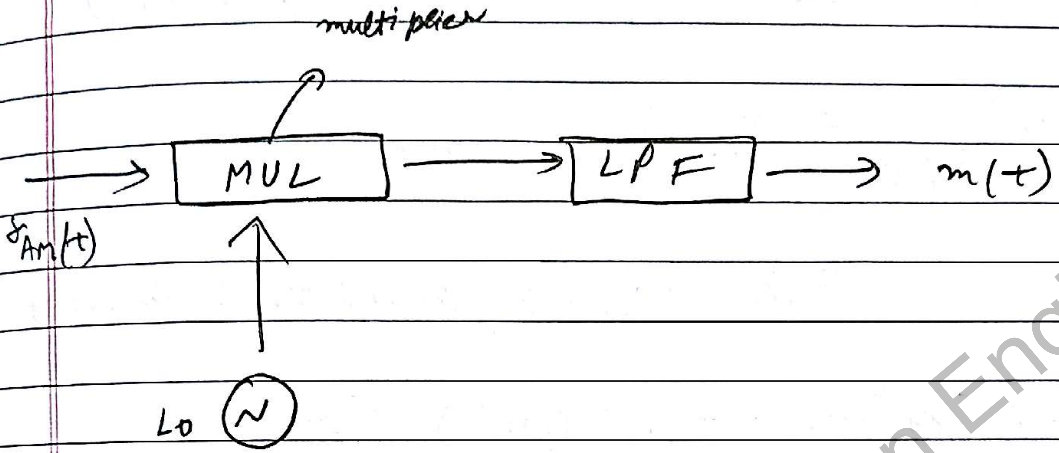
value of K_a is not dependent on A_c.
If, value of K_a is not given, then we take it equal to $K_a = 1/A_c$.

* Generally, if μ is high, quality of reconstructed message signal will be superior.

* For detection of message signal, the μ value required depends on the requirement of the type of modulation i.e. whether we need less power / more power required.

If μ value is high, then side band power will also be high, so reconstruction is better.

SYNCHRONOUS DETECTOR / COHERENT DETECTOR.



$$s_{AM}(t) = A_c\left(1+K_a m(t)\right)\cos 2\pi f_c t.$$

$$= A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t$$

* For proper reconstruction of message signal, local oscillator output should be properly synchronized in both frequency and phase with respect to transmitter carrier.

* Frequency synchronization can be easily maintained but to maintain phase synchronization additional circuitry is required, which makes synchronous detector complex.

classmate
Date
Page

Case I

Assume $(LO)_{output} = A_c \cos 2\pi f_c t$ (perfectly synchronised).

$(MUL)_{output} = s_{AM}(t) \; (LO)_{output}$

$= A_c(1+K_a m(t)) \cos 2\pi f_c t \cdot A_c \cos 2\pi f_c t.$

$= A_c^2 \cos^2 2\pi f_c t + A_c^2 K_a m(t) \cos^2 2\pi f_c t.$

$= \frac{A_c^2}{2}(1 + \cos 4\pi f_c t) + \frac{A_c^2}{2} K_a m(t) [1 + \cos 4\pi f_c t]$

this does not contain m(t)

these both will not pass.

passing this through low pass filter.

bcoz we need message signal.

$\Downarrow$

Output

DC signal so it will also not pass.

$\downarrow$

this does not contain message signal. $\longrightarrow$ so after adding additional circuitry, we will remove this signal

classmate
Date
Page

Output of low pass filter $= \cancel{\frac{A_c^2}{2}} + \frac{A_c^2}{2} k_a m(t)$.

$$(LPF)_{output} = \underbrace{\frac{A_c^2}{2} K_a}_{\text{Const. amplitude}} m(t).$$

Const. amplitude..

Now, final message can be recovered on passing it through the amplifier, which will divide this component by dividing this by $\frac{A_c^2}{2} k_a$ → using circuitry amplifier.

$\boxed{\text{Amplifier.}} \rightarrow m(t).$

Hence, final message signal is obtained.

Case 2

Assume

$$(LO)_{output} = A_c \cos(2\pi f_c t + \phi)$$ (No phase synchronisation)

$$(MUL)_{output} = S_{AM}(t) \; (LO)_{output}.$$

$$= A_c(1 + K_a m(t)) \cos 2\pi f_c t \cdot A_c \cos(2\pi f_c t + \phi)$$

$$= \frac{A_c^2}{2}\left[\cos(4\pi f_c t + \phi) + \cos\phi\right]$$

$$+ \frac{A_c^2}{2} K_a m(t) \left[\cos(4\pi f_c t + \phi) + \cos\phi\right].$$

output of low pass filter.

$$(LPF)_{output} = \frac{A_c^2}{2}\cos\phi + \frac{A_c^2}{2} K_a m(t) \cos\phi.$$

this DC term can also be removed.

(cond^n is ϕ should be const.)

$$(LPF)_{output} = \frac{A_c^2}{2} k_a \, m(t) \cos\phi .$$

ϕ must be constant.

ϕ usually varies with time, in practical.

To recover $m(t)$, ϕ should be constant. To maintain ϕ to be constant, additional circuitry is required, which makes synchronous detector complex.

If $\phi = 90^\circ$, then $(LPF)_{output} = 0$. $(\because \cos 90^\circ = 0)$

$\Downarrow$

and this condn should not be there.

this is called Quadrature Null Effect (QNE).

ADVANTAGES OF AM.

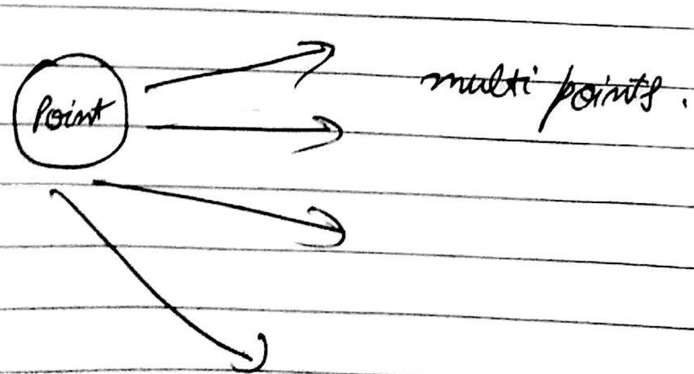
1. Demodulation is simple.
2. Used for long distance communication as compared to fm is used for short distance.

DISADVANGES OF AM.

1. Transmitted power is wasted
2. Demands for high channel bandwidth
3. AM transmission is highly noisy (FM is less noisy).
4. Affected by quadrature null effect (QNE).

APPLICATIONS

1. For broadcasting AM is preferred.



one sender &
many receivers (generally video signals)
e.g. news channels.

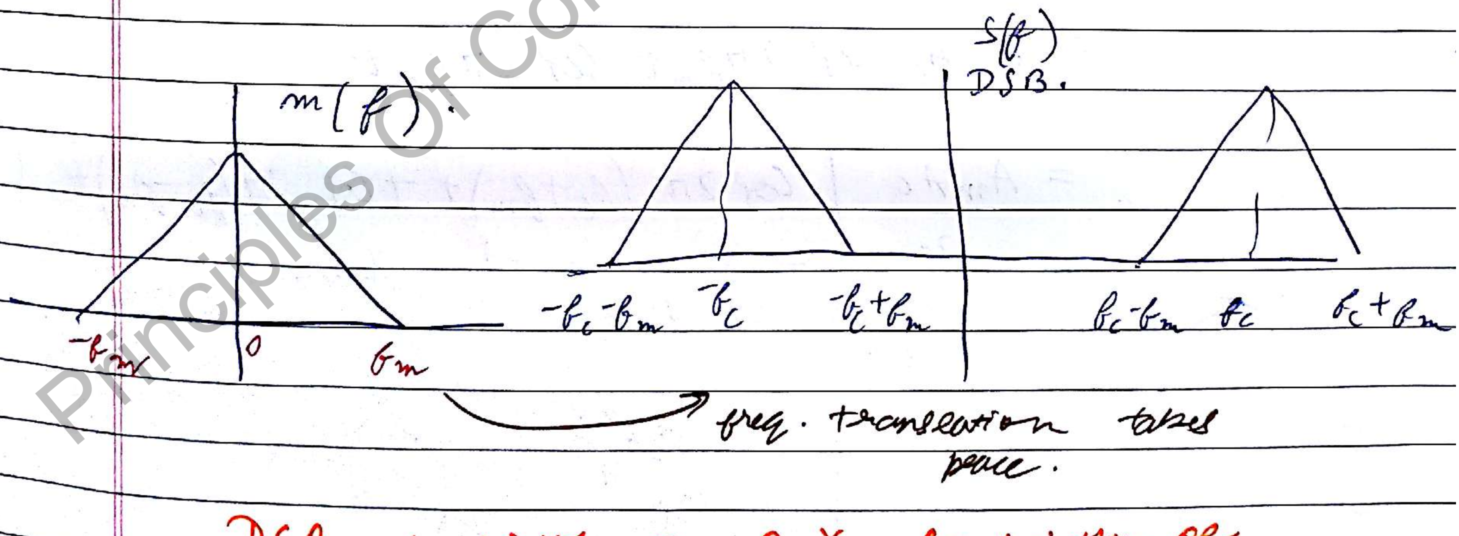
(2) AM receiver is simple and cheaper, so it is highly preferred for broadcasting.

DSB-SC (DOUBLE SIDE BAND - SUPRESSED CARRIER).

General equation of DSB-SC ⇒

$$s_{DSB}(t) = m(t) \cdot c(t)$$

$$= A_c\, m(t) \cos 2\pi f_c t.$$



DSB bandwidth = 2 X Bandwidth of message.

$= 2f_m$.

freq. is same,

but, here advantage is carrier is not present at that point.

So, carrier is not ~~to~~ be sent alongwith, and hence, power requirement is less.

SINGLE TONE DSB.

$$m(t) = A_m \cos 2\pi f_m t$$ → message sig. has only single freq.

$$\underset{DSB}{s(t)} = m(t)\,c(t)$$

$$= A_m A_c \cos 2\pi f_m t \cos 2\pi f_c t$$

$$= \frac{A_m A_c}{2}\Big[\underbrace{\cos 2\pi (f_c+f_m)t}_{USB} + \underbrace{\cos 2\pi (f_c-f_m)t}_{LSB}\Big]$$

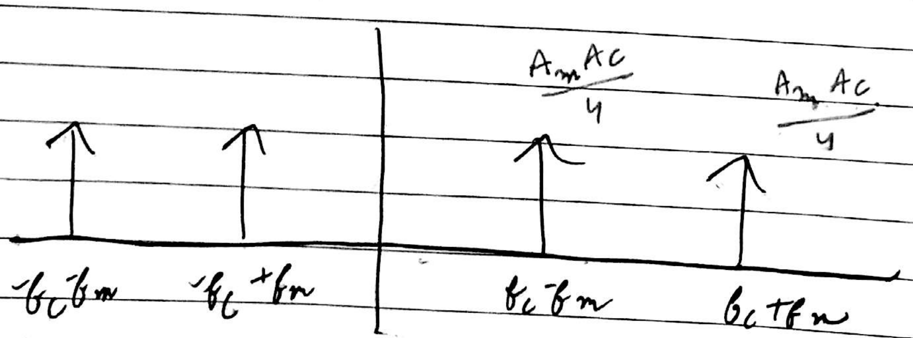
upper side band ← USB

LSB → lower side band

Mag. spectra

↓

we will take fourier transform



→ and fourier transf. of $\cos f_m t$ will be ↑ | ↑.

This can also be called area,
since impulse has prop.
i.e. $\int_{-\infty}^{\infty} \delta(t)\,dt = 1.$

Now, for calculating power, it will be $\frac{(\text{amp.})^2}{2}$.

$P_t = P_{USB} + P_{LSB}$

$= \frac{A_m^2 A_c^2}{8} + \frac{A_m^2 A_c^2}{8} = \frac{A_m^2 A_c^2}{4}.$

If resistance R is taken, then,

\# $\boxed{P_t = \frac{A_m^2 A_c^2}{4R}}$

MULTI - TONE DSB. Same as in case of multi tone AM.

Q) A carrier signal of $c(t) = 20\cos(2\pi \times 10^6 t)$ is DSB modulated by $m(t) = \cos(2\pi \times 10^4 t) + 2\cos(4\pi \times 10^4 t) + 4\cos(6\pi \times 10^4) t$.

i) Find bandwidth, total power & efficiency.

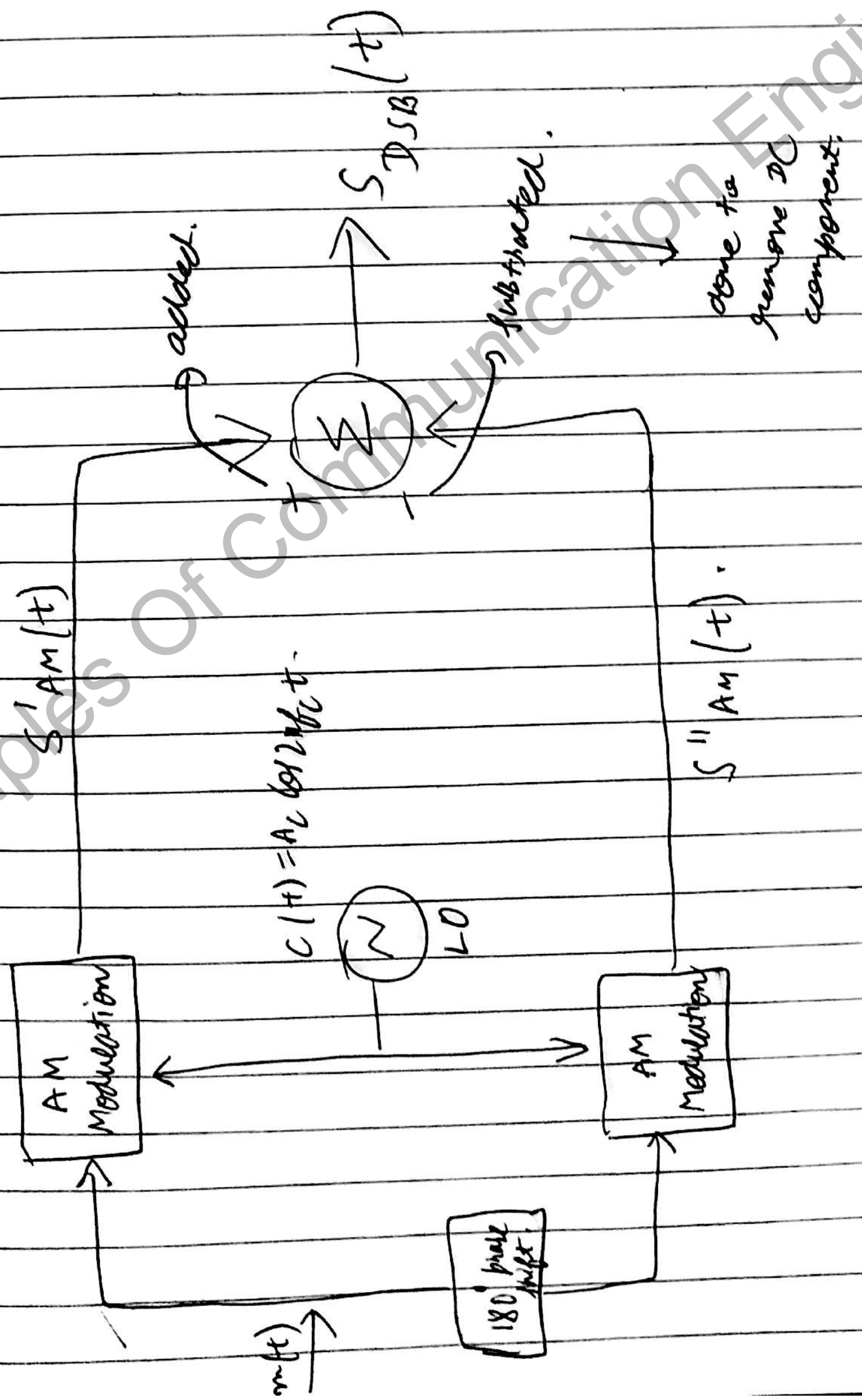
$2 \times$ max freq.

$2 \times 3 = 6.$

$\frac{\text{side band power}}{\text{total power}}$

GENERATION OF DSB

Balanced   Ring.



$$\left(\text{Summer}\right)_{\text{output}} = S'_{AM}(t) - S''_{AM}(t)$$

$$S'_{AM}(t) = A_c\left(1 + k_a m(t)\right)\cos 2\pi f_c t$$

$$S''_{AM}(t) = A_c\left(1 - k_a m(t)\right)\cos 2\pi f_c t.$$

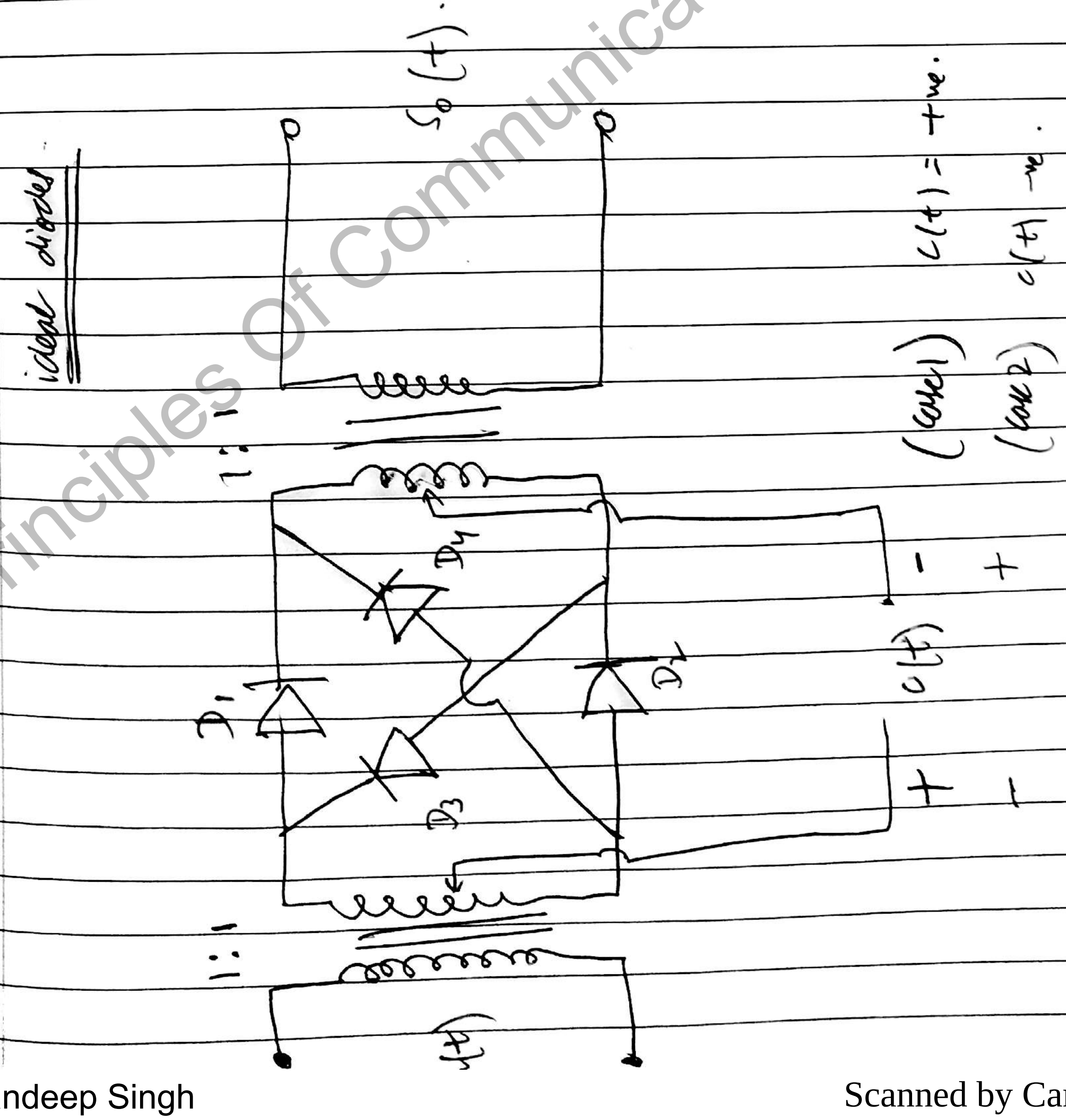
$$S_{DSB} = 2A_c k_a\, m(t)\cos 2\pi f_c t.$$

$$\boxed{S_{DSB} = A_c'\, m(t)\cos 2\pi f_c t.}$$

$A_c' = 2A_c k_a$

Hilbert transf. gives -90° phase shift to any message sig. → so 180° phase shift means Hilbert transf. cascaded 2 times.

RING MODULATOR



In this type of modulator, 4 diodes are connected in the form of a ring to generate DSB signal.

(1) For $c(t) = +ve$.

D_1 & D_2 — work (short circuit)

D_3 & D_4 — not working (open circuit)

$$S_0(t) = +m(t)$$

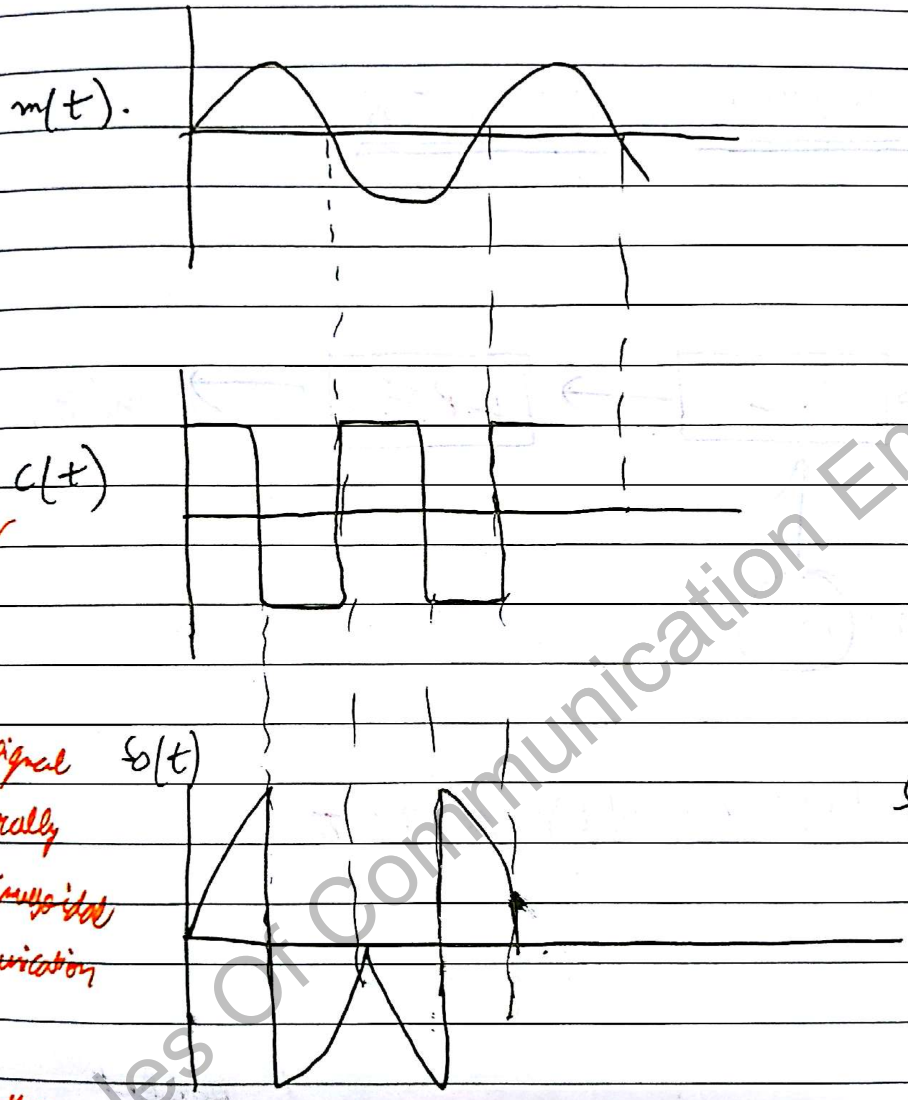
(2) For $c(t) = -ve$.

D_1 & D_2 — off (open circuit).

D_3 & D_4 — on (short circuit).

$$S_0(t) = -m(t).$$

$$S_0(t) = m(t) \cdot c(t).$$



carrier signal is generally taken sinusoidal in communication ⇓ to make the analysis easier.

$$S_0(t) = m(t) \cdot c(t).$$

NOTE:

Transformers are bulky and expensive, so balanced modulator is preferred.

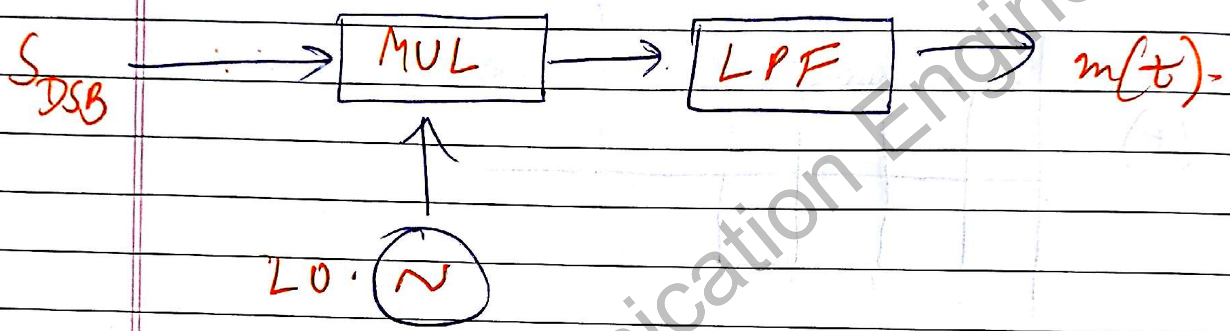
There are even more losses in tranformers.

Product modulator is the other name given to DSB modulator.

DEMODULATION OF DSB.

Only one method :-

SYNCHRONOUS DETECTOR.



$$S_{DSB}(t) = A_c\, m(t) \cos 2\pi f_c t$$

① Assume

$$(LO)_{output} = A_c \cos 2\pi f_c t$$ (perfect synchronisation).

$$(MUL)_{output} = A_c^2\, m(t) \cos^2(2\pi f_c t)$$

$$= \frac{A_c^2\, m(t)}{2} \left(1 + \cos 4\pi f_c t\right)$$

$$(LPF)_{output} = \frac{A_c^2}{2} m(t)$$

$\downarrow$

$\boxed{\text{Amplifier}}$

$\downarrow$

$m(t)$. $\longrightarrow$ message signal is obtained.

(2)

$$(LO)_{output} = A_c \cos(2\pi f_c t + \phi).$$

$$(MUL)_{output} = A_c^2 m(t) \cos 2\pi f_c t \cos(2\pi f_c t + \phi).$$

$$(LPF)_{output} = \frac{A_c^2}{2} m(t) \cos\phi.$$

If $\phi = 90^\circ$, $(LPF)_{output} = 0$,

$\downarrow$

this is called quadrature null effect (QNE).

To recover m(t), ϕ must be constant and to maintain ϕ to be constant additional circuitry is required, which makes synchronous detector complex.

ADVANTAGES OF DSB.

1. Power will be saved
2. Used for long distance communication.

DRAWBACKS OF DSB.

1. Complexity in demodulation.
2. Demand for high channel bandwidth.
3. Affected by QNE (quadrature null effect).

APPLICATIONS OF DSB.

1. DSB-SC is used in quadrature carrier multiplexing.

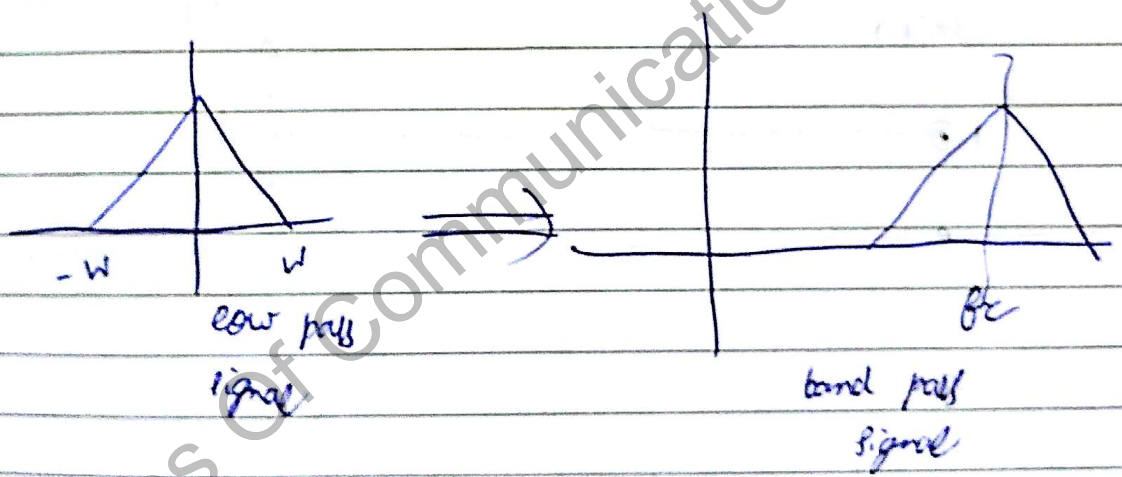
→ more than one message signal is passed through channel at a single time.

HILBERT TRANSFORMATION.

used in generation of single side band.

It takes a time domain signal to other time domain signal.

It is very useful transform for representation of band pass signal.

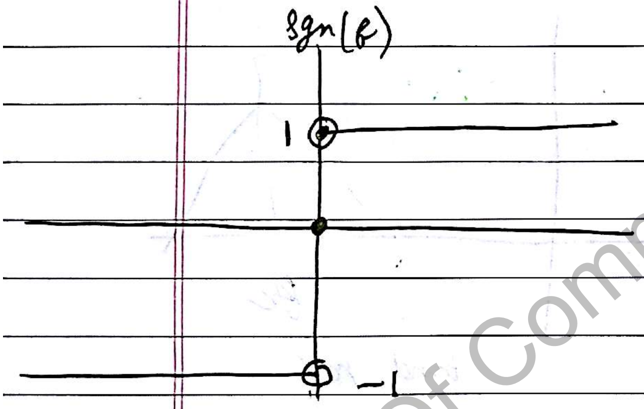


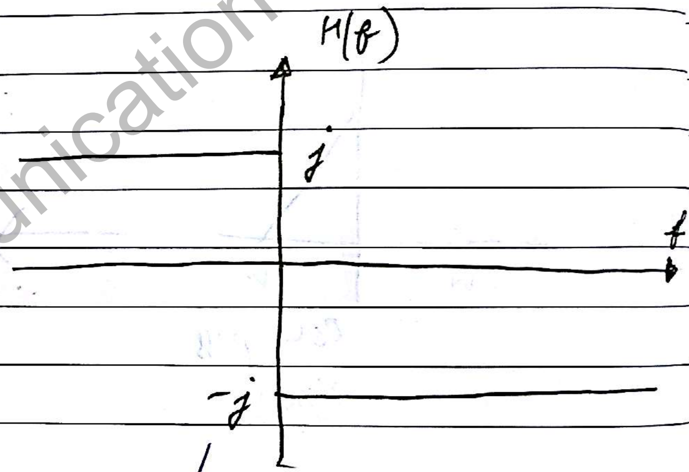
It is also useful for representation of single side band (SSB) signal.

Let a linear filter (LTI system) whose transfer function $H(f)$,

$$H(f) = -j \, sgn(f).$$

$$sgn(f) = 1 \quad (f > 0)$$
$$= 0 \quad (f = 0)$$
$$= -1 \quad (f < 0).$$





imaginary term $\quad j = \sqrt{-1}$

$$j = e^{j\pi/2}$$

$$|j| = |e^{j\pi/2}| = 1 \angle \pi/2$$

amplitude

$$H(f) = -j \, sgn(f).$$

amplitude is 1
angle is $\angle \pi/2$

$$H(f) = -j\, sgn(f).$$

On multiplying any signal with $H(f)$ we are doing phase shift of $-\frac{\pi}{2}$.

$$-j = |e^{-j\pi/2}| = 1 \angle -\pi/2.$$

there is phase shift of any signal by an angle of $-\pi/2$, on doing hilbert transform.

So, on hilbert transform phase of signal decreases by $\pi/2$.

$$H(f) = -j\, sgn(f).$$

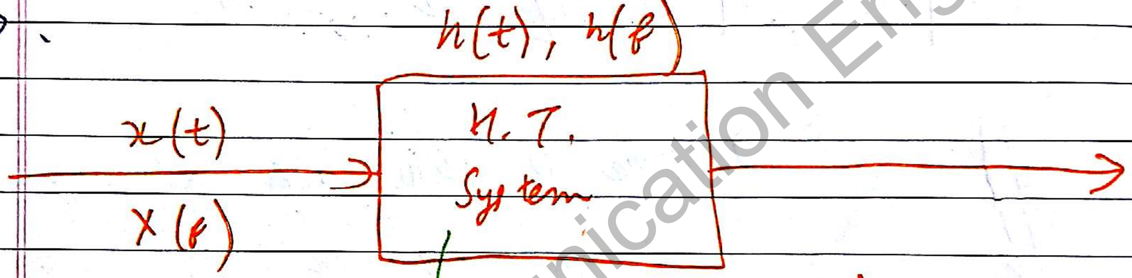
Inverse Fourier Transform.

fourier transform

$$\boxed{\frac{j}{\pi t} \xleftrightarrow{F.T.} sgn(f)}$$

Multiplication in 't' domain $\longleftrightarrow$ Convolution in 'f' domain

$\Rightarrow$



$y(t) = h(t) * x(t)$

$Y(f) = X(f) . H(f)$.

System whole transfer fn t is $-j\, sgn(f)$.

Convolution of 2 signals is output.

$$\boxed{y(t) = x(t) * \frac{1}{\pi t}.}$$

$$\frac{j}{\pi t} \longleftrightarrow sgn(f)$$

$$\boxed{\frac{1}{\pi t} \longleftrightarrow -j\, sgn(f)}$$

So, $y(t) = x(t) * \frac{1}{\pi t}$.

PROPERTIES OF HILBERT TRANSFORM.

1. Hilbert transform is denoted by ^ (cap)

$$x(t) \xrightarrow[\text{Transform}]{\text{Hilbert}} \hat{x}(t).$$

1. $\hat{\hat{x}}(t) = -x(t)$.

double hilbert transform of signal.

$-180°$ phase shift in output.

(2 times phase shift of $-\frac{\pi}{2}$)

2. Relation Between Energy of $x(t)$ and $\hat{x}(t)$.

$$\hat{X}(f) = \underbrace{(-j\,\mathrm{sgn}(f))}_{\text{transfer funct.}}\,X(f)$$

$X(f)$ → fourier of $x(t)$.

mag. of this $= 1$.

taking modulus →

$$|\hat{X}(f)| = |X(f)|$$

⇓

taking energy.

$$\int x^2(t)\,dt = \int |X(f)|^2\,df \quad \text{(Parseval's Theorem)}$$

energy of $x(t)$ and hilbert transform of $x(t)$ is same.

3. $x(t)$ and $\hat{x}(t)$ are ORTHOGONAL.

angle b/w both is of $\pi/2$.

time on taking hilbert transform there is phase shift of 90°.

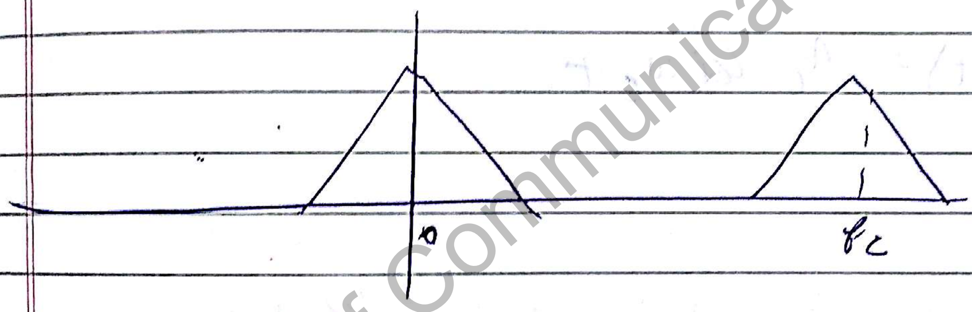
$$\boxed{\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = 0}$$

4. $c(t)$ and $m(t)$ have non overlapping spectra.

$m(t) \rightarrow$ low pass (contains low freq. component)

$c(t) \rightarrow$ high pass (contains high freq component).

$$m(t)\, c(t) = m(t)\, \hat{c}(t).$$



low pass signal | high pass signal.

$\Downarrow$

so that spectra of both does not overlap.

hilbert transform.

$$\widehat{m(t)\,c(t)} = m(t)\,\hat{c}(t)$$

complete signal.

agar dono non overlapping hain toh carrier ka hilbert transform hoga.

this is there if m(t) & c(t) does not OVERLAP.

$$m(t) = A_m \cos \omega_m(t)$$

$$c(t) = A_c \cos \omega_c t$$

$$\widehat{m(t)\,c(t)} = \widehat{A_c A_m \cos \omega_m t \cos \omega_c t}$$

$$= A_m \cos \omega_m(t)\; A_c \cos\left(\omega_c t - \frac{\pi}{2}\right)$$

$$= \left(A_m \cos \omega_m(t)\right)\left(A_c \sin \omega_c t\right)$$

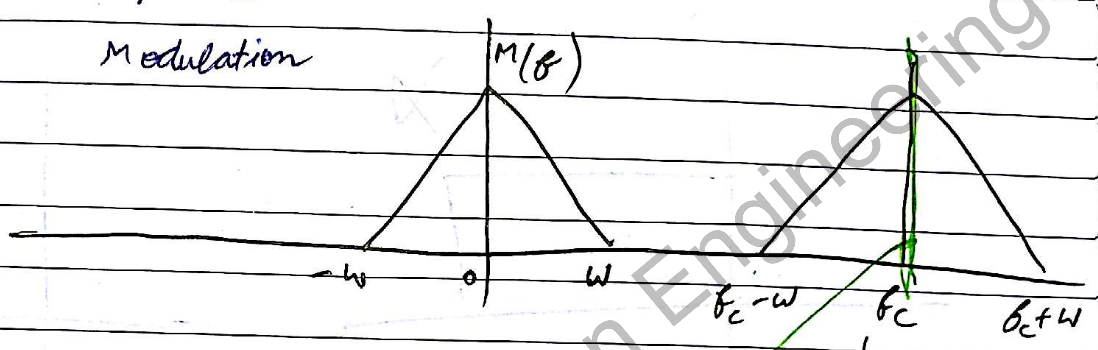
Hilbert transform of complete wave.

SSB - SC

(Single Side Band - Supressed carrier)

Amplitude

Modulation



Spectra of modulated signal. (AM or DSB).

~~both portions~~ are symmetrical about this point

so both have same energy.

So, with SSB - SC we send only carrier of one side and half of the power is saved.

Power in AM or DSB

$\frac{\mu^2}{2} P_c$

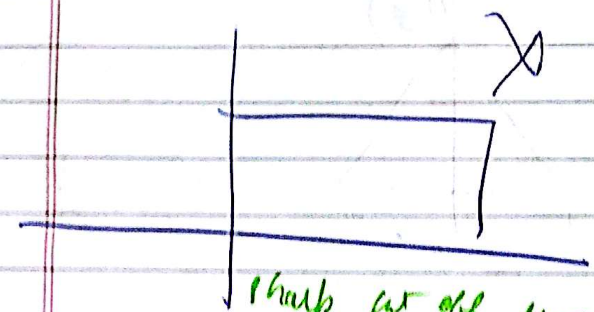
Now, in SSB - SC

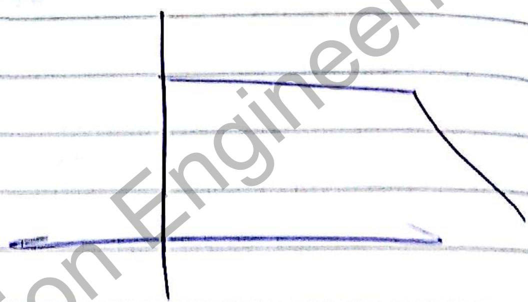
$\frac{\mu^2}{4} P_c$

half of the

Problem is that practical filters do not have very sharp cut off frequency.

So, we are not able to achieve the desired result.

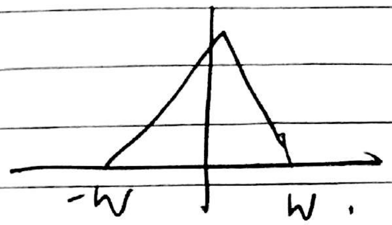




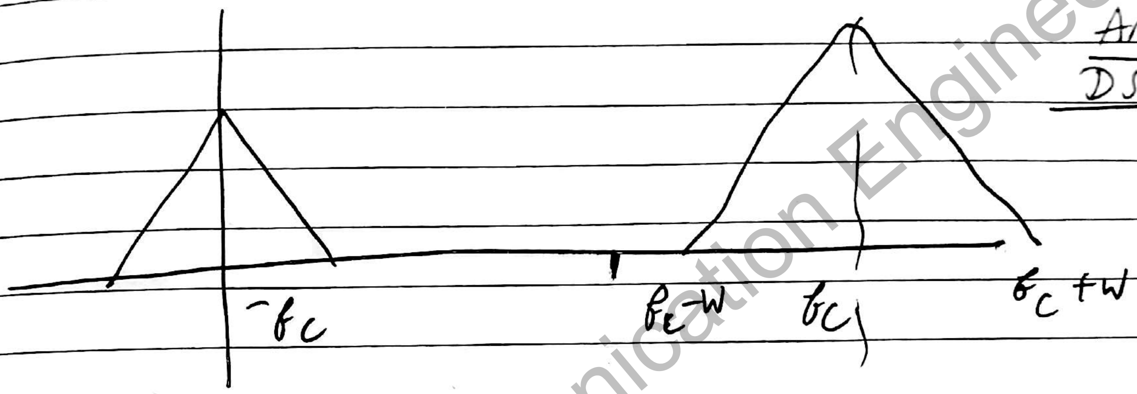
So, we use SSB-SC only for VOICE TRANSMISSION.

advantage of SSB SC is that we can reduce the power through it.

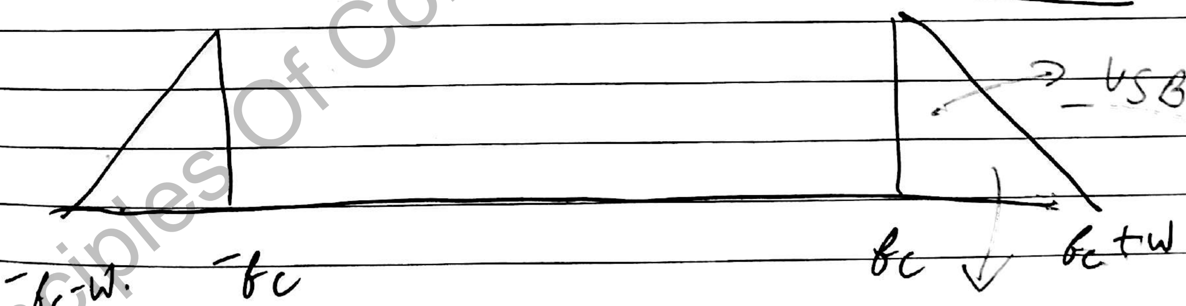
$m(t) \longleftrightarrow m(f)$.



AM or DSB.



SSB.

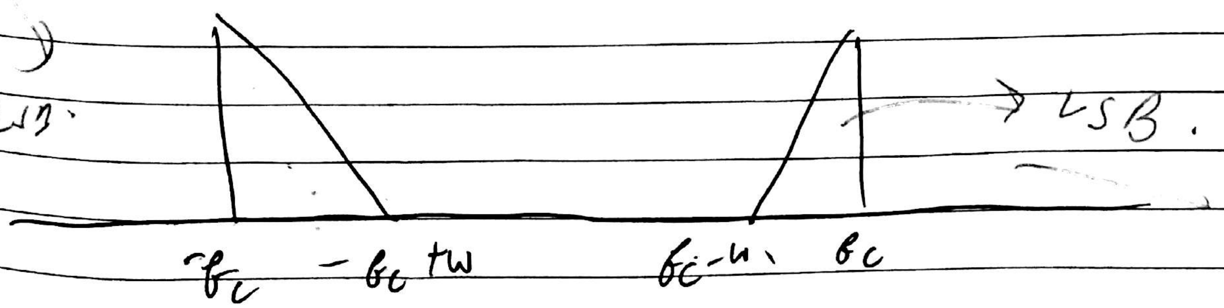


USB.

2 cases are possible in SSB

USB LSB

Only half of the part is sent in SSB.



LSB.

classmate
Date
Page

Bandwidth of the channel is reduced as using SSB.

NOTE: ① 50% of power will be saved in SSB compared to DSB

② 50% of channel bandwidth is saved in SSB compared to AM & DSB.

For SSB detection, coherent detection method is used.

Coherent detector firstly creates the required side band because of symmetry property, then it recover original message.

SINGLE TONE SSB.

Assume

$$m(t) = A_m \cos \omega_m (t).$$

$$s_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} \cos 2\pi (f_c + f_m) t$$

general eq^n of AM.

$$+ \frac{A_c \mu}{2} \cos 2\pi (f_c - f_m) t.$$

Bandwidth : $2f_m$.

general eq^n of DSB.

$$s_{DSB}(t) = \frac{A_c A_m}{2} \cos 2\pi (f_c + f_m) t + \frac{A_c A_m}{2} \cos 2\pi (f_c - f_m) t.$$

Bandwidth : $2f_m$

$$\boxed{s_{SSB}(t) = \frac{A_c A_m}{2} \cos 2\pi (f_c \pm f_m) t.}$$

$+ \Rightarrow$ USB

$- \Rightarrow$ LSB.

dono me se ek hoga ya USB ya LSB.

highest freq. bhi yehi hai
lowest freq. bhi yehi hai.
toh bandwidth = 0

Bandwith of SSB channel is 0.

Yeh band of freq. represent karta hai.

⇓

Iske bandwidth = higher freq − lower freq.

Yeh single freq. hai

⇓

Tone channel ki bandwidth = 0.

Total Power of Multi Tone

$$P_t = P_{LSB} = P_{USB} = \frac{\mu^2}{4} P_c$$

$$= \frac{\mu^2}{4} \times \frac{A_c^2 A_m^2}{2}$$

$$= \frac{\mu^2 A_c^2 A_m^2}{8}$$

$$\boxed{P_t = \frac{A_c^2 A_m^2}{8}}$$

$$\boxed{\eta = \frac{P_{SB}}{P_t} = 1}$$

classmate
Date
Page

GENERAL EXPRESSION FOR SSB-SC.

$$S_{SSB}(t) = \frac{A_c A_m}{2} \cos 2\pi (f_c \pm f_m) t.$$

$$= \frac{A_c A_m}{2} \cos 2\pi f_c t \cos 2\pi f_m t \mp \frac{A_c A_m}{2} \sin 2\pi f_c t \sin 2\pi f_m t.$$

in dono terms ka hilbert transform hoga.

bcoz

$$\widehat{m(t)\, c(t)} = m(t)\, \hat{c}(t).$$

$S_{SSB}(t)$

$$m(t) \xrightarrow[\text{phase shift}]{90^\circ} A_m \sin 2\pi f_m t.$$

phase diff. of 90°.

$$S_{SSB}(t) = \frac{A_c A_m}{2} \cos 2\pi f_c t \cos 2\pi f_m t + \frac{A_c A_m}{2} \sin 2\pi f_c t \sin 2\pi f_m t$$

in phase component of carrier.

phase diff. of 90°.

quadrature phase component of carrier.

$$\text{B.P.S.} = x_I \cos w_c t + x_q \sin w_c t.$$

band pass signal

phase diff. of 90°.

$$x(t) \cos w_c t \pm \hat{x}(t) \sin w_c t.$$

in phase component

quadrature component.

Since hilbert transform is at an angle of 90°.

(quadrature component can be represented in form of hilbert transform) we have $m(t) = A_m \cos 2\pi f_m t$

90° phase shift of $m(t) \Rightarrow A_m \sin 2\pi f_m(t) = \hat{m}(t)$.

$\hat{m}(t) \longrightarrow$ Hilbert transform of signal $m(t)$.

Modulated signals are band pass signals.

USB

$$S_{SSB}(t) = \frac{A_c\, m(t)}{2} \cos 2\pi f_c t \mp \frac{A_c\, \hat{m}(t)}{2} \sin 2\pi f_c t.$$

LSB.

$+ \rightarrow$ LSB

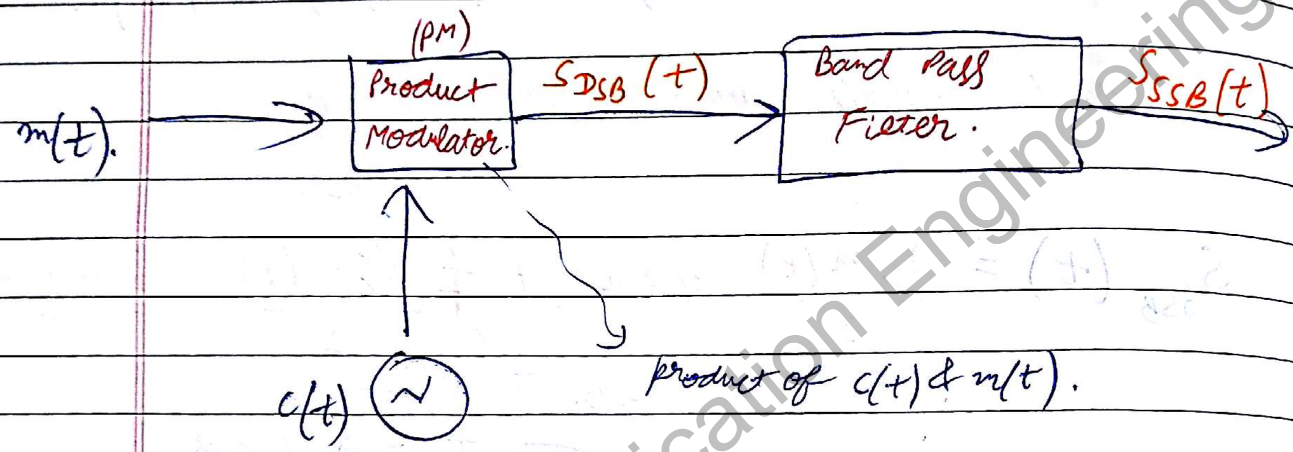
$- \rightarrow$ USB.

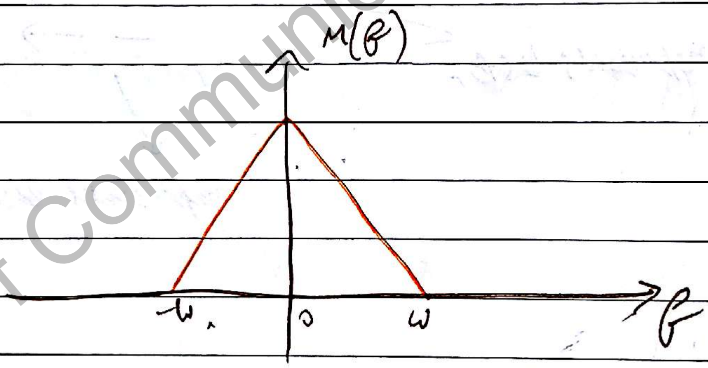
represents LSB.

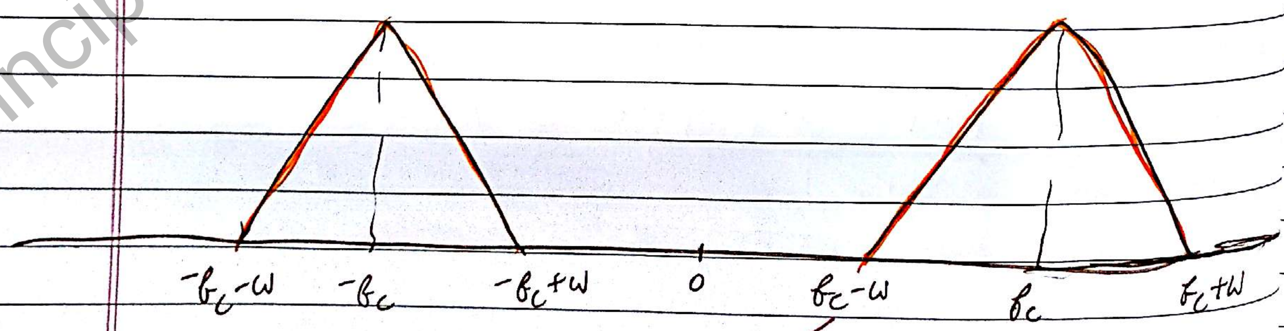
represents USB.

GENERATION OF SSB.

1) FREQUENCY DISCRIMINATION / FILTER METHOD.

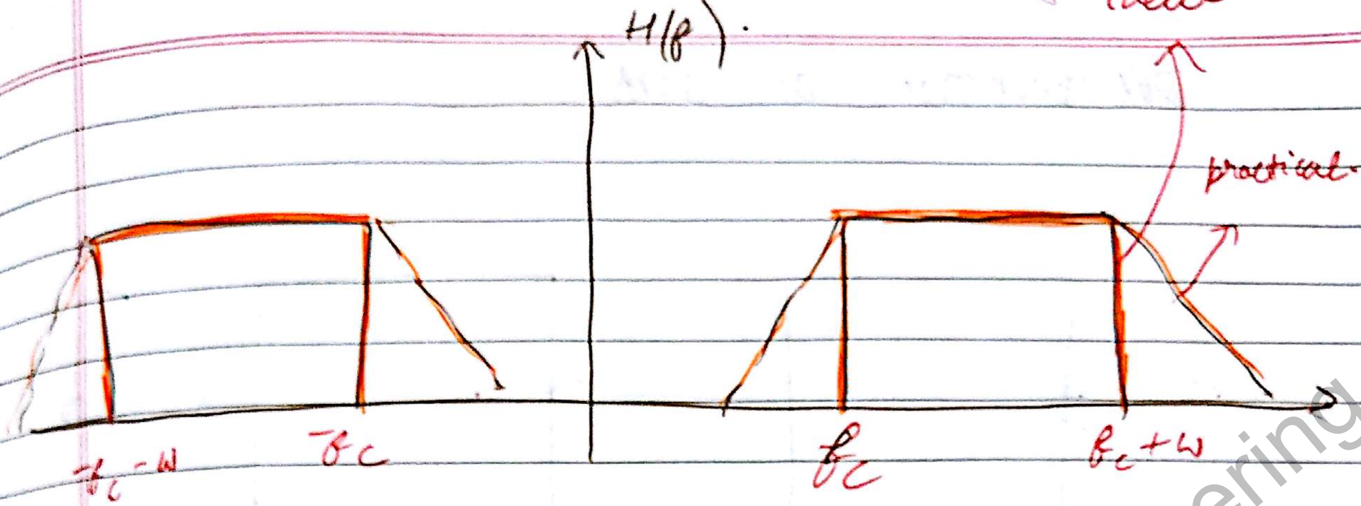


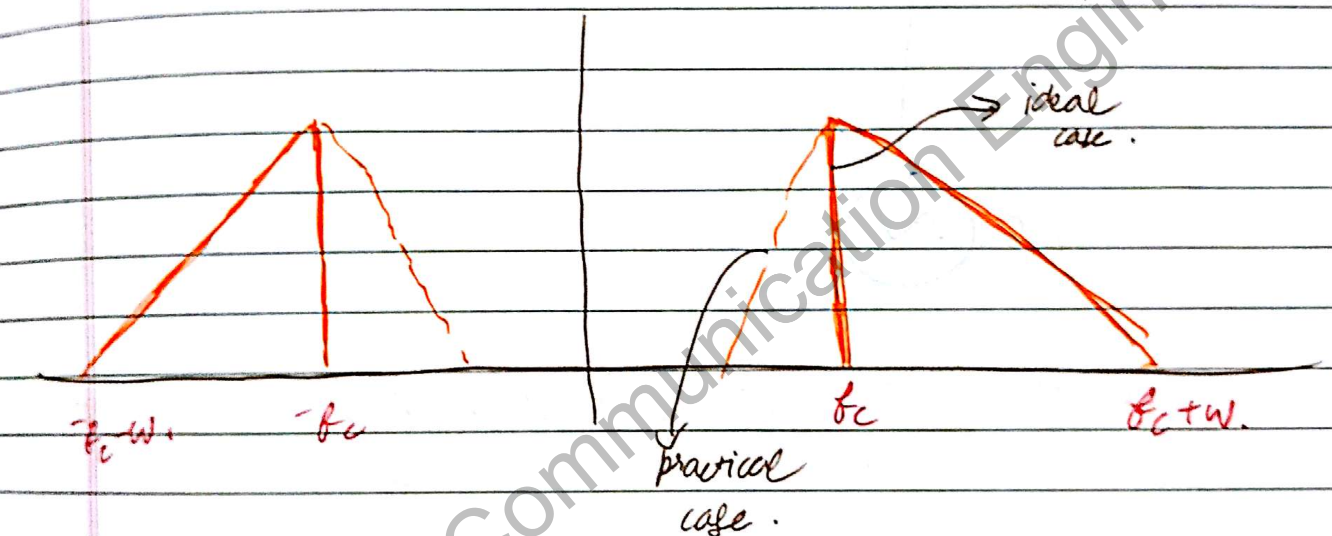




passed through filter whose cut off freq. > f_c.

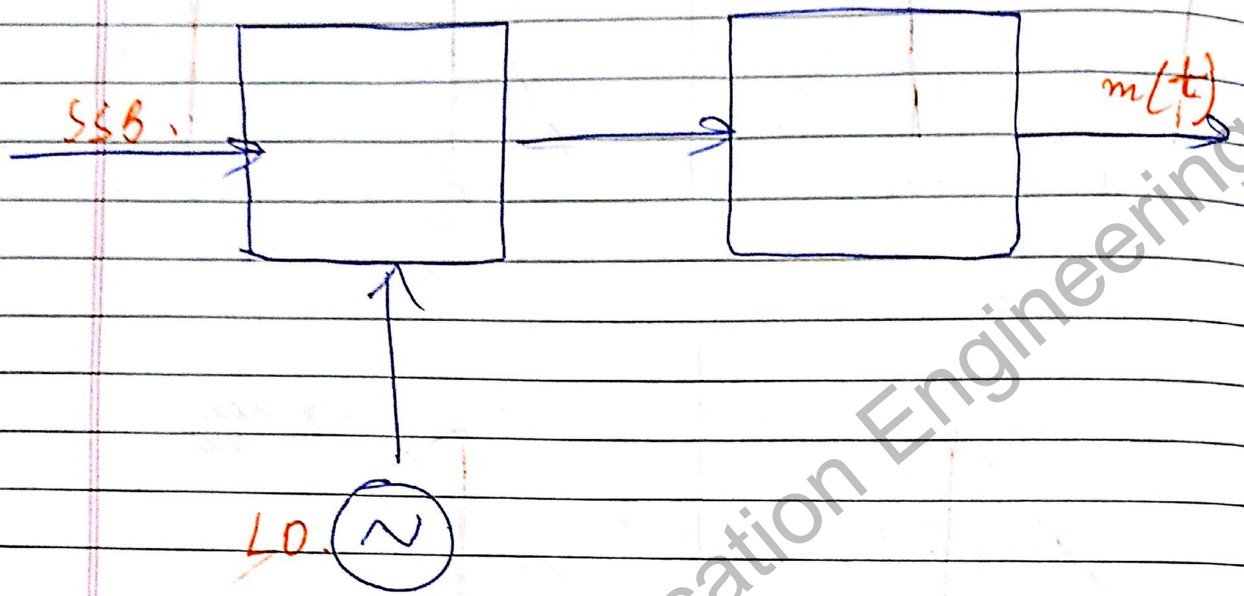
this is a DSB signal (double side band)





Since bandpass filter is not ideal in the resulting SSB alongwith complete side band unwanted frequency will also be present, because of the above drawback SSB is limited to only for VOICE TRANSMISSION.

DEMODULATION OF SSB.



$$s_{SSB}(t) = \frac{A_c m(t)}{2} \cos 2\pi f_c t \mp \frac{A_c \hat{m}(t)}{2} \sin 2\pi f_c t.$$

1) Assume $(LO)_{output} = A_c \cos 2\pi f_c t$. (perfect synchronization)

$$(MUL)_{o/p} = \frac{A_c^2}{2} m(t) \cos^2 2\pi f_c t \mp \frac{A_c^2 \hat{m}(t)}{2} \sin 2\pi f_c t \cos 2\pi f_c t$$

multiplying & dividing by 2.

~~Low pass component~~ will come here (which will be const. term)

factor of $\sin 4\pi f_c t$ will come.

so, it will be high pass signal

$$(LPF)_{output} = \frac{A_c^2 m(t)}{4} \longrightarrow \boxed{\text{Amplifier}} \longrightarrow m(t).$$

2) Assume $(LO)_{output} = A_c \cos(2\pi f_c t + \phi)$.

(no phase synchronization).

$$(MUL)_{output} = s_{SSB}(t) \, (LO)_{output}$$

$$= \left(\frac{A_c m(t)}{2} \cos 2\pi f_c t \mp \frac{A_c \hat{m}(t)}{2} \sin 2\pi f_c t \right) A_c \cos(2\pi f_c t + \phi)$$

$$= \frac{A_c^2}{4} m(t) \left[\cos(4\pi f_c t + \phi) + \cos\phi \right] \mp \frac{A_c^2 \hat{m}(t)}{4} \left[\sin(4\pi f_c t + \phi) - \sin\phi \right].$$

both these are low pass signals.

$$(LPF)_{output} = \frac{A_c^2 m(t)}{4} \cos\phi \mp \frac{A_c^2 \hat{m}(t)}{4} \sin\phi$$

If $\phi = 0$,

$$O/P = \frac{A_c^2 m(t)}{4}$$

If $\phi = 90°$,

$$O/P = \frac{\mp A_c^2 \hat{m}(t)}{4} \neq 0$$

here, there is a phase shift of 90°.

(no quadrature null effect) -

ADVANTAGES:

1) Power is saved
2) Channel bandwidth is saved
3) No quadrature null effect

DISADVANTAGES-

1) Demodulation is complex
2) Limited for voice transmission

APPLICATIONS.

For voice transmission, SSB is preferred (as bandwidth & power both are saved).

In the past, for landline telephonic conversation, SSB was used.

Nowdays, SSB is replaced by PCM.
↓
pulse code modulation.

$A_c \cos 2\pi f_c t$
↓
$A_c [\, 1 + K_a m(t)$ — A_m

$m(t)\, c(t) \longrightarrow$ DSB

→ Band Pass filter $\longrightarrow$ SSB

ANGLE MODULATION.

phase of carrier signal will change acc. to message signal.

Assume carrier signal, $= A_c \cos(2\pi f_c t + \phi)$

$= A_c \cos \theta$.

where $\theta(t) = 2\pi f_c t + \phi$.

If total angle i.e. $\theta(t)$ of carrier signal varies acc. to message signal amplitude variation,

it is called as ANGLE MODULATION.

If angle modulation occurs due to dependence of f_c (carrier frequency) on $m(t)$, then it is called as FREQUENCY MODULATION.

If angle modulation occurs due to dependence of phase ϕ on $m(t)$, then it is called as PHASE MODULATION.

$\frac{d\theta}{dt} = f_c \longrightarrow$ frequency.

if this freq. varies with message signal amplitude.

$\Downarrow$

then it is freq. modulation.

ϕ

$\downarrow$

if the phase varies with the amplitude of message signal.

$\Downarrow$

then it is amplitude modulation.

1) PHASE MODULATION.

Assume carrier signal before modulation,

$$c(t) = A_c \cos 2\pi f_c t.$$

Carrier signal after modulation,

$$S_{PM}(t) = A_c \cos\left(2\pi f_c t + \phi(t)\right).$$

$$\phi(t) = K_p\, m(t).$$

$K_p \longrightarrow$ phase sensitivity of phase modulation.

K_p specifies amount of phase shift in the carrier signal, for 1 V change in message signal.

2) FREQUENCY MODULATION.

Assume frequency of carrier signal before modulation, f_c

Frequency of carrier signal after modulation, f_i

$f_i \rightarrow$ instantaneous frequency.

$$\boxed{f_i = f_c + k_f\, m(t)}$$

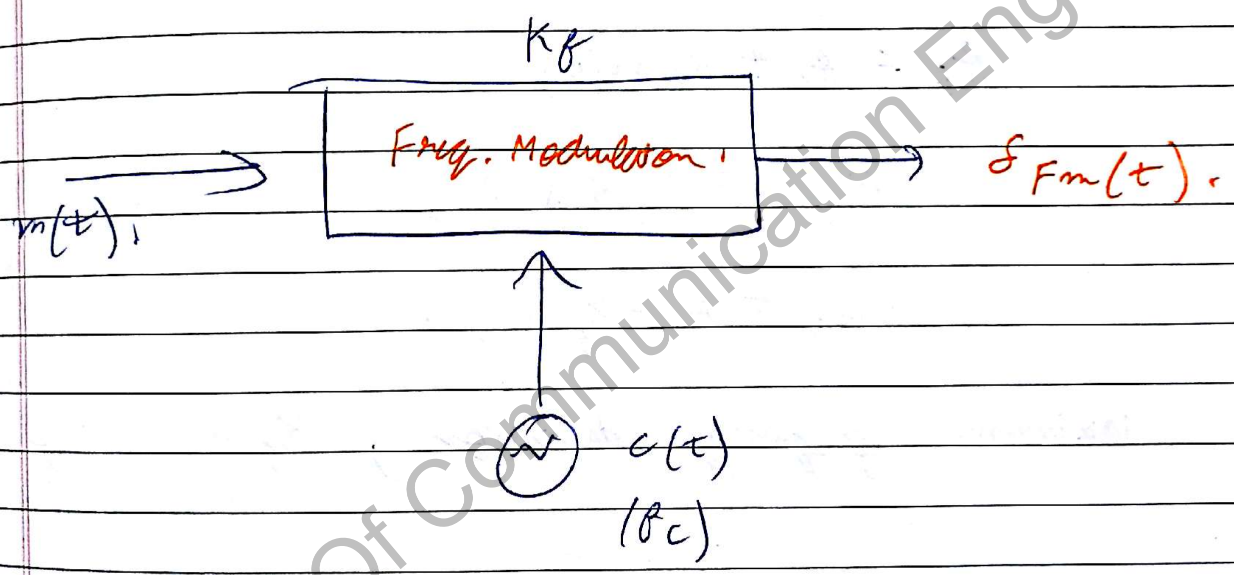
$k_f \rightarrow$ frequency sensitivity of frequency modulator.

k_f specifies amount of frequency change in carrier signal for 1 V change in message signal.

VCO → Voltage control Oscillator.

Input — amplitude / voltage.
→ used to generate freq.

In frequency modulation, message signal voltage variations will be converted as corresponding carrier signal frequency variations, so it is called as Voltage to Frequency Converter.



$$f_i = f_c + K_f \underbrace{m(t)}_{\text{message signal (sinusoidal)}}$$

message signal.
(sinusoidal).

$f_{min} = f_c - K_f \left|m(t)\right|_{max}$ $\quad$ $f_{max} = f_c - K_f \left|m(t)\right|_{min}$.

If $m(t) = 0$ $\quad f_i = f_c$

$m(t) = +ve$ $\quad f_i > f_c$

$m(t) = -ve$ $\quad f_i < f_c$.

Let,

$m(t) = A_m \cos 2\pi f_m t$ or $A_m \sin 2\pi f_m t$.

$$f_i = f_c + K_f\, m(t).$$

$$f_{max} = f_c + K_f A_m$$

$$f_{min} = f_c - K_f A_m.$$

Maximum frequency deviation, $K_f A_m = \Delta f$.

$$f_{max} = f_c + \Delta f$$

$$f_{min} = f_c - \Delta f.$$

Total frequency swing $= f_{max} - f_{min} = 2\Delta f$.

Q) For an FM signal having minimum freq. of 1 MHz, corresponding freq. swing is 1200 kHz. Find Δf, f_c and f_{max}.

$f_{min} = \cancel{100}\ 1\ MHz = 1000\ kHz$.

freq. swing $= 2\Delta f = 1200\ kHz$

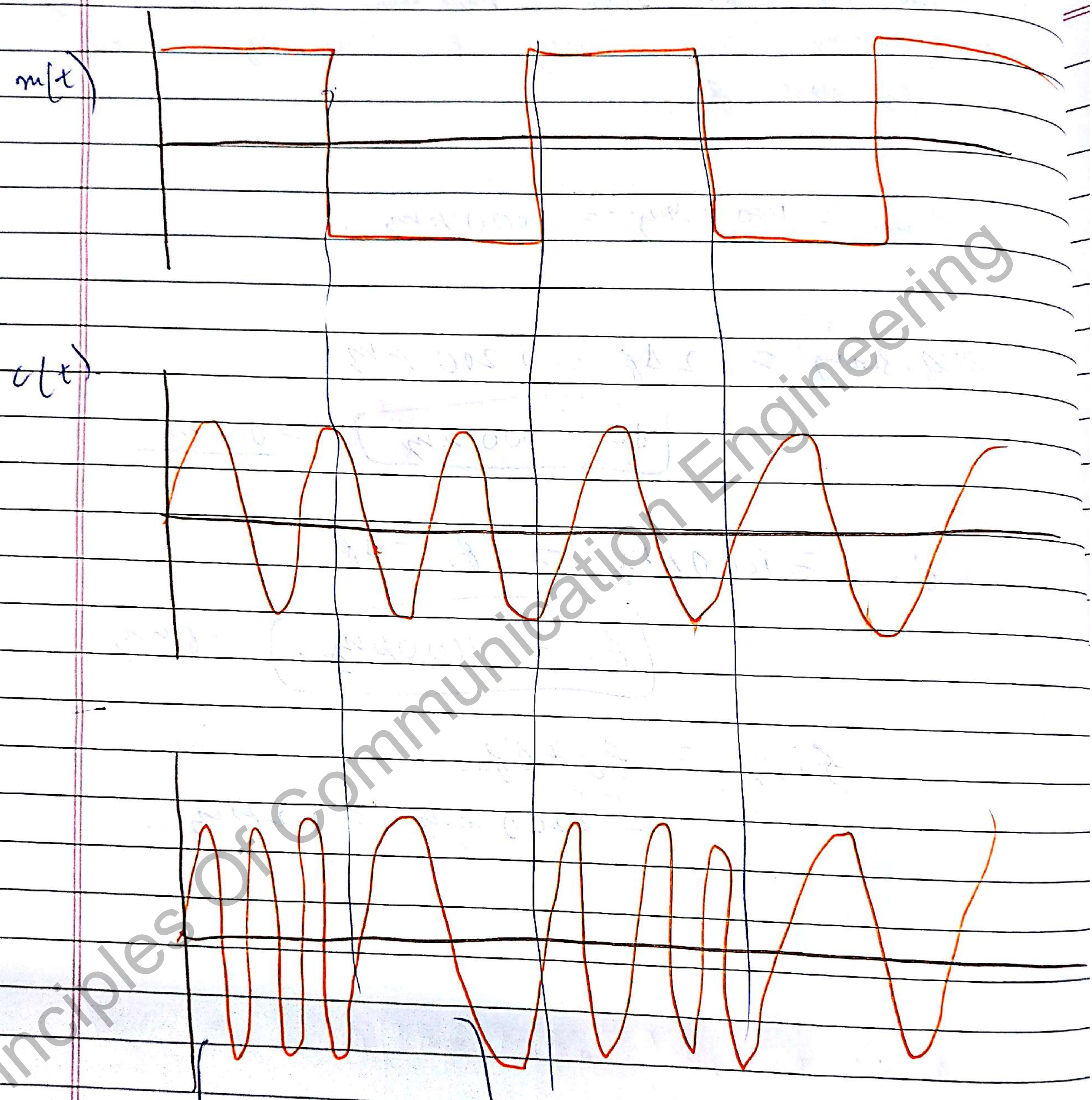
$\boxed{\Delta f = 600\ kHz} = \underline{0.6\ MHz}$.

$f_{min} = 1000\ kHz = f_c - \Delta f$

$\boxed{f_c = 1600\ kHz.} = 1.6\ MHz$.

$f_{max} = f_c + \Delta f$

$= 2200\ kHz = 2.2\ MHz$.



freq. is higher where m(t) +ve.

freq. is lower where m(t) is -ve.

GENERAL EXPRESSION OF FM SIGNAL.

Assume, carrier signal before modulation,

$$c(t) = A_c \cos(2\pi f_c t + \phi)$$

where $\theta(t) = 2\pi f_c t + \phi$.

$$c(t) = A_c \cos(\theta(t))$$

$$\left(\frac{d\theta(t)}{dt} = 2\pi f_c\right).$$

General expression of FM signal,

$$s_{FM}(t) = A_c \cos\theta_i(t). \quad \text{--- (i).}$$

$$\frac{d\theta_i}{dt} = 2\pi f_i \quad , \quad \boxed{f_i = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}}$$

$$\theta_i(t) = 2\pi \int f_i(t)\, dt.$$

$$= 2\pi \int \left[f_c + k_f\, m(t)\right] dt$$

$$= 2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt.$$

$$s_{FM}(t) = A_c \cos[\theta_i(t)]$$

$$= A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right].$$

General expression

SINGLE TONE FM EXPRESSION.

$$\boxed{m(t) = A_m \cos 2\pi f_m t.}$$

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + \frac{k_f \times \cancel{2\pi}}{\cancel{2\pi} f_m} A_m \sin 2\pi f_m(t)\right].$$

IMP.

$$\boxed{\beta = \frac{k_f A_m}{f_m}} \longrightarrow \text{Modulation Index}.$$

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m(t)\right].$$

$$\boxed{\beta = \frac{\Delta f}{f_m}.} \longrightarrow \Delta f = k_f A_m \longrightarrow \text{freq. swing}.$$

2 cases of β.

$\beta > 1$ — Wide Band Frequency Modulation (WBFM)

$\beta < 1$ — Narrow Band Frequency Modulation (NBFM).

NARROW BAND FREQUENCY MODULATION (NBFM).

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta \underbrace{\sin 2\pi f_m t}_{\theta}\right].$$

$$= A_c\left[\cos(2\pi f_c t)\cos\left(\boxed{\beta \sin(2\pi f_m t)}\right) - \sin 2\pi f_c t \times \sin\left(\boxed{\beta \sin 2\pi f_m t}\right)\right].$$

θ.

For NBFM, $\beta \leq 1$ $\quad \sin\theta \approx \theta$, $\cos\theta \approx 1$ (for small θ).

$$s_{NBFM} \cong A_c \cos 2\pi f_c t \cdot 1 - \left(\underbrace{A_c \sin 2\pi f_c t}\right) \times \left(\underbrace{\beta \sin 2\pi f_m t}\right)$$

Expanding these 2 time terms.

$$s_{NBFM} = A_c \cos 2\pi f_c t - \frac{\beta A_c}{2} \cos 2\pi (f_c - f_m) t + \frac{\beta A_c}{2} \cos 2\pi (f_c + f_m) t.$$

NBFM is similar to AM.

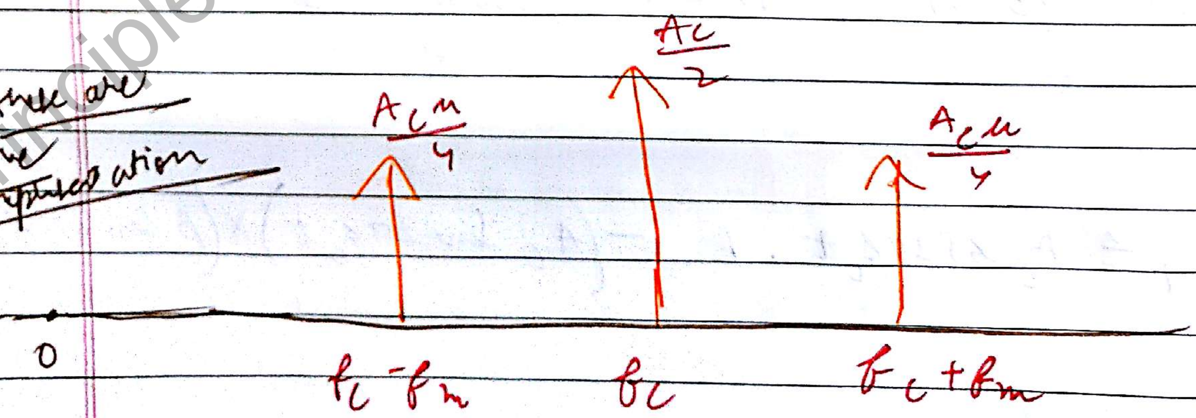
(only diff. of sign).

So, there is 180° phase shift in lower side band.

Single Tone Expression of AM & NBFM will be same except 180° phase shift at LSB freq. component.

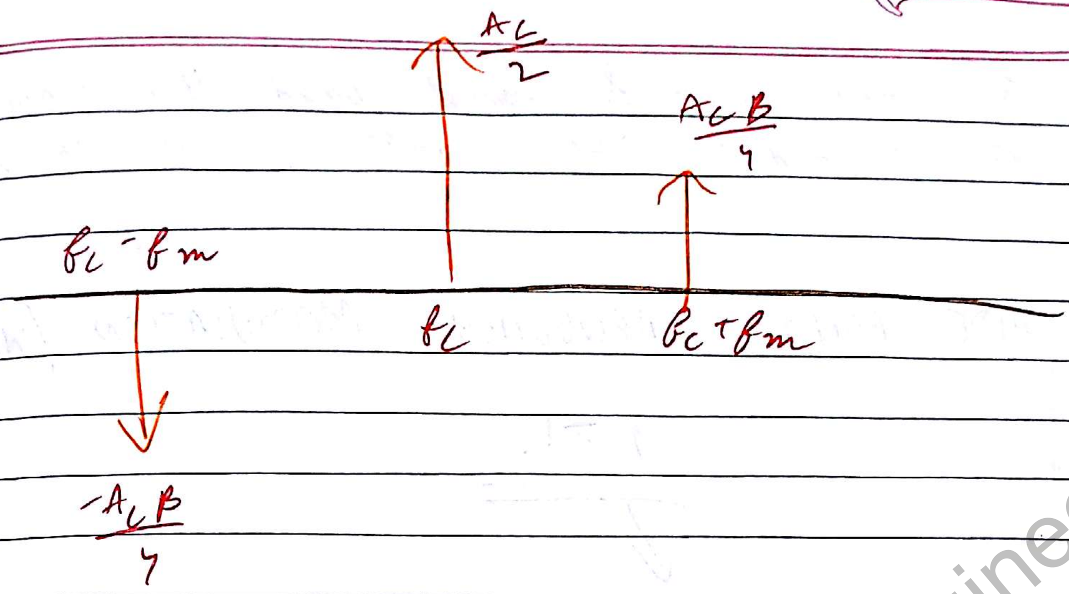
Magnitude spectrums of AM & NBFM will be almost same.

these are the representation



-ve side will be shown here for mathematical representation

$s_{AM}(t)$ AM → Bandwidth = $2F_m$.



NBFM – Bandwidth $= 2f_m$.

Because of its high similarity with AM, NBFM is less practically imp. than WBFM.

POWER OF NBFM.

Total power of NBFM, $P_t = P_c + P_{LSB} + P_{USB}$.

$$P_t = \frac{A_c^2}{2} + \frac{(A_c\beta)^2}{8} + \frac{(A_c\beta)^2}{8}$$

$$\boxed{P_t = \frac{A_c^2}{2}\left(1 + \frac{\beta^2}{2}\right)}$$

power is similar to AM.

Transmitted power & channel bandwidth requirements of AM and NBFM will be almost same.

WIDE BAND FREQUENCY MODULATION (WBFM).

$\beta > 1$.

↓

freq. is continuously changing.

↓

no pure sinusoidal term.

So, bessel function is introduced for const. freq.

Bessel Function.

$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(x\sin\theta - n\theta)}\,d\theta .$$

Properties:

1) $J_n(x)$ ↓ with n ↑ses. so,

$J_0(x) > J_1(x) > J_2(x)$ . . .

2) $J_{-n}(x) = (-1)^n J_n(x)$

$= -J_n(x)$ where $n \to$ odd

$= J_n(x)$ where $n \to$ even.

3) $\sum_{n=-\infty}^{\infty} J_n^2(x) = 1$.

4) $J_n(x)$ always result in real values only.

$e^{j(\theta)}$ → if this is in sine terms.

↓

then fundamental freq. is 2π.

ALTERNATE SINGLE TONE FM EXPRESSION.

$$S_{FM}(t) = A_c \cos\underbrace{\left(2\pi f_c t + \beta \sin 2\pi f_m t\right)}_{\theta}.$$

$$\cos\theta = Re\left[e^{j\theta}\right].$$

$$S_{FM}(t) = A_c \, Re\left[e^{j(2\pi f_c t + \beta \sin 2\pi f_m t}\right].$$

$$= A_c \left[Re\left[e^{j2\pi f_c t} \underbrace{\boxed{e^{j\beta \sin 2\pi f_m t}}}_{f(t)}\right]\right] \text{—(1)}$$

as $\sin 2\pi f_m t$ varies with time period T $\left(\frac{1}{f_m}\right)$.

$\therefore$ $e^{j\beta \sin 2\pi f_m t}$ also varies with same time period. ∴

f(t) has same freq. ...

ALTERNATE SINGLE TONE FM EXPRESSION.

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + \beta \sin 2\pi f_m t\right)$$

$$\cos\theta = Re\left[e^{j\theta}\right].$$

$$S_{FM}(t) = A_c\, Re\left[e^{j[2\pi f_c t + \beta \sin 2\pi f_m t]}\right].$$

$$= A_c\, Re\left[e^{j2\pi f_c t} \cdot \underbrace{\boxed{e^{j\beta \sin 2\pi f_m t}}}_{f(t)}\right] \quad -(1)$$

$e^{j\beta \sin 2\pi f_m t}$ is a periodic signal. $T = \frac{1}{f_m}$.

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{+jn2\pi f_0 t}, \quad f_0 = \frac{1}{T}.$$

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jn2\pi f_0 t}\, dt,$$

$$f(t) = e^{j\beta \sin 2\pi f_m t}.$$

Date ______
Page ______

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} c_n \, e^{jn2\pi f_m (t)} \quad (2)$$

$$c_n = \frac{1}{1/f_m} \int_{-1/2f_m}^{1/2f_m} e^{j\beta \sin 2\pi f_m t} \, e^{-jn2\pi f_m t} \, dt$$

$$= f_m \int_{-1/2f_m}^{1/2f_m} e^{j(\beta \sin 2\pi f_m t - n2\pi f_m t)} \, dt.$$

Assume, $2\pi f_m t = \theta \Rightarrow 2\pi f \, dt = d\theta.$

$$c_n = f_m \int_{-\pi}^{\pi} e^{j(\beta \sin\theta - n\theta)} \frac{d\theta}{2\pi f_m}$$

$$t = \frac{-1}{2f_m} \Rightarrow \theta = -\pi$$

$$t = \frac{1}{2f_m} \Rightarrow \theta = \pi.$$

$$C_n = J_n(\beta) \cdot e^{j\beta \sin 2\pi f_m t}.$$

$$= \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{jn2\pi f_m t}$$

$$S_{FM}(t) = A_c\, Re\left[e^{j2\pi f_c t} \cdot \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{jn2\pi f_m t} \right]$$

$$S_{FM}(t) = A_c\, Re \left| \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi(f_c + nf_m)t} \right|$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi (f_c + nf_m)t.$$

$\beta > 1.$

alternate single tone FM.

J_n terms & f terms are diff. now, 10 power can be analysed.

classmate Date ____ Page ____

SPECTRUM ANALYSIS OF WBFM.

$$S_{WBFM} = A_c \sum_{-\infty}^{\infty} J_n(\beta) \cos 2\pi (f_c + n f_m) t, \quad \beta > 1.$$

Expansion

$= A_c J_0(\beta) \cos 2\pi f_c t +$

$A_c J_1(\beta) \cos 2\pi (f_c + f_m) t +$

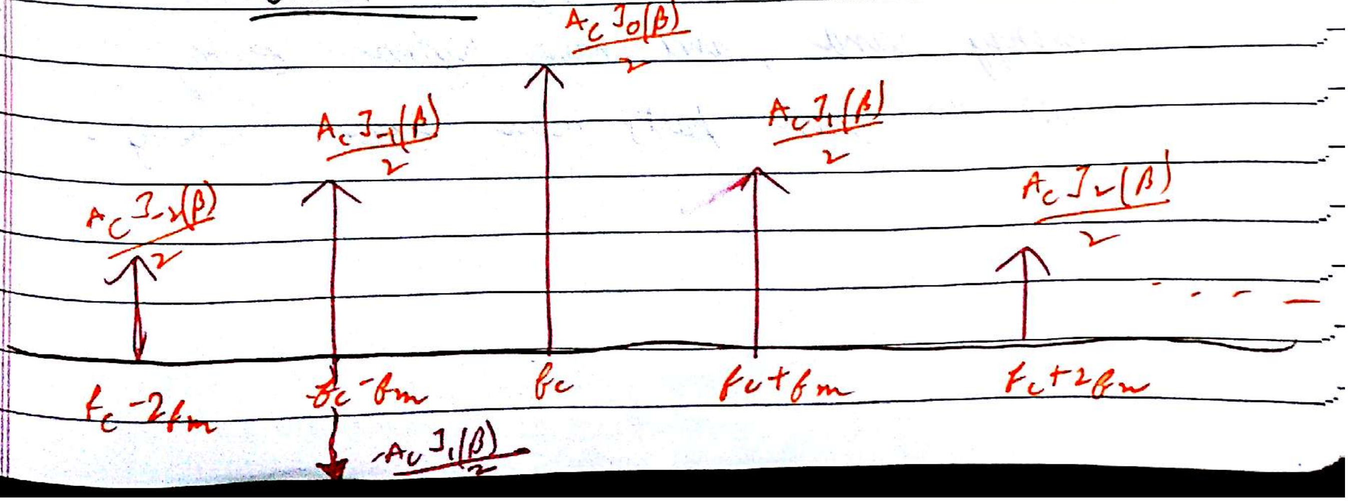
$A_c J_{-1}(\beta) \cos 2\pi (f_c - f_m) t +$

$A_c J_2(\beta) \cos 2\pi (f_c + 2f_m) t +$

$A_c J_{-2}(\beta) \cos 2\pi (f_c - 2f_m) t +$

⋮

SPECTRUM



For small β,

$$J_0(\beta) \cong 1 \qquad J_1(\beta) \cong \frac{\beta}{2}$$

NBFM

$$J_{-1}(\beta) = \frac{-\beta}{2} \quad \ldots\ldots$$

$$J_2(\beta) \cong J_3(\beta) \approx 0 \quad \ldots\ldots$$

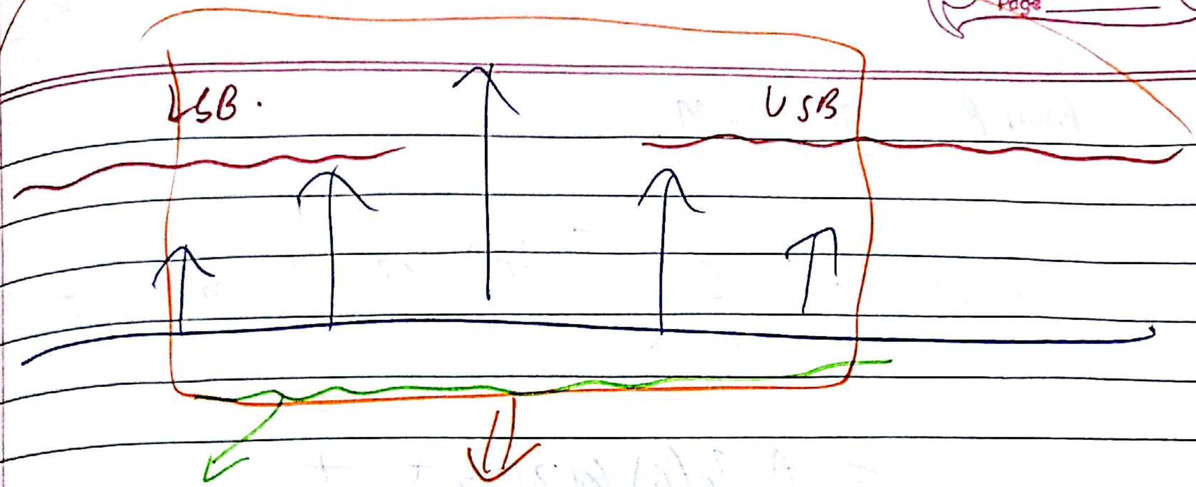
in the case of NBFM, these values are there.

NOTE: We cannot transmit message using WBFM from one to other place because WBFM has infinite bandwidth.

since it is converted to bessel funct (defined from $-\infty$ to ∞).

Here, 95–99% of energy is around the centre energy band, and have higher energy, all the other parts have lower energy.

all these parts have low energy.



significant side bands.

Only this portion has max. amount of energy.

So this portion of the entire signal is transmitted & also there is no loss of imp. information.

WBFM is having carrier frequency component & infinite no. of upper side bands & lower side bands.

For WBFM, most of the strength will be contained by lower order side band, which are called as SIGNIFICANT SIDE BANDS.

POWER OF FM.

$$S_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(f_c + n f_m)t$$

$$= A_c J_0(\beta) \cos 2\pi f_c t +$$

$$A_c J_1(\beta) \cos 2\pi (f_c + f_m)t +$$

$$A_c J_{-1}(\beta) \cos 2\pi (f_c - f_m)t +$$

$$- - - - - -$$

$$P(t) = \ldots + P_{LSB_2} + P_{LSB_1} + P_c + P_{USB_1} + P_{USB_2} + \ldots$$

$$P_c = \frac{A_c^2 J_0^2(\beta)}{2R}$$

Here, P_c is dependent on β.

$$P_{USB_1} = \frac{A_c^2 J_1^2(\beta)}{2R} \qquad P_{LSB_1} = \frac{A_c^2 J_{-1}^2(\beta)}{2R}$$

$$P_{USB_2} = \frac{A_c^2 J_2^2(\beta)}{2R} \qquad P_{LSB_2} = \frac{A_c^2 J_{-2}^2(\beta)}{2R}.$$

$$P_T = \frac{A_c^2}{2R} \sum_{n=-\infty}^{\infty} J_n^2(\beta) = \frac{A_c^2}{2R}$$

Power of a signal before modulation.

$c(t) = A_c \cos 2\pi f_c t$, $P_c = \frac{A_c^2}{2R}$.

For NBFM, $P_t = \frac{A_c^2}{2R}\left(1 + \frac{\beta^2}{2}\right)$.

$\beta \rightarrow$ very small.

Here, β is very small, so it does not affect the total power approximate, $\frac{A_c^2}{2R}$.

* For FM, the power of carrier signal before modulation & after modulation will be same.

* The power of carrier frequency component before & after modulation will be different.

Before modulation $\frac{A_c^2}{2R}$

After Modulation. $\frac{A_c^2 J_0^2(\beta)}{2R}$

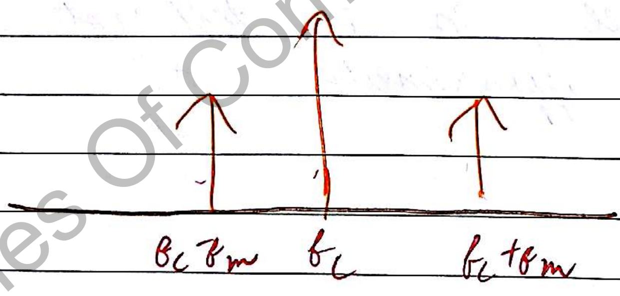
BANDWIDTH OF FM.

Actual bandwidth of WBFM is infinite so that before transmission it should be band limited by using band limiting process.

To bandlimit WBFM signal, lower order significant side band should be retained and higher order insignificant side bands should be eliminated.

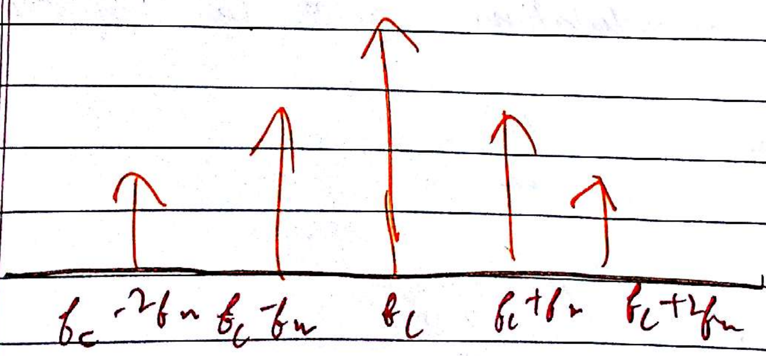
1) Assume, FM having significant side bands upto first order.

After bandlimiting,



$BW = 2f_m$

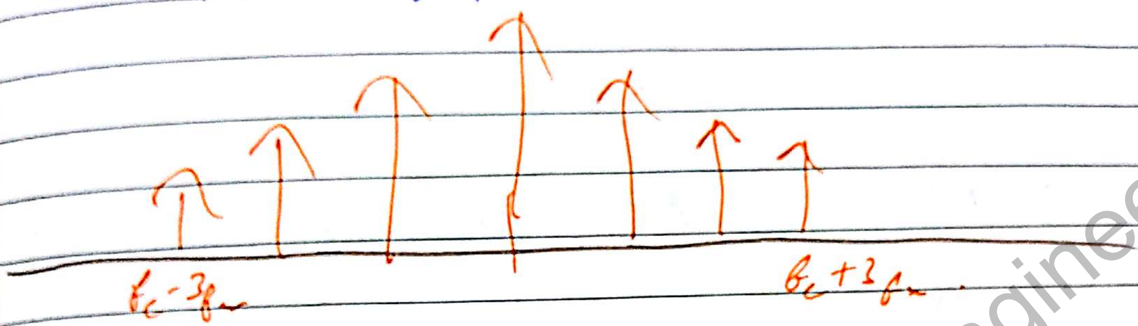
2) Assume, FM having significant side bands upto second order.



$BW = 2(2f_m)$

2) Assume FM having significant side bands upto 3rd order.

$BW = 6f_m = 3(2f_m)$.



According to Carson's Rule, f_m having significant side bands upto $(\beta + 1)$ order is

$$BW = (\beta + 1) 2f_m.$$

$$\beta = \frac{\Delta f}{f_m}$$

So, $$BW = 2(\Delta f + f_m).$$

To save channel bandwidth, AM can be used instead of FM because in AM, bandwidth $= 2f_m$, but in WBFM bandwidth $= 2(\Delta f + f_m)$.

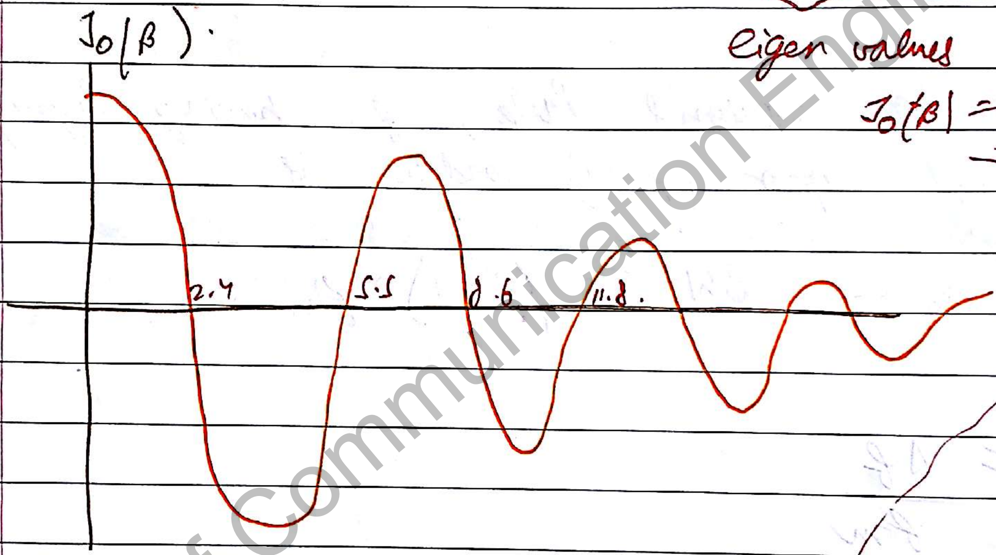
To save power prefer FM over AM.

POWER EFFICIENCY

$$P_c = \frac{A_c^2 J_0^2(\beta)}{2R}$$

$J_0(\beta) = 0, \quad \beta = \underbrace{2.4, 5.5, 8.6, 11.8, \ldots}_{}$

eigen values where $J_0(\beta) = 0$ hence, $P_c = 0$.



Curve of bessel funct.

here, carrier power is dependent on β.

efficiency = $\frac{\text{side band}}{\text{total}}$

$= \frac{\text{side band}}{\text{side band}}$

$= 100\%$

For above values of β, power efficiency of WBFM will become 100%, which are called as EIGEN VALUES of WBFM.

Q) A carrier signal of 20V, 5MHz is frequency modulated by a message signal of 5V, 25kHz with K_f (sensitivity const) = 50 kHz/V

Find Δf, β, bandwidth & power.

Q2) Repeat above if message signal amplitude is doubled ??

Ans: $\Delta f = 250$ kHz

$\beta = 10$ (WBFM)

Bandwidth = 550 kHz

Power = 200 W.

$\Delta f_2 = 500$ kHz

$\beta = 20$.

Bandwidth = 1050 kHz

Power = 200W.

NOTE: If message amplitude is doubled, then both Δf and β will be doubled.

Bandwidth will be almost doubled.

Q) An FM signal is given by

$$s(t) = 10\cos\left(2\pi \times 10^6 t + 8\sin 4\pi \times 10^3 t\right)$$

Find Δf, β, bandwidth & power.

Q2) Repeat above if message signal freq. is doubled.

Ans: 1) $\Delta f = 16\,kHz$, $\beta = 8$

Bandwidth = 36 kHz

Power = 50W

2) $\Delta f = 16\,kHz$, $\beta = 4$

Bandwidth = 40 kHz.

Power = 50W

Note: Frequency deviation Δf is independent of message frequency. If message frequency doubles then β will be halved & bandwidth remains almost same.

classmate

Date ______

Page ______

Q) For an FM signal, having max. freq. deviation Δf, of 16 kHz, corresponding message freq. component is 4 kHz.

1) Find β & bandwidth.

2) Repeat above if message signal amplitude is doubled & freq. is reduced to 1 kHz.

Ans: (1) $\Delta f = 16\,kHz$. $f_m = 4\,kHz$ $\beta = 4$.

Bandwidth $= 40\,kHz$.

(2) $\Delta f = 32\,kHz$ $f_m = 1\,kHz$ $\beta = 32$

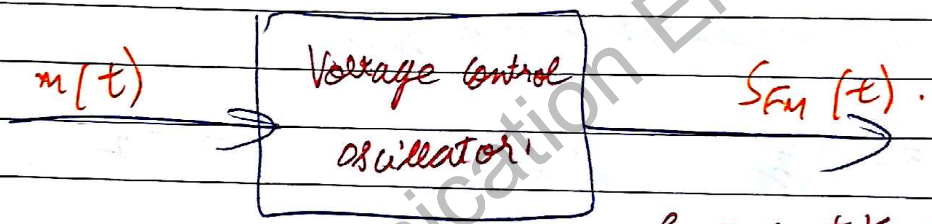
Bandwidth $= 66\,kHz$.

GENERATION OF FM

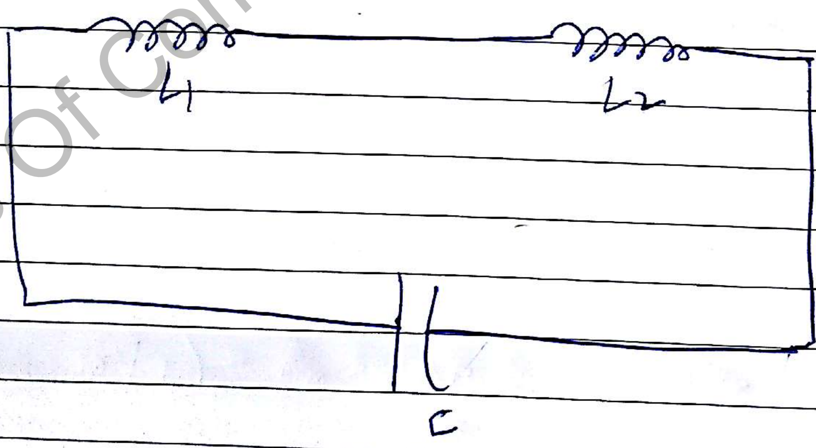
DIRECT METHOD (in syllabus)

INDIRECT METHOD (not in syllabus)

DIRECT METHOD



$f_i = f_c + K_f\, m(t)$.

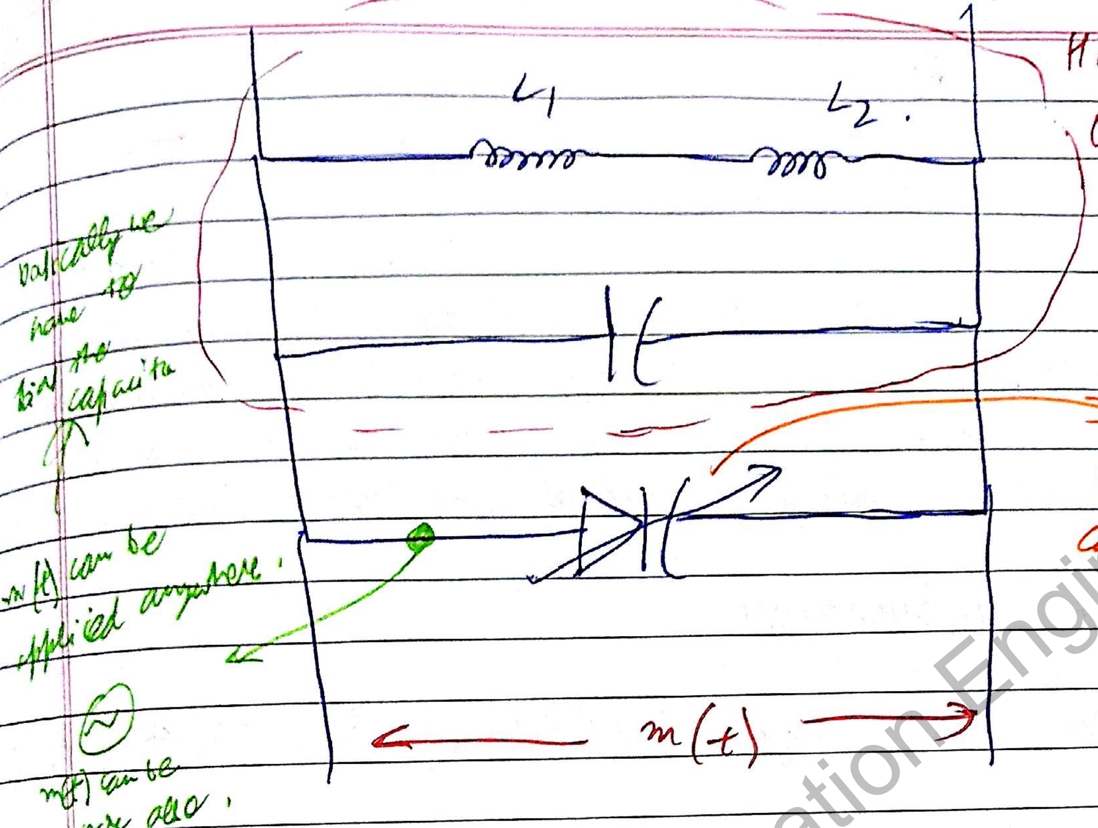


$$f = \frac{1}{2\pi\sqrt{(L_1+L_2)C}}$$

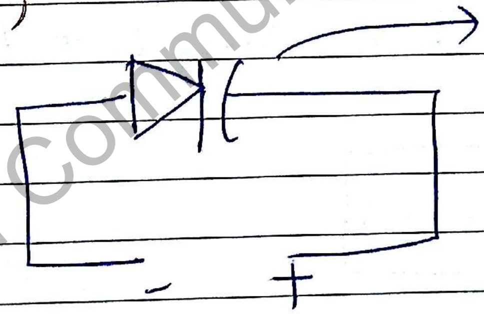
2 inductor ||

inductor.

HERTLEY OSCILLATOR.

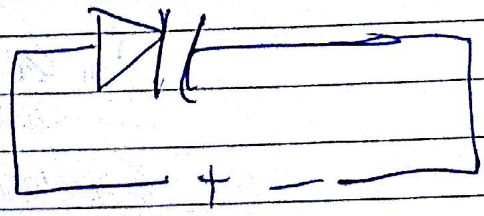


1) m(t) is +ve,



reverse bias ↑ses ⇒ depletion width ↑ses ⇒ capacitance ↓ses ⇒ freq. ↑ses.
(C') → || combination of C & variable capacitor.

2) m(t) is −ve.



forward bias ↑ses ⇒ depletion width ↓ses ⇒ capacitance ↑ses ⇒ f_i ↓ses.

$$f_i = \frac{1}{2\pi\sqrt{(L_1+L_2)(C'+C)}}$$

NOTE: In direct FM, the instantaneous frequency of carrier wave is varying directly in accordance with message signal by means of a device known as voltage control oscillator (VCO).

PHASE MODULATION.

Carrier Signal before Modulation, $c(t) = A_c \cos 2\pi f_c t$

Carrier Signal After Phase Modulation,

$$S_{PM}(t) = A_c \cos(2\pi f_c t + \phi(t))$$

$\phi(t) \rightarrow$ phase deviation.

$$\boxed{\phi(t) = K_p m(t)}$$

$$S_{PM}(t) = A_c \cos(2\pi f_c t + K_p m(t)).$$

let $m(t) = A_m \sin 2\pi f_m t$ or

$A_m \cos 2\pi f_m t.$

Maximum Phase Deviation.

$$\Delta\phi = |\max(\phi(t))|$$

$$= |\max(K_p m(t))| = K_p A_m.$$

$$\boxed{\Delta\phi = K_p A_m \text{ radians.}}$$

SINGLE TONE PM.

$$S_{PM}(t) = A_c \cos(2\pi f_c t + K_p m(t)).$$

Assume, $m(t) = A_m \cos 2\pi f_m t$.

$$S_{PM}(t) = A_c \cos(2\pi f_c t + \underbrace{K_p A_m}_{\beta} \cos 2\pi f_m t).$$

$$\boxed{K_f A_m = \beta}$$

For same message signal expression of FM & PM is,

$$S_{PM}(t) = A_c \cos(2\pi f_c t + \beta \cos 2\pi f_m t)$$

$$S_{FM}(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$$

when 2 signals are given & then we cannot identify which is FM & which is PM?

FM & PM can be interchanged also.

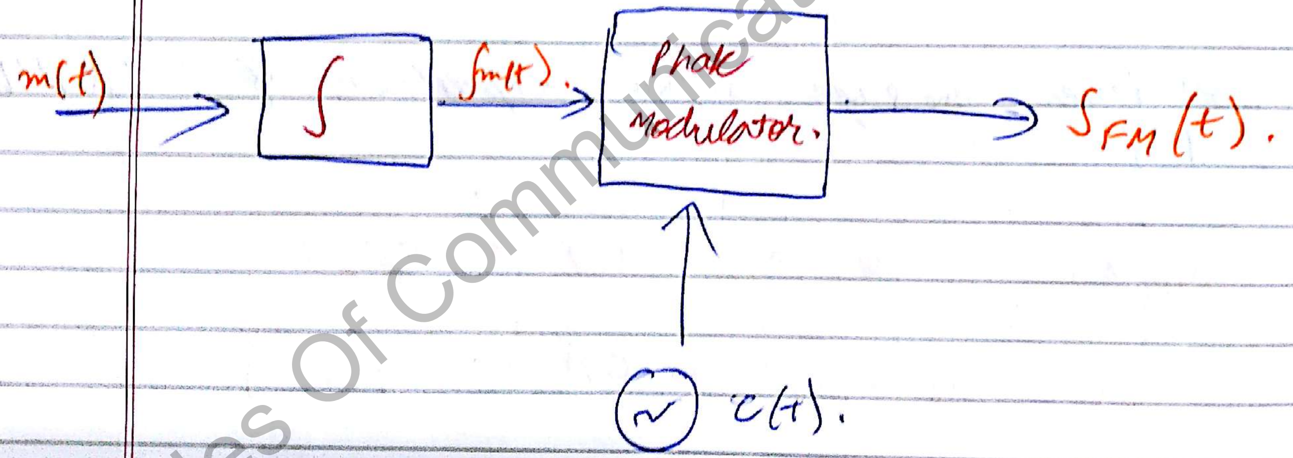
(Koi bhi kuch bhi ho sakta hai).

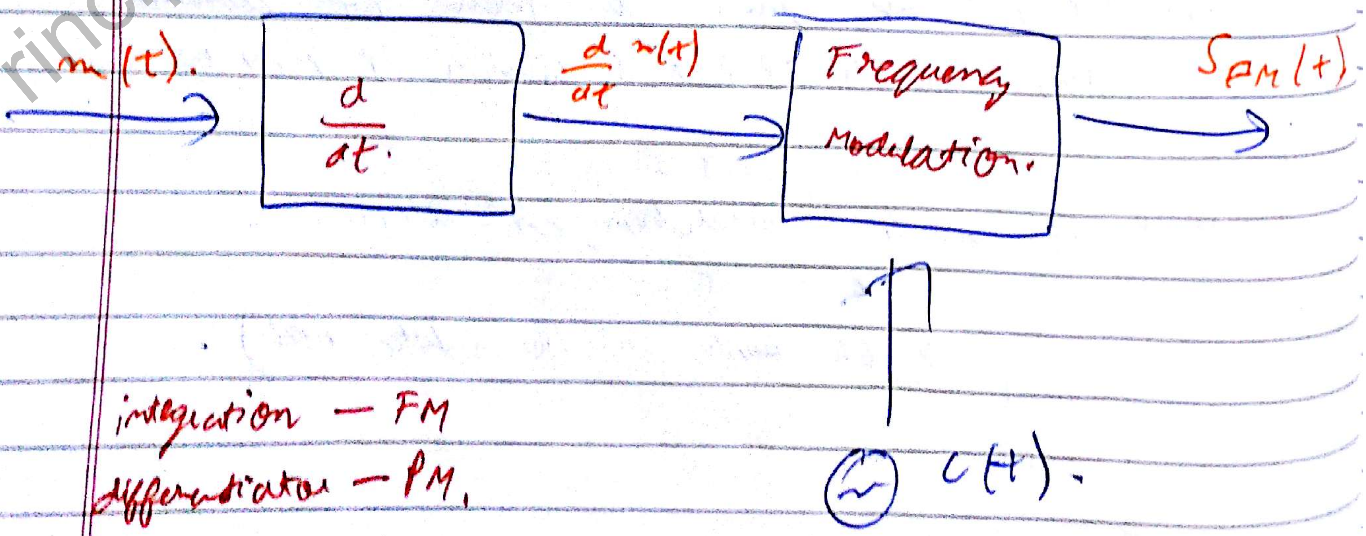
NOTE: SINGLE TONE PM & FM expression will be same for same message signal, except 90° phase shift at message freq. component.

GENERATION OF PM FROM FM.

$$S_{PM}(t) = A_c \cos\left(2\pi f_c t + K_p m(t)\right).$$

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int m(t)\, dt\right).$$





integration — FM
differentiator — PM.

$$FM\left[m(t)\right] = PM\left[\int m(t)\right].$$

$$PM\left[m(t)\right] = FM\left[\frac{d}{dt}m(t)\right].$$

Conclusion: Freq. modulation of one signal is phase modulation of other signal & vice versa.

For some PM signal, it is possible to find freq. deviation & for some FM signal, it is possible to find phase deviation.

⇒ Δf for PM and $\Delta\phi$ for FM.

Q) An angle modulated signal is given by ~~10cos(2π fc~~

$10\cos(2\pi\times10^6 t + 6\sin 4\pi\times10^3 t)$. Find Δf & $\Delta\phi$.

this coeff. $= \beta$

$\underline{\beta = 6}$

$f_m = 2\times10^3$

$\Delta f = \beta f_m$
$= 12\text{ kHz}$

$\Delta f = K_p\, m(t)$

Bandwidth

this single is $\underline{PM}$

then, $\Delta\phi = \beta = 6$

$\beta = K_p A_m = 6.$

$\longrightarrow \Delta\phi$

Ans.

MAX PHASE DEVIATION OF FM SIGNAL.

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t\right) + 2\pi k_f \int m(t)\, dt$$

$$\left[\Delta\phi\right]_{FM} = \left| \max\left(2\pi k_f \int m(t)\, dt\right)\right|$$

$$\left(\Delta\phi\right)_{PM} = \left| \max\left(k_p\, m(t)\right)\right|$$

$$= k_p A_m.$$

for $m(t) = A_m \cos 2\pi f_m t.$

$$\left[\Delta\phi\right]_{FM} = \max\left[\frac{2\pi k_f A_m \sin 2\pi f_m t}{2\pi f_m}\right]$$

$$= \frac{k_f A_m}{F_m}.$$

Max!!

$\Delta\phi_{FM} = \dfrac{k_f A_m}{F_m} = \beta_{FM}.$

$\Delta\phi_{PM} = k_p A_m = \beta_{PM}.$

Max Freq. Deviation of PM signal.

$$s_{(PM)}(t) = A_c \cos\underbrace{(2\pi f_c t + k_p m(t))}_{\theta(t).}$$

$$s_{(PM)}(t) = A_c \cos(\theta_i(t))$$

$$f_p = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = \frac{1}{2\pi}\left[2\pi f_c + k_p \frac{dm(t)}{dt}\right].$$

$$\boxed{F_p = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}}$$

$$(\Delta F)_{PM} = \left| \max\left(\frac{k_p}{2\pi}\frac{d(m(t))}{dt}\right)\right|$$

$$(\Delta F)_{FM} = \left| \max\left(k_F\, m(t)\right)\right|$$

$$s_{FM}(t) = A_c \cos\underbrace{\left(2\pi f_c t + 2\pi k_f \int m(t)\, dt\right)}_{\theta_i(t).}$$

(1) Sanjay Sharma. (2) Shom Series

for random process !!

$$f_i = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

$$= f_c + K_f\, m(t).$$

* $$\boxed{BW}_{PM} = (\beta_{PM} + 1)\, 2f_m$$

$$= 2\left[\Delta f_{PM} + f_m\right].$$

* $$\boxed{BW}_{FM} = (\beta_{FM} + 1)\, 2f_m = 2\left[\Delta f_{FM} + F_m\right]$$

let $m(t) = A_m \cos 2\pi f_m t$.

$$(\Delta f)_{PM} = \left| \max \left(\frac{K_p}{2\pi} \frac{d\, m(t)}{dt} \right) \right|$$

$$(\Delta f)_{PM} = \max \left[\frac{K_p}{2\pi} \left(-A_m \mp 2\pi f_m \sin 2\pi f_m t \right) \right]$$

$$\left[\begin{array}{l} (\Delta f)_{PM} = K_p A_m F_m = \beta_{PM} F_m \\ \\ (\Delta F)_{FM} = K_f A_m \end{array} \right]$$

ASSIG. (not so imp.).

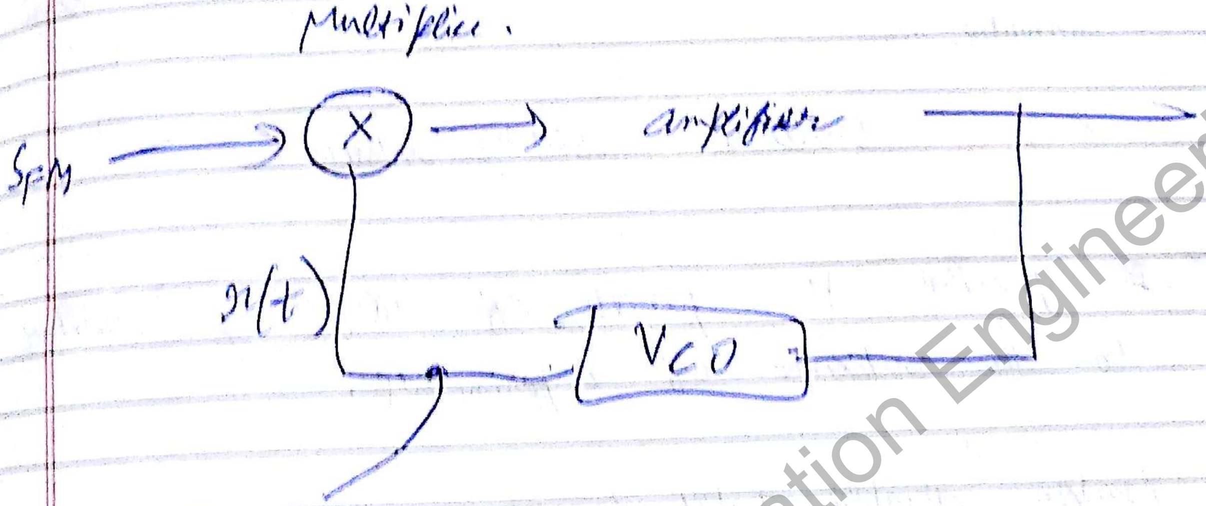
Balance Modulator.

Phase discriminator.

Tuned amplifier

envelope detector.

Simon Haykin.



this signal's phase is made equal to S_{FM} phase.

$$S_{FM} = A_c \cos(2\pi f_c t + \phi_1(t))$$

$$x(t) = A \cos(2\pi f_c t + \phi_2(t)).$$

ϕ_1 & ϕ_2 are made equal.

RANDOM PROCESSES.

Random Variables

Output of an uncertain program is random variable.

e.g. : Outcome throwing a dice.

* It specifies the process of assigning numbers to the outcome of experiments.

Random variables takes finite value set X, is discrete random variable.

X → discrete random variable.

denoted by caps.

Random variable takes ∞ value set.

X → continuous random variable.

Random variable denoted in caps and values taken by RV denoted with small letters.

$X = \{x_i\}$.

RV defined for undeterministic experiment.

A random variable X is defined such that it specifies number of possible heads in the experiment of tossing a coin twice.

$\{S\}$	$X = \{x_i\}$
HH	2
HT	1
TH	1
TT	0

No of heads.

$X = \{x_i\}$ → discrete random variable.

→ finite set.

PROBABILITY MASS FUNCTION (PMF).

It is denoted by: → value taken by x.

$$P_X(x) = P(X = x).$$

P_X → random variable x

$P(X=x)$ → probability of X equals x.

It specifies probability of random variable, taking each of its possible values.

PMF

$x = \{ x_i \}$

HH $\longrightarrow \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ 2

HT $\longrightarrow \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ 1

TH $\longrightarrow \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ 1

TT $\longrightarrow \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ 0.

$P(X=2) = \frac{1}{4}$ $P(X=1) = \frac{1}{2}$ $P(X=0) = \frac{1}{4}$

No head. $P_X(0) = P(X=0) = 1/4$

One head. $P_X(1) = P(X=1) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Two heads. $P_X(2) = P(X=2) = \frac{1}{4}$

CUMULATIVE DISTRIBUTION FUNCTION.

$F_X(x) = P(X \leq x)$.

CDF specifies probability of random variable X taking the values upto x.

It is also called as probability distribution function.

classmate

Date ______

Page ______

$F_X(-\infty) = 0 \qquad F_X(0) = P(X \le 0) = 1/4$

$F_X(0.6) = P(X \le 0.6) = 1/4.$

Q) Construct distribution function for above discrete random variable.

$$F_X(0.9) = P(X \le 0.9) = 1/4$$

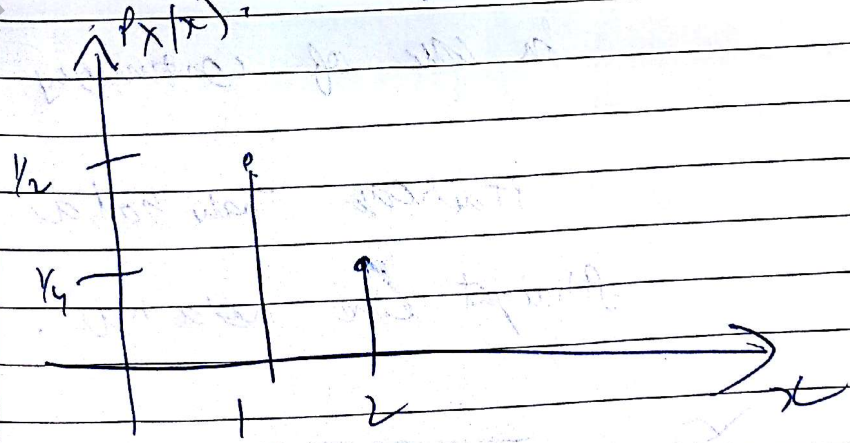
$$F_X(1) = P(X \le 1) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

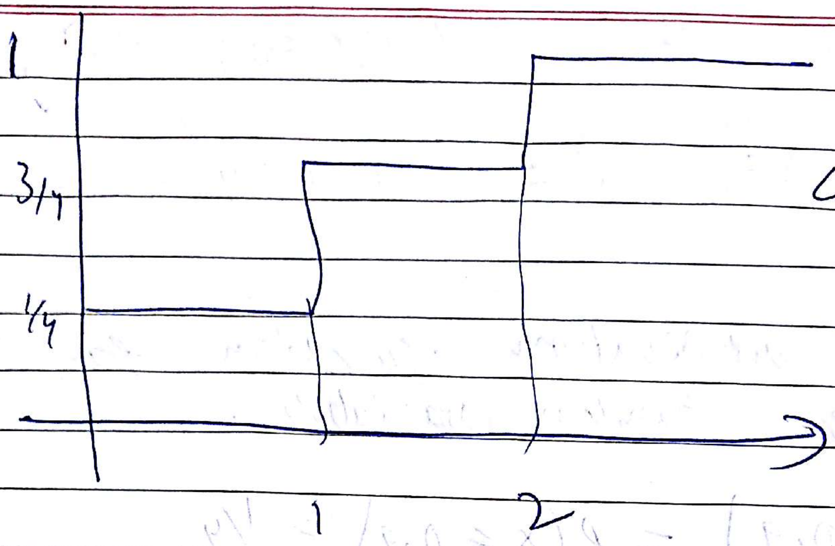
$$F_X(1.5) = P(X \le 1.5) = 3/4$$

$$F_X(2) = P(X \le 2) = 1$$

$$F_X(3) = P(X \le 3) = 1$$

\# $\boxed{0 \le F_X(x) \le 1}$

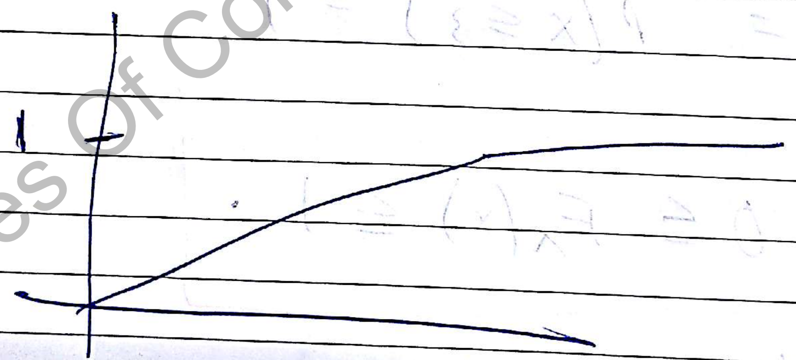




CDF is always staircase.

Distribution function of discrete random variable results in staircase waveform.

in case of CRD



in case of continuous

staircase nahi hota a straight line hota hai.

classmate

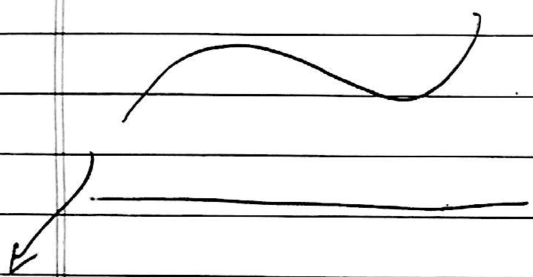
Date

Page

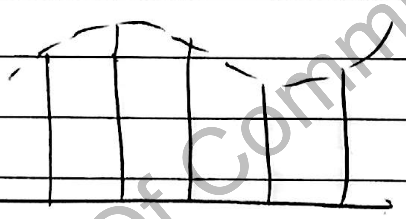
PULSE MODULATION

ANALOG PULSE MODULATION. (PAM, PPM, PMM).

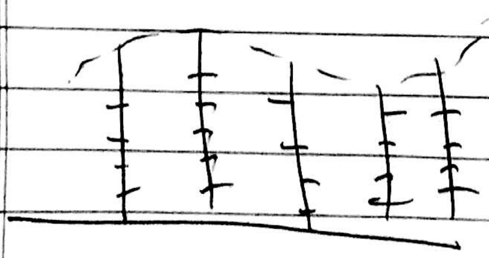
DIGITAL PULSE MODULATION.



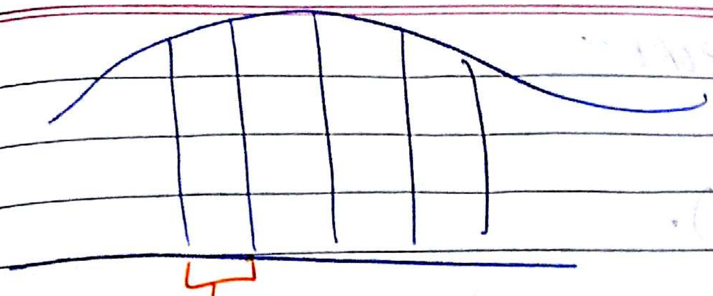
signal is discrete in time.
↓
analog



so we will take samples.



- encoding.



$f_s \geq 2 f_m$

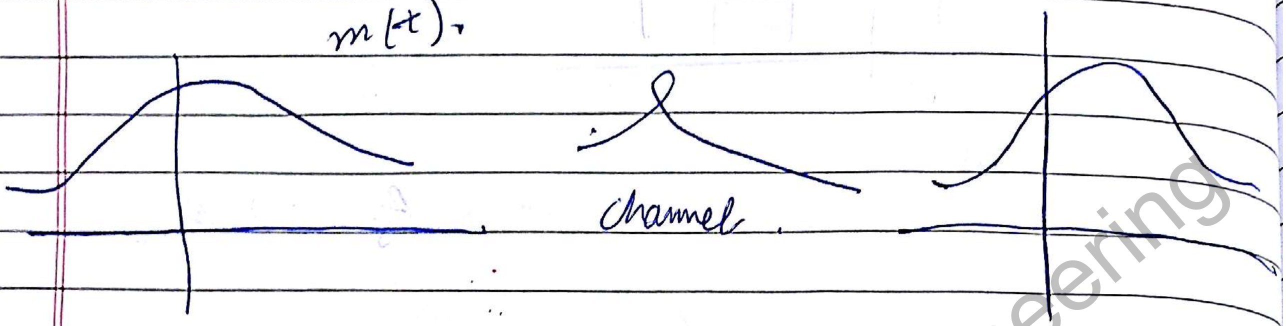
sampling freq.   message freq.

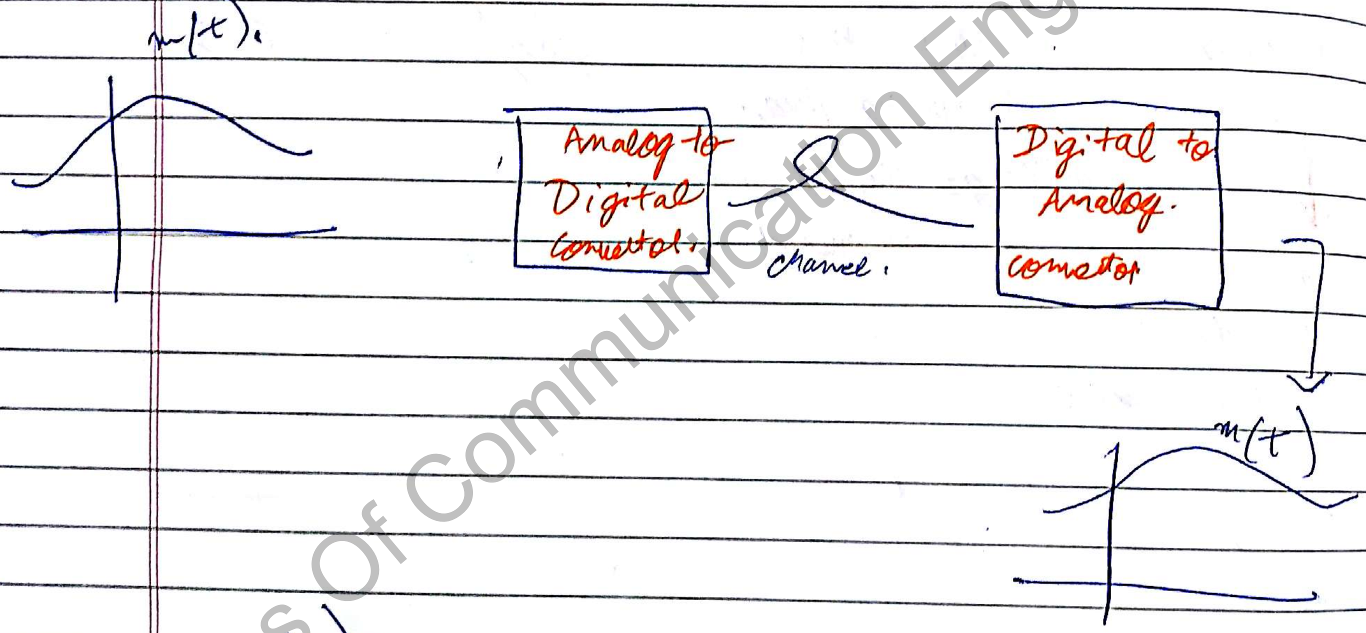
above this sampling freq. message signal can be recovered.

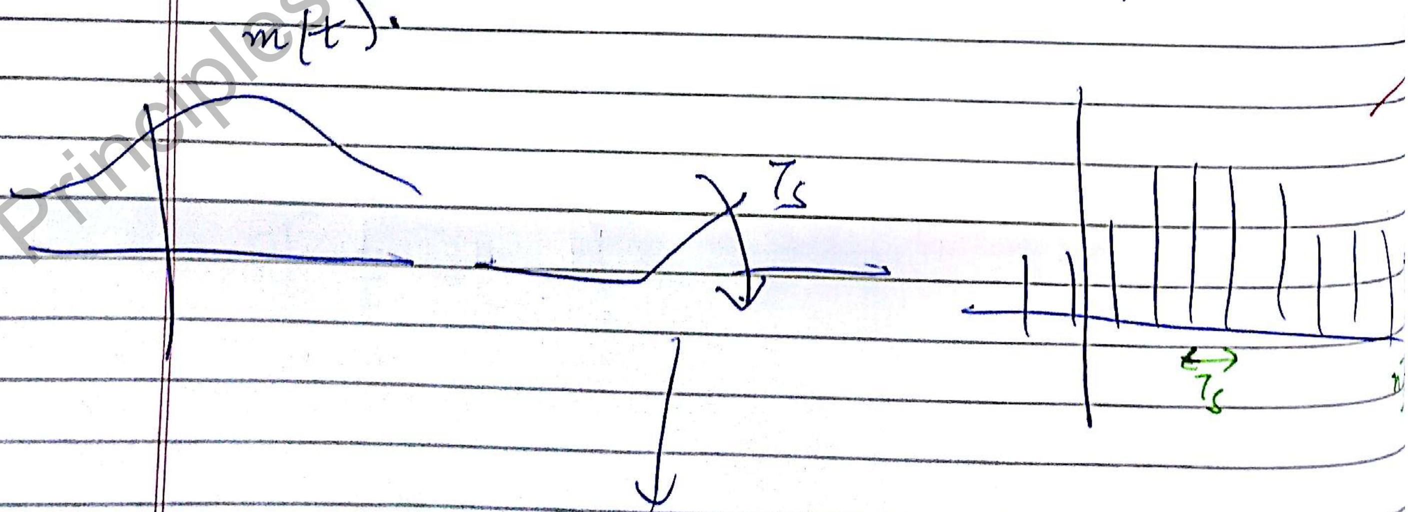
sampling time.

regular interval & should be small.

SAMPLING THEOREM.

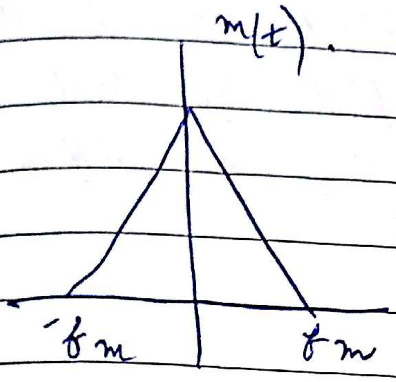






Switch whose
opening & closing interval
is T_s.

$T_s = \frac{1}{f_s}$ $f_s \rightarrow$ sampling frequency $\left(\frac{\text{sample}}{\text{sec}}\right)$.

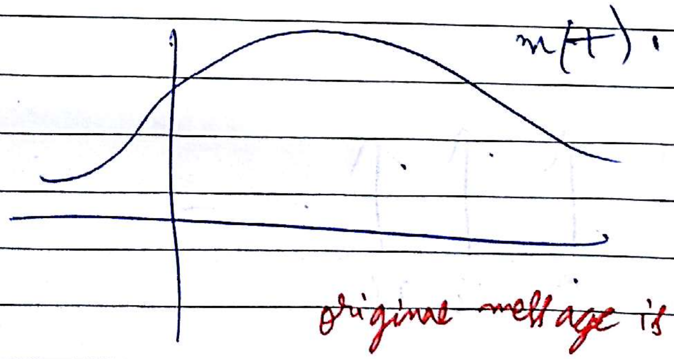


$\boxed{f_s \geq 2 f_m}$

Nyquist Rate

sampled → signal.

reconstruction filter.



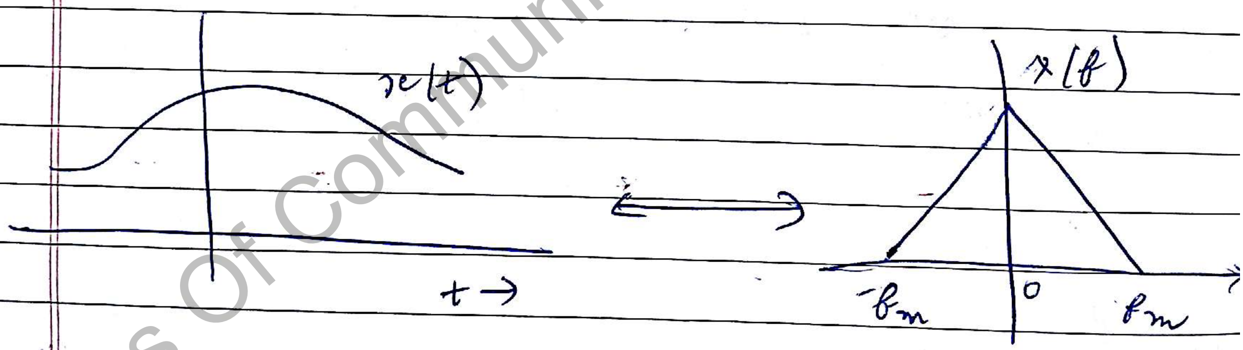
original message is again recovered.

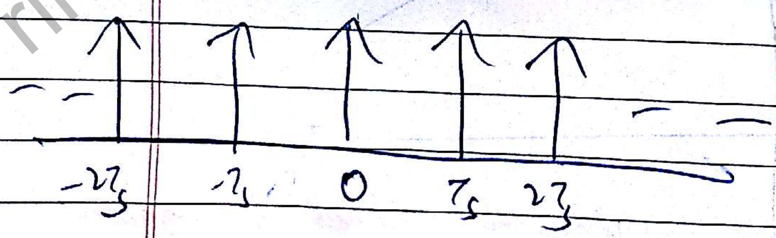
$T_s \rightarrow$ sampling interval.

Compared to transmission of continuous signal, transmitting its digital equivalent provides very much of noise free environment.

A continuous signal band limited to f_m Hz can be converted to its sampled equivalent without information loss provides $f_s \geq 2f_m$ or sampling interval $T_s \leq \frac{1}{2f_m}$ sec.

A continuous signal band limited to f_m Hz can be completely recovered from its sampled equivalent provided $f_s \geq 2f_m$ or $T_s \leq \frac{1}{2f_m}$ sec





$$\sum_{n=-\infty}^{\infty} \delta(t-nT_s) = \ldots + \delta(t+T_s) + \delta(t) + \delta(t-T_s) + \ldots$$

classmate
Date
Page

$x(t) \cdot \delta(t) = x(0)\,\delta(t)$

$x(t) \cdot \delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

properties of impulse fun^t.

$\boxed{x_s(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)}$

$X_s(f) = X(f) \otimes f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$.

Fourier transform

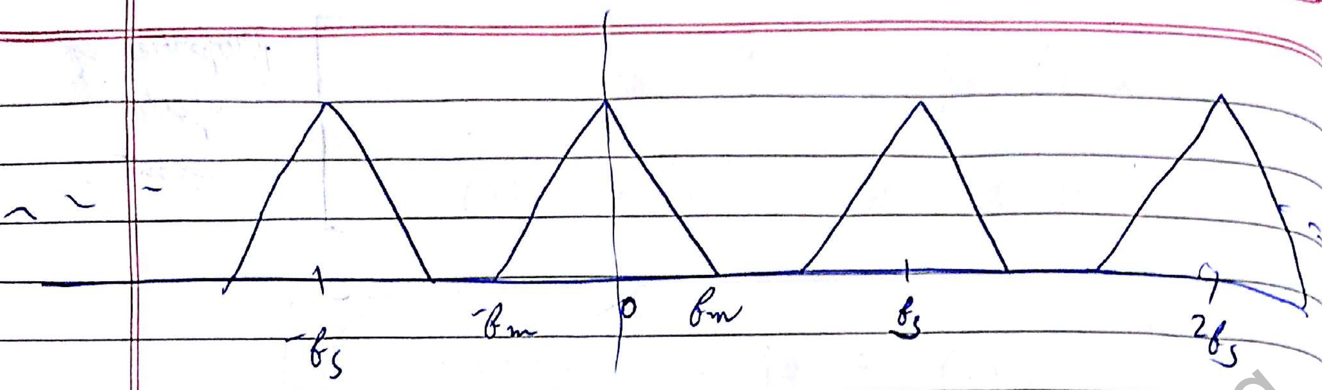
$$\boxed{X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)}$$

$x(t) \otimes \delta(t) = x(t)$

$x(t) \otimes \delta(t-t_0) = x(t-t_0)$.

property of convolution.

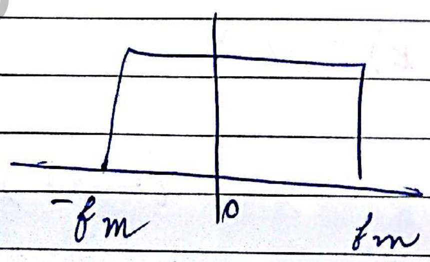
when f is less than $2f_m$ then overlapping will be there & signal | info. will be lost.

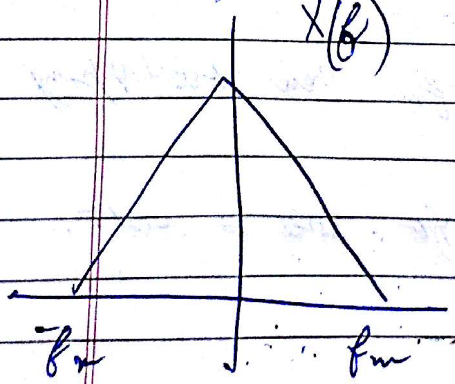


representation of sampled signal.

to recover original signal at receiver's end, we will add a low pass filter of freq. f_m, to recover the original signal.

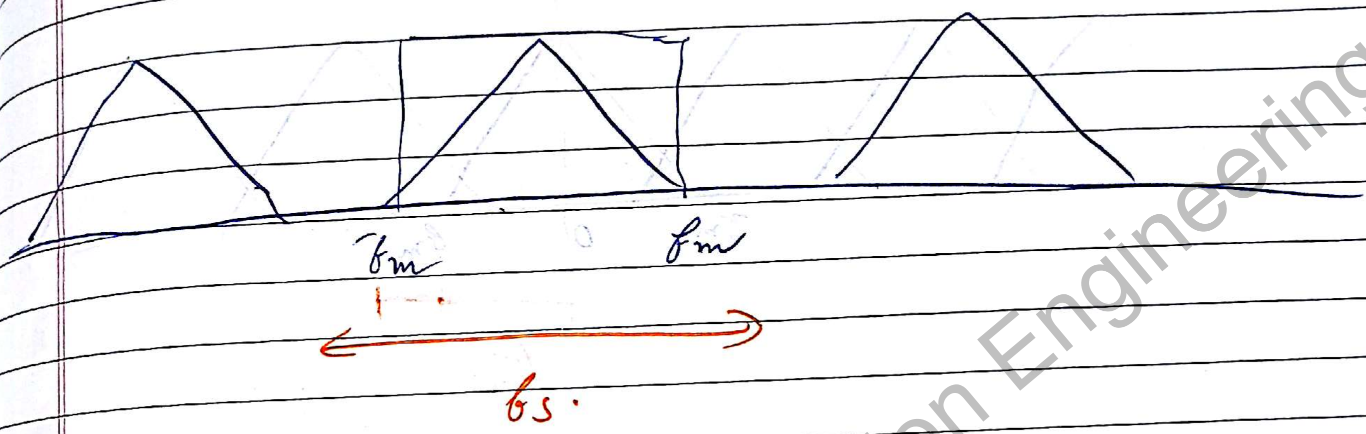
LPF



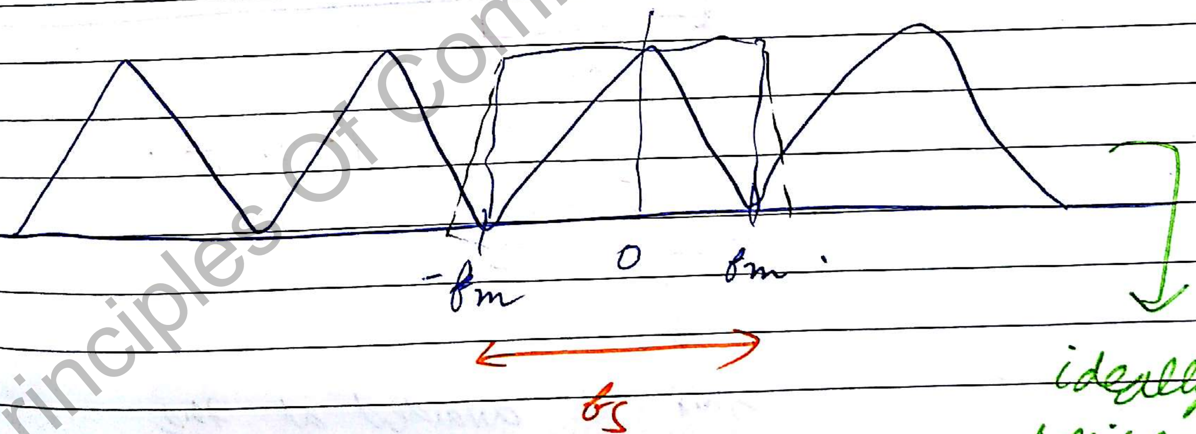


$\xrightarrow[\text{Fourier Transform}]{\text{Inverse}}$ $x(t)$

Case 1) $f_s \geq 2f_m$ or $T_s < \frac{1}{2f_m}$ (Over Sampling) (Practically).

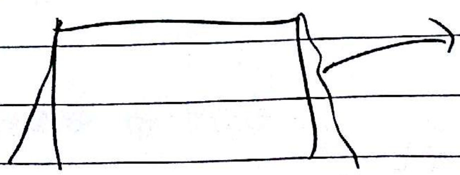


Case 2) $f_s = 2f_m$ or $T_s = \frac{1}{2f_m}$ (Control Sampling).



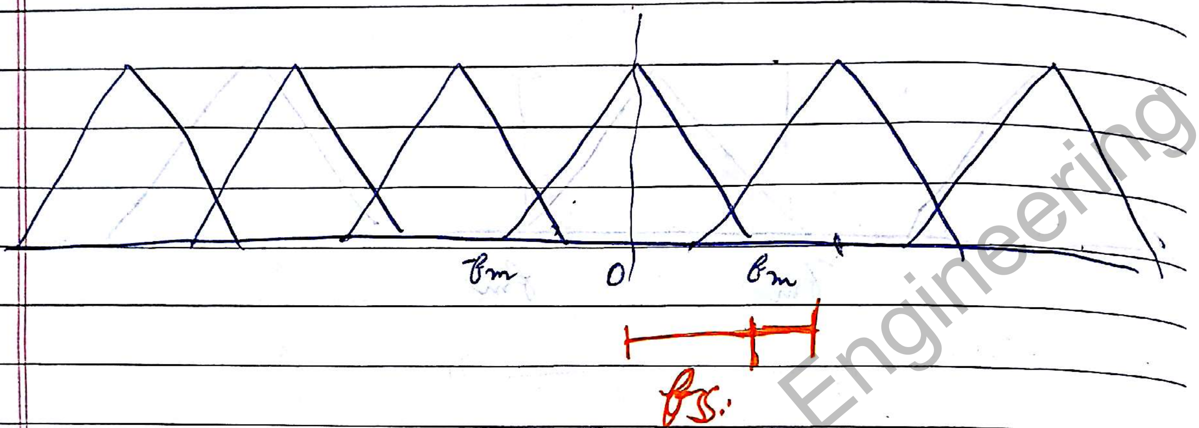
ideally possible.

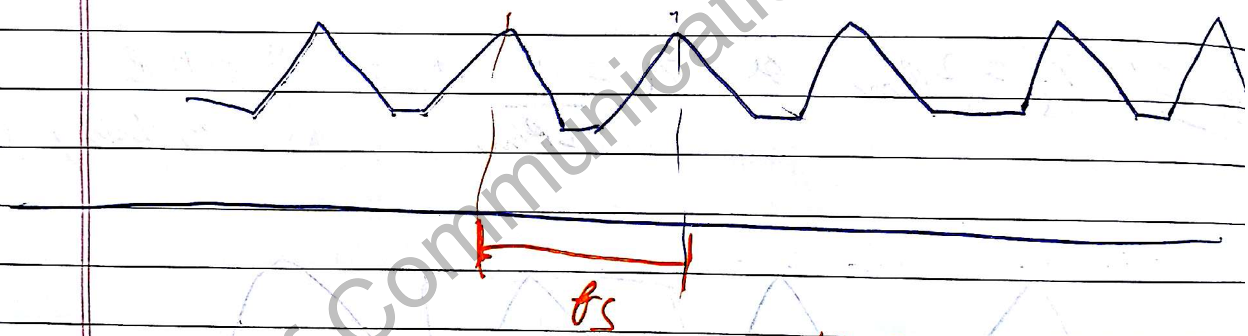
bandpass filter practically.



So practically not possible.

Case 3 $f_s < 2f_m$ or $T_s > \frac{1}{2f_m}$





~~phasing~~.

Aliasing.

∵ info. is lost in this case.

this is avoided at the time of sampling.

The effect of aliasing can be reduced :-

① Pre alias filter must be used to limit band of frequencies of the required signal f_m Hz.

② Sampling frequency f_s must be selected such that $f_s > 2f_m$.

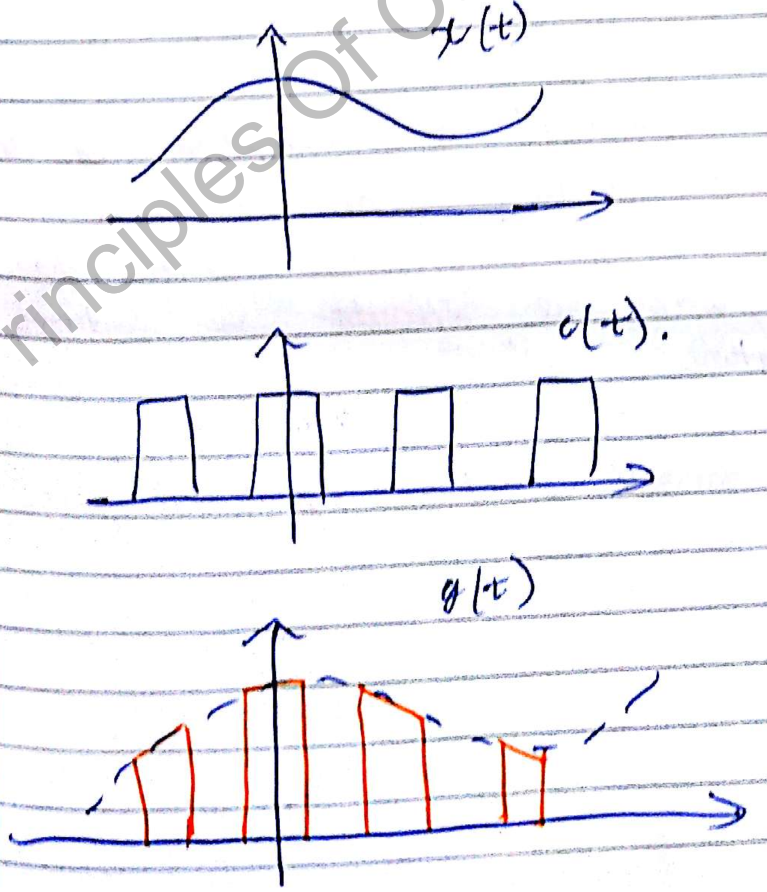
TYPES OF SAMPLING.

① NATURAL

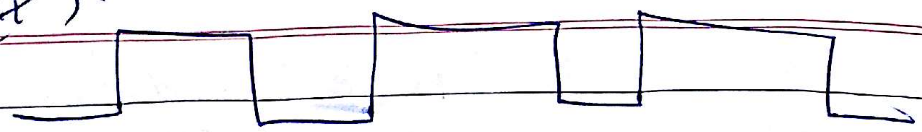
② FLAT TOP

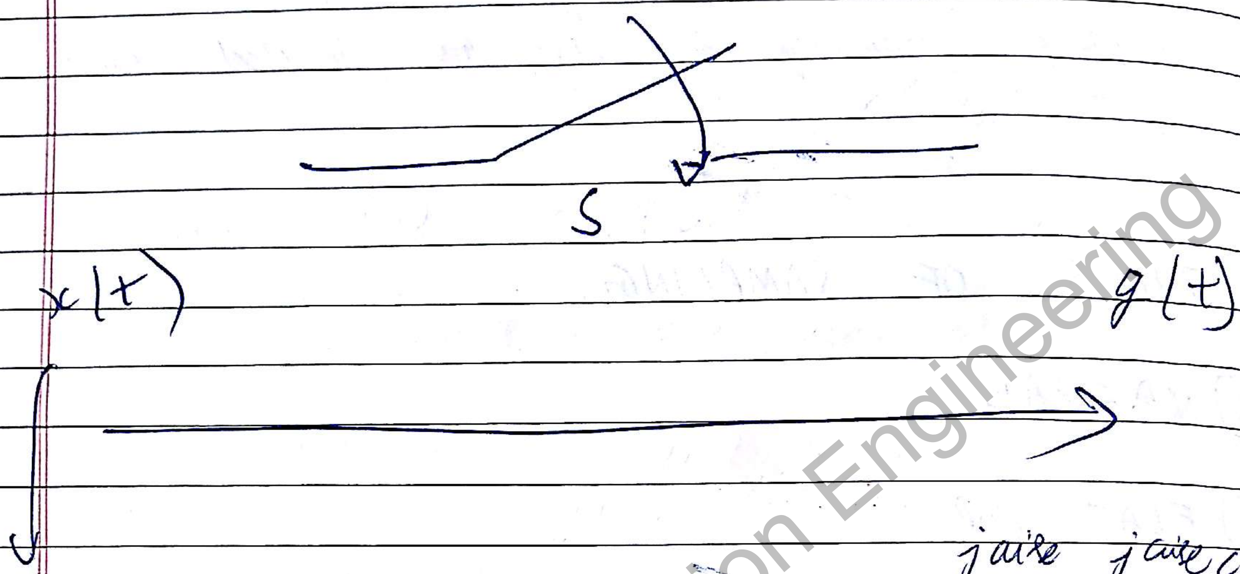
③ IDEAL.

NATURAL SAMPLING.



$c(t)$:





passed through switch controlled by $c(t)$.

jaise jaise $c(t)$ ka amplitude vary karega, vaise vaise $g(t)$ milega.

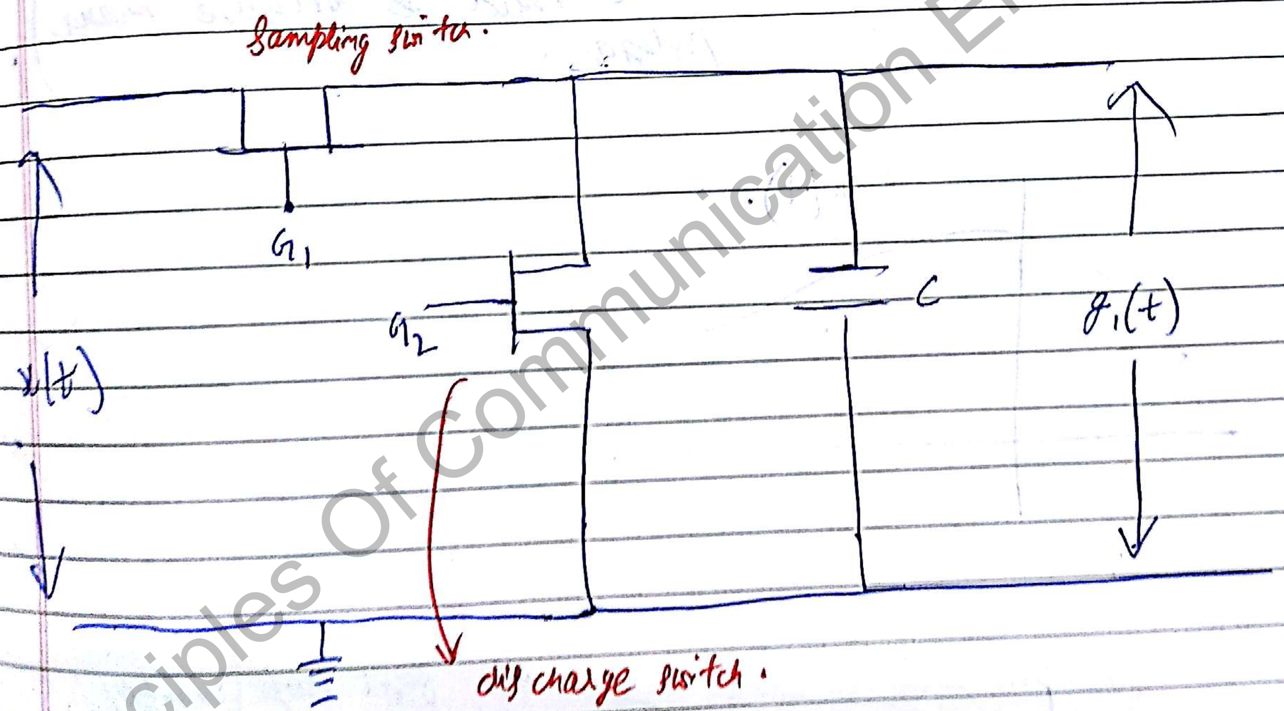
It is a practical method of sampling in which pulse have finite width equal to τ, Sampling is done in accordance with the carrier signal, which is ~~discrete in nature.~~ digital in nature.

Spectrum of natural sample signal is given by:

* $\boxed{G(f) = \frac{A\tau}{T_s}\left(\Sigma\, \text{sinc}(n f_s \tau) \times (f - n f_s)\right).}$

* Minimum sampling rate and maximum sampling interval allow to avoid aliasing are called as NYQUIST RATE and NYQUIST SAMPLING.

FLAT TOP SAMPLING.



(Sample & hold circuit)

n-channel MOSFET.

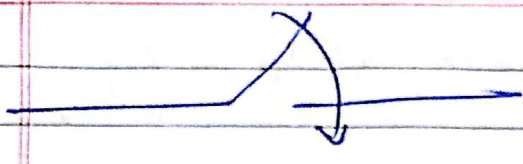
signal pass.

↳ when G_1 will be high ($+V_{cc}$).

3 states.

ON OFF high impedance. (gate voltage = 0).

Switch.

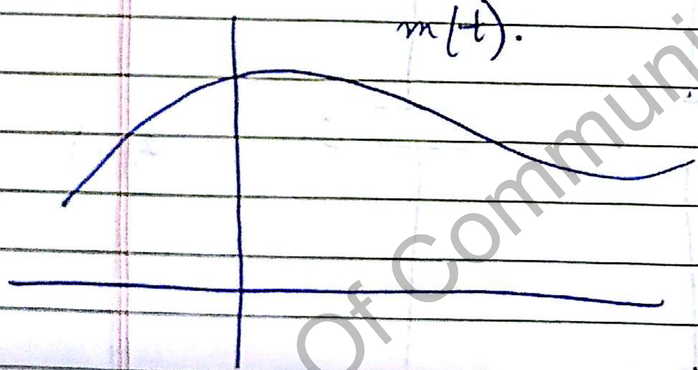


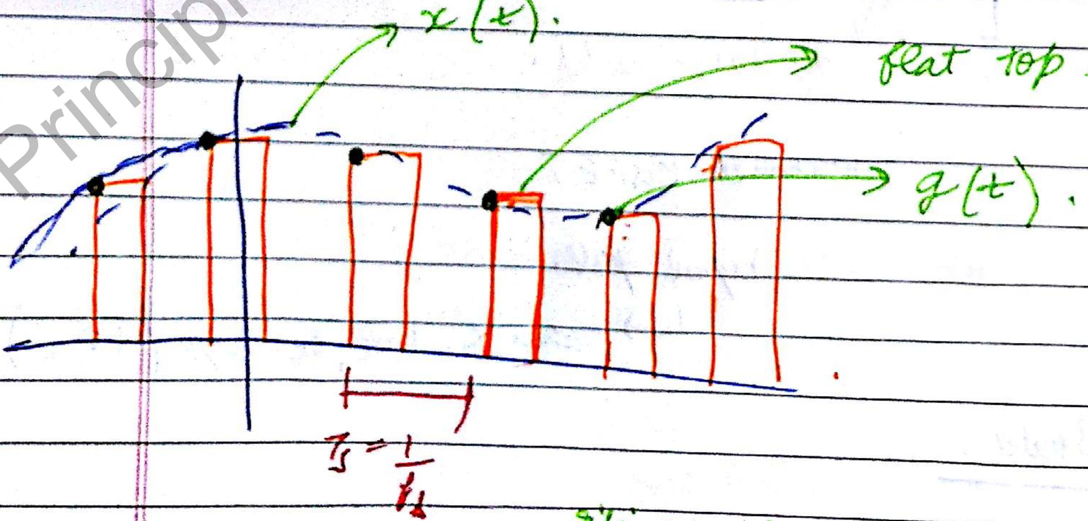
Once the capacitor is charge,

$+V_m$ will be stored.

Q_2 mosfet provides discharge path for the capacitor.

(capacitor ko hume khud se discharge karna padega).





Previously flat wala not there. curvature wala same as message like

sample points.

rising and par ek sample hoga usi ko flat kar dege.

Flat Top Sampling is like natural sampling i.e. practical in nature, in comparison to natural sampling, flat top sampling can be easily obtained.

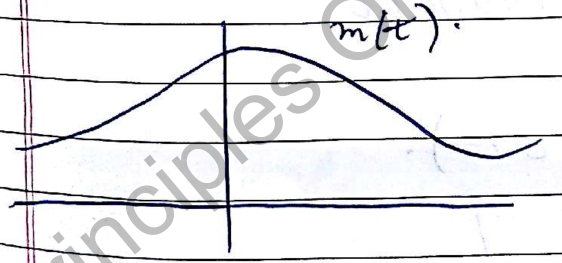
In this technique, the top of the samples remain constant and is equal to the instantaneous value of the message signal $x(t)$, at the start of sampling process.

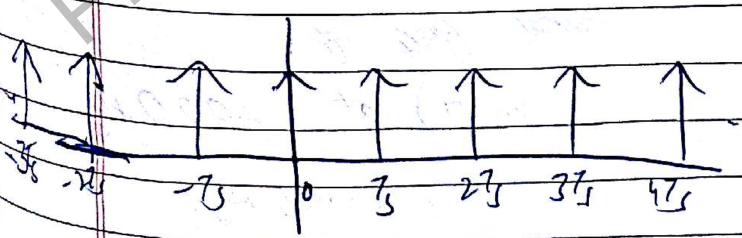
Sample and hold circuit is used in this type of sampling.

Spectrum of flat top sampled signal is,

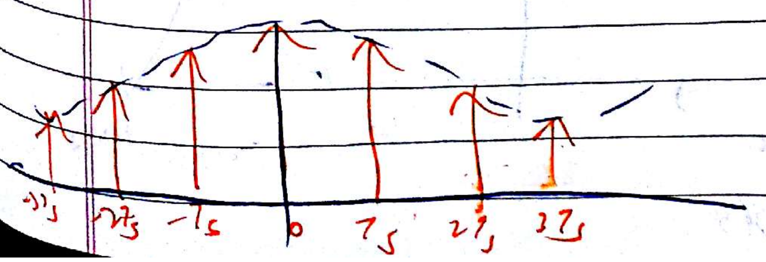
$$G_\delta = f_s \left[\sum X(f - nf_s)\, H(f) \right]$$

IDEAL SAMPLING





Train of impulses.



jahan jahan impulses hongi, wahan wahan message signal ka same amplitude aa jaayega.

Ideal Sampling is also known as INSTANTANEOUS SAMPLING or IMPULSE SAMPLING.

Train of impulses are used as a carrier signal for ideal sampling.

In this sampling technique, the sampling fun^{+} is the train of impulses and the principle used is known as MULTIPLICATION PRINCIPLE.

Spectrum of ideal sampled signal :-

$$G(f) = f_s \left[\Sigma X(f - nf_s) \right].$$

Q) Find the minimum sampling rate required to recover given message signal $m(t)$.

$$y(t) = m(t) \cos 4000\pi t$$

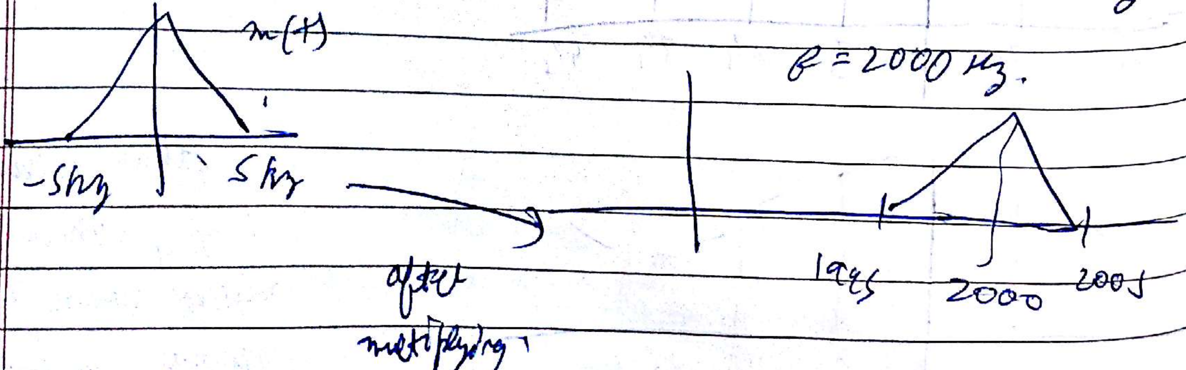
already modulated signal

$m(t)$ → freq. 0.

$c(t)$ → takes freq. of $m(t)$ at 2000 Hz.

$f = 2000$ Hz.

Suppose



sampling freq:

$$f_s \geq 2 f_m \quad (\text{to recover}).$$

$$f_s \geq 2(5)$$

$$f_s \geq 10.$$

$$b(f) = f_s \left[\Sigma X(f - n f_s) \right].$$

if $n=0$, we will get original signal.

$m(t)$ cos $4000\pi t$.

→ it is shifted at 2000 Hz.

So, now we need band pass filter around this freq. So that this can pass.

$f - n f_s$

$$f_s = 2 \times 5$$
$$f_s = 10.$$

$n f_s = 2000$ → its port. par humara signal shifted hai.

$$n = \frac{2000}{10}$$

$n = 200$

Both modulation & demodulation can be done with the help of SAMPLING MECHANISM.

$\boxed{f_s = 2f_m}$ → always find critical f_s in the question.

min. will be $2f_m$ only.

20×1

10×2

5×4 → yeh critical freq. hogi.

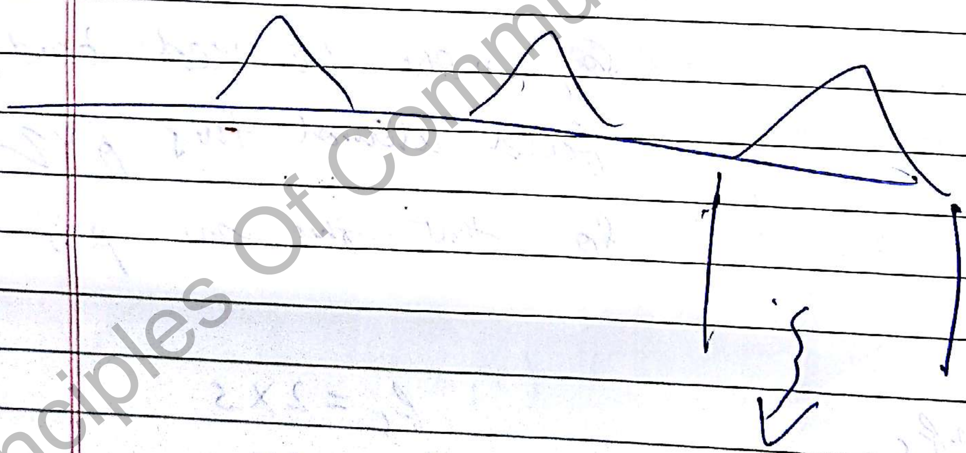


abb humara signal yahan placed hai.

⇓

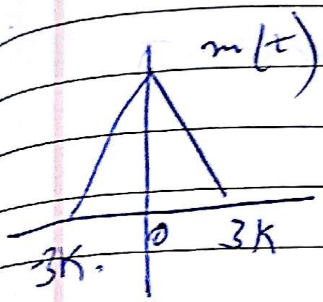
and hume yahan par low pass filter lagana hai.

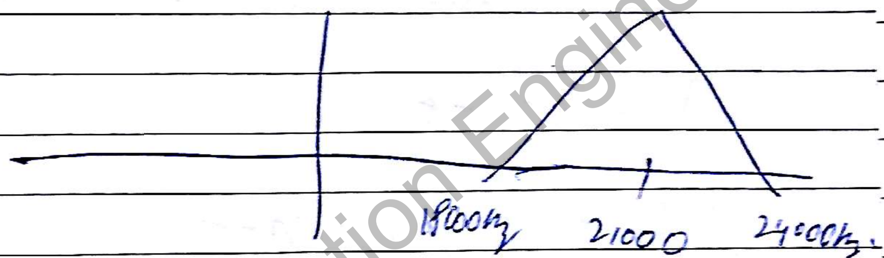
Q) $y(t) = x(t) \cos 42000\pi t$.

$y(t) = x(t) \cos 42000\pi t$

$\downarrow$

$f = 21000\,Hz$.





21×1

7×3.

multiple of 21.

$f_s = 2 \times 3000$

$= 6000$

Sampling freq.

is criteria ko follow karega.

$n f_s = 21000$

$f_s = 7$ $f_s = 3$

$n = \frac{21000}{6000} = \frac{\cancel{21}\ 7}{\cancel{6}\ 2}$

$\cancel{n = 3.5}$

$n = 7$

n ki value multiple me honi hai.

f_s wale criteria ko follow kare & min. value ho wahi wahi humari sampling freq. hogi.

classmate
Date
Page

7 × 3

f_s

~~minimum~~ f_s 6K ho sakta hai but we don't have an option, we have to choose 7K.

Q) Find nyquist. of the following signal:-

(i) $\sin 4000\pi t$.

(ii) $\sin 4000\pi t \cos 6000\pi t$

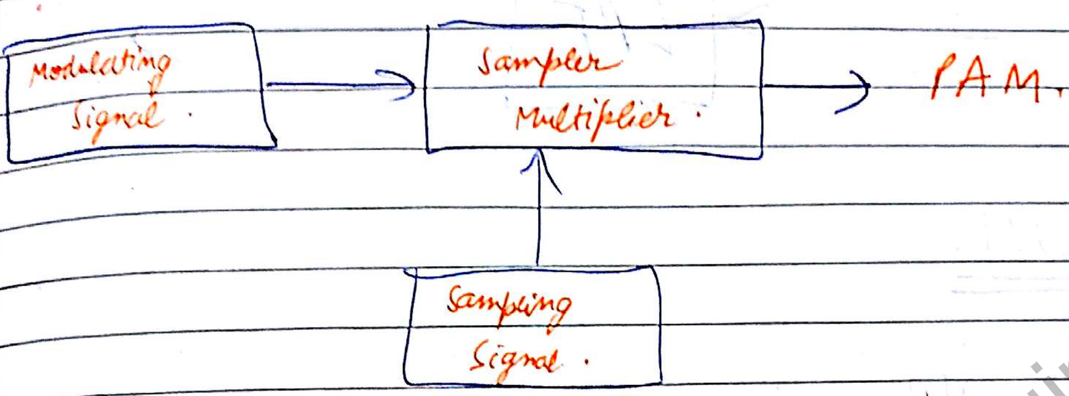
(iii) $\text{sinc}\, 100t$

(iv) $\text{sinc}^2\, 100t$

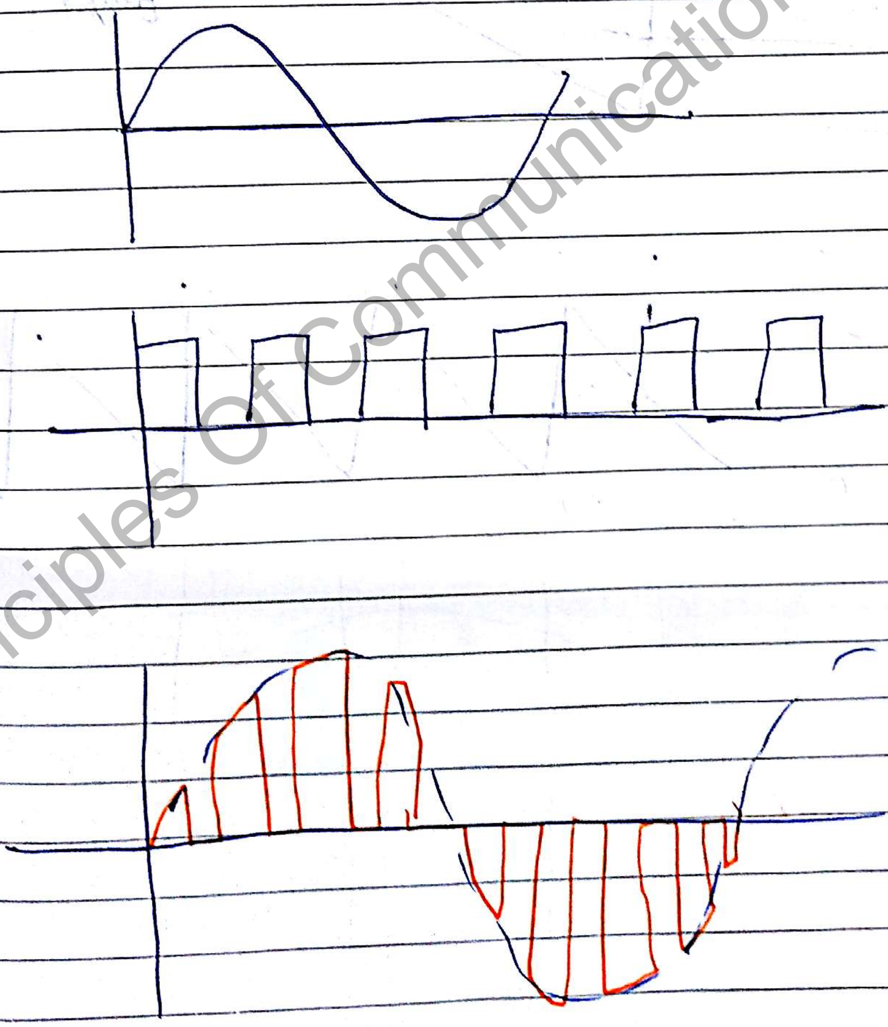
(v) $\text{sinc}\, 400t\ \text{sinc}\, 600t$

(vi) $\text{sinc}\, 400t$ ⓧ $\sin 600t$

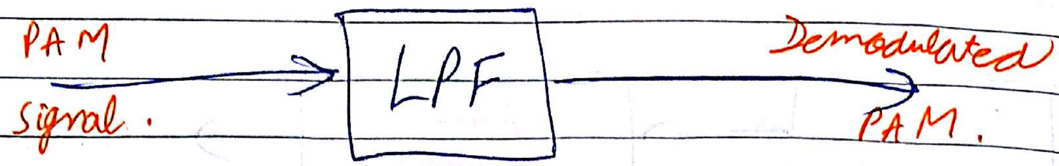
PAM



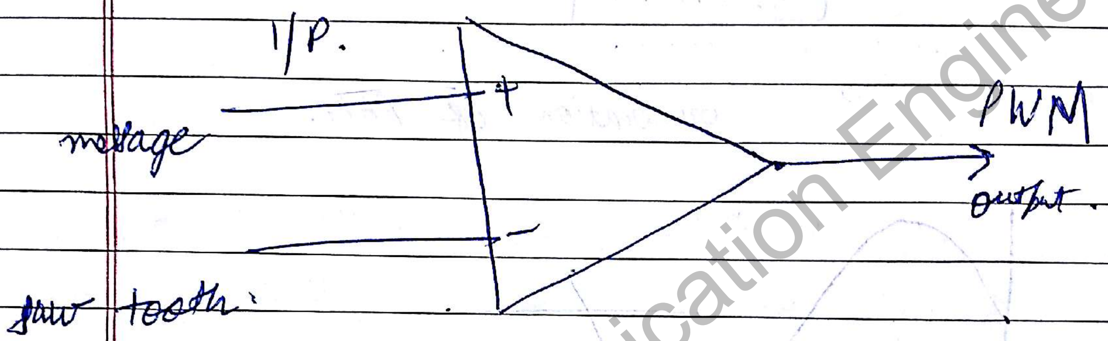
Generation of PAM.



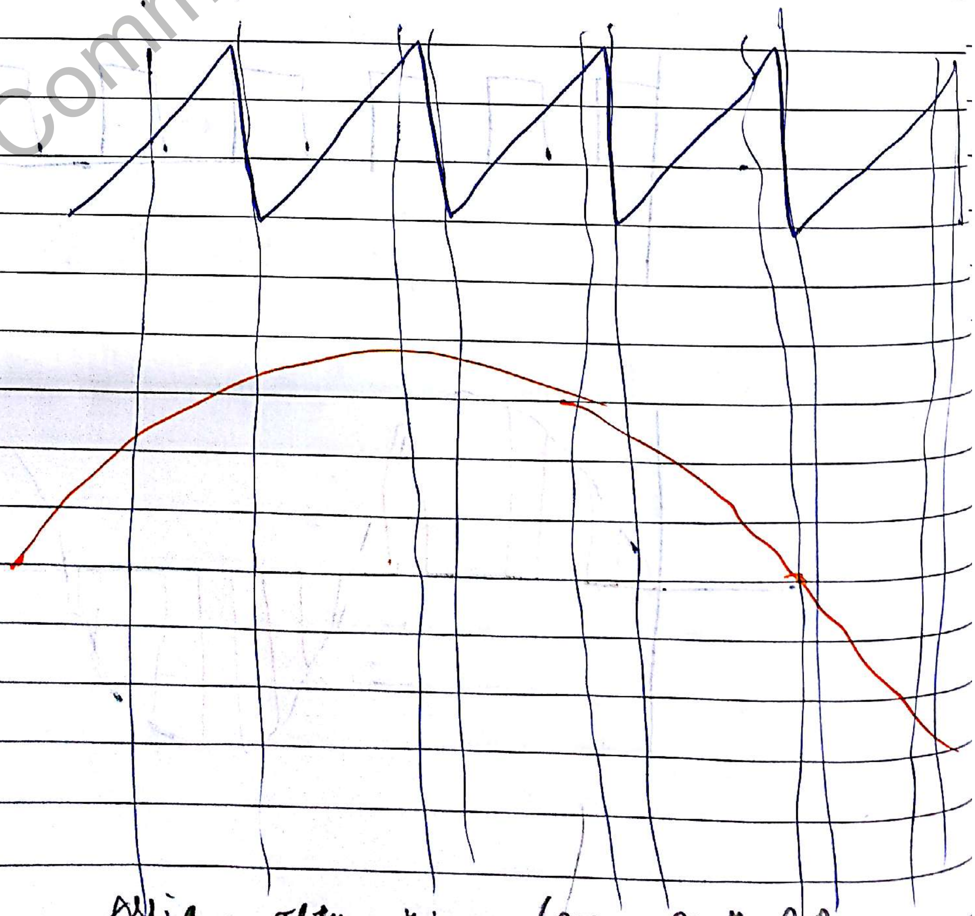
Demodulation of PAM.



PWM.



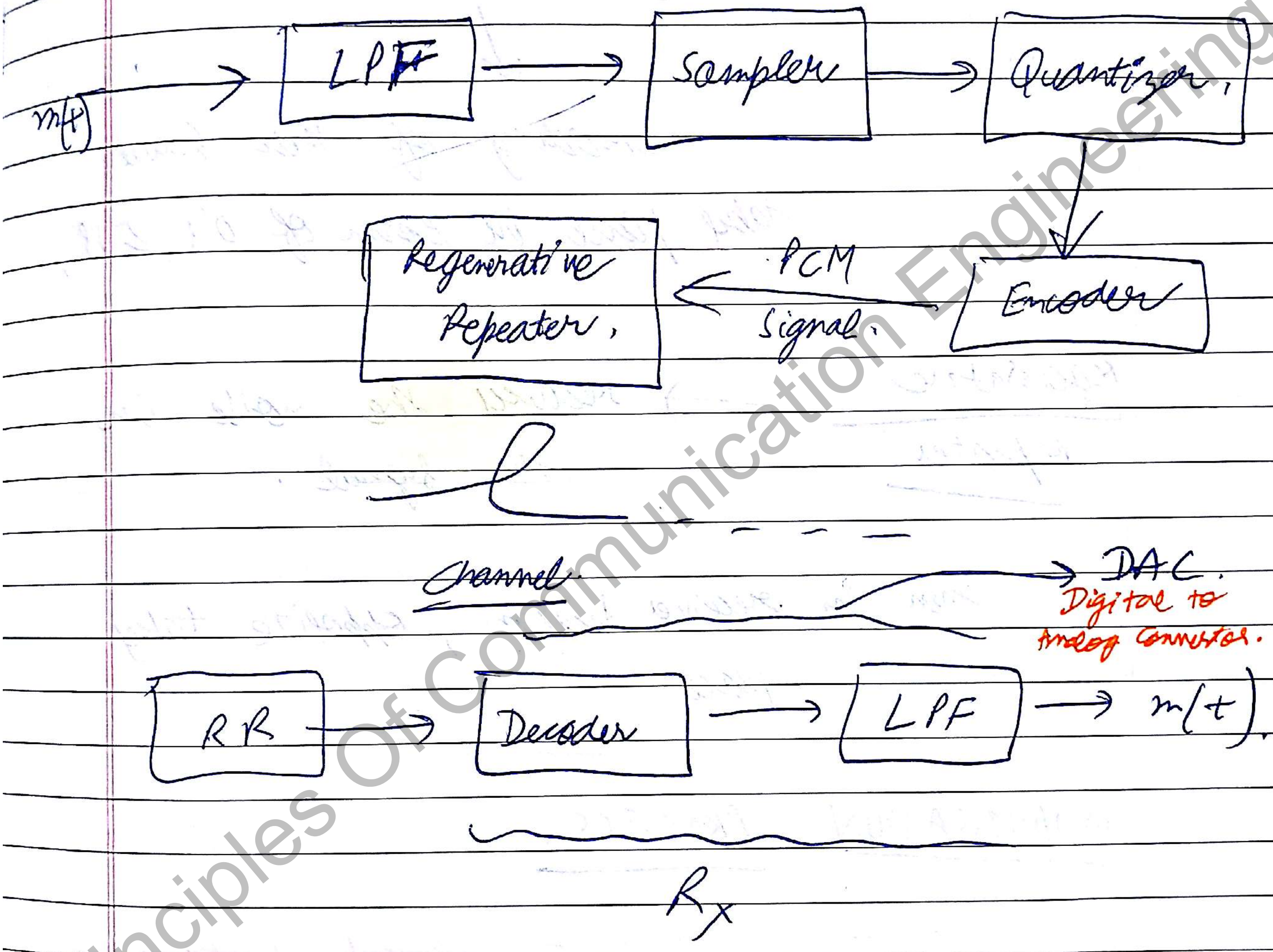
Generation



Assig. after this (PAM, PWM, PPM complete)

PCM (Pulse Code Modulation).

↳ now we will do digitisation of amplitude.



first of all, m(t) is band limited after passing through LPF

and then sampling is done by nyquist rate

Quantizer → brings the time discretized signal.
to amplitude at diff - diff levels.
↓
encoding of these levels.
takes place in form of 0's & 1's.

Regenerative repeater → reduces the noise in the signal.

then in receive section, opposite takes place.

QUANTISATION PROCESS.

Quantisation Error = Sampled Voltage − Quantized voltage.

2.1V after sampling.
and 2V is the sampled value.

So error is,
$2.1 - 2 = 0.1$ V

Resulting band limited signal will be sampled acc. to quantiser rounds off each of the sample voltage to the nearest available quantisation voltage.

Encoder represents each of the quantisation voltage by a unique binary code.

Using regenerative repeaters we can avoid channel noise which is why we transmit data in form of digital., there will be small quantisation error.

Without quantiser numerous no. of binary codes need to be generated, which is not possible for encoder.

$L = 2^n$ $\quad n \rightarrow$ no. of bits per sample.

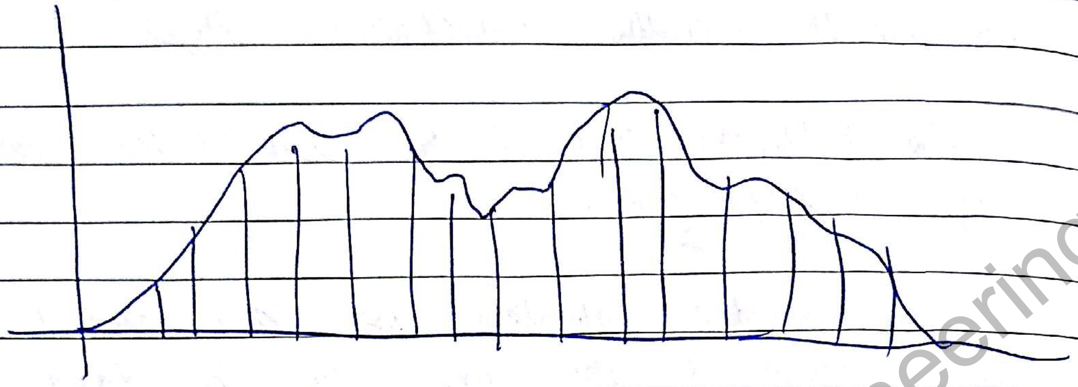
$L \rightarrow$ ~~no. of levels~~.
~~quantisation levels~~.

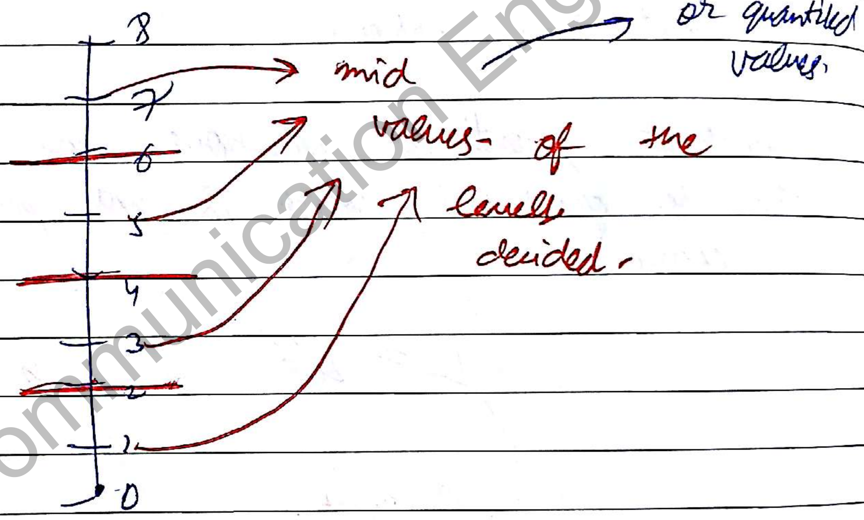
RR (regenerative repeaters) eliminate channel noise completely & transmitted electrical pulses will be regenerated back.

Decoder will do the reverse of encoder

LPF reconstructs message signal from its quantised equivalent, so that finite amount of quantisation error will be retained in the reconstructed message signal.

QUANTISATION PROCESS.





min value = 0 V

max. value = 8 V

quantised at 4 levels.

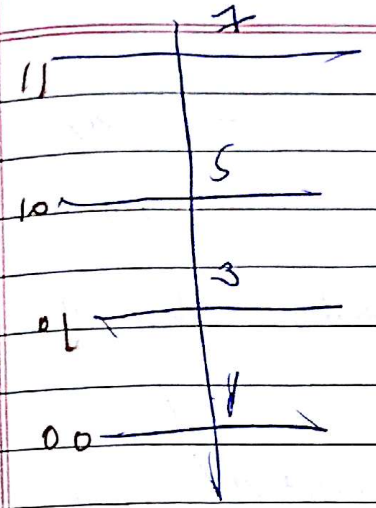
$2^n = 2^2 = 4$ 2 bits encoding is required.

Δ, Step size = $\frac{DR}{L}$ → dynamic range. L → levels.

$= \frac{V_{max} - V_{min}}{L} = \frac{8-0}{4}$

$= 2$.

each quantised level is encoded with 2 bits in this case i.e. 00, 01, 10, 11



→ all these are quantised level.

1.2 $\xrightarrow[\text{to}]{\text{quantised}}$ 1

4.6 $\xrightarrow[\text{to}]{\text{quantised}}$ 5

(nearest quantised level)

4 → depends on the type of quantisation.

Sampled value (SV).	Quantised value (QV).	encoded o/p	$Q_e = SV - QV$
0.5	1V	00	−0.5
2.2	3V	01	−0.8
3.6	3V	01	0.6
4.4	5V	10	−0.6
5.7	5V	10	0.7
7.9	7V	11	0.9

$$(Q_e)_{max/min} = \frac{\pm \Delta}{2}.$$

QE max QE min.

S.V.	Δ		S.V.	Δ	
8V	8−7 = 1	1/2	0V	0−1	−1/2

SV	QV	Q.E.	
8 V	7 V	1V	$\longrightarrow$ max
0 V	1V	−1V	$\longrightarrow$ min

* Total dynamic range of the signal is divided into n equal no. of steps.

* Middle of each step will be selected as a quantisation voltage. Each of the sample corresponds to specific step, will be round off to middle of step or to nearest quantisation voltage.

Formula → Assume no. of bits/sample $= n$.

No. of quantisation voltage $\Rightarrow L = 2^n$

$$\Delta = \frac{D.R.}{L} = \frac{V_{p-p}}{2^n}$$

For $m(t) = A_m \cos 2\pi f_m t$ or $A_m \sin 2\pi f_m t$,

$$\Delta = \frac{2A_m}{L} = \frac{2A_m}{2^n}$$

$$(QE)_{max} = \pm \frac{\Delta}{2}.$$

T_s : Sampled Duration

T_b : Bit Duration.

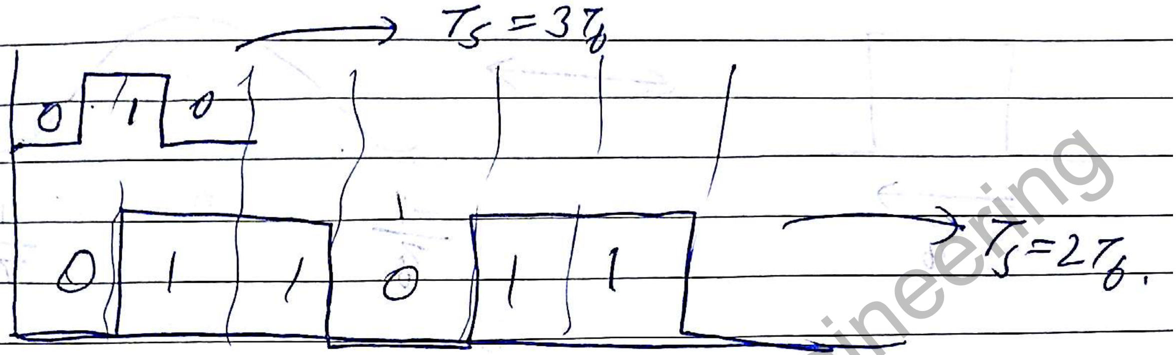
classmate

Date

Page

BIT DURATION (T_b):

$T_s = nT_b$.



$T_s = nT_b$ → time which encoded time takes.

Here, duration of one quantised value $= 2T_b$

∴ there are 2 bits required for quantisation.

BIT RATE (R_b): (bits/sec).

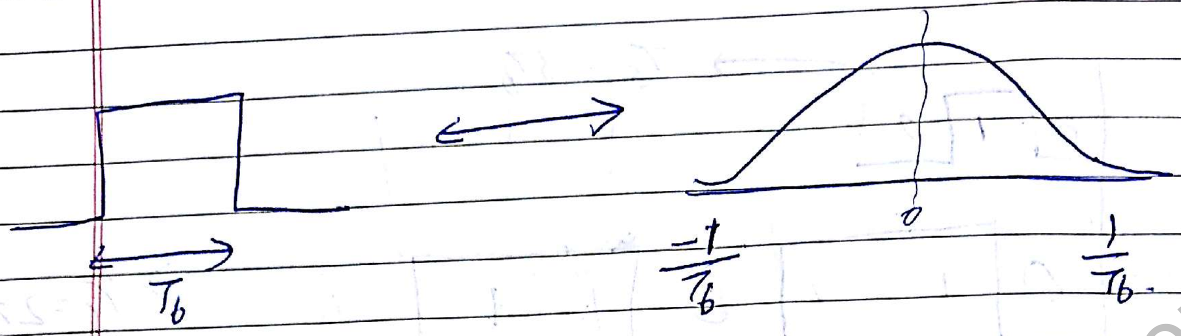
$$R_b = \frac{\text{bits}}{\text{sample}} \times \frac{\text{sample}}{\text{sec}}$$

$$= n f_s. \quad = n \times \frac{1}{T_s} = \frac{1}{T_b}.$$

$$\boxed{R_b = \frac{1}{T_b}}. \qquad (\because T_s = nT_b)$$

Only significant band is sent. Sinc

TRANSMISSION BANDWIDTH



$$S.B. = \frac{1}{T_b} - 0 = \frac{1}{T_b} = R_b = nf_s$$

sampled bandwidth

$$C.B. \geq S.B.$$

$\boxed{\text{channel bandwidth} \geq \text{signal bandwidth.}}$

Q) A message signal of $10\cos 2\pi \times 10^4 t$ is transmitted by using 4 bit PCM system. Find the parameters of PCM.

$10\cos 2\pi 10^4 t$ 4 bit PCM system.

$$\Delta = \frac{DR}{L}$$

$L = 2^n$

$n = 4$ bits/sample

$$\boxed{L = 16}$$

$$\Delta = \frac{\cancel{20}\,5}{\cancel{16}\,4} = 1.25$$

$$QE_{max} = \pm \frac{1.25}{2} = \pm 0.625.$$

$f_s = 2f_m = 23 kHz.$

$R_b = 80 kHz$. (bit rate)

$T_b = \frac{1}{80}$ msec. Bandwidth $= R_b = 80 kHz$

Q) A message signal having peak to peak voltage of 10 V, is band limited to 20kHz is transmitted by using PCM. No. of quantization levels $L = 256$. Sampling rate is 25% more than nyquist rate. Find the parameters of PCM.

$\Delta = \frac{DR}{L} = \frac{10}{256}$

20kHz → band limited.

$2a_m = 10 V$, $f_m = 20 kHz$ → 25% more

Sampling freq. $= 1.25 \times 2f_m$
$= 1.25 \times 2 \times 20$
$= 1.25 \times 40 = 50 kHz.$

$Q.E_{max} = \frac{10}{256 \times 2} = \frac{5}{256}$

$R_b = n f_s$
$= 8 \times 50 kHz = 400 kHz.$

$T_b = \frac{1}{R_b}$

Bandwidth $= 80 kHz$.

classmate
Date
Page

Q) A message signal of $10\cos(\pi \times 10^5 t)$ is transmitted by using PCM. max. quantisation error should be atmost of 0.1% of peak amplitude of message signal. Find parameters of PCM.

$f_m = 2 \times 10^4$

$$\frac{\Delta}{2} = \frac{0.1}{100} \times 10$$

$$\Delta = 0.01 \times 2$$

$$\boxed{\Delta = 0.02}$$

$f_s = 2 f_m$

$\boxed{f_s = 4 \times 10^4}$

$$0.02 = \frac{DR}{L}$$

$Q_e \leq 0.1\%$ of AM

$$L = \frac{\cancel{20}^{10}}{\cancel{0.02}} \times 100$$

$$\boxed{L = 1000}$$

$2^n = 1000$

$n = 10$

$n = 10$

Q) A message signal is transmitted by PCM such that max. quantisation error should be atmost of 2% of peak to peak amplitude of message signal. Find min. no. of bits per sample required.

$$\frac{\Delta}{2} \leq \frac{2}{100} \times A_m$$

$$\Delta \leq 0.04 A_m$$

(Pi M kar sayega) ✗ nahi sayega.

→ kisi bhi signal ka.

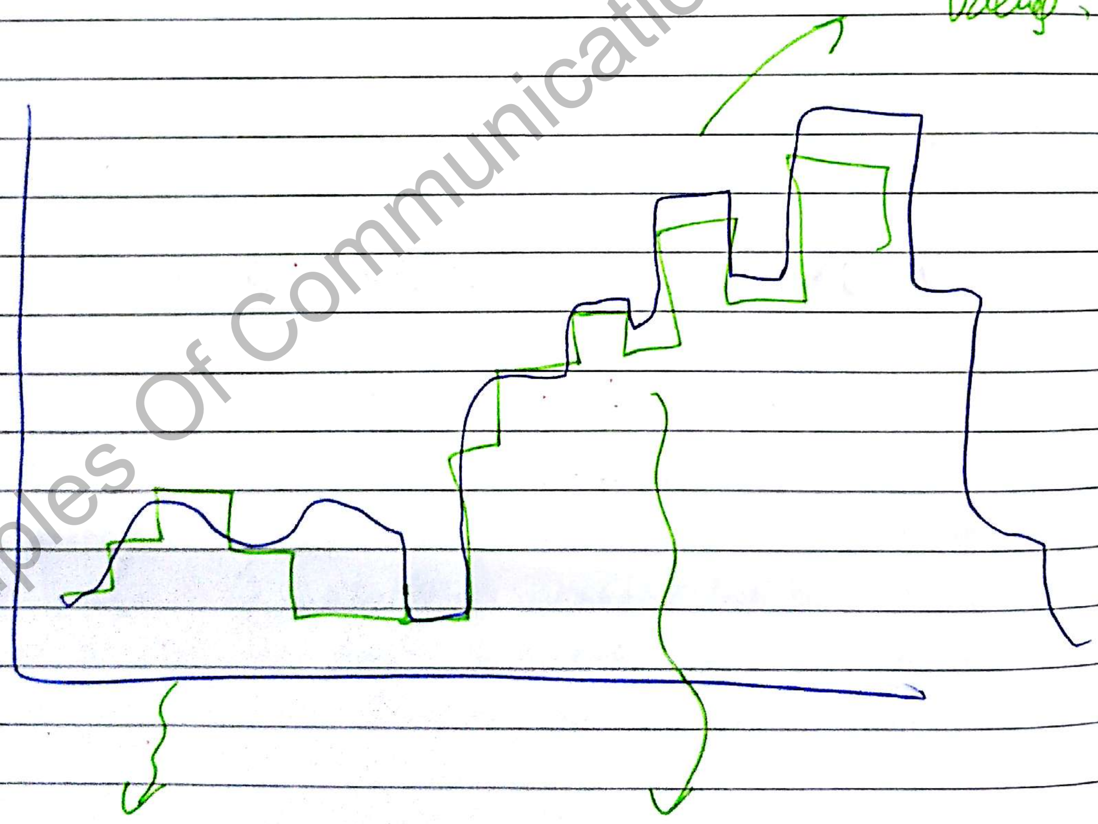
Sampling theorem acche se.

QUANTISATION

linear → step size is fixed.

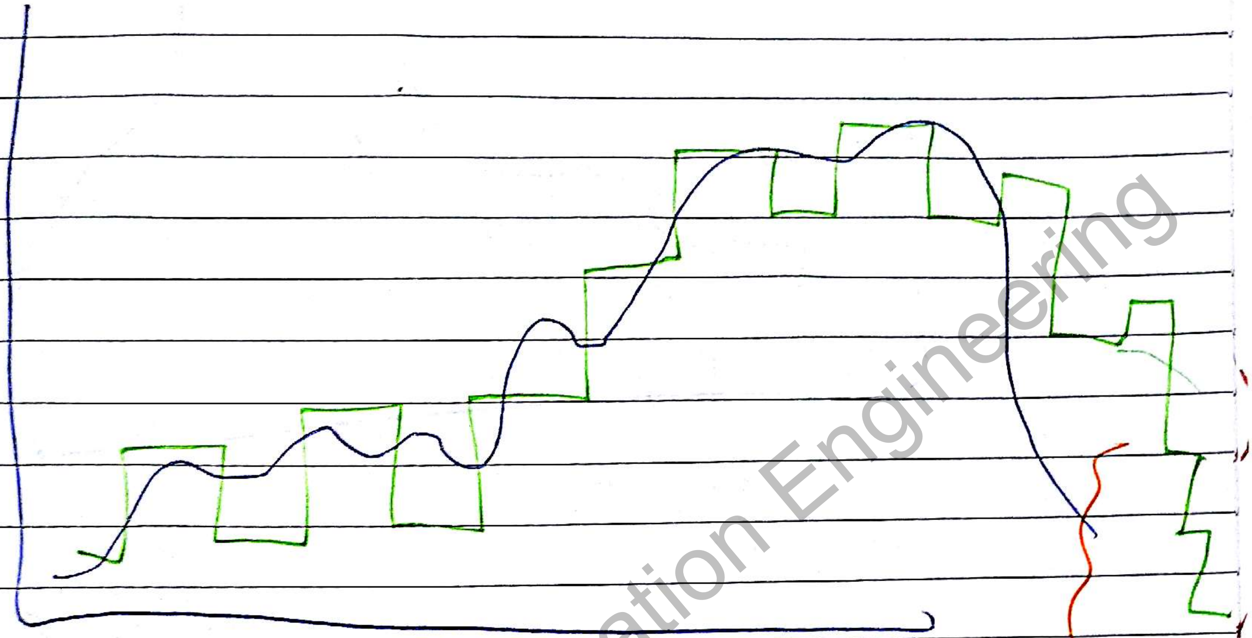
Δ once assigned does not changes.

Non-linear.



Date
Page

Here error will be present when there will be uniform quantisation in this case



in this portion sampling does not track the signal.

So there will be error in this case.

↳ because of uniform quantisation

non linear quantisation

at sender's side,

compression will take place.

at receiver side,

expansion will take place.

not in syllabus

Linear Quantisation

Mid Rise

Mid Tread.

Q) A message signal of $4\cos 4\pi \times 10^3 t$ is transmitted by using 3 bit PCM system.

(i) Find PCM parameters.

(ii) Given sample values -3.5, -2.1, -1.6, 0.8, 2.9, 3.3.

Find corresponding quantiser & encoder output.

(iii) Plot quantiser characteristic.

$A_m = 4V$

$n = 3$

$f_m = 2\,kHz$

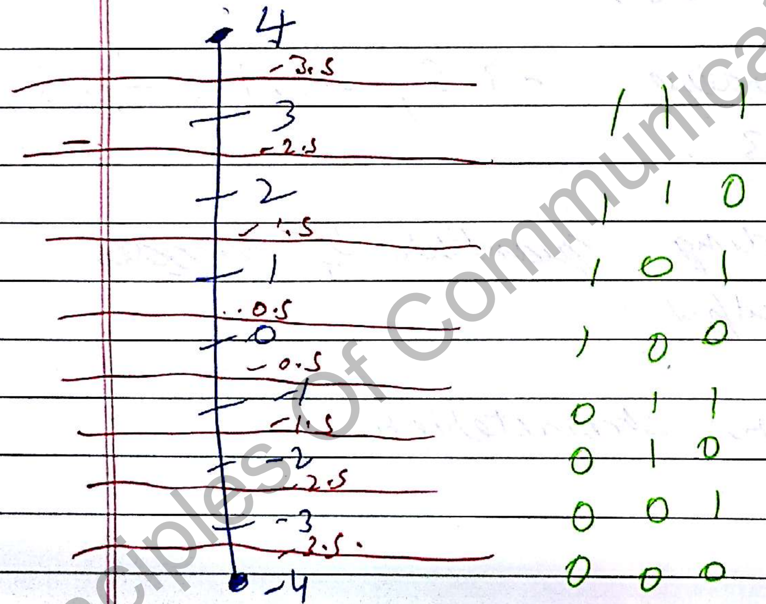
$\boxed{L = 8} = 2^3$

$f_s = 2f_m = 4\,kHz.$

$$\Delta = \frac{8}{8} = 1 = \frac{4 \times 2}{8}.$$

$R_b = 12\,kbps = n f_s = B.W.$

Sampled Value S.V.	Quantised Value Q.V.	Encoder Output
-3.5	-3.5	0 0 0
-2.1	-2.5	0 0 1
-1.6	-1.5	0 1 0
0.8	0.5	1 0 0
2.9	2.5	1 1 0
3.3	3.5	1 1 1



→ Encoding of quantised levels

—— quantised levels.

—— encoded values.

000 --- 111.

encoding hamesha neeche se start hogi......

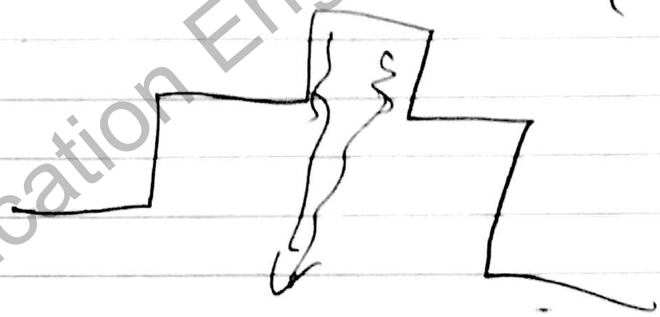
Anatropic will be always staircase.

Mid Tread → Mid Rise

a step size Δ is always fixed.

⇓

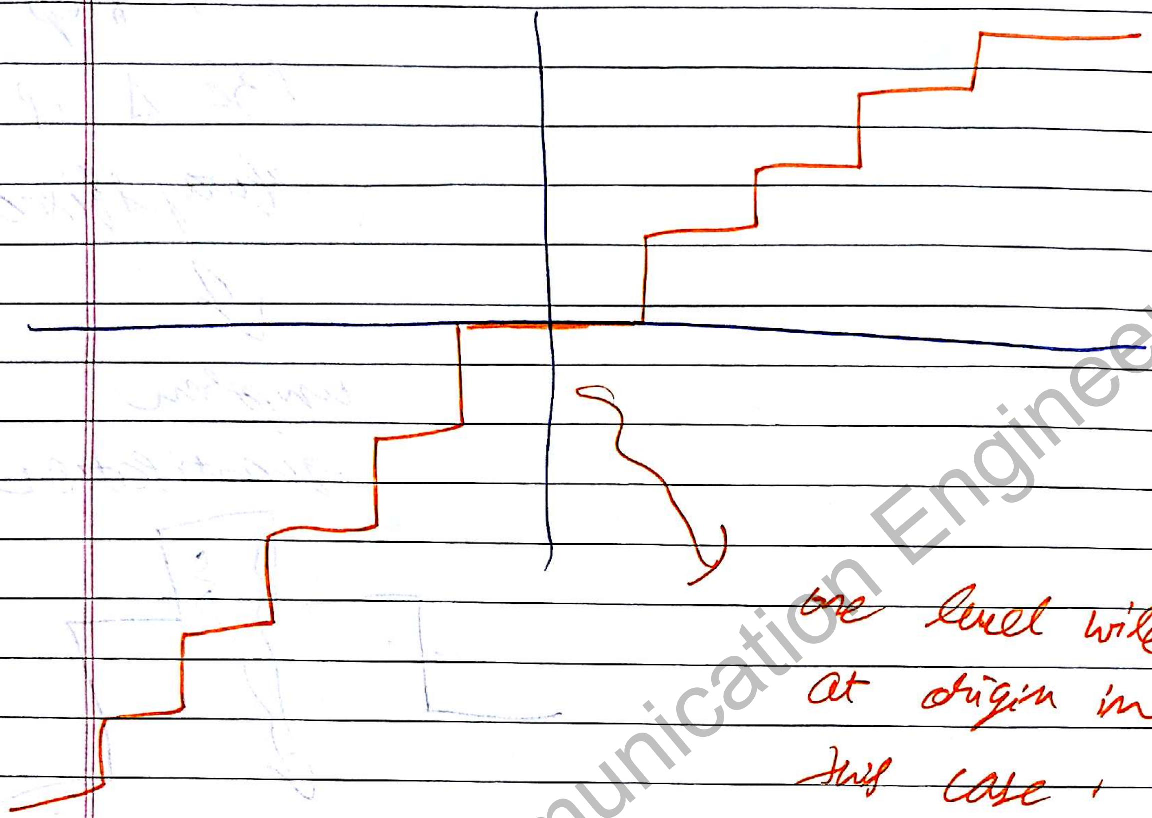
uniform quantisation



level height is fixed.

Mid Rise → pulse is rising at origin.

Mid Tread



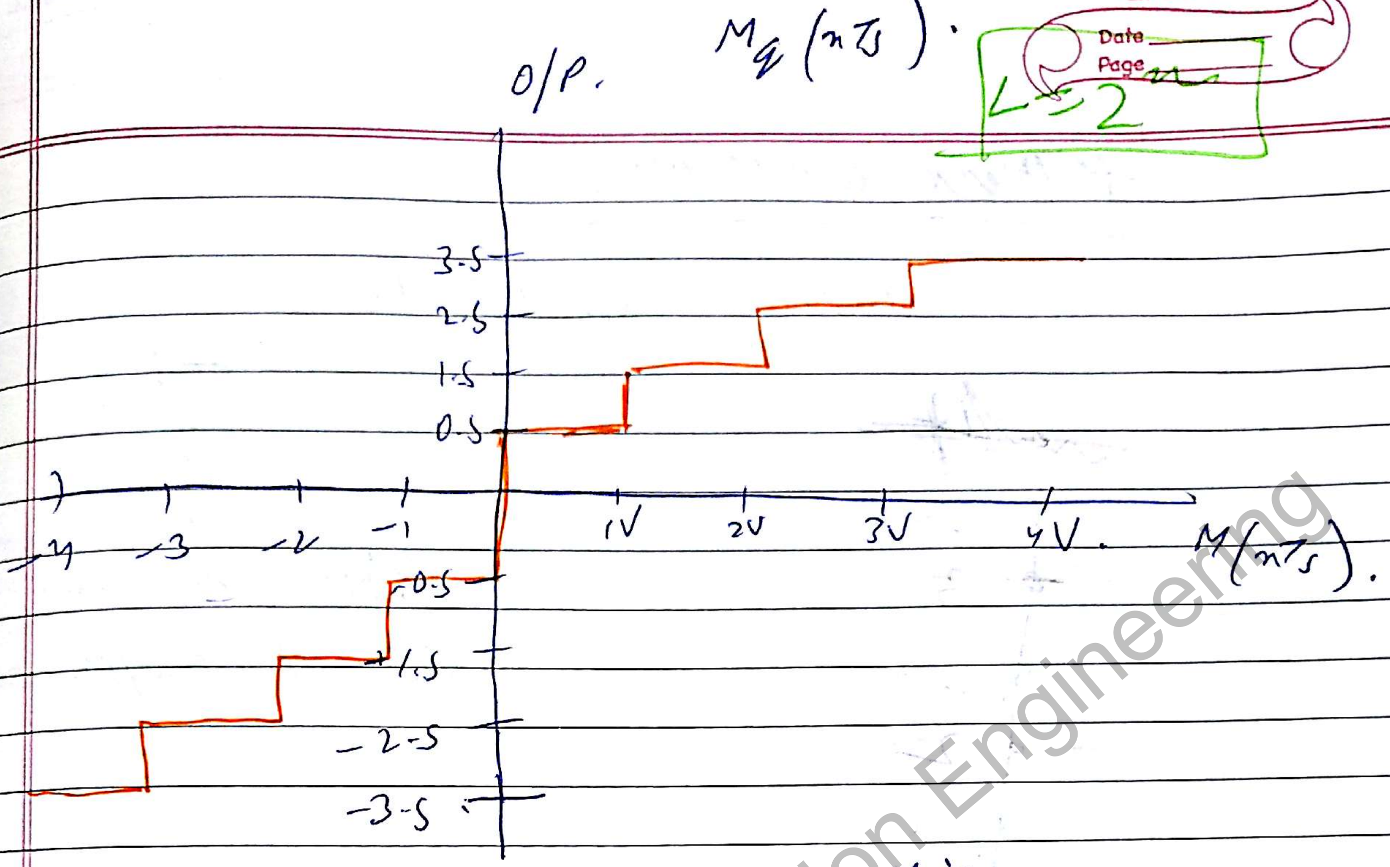
one level will be at origin in this case.

Characteristic For Mid Rise. (for the question given), Not characteristics.

classmate

Date

Page 2



Midrise Quantisation.

Δ is always 1.

We choose mid quantisation.

↓

so unifor division across origin & hence 0.5 and -0.5 pts come.

For it to be symmetric at origin we will ~~di~~ ~~vide~~ divide it by 0.5. start.

and then we will increase by 1.

0 – 1 any value, it will be quantised to 0.5.

1 – 2 ———— 1.5

2 – 3 ———— 2.5

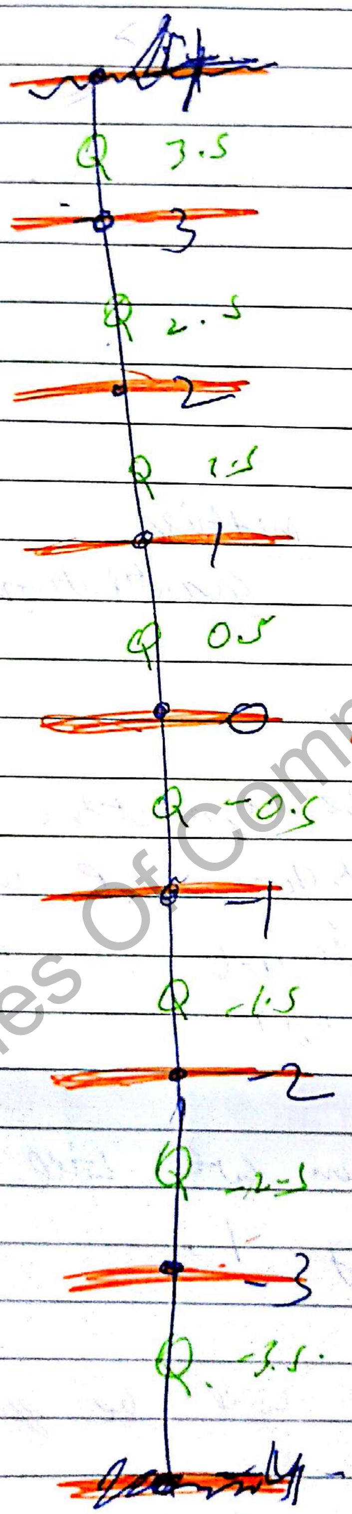
and so on.

Midtread Quantization

here, level will be

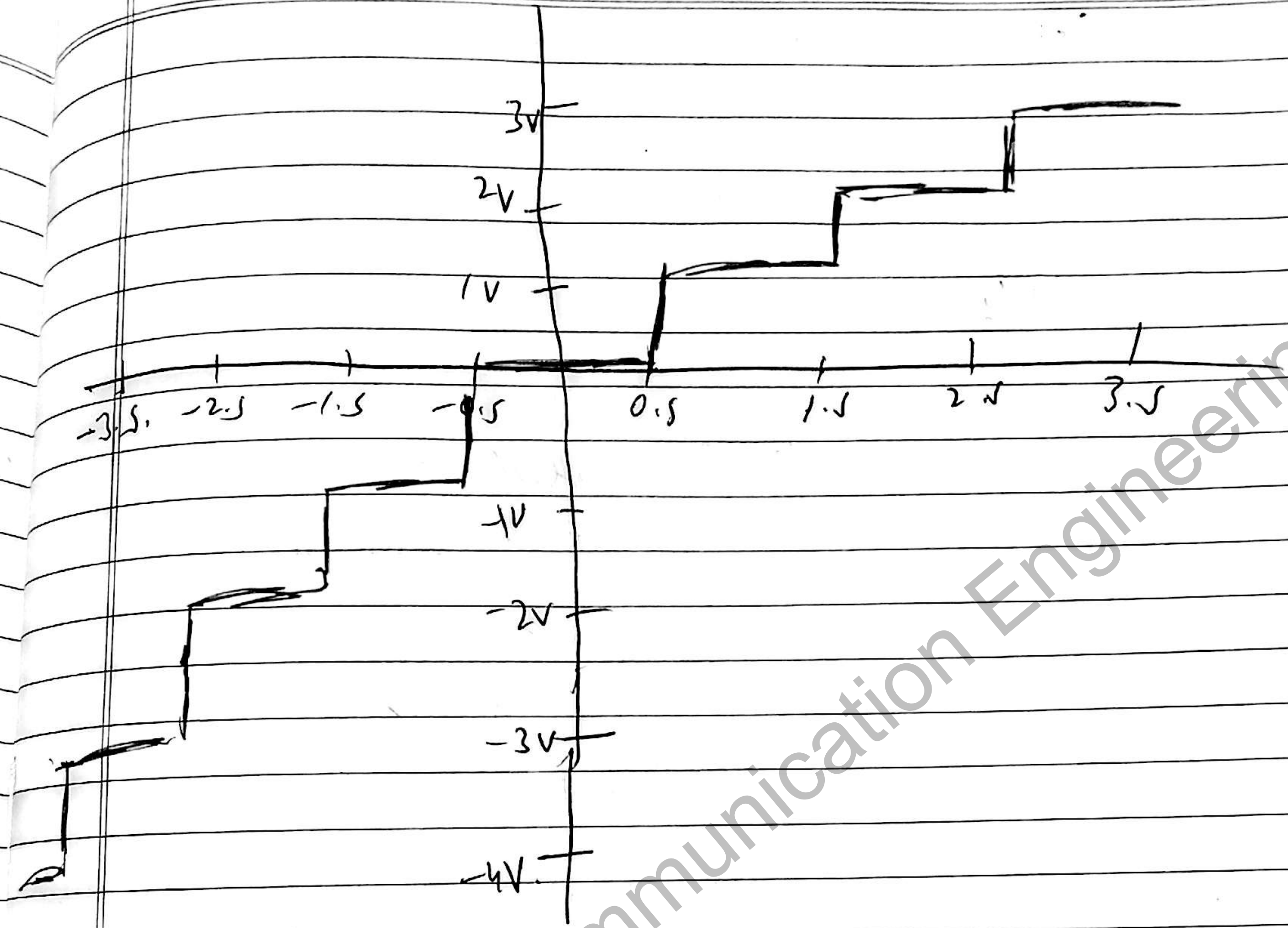
$2^n - 1$

since one level lies at origin only.



Quantisation levels.

0 will always be quantisation level in mid tread.



3V
2V
1V
-1V
-2V
-3V
-4V
-3.5
-2.5
-1.5
-0.5
0.5
1.5
2.5
3.5

Default is MID RISE Quantisation because quantisation error is limited to $\pm \frac{\Delta}{2}$

but in mid tread it can be greater than $\Delta/2$.

e.g. if 4 is sampled value

quantised value is 3.

$\therefore Q_e = 1$

$\therefore$ in mid tread

$(Q_e)_{max} = \pm \Delta$

or $(Q_e)_{max} > \frac{\Delta}{2}$.

$$\boxed{\frac{S}{N} = \frac{\Delta^2}{12}}$$

PCM ka SNR

signal to noise ratio.

Exercise Book
240 Pages
(Total Pages Include Index & Printed Information)
Size : 29.7 x 21 cm
MRP Rs. 75.00
Inclusive of all taxes

Batch : F/Z/SU
02000984
8 902519 009845
©ITC Limited

Type of Ruling : Single Line