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Amath 483 HW2

Extra Credit

1. For $+\infty$: the sign bit is 0
the exponent bits: 11111

So, the 6-bit representation of $+\infty$ is 011111

For $-\infty$: the sign bit is 1
the exponent bits: 11111

So, the 6-bit representation of $+\infty$ is 111111

2. The definition of 1-norm:

$\|v\|_1 = |v_1| + |v_2| + \cdots |v_n|$,

which is the sum of the absolute vector value.
It represents the "distance" along the axes in a Cartesian coordinate system between the origin and the point represented by the vector "v".

The definition of 2-norm:

$\|v\|_2 = \sqrt{v_1^2 + v_2^2 + \cdots v_n^2}$

which is the square root of the square root of the sum of squares.
It represents the length of the vector "v" as the distance from the origin to the point represented by the vector in a Cartesian coordinate systems.

The definition of ∞-norm

$\|v\|_\infty = \max(|v_1, |v_2| \cdots |v_n|)$,

which is the absolute value of the largest component of the vector. It represents the "distance" along the axis with the largest absolute value among all the components of vector "v".

3. $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$

For 1-norm:

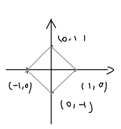
$|x| + |y| = 1$

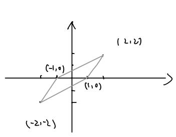
$\therefore$ unit vector along x-axis $[1,0]^T$. $[-1,0]^T$
unit vector along y-axis $[0,1]^T$. $[0,1]^T$

Apply A to these unit vector, we get

$\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\qquad \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ $\qquad \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$



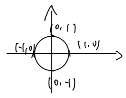


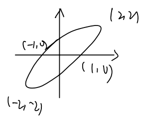
For 2-norm:

$\|v\|_2 = \sqrt{x^2 + y^2} = 1$

$\because$ the definition of 2-norm
$\therefore$ the unit ball is a circle

from $[0,1]^T$, we could get $[2,2]^T$
from $[1,0]^T$, we could get $[1,0]^T$
from $[0,-1]^T$, we could get $[-2,-2]^T$
from $[-1,0]^T$, we could get $[-1,0]^T$





For 3-norm:

$||v||_\infty = \max(|x|,|y|)$.

The same reason as 2-norm,

yet it should be a square.

from $[1,1]^T$, we could get $[3,2]^T$

from $[-1,1]^T$, we could get $[1,2]^T$

from $[1,-1]^T$, we could get $[-1,-2]^T$

from $[-1,-1]^T$, we could get $[-3,-2]^T$